C 2

Working Paper TSRA-90-008
Methodology Branch

USERS' MANUAL OF PROGRAMME
BENCH
to Benchmark, Interpolate and Calendarize
Time Series Data
on Microcomputer

by
Pierre A. CHOLETTE

October 24 2001

Statistics Canada
Time Series Research and Analysis Division
R.H. Coats Building, 3rd floor "H"
Ottawa, Canada, K1A 0T6
(613) 951-1601

TABLE OF CONTENTS

PREFACE

In the 1980s the Advisory Committee on Statistical Methods of Statistics Canada recommended a thorough examination of benchmarking and interpolation practices in the Agency. The ensuing research aimed to develop and explore the feasibility of methods, which would be based on statistical models (rather than often ad hoc numerical approaches); reflect the fact that the survey errors are heteroscedastic and autocorrelated through time; provide the variance of the benchmarked series; perform preliminary benchmarking of current years; and, reflect the fiscal character of benchmarks when applicable.

Programme BENCH is the result of this research carried out at Statistics Canada and elsewhere on benchmarking, interpolation and calendarization (benchmarking can also be used for interpolation and calendarization). It embodies the methods which were deemed most relevant and feasible; future versions of the programme will include more sophisticated methods.

I am indebted to Estela Bee Dagum, Gordon Brackstone and David Binder, for having supported and encouraged the research; to Guy Huot, Christian Nicol, Benoît Quenneville, Marietta Morry and Norma Chhab, for having thoroughly examined and commented earlier versions in this document and tested the programme; and to Hew Gough, Normand Laniel, Milorad Kovačević, Ijaz Mian, Paul Wong, Helen Fung and Kim Chiu, for very appreciated comments.

- résumé -

Le présent document présente un logiciel d'étalonnage et d'interpolation, pour les séries socio-économiques. L'étalonnage est le processus qui combine de manière optimale des chiffres relatifs à une même variable, mais observés à des fréquences différentes (ex. mensuelle et annuelle). L'étalonnage classique, qui ajuste des données mensuelles pour que leurs sommes annuelles soient égales aux jalons annuels, est un cas particulier. L'interpolation consiste à obtenir des estimations plus fréquentes (ex. mensuelles) à partir de données moins fréquentes (ex. annuelles). Le logiciel peut également effectuer la «calendrialisation» des séries socio-économiques, par exemples l'annualisation de chiffres d'années financières, la trimestrialisation de chiffres de trimestres financiers, la mensualisation d'agrégats de chiffres hebdomadaires, etc.

Le programme s'exécute sur les ordinateurs personnels. La méthodologie est tirée de Cholette et Dagum (1991), dont la méthode de Denton (1971) est un cas particulier, et de Mian et Laniel (1991). Le logiciel peut traiter des chiffres décennaux, quinquennaux, annuels, trimestriels, mensuels et quotidiens.

- summary -

This document presents a benchmarking and interpolation programme for socio-economic time series. Benchmarking is the process of combining data, which pertain to a same given variable, but are observed at different frequencies (e.g. annual versus monthly). Classical benchmarking, which adjusts monthly data so that their annual sums are equal to annual benchmarks, is a particular case. Interpolation consists of obtaining more frequent estimates (e.g. monthly) from less frequent data (e.g. annual). The programme can also operate calendarization, that is the conversion of fiscal year data into calendar year values, of fiscal quarter data into calendar quarter values, of aggregates of weekly data into monthly values, etc.

The programme runs on personal computers. The methodology is based on Cholette and Dagum (1989, 1994), of which the Denton (1971) method is a particular case, and on Mian and Laniel (1991). The programme can handle decennial, quinquennial, annual, quarterly, monthly and daily data.

1. INTRODUCTION

Benchmarking and interpolation techniques are widely used in statistical agencies and other organizations producing or using time series. Programme BENCH provides three benchmarking-interpolation methods; calendarization is achieved as a by-product of benchmarking (or interpolation).

Programme BENCH was developed in Fortran 77, under the DOS operating system. However it will work under OS/2 (in the built-in DOS compatibility box) and under UNIX with DOS emulation software (e.g. DOS Merge and VP/IX). It does not require a mathematical co-processor; however, if one is available, it will significantly speed up processing.

The programme diskettes also contain an auxiliary programme LISTR (Buerg, 1990), which is a general browsing utility programme. We recommend LISTR (a public domain software) for browsing, searching and examining the various files produced by BENCH.

To get acquainted with the concepts and notions related to benchmarking, interpolation and calendarization, the reader is advised to examine sections 2 and 3. Section 2 describes the various situations where benchmarking, interpolation and calendarization are applicable. Section 3 presents the statistical methodology and features available in the programme. Sections 4 to 6 explain in detail how to use BENCH. Section 7 is a tutorial which explains how to load and apply the programme to the test examples supplied on the diskettes.

PART 1: FOUNDATIONS OF THE BENCHMARKING-INTERPOLATION METHODS IN THE PROGRAMME

2. OVERVIEW OF BENCHMARKING, INTERPOLATION AND CALENDARIZATION

This section defines and explains the relationships between benchmarking, interpolation and calendarization, and the situations where these techniques are relevant.

- **Benchmarking** - Benchmarking arises in the following situation. Two sources of data are available for the same *target variable* with different frequencies, for example a monthly source versus an annual source; furthermore the two sources of information do not agree, the annual sums of the monthly measurements of a variable are not equal to the corresponding annual measurements. Benchmarking is then the process of optimally combining the two sources of information, to achieve improved monthly and annual estimates of the variable. Similarly benchmarking arises in the presence of quarterly versus annual data, monthly versus quarterly, annual versus quinquennial, daily versus weekly, daily versus monthly, etc.

Generally, one source of information (typically the less frequent) is more reliable than the other, because one originates from a sample survey (for instance) and the other from a larger survey, from a census or exhaustive administrative records. Such is the case of the Canadian Wages and Salaries: The monthly estimates originate from the Survey of Employment, Payrolls and Hours; and the annual *benchmark* measurements of the same variable originate from exhaustive administrative records, namely the Income Tax forms filed by Canadians and (confidentially) compiled by Revenue Canada. These benchmarks are believed to be absolutely reliable, compared to the monthly survey estimates. The benchmarking process consequently adjusts the monthly data so that they annually sum to the annual benchmarks. Under the widely used proportional Denton (1971) benchmarking method, the benchmarked monthly values are as proportional as possible to the *original* monthly values, or put differently preserve the proportional month-to-month movement of the original series.

- **Interpolation** - Benchmarking is also used to interpolate and extrapolate more frequent values from less frequent data. It is common, for instance, to benchmark a quarterly *indicator*, deemed to behave like the target variable, to annual data on the target variable. The resulting benchmarked values are interpolations (and extrapolations), in the sense that no genuine quarterly measurements existed for the target variable. Similarly, monthly interpolations are obtained by benchmarking a monthly indicator to quarterly or annual data; and annual interpolations, by benchmarking an annual indicator to quinquennial data. In some cases, the indicator is in fact a weighted combination of related variables available at the desired frequency (where the weights originate from a regression model, e.g. Chow and Lin, 1971). The indicator may also be a seasonal pattern, a seasonal-trading-day pattern or a daily pattern, in percentage. Some authors (e.g. Ibid.) distinguish between interpolation, where the benchmarks refer to discrete points of the more frequent values, and distribution, where the benchmarks refer to several points collectively, like in the case of annual totals. We use interpolation to cover both cases, because in many applications, the two types of benchmarks are present at once.

- **ARIMA Interpolation** - It is also possible to specify that the desired monthly (say) interpolations follow an AutoRegressive Integrated Moving Average model (Box and Jenkins, 1976) and satisfy some annual, quarterly or other benchmarks (Cholette, 1982; Guerrero, 1989, 1991; Hillmer and Trabelsi, 1989). One can generate ARMA interpolations and extrapolations with programme BENCH, provided that the ARMA model is stationary in mean. However, the model may tend to non-stationarity; one example being a seasonal autoregressive model (1,0)(1,0) with parameter values 0.99 and 0.99. Interpolations which are non-stationary in mean may also be achieved by benchmarking an original series behaving linearly, exponentially, etc.

- **Calendarization** - In situations where the data cover four or five weeks, instead of calendar months, one can use benchmarking and interpolation to calendarize such data. A daily pattern (of relative activity of days within the week) is benchmarked to the data covering four or five weeks; the resulting daily interpolations are then combined into monthly values by taking the monthly sums (Cholette and Chhab, 1991). Similarly, calendar year values may be obtained, by benchmarking a sub-annual indicator (or pattern) to the fiscal year data and by taking the calendar year sums of the interpolations. Calendar quarter values may also be obtained by benchmarking a monthly indicator to fiscal quarter data and then taking the calendar quarter sums. In many cases, calendarization is accomplished as a pure by-product of benchmarking (in the sense that calendarization was not the goal at the outset). This happens when a target variable needs to be benchmarked to benchmarks referring to fiscal periods; the calendarized values are then the appropriate sums of the benchmarked series. This situation occurs with governmental and other institutional data with common fiscal years, e.g. school, hospital, banking data. In all those cases, calendarization may also be achieved by generating more frequent ARIMA interpolations and by recombining these interpolations as desired.

One concludes that benchmarking methods are also applicable for the purposes of interpolation and calendarization. In fact, a variety of benchmarking, interpolation and calendarization techniques - many ad hoc and not even thought as being related to benchmarking and interpolation - are widely used across statistical agencies.

3. METHODOLOGY USED

This section presents the benchmarking methods available in programme BENCH and their features. For simplicity, it is assumed that the original series to be benchmarked is sub-annual and that the benchmarks are annual, unless otherwise specified.

3.1 MODELS FOR BENCHMARKING

Programme BENCH provides three statistical models to benchmark or interpolate: they will be referred to as the additive model, the multiplicative model and the mixed model; their solutions are presented in Appendix A, B and C respectively. All three models improve over most currently used benchmarking methods, because

(1) the variance of the benchmarked series is calculated,
(2) an explicit bias parameter in the sub-annual series is calculated,
(3) the sub-annual error may be heteroscedastic and follow an ARMA process.

- The Additive Model - The Cholette and Dagum (1991) method, developed in the regression framework, consists of two linear equations:

$$s_t = a + \theta_t + e_t, \quad E(e_t) = 0, \; E(e_t\, e_{t-k}) = \sigma_{e_t}\sigma_{e_{t-k}}\, \rho_k, \; t=1,\ldots,T, \tag{1a}$$

$$y_m = (\sum_{t \in m} \theta_t)/p_m + w_m, \quad E(w_m) = 0, \; E(w_m^2) = \sigma^2_{w_m}, \; m=1,\ldots,M. \tag{1b}$$

In equation (1a), the observed sub-annual series s_t is the *sum* of the true but unknown sub-annual values θ_t, an unknown constant bias a and autocorrelated errors e_t. The estimates of θ_t will be the benchmarked (or interpolated) series. The bias parameter may optionally be omitted from the model. In some applications, the errors e_t will be interpreted as the sampling errors. (Generally the latter is just one component of the global error.) These errors may be heteroscedastic, i.e. their variance σ_t^2 may vary with time t. The autocorrelations ρ_k correspond to those of a stationary and invertible AutoRegressive Moving Average (ARMA) processes, supplied by the users with the parameter values. This is equivalent to assuming that e_t follows a process given by

$$e_t = \sigma_t \varepsilon_t,$$

where ε_t follows the selected stationary ARMA process

$$\varepsilon_t = (\eta(B)/\varphi(B))\, v_t,$$

where $\eta(B)$ and $\varphi(B)$ are the moving average and the autoregressive polynomials respectively and v_t the white noise generating the ARMA process. (For simplicity, the rest of this document will refer to the ARMA process followed by e_t, although strictly speaking the process is that of ε_t.)

The benchmarks y_m in equation (1b) correspond to the sums of the true sub-annual values over the reference periods of each benchmark. For the time being, the values of p_m are set to 1 and will be discussed again in connection to flow, stock and index series. The benchmarks are subject to uncorrelated errors w_m, with heteroscedastic variance $\sigma^2_{w_m}$. The errors e_t and w_m are mutually uncorrelated.

Under model (1), benchmarking optimally combines the sub-annual observations s_t and the benchmark observations y_m to estimate the true values θ_t.

- The Multiplicative Model - The multiplicative model is a logarithmic variant of model (1). The model states that the bias, the errors and the true sub-annual values *multiply* to generate the sub-annual observations: $s_t = a \times \theta_t \times e_t$. Taking the logarithmic transformation, the model becomes

$$ln\ s_t = ln\ a + ln\ \theta_t + ln\ e_t,$$

$$E(ln\ e_t) = 0,\ E(ln\ e_t\ ln\ e_{t-k}) = \sigma_{ln\, e_t} \sigma_{ln\, e_{t-k}}\, \rho_k,\ t=1,...,T, \tag{2a}$$

$$y_m = (\sum_{t \in m} \theta_t)/p_m + w_m,\ E(w_m) = 0,\ E(w_m^2) = \sigma^2_{w_m},\ m=1,...,M, \tag{2b}$$

where ρ_k's are as in model (1) and where $\sigma_{ln\, e_t} = \sigma_{e_t}/s_t$. The errors in the benchmarks are additive, but may in fact behave multiplicatively if their variance is defined in terms of coefficients of variation.

- The Mixed Model - In the mixed model proposed by Laniel and Fyfe (1990) and Mian and Laniel (1991), the bias is multiplicative but the error is additive. Subject matter experts of series often think of the bias as a percentage of the sub-annual series. The model is as follows:

$$s_t = a \times \theta_t + e_t,\ E(e_t) = 0,\ E(e_t\ e_{t-k}) = \sigma_{e_t} \sigma_{e_{t-k}}\, \rho_k,\ t=1,...,T, \tag{3a}$$

$$y_m = (\sum_{t \in m} \theta_t)/p_m + w_m,\ E(w_m) = 0,\ E(w_m^2) = \sigma^2_{w_m},\ m=1,...,M, \tag{3b}$$

where the autocorrelations ρ_k are as in model (1). The additive errors may in fact behave multiplicatively, if their variance is defined in terms of coefficients of variation. (In that case, the estimates could be very close to those of the multiplicative model.)

- The Hillmer and Trabelsi Benchmarking Method - In the Hillmer and Trabelsi benchmarking (1987) method, the true series follows an ARIMA process: $(1-B)^d(1-B_s)^D[\theta_t - \mu_t] = [\eta^*(B)/\varphi^*(B)]\, v_t^*$, where μ_t incorporates a known deterministic behaviour of θ_t (e.g. deterministic trend and seasonality, trading-day variations) and where d and D are the degrees of regular and seasonal differencing. This feature can be incorporated in the additive model, by adding the following equation to (1a) and (1b):

$$(1-B)^d(1-B_s)^D \mu_t = (1-B)^d(1-B_s)^D \theta_t + e_t^*,\ E(e_t^*\ e_{t-k}^*) = \sigma^2_{v^*} \rho_k^*,$$

where $e_t^* = [\eta^*(B)/\varphi^*(B)]\, v_t^*$ and the autocorrelations ρ_k^* are given by the ARMA process of e_t^* and where $\sigma^2_{v^*}$ is the variance of input white noise of the process. In principle, this ARIMA feature improves the estimates, namely by giving structure to the series to be estimated and by increasing the degrees of freedom in the estimation. It is hoped that future versions of programme BENCH will incorporate the Hillmer and Trabelsi method as another optional model.

Note however that, even without the ARIMA process for θ_t, the additive model (1) already implies an indirect (so to speak) stochastic behaviour for θ_t. Indeed, equation (1a) can be written $s_t = a + \theta_t + \sigma_t(\eta(B)/\varphi(B))\, v_t$, which implies the heteroscedastic process with non-stationary mean $(s_t - a)$: $\theta_t = (s_t - a) - \sigma_t(\eta(B)/\varphi(B))\ v_t$.

3.2 FEATURES OF THE MODELS

Most of the features described here are shared by the three benchmarking models in the programme, for instance the ability to deal with flows, stock and index series, with fiscal benchmarks; a few features are particular to individual methods.

- Flexible Reference Periods for the Benchmarks - In order to accommodate all the situations described in Section 2, the three models allow for benchmarks of *any* frequency, e.g. annual, bi-annual, quinquennial, quarterly, monthly, weekly, daily, etc. This is achieved by *reference periods*, specified by the starting year, the starting month, the ending year and the ending month, covered by each benchmark. Thus, one benchmark may cover 15 months and the next benchmark 12 months; benchmarks may be bi-annual and suddenly quinquennial. Some sub-annual periods may even be covered by more than one benchmark, for instance by a quarterly benchmark and an annual benchmark.

- Fiscal Benchmarks - As a particular case of this feature, the benchmarks may be *fiscal*, i.e. refer to fiscal (instead of calendar) years, to fiscal quarters, and to periods of four or five weeks. In the latter case the reference periods are specified by the starting year, the starting month, the starting day, the ending year, the ending month and the ending day of each benchmark.

- Sub-Annual Benchmarks - Benchmarks covering only one sub-annual period of the original series will be referred to as sub-annual benchmarks. Such benchmarks are very useful:

(1) To force a benchmarked series to start from a specific value, in order to avoid revisions to values deemed "historical" or "final" prior to a certain date (Helfand, Monsour and Trager, 1977).
(2) To force a benchmarked series to run through a specific value.
(3) To force a benchmarked series to end at a specific value. This feature may be used to link an "old" survey with a "new" survey, pertaining to the same socio-economic variable, by specifying one new value (or the average of a few new values) as a sub-annual benchmark on the overlapping periods of the two surveys.
(4) To accommodate stock series.

- The Types of Series - Flow, stock and index series are now defined for the purpose of benchmarking, interpolation and calendarization.

. Flow Series - A *flow series* is a series whose annual values correspond to the annual sums of quarterly (or monthly) values in the year and the quarterly values correspond to the sum of the monthly values in the quarter, etc. For example, the amount of gasoline sold in July 1990 is the sum of the amounts sold on each day in that month.

. Stock Series - A *stock series* is a series whose annual values correspond to only one sub-annual value. For instance, the annual values of inventories usually correspond to the December 31st values; the Canadian annual population values correspond to the July first values. The reference periods of each benchmark determine whether the series is a flow or a stock: if the benchmarks refer to one sub-annual period, the series is a stock.

. Index Series - An *index series* is a series whose annual values correspond to the average of the sub-annual values. For instance, the annual values of the Consumer Price Index, of the Index of Industrial Production and of Unemployment are all defined as the averages of the monthly values; the quarterly values of Interest Rates may be defined as the average of the corresponding monthly values. Note that the Consumer Price Index, Unemployment and Interest Rates are *conceptually* stock series, i.e. levels; and the Index of Industrial Production, a flow series; but all are index series for the purpose of benchmarking, because the annual values are expressed in terms of averages of the corresponding sub-annual values.

For flow series, the p_m's of equation (1b), (2b) and (3b) are equal to 1, and the summations take place over more than one sub-annual period t ; for stock series, the p_m's are equal to 1 and the summation takes place over only one sub-annual period t (i.e. the summations are trivial). For index series,the p_m's are the number of sub-annual periods covered by each benchmark (e.g. for annual benchmarks and monthly sub-annuals, the p_m's would all be equal to 12).

- **Heteroscedasticity** - Models (1) to (3) weight each sub-annual and annual observation with the inverse of its variance. In programme BENCH, this variance may also be supplied in the form of the standard deviation or the coefficient of variation (which are internally transformed into variance). As a rule, the benchmarked series will run closer to the observations with lower variance.

- **Binding and Non-Binding Benchmarks** - A benchmark is *binding* (classical case) when its variance is zero; and *non-binding*, otherwise. The benchmarked series exactly sums (or averages) to a binding benchmark over the reference periods of the benchmark. Some values in a set of benchmarks may thus be binding; and others, non-binding. Benchmarks should be specified as binding when they provide fully reliable measurement of both the level and of the super-annual (trend-cyclical) movement, of the target variable. On the other hand, benchmarks should be specified as non-binding, when they *collectively* provide a reliable measurement of the (average) level of the target variable but not of the super-annual movement. A benchmark which do not measure the level nor the movement reliably should not be used. In the example of Wages and Salaries in section 2, all the benchmarks were binding; the annual sums of the benchmarked series therefore summed exactly to the annual benchmarks.

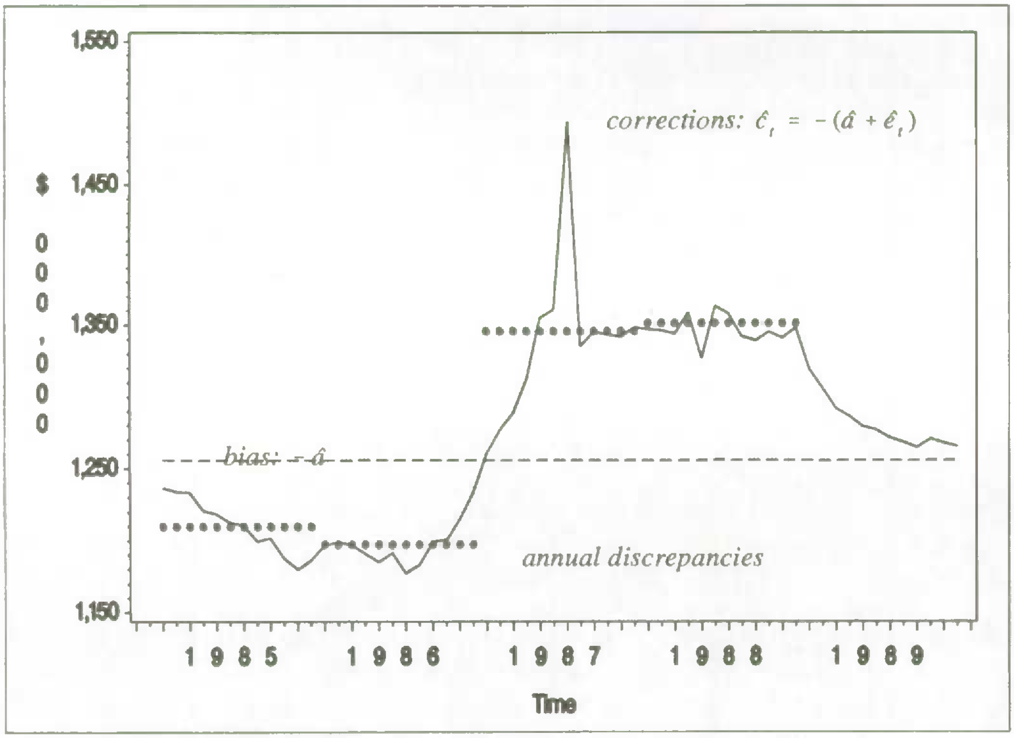

Figure 1: Example of Monthly Corrections Made to an Original Series under the Additive Model (1) with ARMA Process $e_t = \sigma_t[1/(1-0.8B)]v_t$ and Heteroscedasticity

- **Preliminary Benchmarking of Current Values** - The annual benchmark of a given year usually becomes available some months after that year is over; thus, there is typically no benchmark for the current (most recent) sub-annual values. The expression preliminary benchmarking refers to the adjustment made to the current sub-annual observations. Preliminary benchmarking should be such that

(1) the preliminary benchmarked values are in continuity with past benchmarked values and

(2) the revisions of the preliminary benchmarked values, when the benchmark does become available, are minimal.

. **Treatment of Years without Benchmark in General** - First it must be pointed out that the benchmarking models (1) to (3) "spontaneously" find estimates for all years, whether they have benchmarks or not, and whether they are current or not. (In other words, no special treatment is required for years without benchmark.) The *corrections*

$\hat{c}_t$, made to the sub-annual series to obtain the benchmarked series, are equal to minus the estimated bias $\hat{a}$ plus the fitted ARMA values $\hat{e}_t$: $\hat{c}_t = -(\hat{a}+\hat{e}_t)$. The corrections are

(1) such that $\hat{e}_t$ follows its ARMA process and

(2) additionally for years with benchmarks, such that the benchmarked series satisfies the benchmark (if binding). For years without benchmark, the second requirement simply does not apply.

. Treatment of Current Years without Benchmark - The treatment of the current years without benchmark is just a particular case of what was stated in the previous paragraph. Furthermore, the corrections $\hat{c}_t = -(\hat{a}+\hat{e}_t)$ eventually converge to minus the bias value $-\hat{a}$, because the ARMA process is stationary. This convergence, illustrated in Figure 1, assumes that the expectation of next *annual discrepancy*, between the next benchmarks and the annual sum of the sub-annual series, is given by the bias. This assumption makes sense since the bias is an average of the past discrepancies; and, of the last few discrepancies, if moving estimation intervals were used (see next). (The corrections do not behave very smoothly in the figure, because the errors were heteroscedastic.)

- Estimation Intervals - Programme BENCH can process all the sub-annual and annual observations of the series as a whole, or a subset of the observations in a moving average manner.

. Estimation over the Whole Series - Models (1), (2) and (3) may be specified for the whole length of the series. This option should be used

a) for short series or

b) when the reference periods of some benchmarks overlap or

c) when choosing a seasonal ARMA process for the error e_t (otherwise the seasonal pattern would be dropped between the intervals or a spurious seasonal pattern can be introduced).

. Estimation over Moving Estimation Intervals - Programme BENCH optionally performs benchmarking in a moving manner, i.e. on successive *estimation intervals*. By default, each estimation interval covers five benchmarks; for shorter series, the interval covers the whole series. For example, for a 20-year monthly series with annual benchmarks, the 5-year moving estimation intervals operate as follows:

(1) The first 5-year estimation interval covers years 1 for 5 (whether calendar or fiscal) and provides estimates for years 1 for 3.
(2) The second estimation interval covers years 2 for 6 and provides estimates for year 4.
(3) The third estimation interval covers years 3 for 7 and provides estimates for year 5. And so on.
(4) Finally the last estimation interval covers years 16 for 20 and provides estimates for years 18 for 20.

When sub-annual values are not covered by benchmarks at the start or the end of the series, the first and last estimation intervals respectively begin and end with the first and last time periods of the sub-annual series.

For long series, this moving estimation greatly reduces the calculations, without noticeable effect on the estimates if the benchmarks are binding or nearly binding. Moving estimation intervals also reduce revisions: with a 5-year moving interval for instance, the estimates of year m automatically become final when the benchmark of year m+2 is incorporated in the estimation.

With moving estimation, the estimate of the bias becomes stochastic, i.e. evolves through time according to the innovations (information) incorporated in, and dropped from, the estimation interval.

. Links between the Moving Estimation Intervals - Links between the estimation intervals can optionally be imposed to ensure continuity between the estimates from the different intervals. Reverting to the 5-year moving estimation example:

(1) The first estimation interval covers years 1 to 5 and provides estimates to years 1 to 3 (as before). The last monthly estimate of year 3 is set aside for the second interval.
(2) The second estimation interval covers the last month of year 3 (whose estimate was set aside) and years 4 to 6 and provides estimates to year 4. The last monthly estimate of year 3 is specified as a sub-annual binding

(see above) benchmark, which forces the monthly estimates to start from the last monthly estimate of year 3. The last monthly estimate of year 4 is set aside for the third interval.

(3) The third estimation interval covers the last month of year 4 (whose estimate was set aside) and years 5 to 7 and provides estimates to year 5. The last monthly estimate of year 4 is specified as a sub-annual binding benchmark, which forces the monthly estimates to start from the last monthly estimate of year 4. The last monthly estimate of year 5 is set aside for the fourth interval. And so on.

(4) Finally, the last estimation interval covers the last month of year 17 and years 18 to 20 and provides estimates to years 18 to 20. The last monthly estimate of year 17 is specified as a sub-annual binding benchmark, which forces the monthly estimates to start from the last month estimate of year 17.

These links and the moving estimation are described in Cholette and Chhab (1988, 1991). Links are desirable when the moving interval is shorter than five benchmarks. With daily data such linked intervals reduce the calculations. However with these links, no variance is estimated for the benchmarked series, because the variance would be underestimated in the neighbourhood of the link points.

- Preserving the Movement of the Original Series - As just explained, links between moving estimation intervals ensure continuity between the estimates from the different intervals; similarly preliminary benchmarking preserves continuity between past years with benchmarks and the current year. Generally speaking, the benchmarks tend to drive the level of the benchmarked series; and the ARMA process chosen for e_t, to drive the modification of the sub-annual movement of s_t. The reasons are that the e_t's are usually correlated and the w_m's are not, and the s_t's are the ones subject to the bias (when specified). The benchmarking methods of the Denton type were specifically designed to preserve the movement of s_t as much as possible.

- Benchmarking Methods of the Denton type - Results virtually identical to those of both the additive and the proportional Denton methods can be achieved with the additive model (1), by selecting the appropriate options. Two things should be pointed out here. First in the Denton approach, the e_t's in (1) should be interpreted as the bias in the original series which evolves through time like a first difference process (the "random walk"), $e_t = e_{t-1} + v_t$, or as a second difference process, $e_t = 2e_{t-1} - e_{t-2} + v_t$; and, that there is no survey error as such. (This does not mean however that the bias is out of control, since the innovations to the random walk are in fact the annual discrepancies, which benchmarking distributes over the sub-annual values.) Second, under its Denton variant, as well as under its growth preservation and pro-rating variants described below, model (1) collapse to a numerical (as opposed to statistical) method, because it does not use all the statistical information (variances) of the original series and merely preserves its movement. However this Denton "option" in the programme should satisfy many practitioners who need more general benchmarking-interpolation methods without *having* to give up the widely used Denton approach.

. The Additive Denton Method - The additive model (1) approximates the additive Denton results with first differences, if (i) the constant bias parameter is omitted, (ii) the benchmarks are binding (with zero variances), (iii) the variances of the sub-annual series are constant and (iv) the ARMA chosen for e_t (which now plays the role of the bias) approximates a random walk process. In fact, if the user supplies no ARMA process, the programme uses the virtual random walk process, $e_t = \sigma[0.999999 e_{t-1} + v_t]$. (This idea originates from Bournay and Laroque (1979)). Like in the additive Denton method, the resulting benchmarked series is as parallel as possible to the original series and therefore preserves the period-to-period movement of the original series.

. The Proportional Denton Method - The additive model (1) above approximates the results of the proportional Denton results with first differences if: (i) the bias parameter is omitted, (ii) the benchmarks are binding (have zero variances) (iii) the coefficients of variation (**CVs**) of the sub-annual series are constant and (iv) the ARMA chosen approximates a random walk process, for instance $e_t = \sigma_t[0.99999 e_{t-1} + v_t]$, where the values of σ_t are given by the CVs. The resulting benchmarked series is then as proportional as possible to the original series and therefore preserves the proportional period-to-period movement of the original series. The mixed model (2) also reproduces the results of the proportional Denton method, under the same conditions.

. The Denton Methods with Second Differences - Both the additive and the proportional variants of the Denton approach with second differences can be approximated by the additive model (1) by selecting a stationary process which approximates second differences instead of the random walk, for instance $e_t = \sigma_t [(0.99+0.99)e_{t-1} - (0.99 \times 0.99)e_{t-2} + v_t]$.

- Growth Rate Preservation - It has been argued that a benchmarking method should preserves the *period-to-period growth rates* (Smith, 1977) of the original series. The multiplicative model (2) achieves this (i) if the CVs of the sub-annual series are constant and (ii) if the ARMA chosen for e_t approximates a random walk, for instance $e_t = \sigma_t [0.99999 e_{t-1} + v_t]$, where the values of σ_t are given by the constant CVs. The ratios of the growth rates of the benchmarked series to those of the original series are then as close to 1.0 as possible. The results are also similar to those obtained with the proportional Denton method (Helfand, Monsour and Trager, 1977). Proportional benchmarking has in fact been seen as an approximation to the growth rate approach.

- Pro-Rating - The simplest benchmarking method of all is pro-rating, which consists of applying the proportional annual discrepancy to the sub-annual values in the year. For instance, if the annual benchmark of one year is 10% higher than the sum of the monthly values in the year, these monthly values are raised by 10%. Pro-rating can be carried out by programme BENCH, by (i) selecting the additive benchmarking model above, (ii) omitting the bias parameter, (iii) specifying constant CVs for the sub-annual series and zero CVs the benchmarks and (iv) selecting a first order autoregressive process for the error with parameter equal to zero: $e_t = \sigma_t [0.0\, e_{t-1} + v_t]$.

3.3 ASSESSMENT OF THE BENCHMARKED SERIES

In order to assess the results of benchmarking or interpolation, programme BENCH produces various statistics and plots.

- Significance of the Estimated Bias Parameter - For the three models discussed in section 3.1, the programme displays the estimate of the bias and its calculated standard deviation. A large standard deviation indicates that the bias estimate is not significantly different from zero (or 1 in the multiplicative and mixed models) and could be removed from the model.

- Statistics on the Discrepancies and Movement Preservation - The statistics on the discrepancies, between the annual benchmarks and the annual sums (or averages) of the sub-annual series, and on movement preservation are rather empirical. They are especially meaningful with the methods of the Denton type, which aim at preserving movement. They retain usefulness, as long as the ARMA process chosen preserves some of the movement from the original series, e.g. any first order autoregressive processes with positive parameter, with constant variance or CVs.

. Statistics on the Discrepancies

The statistics on the discrepancies are displayed with respect to both the original and the benchmarked series. The *average of the discrepancies* measures the level difference between the benchmarks and the original series, or between the benchmarks and the benchmarked series. In the latter case, the discrepancies are actually residual discrepancies, which are null in case of binding benchmarks (and this should be monitored). Benchmarking with the Denton methods *may* be successful despite very large discrepancies with respect to the original series, because two series with different frequency may display perfectly compatible movements but very different levels.

The *minimum* and the *maximum of the discrepancies* measure the dispersion of the discrepancies. A low dispersion of the discrepancies thus indicates that benchmarking with the Denton methods is probably successful. The occurrence of scattered, i.e. erratic, discrepancies implies that in the process of satisfying the benchmarks, benchmarking distorts the movement of the original series. The reason is that benchmarking consists of distributing these discrepancies over the original series. Scattered discrepancies may mean one of the following:

(1) Some of the benchmarks are not reliable and should be specified as non-binding.
(2) The super-annual (e.g. trend-cyclical) movement of the original series is not reliable. If the benchmarks are reliable and binding, the benchmarked series would incorporate the super-annual movement of the benchmarks, thus producing an improvement over the original.

(3) Both the benchmarks and the original series are not reliable. In this case, the resulting benchmarked series will be an optimal combination of the information available at all frequencies (if the appropriate variances are specified for each annual and sub-annual observation).

. Statistics on Movement Preservation

- The information provided by the discrepancies is reflected by the *average absolute change in movement* between the original and the benchmarked series. The two following statistics are calculated:

$$c_m = \sum_{t=2}^{T} \left| [(\hat{\theta}_t/\hat{\theta}_{t-1}) / (s_t/s_{t-1})] - 1.0 \right| / (T-1),$$

$$c_a = \sum_{t=2}^{T} \left| (\hat{\theta}_t - \hat{\theta}_{t-1}) - (s_t - s_{t-1}) \right| / (T-1),$$

The first statistic c_m measures the change in the period-to-period growth rates; and the second c_a, the change in the period-to-period differences. The former is more relevant to multiplicative and proportional benchmarking; and the latter, to additive benchmarking.

High values of these statistics correspond to erratic discrepancies and low values to relatively constant discrepancies. Low values mean that the original series does not have to be distorted much to satisfy the benchmarks (if binding) or put differently the movements of the benchmarks are consistent with those in the original series.

- **Plots** - Time series analysts like to examine series graphically. A first plot displays the original and the benchmarked series, accompanied by the benchmarks. A second plot displays the corrections made to the original series to obtain the benchmarked series, along with the discrepancies. The flatter these corrections, the more the benchmarked series preserves the original movement of the sub-annual series.

- **Interpreting the Statistics and Plots** - The interpretation of the above statistics and plots should vary with the situation considered. If the original series and the set of benchmarks are both supposed to directly measure the same *target variable*, scattered discrepancies and poor movement preservation should be interpreted more critically. (The target variable is that measured by the benchmarks and eventually by the benchmarked series.) A large dispersion of the discrepancies and a large change in movement implies a contradiction between the less frequent benchmark measurements and the more frequent measurements of the variable.

If, on the other hand, the original series does not measure the target variable and is a seasonal pattern (e.g. percentages) used for interpolation, scattered discrepancies and poor movement preservation should not be interpreted as critically. The statistics originate from the fact that the benchmarks - and not the original series - contain the super-annual movement (e.g. trend and business cycle); benchmarking then forces the super-annual movement onto the sub-annual series. Or put differently, the seasonal pattern provides only the sub-annual movement; and the benchmarks, the level and the super-annual (e.g. trend-cycle) movement.

In some situations, the original series is an indicator or *proxy* variable deemed to behave more or less like the target variable. The proxy is the best substitute measurement of the target variable and is the best available. It is important to note that in such cases, the benchmarks are deemed to contain the business cycle (and the super-annual) information and not the proxy, because only the benchmarks directly measure the target variable. We would then recommend the following. The proxy indicator variable should be transformed into a seasonal-trading-day pattern (possibly including an irregular component reflecting special events). This transformation can be achieved with time series decomposition methods, like the X-11-ARIMA/88 seasonal adjustment method (e.g. Dagum, 1980, 1988) and the X-11 procedure in SAS/ETS (SAS Institute, 1984). The resulting benchmarked series then adopts the sub-annual (e.g. seasonal, trading-day) movement of this transformed proxy and the super-annual movement of the benchmarks. Without such a transformation, the benchmarked series combines the business cycle movements present in the proxy (which may either be absent in the benchmarks or be present with a different timing) with those in the benchmarks. In other words, the business-cycle in the benchmarked series may be more complex than that in the benchmarks, because it combines two *different* trend-cycles into one. This recommendation is even more crucial if the proxy is monthly and the benchmarks of the target variable are quarterly.

For the same reasons, if the original series is a non-seasonal proxy indicator (with no trading-day variations either), we recommend replacing the proxy indicator by any constant. Such an "indicator" combined with a first order autoregressive ARMA process (with positive parameter) yield a smooth benchmarked series which satisfies the benchmarks and incorporates their level and super-annual (e.g. business cycle) movement. (See Cohen, Müller and Padberg, 1971.)

3.4 CRITERIA FOR SELECTING THE BENCHMARKING MODELS AND THE ARMA PROCESSES

This sub-section provides broad guidelines regarding the appropriateness of the three benchmarking models under various situations and regarding the selection of the ARMA processes for e_t.

- Choosing between the Additive, the Multiplicative and the Mixed Models - The following criteria could be used to choose among the additive, the multiplicative or the mixed models .

(1) If the original series and the benchmarks measure the same target variable, the choice of the model could be governed by the following criteria:

a) If a bias parameter is desired, choose the model for which the bias parameter is most significant, i.e. has lowest standard deviation.

b) If movement preservation is a goal, choose the model which minimizes the movement preservation statistics.

c) If the variances of the original series are rather constant through time, the additive model is usually appropriate. If the CVs are constant (which implies moving variances), the multiplicative or the mixed models are probably appropriate.

(2) If the original series is a seasonal and/or trading or a weekly pattern in percentage, the multiplicative model is appropriate. The resulting interpolated series displays the desired seasonal pattern with amplitude proportional to the level of the series.

(3) If the original series is trivial (any constant), like in Boot, Feibes and Lisman (1967) and in Cohen, Müller, and Padberg (1971), the additive model is appropriate. The resulting interpolated series displays no sub-annual (e.g. seasonal) movement, just a smooth trend-cycle movement compatible with the benchmarks.

- Choosing the ARMA processes - In principle the survey, which produces the values of the sub-annual series, also provides the autocovariance of the errors, along with their variances. From the autocovariance, an ARMA model is derived and used in the benchmarking procedure. In many situations however, the variances are available (typically in the form of CVs) but not the autocovariances (in some cases they may occasionally be available and for a few selected lags only). Our recommendation would be the following.

(1) Use the variances available from the survey, so that they are reflected in the benchmarked series.

(2) Choose an ARMA model which preserves the month-to-month or same-month-to-same-month movement of the original series. An autoregressive model of order 1 with positive parameter (lower than 1.0) preserves the month-to-month movement. The closer to 1.0 the parameter, the more movement preservation occurs. When a bias is present in the model, we would recommend a value around 0.8 and 0.9, because the resulting benchmarked series retains much of the original movement and the preliminary benchmarked values converge to the original series corrected for bias (if applicable), as displayed in Figure 1. (See discussion of preliminary benchmarking.) This implicity assumes that the next annual discrepancy will be close to the average of the actual annual discrepancies, which is measured by the bias.

PART 2: USERS MANUAL OF THE PROGRAMME

4. DESCRIPTION OF FILES AND OPTIONS IN PROGRAMME BENCH

This section describes the input files required and the output files produced by the benchmarking and interpolation programme BENCH.

Programme BENCH requires up to five input files (depending on options selected):

(1) a file called "`bench.fil`", which contains the file names and record layouts (formats) of the other files used by the programme,
(2) the *option file*, which contains the options used for each *pair* of original series and set of benchmarks,
(3) the *file of the original series*, which contains the values to be benchmarked,
(4) the *file of the benchmarks*,
(5) and - if applicable - the *file of the calendarization dates*, which contains the dates for which calendarized values are desired.

The programme produces up to three output file:

(1) a *log output* file called "`bench.log`", which contains statistics, plots and error messages,
(2) the *file of the benchmarked series*, which contains the benchmarked (or the interpolated) values,
(3) and the *file of the calendarized values* if applicable.

This section will rely heavily on one example - that of the Canada Total Retail Trade series - to illustrate the programme. A more typical job would process more than one series; such examples will be provided in the tutorial section 7.

4.1 THE FILE "bench.fil"

The file called "`bench.fil`" supplies the file names and record layouts of all files used by programme BENCH. This file *must* exist. The record layouts are described by means of Fortran format statements. (More information about these can be found in a Fortran manual, e.g. MicroSoft, 1985).

Table 4.1 displays an example of file "`bench.fil`". Each record of this file has 80 columns. The first record (line) of the file pertains to the option file. The next three records pertain to the file of the original series, the file of the benchmarks and the file of the benchmarked series, respectively. The last two records pertain to the file of the calendarization dates and the file of the calendarized values calculated. These two records are not mandatory; their absence will indicate to the programme that no calendarization is to be performed.

Table 4.1: Example of File "`bench.fil`"

```
retail.opt
retail.ori  (a10,2i5,t16,i5,t11,2i5,t16,i5,f10.0,f5.0)
retail.ben  (a10,2i5,t16,i5,t21,2i5,t26,i5,f15.0,f10.0)
retail.out  (a10,6i4,f10.0,f5.2)
retail.cai  (a10,2i5,t16,i5,t21,2i5,t26,i5)
retail.cao  (cansimA)
```

- The File Names

The first part of each record of file "`bench.fil`" provides the file names of each file to be processed by the programme. The file name must start in column 1 and may consist of up to 20 characters. The file name may contain directory information. For instance in the file name "`c:\usera\retail.opt`", "`c:`" means disk drive C, the first "`\`" means the root directory, "`usera\`" means sub-directory usera and "`retail.opt`" is the actual file name. In most cases however, such information should not be necessary if the computer is appropriately configured. (Tips of configuration are given in section 7.)

- The Record Layouts or Formats

The second part of each record of file "`bench.fil`" provides the record layout or *format* of each file to be processed. The format statement starts with a left bracket ("`(`"), located anywhere after the file name, and may contain up to 60 characters. For the option file, no format is required, as it is internally set to "`(80a1)`"). For the other files, the formats supplied must meet the following requirements:

(1) each record must contain only one observation of the input or output series (as opposed to 12 for instance);
(2) each record must bear a series identifier; the reference periods covered by the observation on the record, namely the *starting year*, the *starting month*, the *starting day* (if applicable), the *ending year*, the *ending month* and the *ending day*; the observation of the series; and other information (e.g. CV's) depending on the file.

This characterization of the reference periods (in terms of starting and ending year, month and day) allows the programme to process a wide range of socio-economic data: daily, monthly, quarterly, semi-annual, annual, bi-annual, quinquennial, decennial data, etc.; and even data with capricious reference periods, like data covering sometimes 4 and sometimes 5 weeks (such examples are given in section 7).

For quarterly data, the term "month" may actually mean quarter. Indeed, the reference periods of quarterly data may be specified in terms of quarters or months. For example a quarterly observation for the third quarter of 1981 could have the following starting year, starting "month", ending year and ending "months": 1981, 3, 1981, 3; or the following starting year, starting month, ending year and ending months: 1981, 7, 1981, 9. The reference periods of annual (quinquennial, etc.) data, may similarly be specified in terms of months or quarters.

In some applications, the starting and ending days are irrelevant, like in the two examples just given. Although programme BENCH requires them, they may be set equal to the starting and ending months respectively. In other applications, the ending year and month coincide with the starting year and month, like in the first of the two examples just given. The coding of unnecessary or irrelevant information may be avoided by specifying the format statement appropriately (this will be illustrated in section 4.2).

Programme BENCH can interface with Statistics Canada's **CANSIM** (Canadian Socio-Economic Information Management System) databank and **X-11-ARIMA** seasonal adjustment programme (see section 6). This feature comes at a price however, because CANSIM and X-11-ARIMA do not describe the reference periods of the data as fully as BENCH, among other reasons.

4.2 THE OPTION FILE

The first record of file "`bench.fil`" pertains to the option file. The first 20 columns of the record provide the file name of the option file, "`retail.opt`" in the example of Table 4.1. This file contains the options to be used in benchmarking each pair of original and benchmark values. Each record of the option file has 80 columns. The format is always "`(80a1)`" and is not supplied after the file name, as in the case of the other input and output files. Table 4.2 displays the option file "`retail.opt`" of the Canada Total Retail Trade series.

Table 4.2: The options file "retail.opt" for the Retail Trade Series

```
tit=Canada Total Retail Trade - link to October 1989 - Fiscal Years.
    ids=Retail, ysf=1984, msf=1,
    idb=Retail, idi=Retail2, idc=RetailC,
Basic options:   met=0, int=0, cvs=0, cvb=0, cvi=0, cvc=0,
                 bia=1, pls=1, plc=1,
ARMA options:    p-r=1, 0.80;
```

- Syntax of the Keywords

The options in the option file are keyword-driven. The syntax of the keywords is the following.

(1) Each *set of options*, used for one pair of original and benchmark values, may occupy up to 6 records and must end with a semi-colon ("`;`"), which marks the end of the set of options.
(2) Each keyword in the set of options consists of a string of 4 characters ending with the equal sign ("`=`"). As much as possible, the first 3 characters correspond to the 3 first letters of the option considered, for instance the keyword "`bia=`" controls the **bias** option. For the keywords, the programme *does not* distinguish between lower-case and upper case characters. Thus the following keywords are all valid and equivalent: "`bia=`", "`BIA=`", "`Bia=`", "`bIa=`", etc.
(3) Each keyword is followed by an *entry*. The entry starts with the first character following the "`=`" sign of the keyword and ends with the character preceding the first comma "`,`" encountered. Depending on the keyword, the entries may be alpha-numeric or numeric. All entries may contain blanks; thus the following keywords and entries are all valid (and equivalent): "`bia= 1,  `", "`bia=1,`", "`bia=01,`". In the case of alpha-numeric

entries, the programme *does* distinguish between lower-case and upper-case characters; thus "IdS=First Series," is *not* equivalent to "IdS=first series,".

(4) A few keywords may require more than one entry; the number of extra entries is determined by the value of the first entry. In such cases, the extra entries follow the first entry and are separated by commas, for instance "p-r=2, 1.10, -0.20, ".

(5) Only 4-character strings ending with "=" are examined as potential keywords. Strings ending with "=", but not corresponding to any valid keyword, will generate an error message and abort the programme. However, strings of any length not containing "=" are acceptable and may be used to document the file, e.g. "ARMA options: " in Table 4.2);

(6) The last entry of a set of keywords may end with a semi-colon instead of a comma.

(7) When applicable a numerical entry of "0" means "no"; and of "1", "yes". Thus "bia=1," and "bia=0," respectively mean do calculate and do not calculate the bias parameter.

(8) The maximum length of the entries is 10 for numeric entries, 20 for alpha-numeric entries, and 70 for the entry of the title keyword "tit=". Entries of length 0, e.g. as in "bia=,", are interpreted as zero or blank depending on the keyword.

(9) For numerical entries, blanks are interpreted as zeroes. Thus "int=1 ," would be interpreted as "int=10,"; and "bia= 1,", as "bia=1,".

The options are now presented in four groups: the *identifier options*, the *control date options*, the *basic options* and the *ARMA options*. They are displayed under those headings in the log output file.

- The Identifier Options

The identifier keywords are used to identify the pairs of series to be processed and to match the set of options to the appropriate pairs of series in the input files, or *triplets* of series if calendarization is requested.

. The Title Option: "tit="

The keyword "tit=" sets the title (displayed on the log output file) for the pair of series being processed. The entry of keyword "tit=", i.e. the title, may contain up to 70 alpha-numeric characters and blanks. Table 4.2 provides an example.

Note: The title may not contain commas as this would truncate the entry.

. The Control Series Identifiers: "ids=", "idb=", "idi", "idc=" and "ida="

The keyword "ids=" provides the control identifier for the original series; the keyword "idb=", for the set of benchmarks; "idi=", for the benchmarked series; "idc=", for a set of calendarization dates; and "ida", for a set of calendarized values. The entry, i.e. the control series identifier, may contain up to 20 alpha-numeric characters including blanks, but no commas. Valid examples are: "IdS=Monthly Autos,", "IdB=FisYear Autos,", "IdI=Bench. Autos," "IdC=CalYear Autos,".

Note: If the identifiers of the benchmarked series and of the calendarized values are not supplied, they are set to the identifiers of the original series and the calendarization dates respectively.

. The Search Option: "sea="

The search option determines whether only some series or all series in the input files will be processed. The search option is grouped with the identifier options, because the series identifiers are the key to any file search.

When "sea=1," is used, (i) *each* pair (or triplet) of control identifiers in the options file is read, (ii) the input files are searched for the series with matching series identifiers and (iii) each pair is processed according to the *specific set of options* chosen for that pair in the option file. Processing stops when the programme hits the "end-of-file" in the option file.

The search option ("sea=1,") is appropriate when the user wants to (i) process only a subset of the series in the input files or (ii) process the same pair of series more than once (presumably with different options) and/or (iii) process the same original series with different sets of benchmarks.

When "sea=0," is used, (i) the first original series is paired with the first set of benchmarks, (ii) the second original series is paired with the second set of benchmarks, etc. Each pair (or triplet) is processed according to *default options* (see below); or according to specific options, with matching control identifiers, *provided the specific sets of options are in the same order* in the option file as the pair of series in their respective input files. Processing stops when the programme hits the "end-of-file" on any one of the input files, except for the option file.

The default entry for the search option is "0", i.e. no searching. This option is appropriate in a production environment where all series are in the same pre-determined order and require more or less the same processing options. The example of the Wholesale Trade system of series presented in section 7 illustrates this.

Note: Matching occurs when the series identifiers in the input files are identical to the corresponding control identifiers in the option file. Since the programme distinguishes between lower and upper-case, the series identifier "Series 1" (say) does *not* match the control identifier "series 1"

Note: The search option may only be chosen on the first set of options in the option file. Once chosen, it cannot be changed. The programme will ignore any attempt to change it.

Note: When the search option is used ("sea=1,"), the sets of specific options may be in any order in the option file, because the input files are rewound after the processing of each pair of series.

- The Control Date Options

The control date options are used to select a range of dates (from the available dates) for the series in the input files.

. The Control Dates for the Original Series: "ysf=", "msf=", "dsf=", "ysl=", "msl=" and "dsl="

The keywords "ysf=", "msf=", "dsf=", "ysl=", "msl=" and "dsl=" respectively specify the control starting year, starting month, starting day and the control ending year, ending month and ending day to be processed for the original series. For example "ysf=81, msf=3, dsf=1, ysl=91, msl=9, dsl=30," processes the original series from March first 81 to September 30 91. One may specify only the ending or the starting dates; in the example of Table 4.2, the Retail Trade series is processed starting in January 1984 onwards. The default value of the control dates is "0"; the programme processes the observations for all dates.

Note: When the day identifiers are absent on the input file (see section 4.3), the control starting days and ending days must be left unspecified (or set to "0").

Note: When the control starting or ending years are equal to a number *N* lower than 100, they are interpreted as (1900+*N*). For instance the entry "1" in "ysf=1," is interpreted as 1901.

. The Control Dates for the Benchmarks: "ybf=", "mbf=", "dbf=", "ybl=", "mbl=" and "dbl="

The keywords "ybf=", "mbf=", "dbf=", "ybl=", "mbl=" and "dbl=" respectively specify the control starting year, starting month, starting day and the control ending year, ending month and ending day to be processed for the benchmarks. The control dates of the benchmarks work exactly like those of the original series.

- The Basic Options

The basic options are mainly methodological in nature. They specify which of the three benchmarking models to use, whether to estimate the bias parameter, etc.

. The Benchmarking Model Option: "met="

The keyword "met=" determines which of the three benchmarking models described in Section 3 to estimate. An entry of "0" ("met=0,") requests the additive model; an entry of "1", the mixed model; and an entry of "2", the multiplicative (log-additive) model.

Note: Non-positive values in the original series or in the benchmarks, combined with the multiplicative or the mixed model ("met=2," or "met=1,"), produce an error message and programme abortion.

. The Index Option: "ind="

The index option specifies whether the series is an index series or not. For an index series, the benchmarked series averages to the benchmark values over the reference periods of the benchmarks, instead of summing. An entry of "1" ("ind=1,") indicates an index series; and other numeric entries, a non-index series.

Note: No stock series option is required: if the reference periods of the benchmarks cover only one period of the original series, the series is a stock series. (More details in section 4.4)

. The Moving Estimation Option: "int="

The moving estimation option determines the length of the moving estimation intervals, in number of benchmarks, as described in section 3. An entry of "0" ("int=0,") specifies no moving interval; in other words, the interval covers the whole series. An entry of "7" ("int=7,") specifies that each estimation interval covers the reference periods of 7 benchmarks: the interval begins with the starting year, month and day (if applicable) of the first benchmark and finishes with the ending year, month and day of the seventh benchmark in the interval. At the end of the series however, the interval finishes with the last time period of the original series; and at the start of the series, the interval begins with the first period. The default value is 5.

The entry would normally be an odd number; for series shorter than the length specified, the estimation interval will cover the whole series. A moving estimation interval of 2 is appropriate for stock series, if the ARMA model selected (see below) is first order autoregressive. The estimates are then identical to those obtained on longer intervals.

The maximum interval depends on the capacity of the programme (see sub-section 4.3 §*Maximum number of observations*). At the time of writing, the maximum interval is 42. Entries larger than capacity are replaced by the maximum interval, e.g. "42".

Note: If the ARMA model chosen for the error is seasonal, the moving estimation interval should be set to cover the whole series, e.g. by entering "int=0,".

Note: If the reference periods of two or more benchmarks overlap, that is some periods or the original series are covered by more than one benchmark, e.g. one annual and one sub-annual benchmark, the moving estimation interval should be set to cover the whole series (e.g. "int=0,").

. The Link Option: "lin="

The link option determines whether links are made between estimation intervals, as described in section 3. An entry of "1" ("lin=1,") specifies a link; and "0", no link.

Note: With the link option, no dispersion (e.g. variance) is calculated for the benchmarked series. The reason is that the dispersion of the benchmarked series would be under-estimated at - and in the neighbourhood of - the link points.

. The Measure of Dispersion Options: "cvs=", "cvb=", "cvi=" and "cvc="

As explained in section 4.3, each observation is read (and written) with a corresponding measure of dispersion (i.e. of reliability). The measure of dispersion options determine whether the *dispersion values* read from the files are coefficients of variation, standard deviations or variances. With keyword "cvs=", an entry of "0" specifies that the dispersion values read for the original series are coefficient of variations; an entry of "1", standard deviations; and "2", the variances. Keyword "cvb=" controls the measure of dispersion of the benchmarks in the same manner; keyword "cvi=", that of the benchmarked (interpolated) series; and "cvc=", that of the calendarized values. The *coefficient of variation* is defined as the standard deviation of the observation divided by the value of the observation, times *100%*.

Note: Non-positive values in the original series, combined with the coefficient of variation as measure of dispersion ("cvs=0,"), produce an error message and abortion of the programme; and similarly negative values in the benchmarks. (The benchmarks may have zero dispersion, but the original series must have positive dispersion.)

. The Bias Option: "bia="

The bias option determines whether the programme estimates a bias parameter on each interval. An entry of "1" ("bia=1,") requests bias estimation; and of "0" , no such estimation.

. The Plot Options: "pls=", "plc="

The plot options determines whether plots will be produced. Keyword "pls=" controls the plot of the original and benchmarked series; and "plc=", that of the corrections made to original series to obtain the benchmarked series.

If "pls=1," is used, the original and the benchmarked series are plotted along with the benchmarks.

If "plc=1," is used, the corrections made to the original series are plotted along with the discrepancies between the benchmarks and the corresponding sums of the original series. If the multiplicative or the mixed benchmarking model ("met=2," or "met=1,") were used, the corrections and the discrepancies are proportional, i.e. the corrections are then the ratios (instead of the differences) of the benchmarked to the original series. The corrections and the discrepancies are also proportional, if the additive benchmarking model was used, without bias estimation and with coefficient of variation as measure of dispersion of the original series ("bia=0, cvs=0,") .

The graphical examination of the corrections is often more informative about the effect of benchmarking, than that of the original and benchmarked series, because the seasonal, trend-cyclical and irregular fluctuations are usually absent from the corrections. The default values of the plot options are therefore "pls=0," and "plc=1,".

Table 4.9 and 4.10 of section 4.8 illustrate and explain the plots in more detail.

- The ARMA Options: "p-r=", "q-r=", "p-s=", "q-s=" and "n-s=",

The ARMA options set the ARMA model (not to be confused with the benchmarking model) for the error in the original series. The ARMA model is specified by 5 keywords, "p-r=", "q-r=", "p-s=", "q-s=" and "n-s=", which respectively control the number of *regular* autoregressive parameters (maximum 25), the number of regular moving average parameters (max. 25), the number of *seasonal* autoregressive parameters (max. 3), the number of seasonal moving average parameters (max. 3) and the periodicity of seasonality (i.e. the number of seasons). The keywords "p-r=", "q-r=", "p-s=" and "q-s=" may require more than one entry: the number of parameters and the values of those parameters. In this example, "p-r=2, 1.10, -0.20,"., the first entry is the number (2) of regular autoregressive parameters and the second and third entries are the values of the two parameters, 1.10 and -0.20. The following exemplifies how a full seasonal autoregressive moving average model with seasonal periodicity 12 could be entered: "p-r=2, 1.10, -0.15, q-r=1, 0.30, p-s=1, 0.95, q-s=1, 0.40, n-s=12,".

ARMA models are often denoted as $(p,q)(P,Q)_s$; and the parameters, as $\varphi_1,..., \varphi_p$, θ_1 ,..., θ_p, Φ_1 ,..., Φ_p, and Θ_1 ,..., Θ_p. (Symbol θ does not stand for the true original series in this sub-section.) The entries to keywords "p-r=", "q-r=", "p-s=", "q-s=" and "n-s=" respectively set the values of p, q, P, Q and s.

Note: Programme BENCH allows only for seasonal ARMA models which are multiplicative (the regular part multiplying the seasonal part). However, non-multiplicative seasonal models may be achieved by specifying regular autoregressive (and/or moving average) polynomials of orders larger than s (p ,q>s).

Note: Parameter φ_k (or θ_k) is assumed to be of order k; however the value of φ_k may be set equal to zero.

Note: For seasonal models, the estimation should not be carried out on moving intervals; this would cause discontinuities in the estimated seasonal pattern. One should use "`int=0,`".

- The Internal Default Options
The option file may be empty (or blank), but it must exist. If the file is empty, *internal default options* are used by BENCH. If the file is not empty, but some of the options are not specified, the internal default values are used for these options. For some of the options, unacceptable entries will be replaced by default values.

The internal default identifier options are:
. no title ("`tit=,`"), i.e. blank title,
. control series identifiers equal to the series identifiers read from the various input files (e.g. the control identifiers of the original series are set equal to the series identifiers on the input file of the original series),
. no search of the input files ("`sea=0,`").

The internal default control dates are all equal to zero, which means the series are processed for all available dates.

The internal default basic options are
. additive benchmarking model ("`met=0,`"),
. moving estimation interval equal to 5 ("`int=5,`"),
. no link between the estimation intervals ("`lin=0,`"),
. coefficients of variation as measures of dispersion ("`cvs=0, cvb=0, cvi=0, cvc=0,`"),
. no bias estimation ("`bia=0,`"),
. non-index (i.e. flow) series ("`ind=0,`"),
. no plot of the series ("`pls=0,`"),
. plot of the corrections ("`plc=1,`").

The internal built-in default ARMA options are
. one regular autoregressive parameter, with value 0.999999 ("`p-r=1, 0.999999,`")
. no regular moving average parameter ("`q-r=0,`"),
. no seasonal autoregressive parameter ("`p-s=0,`"),
. no seasonal moving average parameter ("`q-s=0,`"),
. number of seasons equal to 1 ("`n-s=1,`").

In the $(p,q)(P,Q)_s$ mathematical notation, the internal default ARMA model is $(1,0)(0,0)_1$ with autoregressive parameter $\varphi_1 = 0.999999$. The resulting ARMA model $e_t = 0.999999 e_{t-1} + v_t$ is virtually equivalent to the "random walk" model. If the coefficients of variation are constant and the benchmarks binding (with dispersion values equal to 0), the benchmarked series obtained, on one estimation interval, with the internal default options, virtually coincides with the benchmarked series obtained with the Denton first difference proportional variant.

- The User-Supplied Default Options
Users may supply their own default options, by entering a set of options with control series identifier equal to "`default`" for the original series or for the benchmarks or for both (e.g. "`ids=default,`"), as the *first* set of options in the option file.

If the search option is not used ("`sea=0,`"), the series with no matching set of options are benchmarked according to those user-supplied default options. This is particularly useful when all, or most, series are to be benchmarked using the same options. The user-supplied default options are entered once at the beginning of the option file;

followed by the specific sets of options, with matching control series identifiers, for the few "atypical" series, if applicable. However, the specific sets of options must be in the same order as the original series. This will be illustrated with the system of Wholesale Trade series.

If all series are to be processed with different options, users-supplied default options may still be useful. The user-supplied default options reflect the options most common to the various series; and the specific sets of options, need only specify the options which depart from the user-supplied options. Indeed, the options not entered in the specific sets of options would default to the corresponding user-supplied default values.

The options and the keywords and the default values are tabulated in Table 4.3 for quick reference.

Table 4.3: Summary of the Options in Programme BENCH, with the Internal Default Values

Table 4.3 A: The Identifier Options

Keyword	Entry	Description	Internal Default Values
"tit="	any	Title for the benchmarking problem considered	"tit=,"
"ids="	any	Control identifier for the original series	"ids=,"
"idb="	any	Control identifier for the benchmarks	"idb=,"
"idi="	any	Control identifier for the benchmarked series	entry of "ids="
"idc="	any	Control identifier for the calendarization dates	"idc=,"
"ida="	any	Control identifier for the calendarized values	entry of "idc="
"sea="	"0"	No input file search for control identifiers	"sea=0,"
	"1"	Input file search for control identifiers	

Table 4.3 B: The Control Date Options

"ysf="	numeric	Control starting year for the original series	"ysf=0,"*
"msf="	numeric	Control starting month " " " "	"msf=0,"*
"dsf="	numeric	Control starting day " " " "	"dsf=0,"*
"ysl="	numeric	Control ending year for the original series	"ysl=0,"*
"msl="	numeric	Control ending month " " " "	"msl=0,"*
"dsl="	numeric	Control ending day " " " "	"dsl=0,"*
"ybf="	numeric	Control starting year for the benchmarks	"ybf=0,"*
"mbf="	numeric	Control starting month " " "	"mbf=0,"*
"dbf="	numeric	Control starting day " " "	"dbf=0,"*
"ybl="	numeric	Control ending year for the benchmarks	"ybl=0,"*
"mbl="	numeric	Control ending month " " "	"mbl=0,"*
"dbl="	numeric	Control ending day " " "	"dbl=0,"*

* entry "0" means all years, or all months or all days

Table 4.3 C: The Basic Options

Keyword	Entry	Description	Internal Default Values
"met="	"0"	Additive model	"met=0,"
	"1"	Mixed model	
	"2"	Multiplicative (log-additive) model	
"int="	numeric	Length of the moving estimation interval	"int=5,"
"lin="	"0"	No link between estimation intervals	"lin=0,"
	"1"	Link between estimation intervals	
"cvs="	"0"	Coeff. of var. as dispersion of the original series	"cvs=0,"
	"1"	Standard deviation " "	
	"2"	Variance " "	
"cvb="	"0"	Coeff. of var. as dispersion of the benchmarks	"cvb=0,"
	"1"	Standard deviation " "	
	"2"	Variance " "	
"cvi="	"0"	Coeff. of var. as dispersion of the benchmarked series	"cvi=0,"
	"1"	Standard deviation " " "	
	"2"	Variance " " "	
"cvc="	"0"	Coeff. of var. as dispersion of the benchmarked series	"cvc=0,"
	"1"	Standard deviation " " "	
	"2"	Variance " " "	
"bia="	"0"	No calculation of a bias parameter	"bia=0,"
	"1"	Calculation of a bias parameter on each estimation interval	
"ind="	"0"	Non-index series	"ind=0,"
	"1"	Index series	
"pls="	"0"	No plot of the benchmarked series	"pls=0,"
	"1"	Plot of the benchmarked series	
"plc="	"0"	No plot of the corrections	
	"1"	Plot of the corrections	"plc=1,"

Table 4.3 D: The ARMA Options

"p-r="	numeric#	Number of regular autoregressive parameters	"p-r=1, 0.999999,"
"q-r="	numeric#	Number of regular moving average parameters	"q-r=0,"
"p-s="	numeric#	Number of seasonal autoregressive parameters	"p-s=0,"
"q-s="	numeric#	Number of seasonal moving average parameters	"q-s=0,"
"n-s="	numeric	Number of seasons	"n-s=1,"

Numeric entry, followed by the number of specified parameters, e.g. "p-r=2, 1.10, -0.20,"

4.3 THE FILE OF THE ORIGINAL SERIES

The second record in file "`bench.fil`" pertains to the file of the original series to be benchmarked. The explanations given here also hold for the remaining input and output files and will not be repeated in as much detail.

According to the file "`bench.fil`" of Table 4.1 for instance, the original Retail Trade series is read from a file named "`retail.ori`". As explained in section 4.2, the file name may consist of up to 20 characters and may contain directory information.

. The Record Layout

The record layout must be such that on each record the information is read in the following order: series identifier; reference periods, i.e. starting year, starting month, starting day, ending year, ending month and ending day; value of the original series and dispersion (standard deviation, variance or coefficient of variation) of the original value. For example, the format statement "`(a20,6i5,f10.0,f5.0)`" would cause BENCH to read the series identifier in columns 1 to 20 (item "`a20`" in the format statement); the starting year, month and day and the ending year, month and day, in the following 6 fields of 5 columns ("`6i5`"); the original value, in the following field of 10 columns ("`f10.0`"); and the dispersion value, in the following field of 5 columns ("`f5.0`"). Each entry must be right-justified in its field. As will be illustrated, the information on the records may, in fact, be in any order, provided it is *read* in the expected order.

. The Series Identifier

The maximum length of the series identifier is 20 characters. The identifiers *may* contain blanks. The identifiers must be different from one series to the next, because the programme detects the end of one series and the start of another series, by the change in the identifiers.

. The Starting and Ending Years, Months and Days

As explained in section 4.1, programme BENCH requires that the reference of each value (of a series) be identified by a starting year, month and day and by an ending year, month and day. In practice, the starting year and month are often equal, as in the case of a monthly value covering from February 92 to February 92 for instance. In such cases, one can select a format which reads the fields of the year and month twice, the first time as the starting year and month and the second time as the ending year and month. Similarly, the starting and ending days are often irrelevant, as in the case of monthly values always covering from the first day to the last day of the month. In such cases, the starting and ending days *must* be set equal to the starting and ending months respectively. In order to do that, one can select a format statement which reads the field of the starting month twice, the first time as the starting month and the second time as the starting day; and similarly for the field of the ending month. Both these format specification techniques are now illustrated, with the example of Table 4.4.

Table 4.4: Sample of File of the Original Series ("retail.ori), for Retail Trade

```
Retail    1980    1   5651446  0.8
Retail    1980    2   5761119  0.7
Retail    1980    3   6127963  0.8
Retail    1980   12   9081377  0.7
                .
                .
                .
Retail    1989    8  14584459  0.9
Retail    1989    9  14521628  0.8
Retail    1989   10  14297343  1.7
Retail    1989   11  15182860  1.6
Retail    1989   12  17909814  1.4

123456789 123456789 123456789 123456789
```

Table 4.4 displays a sample of the file of the original Retail Trade series. (The full file is available with the programme diskettes.) According to the format statement "`(a10,2i5,t16,i5,t11,2i5,t16,i5,f10.0,f5.0)`" in the second line of Table 4.1, the record layout of file "`retail.ori`" of Table 4.4 is the following. The series identifier occupies the first 10 columns of each input record (item "`a10`" in the format statement). The starting year and the starting month occupy the next two fields of 5 columns ("`2i5`"), i.e. columns 11 to 15 and 16 to 20. The starting day is set equal to the starting month by reading the field of the starting month (columns 16 to 20) a second time ("`t16,i5`"). The ending year, the ending month and the ending day are set equal to the starting year, month and day respectively by reading the latter fields a second time ("`t11,2i5,t16,i5`"). The value of the original series occupies the following 10 columns ("`f10.0`"), i.e. columns 21 to 30; and its dispersion value, the following five columns ("`f5.0`").

Note: The information on the records may be in any order, provided it is *read* in the expected order. This is accomplished by means of "tabs" (i.e. the "t"'s) in the format statement.

. The Maximum Number of Original Observations

In principle, the capacity of the programme is tailored to the need of the user. The programme displays its capacity (on the log output file) after the heading. The version which produced Table 4.8 allowed for 732 original observations in any one series; and for 190 original observations in any moving estimation interval. (This implied that series comprising more than 190 observations must be processed in a moving manner). Users requiring more (or less) capacity may contact the author.

. The Maximum Number of Series

In principle, programme BENCH may process any number of series.

. The Year, Month and Day Identifiers

The year, month and day identifiers must be entered in integer numeric values, for instance "`88 11 21`" or "`1988 11 21`" meaning year 1988, month 11 and day 21. As explained earlier in this section, the day identifier may be irrelevant; in such cases, the user should specify its field to coincide with that of the month identifier.

. The Dispersion Values of the Original Series

Depending on the entry of keyword "`cvs=`" in the option file for the series considered (see section 4.2), the input dispersion values are either coefficients of variation, standard deviations or variances. In the example of Table 4.4, the dispersion values are coefficients of variations, read in columns 31 to 35. The benchmarked series tends to run closer to the original values with relatively lower dispersion. If no dispersion values are available, a zero or blank field may be specified in the format statement; the resulting zero entries are interpreted as 1.0 (e.g. the coefficient of variation is 1.0%).

Note: If some of the observations are equal to zero and the coefficient of variation was used ("`cvs=0,`"), the programme prints an error message and stops.

. Order of the Records

Programme BENCH requires the records to be in chronological order. The programme performs checks for ascending chronological order, but not for missing records, e.g. a missing day in a daily series, or for duplicate records.

4.4 THE FILE OF THE BENCHMARKS

The third record of file "`bench.fil`" pertains to the file of the benchmarks. According to Table 4.1, the benchmarks of Retail Trade are located in the file named "`retail.ben`". The record layout must be such that the information on each record is read in the following order: series identifier; reference periods of the benchmark, i.e. starting year, starting month, starting day, ending year, ending month and ending day; value of the benchmark and dispersion of the benchmark. For example, the format statement "`(a10,6i4,f10.0,f5.0)`" would cause BENCH to read the series identifier in columns 1 to 10 (item "`a10`" in the format statement); the starting year, month and day and the ending year, month and day, in the following 6 fields of 4 columns ("`6i4`"); the value of the benchmark, in the following field of 10 columns ("`f10.0`"); and the dispersion value, in the following field of 5 columns ("`f5.0`"). More details are given in section 4.3.

Table 4.5: File "retail.ben" referred to in Table 1

```
Retail    1985    2 1986    1      143965400  0.03276
Retail    1986    2 1987    1      154377100  0.03055
Retail    1987    2 1988    1      169944600  0.19263
Retail    1988    2 1989    1      181594000  0.13728
Retail    1989   10 1989   10       15584920  0.5
Retail    1989   11 1989   11       16430621  0.6
Retail    1989   12 1989   12       19182630  0.5

123456789 123456789 123456789 123456789 123456789          (col. no)
```

Table 4.5 displays the file of the benchmarks for the Retail Trade series. According to the format of the file "`(a10,2i5,t16,i5,t21,2i5,t26,i5,f15.0,f10.0)`" (on 3rd line of Table 4.1), the series identifier occupies the first 10 columns of each record ("`a10`"); the starting year and month occupy the next 2 fields of 5 columns ("`2i5`"); the starting day coincides with the starting month ("`t16,i5`"); the ending year and month occupy the next 2 fields of 5 columns ("2i5"); the ending day coincides with the ending month ("t26, i5"); the value of the benchmark occupies the next 15 columns ("f15.0"), and its dispersion, the next 10 columns ("f10.0").

In the case of Table 4.5, the first four benchmarks are annual, because they cover 12 consecutive months. However the years are fiscal, covering from February of one year to January of the following year. The three last benchmarks are monthly, covering the months of October, November and December respectively.

. The Series Identifiers

The rules set for the identifiers of the original series still apply for the benchmarks. Namely, the identifiers of the benchmarks must be different from one series to the next.

Note: The series identifiers of the benchmarks need not be equal to those of the original series, as in the example of Retail Trade.

. The Starting and Ending Years, Months and Days

The rules set for the reference periods of the original series still prevail. For example a fiscal benchmark could have reference periods "88 4 4 89 3 3" (where " 4" and " 3" refer to April and March); a weekly benchmark covering five weeks could have reference periods "1988 10 31 1988 11 27", where "10" and "11" refer to October and November and "31" and "27" refer to the day of each month.

Note: The reference periods of the benchmarks must be embedded in those of the original series. In other words, the periods covered by the benchmarks must not extend beyond the periods covered by the original series.

Note: The reference periods specified for each benchmark must exist for the corresponding original series. Consequently, if the reference periods of the original values are described in terms of years, months and genuine days, this must also be the case for the benchmarks.

Note: The records must be in chronological order. If one period is covered by more than one benchmark (e.g. by an annual and sub-annual benchmarks), the benchmark with the earliest starting reference periods must come first in the file.

Note: For a stock series, the reference periods of the benchmarks cover only one time period of the original series. If the benchmark is binding (zero dispersion), the benchmarked series will coincide with it at that time period.

Note: Two or more benchmarks may cover exactly the same reference periods, if they have non zero dispersion, i.e. if they are non-binding. (See next note below.)

. The Dispersion Values of the Benchmarks

Depending on the entry of keyword "cvb=" in the option file for the series considered (see section 4.2), the input dispersion values are either coefficients of variation, standard deviations or variances. If no dispersion values are available, a blank field may be specified in the format statement; the resulting entries are interpreted as 0.0 (e.g. the coefficient of variation is 0.0%). A benchmark with dispersion equal to 0.0 is binding; the benchmarked series fully complies with a binding benchmark, in other words the discrepancy between a binding benchmark and the benchmarked series is zero.

Note: Two or more benchmarks may cover exactly the same reference periods, if they have non zero dispersion, i.e. if they are non-binding. However the estimation interval should then be set to cover the whole series. If more than one such benchmark has zero dispersion a Fortran "run-time error" message will occur as a result of a singular matrix.

. The Maximum Number of Benchmarks

The version which produced Table 4.8 allowed for 106 benchmarks; and, for 42 benchmarks on any estimation interval. (This implied that series comprising more than 42 benchmarks must be processed in a moving manner). More details in Section 4.3 under Maximum Number of Original Observations.

4.5 THE FILE OF THE BENCHMARKED SERIES

The fourth record of file "bench.fil" pertains to the file of the benchmarked series, which contains the estimated benchmarked (or interpolated) series. According to Table 4.1, the benchmarked Retail Trade values are located in file "retail.out". Programme BENCH writes the information on each record in the following order: series identifier, reference periods (starting year, month and day and ending year, month and day), the value of the benchmarked series and its dispersion. More details are given in section 4.3.

In the case of the Retail Trade series, the format statement "(a10,6i4,f10.0,f5.2)" (on line 4 of Table 4.1) causes BENCH to write the series identifier in columns 1 to 10 (item "a10" in the format); the starting year, month and day and the ending year, month and day, in the following 6 fields of 4 columns ("6i4"); the benchmarked value, in the following field of 10 columns ("f10.0"); and the dispersion value, in the following field of 5 columns ("f5.2") with 2 decimals. In the example, the dispersion values are coefficients of variation.

Note: The series identifier of the benchmarked series is given by the control series identifier of the benchmarked series (by means of keyword "idi=") in the option file. If no such control series identifier was entered, the series identifier is set identical to that of the original series.

Table 4.6: Sample of the File of the Bench-marked Series ("retail.out") for Retail Trade

```
Retail    1984   1   11984   1   1  8948998.  .85
Retail    1984   2   21984   2   2  9125989.  .91
Retail    1984   3   31984   3   3 10359992.  .97
            .
            .
            .
Retail    1989   3   31989   3   3 14923096.  .76
Retail    1989   4   41989   4   4 15654382.  .78
Retail    1989   5   51989   5   5 16737529.  .70
Retail    1989   6   61989   6   6 16845388.  .77
Retail    1989   7   71989   7   7 15547353.  .73
Retail    1989   8   81989   8   8 15825313.  .67
Retail    1989   9   91989   9   9 15757788.  .50
Retail    1989  10  101989  10  10 15577800.  .45
Retail    1989  11  111989  11  11 16440204.  .47
Retail    1989  12  121989  12  12 19175235.  .43

123456789 123456789 123456789 123456789 123456789
```

4.6 THE FILE OF THE CALENDARIZATION DATES

In programme BENCH calendarization merely consists of taking the sums - or the averages in the case of index series - of the benchmarked or interpolated series, over the dates specified in the file of the calendarization dates. The fifth record of file "bench.fil" pertains to the file of the calendarization dates. According to Table 4.1, the calendarization dates of the Retail Trade example are located in file "retail.cai". If the file of the calendarization dates is absent from file "bench.fil", i.e. the file contains only four records, no calendarization is performed.

The programme expects the information on each record of the file in the following order: series identifier; the reference periods of the calendarized value, i.e. starting year, starting month, starting day, ending year, ending month and ending day. For example, the format statement "(a20,6i5,f10.0,f5.0)" would cause BENCH to read the series identifier in columns 1 to 20 (item "a20" in the format statement); the starting year, month and day and the ending year, month and day, in the following 6 fields of 5 columns ("6i5").

Table 4.7: File of the Calendarization Dates ("retail.cai") for Retail Trade

```
RetailC   1981    1 1981   12
... typ=m, pro=12.
RetailC   1992    1 1992   12

123456789 123456789 123456789   (col no.)
```

The file of the calendarization dates of Retail Trade is reproduced in Table 4.7. According to its format "(a10,2i5,t16,i5,t21,2i5,t26,i5)" (given by the fifth line of Table 4.1), the series identifier is read in columns 1 to 10 (item "a10" in format statement); the starting year and month are read in the next 2 fields of 5 columns ("2i5"); the starting day is read in the same field as the starting month ("t16,i5"), i.e. is set equal to the starting month; the ending year and month are read in the next 2 fields of 5 columns ("t21,2i5"); the ending day is read from the same field as the ending month ("t26,i5"). The second record of file "retail.cai" in Table 4.7, starting with "...", specifies that records are to be inserted between the first record and the third record; that the reference periods are in terms of months ("typ=m,") and that the progression will be 12 months ("pro=12,"). This feature of the programme, designed to avoid repetitious coding, will be fully explained in section 5.

The calendarization dates in file "retail.cai" specify that calendar year values of the benchmarked series are to be taken. The annual benchmarks of Retail Trade (Table 4.5) reflected fiscal years covering from February of one year to January of the following year; the calendarized values will reflect the calendar year. The calendarization of the fiscal year values emerges as a by-product of benchmarking.

4.7 THE FILE OF THE CALENDARIZED VALUES

The sixth record of file "bench.fil", pertains to the file of calendarized values. According to Table 4.1, the calendarized values of the Retail Trade are located in file "retail.cao". Programme BENCH writes the information on each record in the following order: series identifier, reference periods (starting year, starting month, starting day, ending year, ending month and ending day), calendarized value and its dispersion. For instance, the format statement "(a10,6i5,f12.0,f5.2)" would cause programme BENCH to write the series identifier in columns 1 to 10; the reference periods, in the next 6 fields of 5 columns; the calendarized values, in the next field of 12 and its dispersion in the next field of 5 with 2 decimals ("f5.2").

Note: At the time of writing, no dispersion is calculated for the calendarized values, zeroes will appear in the field.

In the case of the Retail Trade series, a built-in CANSIM format is used. The statement "(cansimA)" of the sixth line of Table 4.1 specifies that the calendarized values will be written in the CANSIM format. The CANSIM and X-11-ARIMA formats will be explained in section 6.

4.8 THE LOG OUTPUT FILE

The log output file "bench.log" produced by programme BENCH basically contains statistics and plots about the benchmarking-interpolation exercise. Table 4.8 shows a sample of the file for the Retail Trade series. The screen monitor displays a subset of the file during execution.

- Options Used and Statistics

The log output file displays a heading, the names and record layout of the files processed (i.e. the contents of file "bench.fil"), the options used for each pair of original and benchmark series processed, the estimation intervals processed with the bias estimate on the interval, the assessment statistics described in section 3 and error and warning messages if applicable.

The keywords and options displayed in the log output file reflect the *interpretation* by the programme of the keywords and entries supplied by the user; the options not used appear with their (internal or user-supplied) default values

Like in any other programme, the error messages do not anticipate all potential errors; also a message may be the result of a previous detected or undetected error. Con-sequently, the error messages should not always be interpreted too literally. The error messages are presented in Appendix E, with the possible remedy.

Table 4.8: Sample of the log output file ("BENCH.LOG") for Retail Trade

```
                         *** Programme BENCH ***

          to Benchmark, Interpolate and Calendarize Time Series Data

                          by Pierre A. Cholette
               Time Series Research and Analysis Division
                          STATISTICS CANADA
                       Ottawa, Canada, K1A 0T6
                           (613) 951-1601

                            Version 1.00
                          December 23, 1992

 Capacity of version:   732 original obs. and 190 per estimation interval
                        106 benchmarks and 42 per estimation interval

                           December  23 1992; 14 hr  6 min.  50.53 sec.

---------------------------------------------------------------------------
Files and formats in file Bench.Fil:
retail.opt
retail.ori    (a10.2i5.t16.i5.t11.2i5.t16.i5.f10.0.f5.0)
retail.ben    (a10.2i5.t16.i5.t21.2i5.t26.i5.f15.0.f10.0)
retail.out    (a10.6i4.f10.0.f5.2)
retail.cai    (a10.2i5.t16.i5.t21.2i5.t26.i5)
retail.cao    (cansimA)

---------------------------------------------------------------------------
                                                          Problem    1

Original series      Retail             available from 1980    1    1
                                                   to   1989   12   12

Set of benchmarks    Retail             available from 1985    2    2
                                                   to   1989   12   12

  Identifier options:

    tit=   Canada Total Retail Trade - link to October 1989 - Fiscal Years


      ids=Retail              . idb=Retail              .
      idc=RetailC             . idi=Retail2             . sea=0.

  Control date options:
      ysf=1984, msf=   1, dsf=   1, ysl=   0, msl=   0, dsl=   0.
      ybf=   0, mbf=   0, dbf=   0, ybl=   0, mbl=   0, dbl=   0.

  Basic options:
      met=   0, ind=   0, int=  42, lin=   0, cvs=   0, cvb=   0.
      cvi=   0, cvc=   0, bia=   1, pls=   1, plc=   1.

  ARMA  options:
      p-r=   1, q-r=   0, p-s=   0, q-s=   0, n-s=   1.
      model: (1 0)(0 0) 1.
      parameters in p q P Q order:   .800

                                     --------------------------------------

     Additive model              No.    Additive bias
  |-- Estimation interval --|   iter.   estimate and std dev
  1: 1984  1  1 to 1989 12 12    1   -.1199291E+07  .4443698E+05

  Average, min, and max. of the proport. discrepancies of the
       original series:               .1091553E+01  .1071068E+01  .1103098E+01
       benchmarked series:            .1000149E+01  .9994171E+00  .1000457E+01

  Average, min, and max. of the additive discrepancies of the
       original series:               .8923250E+07  .1247761E+07  .1600019E+08
       benchmarked series:            .2057219E+05 -.9582781E+04  .8304106E+05

  Avg. abs. change between the original and the benchmarked series
       in period-to-period growth rates:             .8191986E-02
       in period-to-period differences:              .1370040E+05

---------------------------------------------------------------------------
Benchmarked values in file:  retail.out
Calendarized values (if applic.) in file:  retail.cao
Statistics in file:  bench.log

---------------------------------------------------------------------------

                          December  23 1992; 14 hr  6 min.  51.57 sec.

                               Programme took  0 hr  0 min.   0.88 sec.
```

- **Plot of the Series**

The log output file "bench.log" also contains the plots produced by programme BENCH. As shown in Table 4.9, they are "character" or "printer" plots (as opposed to screen plots with continuous curves). Time is displayed on the vertical axis and the values of the variables on the horizontal axis. Although not very sophisticated, such plots are useful here, because the time axis is never compressed, since each period of time occupies one line and the plot occupies as many lines as necessary. (The compressing of both axes in screen plots often requires that each plot be done ad hoc, i.e. separately.)

In order to maximize the visual impact, the values are displayed in "pages" containing 49 periods of time (each page starts with "`Units=`"), and all pages have a common scale but different locations. (This is like a set of maps showing different geographical areas with a common scale, e.g. 1 cm^2 for 1 km^2.) In order to link the pages together, each page repeats the last time period of the previous page: thus in Table 4.9, the first page ends with January 88; and the second page starts with January 88. (This is like the set of maps overlapping at the edges.)

Table 4.9: Plot of Original and Benchmarked Series produced by BENCH

```
Plot of the series  idi=Retail2              . idb=Retail

  Original: r; Benchmarked: 1.2....: Benchmarks: b; Coincidence: =

   Units=10**( 4), idi=Retail2                  idb=Retail
 775.153   915.289  1055.420  1195.551  1335.682  1475.813  1615.944  1756.075
     |         |         |         |         |         |         |         |
84  1r------1  |         |         |         |         |         |         |
84  2|r-------2|         |         |         |         |         |         |
84  3|---------r|-----3  |         |         |         |         |         |
84  4|---------r------4  |         |         |         |         |         |
84  5|---------|-----r---|--5      |         |         |         |         |
84  6|---------|------r--|----6    |         |         |         |         |
84  7|---------|r------7 |         |         |         |         |         |
84  8|---------|--r------|8        |         |         |         |         |
84  9|---------r------9  |         |         |         |         |         |
84 10|---------|---r-----|0        |         |         |         |         |
84 11|---------|-------r-|----N    |         |         |         |         |
84 12|---------|---------|-----r---|--D      |         |         |         |
85  1|-----r---|--1      |         |         |         |         |         |
85  2|---r-----|2        |   b     |         |         |         |         |
85  3|---------|----r----|-3  b    |         |         |         |         |
85  4|---------|------r--|----4 b  |         |         |         |         |
85  5|---------|---------|----r-b--|-5       |         |         |         |
85  6|---------|---------|r-----b6 |         |         |         |         |
85  7|---------|--------r|-----7b  |         |         |         |         |
85  8|---------|---------|-r----b-8|         |         |         |         |
85  9|---------|-----r---|--9   b  |         |         |         |         |
85 10|---------|---------|-r----b0 |         |         |         |         |
85 11|---------|---------|-----rb--|--N      |         |         |         |
85 12|---------|---------|------b--|----r----|-D       |         |         |
86  1|---------|-r-------|      b  |         |         |         |         |
86  2|--------r|-----2   |         b         |         |         |         |
86  3|---------|-------r-|----3    b         |         |         |         |
86  4|---------|---------|---r-----|4b       |         |         |         |
86  5|---------|---------|---------|rb-----5 |         |         |         |
86  6|---------|---------|----r----|-b6      |         |         |         |
86  7|---------|---------|-----r---|-b7      |         |         |         |
86  8|---------|---------|-----r---|-b8      |         |         |         |
86  9|---------|---------|---r-----|9b       |         |         |         |
86 10|---------|---------|-------r-|-b--0    |         |         |         |
86 11|---------|---------|-------r-|-b--N    |         |         |         |
86 12|---------|---------|---------|-b-------|-r------D         |         |
87  1|---------|-----r---|--1      |b        |         |         |         |
87  2|---------|---r-----|2        |         b         |         |         |
87  3|---------|---------|--r------|3        b         |         |         |
87  4|---------|---------|---------|-r-------=         |         |         |
87  5|---------|---------|---------|-----r---b---5     |         |         |
87  6|---------|---------|---------|------r--b----6    |         |         |
87  7|---------|---------|---------|----r----b--7      |         |         |
87  8|---------|---------|---------|r-------8b         |         |         |
87  9|---------|---------|---------r-------9 b         |         |         |
87 10|---------|---------|---------|-------r-b----0    |         |         |
87 11|---------|---------|---------|----r----b--N      |         |         |
87 12|---------|---------|---------|---------b---------|--r------|D        |
88  1|---------|---------|r-------1|         b         |         |         |

   Units=10**( 4), idi=Retail2                  idb=Retail
1055.420  1195.551  1335.682  1475.813  1615.944  1756.075  1896.206  2036.337
     |         |         |         |         |         |         |         |
88  1|---r-----|-1       |  b      |         |         |         |         |
88  2|--r------|2        |        b|         |         |         |         |
88  3|---------|-----r---|----3   b|         |         |         |         |
88  4|---------|--------r|-------4b|         |         |         |         |
88  5|---------|---------|--r-----b|5        |         |         |         |
88  6|---------|---------|---r----b|--6      |         |         |         |
88  7|---------|---------|r-------b7         |         |         |         |
88  8|---------|---------r-------8b|         |         |         |         |
88  9|---------|---------r-------9b|         |         |         |         |
88 10|---------|---------|r-------=|         |         |         |         |
88 11|---------|---------|---r----b|--N      |         |         |         |
88 12|---------|---------|--------b|---------|----r----|---D     |         |
89  1|-----r---|---1     |        b|         |         |         |         |
89  2|----r----|--2      |         |         |         |         |         |
89  3|---------|--------r|------3  |         |         |         |         |
89  4|---------|---------|---r-----|-4       |         |         |         |
89  5|---------|---------|---------|r-------5|         |         |         |
89  6|---------|---------|---------|r-------6|         |         |         |
89  7|---------|---------|--r------|7        |         |         |         |
89  8|---------|---------|----r----|--8      |         |         |         |
89  9|---------|---------|----r----|-9       |         |         |         |
89 10|---------|---------|--r------|=        |         |         |         |
89 11|---------|---------|--------r|------=  |         |         |         |
89 12|---------|---------|---------|---------|-----r---|---=     |         |
```

Under the plot series option ("`pls=1,`"), the programme displays the original series (symbol "`r`"), the benchmarked series and the benchmarks against time. The benchmarked series uses the ending month as symbol ("`1 2 ... 9 O N D 1 2...`"). The area under the benchmarked series is "shaded" with dashes ("`-`"); this is a fairly good substitute to joining the points. The benchmarks (symbol "`b`") are represented by their average over their reference periods. This confers the benchmarks the same order of magnitude as the benchmarked series and also indicates their reference periods, namely that the benchmarks cover fiscal years covering from February to January in the example. Benchmarks with value zero are not displayed. The time axis is labelled with the *ending* year, month and day (if applicable) of each time period.

- Plot of the Corrections

When the plot correction option is chosen ("`plc=1,`"), the programme displays the corrections made to the original series to obtain the benchmarked series. Flatter corrections imply stronger movement preservation from the original series. Table 4.10 contains such a plot. At the start and at the end of the series, the corrections converge to the estimated bias parameter (1,199,291 according to Table 4.8). That convergence takes place according to the ARMA model chosen for the error (a heteroscedastic auto-regressive model of order 1, with parameter 0.8 in the example).

The plot also exhibits the discrepancies between the benchmarks and the original series. For binding benchmarks, the surface under the corrections is equal to that under the discrepancies (and this also holds for Table 4.9 mutatis mutandis). Discrepancies equal to zero are not plotted.

As in Table 4.9, the corrections are displayed in pages, with common scale but different location. (For more explanation refer to previous sub-section.)

Table 4.10: Plot of the Corrections produced by BENCH

```
Plot of the corrections:  Retail2                  Retail

        Additive corrections: 1,2,3....: Discrepancies: b: Coincidence: =

      Units=10**( 3), idi=Retail2               idb=Retail
   1086.274  1128.978  1171.681  1214.385  1257.089  1299.792  1342.496  1385.199
84  1   |---------|---------|--1      |         |         |         |         |
84  2   |---------|---------|--2      |         |         |         |         |
84  3   |---------|---------|-3       |         |         |         |         |
84  4   |---------|---------|-4       |         |         |         |         |
84  5   |---------|---------|-5       |         |         |         |         |
84  6   |---------|---------|6        |         |         |         |         |
84  7   |---------|---------|7        |         |         |         |         |
84  8   |---------|--------|8         |         |         |         |         |
84  9   |---------|---------9         |         |         |         |         |
84 10   |---------|--------0|         |         |         |         |         |
84 11   |---------|--------N|         |         |         |         |         |
84 12   |---------|------D  |         |         |         |         |         |
85  1   |---------|------1  |         |         |         |         |         |
85  2   |------b--|-----2   |         |         |         |         |         |
85  3   |------b--|----3    |         |         |         |         |         |
85  4   |------b--|4        |         |         |         |         |         |
85  5   |------b--5         |         |         |         |         |         |
85  6   |------b6 |         |         |         |         |         |         |
85  7   |------=  |         |         |         |         |         |         |
85  8   |---8  b  |         |         |         |         |         |         |
85  9   |----9 b  |         |         |         |         |         |         |
85 10   |0     b  |         |         |         |         |         |         |
85 11   N      b  |         |         |         |         |         |         |
85 12   |---D  b  |         |         |         |         |         |         |
86  1   |------b1 |         |         |         |         |         |         |
86  2   |---------2   b     |         |         |         |         |         |
86  3   |---------|3  b     |         |         |         |         |         |
86  4   |---------|4  b     |         |         |         |         |         |
86  5   |---------|5  b     |         |         |         |         |         |
86  6   |---------|--6b     |         |         |         |         |         |
86  7   |---------|7  b     |         |         |         |         |         |
86  8   |---------|-8 b     |         |         |         |         |         |
86  9   |---------|---b9    |         |         |         |         |         |
86 10   |---------|---b0    |         |         |         |         |         |
86 11   |---------|---b--N  |         |         |         |         |         |
86 12   |---------|---b----D|         |         |         |         |         |
87  1   |---------|---b-----|-1       |         |         |         |         |
87  2   |---------|---------|----2    |         b         |         |         |
87  3   |---------|---------|------3  |         b         |         |         |
87  4   |---------|---------|---------|4        b         |         |         |
87  5   |---------|---------|---------|-------5 b         |         |         |
87  6   |---------|---------|---------|---------=         |         |         |
87  7   |---------|---------|---------|---------b---------|---------7         |
87  8   |---------|---------|---------|-----8   b         |         |         |
87  9   |---------|---------|---------|--------9b         |         |         |
87 10   |---------|---------|---------|--------0b         |         |         |
87 11   |---------|---------|---------|---------=         |         |         |
87 12   |---------|---------|---------|---------b-D       |         |         |
88  1   |---------|---------|---------|---------b--1      |         |         |

      Units=10**( 3), idi=Retail2               idb=Retail
   1214.385  1257.089  1299.792  1342.496  1385.199  1427.903  1470.606  1513.310
88  1   |---------|--b---1  |         |         |         |         |         |
88  2   |---------|-------2 |   b     |         |         |         |         |
88  3   |---------|--------3|   b     |         |         |         |         |
88  4   |---------|---------|---=     |         |         |         |         |
88  5   |---------|-----5   |   b     |         |         |         |         |
88  6   |---------|---------|---b--6  |         |         |         |         |
88  7   |---------|---------|---b-7   |         |         |         |         |
88  8   |---------|---------|-8 b     |         |         |         |         |
88  9   |---------|---------|-9 b     |         |         |         |         |
88 10   |---------|---------|---b0    |         |         |         |         |
88 11   |---------|---------|---bN    |         |         |         |         |
88 12   |---------|---------|---b----D|         |         |         |         |
89  1   |---------|---------|1  b     |         |         |         |         |
89  2   |---------|------2  |         |         |         |         |         |
89  3   |---------|3        |         |         |         |         |         |
89  4   |---------4         |         |         |         |         |         |
89  5   |------5  |         |         |         |         |         |         |
89  6   |------6  |         |         |         |         |         |         |
89  7   |-----7   |         |         |         |         |         |         |
89  8   |----8    |         |         |         |         |         |         |
89  9   |---9     |         |         |         |         |         |         |
89 10   |---------|---0b    |         |         |         |         |         |
89 11   |------b-N|         |         |         |         |         |         |
89 12   |---------D b       |         |         |         |         |         |
```

5. AUTOMATED DATA GENERATION

In order to minimize coding, programme BENCH provides a way to automatically generate input data. This is especially useful when the needed data consist of a seasonal pattern or a daily pattern to be repeated. The feature was used to generate the calendarization dates in section 4.6, without much explanation. The automatic generation cannot be used with the CANSIM and X-11-ARIMA formats.

The automatic generation occurs when a record in any one of the input files starts with " . . ."; the programme then inserts records between the previous record of the file and the next record. The insertion begins with the reference periods (starting year, month and day and ending year, month and day) of the previous record and ends with those of the next record. The insertion requires a certain number of parameters, which specify the *type* of dates desired, the *progression* of the dates, the *repetition* factor, and the *slope* of the series values; these parameters are entered by means of keywords "`typ=`", "`pro=`", "`rep=`" and "`slo=`" respectively, which obey the syntax established in section 4.2.

. The Type of Dates Desired: "typ="

Keyword "`typ=`" specifies the type of reference dates desired for the data to be generated. Keyword and entry "`typ=m,`" specifies the reference dates to be monthly; "`typ=q,`", quarterly; and "`typ=d,`", daily. The default entry is "`m`". The type of dates may be easier to understand as that to which the progression of the dates applies.

. The Progression of the Dates: "pro="

Keyword "`pro=`" determines the progression of the dates. An entry of "`1`" specifies that the progression of the dates (of the type specified by keyword "`typ=`") is 1. For instance, if the type of dates is monthly and the progression is 1 ("`typ=m, pro=1,`"), the starting and ending months on the previous record are incremented by 1; if the progression is 3, the starting and ending months are incremented by 3; if the type of dates is daily and the progression is 7 ("`typ=d, pro=7,`"), the starting and ending days on the previous record are incremented by 7. In the example of Table 4.7, the type of date was monthly and the progression was 12 ("`typ=m, pro=12,`"); given that the starting and ending months on the previous record referred to the calendar year ("`1981  1 1981 12`"), the procedure inserted the reference dates of calendar years from 1982 to 1991. The default entry for the progression is "`1`".

. The Repetition Factor: "rep="

Keyword "`rep=`" sets the repetition factor, which determines how many of the preceding records are repeated. The example in Table 5.1 repeats a percent seasonal pattern and the coefficients of variation from 1981 to 1992 for series (with identifier) "`Series1`"; and a different seasonal pattern for series "`Series2`". Note that although the data generated are quarterly, their reference periods are expressed monthly. The default entry for the repetition factor is "`1`".

Note: When attempting to repeat 12 records (say), at least 12 records must precede the record bearing " . . .". If this is not the case, the programme prints an error message and aborts.

Table 5.1: Repeating a Quarterly Seasonal Pattern

```
Series1  81   1   1  81   3  31    105  0.7
Series1  81   4   1  81   6  30    110  0.8
Series1  81   7   1  81   9  30     95  0.7
Series1  81  10   1  81  12  31     90  0.8
... typ=m, pro=3, rep=4,
Series1  92  10   1  92  12  31     90  0.8
Series2  81   1   1  81   3  31    103  0.6
Series2  81   4   1  81   6  30    107  0.5
Series2  81   7   1  81   9  30     93  0.5
Series2  81  10   1  81  12  31     98  0.6
... typ=m, pro=3, rep=4,
Series2  92  10   1  92  12  31     98  0.6
```

Note: The automatic data generation may be used more than once within a given series. For instance, one could generate one seasonal pattern from 1981 to 1987, then a (presumably slightly) different seasonal pattern from 1988 to 1992.

. **The Slope: "slo="**

Keyword "slo=" sets the slope of the series values to be generated, that is the change from one time period to the next. In the example of Table 5.2, the data generated, 101, 102, 103, ..., 124, from January 1981 to December 1982 have a slope of 1.0 (and dispersion 0.9); and the data generated, 125.5, 127.0, 128.5, ..., from January 1983 to December 1992 have a slope of 1.5 and dispersion 0.8. (The programme will put the right values in December 1992.) The default entry for keyword "slo=" is "0".

Table 5.2: Generating Values Evolving Linearly

```
Series2  81   1   1  81   1  31    101.0  0.9
... typ=m, slo=1.
Series2  82  12   1  82  12  31    124.0  0.9
Series2  83   1   1  83   1  31    125.5  0.8
... typ=m, pro=1, slo=1.5,
Series2  92  12   1  92  12  31      0    0.0
```

Note: It is possible to combine the repeat factor and the slope to generate data with linear trend and additive seasonality.

Table 5.3: Generating Monthly Data Pertaining to One Day in the Month

```
SeriesId  81   1  15  81   1  15     500  0.7
... typ=m, pro=1, slo=50,
SeriesId  92  12  15  92  12  15     500  0.7
```

Table 5.3 illustrates the generation of monthly data which pertain to the 15th of each month. However if that day is greater or equal than 28, it will be set to the last day of the month. (In combination with "typ=m", the programme resets a starting or an ending day value greater or equal to 28 to the last day in the month.)

Table 5.4 illustrate the generation of weekly data. The second point will cover from January 10 1981 to January 16 1981; the programme will put the right dates on the last record generated. With "pro=28," the data would cover 4 weeks instead of 1.

Table 5.4: Generating Weekly Data

```
SeriesId  81   1   3  81   1   9     100
... typ=d, pro=7,
SeriesId  92  12  15  92  12  21     100
```

Table 5.5: Generating a Daily Pattern

```
DailyAS      88   10   31              1.400
DailyAS      88   11    1              1.400
DailyAS      88   11    2              1.400
DailyAS      88   11    3              1.400
DailyAS      88   11    4              1.400
DailyAS      88   11    5              0.001
DailyAS      88   11    6              0.001
... typ=d, pro=1, rep=7;
DailyAS      89   10    1              0.001
```

Table 5.5 illustrates the generation of daily data, by repeating a daily pattern over the range selected.

The test data on the diskettes provide more examples of data generation. Table 5.6 summarizes the options related to data generation.

Table 5.6: Summary of the Data Generating Options in Programme BENCH, with the Default Values

Keyword	Entry	Description	Default values
"typ="	"m"	Type of reference dates: monthly	"typ=m,"
	"q"	Type of reference dates: quarterly	
	"d"	Type of reference dates: daily	
"pro="	numeric	Progression of the dates	"pro=1,"
"rep="	numeric	Repetition Factor	"rep=1,"
"slo="	numeric	Slope of series values	"slo=0.0,"

Table 6.1: Summary of the CANSIM and X-11-ARIMA formats and keywords

Keyword	Entry	Description	Fortran statement
"(cansimm)"	none	CANSIM format and monthly data	(a8,6i2,12e16.10,18x)
"(cansimq)"	none	CANSIM format and quarterly data	"
"(cansima)"	none	CANSIM format and annual data	"
"(x11m0)"	none	X-11-ARIMA monthly format "0"	(12f6.0,i2,a6)
"(x11m2)"	none	X-11-ARIMA monthly format "2"	(6f12.0,/,6f12.0,i2,a6)*
"(x11m3)"	none	X-11-ARIMA monthly format "3"	(a6,i2,12f6.0)
"(x11m4)"	none	CANSIM format and monthly data	
"(x11m5)"	none	X-11-ARIMA monthly format "5"	(a6,i2,6f12.0,/,8x,6f12.0)*
"(x11q0)"	none	X-11-ARIMA quarterly format "0"	(4(12x,f6.0),i2,a6)
"(x11q2)"	none	X-11-ARIMA quarterly format "2"	(4f12.0,24x,i2,a6)
"(x11q3)"	none	X-11-ARIMA quarterly format "3"	(a6,i2,4(12x,f6.0))
"(x11q4)"	none	CANSIM format and quarterly data	
"(x11q5)"	none	X-11-ARIMA quarterly format "5"	(a6,i2,4f12.0)

			Default value
"ref="	"ymm"	Reference periods in terms of year, month and month	"ref=ymm,"
	"ymd"	Reference periods in terms of year, month and day	
	"yqq"	Reference periods in terms of year, quarter and quarter	
"lag="	numeric	Lag or lead (-) of the data in number of months or quarters (for a lead a negative number is entered)	"lag=0,"
"sto="	"0"	Not a stock series	"sto=0,"
	"1"	Stock series	
"dis="	numeric	Dispersion value to be assigned to the input series	"dis=1,"
"dec="	numeric	Number of decimal	"dec=1,"

* These formats imply two records per year - even for incomplete years; and consequently, an even number of records in the file

6. CANSIM AND X-11-ARIMA INTERFACE

As mentioned earlier, programme BENCH has a few built-in formats to interface with Statistics Canada's CANSIM databank and the X-11-ARIMA seasonal adjustment programme. The use of these formats entails some loss in the flexibility of the programme, because they only allow monthly, quarterly and annual data, and for other reasons to be mentioned.

- The CANSIM and X-11-ARIMA Formats Keywords

As explained in section 4.1, each record of file "bench.fil" provides a file name and a format statement. If the file name is followed by one of the first 13 keywords displayed in Table 6.1, the file is deemed to be in a CANSIM or X-11-ARIMA format. The keywords starting with "(cansim" specify the CANSIM format; and the keywords starting with "(x11", one of the X-11-ARIMA formats.

In the case of the CANSIM format, the ending of the keyword (starting with "(cansim") specifies the periodicity of the data: ending "m)" specifies monthly data; ending "q)", quarterly data ; and ending "a)", annual data. With the CANSIM formats, the series identifier can only have 8 (left-most) characters; and each record contains 12 series values. (For readers who need to know, the CANSIM Fortran format statement is (a8,6i2,12e16.10,18x). The item "6i2" is used to read the year, month and day of the first value on the record and the year, month and day of the last value.)

In the case of the X-11 (-ARIMA) formats, the letter following the first part of the keyword "(x11" specifies the periodicity of the data: letter "m" specifies monthly data; and "q" quarterly data. The number following "m" or "q", specifies which of the 5 monthly or quarterly X-11 formats displayed in Table 6.1 is used. With the X-11 formats, the series identifier can only have 6 (left-most) characters; and each record contains 12, 6 or 4 series values (depending on the exact format).

Note: The use of the X11 monthly or quarterly formats implies that the data are monthly or quarterly respectively; the use of the CANSIM format implies monthly, quarterly or annual data. This holds in both input and output. For instance, attempts to output monthly data in a quarterly format will result in errors in the output files, which will be unmonitored by the programme.

Note: One can use uppercase or lowercase characters, or both, for the keywords and their entries.

- Related Options

For CANSIM and X-11-ARIMA each series values technically refers to one time period, whereas in programme BENCH each value generally covers a number of periods beginning with the starting year, month and day and ending with the ending year, month and day (although the starting and ending periods may coincide). The CANSIM and X-11 records do not carry dispersion values (e.g. coefficients of variation). In order to circumvent some of these limitations, the format keywords may be followed by option keywords.

. The Reference Period Options: "ref=", "lag=", "sto=",

Keywords "ref=", "lag=" and "sto=" assign more descriptive reference periods to the *input* data.

Keyword "ref=" determines the kind of the reference periods assigned: entry of "ymd" (i.e. "ref=ymd,") assigns reference periods in terms of year, month and day; entry of "ymm", in terms of year, month, month; and "yqq", in terms of year, quarter, quarter. With "typ=ymd," (and with "lag=0, sto=0,") a monthly value for November 1981 (say) would receive starting year 1981, starting month 11, starting day 1, ending year 1981, ending month 11 and ending day 1; and, 1981, 11, 11, 1981, 11, 11, with "typ=ymm,". With "typ=ymd,", a quarterly value for the four quarters of 1981, would receive starting year 1981, starting month 10, starting day 1, ending year 1981, ending month 12 and ending day 31; and 1981, 4, 4, 1981, 4, 4 with "typ=yqq,".

Keyword "lag=" determines the number of months or quarters (depending on the entry on keyword "ref=") by which the input data should be lagged. The combination "ref=ymd, lag=1, sto=0," would cause quarterly data to be lagged by 1 month. Under that combination a value for the first quarter of 1992 (say) would receive reference periods: 1991 12 1 to 1992 2 29; under combination "ref=ymd, lag=-1, sto=0,", the same value would receive reference periods 1992 2 1 to 1992 4 30. Under combination "ref=yqq, lag=-1, sto=0" a value for year 1991 would receive reference periods 1991 2 2 to 1992 1 1 (from second quarter of 91 to first quarter of 92); under combination "ref=ymm, lag=-3, sto=0," the same value would receive 1991 4 4 to 1992 3 3 (from April 1991 to March 1992) .

Note: The lag option is relevant in the case of quarterly and annual data only. Furthermore quarterly data cannot be lagged if their reference periods were specified in terms of year, quarter, quarter ("ref=yqq,")

Note: For annual data the acceptable entries of the lag option range from -11 to 11 (months) with "`ref=ymm,`" and "`ref=ymd,`"; and from -3 to 3 (quarters) with "`ref=yqq,`". For quarterly data the acceptable range is -2 to 2 (months).

Keyword "sto=" determines whether the input series is a stock, i.e. whether it covers only one time period. Programme BENCH sets the starting years, months and days of stock series equal to the ending years, months and days. For instance the combination "`ref=ymd, lag=0, sto=1,`" would cause data for year 1992 to receive reference periods 1992 12 31 to 1992 12 31. The combination "`ref=ymd, lag=0, sto=1,`" would cause data for February 1992 to receive reference periods 1992 2 29 to 1992 2 29.

. The Dispersion Option: "`dis=`"

The dispersion option attributes to CANSIM and X-11 input series a dispersion value. The dispersion is the same for all observations in the series and for all series in the file. For instance, the combination "`dis=1.5,`" in the file of the original series, with "`cvs=0,`" in the option file, would assign a coefficient of variation of 1.5% to the original series. The default value is 1.0.

Note: The CANSIM and X-11 formats do not work properly on output, if the data refer to fiscal periods (e.g. fiscal quarters, aggregates of weekly data), or if the reference periods of the data are irregular (e.g. some points referring to years, other points referring to months), or if the data are quarterly with reference periods in terms of months (e.g. October to December, instead of 4th Quarter).

Note: The options just described apply on input only and are ignored on output. This implies for instance that output fiscal year or fiscal quarter values (using the appropriate CANSIM formats) would have the wrong year and wrong month identifier. The year and month identifiers given on output are the ending year and ending month.

. The Decimal Option: "`dec=`"

The decimal option sets a number of decimals to the X-11 input and output files. Keyword and entry "`dec=2,`" would cause the values to be read or written with 2 decimal places.

7. TUTORIAL: INSTALLING AND RUNNING THE PROGRAMMES

This section shows how to load programme BENCH onto the hard disk of a microcomputer operating under DOS. The section is also a tutorial through which the user applies the programme to sets of examples provided.

7.1 INSTALLING THE PROGRAMME

First make sure there is at least two megabytes of space on the hard disk. The *ASCII* files produced by BENCH may be quite large. The space occupied by the programme itself depends on the version.

- Installing BENCH

The following steps load programme BENCH on the hard disk, in a directory called "BENCHMAR":

On the hard disk, create directory "BENCHMAR", by entering the following DOS commands:

"`c:`", "`cd \`" and "`md benchmar`",

where only the characters between quotes are entered (by hitting key <Enter> on the keyboard).

Load the programme in directory "BENCHMAR" by entering

"`c:`", "`cd \benchmar`" and "`b:program`",

where "`b:`" stands for floppy disk drive B. (If you are using drive A, replace all "`b:`" by "`a:`" in this section.)

- Installing the Browsing Utility Programme LISTR
The utility programme LISTR is a public domain programme by V.D. Buerg (1990). LISTR allows one to quickly browse and examine very large *ASCII* files (up to 16 MB) of any width, to search them for strings of characters, to split them, to print part of them, etc. The following steps load LISTR onto the hard disk drive in a directory called "BROWSE":

On your hard disk, create directory BROWSE, by entering the following DOS commands:

"`c:`", "`cd \`", and "`md browse`".

Load the browsing utility in directory BROWSE, by entering

"`c:`", "`cd \browse`" and "`b:browse`".

You have now loaded LISTR and the supporting documentation and other accessory files and programmes which came with LISTR. Although only the file called "LISTR.COM" is used in this tutorial, you may want to examine the other programmes as well; if this is not the case, you may eventually delete the other files.

- Making the Programmes Accessible
The following steps make the programmes BENCH and LISTR available from any other directory, by creating "batch files". (Another result will be to make it unnecessary to supply the paths of the files, if the files are all located in the directory in which the programmes are executed.)

On your hard disk, go to the DOS directory (or preferably to an existing pathed directory containing batch files accessible under any configuration), by entering:

"`c:`" and "`cd \dos`".

Load the batch files in the DOS directory by entering:

"`c:`" and "`b:batches`".

7.2 INSTALLING AND RUNNING THE EXAMPLES
The programmes are now loaded and accessible; you are now ready to load and run the examples.

- Installing the Examples
The following steps load the example files on to the hard disk drive C into a personal sub-directory:

On your hard disk, go to a personal (as opposed to a programme) directory, by entering:

"`c:`" and "`cd \`*`Person`*`\`*`DirName`*",

where "*`Person`*`\`*`DirName`*" is an existing directory (e.g. "`userA\project1`")

Load the examples, by entering:

"`c:`" and "`b:examples`" .

The examples are now loaded; you are now ready to run them, in order to test programme BENCH and to learn how to use it.

- First Exercise: Benchmarking the Canadian Retail Trade Series
The first exercise consists of running the Canadian Retail Trade series used as example in many of the tables in the above sections.

Examine file "retail.fil", by entering "listr retail.fil". The content of Table 4.1 should appear on the screen. The file contains the names and the formats of the input and output file required by programme BENCH: File "retail.opt" is the (name of the) option file; file "retail.ori" is the file of the original series; "retail.ben", the file of the benchmarks; "retail.out", the file of benchmarked series; "retail.cai", the file of the calendarization dates; and file "retail.cao", the file of the calendarized values.

Make the files named in file "retail.fil" available to programme BENCH by entering:

"copy retail.fil bench.fil".

Do not rename "retail.fil" as this would eventually lose the file.

Examine the file of the original series by entering "listr retail.ori". The content of Table 4.3 should appear on the screen. The right-most column contains the coefficients of variation of each original value; the original values are in the second right-most column. Browse through the file by entering "<Pg Dn>" (page down), "<Pg Up>" (page up), "<→>" (right arrow), "<←>" (left arrow), "<↓>" (down arrow), "<↑>" (up arrow), "<End>" (end of document), "<Home>" (start of document), etc. (Key "<F1>" triggers on-line help; key "<Enter>" terminates the help.) Enter "<Esc>" to exit the utility programme.

Examine the file of the benchmarks by entering "listr retail.ben". The content of Table 4.5 should appear on the screen. The right-most column contains the coefficients of variation of each benchmark; the benchmarks are located in the second right-most column. Notice that the first four benchmarks refer to fiscal years ranging from February to January of the following year; and that the last three benchmarks are monthly benchmarks referring to only one month. Enter "<Esc>" to exit LISTR.

Examine the option file by entering "listr retail.opt". The content of Table 4.2 should appear on the screen. The entry following "tit=" determines the title which will appear on the log output.; "ysf=1984, msf=1," means that only the original values starting from January 1984 will be used. Keyword "met=0," requests the additive benchmarking model. Keywords "cvs=0, cvb=0, cvi=0," specify the measure of dispersion of the original series, of the benchmarks and of the benchmarked series to be the coefficient of variation. Keyword "bia=1," means a bias parameter will be estimated; "int=0,", that the estimation will not be done in on moving intervals but on the whole series. The options not specified will be set equal to their internal default values displayed in Table 4.3. Enter "<Esc>" to exit LISTR.

Benchmark the Retail Trade series by entering:

"bench".

Programme BENCH then reads the input files and creates the output files referred to in file "bench.fil". The programme also produces the log output file "bench.log" containing the statistics and plots.

Examine the file of the benchmarked series by entering "listr retail.out". The content of Table 4.6 should appear on the screen. The right-most column contains the coefficients of variation of each benchmarked values; these are located in the second right-most column. Enter "<Esc>" to exit LISTR.

Examine the log output file by entering:

"listr bench.log".

The content of Table 4.8 should appear on the screen, namely the title page of programme BENCH, the names and formats of the files processed, the options used, the statistics and plots, as described in section 4.8. Find the plot of the series by entering "f plot" (find string "plot"). The content of Table 4.9 should appear on the screen. Display 43 lines on the screen (instead of 25) by entering "<Alt> e"; this mode is more appropriate to examine the plots. Note that the benchmarked series leads to the monthly benchmarks at the end of the series. Enter "<Esc>" to exit LISTR.

You may want to preserve file "bench.log" ; this may be achieved by entering "copy bench.log retail.log" (for instance).

You may want to examine the calendarized values in file "retail.cao", i.e. the calendar year sums of the benchmarked series. They were written in the CANSIM format.

Part of any of the files may be printed within programme LISTR by using the "<Alt> m", "<Alt> b" to mark the start and the end of an area; and "<Alt> p" to print the marked area.

- Second Exercise: Benchmarking a System of Series

The second exercise consists of benchmarking the system of Canadian Wholesale Trade series, which comprises nine "Trade Group" Totals and eleven Province Totals and the grand Canada Total. The series in the system are benchmarked individually and not as a whole. As a result, the sum of the nine benchmarked Trade Group values for a given month may differ slightly from the benchmarked grand Canada Total value for that month; and similarly, for the eleven Province Totals. (This problem also arises in seasonal adjustment. Future versions of this programme may provide options for benchmarking a system of series as whole in order to maintain additivity.)

Examine file "whole.fil", by entering "`listr whole.fil`". The file contains the names and formats of the input and output file required by programme BENCH. File "whole.opt" is the (name of the) option file; file "whole.ori" is the file of the original series; "whole.ben", the file of the benchmarks; "whole.out", the file of benchmarked series; "whole.cai", the file of the calendarization dates; and file "whole.cao", the file of the calendarized values.

Make the files named in file "whole.fil" available to programme BENCH by entering

"`copy whole.fil bench.fil`".

Examine the file of the original series by entering "`listr whole.ori`". Notice that the series identifiers (e.g. "`   0     0`") have blanks; the first digit indicates the trade group and the other digit(s) the province. According to the format of the file, `(A10,2I5,T16,I5,T11,2I5,T16,I5,F20.0,F2.0)`, the dispersion values, coefficients of variation in this case, will be read in the empty field following the original values in the right-most column. This will result in a default dispersion value of to 1.0%.

Examine the option file by entering "`listr whole.opt`". Notice that the user-supplied default options at the start of the file, namely mixed benchmarking ("`met=1,`") and no plots ("`pls=0, plc=0,`"). These user-supplied default values will be used for the 21 series, except for 3 series. Canada Total with identifier "`   0    0`" (see "`ids=    0    0`") will be plotted ("`pls=1, plc=1,`"), Prince Edward Island and Yukon and North-West Territories with identifier "`   0 11`" and "`   0 60`" will use multiplicative benchmarking ("`met=2,`"). (If the search feature were used in the default set of options at the beginning (by having "sea=1," instead of "sea=0,") then only the three series with specific options would be processed.)

Examine the file of the calendarization dates by entering "listr whole.cai". Notice the data generation commands "`... typ=m, pro=3,`"; the dates generated will pertain to calendar quarters. Thus calendar quarter values will be produced as a by-product of benchmarking.

Benchmark the system of series, by entering

"`bench`".

Programme BENCH then reads the input files and creates the output files referred to in file "bench.fil". The programme also produces the log output file "bench.log" containing the statistics and plots.

Examine the file of the benchmarked series by entering "`listr whole.out`". The right-most column contains the coefficients of variation of each benchmarked value located in the second rightmost column.

Examine the log output file by entering:

"listr bench.log".

Notice the user-supplied default options, by entering "f user-sup". Find the first pair of original series and benchmarks processed, by entering "f problem"; string "Problem 1" should appear at the top of the screen. The first pair of series pertains to Canada Total. Examine the statistics for Canada Total (by entering "<↓>"), namely the estimated bias, the annual discrepancies before and after benchmarking. Enter "<Alt> e", to get more lines on the screen, and examine the plot for Canada Total. Find the plot of the proportional corrections, by entering "<Home>", "f corrections". Notice that the corrections are smooth; this is due to the constant coefficients of variation.

Find the second pair of series processed, by entering "f problem"; string "Problem 2" should appear at the top of the screen. The second pair of series pertains to the province of Newfoundland for which the user-supplied default options were used (see "WARNING" message).

Find the third pair of series, by entering "f problem" or "<F3>" (find again). The third pair pertains to the province of Prince Edward Island. Notice that the multiplicative model was used instead of the mixed, because this province had specific options in the option file.

Find the fourth pair of series processed, by entering "f problem". String "Problem 4" should appear at the top of the screen. The fourth pair pertains to the province of Nova Scotia; notice that the programme has now reverted to the (user-supplied) default options (see "WARNING" message). Find the fifth pair of series by entering "<F3>" (find again). Find the sixth and the seventh pairs, by entering "<F3>" and "<F3>". Enter "<Esc>" to exit browsing utility LISTR. You may want to preserve file "bench.log", by entering "copy bench.log whole.log" (for instance).

- Third Exercise: Transforming Aggregate of Weekly Data into Monthly Values

The third set of examples consists of six cases of conversion of data covering 4 and 5 weeks into monthly values. The calendarization will be carried out in two different ways, directly and indirectly. In the direct approach, programme BENCH benchmarks a daily pattern to the weekly aggregates (covering 4 or 5 weeks), which produces daily interpolations; these interpolations are then converted into monthly values (by taking the monthly sums). The indirect calendarization consists of two parts, first a weekly pattern is benchmarked to the weekly aggregates, which produces weekly interpolations; second, the daily pattern is benchmarked to the weekly interpolations, which produces daily interpolations, then these daily interpolations are converted into monthly values.

. Direct Calendarization

The following steps perform the direct calendarization of the weekly aggregates.

Examine file "weekly.fil", which contain the names of the files involved in the exercise, by entering "listr weekly.fil". Make those files available to programme BENCH, by entering

"copy weekly.fil bench.fil"

Examine the daily pattern to be benchmarked by entering, "listr daily.pat", by entering "listr daily.pat". Notice the automatic data generation commands "... typ=d, pro=1, rep=7;" designed to repeat the daily pattern.

Examine the benchmarks, which cover 4 or 5 weeks, by entering "listdr weekly.agg". Notice that the starting years, months and days and the ending years, months and days are essential to exactly describe the reference periods of each benchmark.

Examine the options, by entering "listdr weekly.opt". The estimation intervals will cover 3 benchmarks ("int=3,"), and there will be links between the intervals ("lin=1,"). The multiplicative model will be used ("met=2,").

Examine the calendarization dates, by entering, "listr month.ref". Notice the automatic data generation commands "... typ=m, pro=1,", designed to generate the reference periods of months.

Interpolate the daily values from the weekly aggregates, by entering

"bench".

Examine the daily interpolations, by entering "listdr daily.int". The coefficients of variation of the interpolations in the last column were set to zero, because the link option was used (see section 4.2).

Examine the monthly values by entering "listdr month.val". These values were written in X-11-ARIMA monthly format "5". The asterisks mean missing values.

Examine the log output file by entering:

"listr bench.log".

Notice the user-supplied default options, by entering "f user-". Find the first pair of series processed, by entering "f problem"; string "Problem 1" should then appear at the top of the screen. The first pair of series pertains to respondent A with identifier "RESP_A". Notice the reference periods of each estimation interval displayed under "|-- Estimation interval --|". Examine the plot of the daily interpolations, by entering "<Alt> e" and "<Pg Dn>" or "<↓>"; the interpolations are smooth and behave proportionally to the daily pattern. Find the second pair of series, by entering "<Alt> e" and "f problem". Find the next pair of series by entering "<F3>" (find again). Enter "<Esc>" to exit LISTR.

. Indirect Calendarization

The following steps perform the indirect calendarization of the weekly aggregates, in two parts.

Examine file "weeklyw1.fil", which contains the names of the files involved in the first part of the indirect calendarization, by entering "listr weeklyw1.fil". No calendarization will be done in this part because the file contains no file name for the calendarization dates (i.e. contains only 4 instead of 6 records). Make those files available to programme BENCH, by entering

"copy weeklyw1.fil bench.fil".

Examine the weekly pattern to be benchmarked by entering "listr weekly.pat". Notice the automatic data generation commands "... TYP=D, PRO=7,". Coding these by hand would be extremely laborious and subject to error.

Benchmark the weekly pattern in file "weekly.pat" to the weekly aggregate in file "weekly.agg", by entering

"bench".

Examine the log output file by entering:

"listr bench.log".

Notice that the estimation interval is 5 ("int=5") instead of 3 in the direct calendarization. Notice the smoothness of the weekly interpolations, by entering "<Alt> e" and "<Pg Dn>" or "<↓>";

Examine file "weeklyw2.fil", which contains the names of the files involved in the second part of the indirect calendarization, by entering "listr weeklyw2.fil". Make those files available to programme BENCH, by entering

"copy weeklyw2.fil bench.fil"

Benchmark the daily pattern in file "daily.pat" to the weekly interpolations in file "weekly.int", by entering "bench".

Examine the log output file by entering:

"listr bench.log".

Notice the smoothness of the daily interpolations, by entering "<Alt> e" and "<Pg Dn>" or "<↓>".

Examine the indirectly obtained calendarized values, by entering "listr month2.val".

The indirect calendarization offers the opportunity to specify a weekly pattern analogous to a 12-month seasonal pattern. For some retail trade series for instance, that 52-week weekly pattern could have its peak in the weeks preceding Christmas and its trough in January. This should improve the calendarization. (In the examples supplied, the weekly pattern consisted merely of values equal to 100.)

- Fourth Exercise: Calendarizing Fiscal Year Data

The fourth exercise consists of calendarizing fiscal year data on capital expenditures, covering from the second quarter of one year to the first quarter of the following year. These fiscal year data will be calendarized in three different manners (in the same job). First a quarterly original series equal to 1000 will be benchmarked to the fiscal year data; second, a quarterly original series equal to a straight line will be benchmarked to the same fiscal year data; and finally a quarterly seasonal pattern will be benchmarked; in each case, the calendarized yearly values will be the calendar year sums of the quarterly interpolations obtained by benchmarking.

Examine file "capital.fil", which contains the names of the files involved in the exercise, by entering "listr capital.fil". Make those files available to programme BENCH, by entering

"copy capital.fil bench.fil"

Examine the options, by entering "listr capital.opt". User-supplied default options are specified by "ids=default, met=0, int=5, bia=1, p-r=1, 0.95, pls=1, plc=0, sea=1,". The search option used ("sea-=1,") will cause the benchmarks with identifier "FisYearVal" to be processed three times, the first time with original series with identifier "QuartInt1", the second time with original series "QuartInt2", and finally with original series "QuartInt3".

Examine the file of the original series by entering "listr capital.ori". Notice the three original series with identifiers "QuartInt1", "QuartInt2" and "QuartInt3" and the data generation commands (e.g. "... typ=q,").

Examine the file of the benchmarks by entering "listr capital.ben". Notice their fiscal reference periods.

Calendarize the fiscal year data by entering

"bench".

Examine the plots of the quarterly interpolations in the log output file, by entering

"listr bench.log",

and "f plot" (find plot), "<Alt> e" (43 lines on screen), "<↓>", examine the 1992-1993 values of the first interpolations, do the same for the second interpolations and third interpolations, by entering "<F3>" (find again) "<↓>", etc. Exit LISTR, by entering <Esc>.

Examine the calendarized values by entering "listr capital.cao".

- Other Exercises

The programme diskette provides a few more examples, namely a case of ARIMA interpolation (files starting with "`arimaint.`"), a case of combining monthly ARIMA forecasts of unemployment with an annual econometric forecast (files starting with "`unemploy.`"), cases of interpolation and benchmarking annual and monthly demographic values to quinquennial benchmarks (files starting with "demog"), cases of calendarization of banking data covering from November to October of the following year (files starting with "`banks`")

In order to run those examples, copy the appropriate file ending with "`.fil`" to "`bench.fil`" and then execute programme BENCH.

REFERENCES

Bassie, V.L (1958), **Economic Forecasting**, Appendix A, "Interpolation Formulas for the Adjustment of Index Numbers", McGraw-Hill

Bell, W., Wilcox, D. (1990), "The Effect of Sampling Error on the Time Series Behavior of Consumption Data", Research Paper, Bureau of the Census, Washington, D.C. 20233, U.S.A., Revised April 6, 1990.

Boot, J.C.G., Feibes, W., Lisman, J.H.C. (1967), "Further Methods of Derivation of Quarterly Figures from Annual Data", **Applied Statistics**, Vol. 16, no. 1, pp. 65-75.

Bournay, J., Laroque, G. (1979), Réflexions sur la méthode d'élaboration des comptes trimestriels", Annales de l'I.N.S.E., Vol. 36, pp. 3-30.

Box, G.E.P., Jenkins, G.M. (1970, 1976), Time Series Analysis, Forecasting and Control, Holden-Day.

Cholette, P.A. (1982), "Prior Information and ARIMA Forecasting", **Journal of Forecasting**, Vol. 1, pp. 375-383.

Cholette, P.A. (1985), "Estimating Sub-Annual Series from Growth Rates and Annual Benchmarks", Time Series Research and Analysis, **Statistics Canada**, Research Paper TSRA-85-018B.

Cholette, P.A., Higginson, J. (1987), "A Note on Aggregating Data Referring to Different Financial Years", **Statistics Canada**, Time Series Research and Analysis Division, Research Paper No. TSRA-87-015E.

Cholette, P.A., Baldwin, A. (1988), "Converting Fiscal Year Data into Calendar Year Values", Time Series Research and Analysis, **Statistics Canada**, Ref No. TSRA-88-012E.

Cholette, P.A. (1988), "Weights to Calendarize Fiscal Year Data Referring to any Consecutive 12 Months or 4 Quarters", **Statistics Canada**, Time Series Research and Analysis Division, Research Paper No. TSRA-88-023E.

Cholette, P.A., Chhab, N. B. (1988; 1991), "Converting Aggregates of Weekly Data into Monthly Values", **Applied Statistics** (1991), Vol. 40, No. 3, pp. 411-422.

Cholette, P.A., (1989, 1990), "Transforming Fiscal Quarter Data into Calendar Quarter Values", **Statistics Canada**, Time Series Research and Analysis Division, Working Paper No. TSRA-89-017EF; in Sing, A.C and Withridge, P. eds. **Analysis of Data in Time, Proceedings of the 1989 International Symposium**, Statistics Canada, 1990

Cholette, P.A, Dagum, E. Bee (1989, 1994), "Benchmarking Socio-Economic Time Series Data: A Unified Approach", **Statistics Canada**, Time Series Research and Analysis Division, Working Paper No. 89-006E; forthcomming in **The International Statistical Review**.

Cholette, P.A. (1990), «L'annualisation des chiffres d'exercices financiers», **L'Actualité économique**, Vol. 66, pp. 218-230; **Statistique Canada**, Division de la recherche et de l'analyse des chroniques, Document de recherche no. TSRA-89-009F;

Cholette, P.A. (1990), "Linking and Benchmarking the Redesigned Retail and WholeSale Trade Series, Time Series Research and Analysis, **Statistics Canada**, Ref No. TRSRA-90-004E.

Cholette, P.A. and Dagum, E. Bee (1991), "Benchmarking Time Series with Autocorrelated Sampling Errors", Time Series Research and Analysis, **Statistics Canada**, Ref No. TRSRA-91-005E.

Chow, G.C., Lin, An-loh (1971), "Best linear Unbiased Interpolation, Distribution and Extrapolation of Time Series by Related Series", **Review of Economics and Statistics**, Vol. 53, No. 4, pp. 372-375.

Cohen, K.J., Müller, W., Padberg, M.W. (1971) "Autoregressive Approaches to the Disaggregation of Time Series Data", **Applied Statistics**, Vol. 20, pp 119-129.

Dagum, E. Bee (1980), The X-11-ARIMA Seasonal Adjustment Programme, **Statistics Canada**, Cat. 12-564E; La Méthode de désaisonnalisation X-11-ARIMA, **Statistique Canada**, Cat. 12-564F.

Dagum, E. Bee (1988), The X-11-ARIMA/88 Seasonal Adjustment Method, **Statistics Canada**, Time Series Research and Analysis Division, Ottawa, Canada K1A 0T6.

Denton, F.T. (1971), "Adjustment on Monthly or Quarterly Series to Annual Totals: An Approach Based on Quadratic Minimization", **Journal of the American Statistical Association**, Vol. 66, No. 333, pp. 99-102.

Guerrero, V. M. (1989) "Optimal Conditional ARIMA Forecast", **Journal of Forecasting**, Vol. 8, pp. 215-229.

Guerrero, V. M. (1991), "ARIMA Forecasts with Restrictions derived from a Structural Change", **International Journal of Forecasting**, Vol. 7, pp. 339-347.

Harvey, A.C. (1981), **The Econometric Analysis of Time Series**, Philip Allan, Oxford.

Helfand, S.D., Monsour, J.J., Trager, M.L. (1977), "Historical Revision of Current Business Survey Estimates", **Proceedings of the Business and Economic Statistics Section, American Statistical Association**, pp. 246-250.

Hillmer, S.C., Trabelsi, A. (1987), "Benchmarking of Economic Time Series", **Journal of the American Statistical Association**, Vol. 82, pp. 1064-1071.

Laniel, N., Fyfe, K. (1990), "Benchmarking of Economic Time Series", **Survey Methodology**, Vol. 16, pp. 271-277.

Lisman, J.H.C. and Sandee, J. (1964),"Derivation of Quarterly Figures From Annual Data", **Applied Statistics**, Vol. 13, pp.87-90.

McLeod, I., (1975), "Derivation of the Theoretical Autocovariance Function of Autoregressive-Moving Average Time Series", *Applied Statistics*, Vol. 24, No. 2, 255-256.

Mian, I.U.H., Laniel, N. (1991), "Maximum Likelihood Estimation for the Constant Bias Benchmarking Model", **Statistics Canada**, Social Survey Method Division, September 12, 1991.

MicroSoft (1987), **MicroSoft Fortran 4.1, Optimizing Compiler**, MicroSoft Corporation, 16011, 36th Way, Box 97017, Richmond, USA, WA 98073-9717.

Pankratz, A. (1989), "Time Series Forecasts and Extra-Model Information", **Journal of Forecasting**, Vol. 8, pp. 75-83.

SAS Institute Inc. (1984) **SAS/ETS User's Guide, Version 5 Edition**, Cary, NC: SAS Institute, Inc.

SAS Institute Inc. (1985) **SAS User's Guide: Basics, Version 5 Edition**, Cary, NC: SAS Institute, Inc.

SAS Institute Inc. (1985b) **SAS/IML User's Guide, Version 5 Edition**, Cary, NC: SAS Institute, Inc.

Smith, P. (1977), "Alternative Methods for Step-Adjustment", **Statistics Canada**, Econometrics Section, Current Economic Analysis Division.

Trabelsi, A., Hillmer, S.C. (1989), " A Benchmarking Approach to Forecast Combination", **Journal of Business and Economic Statistics**, Vol. 7, No. 3, pp. 353-362.

Trabelsi, A., Hillmer, S.C. (1990), "Benchmarking Time Series with Reliable Benchmarks", **Applied Statistics**, Vol. 39, No. 3, pp. 367-379.

Appendix A: Estimation of the Additive Benchmarking Model

This appendix presents the solution of the additive benchmarking model. The solution found for the additive model is to a large extent applicable to the other models.

The two equations (1a) and (1b) of section 3 are written in matrix algebra

$$s \;=\; \iota\, a \;+\; I_T\, \theta \;+\; e,\;\; \mathscr{E}(e) = 0,\; \mathscr{E}(e\, e') = V_e, \tag{A.1}$$

$$y \;=\; J\, \theta \;+\; w,\;\; \mathscr{E}(w) = 0,\; \mathscr{E}(w\, w') = V_w, \tag{A.2}$$

where I_T is a T by T identity matrix , ι is a T by 1 vector of ones, J is a M by T design matrix of zeroes and ones:

$$\underset{M\times T}{J} \;=\; \left[\, j_{m,t}/p_m,\;\; j_{m,t} = \begin{cases} 1, & t\in m, \\ 0, & t\notin m, \end{cases} \right], \tag{A.3}$$

where p_m is equal to 1 for flow and stock series and equal to the number of periods covered by benchmark y_m for index series.

Equations (A.1) and (A.2) can be written as a single equation

$$\begin{bmatrix} s \\ y \end{bmatrix} = \begin{bmatrix} \iota & I_T \\ 0 & J \end{bmatrix} \begin{bmatrix} a \\ \theta \end{bmatrix} + \begin{bmatrix} e \\ w \end{bmatrix}$$

or

$$Y \;=\; X\,\beta + u,\; \mathscr{E}(u) = 0,\; \mathscr{E}(u\, u') = V = \begin{bmatrix} V_e & 0 \\ 0 & V_w \end{bmatrix}, \tag{A.4}$$

where Y, X, β and u are defined implicitly. The General Least Squares solution of model (A.4) is

$$\hat{\beta} = \begin{bmatrix} \hat{a} \\ \hat{\theta} \end{bmatrix} = (X'V^{-1}X)^{-1}\, X'V^{-1}Y,\;\; cov(\hat{\beta}) = (X'V^{-1}X)^{-1} = \begin{bmatrix} v_a & v_{a\theta} \\ v_{\theta a} & V_\theta \end{bmatrix}. \tag{A.5}$$

Defining $Q = (J\, V_e J' + V_w)^{-1}$, Cholette and Dagum (1991) show that

$$v_a \;=\; 1\,/\,(\iota' J' Q\, J\, \iota), \tag{A.6}$$

$$v_{\theta a} = v_{a\theta}' \;=\; -(I_T - V_e J' Q\, J)\,\iota\, v_a, \tag{A.7}$$

$$V_\theta \;=\; (V_e - V_e J' Q\;\; J V_e) + (I_T - V_e J' Q\, J)\,\iota\, v_a\, \iota' (I_T - V_e J' Q\, J)', \tag{A.8}$$

$$\hat{a} \;=\; -\, v_a\, \iota' J'\, Q\, (y - J s), \tag{A.9}$$

$$\hat{\theta} \;=\; s^* + V_e J'\, Q\, (y - J s^*),\;\; s^* = s - \iota\hat{a}, \tag{A.10}$$

where s^* contains the sub-annual observations corrected for the estimated bias. The calculation of matrix Q requires the inversion of a M by M matrix (M«T); this inversion is the only one needed. The only matrices actually stored are $(V_e J')$, Q and $(J\,\iota)$.

When the bias parameter is dropped from the model, equations (A.8) to (A.10) remain valid, if the terms in v_a and $\hat{a}$ are ignored.

Appendix B: Estimation of the Multiplicative Benchmarking Model

This appendix presents the solution of the multiplicative benchmarking model. The two equations (2a) and (2b) of section 3 are written in matrix algebra as

$$ln\, s \;=\; \iota\, ln\, a \;+\; I_T\, ln\, \theta \;+\; ln\, e,\;\; \mathscr{E}(ln\, e) = 0,\; \mathscr{E}(ln\, e\;\, ln\, e') = V_e, \tag{B.1}$$

$$y \;=\; J\,\theta \;+\; w,\;\; \mathscr{E}(w) = 0,\; \mathscr{E}(w\; w') = V_w, \tag{B.2}$$

where I_T, ι and J are as in Appendix A.

Equations (B.1) and (B.2) can be written as a single equation

$$Y \;=\; f(\beta) + u,\; \mathscr{E}(u) = 0,\; \mathscr{E}(u\; u') = V = \begin{bmatrix} V_e & 0 \\ 0 & V_w \end{bmatrix}, \tag{B.3}$$

where

$$Y = \begin{bmatrix} ln\, s \\ y \end{bmatrix},\; \beta = \begin{bmatrix} ln\, a \\ ln\, \theta \end{bmatrix},\; f(\beta) = \begin{bmatrix} \iota\, ln\, a + ln\, \theta \\ J\,\theta \end{bmatrix},\; u = \begin{bmatrix} ln\, e \\ w \end{bmatrix}. \tag{B.4}$$

If the disturbances are normally distributed, the solution to (B.3) may be obtained by maximizing the log-likelihood function

$$ln\; p(\beta) = (N/2) ln\, 2\pi - (N/2)\, ln\, |V| - SS(\beta)/2, \tag{B.5}$$

where

$$SS(\beta) = [Y - f(\beta)]'\, V^{-1} [Y - f(\beta)], \tag{B.6}$$

is the minimized sum of squares of the residuals.

In the additive model $f(\beta)$ was linear and equal to $X\beta$. Under the Gauss-Newton method (Harvey, 1981), $f(\beta)$ is linearized by first degree Taylor expansion at the initial values of the parameters, $\beta_0 = [ln\, a_0 \;\; ln\, \theta_0']'$ (by taking the first derivatives):

$$f(\beta) \approx f_0 + X_0 (\beta - \beta_0), \tag{B.7}$$

where

$$f_0 = f(\beta_0) = \begin{bmatrix} \iota\, ln\, a_0 + ln\, \theta_0 \\ J\,\theta_0 \end{bmatrix},\; X_0 = \begin{bmatrix} \iota & I_T \\ 0 & J_0 \end{bmatrix},\; J_0 = \left[j_{m,t}/p_m,\;\; j_{m,t} = \begin{cases} \theta_{0,t}, & t \in m, \\ 0, & t \notin m, \end{cases} \right]. \tag{B.8}$$

where p_m is equal to 1 for flow and stock series and equal to the number of periods covered by benchmark y_m for index series.

Substituting (B.7) in (B.6) gives a linearized sum of squares:

$$SS(\beta) \approx \{Y - [f_0 + X_0(\beta - \beta_0)]\}' V^{-1} \{Y - [f_0 + X_0(\beta - \beta_0)]\}$$
$$= \{Y^{\dagger} - X_0 \delta\}' V^{-1} \{Y^{\dagger} - X_0 \delta\}, \quad \text{(B.9)}$$

where

$$Y^{\dagger} = Y - f_0 = \begin{bmatrix} \ln s - \iota \ln a_0 - \ln \theta_0 \\ y - J\theta_0 \end{bmatrix}, \; \delta = [\beta - \beta_0] = \begin{bmatrix} (\ln a - \ln a_0) \\ (\ln \theta - \ln \theta_0) \end{bmatrix} = \begin{bmatrix} \delta_a \\ \delta_\theta \end{bmatrix}. \quad \text{(B.10)}$$

Equation (B.9) shows that the sum of squares may be minimized with respect to δ; the generalized least squares estimate of δ is

$$\hat{\delta} = (X_0' V^{-1} X_0)^{-1} X_0' V^{-1} Y^{\dagger} \quad \text{(B.11)}$$

Since $\delta = (\beta - \beta_0)$, the revised estimates of the parameters are given by $\hat{\beta} = \beta_0 + \hat{\delta}$. Letting the revised estimates play the role of the initial estimates, β_0, matrix X_0 and vector $Y^{\dagger}$ are revised and the next step, δ, is estimated. This process is carried out until the steps $\hat{\delta}$ are arbitrarily small.

Since matrix X_0 has the same structure (and to a large extent the same content) as X in the additive model, the matrix formulae developed for the latter hold for the multiplicative model. Defining $Q^{\dagger} = (J_0 V_e J_0' + V_w)^{-1}$, $s^{\dagger} = (\ln s - \ln \theta_0 - \ln a_0)$ and $y^{\dagger} = (y - J\theta_0)$, the formula (A.6) to (A.10) are usable with a few modifications

$$v_a = 1 / (\iota' J_0' Q^{\dagger} J_0 \iota), \quad \text{(B.12)}$$

$$v_{\theta a} = v_{a\theta}' = -(I_T - V_e J_0' Q^{\dagger} J_0) \iota v_a, \quad \text{(B.13)}$$

$$V_\theta = (V_e - V_e J_0' Q^{\dagger} J_0 V_e) + (I_T - V_e J_0' Q^{\dagger} J_0) \iota v_a \iota' (I_T - V_e J_0' Q^{\dagger} J_0)', \quad \text{(B.14)}$$

$$\hat{\delta}_a = - v_a \iota' J_0' Q^{\dagger} (y^{\dagger} - J_0 s^{\dagger}), \quad \text{(B.15)}$$

$$\hat{\delta}_\theta = s^* + V_e J_0' Q^{\dagger} (y^{\dagger} - J_0 s^*), \quad s^* = s^{\dagger} - \iota \hat{\delta}_a, \quad \text{(B.16)}$$

Equations (B.12), (B.15) and (B.16) are used to estimate the steps, i.e. the change in the parameters; and after convergence, (B.14) is used to estimate the variance of the benchmarked series. If the bias is omitted from the model, only (B.16) with $\hat{\delta}_a = 0$ is used to estimate the steps; and only the first term of (B.14), to estimate the variance.

Appendix C: Estimation of the Mixed Benchmarking Model

This appendix presents the solution of the mixed benchmarking model.

The two equations (3a) and (3b) of section 3 are written in matrix algebra as

$$s = \theta a + e, \; \mathscr{E}(e) = 0, \; \mathscr{E}(e e') = V_e, \quad \text{(C.1)}$$

$$y = J \theta + w, \; \mathscr{E}(w) = 0, \; \mathscr{E}(w w') = V_w, \quad \text{(C.2)}$$

where J is as in Appendix A.

Equations (C.1) and (C.2) can be written as a single equation

$$Y = f(\beta) + u,\ \mathscr{E}(u) = 0,\ \mathscr{E}(u\,u') = V = \begin{bmatrix} V_e & 0 \\ 0 & V_w \end{bmatrix}, \tag{C.3}$$

where

$$Y = \begin{bmatrix} s \\ y \end{bmatrix},\ \beta = \begin{bmatrix} a \\ \theta \end{bmatrix},\ f(\beta) = \begin{bmatrix} \theta\,a \\ J\,\theta \end{bmatrix},\ u = \begin{bmatrix} e \\ w \end{bmatrix}. \tag{C.4}$$

If the disturbances are normally distributed, the solution to (C.3) may be obtained by maximizing the log-likelihood function (B.5), which is achieved by minimizing the sum of squares of the residuals:

$$SS(\beta) = [Y - f(\beta)]'\,V^{-1}[Y - f(\beta)], \tag{C.5}$$

Following the same approach as for the multiplicative model, $f(\beta)$ is linearized by first degree Taylor expansion at the initial values of the parameters, $\beta_0 = [\,a_0' \ \ \theta_0']'$:

$$f(\beta) \approx f_0 + X_0\,(\beta - \beta_0), \tag{C.6}$$

where

$$f_0 = f(\beta_0) = \begin{bmatrix} \theta_0\,a_0 \\ J\,\theta_0 \end{bmatrix},\ X_0 = \begin{bmatrix} \theta_0 & a_0 \otimes I_T \\ 0 & J \end{bmatrix} \tag{C.7}$$

Substituting (C.6) in (C.5) gives a linearized sum of squares:

$$\begin{aligned} SS(\beta) &\approx \{Y - [f_0 + X_0(\beta - \beta_0)]\}'\,V^{-1}\{Y - [f_0 + X_0(\beta - \beta_0)]\} \\ &= \{Y^{\dagger} - X_0\delta\}'\,V^{-1}\{Y^{\dagger} - X_0\delta\}, \end{aligned} \tag{C.8}$$

where

$$Y^{\dagger} = Y - f_0 = \begin{bmatrix} s - \theta_0\,a_0 \\ y - J\theta_0 \end{bmatrix},\ \delta = [\beta - \beta_0] = \begin{bmatrix} (a - a_0) \\ (\theta - \theta_0) \end{bmatrix} = \begin{bmatrix} \delta_a \\ \delta_\theta \end{bmatrix}. \tag{C.9}$$

Equation (C.8) shows that the sum of squares may be minimized with respect to δ; the generalized least square estimates of δ is

$$\hat{\delta} = (X_0' V^{-1} X_0)^{-1}\, X_0' V^{-1} Y^{\dagger} \tag{C.10}$$

Since $\delta = (\beta - \beta_0)$, the revised estimates of the parameters are given by $\hat{\beta} = \beta_0 + \hat{\delta}$. Letting the revised estimates play the role of the initial estimates, β_0, matrix X_0 and vector $Y^{\dagger}$ are revised and the next step, δ, is estimated. This process is carried out until the steps $\hat{\delta}$ are arbitrarily small.

Since matrix X_0 has the same structure as X in the additive model, the matrix formulae developed for the latter hold for the multiplicative model. Defining $V_e^{\dagger} = ((1/a_0^2) \otimes V_e)$, $Q^{\dagger} = (J\,V_e^{\dagger} J' + V_w)^{-1}$, $\iota^{\dagger} = ((1/a_0) \otimes \theta_0)$, $s^{\dagger} = (s - \theta_0\,a_0)$ and $y^{\dagger} = (y - J\theta_0)$, the formula (A.6) to (A.10) are usable with a few modifications

$$v_a = 1 / (\boldsymbol{\iota}^{\dagger\prime} \boldsymbol{J}' \boldsymbol{Q}^{\dagger} \boldsymbol{J} \boldsymbol{\iota}^{\dagger}), \quad \text{(C.11)}$$

$$v_{\theta a} = v_{a\theta}' = -(\boldsymbol{I}_T - \boldsymbol{V}_e^{\dagger} \boldsymbol{J}') \boldsymbol{Q}^{\dagger} \boldsymbol{J}) \boldsymbol{\iota}^{\dagger} v_a, \quad \text{(C.12)}$$

$$\boldsymbol{V}_\theta = (\boldsymbol{V}_e^{\dagger} - \boldsymbol{V}_e^{\dagger} \boldsymbol{J}' \boldsymbol{Q}^{\dagger} \boldsymbol{J} \boldsymbol{V}_e^{\dagger}) + (\boldsymbol{I}_T - \boldsymbol{V}_e^{\dagger} \boldsymbol{J}' \boldsymbol{Q}^{\dagger} \boldsymbol{J}) \boldsymbol{\iota}^{\dagger} v_a \boldsymbol{\iota}^{\dagger\prime} (\boldsymbol{I}_T - \boldsymbol{V}_e^{\dagger} \boldsymbol{J}' \boldsymbol{Q}^{\dagger} \boldsymbol{J})', \quad \text{(C.13)}$$

$$\hat{\delta}_a = - v_a \boldsymbol{\iota}' \boldsymbol{J}_0' \boldsymbol{Q}^{\dagger} (\boldsymbol{y}^{\dagger} - \boldsymbol{J}_0 \boldsymbol{s}^{\dagger}), \ \hat{a} = a_0 + \hat{\delta}_a, \quad \text{(C.14)}$$

$$\hat{\boldsymbol{\delta}}_\theta = \boldsymbol{s}^* + \boldsymbol{V}_e^{\dagger} \boldsymbol{J}_0' \boldsymbol{Q}^{\dagger} (\boldsymbol{y}^{\dagger} - \boldsymbol{J}_0 \boldsymbol{s}^*), \ \ \boldsymbol{s}^* = \boldsymbol{s} - \boldsymbol{\theta}_0 \hat{a}, \quad \text{(C.15)}$$

Equations (C.11), (C.14) and (C.15) are used to estimate the steps, i.e. the change in the parameters; and after convergence, (C.13) is used to estimate the variance of the benchmarked series. If the bias is omitted from the model, the additive model is used without bias.

Appendix D: Perpetual Calendar

Calendars from 1871 to 2050

Look for the year required in the index below. The number opposite each year is the number of the calendar to use for that year. The last numbers provide the date of Easter, when available.

Calendriers de 1871 à 2050

Choisir l'année désirée dans l'index ci-dessus. Le numéro du calendrier approprié apparaît après chaque année. Les derniers chiffres donne la date de Pâques, lorsque disponible.

1871: 1	1901: 3 07/4	1931: 5 05/4	1961: 1 02/4	1991: 3 31/3	2021: 6 04/4
1872: 9	1902: 4 30/3	1932: 13 27/3	1962: 2 22/4	1992: 11 19/4	2022: 7 17/4
1873: 4	1903: 5 12/4	1933: 1 16/4	1963: 3 14/4	1993: 6 11/4	2023: 1 09/4
1874: 5	1904: 13 03/4	1934: 2 01/4	1964: 11 29/3	1994: 7 3/4	2024: 9 31/3
1875: 6	1905: 1 23/4	1935: 3 21/4	1965: 6 18/4	1995: 1 16/4	2025: 4 20/4
1876: 14	1906: 2 15/4	1936: 11 12/4	1966: 7 10/4	1996: 9 7/4	2026: 5
1877: 2	1907: 3 31/3	1937: 6 28/3	1967: 1 26/3	1997: 4 30/3	2027: 6
1878: 3	1908: 11 19/4	1938: 7 17/4	1968: 9 14/4	1998: 5 12/4	2028: 14
1879: 4	1909: 6 11/4	1939: 1 09/4	1969: 4 6/4	1999: 6 4/4	2029: 2
1880: 12	1910: 7 27/3	1940: 9 24/3	1970: 5 29/3	2000: 14 23/4	2030: 3
1881: 7	1911: 1 16/4	1941: 4 13/4	1971: 6 11/4	2001: 2 15/4	2031: 4
1882: 1	1912: 9 07/4	1942: 5 05/4	1972: 14 2/4	2002: 3 31/3	2032: 12
1883: 2	1913: 4 23/4	1943: 6 25/4	1973: 2 22/4	2003: 4 20/4	2033: 7
1884: 10	1914: 5 12/4	1944: 14 09/4	1974: 3 14/4	2004: 12 11/4	2034: 1
1885: 5	1915: 6 04/4	1945: 2 01/4	1975: 4 30/3	2005: 7 27/3	2035: 2
1886: 6	1916: 14 23/4	1946: 3 21/4	1976: 12 18/4	2006: 1 16/4	2036: 10
1887: 7	1917: 2 08/4	1947: 4 06/4	1977: 7 10/4	2007: 2 08/4	2037: 5
1888: 8	1918: 3 31/3	1948: 12 28/3	1978: 1 26/3	2008: 10 23/3	2038: 6
1889: 3	1919: 4 20/4	1949: 7 17/4	1979: 2 15/4	2009: 5 12/4	2039: 7
1890: 4	1920: 12 04/4	1950: 1 09/4	1980: 10 6/4	2010: 6 04/4	2040: 8
1891: 5	1921: 7 27/3	1951: 2 25/3	1981: 5 19/4	2011: 7 24/4	2041: 3
1892: 13	1922: 1 16/4	1952: 10 13/4	1982: 6 11/4	2012: 8 08/4	2042: 4
1893: 1	1923: 2 01/4	1953: 5 05/4	1983: 7 3/4	2013: 3 31/3	2043: 5
1894: 2	1924: 10 20/4	1954: 6 18/4	1984: 8 22/4	2014: 4 20/4	2044: 13
1895: 3	1925: 5 12/4	1955: 7 18/4	1985: 3 7/4	2015: 5 05/4	2045: 1
1896: 11	1926: 6 04/4	1956: 8 01/4	1986: 4 30/3	2016: 13 27/3	2046: 2
1897: 6	1927: 7 17/4	1957: 3 21/4	1987: 5 19/4	2017: 1 16/4	2047: 3
1898: 7	1928: 8 08/4	1958: 4 06/4	1988: 13 3/4	2018: 2 01/4	2048: 11
1899: 1	1929: 3 31/3	1959: 5 29/3	1989: 1 26/3	2019: 3 21/4	2049: 6
1900: 2	1930: 4 20/4	1960: 13 17/4	1990: 2 15/4	2020: 11 12/4	2050: 7

CALENDAR 1 / CALENDRIER 1

January / Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

February / Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28				

March / Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

April / Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

May / Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

June / Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

July / Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

August / Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

September / Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

October / Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

November / Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

December / Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

CALENDAR 2 / CALENDRIER 2

January / Janvier

S D	M L	T M	W M	T J	F V	S S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

February / Février

S D	M L	T M	W M	T J	F V	S S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28			

March / Mars

S D	M L	T M	W M	T J	F V	S S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

April / Avril

S D	M L	T M	W M	T J	F V	S S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

May / Mai

S D	M L	T M	W M	T J	F V	S S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

June / Juin

S D	M L	T M	W M	T J	F V	S S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

July / Juillet

S D	M L	T M	W M	T J	F V	S S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

August / Août

S D	M L	T M	W M	T J	F V	S S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

September / Septembre

S D	M L	T M	W M	T J	F V	S S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

October / Octobre

S D	M L	T M	W M	T J	F V	S S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

November / Novembre

S D	M L	T M	W M	T J	F V	S S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

December / Décembre

S D	M L	T M	W M	T J	F V	S S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

CALENDAR 3 / CALENDRIER 3

January / Janvier

S D	M L	T M	W M	T J	F V	S S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

February / Février

S D	M L	T M	W M	T J	F V	S S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28		

March / Mars

S D	M L	T M	W M	T J	F V	S S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

April / Avril

S D	M L	T M	W M	T J	F V	S S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

May / Mai

S D	M L	T M	W M	T J	F V	S S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

June / Juin

S D	M L	T M	W M	T J	F V	S S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

July / Juillet

S D	M L	T M	W M	T J	F V	S S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

August / Août

S D	M L	T M	W M	T J	F V	S S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

September / Septembre

S D	M L	T M	W M	T J	F V	S S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

October / Octobre

S D	M L	T M	W M	T J	F V	S S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

November / Novembre

S D	M L	T M	W M	T J	F V	S S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

December / Décembre

S D	M L	T M	W M	T J	F V	S S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

CALENDAR 4 / CALENDRIER 4

January — Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

February — Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

March — Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

April — Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May — Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

June — Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

July — Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

August — Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

September — Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

October — Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

November — Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

December — Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

CALENDAR 5 / CALENDRIER 5

January — Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

February — Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

March — Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

April — Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

May — Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

June — Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

July — Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

August — AoÛt

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

September — Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

October — Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

November — Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

December — Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

CALENDAR 6 / CALENDRIER 6

January / Janvier

S / D	M / L	T / M	W / M	T / J	F / V	S / S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

February / Février

S / D	M / L	T / M	W / M	T / J	F / V	S / S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28						

March / Mars

S / D	M / L	T / M	W / M	T / J	F / V	S / S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

April / Avril

S / D	M / L	T / M	W / M	T / J	F / V	S / S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

May / Mai

S / D	M / L	T / M	W / M	T / J	F / V	S / S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

June / Juin

S / D	M / L	T / M	W / M	T / J	F / V	S / S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

July / Juillet

S / D	M / L	T / M	W / M	T / J	F / V	S / S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

August / Août

S / D	M / L	T / M	W / M	T / J	F / V	S / S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

September / Septembre

S / D	M / L	T / M	W / M	T / J	F / V	S / S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

October / Octobre

S / D	M / L	T / M	W / M	T / J	F / V	S / S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

November / Novembre

S / D	M / L	T / M	W / M	T / J	F / V	S / S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

December / Décembre

S / D	M / L	T / M	W / M	T / J	F / V	S / S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

CALENDAR 7 / CALENDRIER 7

January / Janvier

S / D	M / L	T / M	W / M	T / J	F / V	S / S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

February / Février

S / D	M / L	T / M	W / M	T / J	F / V	S / S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28					

March / Mars

S / D	M / L	T / M	W / M	T / J	F / V	S / S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

April / Avril

S / D	M / L	T / M	W / M	T / J	F / V	S / S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

May / Mai

S / D	M / L	T / M	W / M	T / J	F / V	S / S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

June / Juin

S / D	M / L	T / M	W / M	T / J	F / V	S / S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

July / Juillet

S / D	M / L	T / M	W / M	T / J	F / V	S / S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

August / Août

S / D	M / L	T / M	W / M	T / J	F / V	S / S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

September / Septembre

S / D	M / L	T / M	W / M	T / J	F / V	S / S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

October / Octobre

S / D	M / L	T / M	W / M	T / J	F / V	S / S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

November / Novembre

S / D	M / L	T / M	W / M	T / J	F / V	S / S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

December / Décembre

S / D	M / L	T / M	W / M	T / J	F / V	S / S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

CALENDAR 8 / CALENDRIER 8

January Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

February Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29			

March Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

April Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

May Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

June Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

July Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

August Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

September Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

October Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

November Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

December Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

CALENDAR 9 / CALENDRIER 9

January Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

February Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29		

March Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

April Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

May Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

June Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

July Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

August Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

September Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

October Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

November Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

December Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

CALENDAR 10 / CALENDRIER 10

January Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

February Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	

March Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

April Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

May Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

June Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

July Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

August Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

September Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27

October Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25

November Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22

December Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27

28 29 30 | 26 27 28 29 30 31 | 23 24 25 26 27 28 29 30 | 28 29 30 31

CALENDAR 11 / CALENDRIER 11

January / Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

February / Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

March / Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

April / Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

May / Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

June / Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

July / Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

August / Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

September / Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

October / Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

November / Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

December / Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

CALENDAR 12 / CALENDRIER 12

January / Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

February / Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29						

March / Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

April / Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

May / Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

June / Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

July / Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

August / Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

September / Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

October / Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

November / Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

December / Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

CALENDAR 13 / CALENDRIER 13

January / Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

February / Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29					

March / Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

April / Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

May / Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

June / Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

July / Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

August / Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

September / Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

October / Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

November / Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

December / Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

CALENDAR 14 / CALENDRIER 14

January / Janvier

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

February / Février

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29				

March / Mars

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

April / Avril

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

May / Mai

S	M	T	W	T	F	S
D	L	M	M	J	V	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

June / Juin

S	M	T	W	T	F	S
D	L	M	M	J	V	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

July / Juillet

S	M	T	W	T	F	S
D	L	M	M	J	V	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

August / Août

S	M	T	W	T	F	S
D	L	M	M	J	V	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

September / Septembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

October / Octobre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

November / Novembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

December / Décembre

S	M	T	W	T	F	S
D	L	M	M	J	V	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						