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AN EXACT TEST FOR THE PRESENCE OF STABLE SEASONALITY WITH APPLICATIONS

by

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SUMMARY

The X11ARIMA seasonal adjustment method and the Census X11 variant use a standard ANOVA-F-test to assess the presence of stable seasonality. This F-test is applied to a series consisting of estimated seasonals plus irregulars (residuals) which may be (and often are) autocorrelated, thus violating the basic assumption of the F-test. This limitation has long been known by producers of seasonally adjusted data and the nominal value of the F statistic has been rarely used as a criterion for seasonal adjustment. Instead, producers of seasonally adjusted data have used rules of thumb, such as, F equal to or greater than 7.

This paper introduces an exact test which takes into account autocorrelated residuals following a SMA process of the (0,q)(0,Q) type. Comparisons of this modified F-test and the standard ANOVA test of X11ARIMA are made for a large number of Canadian socioeconomic series.

KEYWORDS: Standard Anova, Autocorrelated Residuals, Seasonality.

1. INTRODUCTION

In the analysis of social and economic time series, it is traditional to decompose the observed series into four unobserved components, namely the trend, the cycle, the seasonal variations, and the irregulars.

Socioeconomic time series are often presented in seasonally adjusted form so that the underlying short-term trend can be more easily analysed and current socioeconomic conditions can be assessed. There are several seasonal adjustment methods available which estimate the seasonal component present in a time series, but the Census X-11 variant (Shiskin, Young and Musgrave 1967) and the X-11 ARIMA method (Dagum 1980) are the most widely applied. To identify the presence of stable seasonality in a time series, the X-11-ARIMA method as well as the Census X-11 variant use the results of the usual F-test in a one-way ANOVA between monthly seasonal variations and the residuals. However, the residuals in this ANOVA are often autocorrelated, so the nominal significance level of the F-test may not be valid. Aware of this limitation, producers of seasonally adjusted data, do not guide themselves by the nominal significance level of the F-test for presence of stable seasonality but by some rule of thumb based on empirical knowledge (see e.g. Shiskin and Plewes (1978)). In fact, implicit in the X-11-ARIMA test for the presence of 'identifiable seasonality' is that the F-value for stable seasonality should be greater or equal to 7 if missing seasonality is not present.

The testing for stable seasonality (similarly for annual seasonal shifts) can be approached as a test for the significance of certain regression coefficients in a linear model with autocorrelated errors. The traditional Wald's test, the likelihood ratio test, and the tests falling within a generalized least squares framework, all run into convergence problems in testing such a linear model with highly autocorrelated errors [cf. Sutradhar and Bartlett (1990)]. Pierce (1978) constructed an F-test based on transformed residuals which are approximately white noise. The transformation suggested in Pierce (1978) is equivalent to use the inversion of the error covariance matrix which may not be obtained when error covariance matrix is too weak for highly autocorrelated errors. Recently Sutradhar, MacNeill and Dagum (1991) proposed a modified F-test, within a linear model framework for testing for stable seasonality. Their modified F-test is derived following Sutradhar, MacNeill and Sahrmann (1987), and the test accounts for the presence of autocorrelations in the residuals. The test does not require any transformation or any inversion of the error covariance matrix.

Exact tests for testing the null hypothesis that the seasonal pattern changes over time against the alternative that the seasonal pattern is constant had been developed by Franzini and Harvey (1983). Unlike Franzini and Harvey, the present approach assumes that the seasonal pattern is stable over time possibly at different levels (due to annual shifts) and then tests for the presence of significance stable seasonality.

In most empirical cases, a seasonal moving average (SMA) error model of the $(0,q)\ (0,Q)_s$ type is sufficient. In this investigation we simplify the exact test proposed by Sutradhar, MacNeill and Dagum (1991), for such error models. The test is applied to examine for the presence of stable seasonality as well as of annual seasonal shifts in a number of socioeconomic series.

The plan of this paper is as follows. Section 2 presents the exact test. Section 3 analyses the results from the application of the modified F-test to a set of socioeconomic time series and compares them with the values given by the X-11-ARIMA method. Section 4 gives the conclusions.

2. MODIFIED F-TEST

2.1 Selected Model

Consider a stationary seasonal time series $\{Z_t\}$, given by

$$Z_t=S_t+U_t, \qquad (2.1)$$

where Z_t is the observed series at time t, S_t is the seasonal component, and U_t the irregulars. If the time series contains trend, which is most likely, it is assumed that a suitable detrending technique will yield the model (2.1). In the later case, the detrended series may be obtained from the original series by taking appropriate differences as in ARIMA modelling (Box and Jenkins, 1970) or as traditionally done by statistical agencies, using the X-11-ARIMA method or Census X-11 variant.

Next, suppose there are k seasons in a year and there are kn observations in a time series of n years. Let $Z\{(i-1)n+j\}$ be the jth (j=1,...,n) observation under the ith season (i=1,...,k), which corresponds to Z_t in (2.1). We shall denote in similar manner the (i,j)th components of S_t and U_t, for all t=1,...,kn. Then, the model assumed for S_t is:

$$S((i-1)n+j) = \mu+\alpha_i+\beta_j \qquad (2.2)$$

with $\sum_{i=1}^{k} \alpha_i=0, \quad \sum_{j=1}^{n} \beta_j = 0.$

The α's and β's in (2.2) represent, respectively, the stable seasonality and annual seasonal shifts in the seasonal time series. Thus, when testing for the presence of stable seasonality, we test the hypothesis

$$H_0: \ \alpha_i=0 \text{ vs. } H_1: \alpha_i\neq 0 \text{ for at least one } i; \qquad (2.3)$$

and when testing for the presence of annual seasonal shifts, we test the hypothesis

$$H_0: \ \beta_j=0 \text{ vs. } H_1: \ \beta_j\neq 0 \text{ for at least one } j. \qquad (2.4)$$

Consequently, the rejection of H_0 in (2.3) and (2.4) would indicate that the series contains significant stable seasonality as well as annual seasonal shifts.

Taking into account model (2.2), the model (2.1) can be written as follows:

$$Z^* = X\gamma + U^* \qquad (2.5)$$

where

$$Z^* = [Z(1),\ldots, Z(n), Z(n+1),\ldots, Z(kn)]',$$

$$U^* = [U(1),\ldots, U(n), U(n+1),\ldots, U(kn)]',$$

$$\gamma = [\mu,\alpha_1,\ldots, \alpha_{k-1}, \beta_1,\ldots, \beta_{n-1}]',$$

and X is the appropriate $kn\times(k+n-1)$ design matrix.

2.2 Test Statistics

U^* in (2.5) can be represented by seasonal autoregressive moving average stationary (SARMA) process $(p,q)(P,Q)_s$. In most empirical cases we found, however, that a $(0,q)(0,Q)_s$ model is sufficient. Let Σ^* denote the $kn \times kn$ covariance matrix of U^*. Naturally, Σ^* will contain θ and Θ, where θ and Θ's are the parameters associated with the SARMA $(0,q)$ $(0,Q)_s$ process. Now, we test the null hypotheses $\beta_j=0$, and $\alpha_i=0$, by using the modified F-statistics F_{M1} and F_{M2} respectively, given by [cf. Sutradhar, MacNeill and Dagum (1991)].

$$F_{M1} = d_1(\hat{\theta},\hat{\Theta})\ F_{A1}, \tag{2.6}$$

$$F_{M2} = d_2(\hat{\theta},\hat{\Theta})\, F_{A2}\ , \tag{2.7}$$

where F_{A1} and F_{A2} are the corresponding standard ANOVA F-statistics, and $d_1(\theta,\Theta)=c_3(\theta,\Theta)/c_1(\theta,\Theta)$, $d_2(\theta,\Theta)=c_3\ (\theta,\Theta)/c_2(\theta,\Theta)$. For example, for the SARMA $(0,1)(0,1)_{12}$ process, $c_1(\theta,\Theta)$, $c_2(\theta,\Theta)$ and $c_3(\theta,\Theta)$ are given by

$$c_1(\theta,\Theta)=(1+\theta^2)(1+\Theta^2) - (\theta/6)(1+\Theta^2)(11-1/n)+(2\Theta/n)(1+\theta^2)$$
$$+(\theta\Theta/6)\{1-22/n - (n-2)/n(n-1)\},$$

$$c_2(\theta,\Theta) = (1+\theta^2)(1+\Theta^2) - 2(1-1/n)\Theta(1+\theta^2)$$
$$+1/6\{1+(1-1/n)/11\}\theta(1+\Theta^2)-(4/11)(1-1/n)\theta\Theta,$$

$$c_3(\theta,\Theta) = (1+\theta^2)(1+\Theta^2)+(2\Theta/n)(1+\theta^2)+(\theta/6)(1+\Theta^2)(1-1/11n)$$
$$- (\theta\Theta/6n)\ [n/11-2(n-2)/11(n-1)-2].$$

Notice that in the independence case when $\theta=0$, $\Theta=0$, $c_1(.)=c_2(.)=c_3(.)=1$, which is obvious. Consequently, one may test the hypotheses by using standard ANOVA F statistics.

2.3 Computation of p-value

A simulation study (cf. Sutradhar and Bartlett (1989, Table IV, p. 1587)) indicates that for the cases when k groups are independent, the distribution of the modified F-statistics for the SARMA $(0,q)(0,Q)_s$ process, may be approximated by the usual F-distribution. In general, the F approximation to the modified F-statistic would be inappropriate, specially when k groups are correlated and n is small.

In this paper we use the well known Satterthwaite (1946) approximation [cf. Sutradhar, MacNeill and Dagum(1991)] to calculate the p-value, namely, $P_r(F_{M1}\geq f_{M1})$, where f_{M1} is the data based value of F_{M1}. In order to do it, we first compute the eigenvalues $\lambda_1\geq\lambda_2\geq\ldots\geq\lambda_r>0=\lambda_{r+1}=\ldots=\lambda_s>\lambda_{s+1}\geq\ldots\geq\lambda_n$ of

$$\Sigma^{*1/2}\ [d_1(\theta,\Theta)D_1-f_{M1}(I_{kn}-D_2)]\Sigma^{*1/2}, \qquad (2.8)$$

where $d_1(.)$ is given in equation (2.6), $D_1=R(RR')^{-1}R'$, with $R=C(X'X)^{-1}X'$, $D_2=X(X^1X)^{-1}X'$, C being a suitable matrix obtained by expressing the: $H_0:\beta_j=0$ in the form $C\gamma=0$, where γ is given in model (2.5). In equation (2.8) I_{kn} is the $kn\times kn$ identity matrix. Then the Satterthwaite approximation yields

$$P_r(F_{M1}\geq f_{M1}) = P_r[F_{a,b}\geq bd/ac], \qquad (2.9)$$

where $F_{a,b}$ denotes the usual F-ratio with degrees of freedom a and b, with

$$a=\left(\sum_{j=1}^{r}\lambda_j\right)^2/\sum_{j=1}^{r}\lambda_j^2, \qquad b=\left(\sum_{j=s+1}^{n}\lambda_j\right)^2/\sum_{j=s+1}^{n}\lambda_j^2.$$

In equation (2.9),

$$c=\sum_{j=1}^{r}\lambda_j^2/\sum_{j=1}^{r}\lambda_j, \qquad d=\sum_{j=s+1}^{n}\lambda_j^2/\sum_{j=s+1}^{n}|\lambda_j|.$$

Similarly, $P_r(F_{M2}\geq f_{M2})$ may be calculated by using $d_2(.)$ and f_{M2} in place of $d_1(.)$ and f_{M1} respectively in equation (2.9). The construction of D_1 will now depend on a different C matrix which will be obtained by expressing the H_0: $\alpha_i=0$ in the form $C\gamma=0$.

3. APPLICATIONS

3.1 Monthly Series

The modified F statistics F_{M1} and F_{M2} of equations (2.6) and (2.7) are calculated for a set of 26 monthly series obtained from various economic sectors, namely, Imports, Exports, Consumer Prices and Labour. All series cover the period January 1979 till December 1988 inclusive.

Since the modified F-test is not valid when moving seasonality is present (except for annual seasonal shifts), none of the series selected are affected by moving seasonality according to certain preliminary tests available in X11ARIMA. (We also looked at the plots of the seasonal-irregular ratios.)

The X-11-ARIMA method was applied to obtain the detrended series $\{Z_t: t=1,...,120\}$. Diagnostic checks show that the errors of the detrended series, U_t (see equation 2.1) follow a

$(0,1)(0,1)_{12}$ ARMA model for each of the monthly series. The estimates $\hat{\theta}$ and $\hat{\Theta}$ are used to compute the modified F-statistics F_{M1} and F_{M2}.

In testing for the presence of annual seasonal shifts, the p-values for the modified F-test based on the Satterthwaite approximation and on the standard ANOVA F-test were found to be generally different. For both cases, however, the P-values were very large for each of the series indicating that there is no moving seasonality in the form of annual shifts.

To test for the presence of stable seasonality, we computed the p-values of the modified F-statistic F_{M2} (2.7) by using the Satterthwaite approximation and compared to those given by the X-X-11-ARIMA F-test (which is equivalent to standard ANOVA F_{A2}) for the 26 monthly series. The results are shown in Table 1.

(INSERT TABLE 1 HERE)

The p-values of the modified F-statistic in Table 1 show that among the 9 import series, 3 series do not have significant stable seasonality at the 1% significance level (The critical value of $F(11,99;\ 0.01)=2.47$.) Among the 7 exports series, only one series, namely Wheat, appears to have no seasonality. All 6 CPI series have significant stable seasonality and similarly for the 4 Labour series.

The X-11-ARIMA F-test values give same results (either rejection or acceptance of the null hypothesis) as the modified F-test for a large number of series. It seems that for most of the monthly series, under the SARMA $(0,q)(0,Q)_s$ error structure,

TABLE 1. DIAGNOSTICS OF STABLE SERASONALITY IN MONTHLY SERIES

	Series	Parameter estimates θ	Θ	X-11ARIMA F-Test[a]	Modified F F_{M2}(p-value in %)	Final Diagnostic[c]
	IMPORTS					
1.	Fodder and feed	-0.09*	-0.01	3.68	3.43(0.06)	Y
2.	Coal & related materials	0.02	-0.01	64.40	58.76(0.00)	Y
3.	Crude vegetable products	0.02	-0.07*	3.48	2.94(0.27)	Y
4.	Wool & man made materials	0.02	0.29*	10.98	20.63(0.00)	Y
5.	Precious Metals	0.27*	0.01	1.25	1.20(31.10)	N
6.	Oils & fats	0.41*	0.01	8.59	8.22(0.00)	Y
7.	Non-metal minerals	0.04	0.02	16.50	16.68(0.00)	Y
8.	Aircraft engines	0.32*	0.00	2.53[b]	2.36(1.79)	N
9.	Other trans.eq	0.19*	-0.18*	3.48[b]	2.43(1.31)	N
	EXPORTS					
10.	Wheat	0.04	-0.03	1.89	1.71(8.71)	N
11.	Asbestos	0.13*	-0.03	6.83	6.15(0.00)	Y
12.	Wood pulp	-0.27*	0.20*	6.45	9.61(0.00)	Y
13.	Textile fabrics	0.52*	0.13*	12.05	15.06(0.00)	Y
14.	Other fabrics	0.04	0.11*	5.03	6.19(0.00)	Y
15.	Television & telecom.	0.12*	0.01	9.26	8.99(0.00)	Y
16.	Domestic export pass.	-0.30*	-0.14*	24.50	18.52(0.00)	Y
	CPI					
17.	Eggs	-0.04	-0.01	6.90	6.50(0.00)	Y
18.	Pasta	-0.05*	-0.04	3.69	3.24(0.10)	Y
19.	Onions	-0.42*	-0.03	26.90	23.49(0.00)	Y
20.	Housing	0.11*	-0.34*	19.02	9.28(0.00)	Y
21.	Clothing	0.03	-0.42*	47.42	24.30(0.00)	Y
22.	Transport	-0.09*	-0.02	4.21	3.74(0.02)	Y

	LABOUR					
23.	Sask. employment(25-34)	-0.19*	-0.11*	67.40	52.35(0.00)	Y
24.	Sask.not in labour force	0.12*	-0.36*	22.98	12.69(0.00)	Y
25.	Ont. unemployment(25-44)	-0.21*	0.07*	31.4	34.23(0.00)	Y
26.	Ont. unemploy. male & female (20-24)	-0.02	0.19*	24.27	34.78(0.00)	Y

[a] Critical value is F(11,99;0.01)=2.47

[b] X-11-ARIMA and Modified F give conflicting inference.

[c] Y (Yes) - stable seasonality is significant
N (No) - stable seasonality is not present.

* Significant values at 5% level.

the X-11-ARIMA F-test (or equivalently standard ANOVA F-test) is more affected by large negative values of θ, i.e. when there is seasonal autocorrelation in the residuals. Only 2 series, namely, Imports Aircraft Engines and Imports other transportation Equipments, have standard F-test values which lead to contradictory conclusions with respect to the modified F-test. On the other hand, if we would follow the rule of thumb of $F \geq 7$ to justify seasonal adjustment, then the modified F-test would be in contradiction for 8 out of 12 series. We then seasonally adjusted these 8 series with the X11ARIMA method and found that the quality of the adjustment was acceptable for 6 out of the 8 cases. All series passed the extrapolation ARIMA model automatically chosen for the program, 6 out of the 8 series passed the X11ARIMA guidelines criteria for acceptance; and the 4 series for which the F_{M2} values were relatively small, that is, falling between 3.24 and 3.74 were really strongly affected by trading-day variations. Only Imports Fodder and Feed and Imports Crude Vegetable products gave a seasonally adjusted output that could not be considered reliable.

3.2 <u>Quarterly Series</u>

The X-11-ARIMA method was applied to 4 quarterly series of the System of National Accounts to obtain the detrended values $\{z_t, t=1,...,40\}$. It was found that all 4 series U_t follow a $(0,1)(0,1)_4$ model. The computation for the modified F-test is quite similar to the case for monthly series but since the covariance matrix Σ^* is different, the formulas for $C_1(.)$, $C_2(.)$,

TABLE 2. DIAGNOSTICS OF STABLE SEASONALITY IN QUARTERLY SERIES

Series	Parameter estimates		X-11-ARIMA	Modified F	Final
	θ	Θ	F-Test[a]	F_{M2}(p-value in %)	Diagnostic[c]
1. Deposits in other Institutions	0.53*	0.11*	9.03	9.67(0.04)	Y
2. Net Financial Investments	0.77*	-0.37*	4.86[b]	2.56(8.16)	N
3. Small mortgages	0.17*	-0.01	6.65	4.88(1.02)	Y
4. Corporate Claims	0.77*	-0.31*	7.88[b]	3.58(3.20)	N

[a] Critical value is F(3,27;0.01) = 4.51

[b] X11ARIMA and Modified F give conflicting inference.

[c] Y (yes) - Stable seasonality is significant.

N (no) - Stable seasonality is not present.

* Significant values at 5% level.

and $C_3(.)$ in equations (2.6) and (2.7) were adjusted accordingly.

Similarly to the monthly series, the p-values for testing the presence of annual shifts based on the F_{M1} test were found very large and thus rejecting this pattern of moving seasonality.

The results of the modified F_{M2} test and the X-11-ARIMA F-test for testing for the presence of stable seasonality in each of the 4 series, are given in Table 2. The p-value for two series,

(INSERT TABLE 2 HERE)

namely, Deposits in other Institutions and Small Mortgages are not significant and in agreement with those obtained from X-11-ARIMA. Thus we conclude that these two series contain significant stable seasonality. On the other hand, the modified F-test values for the remaining two quarterly series give conflicting inference results and conclude that they should not be seasonally adjusted.

4. CONCLUSIONS

This paper has introduced an exact test for the presence of stable seasonality and annual seasonal shifts based on the modified F-test by Sutradhar, MacNeill and Sahrmann (1987). The new test takes into account the possibility of autocorrelated residuals in the seasonal-irregular ratios of the X-11-ARIMA method. The residuals are assumed to follow a simple Seasonal Moving Average (SMA) model $(0,q)(0,Q)_s$. This test is applied to a set of quarterly and monthly series from the system of National

Accounts, Imports, Exports, Consumer Prices and Labour. The residuals from the X11ARIMA method are found to follow seasonal moving average models (SMA) where either $\hat{\theta}$ and/or $\hat{\Theta}$ were significant. The exact F-test gives values very different from those of the F-test in X11ARIMA (also in the Census X11 variant) when the autocorrelation of the residuals is of a seasonal character, i.e., whenever $\hat{\Theta}$ is significantly different from zero.

Among the 26 monthly series analysed, only in two cases, the standard F-test values gave conflicting inference conclusions with respect to the modified F-test. On the other hand, if we would follow the common rule of thumb of $F \geq 7$ to justify seasonal adjustment, then the modified F-test gave contradictory results for 8 out of 12 series.

By looking at the seasonal adjustment output of these 8 series we found that 6 can be soundly seasonally adjusted by the X11ARIMA method.

Concerning the quarterly series, the modified F-test indicates there is no stable seasonality in 2 out of the 4 series analysed. Furthermore, in one case, the F-test of X11ARIMA gives an F value greater than 7 whereas the modified F accepts the null hypothesis.

It has been assumed throughout the paper that moving seasonality may be present in the series only in the form of annual shifts. The present test is not suitable to detect other types of moving seasonal patterns in the series. This raises the necessity of further investigations in this direction.

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