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# ON TESTS FOR DETERMINISTIC AND STOCHASTIC SEASONALITIES

by

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# ON TESTS FOR DETERMINISTIC AND STOCHASTIC SEASONALITIES

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## RÉSUMÉ

La désaisonnalisation des séries socio-économiques continue d'être un sujet qui attire beaucoup d'attention.

Une étape éssentielle avant toute désaisonnalisation est de tester les séries pour la présence significative de saisonnalité soit detérministe ou stochastique. Les tests standard du type ANOVA ne sont pas adéquats cars il ignorent la possibilité de résidus autocorrelés.

Cet article introduit un test de F modifié pour la présence de saisonnalité déterministe. Des tests pour la saisonnalité stochastique sont aussi discutés. Les divers tests sont illustrés avec plusieurs exemples.

## ABSTRACT

The adjustment of economic and social time series for seasonal variation has been and continues to be the subject of much attention. As a first step towards seasonally adjusting a series, it is essential to test for the presence of deterministic as well as stochastic seasonalities in time series. The standard ANOVA tests for seasonalities are inappropriate, as they do not allow for the likely possibility that observations are autocorrelated This paper discusses certain modified F-tests for deterministic seasonalities. Tests for stochastic seasonalities are also discussed. The test procedures are illustrated by several numerical examples.

Key Words and Phrases: Two-way correlations; stable and moving seasonalities; Dependent quadratic forms; Modified F-tests; Seasonal adjustments.

## I. INTRODUCTION

Seasonal adjustment procedures are widely employed in the analysis of economic data. One of the main reasons for the adjustment of economic and social time series for seasonal variation is that seasonal components represent the effects of non-economic factors that are exogeneous to the economic system and hence are uncontrollable.

There are various methods available to deseasonalize a time series. The U.S. Bureau of the Census Method II-X-11 variant developed by Shiskin, Young and Musgrave (1967) and the X-11-ARIMA version developed by Dagum (1975, 1980) are widely used by government agencies and statistical bureaus. These seasonal adjustment methods are based on moving average techniques. Kenny and Durbin (1982) provided some methods of improving the performance of the X-11 seasonal adjustment procedures. Wallis (1982) suggested procedures for seasonal adjustment and revision of current data by linear filter methods. The seasonal adjustment procedures links to Box-Jenkins ARIMA modelling has been discussed, among others, by Box, Hillmer and Tiao (1978), Pierce (1978), and Burman (1980). Recently Box, Pierce and Newbold (1987) focussed on current and projected trends in order to deseasonalize a time series. Burridge and Wallis (1985) presented a Kalman filter formulation of the "model-based" methods to perform seasonal adjustment. However, there are situations where the seasonality in a time series may not be significant. In such cases, the adjustment for seasonality is unnecesary. This suggests testing for the presence of seasonality in a time series before making seasonal adjustments. But, very little discussion has appeared in the literature on such testing problems.

The X-11-ARIMA procedure developed by Dagum tests for seasonality and for moving seasonality before making any seasonal adjustment. Pierce (1978) has also discussed tests for deterministic as well as stochastic seasonalities. But, both X-11-ARIMA and Pierce's procedures test for seasonality use standard F-tests. The use of standard F-tests, however, may give misleading results as the auto-correlation in the series used to define the numerator and denominator of the F-ratios invalidate the standard distribution theory. In an effort to come to grips with this distributional problem, Dagum, Huot, and Morry (1988) discuss tests for empirical seasonality. In the sequel we present modifications to F-tests for the presence of deterministic and stochastic seasonalities and discuss distribution theory for our tests.

In Section 2, we discuss the model for seasonal components. A four moment approximation to the distribution of a modified F statistic for testing the deterministic seasonality is discussed in Section 3. An alternate approximation using standard F-ratios with modified degrees of freedom is discussed in Section 4. In Section 5, tests for stochastic seasonalities are given. An example illustrating the use of the tests is discussed in the last section.

## 2. THE MODEL

The multiplicative seasonal model for a time series $\{Y_t\}$ is

$$Y_t = C_t X_t \varepsilon_t , \qquad (2.1)$$

where $C_t$, $S_t$ and $\varepsilon_t$ are, respectively, the trend-cycle, seasonal, and irregular factors of $Y_t$, all at time $t$. Many economic series exhibit exponential growth and, for these, the multiplicative model is most appropriate. For other series, however, an additive model may be more appropriate and this is directly related to the multiplicative model by taking logarithms. If $y_t = \log Y_t$, $c_t = \log C_t$, etc., then (2.1) becomes

$$y_t = c_t + s_t + e_t \qquad (2.2)$$

which is the additive seasonal model.

Suppose $y_t$ follows a $SARMA(p, q)(P, Q)_s$ process. That is,

$$\phi_p(B)\ \Phi_p(B_s)y_t = K^* + \theta_q(B)\ \Theta_Q(B^s)\ a_t, \tag{2.3}$$

where $\phi_p(B)$, $\theta_q(B)$ are polynomials in $B$ of degrees $p$ and $q$ respectively; $\Phi_P(B^s)$, $\Theta_Q(B^s)$ are polynomials in $B^s$ of degrees $P$ and $Q$ respectively, $K^*$ is a constant and $a_t$ is a component of a white noise process with zero mean and variance $\sigma_a^2$. If deterministic and stochastic trend and seasonality are both present, the trend and seasonal components of the observable series $y_t$ can be written as follows [cf. Box, Hillmar and Tiao (1978), Pierce (1978)]:

$$\phi_p(B)\ \Phi_P(B^s)c_t = \sum_i \alpha_i a_{it} + \psi_u(B)\xi_t, \tag{2.4}$$

$$\phi_p(B)\ \Phi_P(B^s)s_t = \sum_j \beta_j b_{jt} + \eta_v(B)d_t\ ,$$

where: the elements $\{a_{it}\}$, and $\{b_{jt}\}$ are respectively trend and seasonal dummy variables; $\{\xi_t\}$ and $\{d_t\}$ are two independent Gaussian white-noise processes with zero means and variances $\sigma_\xi^2$ and $\sigma_d^2$ respectively; $\psi_u(B)$ and $\eta_v(B)$ are polynomials in $B$ of degrees less than or equal to $\max(p + Ps,\ q + Qs)$. It then follows from (2.2), (2.3) and (2.4) that

$$\theta_q(B)\ \Theta_Q(B^s)\ a_t = \psi_u(B)\xi_t + \eta_v(B)\ d_t + \phi_p(B)\ \Phi_P(B^s)\ e_t\ , \tag{2.5}$$

and

$$K^* = \sum_i \alpha_i a_{it} + \sum_j \beta_j\ b_{jt}\ .$$

Thus, the time series $\{y_t\}$ may be modelled as

$$\phi_p(B)\ \Phi_P(B^s)y_t = c_{1t} + s_{1t} + u_t\ , \tag{2.6}$$

where $c_{1t} = \sum_i \alpha_i\ a_{it}$ is the deterministic trend component, $s_{1t} = \sum_j \beta_j b_{jt}$ is the deterministic seasonal component, and

$$u_t = \theta_q(B)\,\Theta_Q(B^s)a_t\,, \tag{2.7}$$

is the stochastic component. A series displays deterministic seasonality if $s_{1t}$ is nonzero; it possesses stochastic seasonality if $\eta_v(B)d_t$ is nonzero. As it is unnecessary [cf. Pierce (1978, p.247)] to separate stochastic trend from the irregular component in order to seasonally adjust a series, we will concentrate on the deterministic trend free series for testing the presence of deterministic seasonality components.

Let $z_t = \phi_p(B)\Phi_P(B^s)y_t - \hat{c}_{1t}$ be the deterministic trend free series. This $\{z_t\}$ series will be referred to as the series of seasonal irregular differences. For the multiplicative model, the $\{z_t\}$ series will be referred to as the series of seasonal irregular ratios. Hence, to test for seasonality, we consider the model

$$z_t = s_{1t} + u_t\,, \tag{2.8}$$

where $s_{1t}$ is the deterministic seasonal component, and $u_t$ is the residual which, at least, contains stochastic seasonal and irregular components.

## 3. TESTS OF DETERMINISTIC SEASONALITY

Suppose there are $k$ seasons in a year and there are $kn$ observations in a time series of $n$ years. Writing $z(t)$ for $z_t$ in (2.8), one may arrange the $kn$ observations in a stacked vector $Z^* = (z_1', \ldots, z_i', \ldots, z_k')'$, where $z_i = [z\{(i-1)n+1\}, \ldots, z\{(i-1)\,n+j\}, \ldots, z\{in\}]'$ is an $n \times 1$ column vector containing the $n$ seasonal-irregular differences (ratios) under the ith season $(i = 1, \ldots, k)$, $z\{(i-1)n+j\}$ being the jth observation $(j = 1, \ldots, n)$ under the same (ith) season. Then for $t = \{(i-1)n+j\}$, the model

(2.8) may be written as

$$z\{(i - 1)n + j\} = s_1\{(i - 1)n + j\} + u\{(i - 1) n + j\}. \tag{3.1}$$

The equation (3.1) may be re-expressed as

$$z_i(j) = s_{1i}(j) + u_i(j) \tag{3.2}$$

where $z_i(j)$ is the seasonal-irregular difference for the ith season $(i = 1, \ldots, k)$ in the jth year $(j = 1, \ldots, n)$. For much economic seasonal data, the deterministic seasonal component $s_{1i}(j)$ in (3.2) contains additive seasonal and annual effects. Then, the model (3.2) may be written as

$$z_i(j) = \mu + \alpha_i + \beta_j + u_i(j) \tag{3.3}$$

where $\mu$ is a suitable constant, $\alpha_i$ is the ith seasonal effect and $\beta_j$ is the jth annual effect. Thus the $\alpha$'s and $\beta$'s in (3.3) represent, respectively, the stable and moving seasonalities of the series. To test for the presence of deterministic seasonality, one tests the hypothesis that $s_{1i}(j) = 0$ in (3.2). In the notation of (3.3) this test is equivalent to testing the hypotheses:

$$H_{01}:\alpha_i = 0 \quad \text{vs} \quad H_{11}:\alpha_i \neq 0 \quad \text{for at least one } i, \tag{3.4}$$

and

$$H_{02}:\beta_j = 0 \quad \text{vs} \quad H_{12}:\beta_j \neq 0 \quad \text{for at least one } j. \tag{3.5}$$

The purpose of the next Section is to discuss test criteria analogous to the classical F-tests for testing the hypotheses in (3.4) and (3.5).

### 3.1 Tests for Stable Seasonality

Denote by $\Sigma^*$ the $kn \times kn$ covariance matrix of $Z^*$, where $Z^*$ is a kn-dimensional vector containing seasonal-irregular differences (ratios). Let $\sigma_a^2 \sigma_{jj'}$ denote the covariance between the jth and j'th years for season $i$ $(i = 1, \ldots, k)$. Also let $\sigma_a^2 \lambda_{jj'}^{(h)}$, denote the covariance between the j-th and j'th years when the months are separated by $h = |i - r|$ for $i \neq r$, $i, r = 1, \ldots, k$. For instance, if $i$ denotes the March season and $r$ denotes the June season, then $h = |i - r| = |3 - 6| = 3$. Thus, $\sigma_a^2 \lambda_{jj'}^{(3)}$ would mean the covariance between the jth and j'th years for any two seasons lagged by 3. Then the covariance matrix $\Sigma^*$ of $Z^*$ may be written as

$$\Sigma^* = \begin{bmatrix} \Sigma & \Lambda_1 & \Lambda_2 \cdots & \Lambda_{k-1} \\ \Lambda_1' & \Sigma & \Lambda_1 \cdots & \Lambda_{k-2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \Lambda_{k-1}' & \Lambda_{k-2}' & \Lambda_{k-3}' & \Sigma \end{bmatrix}, \qquad (3.6)$$

where $\Sigma = \sigma_a^2 (\sigma_{jj'})$ and $\Lambda_h = \sigma_a^2(\lambda_{jj'}^{(h)})$ for $j, j' = 1, \ldots, n$ and $h = |i - r| > 0$, $i, r = 1, \ldots, k$.

Now when testing for the presence of stable seasonality, one tests the hypothesis $H_0 : \alpha_i = 0$ (3.4), where $\alpha_i$ is the ith season effect as in (3.3). In connection with inferences about parameters in ANOVA models with autocorrelated errors, Brillinger (1980) used the Fourier transform of the data and proposed modified F-statistics as functions of the frequency $\lambda$ for testing the row or column effects. Sutradhar and MacNeill (1989) use modified time domain F-statistics in testing for row and/or column effects. Since, in practice, time series data are collected sequentially in time or space, we follow Sutradhar and MacNeill (1989) and test the above hypothesis by using a modified F-statistic $F^*$ defined by

$$F^* = (Q_1/Q_3)\{(n-1)C_3/C_1\}, \qquad (3.7)$$

where:

$$Q_1 = n \sum_{i=1}^{k} (\bar{z}_i(\bullet) - \bar{z}_\bullet(\bullet))^2$$

$$Q_3 = \sum_{i=1}^{k} \sum_{j=1}^{n} (z_i(j) - \bar{z}_i(\bullet) - \bar{z}_\bullet(j) + \bar{z}_\bullet(\bullet))^2 ,$$

with $\bar{z}_i(\bullet) = \sum_{j=1}^{n} z_i(j)/n$, $\bar{z}_\bullet(j) = \sum_{i=1}^{k} z_i(j)/k$, and $\bar{z}_\bullet(\bullet) = \sum_{i=1}^{k} \sum_{j=1}^{n} z_i(j)/kn$;

and

$$C_1 = \frac{1}{n} \sum_{j=1}^{n} \sum_{j'=1}^{n} \left\{\sigma_{jj'} - \frac{2}{k(k-1)} \sum_{h=1}^{k-1} \lambda_{jj'}^{(h)}\right\},$$

$$C_3 = \frac{1}{n} \left[\left\{\sum_{j=1}^{n} \sigma_{jj} - \sum_{j\neq j'} \sigma_{jj'}/(n-1)\right\}\right.$$

$$\left. - \frac{2}{k(k-1)} \left\{\sum_{h=1}^{k-1} \sum_{j=1}^{n} \sum_{j'=1}^{n} (k-h)\, \lambda_{jj'}^{(h)} - \sum_{h=1}^{k-1} \sum_{j\neq j'} (k-h)\, \lambda_{jj'}^{(h)}/(n-1)\right\}\right],$$

where $\sigma_{tt'}$ and $\lambda_{tt'}^{(h)}$ are given by (3.6)

For significance testing one needs to know the p-value of the statistic, namely, $\Pr(F^* \geq f^*)$, where $f^*$ is the data based value of $F^*$. Note that the quadratic forms $Q_1$ and $Q_3$ are not independent. This dependence between the quadratic forms makes the derivation of the exact distribution of the test statistic $F^*$, mathematically intractable; hence we seek a suitable approximation. We consider

$$\Pr(F^* \geq f^*) = \Pr[\{d_1 Q_1 - f^* Q_3\} \geq 0], \tag{3.8}$$

where $d_1 = (n - 1)C_3/C_1$. Sutradhar and MacNeill (1989) give a Gaussian approximation to the distribution of

$$Q = d_1 Q_1 - f^* Q_3, \tag{3.9}$$

for the case when $k$ and $n$ are sufficiently large. In the following sub-section, we provide a finite sample approximation based on the first four moments of $Q$.

### 3.1.1 A Four Moment Approximation

Rewrite $Q$ in (3.9) as

$$Q = m_1 q_1 + m_2 q_2 , \tag{3.10}$$

where $m_1 = d_1 = (n - 1)C_3/C_1$, $m_2 = f^*$, $q_1 = Q_1 = Z^{*\prime} A_1 Z^*$, and $q_2 = Q_3 = Z^{*\prime} A_2 Z^*$, and where

$$A_1 = [(I_k - k^{-1} U_k) \otimes U_n]/n,$$

$$A_2 = [(I_k - k^{-1} U_k) \otimes I_n - \frac{1}{n} \{(I_k - k^{-1} U_k) \otimes U_n\}],$$

where $I_k$ is the $k \times k$ identity matrix, $U_k$ is the $k \times k$ unit matrix, and $\otimes$ is the Kronecker product. Under the assumption tht $u_i(j)$ in (3.3) follow a SARMA (p, q) $(P, Q)_s$ process, $Z^*$ has a kn-dimensional normal distribution with mean vector $m^*$ and covariance matrix $\Sigma^*$ (3.6), where $m^* = (m_{11}, \ldots, m_{ij}, \ldots, m_{kn})'$ with $m_{ij} = \mu + \alpha_i + \beta_j$. For $h, \ell, u, v = 1, 2,$ let

$s(h) = \text{trace } A_h \Sigma^*$ ,

$$s(h, \ell) = \text{trace } A_h\Sigma^* A_\ell\Sigma^* ,$$

$$s(h, \ell, u) = \text{trace } A_h\Sigma^* A_\ell\Sigma^* A_u\Sigma^* ,$$

$$s(h, \ell, u, v) = \text{trace } A_h\Sigma^* A_\ell\Sigma^* A_u\Sigma^* A_v\Sigma^*,$$

and

$$T(h) = \text{trace } m^{*\prime}A_h m^* ,$$

$$T(h, \ell) = \text{trace } m^{*\prime}A_h\Sigma^*A_\ell m^* ,$$

$$T(h, \ell, u) = \text{trace } m^{*\prime} A_h\Sigma^* A_\ell\Sigma^* A_u m^* ,$$

and

$$T(h, \ell, u, v) = \text{trace } m^{*\prime}A_h\Sigma^* A_\ell\Sigma^* A_u\Sigma^* A_v m^* .$$

Then the mixed cumulants of $q_1$ and $q_2$ up to order four are as follows:

$$K_1(h) = S(h) + T(h) ,$$

$$K_2(h, \ell) = 2S(h, \ell) + 4T(h, \ell) ,$$

$$K_3(h, \ell, u) = 8S(h, \ell, u) + 8[T(h, \ell, u) + T(h, u, \ell) + T(u, h, \ell] ,$$

and

$$\begin{aligned} K_4(h, \ell, u, v) = {} & 16[S(h, \ell, u, v) + S(h, \ell, v, u) + S(h, v, \ell, u)] \\ & + 16[\{T(h, \ell, u, v) + T(h, \ell, v, u) \\ & + T(h, v, \ell, u) + T(v, h, \ell, u)\} \\ & + \{T(h, u, \ell, v) + T(h, u, v, \ell) \\ & + T(h, v, u, \ell) + T(v, h, u, \ell)\} \\ & + \{T(u, h, \ell, v) + T(u, h, v, \ell) \end{aligned}$$

$$+ T(u, v, h, \ell) + T(v, u, h, \ell)\}].$$

Next, one obtains the first four cumulants of $Q$ (3.10) as:

$$w_1 = \sum_{h=1}^{2} m_h k_1(h) ,$$

$$w_2 = \sum_{h=1}^{2} \sum_{\ell=1}^{2} m_h m_\ell K_2(h, \ell) ,$$

$$w_3 = \sum_{h=1}^{2} \sum_{\ell=1}^{2} \sum_{u=1}^{2} m_h m_\ell m_u K_3(h, \ell, u) ,$$

and

$$w_4 = \sum_{h=1}^{2} \sum_{\ell=1}^{2} \sum_{u=1}^{2} \sum_{v=1}^{2} m_h m_\ell m_u m_v K_4(h, \ell, u, v).$$

The mean, standard deviation, skewness and kurtosis of the distribution of $Q$ are as follows:

$$\mu_1'(Q) = w_1 ,$$

$$\{\mu_2(Q)\}^{\frac{1}{2}} = w_2^{\frac{1}{2}} ,$$

$$\{\beta_1(Q)\}^{\frac{1}{2}} = w_3/w_2^{\frac{3}{2}} ,$$

and

$$\beta_2(Q) = (w_4/w_2^2) + 3.$$

The distribution of $Q$ is approximated by a Johnson (1949) curve which has the same first four moments, and then $P_r(Q \geq 0)$ is computed. The algorithm AS99 due to Hill, Hill and Holder (1976), among others, is available for fitting the Johnson curve by moments.

### 3.1.2 Test for Stable Seasonality for $(0, 1)(0, 1)_{12}$ Error Process: A Special Case

Let $\delta_r$ be the covariance between any two seasonal-irregular differences lagged by $r$. When seasonal-irregular differences $(Z^*)$ follow a $(0, 1)(0, 1)_s$ process, the covariances are:

$$\delta_0 = \sigma_a^2 (1 + \theta^2)(1 + \Theta^2), \quad \delta_1 = -\sigma_a^2 (1 + \Theta^2),$$

$$\delta_{s-1} = \sigma_a^2 \, \theta \, \Theta, \quad \delta_s = \sigma_a^2 \, \Theta(1 + \theta^2), \quad \delta_{s+1} = \delta_{s-1} ,$$

with the remaining covariances being zero. The computations of the first four moments of $Q$ require $\Sigma^*$ (3.6) to be known, where $\Sigma^*$ is the $kn \times kn$ covariance matrix of $Z^*$. For $n$ years and $k = 12$ seasons, the component matrices of $\Sigma^*$ are:

$$\Sigma = \begin{bmatrix} \delta_0 & \delta_s & 0 & \cdots & 0 & 0 \\ \delta_s & \delta_0 & \delta_s & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & \delta_0 & \delta_s \\ 0 & 0 & 0 & \cdots & \delta_s & \delta_0 \end{bmatrix}_{n \times n} ,$$

$$\Lambda_1 = \begin{bmatrix} \delta_1 & \delta_{s+1} & 0 & \cdots & 0 & 0 \\ \delta_{s-1} & \delta_1 & \delta_{s+1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \delta_1 & \delta_{s+1} \\ 0 & 0 & 0 & \cdots & \delta_{s-1} & \delta_1 \end{bmatrix}_{n \times n} ,$$

$$\Lambda_{11} = \begin{bmatrix} \delta_{s-1} & 0 & 0 & \cdots & 0 & 0 \\ \delta_1 & \delta_{s-1} & 0 & \cdots & 0 & 0 \\ \delta_{s+1} & \delta_1 & \delta_{s-1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \\ 0 & 0 & 0 & \cdots & \delta_{s-1} & 0 \\ 0 & 0 & 0 & \cdots & \delta_1 & \delta_{s-1} \end{bmatrix} ,$$

and $\Lambda_i = 0$ for $i = 2, \ldots, 10$.

The modified F-test statistic $F^*$ (3.7) or equivalently the statistic $Q$ in (3.10) requires $C_3$ and $C_1$ to be known. For the special $(0, 1)(0, 1)_{12}$ process, the appropriate formulas for $C_1$ and $C_3$ are as follows:

$$C_1 = (1 + \theta^2)(1 + \Theta^2) - 2(1 - \frac{1}{n})\,\Theta(1 + \theta^2)$$

$$+ \frac{1}{6}\{1 + (1 - \frac{1}{n})/11\}\,\theta(1 + \Theta^2) - (4/11)(1 - \frac{1}{n})\,\theta\,\Theta \quad (3.11)$$

and

$$C_3 = (1 + \theta^2)(1 + \theta^2) + (2\,\Theta/n)(1 + \theta^2)$$

$$+ (\theta/6)(1 + \Theta^2)\,(1 - 1/11n)$$

$$- (\theta\,\Theta/6n)\,[n/11 - 2(n - 2)/11(n - 1) - 2]. \quad (3.12)$$

### 3.2 Tests for Moving Seasonality

To test for the presence of moving seasonality, one tests the hypothesis (3.5), namely, $H_0\!:\beta_j = 0$, against $H_1\!:\beta_j \neq 0$ for at least one $j$, $j = 1, \ldots, n$. By calculations similar to those for $F^*$, the appropriate modified F-statistic for testing the above hypothesis can be shown to be:

$$F^{**} = (Q_2/Q_3)\,\{(k - 1)\,C_3/C_2\}, \quad (3.13)$$

where

$$Q_2 = k \sum_{j=1}^{n} (\bar{z}_{\bullet}(j) - \bar{z}_{\bullet}(\bullet))^2 ,$$

$$C_2 = \frac{1}{n}\,[\sum_{j=1}^{n} \{\sigma_{jj} + (2/k) \sum_{h=1}^{k-1} \sum_{j'=1}^{n} (k - h)\,\lambda_{jj'}^{(h)}\}$$

$$- \frac{1}{n - 1} \sum_{j \neq j'}^{n} \{\sigma_{jj'} + (2/k) \sum_{h=1}^{k-1} (k - h)\lambda_{jj'}^{(h)}\}],$$

and $Q_3$ and $C_3$ are as in (3.7).

Let $f^{**}$ be the data based value of $F^{**}$. In order to render a decision concerning the null hypothesis $H_0\!:\beta_j = 0$, we require

$P_r(F^{**} \geq f^{**})$. That is, we compute

$$Pr\{Q^* \geq 0\}, \tag{3.14}$$

where $$Q^* = b_1 q_1^* + b_2 q_2^*,$$ and where

$$b_1 = (k - 1)C_3/C_2,$$

$$b_2 = -f^{**},$$

$$q_1^* = Z^{*\prime} B_1 Z^*,$$

$$q_2^* = z^{*\prime} \beta_2 z^*,$$

with

$$B_1 = [U_k \otimes (I_n - n^{-1} U_n)]/K, \quad B_2 = A_2,$$

where $A_2$ is given by (3.10). Replacing $A_1$, $A_2$ by $B_1$, $B_2$, and $m_1$, $m_2$ by $b_1$, $b_2$ the four moments of $Q^*$ follow from the four moments of $Q$. Then the distribution of $Q^*$ is approximated as before by a Johnson (1949) curve which has the same first four moments.

### 3.2.1 Test for Moving Seasonality for the $(0, 1)(0, 1)_{12}$ Error Process: A Special Case (continued)

The $kn \times kn$ covariance matrix $\Sigma^*$ of $Z^*$ remains the same as for the test of stable seasonality. The modified F-statistic $F^{**}$ (3.13), or equivalently the statistic $Q^*$ in (3.14), requires $C_3$ and $C_2$ to be known. For the $(0, 1)(0, 1)_{12}$ process $C_3$ is already given in (3.12). The appropriate formula for $C_2$ is given by

$$C_2 = (1 + \theta^2)(1 + \Theta^2) - (\theta/6)(1 + \Theta^2)(11 - 1/n)$$

$$+ (2\Theta/n)(1 + \theta^2) + (\theta\ \Theta/6)\{1 - 22/n - (n - 2)/n(n - 1)\}. \quad (3.15)$$

## 4. TEST OF A GENERAL LINEAR HYPOTHESIS

Using the notation of (3.1), we rewrite model (3.3) as

$$z\{(i - 1)n + j\} = \alpha_i + \beta_j + u\{(i - 1)n + j\}, \quad (4.1)$$

where $z\{(i - 1)n + j\}$ are the stacked observations read as $z(1), z(2), \ldots, z(n), z(n + 1), \ldots, z(kn)$. Equation (4.1) can be expressed as

$$Z^* = X\gamma + U^* \quad (4.2)$$

where

$$Z^* = (z(1), z(2), \ldots, z(n), z(n + 1), \ldots, z(kn))',$$

$$U^* = (u(1), u(2), \ldots, u(n), u(n + 1), \ldots, u(kn))',$$

$$\gamma = (\alpha_1, \ldots, \alpha_k, \beta_1, \ldots, \beta_n)',$$

and $X$ is the $kn \times (k + n)$ design matrix given by

$$X = \begin{bmatrix} 1_n' & 0' & \cdots & 0' & I_n \\ 0 & 1_n' & \cdots & 0' & I_n \\ \vdots & \vdots & & \vdots & \vdots \\ 0' & 0' & \cdots & 1_n' & I_n \end{bmatrix}$$

where $1_n = (1, 1, \ldots, 1)_{1\times n}$, $0 = (0, 0, \ldots, 0)_{1\times n}$, and $I_n$ is the $n \times n$ identity matrx.

We consider the test of a general linear hypothesis

$$H_0: C\gamma = 0 , \tag{4.3}$$

where $C$ is a suitable matrix corresponding to certain given constraints. For the case when $U^*$ in (4.2) has normal distribution with mean 0 and covariance matrix $\Sigma^*$, Berndt and Savin (1975) have discussed Wald, likelihood ratio (LR), Lagrange multiplier (LM) and max-root (MR) tests for testing the linear hypothesis (4.3). In the context of seasonal analysis of Canadian murders, McLeod, MacNeill and Bhattacharyya (1985) use the Wald, and LR tests for testing a linear hypothesis similar to (4.3). Berndt and Savin (1975) have shown that the Wald, LR, LM, and MR tests based on exact distributions conflict with each other when applied to a given data set. In a later paper, Berndt and Savin (1977) showed that even in the asymptotic case, the Wald, LR and LM tests yield conflicting inferences. For the case when $U^*$ follows the AR(1) process, Sutradhar and Bartlett (1989) have shown by a simulated experiment that the Wald test is too liberal and the LR test has convergence problems for certain values of $\phi$, where $\phi$ is the coefficient of AR(1) process. More specifically, Sutradhar and Bartlett consider the model

$$z_i(j) = \mu + \alpha_i + u_i(j),$$

$$u_i(j) = \phi u_i(j - 1) + a_i(j)$$

where $a_i(j)$ is the component of a white noise series and test $H_0: \alpha_i = 0$ for all $i = 1, \ldots, k$, for $k = 2, 3$. Under the null hypothesis, the Wald and the LR statistics have asymptotically chi-square distributions with $k - 1$ degrees of freedom. Based on 2000 simulations, they have shown that for $n = 100$, for example, the Wald test overestimates the significance levels. The amount of bias increases as $|\phi|$ increases.

It is shown in Sutradhar and Bartlett (1989) that for $\phi = -0.8(0.1)0.6$, the LR test is almost unbiased in estimating the significance level. The LR test overestimates the significance level for the cases when $\phi = 0.7$ and 0.8. For $\phi = -0.9$, and 0.9, the test has convergence problems and highly overestimates significance levels, since they are based only on cases where convergence was achieved. Thus, the use of the Wald and the LR tests (specially Wald) for testing the linear hypothesis with SARMA errors may not be reliable in general, and these tests will not be discussed further in this paper.

For the regression model when the components of $U^*$ are uncorrelated, one tests the linear hypothesis (4.3) by using the classical F-statistic given by

$$F = \frac{(C\bar{\gamma} - C\gamma)'(C(X'X)^{-1}C')^{-1}(C\hat{\gamma} - C\gamma)/q}{(Z^* - X\hat{\gamma})'(Z^* - X\hat{\gamma})/(kn - k - n - 1)}, \tag{4.4}$$

where $\bar{\gamma}$ is the ordinary least square estimate of $\gamma$, and $q$ is the rank of $C$. For the model (4.2) when the error term $U^*$ has covariance $\Sigma^*$, the F-statistic (4.4) is inappropriate, since the dependence among observations implied by the model alters the amount of information provided by the observations. Similarly to (3.7) and (3.13), one may modify the F-statistic (4.4) by adjusting the biases induced by correlations among observations. The modified F-statistic is given by

$$\tilde{F} = \frac{q\{\text{trace}(I - D_2)\Sigma^*\}F}{(kn - k - n - 1)(\text{trace } D_1\Sigma^*)}, \tag{4.5}$$

where: $F$ is given by (4.4), $\Sigma^*$ is the covariance of $U^*$, and

$D_1 = R'(RR')^{-1}$, with $R = C(X'X)^{-1}X'$, $D_2 = X(X'X)^{-1}X'$ .

The statistic $\tilde{F}$ in (4.5) may be expressed as

$$\tilde{F} = \frac{\text{trace}(I - D_2)\Sigma^*}{\text{trace } D_1\Sigma^*} \left\{ \frac{U^{*\prime}D_1U^*}{U^{*\prime}(I - D_2)U^*} \right\} . \qquad (4.6)$$

Now to find the p-value, one may use the four moment approximation discussed in Section 3.1.1. Alternately, one may compute the significance level by using the well-known Satterthwaite approximation as discussed below.

## 4.1 Satterthwaite Approximation

Let $\tilde{f}$ be the observed value of $\tilde{F}$. We need to compute

$$Pr(\tilde{F} \geq \tilde{f}) = Pr[U^{*\prime}\{d^*D_1 - \tilde{f}(I - D_2)\} U^* \geq 0], \qquad (4.7)$$

where $U^* \sim N(0, \Sigma^*)$, $d^* = \text{trace } (I - D_2)\Sigma^*/\text{trace } D_1\Sigma^*$. Let $\Sigma^{*\frac{1}{2}} U^* = \delta^*$ so that $\delta^* \sim N(0, I)$. The the probability in (4.7) reduces to

$$Pr[\delta^{*\prime}\Sigma^{*\frac{1}{2}} (d^*D_1 - \tilde{f}(I - D_2))\Sigma^{*\frac{1}{2}} \delta^* \geq 0]. \qquad (4.8)$$

Further, this probability is equivalent to

$$Pr[\{ \sum_{j=1}^{r} \lambda_j\chi_j^2 / \sum_{j=s+1}^{n} |\lambda_j|\chi_j^2 \} \geq 1], \qquad (4.9)$$

where $\lambda_1 \geq \ldots \geq \lambda_r > 0 = \lambda_{r+1} = \ldots = \lambda_s > \lambda_{s+1} \geq \lambda_{s+2} \geq \ldots \geq \lambda_n$ are the eigenvalues of $(d^*D_1 - \tilde{f}(I - D_2))\Sigma^*$. In (4.9), $\chi_j^2$ denotes chi-square variable with 1 degree of freedom. The probability in (4.9) may be expressed as

$$Pr\left[F_{\zeta_1,\zeta_2} \geq \frac{b\zeta_2}{a\zeta_1}\right] , \qquad (4.10)$$

where $F_{\zeta_1,\zeta_2}$ denotes the usual F-ratio with degrees of freedom $\zeta_1$ and $\zeta_2$, with

$$\zeta_1 = (\sum_{j=1}^{r} \lambda_j)^2 / \sum_{j=1}^{r} \lambda_j^2 ,$$

$$\zeta_2 = (\sum_{j=s+1}^{n} \lambda_j)^2 / \sum_{j=s+1}^{n} \lambda_j^2 ,$$

and where

$$a = \sum_{j=1}^{r} \lambda_j^2 / \sum_{j=1}^{r} \lambda_j ,$$

$$b = \sum_{j=s+1}^{n} \lambda_j^2 / \sum_{j=s+1}^{n} |\lambda_j| .$$

If $\zeta_1$ and $\zeta_2$ are fractions, the probability is computed through interpolation.

## 5. TESTS OF STOCHASTIC SEASONALITY

Recall model (2.8), $z_t = s_{1t} + u_t$, for seasonal irregular differences. In this model, $s_{1t}$ is the deterministic seasonal component and $u_t$ is the stochastic component. However, by (2.5) and (2.7), $u_t$ has the form

$$u_t = \eta_v(B) d_t + \psi_u(B)\xi_t + \phi_p(B)\ \Phi(B^s) e_t , \qquad (5.1)$$

where $\eta_v(B) d_t$ is the seasonal component of $u_t$, and where $\psi_u(B)\xi_t$ and $\phi_p(B)\Phi(B)e_t$ are nonseasonal components. Since $\psi_u(B)\xi_t + \phi_p(B)\Phi(B^s)\ e_t = r_t$, $u_t$ is the sum

$$u_t = s_{2t} + r_t , \qquad (5.2)$$

of two components, seasonal (signal) and nonseasonal (noise).

Now, the stochastic properties of the stationary series $u_t$ can be assessed via its sample autocorrelation function. In general, there will be a nonseasonal filter that will eliminate whatever low-order autocorrelation (trend) from $u_t$ that was not eliminated previously. Let h(B) be the appropriate nonseasonal filter. Then

$$\tilde{u}_t = \tilde{s}_{2t} + \tilde{v}_t \,, \qquad (5.3)$$

where

$$\tilde{u}_t = h(B)u_t,$$

$$\tilde{s}_{2t} = h(B)\eta_v(B)dt = h(B)s_{2t} \,,$$

and

$$\tilde{v}_t = h(\beta)r_t \,.$$

In (5.3), $\tilde{v}_t$ is white noise and $\tilde{s}_{2t}$ is autocorrelated only at seasonal lags. It then follows from (2.7) that

$$\tilde{u}_t = h(B)\ \theta_q(B)\ \Theta_Q(B^s)a_t \,, \qquad (5.4)$$

where $a_t$ is the white noise. Thus one may write

$$\tilde{u}_t = g_u(B)a_t \,, \qquad (5.5)$$

and

$$\tilde{s}_{2t} = g_s(B)d_t, \qquad (5.6)$$

where $g_u(B) = h(B)\ \theta_q(B)\ \Theta_Q\ (B^s)$, and $g_s(B) = h(B)\ \eta_v(B)d_t$, with $d_t$ being white noise. Then the estimate $\hat{\tilde{s}}_{2t}$ that minimizes $E(\tilde{s}_{2t} - \hat{\tilde{s}}_{2t})^2$ is

$$\hat{\tilde{s}}_{2t} = \frac{\hat{\sigma}_d^2 \, \hat{g}_s(B)\hat{g}_s(F)}{\hat{\sigma}_a^2 \, \hat{g}_u(B)\hat{g}_u(F)} , \tag{5.7}$$

[cf. Pierce (1978, p.244)] where $F = B^{-1}$ is the forward shift operator. Next, the seasonal component $s_{2t}$ of $u_t$ in (5.2) is estimated as

$$\hat{s}_{2t} = h^{-1}(B)\, \hat{\tilde{s}}_{2t} . \tag{5.8}$$

Roughly speaking, the non-zero $s_{2t}$ in (5.2) indicates the presence of stochastic seasonality. Since $\tilde{s}_{2t}$ is autocorrelated only at seasonal lags, one may detect the presence of stochastic seasonality by testing $\tilde{\rho}_s = 0$, $\tilde{\rho}_{2s} = 0$, ..., etc, where $\tilde{\rho}_s$ is the autocorrelation of $\tilde{s}_{2t}$ at lag $s$, $s$ being the seasonal period. The null hypothesis $H_0 : \tilde{\rho}_s = 0$ may be tested by using the normal statistic

$$z_s = \frac{\sqrt{n}\, \hat{\tilde{r}}_s}{\text{est. s.d.}(\hat{\tilde{r}}_s)} , \tag{5.9}$$

[cf. McLeod (1978), Ansley and Newbold (1979)]. Similarly, one may test $\tilde{\rho}_{2s} = 0$, $\tilde{\rho}_{3s} = 0$, ... etc. The rejection of the null hypothesis would lead to the conclusion that the series contains stochastic seasonality.

## 6. NUMERICAL DISCUSSION

Airline Series

Consider the airline series of monthly passenger totals for international air travel for the period from 1949 to 1960. This series has been analyzed by Box and Jenkins (1976, Tables 9.1, p. 304), among many others. Box and Jenkins parameterized the data in terms of the seasonal multiplicative model $(0, 1, 1) \times (0, 1, 1)_{12}$. Let $\{w_t\}$ denote the original monthly passenger totals and $\{z_t\}$ denote the deterministic trend free series, i.e., $z_t = \nabla\nabla_{12} w_t$. By expressing $z_t$ in the form of (2 · 8), namely, $z_t = s_{1t} + u_t$, and by using a Gaussian approximation for large k and n, Sutradhar and MacNeill (1989) have shown that the $\{z_t\}$ series contains neither stable nor moving seasonalities. In practice, the season size k may be very small. For example, in a quarterly series k = 4. In such cases, it may be appropriate to use a finite sample approach for testing for the presence of stable and moving seasonalities. Since the approximations developed in Section 3.1 and 3.2 do not require large k or n, we use these approximations in order to test for the presence of stable as well as moving seasonalities in the airline series, where k = 12 and n = 12.

In testing for the presence of stable seasonalities, we find $f^* = 0.6588$ by (3.7), for $\theta = 0.4129$, $\Theta = 0.4503$, and $\sigma_a^2 = 0.00117$. We then compute $p_r(Q \geq 0)$ by (3.10), where Q is the linear combination of the two quadratic forms in normal variables. This probabilty calculation requires the first four moments of Q. The moments calculation is straight-forward as all T(.)s vanish in the equations for mixed cumulants of the two quadratic forms. This is because, under the $H_0$: $\alpha_i = 0$ (3.4), $m^{*'} A_1 = m^{*'} A_2 = 0$ where $m^*$, $A_1$, and $A_2$ are given in section 3.1.1. The first four moments of Q are:

$$\mu_1'(Q) = 0.0640, \ \{\mu_2(Q)\}^{\frac{1}{2}} = 0.0929,$$
$$\{\beta_1(Q)\}^{\frac{1}{2}} = 0.9876, \text{ and } \beta_2(Q) = 4.6421.$$

Then we use the algorithm AS99 due to Hill, Hill and Holder (1976) to approximate the distribution of Q by a Johnson (1949) curve which has the same first four moments and obtain

$$Pr(Q \geq 0) = 0.7370 \ . \qquad (6.1)$$

Since this p-value is very large, we do not reject the null hypothesis that the series does not contain stable seasonality.

In testing for the presence of moving seasonality, we find $f^{**} = 1.1308$ by (3.13) for $\theta = 0.4129$, and $\Theta = 0.4503$. In computing $Pr(F^{**} > f^{**})$, or $Pr(Q^{*} \geq 0)$ (3.14), the four moments of $Q^{*}$ are calculated in a manner similar to that of Q. But unlike the mixed cumulants of the two quadratic forms of the linear combination Q, all T(.)s do not vanish in the equations for mixed cumulants of the two quadratic forms of the linear combination $Q^{*}$. The non-zero T(.)s are: T(2), T(2,2), T(2,1,2), T(2,2,2), T(2,1,2), T(2,1,2,2), T(2,2,1,2). The first four moments of $Q^{*}$ are computed as

$$\mu_1'(Q^{*}) = -0.0303, \quad \{\mu_2(Q^{*})\}^{\frac{1}{2}} = 0.1011,$$
$$\{\beta_1(Q^{*})\}^{\frac{1}{2}} = 0.932, \text{ and } \beta_2(Q^{*}) = 4.5445.$$

Then by fitting the Johnson curve as before we obtain

$$Pr(Q^{*} \geq 0) = 0.3297. \tag{6.2}$$

Since this p-value is also large, we do not reject the null hypothesis that the series does not contain moving seasonality. Thus the moment approxiamtion, and the asymptotic normal test in Sutradhar and MacNeill (1989) reach the same decision in favour of the null hypothesis, namely, that there is no deterministic seasonality present in the airline series.

Canadian Export Series

The data file for this study contains export records of Canada from 1972 to 1981. The purpose of this section is to test for the presence of seasonality in the export data, where seasonality represents the composite effect of climatic and institutional events which repeat more or less regularly. In order to test for the presence of seasonality, it is necessary to detrend the data. The Canadian export data $\{w_t\}$ and its ACF and PACF are shown in Figures 1(a), 1(b), and 1(c) respectively. The ACF and PACF suggest

[INSERT FIGURES 1(a), 1(b), 1(c) HERE]

differencing to make the data trend free. We find that the data $\{w_t\}$ can be adequately modelled as a (0,1,1) process with $\theta = 0.8083$, and $\sigma_a^2 = 105.5215$. The $\{z_t\}$ can be adequately modelled as a (0,0,1) process with $\theta = 0.8083$, and $\sigma_a^2 = 105.5215$. The $\{z_t\}$ series, where $z_t = \nabla w_t$, and its ACF and PACF are plotted in figures 2(a), 2(b), and 2(c) respectively. Since $\{z_t\}$ is stationary, we can assume that $z_t = s_{1t} + u_t$ as in (2.8), where $s_{1t}$ is the deterministic seasonal component consisting of stable and moving seasonalities, and $u_t$ is the

[INSERT FIGURES 2(a), 2(b), 2(c) HERE]

residual which may contain stochastic seasonal, and irregular components. Re-express $z_t$ in the form of (3.3) , that is,

$$z_i(j) = \mu + \alpha_i + \beta_i + \mu_i(j),$$

where $\alpha_i$ is the ith season effect and $\beta_j$ is the jth annual effect, and where $z_i(j)$ is the detrended export for the ith ($i = 1,...,12$) season of the jth ($j = 1,2,..., 10$) year. For $\theta = 0.8083$, the modified F-statistics for testing the $H_0$: $\alpha_i = 0$, and $H_0$: $\beta_j = 0$ were found, respectively, to be

$$f^* = 2.4283, \text{ and } f^{**} = 0.9338$$

By calculations similar to those for the analysis of the airline series, we find

$$\Pr(F^* \geq f^*) = \Pr(Q \geq 0) = 9.64 \times 10^{-34},$$

$$Pr(F^{**} \geq f^{**}) = Pr(Q^{*} \geq 0) = 0.4604$$

Since the p-value of the $F^{*}$-test is very small, we conclude that the stable seasonality in the Canadian export data is highly significant. The $\alpha$-values represent the magnitude of the seasons effect. The general effect $\mu$ is found to be 0.0995, and the 12 $\alpha$-values of the export data are:

$$\alpha_i = 5.27,\ -9.12,\ 0.78,\ -1.44,\ 9.63,\ 1.07,$$
$$-4.08,\ 0.42,\ 2.07,\ 2.11,\ 2.32,\ -8.94.$$

Furthermore, as the p-value of the $F^{**}$-test is large, we do not reject $H_0$: $\beta_j = 0$, and conclude that there is no moving seasonality in the series. The $\beta$-values represent the years effect. The 10 $\beta$-values of the export data are:

$$\beta_j = -0.16,\ 0.82,\ 0.94,\ -0.47,\ -0.82,\ -0.53,$$
$$0.67,\ -1.09,\ -0.18,\ 0.93.$$

Since the stable seasonality is significant, one should seasonally adjust the Canadian export series before using it for further economic planning. The seasonal adjustment can be done by using the X-11-ARIMA method developed by Dagum or by using the model building approaches of Box, Hillmer, and Tiao (1978), Pierce (1978), Burman (1980), for example. However, discussion of various seasonal adjustment procedures is not the concern of the present paper, as we are mainly concerned about the testing procedures for the detection of the presence of stable and moving seasonalities in the data.

We also apply the Satterthwaite approximation described in Section 4.1 to test for the presence of stable and moving seasonalities in the Canadian export data. Testing for the presence of stable seasonality is equivalent to testing the $H_0$: $C\gamma = 0$(4.3), where $\gamma = (\alpha_1 \ldots, \alpha_k, \beta_1 \ldots, \beta_n)'$ and C is the $(k - 1) \times (k + n)$ matrix given by $C = [G, 0]$, where

$$G = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & -1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & -1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ 1 & 0 & 0 & 0 & \cdots & -1 \end{bmatrix} \quad (k - 1) \times k$$

and '0' is the $(k - 1) \times n$ null matrix. Equations (4.4) through (4.10) yield $\zeta_1 = 19.296$, $\zeta_2 = 71.36$, and $a = 0.878$, $b = 6.161$. Since $Pr(F_{19,71} > 25.96) < 0.01$, we reject the null hypothesis and conclude that the series contains significant stable seasonality. Similarly, we test for the presence of moving seasonality which is equivalent to testing the hypothesis $H_0$: $C\gamma = 0$, where $\gamma$ is a $(k + n) \times 1$ vector as before and C is the $(n - 1) \times (k + n)$ matrix given by $C = [0, H]$, where

$$H = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & -1 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & \cdot & & \\ 1 & 0 & 0 & 0 & \cdots & -1 \end{bmatrix} \quad (n - 1) \times n$$

and '0' is the $(n - 1) \times k$ null matrix. In this case we obtain $\zeta_1 = 8.37$, $\zeta_2 = 73.40$, and $a = 15.53$, $b = 2.41$, and compute

$\Pr(F_{8,73} > 1.3589) = 0.2291$. Since this p-value is large, we conclude there is no significant moving seasonality in the data. Thus the four moment approximation and the Satterthwaite approximation provide the same conclusions regarding the significance of stable as well as moving seasonalities of the Canadian export series.

Notice that in testing for the presence of stable and moving seasonalities in the airline as well as the Canadian export data, both series were detrended by taking appropriate differences. The differencing technique to detrend the data is quite suitable when data contain mostly stochastic trend. In a general situation, one may use other detrending techniques which are suitable for removing both deterministic and stochastic trend from the data. The X-11-ARIMA procedure developed by Dagum (1975, 1980), for example, may be used for detrending the data. We detrend the Canadian export data by using the appropriate steps of the X-11-ARIMA procedure and find that the detrended data follow a $(0,1)(0,1)_{12}$ process with

$$\theta_1 = 0.2765,\ \Theta_1 = 0.2995 \text{ and } \sigma_a^2 = 37.1523 .$$

The model (2.8), namely, $z_t = s_{1t} + u_t$, is then fitted. The ordinary least squares estimates of $\mu$, $\alpha$ and $\beta$'s are found to be:

$$\mu = 0.4423 ,$$

$$\alpha_i = 2.96, -6.22, -5.49, -7.00, 2.57, 3.59, -0.51, -0.09$$
$$1.97, 4.07, 6.37, -2.21 ,$$

and

$$\beta_j = -0.95, -0.17, 1.16, -0.56, -0.70, -0.31, 0.35,$$
$$0.28, 1.45, -0.55 .$$

Now to test the $H_0:\alpha_i = 0$, we use, for example, the Satterthwaite approximation and obtain

$$\zeta_1 = 22.56,\ \zeta_2 = 58.83, \text{ and } b\zeta_2/a\zeta_1 = 8.89 .$$

Since $\Pr(F_{23,59} \geq 8.59) \simeq 0$, we reject $H_0:\alpha_i = 0$ and conclude that the series contains significant stable seasonality. We also test $H_0:\beta_j = 0$ by using the Satterthwaite approximation. The modified degrees of freedom for the F-statistic were found to be $\zeta_1 = 11.49$ and $\zeta_2 = 80.29$. Since $\Pr(F_{11,80} \geq 0.174) = 0.0085$, we conclude that there is no moving seasonality in the series. Note that these decisions regarding the significance of stable and moving seasonalities remain the same as in the case where the data were detrended by using differencing.

Canadian Homicide Data (TOT Series)

The TOT data refer to all murders in Canada for the period from 1961 to 1980, excluding the manslaughter and infanticide as they are not considered murder and are not available for the entire period. McLeod, MacNeill and Bhattacharyya (1985) and Dagum, Huot and Morry (1988) have studied this TOT series, among others. McLeod et al concluded that the TOT series (monthly) is not significant at 5%. Dagum et al concluded that there is no identifiable seasonal movement in the monthly murder series. We use the Satterthwaite approximation to examine the presence of seasonalities in the TOT series. Figures 3(a), 3(b), and 3(c) show the monthly TOT data and its ACF, and PACF respectively. As these figures

[INSERT FIGURES] 3(a), 3(b), and 3(c) HERE] suggest differencing, we consider d = 1 and find that the TOT series may be modelled as an (0,1,1) with $\theta = 0.8911$, and $\sigma_a^2 = 309.233$. The graphs for the differenced ($z_t = \nabla w_t$) series along with its ACF and PACF are shown in Figures 4(a), 4(b), and 4(c) respectively. In testing

[INSERT FIGURES 4(a), 4(b), and 4(c) HERE]

for the presence of stable seasonalities, we find $\zeta_1 = 116$, $\zeta_2 = 98$, and $b\zeta_2/a\zeta_1 = 0.6829$. Since $Pr(F_{116,98} \geq 0.6829) = 0.9756$, we decide in favour of $H_0$: $\alpha_i = 0$, i.e., that there is no significant stable seasonality in the TOT series. However, in testing for the presence of moving seasonality, we find $\zeta_1 = 14$, $\zeta_2 = 157$ and $b\zeta_2/a\zeta_1 = 4.22$, which leads to the rejection of the $H_0$: $\beta_j = 0$. Thus the test suggests there is a significant moving seasonality in the TOT series.

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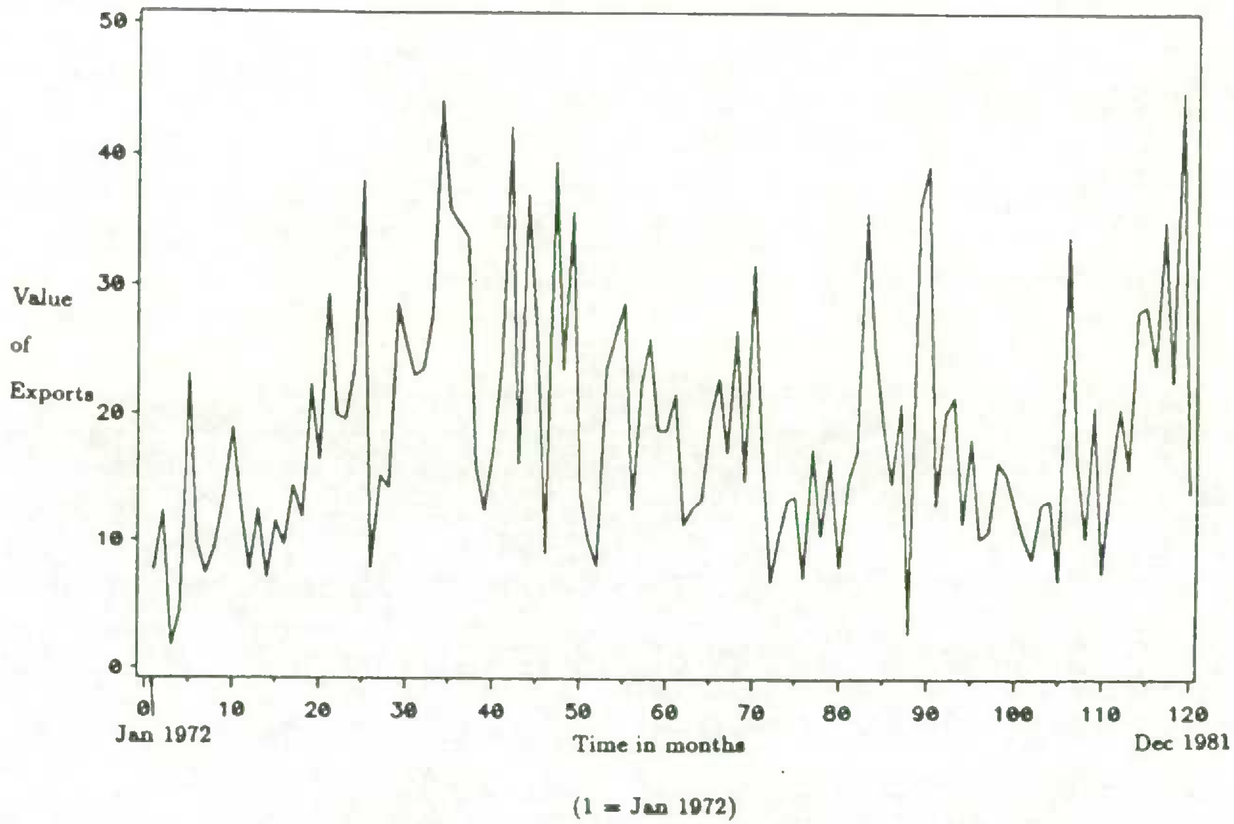


Figure 1(a) Canadian Monthly Exports (1972–1981)

```
     AUTO.  STAND.
LAG  CORR.   ERR.  -1  -.75 -.5 -.25   0   .25  .5   .75   1
                   :----:----:----:----:----:----:----:----:
  1  0.289  0.090                  .   :***.**
  2  0.283  0.089                  .   :***.**
  3  0.182  0.089                  .   :****
  4  0.101  0.089                  .   :**  .
  5  0.199  0.088                  .   :****
  6  0.164  0.088                  .   :***.
  7  0.086  0.087                   .  :**.
  8  0.115  0.087                   .  :**.
  9  0.008  0.087                   .  *  .
 10  0.095  0.086                   .  :**.
 11  0.115  0.086                   .  :**.
 12  0.102  0.085                   .  :**.
 13  0.185  0.085                   .  :**.*
 14 -0.044  0.085                   . *:  .
 15 -0.071  0.084                   . *:  .
 16 -0.055  0.084                   . *:  .
 17 -0.076  0.083                   .**:  .
 18 -0.236  0.083                 **.**:  .
 19 -0.042  0.083                   . *:  .
 20 -0.080  0.082                   .**:  .
 21 -0.136  0.082                   ***:  .
 22 -0.062  0.081                   . *:  .
 23  0.012  0.081                   .  *  .
 24  0.086  0.081                   .  :**.
 25 -0.003  0.080                   .  *  .
```

Figure 1(b)

```
      PR-AUT STAND.
LAG   CORR.   ERR. -1   -.75 -.5  -.25 0    .25  .5   .75  1
                   :----:----:----:----:----:----:----:----:
  1   0.289  0.091                 .   :***.**
  2   0.218  0.091                 .   :****
  3   0.063  0.091                 .   :*  .
  4  -0.016  0.091                 .   *   .
  5   0.144  0.091                 .   :***.
  6   0.077  0.091                 .   :** .
  7  -0.045  0.091                 .  *:   .
  8   0.032  0.091                 .   :*  .
  9  -0.058  0.091                 .  *:   .
 10   0.062  0.091                 .   :*  .
 11   0.068  0.091                 .   :*  .
 12   0.035  0.091                 .   :*  .
 13   0.111  0.091                 .   :** .
 14  -0.170  0.091                 .***:   .
 15  -0.129  0.091                 .***:   .
 16  -0.034  0.091                 .  *:   .
 17  -0.025  0.091                 .  *:   .
 18  -0.296  0.091               **.***:   .
 19   0.106  0.091                 .   :** .
 20   0.099  0.091                 .   :** .
 21  -0.136  0.091                 .***:   .
 22   0.022  0.091                 .   *   .
 23   0.191  0.091                 .   :****
 24   0.135  0.091                 .   :***.
 25  -0.150  0.091                 .***:   .
```

Figure 1(c)

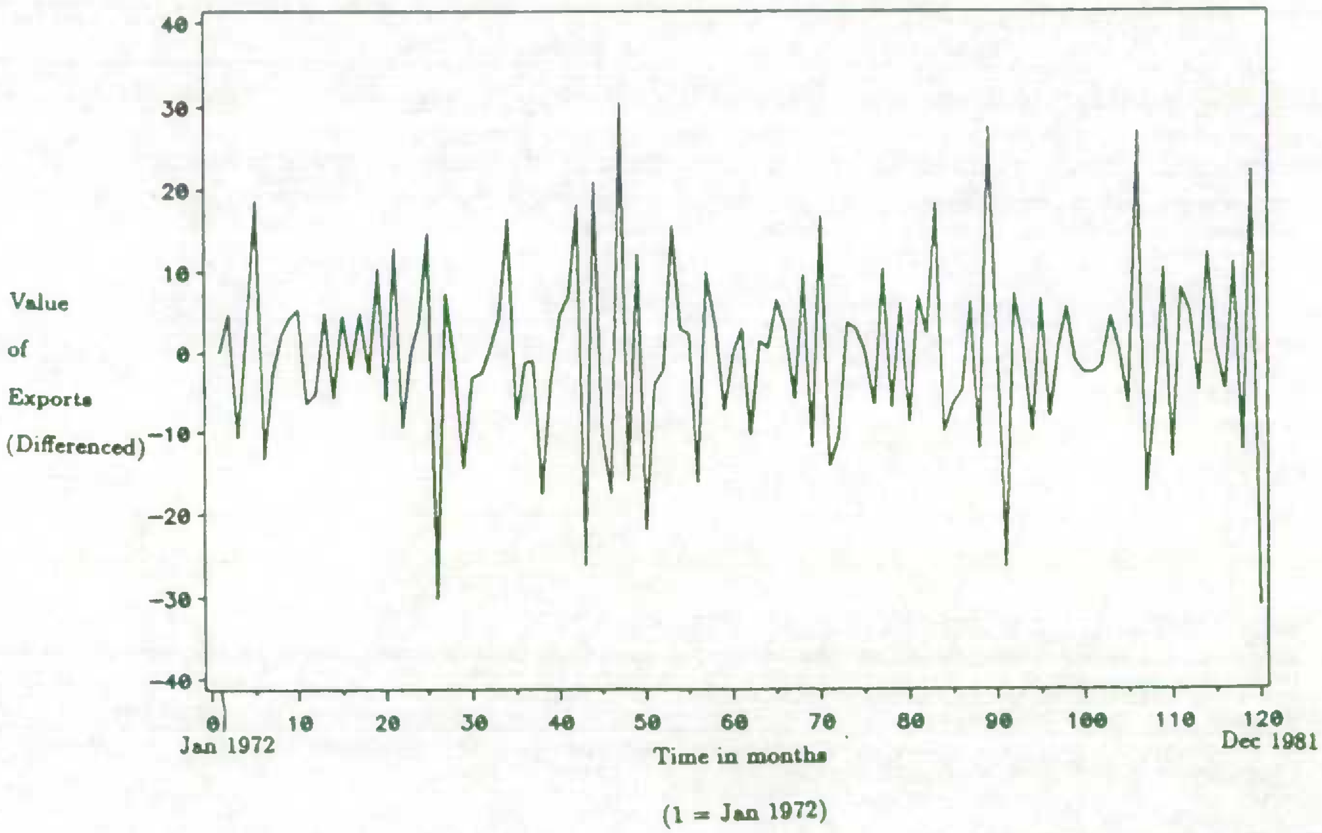


Figure 2(a) Trend-free Canadian monthly export series (1972-1981)

```
      AUTO.  STAND.
LAG   CORR.    ERR.  -1   -.75   -.5 -.25  0   .25  .5   .75   1
                     :----:----:----:----:----:----:----:----:
  1  -0.487   0.090            ******.***:   .
  2   0.052   0.089                  .   :*  .
  3  -0.008   0.089                  .   *   .
  4  -0.117   0.089                  . **:   .
  5   0.086   0.088                  .   :** .
  6   0.028   0.088                  .   :*  .
  7  -0.079   0.087                   .**:  .
  8   0.102   0.087                   .  :**.
  9  -0.135   0.087                   ***:  .
 10   0.040   0.086                   .  :* .
 11   0.023   0.086                   .  *  .
 12  -0.067   0.085                   . *:  .
 13   0.219   0.085                   .  :**.*
 14  -0.134   0.085                   ***:  .
 15  -0.040   0.084                   . *:  .
 16   0.031   0.084                   .  :* .
 17   0.097   0.083                   .  :**.
 18  -0.245   0.083                 **.**:  .
 19   0.161   0.083                   .  :***
 20   0.023   0.082                   .  *  .
 21  -0.098   0.082                   .**:  .
 22   0.001   0.081                   .  *  .
 23   0.001   0.081                   .  *  .
 24   0.127   0.081                   .  :***
 25  -0.052   0.080                   . *:  .
```

Figure 2(b)

```
     PR-AUT  STAND.
LAG   CORR.    ERR.  -1   -.75  -.5  -.25   0   .25   .5   .75   1
                      :----:----:----:----:----:----:----:----:
  1  -0.487   0.091             ******.***:   .
  2  -0.243   0.091                  *.***:   .
  3  -0.132   0.091                   .***:   .
  4  -0.241   0.091                  *.***:   .
  5  -0.147   0.091                   .***:   .
  6  -0.030   0.091                   .  *:   .
  7  -0.108   0.091                   . **:   .
  8  -0.003   0.091                   .   *   .
  9  -0.120   0.091                   . **:   .
 10  -0.123   0.091                   . **:   .
 11  -0.088   0.091                   . **:   .
 12  -0.154   0.091                   .***:   .
 13   0.126   0.091                   .   :***.
 14   0.077   0.091                   .   :** .
 15  -0.028   0.091                   .  *:   .
 16  -0.036   0.091                   .  *:   .
 17   0.212   0.091                   .   :****
 18  -0.184   0.091                   ****:   .
 19  -0.144   0.091                   .***:   .
 20   0.092   0.091                   .   :** .
 21  -0.077   0.091                   . **:   .
 22  -0.206   0.091                   ****:   .
 23  -0.130   0.091                   .***:   .
 24   0.155   0.091                   .   :***.
 25   0.005   0.091                   .   *   .
```

Figure 2(c)

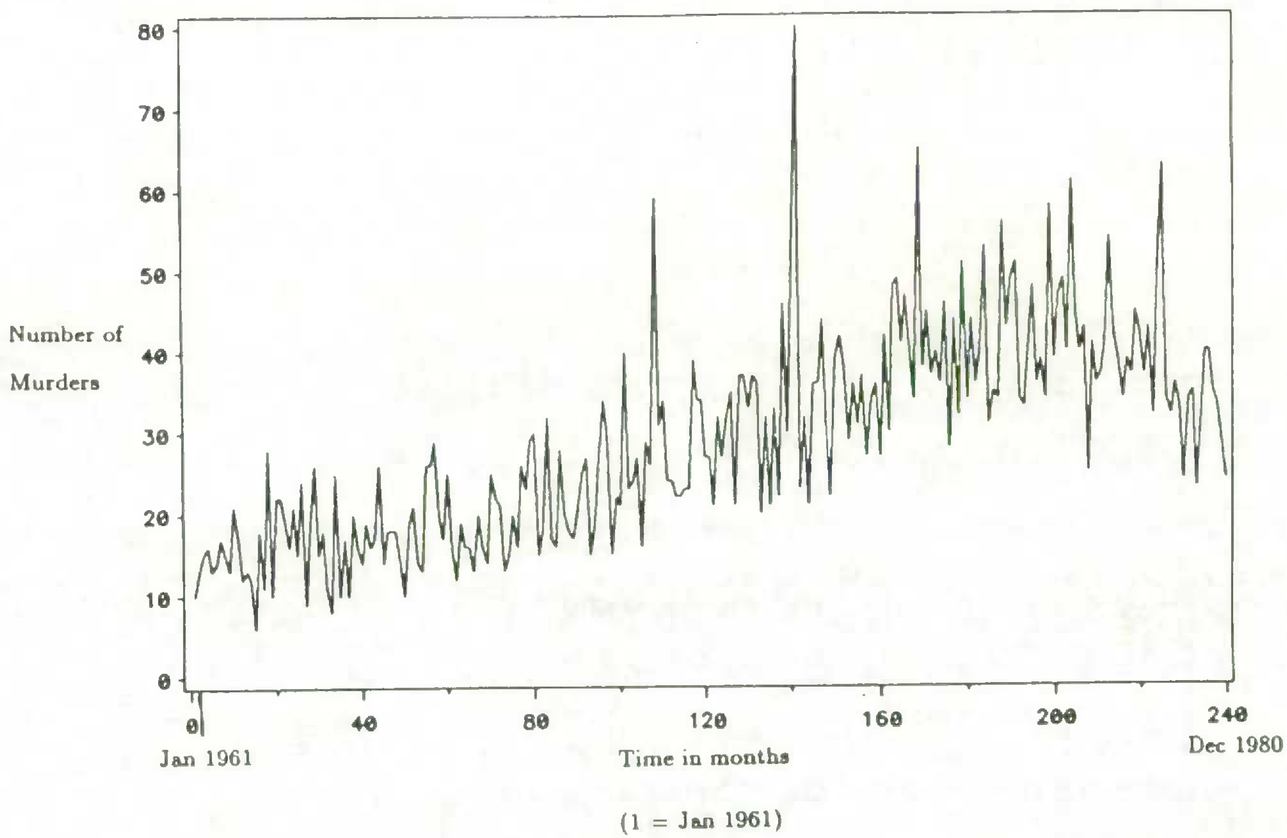


Figure 3(a) Canadian monthly homocide data (TOT series) (1961-1980)

```
      AUTO.  STAND.
 LAG  CORR.   ERR.  -1  -.75 -.5  -.25  0   .25  .5   .75   1
                    :----:----:----:----:----:----:----:----:
   1  0.615  0.064                   .  :**.*********
   2  0.633  0.064                   .  :**.**********
   3  0.576  0.064                   .  :**.*********
   4  0.573  0.064                   .  :**.********
   5  0.593  0.063                   .  :**.*********
   6  0.573  0.063                   .  :**.********
   7  0.577  0.063                   .  :**.*********
   8  0.546  0.063                   .  :**.********
   9  0.586  0.063                   .  :**.*********
  10  0.597  0.063                   .  :**.*********
  11  0.608  0.063                   .  :**.*********
  12  0.577  0.063                   .  :**.*********
  13  0.582  0.062                    . :*.**********
  14  0.515  0.062                    . :*.********
  15  0.551  0.062                    . :*.*********
  16  0.533  0.062                    . :*.*********
  17  0.483  0.062                    . :*.********
  18  0.552  0.062                    . :*.*********
  19  0.467  0.062                    . :*.*******
  20  0.528  0.061                    . :*.*********
  21  0.519  0.061                    . :*.********
  22  0.561  0.061                    . :*.*********
  23  0.554  0.061                    . :*.*********
  24  0.508  0.061                    . :*.********
  25  0.492  0.061                    . :*.********
```

Figure 3(b)

```
         PR-AUT  STAND.
LAG      CORR.    ERR.   -1  -.75 -.5  -.25 0    .25  .5   .75  1
                         :----:----:----:----:----:----:----:----:
  1      0.615    0.065                   .  :**.**********
  2      0.410    0.065                   .  :**.*****
  3      0.183    0.065                   .  :**.*
  4      0.155    0.065                   .  :***
  5      0.194    0.065                   .  :**.*
  6      0.113    0.065                   .  :**.
  7      0.103    0.065                   .  :**.
  8      0.040    0.065                   .  :* .
  9      0.139    0.065                   .  :***
 10      0.147    0.065                   .  :***
 11      0.127    0.065                   .  :***
 12      0.036    0.065                   .  :* .
 13      0.067    0.065                   .  :* .
 14     -0.085    0.065                   .**: .
 15      0.029    0.065                   .  :* .
 16      0.007    0.065                   .  *  .
 17     -0.111    0.065                   .**: .
 18      0.096    0.065                   .  :**.
 19     -0.094    0.065                   .**: .
 20      0.031    0.065                   .  :* .
 21      0.046    0.065                   .  :* .
 22      0.104    0.065                   .  :**.
 23      0.078    0.065                   .  :**.
 24     -0.029    0.065                   . *:  .
 25     -0.048    0.065                   . *:  .
```

Figure 3(c)

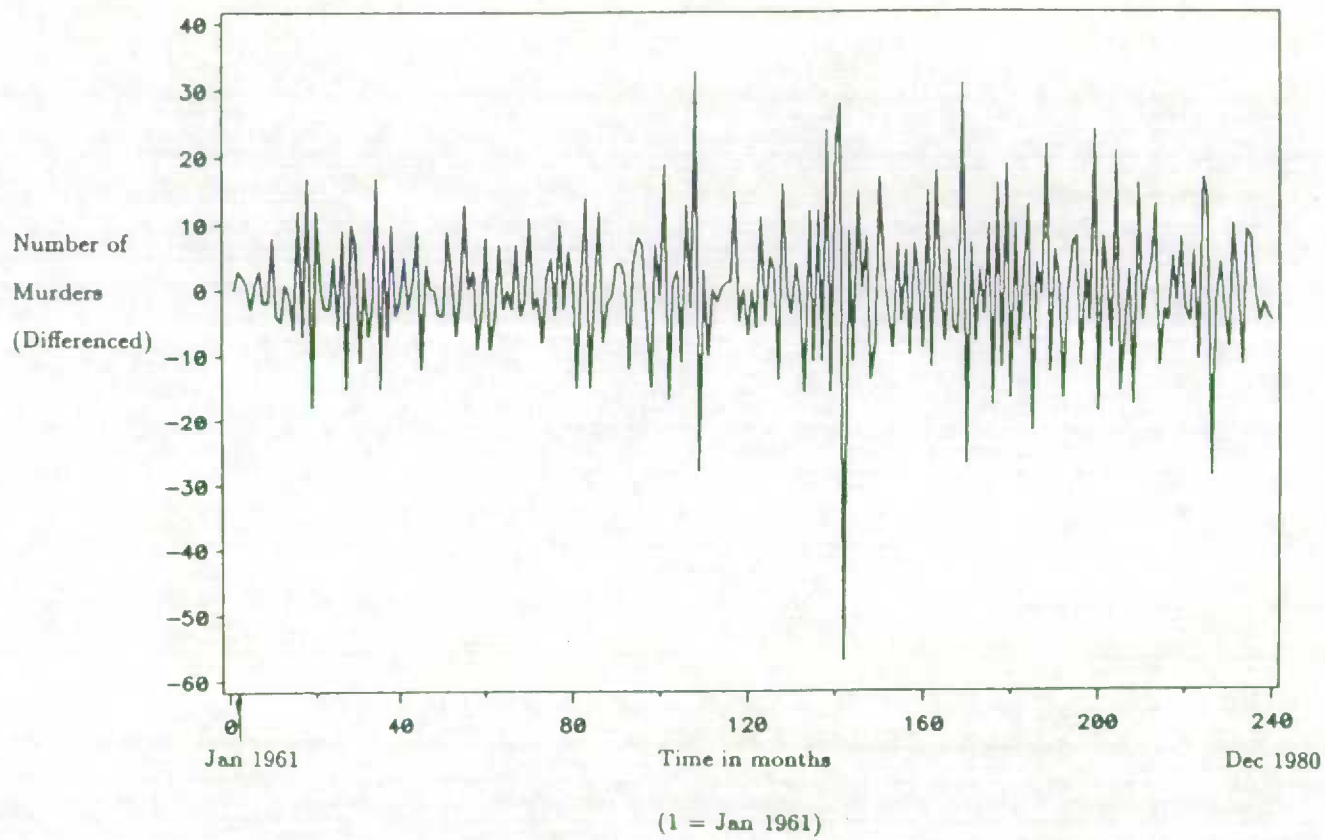


Figure 4(a) Trend-free Canadian homicide data (TOT series)

```
     AUTO.  STAND.
LAG  CORR.    ERR.  -1  -.75 -.5 -.25   0   .25  .5  .75   1
                    :----:----:----:----:----:----:----:----:
  1 -0.528   0.064           ********.**:  .
  2  0.100   0.064                   .  :**.
  3 -0.070   0.064                   . *:  .
  4 -0.032   0.064                   . *:  .
  5  0.054   0.063                   .  :* .
  6 -0.031   0.063                   . *:  .
  7  0.043   0.063                   .  :* .
  8 -0.092   0.063                   .**:  .
  9  0.042   0.063                   .  :* .
 10 -0.003   0.063                   .  *  .
 11  0.053   0.063                   .  :* .
 12 -0.046   0.063                   . *:  .
 13  0.095   0.062                    . :**
 14 -0.140   0.062                   *.*: .
 15  0.083   0.062                    . :**
 16  0.035   0.062                    . :*.
 17 -0.148   0.062                   *.*: .
 18  0.192   0.062                    . :*.**
 19 -0.186   0.062                  **.*: .
 20  0.094   0.061                    . :**
 21 -0.070   0.061                    .*: .
 22  0.062   0.061                    . :*.
 23  0.055   0.061                    . :*.
 24 -0.046   0.061                    .*: .
 25  0.013   0.061                    . * .
```

Figure 4(b)

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```
     PR-AUT  STAND.
LAG  CORR.   ERR.  -1  -.75 -.5 -.25  0  .25  .5  .75  1
                   :----:----:----:----:----:----:----:----:
  1 -0.528  0.065           ********.**:  .
  2 -0.247  0.065                 **.**:  .
  3 -0.198  0.065                  *.**:  .
  4 -0.226  0.065                 **.**:  .
  5 -0.139  0.065                   ***:  .
  6 -0.123  0.065                   .**:  .
  7 -0.061  0.065                   . *:  .
  8 -0.161  0.065                   ***:  .
  9 -0.161  0.065                   ***:  .
 10 -0.144  0.065                   ***:  .
 11 -0.058  0.065                   . *:  .
 12 -0.090  0.065                   .**:  .
 13  0.064  0.065                   .  :* .
 14 -0.058  0.065                   . *:  .
 15 -0.020  0.065                   .  *  .
 16  0.103  0.065                   .  :**.
 17 -0.092  0.065                   .**:  .
 18  0.092  0.065                   .  :**.
 19 -0.024  0.065                   .  *  .
 20 -0.034  0.065                   . *:  .
 21 -0.093  0.065                   .**:  .
 22 -0.077  0.065                   .**:  .
 23  0.043  0.065                   .  :* .
 24  0.057  0.065                   .  :* .
 25  0.042  0.065                   .  :* .
```

Figure 4(c)