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Direction de la méthodologie

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Benchmarking Time Series with Autocorrelated Sampling Errors

by
Pierre A. CHOLETTE
and
Estela Bee DAGUM

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Statistics Canada
Time Series Research and Analysis Division
R.H. Coats Building, 7th floor "H"
Ottawa, Canada, K1A 0T6
(613) 951-1601

abstract

The Denton method is widely used by statistical agencies to benchmark time series (i.e. to adjust them to annual benchmarks). This method does not take into account the presence of autocorrelated sampling errors in the original data. This paper investigates to which extent this omission affects the efficiency of the method relative to optimal regression models that incorporates various types of ARMA processes for autocorrelated sampling errors.

KEYWORDS: Denton model, Sampling error, ARMA process, Relative efficiency

résumé

La méthode de Denton (1971) est très utilisée par les instituts de statistiques pour l'étalonnage des séries chronologiques (c-à-d l'ajustement aux jalons annuels). Cette méthode ne tient pas compte de la présence d'erreurs d'échantillonnage autocorrélées dans les données originales. Ce travail examine dans quelle mesure cette omission affecte l'efficience de la méthode par rapport à des modèles optimaux de régressions qui reflètent divers comportements ARMA des erreurs d'échantillonnage.

MOTS CLÉS: Modèle de Denton, Erreur d'échantillonnage, Processus ARMA, Efficience relative

1. INTRODUCTION

Benchmarking is a procedure very widely used in statistical agencies. Benchmarking situations arise whenever two (or more) sources of data are available for the same *target variable* with different frequencies, e.g. monthly versus annually, monthly versus quarterly. Generally, the two sources of data do not agree; for example, the annual sums of monthly measurements of a variable are not equal to the corresponding annual measurements. Furthermore, one source of data, typically the less frequent, is more reliable than the other, because it originates from a census, exhaustive administrative records or a larger sample. The more reliable measurements are considered as *benchmarks*. Traditionally, benchmarking has consisted of adjusting the less reliable series to make it consistent with the benchmarks. Benchmarking, however, can be defined more broadly as the process of optimally combining two sources of measurements, in order to achieve improved estimates of the series under investigation. Under such a definition, benchmarks are treated as auxiliary observations (Cholette and Dagum, 1989).

A typical example of benchmarking is the following. In Statistics Canada, the monthly estimates of Wages and Salaries originate from the Survey of Employment, Payrolls and Hours, whereas the annual benchmark measurements of the same variable originate from exhaustive administrative records, namely the Income Tax forms filed by Canadians and compiled by Revenue Canada. Benchmarking adjusts the monthly data so that they conform to the benchmarks and preserve the original month-to-month movement as much as possible.

Statistical agencies also use benchmarking to interpolate (and extrapolate) more frequent values from less frequent data. It is common, for instance, to benchmark a quarterly indicator, deemed to behave like a target variable, to annual data.

The resulting benchmarked values are interpolations (and extrapolations), in the sense that no original monthly measurements existed for the target variable. Similarly, monthly interpolations are obtained by benchmarking a monthly indicator to quarterly or annual data; and annual interpolations, by benchmarking an annual indicator to quinquennial data. In some cases, the indicator is in fact a mere pattern in percentages, possibly a seasonal-trading-day pattern.

It is also common to benchmark a daily pattern (of relative activity of days within the week) to data which cover four or five weeks; the resulting daily interpolations are then combined into monthly values by taking the monthly sums (Cholette and Chhab, 1991). Similarly, calendar year values may be obtained, by benchmarking a subannual series to the fiscal year data and by taking the calendar year sums (Cholette and Baldwin, 1989; Cholette, 1990); calendar quarter values, by benchmarking a monthly indicator to fiscal quarter data (Cholette, 1989). In many of these cases, the interpolations are of no interest per se, and the process is referred to as calendarization.

Without loss of generality, it is henceforth assumed that the original values are monthly and the benchmarks annual. The benchmarking models most widely used by statistical agencies are of the Denton (1971) type. Under these models, the benchmarked series fully conforms to the benchmarks, which are considered as *binding*, and the month-to-month movement of the original series is preserved as much as possible.

One current preoccupation among statisticians is that for repeated surveys, estimation procedures - and benchmarking procedures in particular - should reflect the sample design. This was discussed by Hillmer and Trabelsi (1987) and by Trabelsi and Hillmer (1990) in relation to their ARIMA model-based benchmarking method.

The main purpose of this paper is to estimate the relative efficiency of the Denton model, when the original series are contaminated with bias and autocorrelated sampling errors. According to the results presented in Section 6, taking into account bias and the behaviour of the sampling error reduces the variances of the estimates. The improvement varies with the benchmarking situation and the type of ARMA model followed by the sampling error.

Section 2 presents a benchmarking regression model which allows for bias in the original series and for a very general covariance structure of the sampling error. Section 3 discusses the relationship between this regression model and both the Denton model and the ARIMA model-based approach. Section 4 shows how the covariance structure of the sampling error in the regression model can reflect the ARMA behaviour of the error. Section 5 describes how the relative efficiencies are calculated. Section 6 discusses the results, and finally, Section 7 gives the conclusions.

2. A REGRESSION MODEL FOR BENCHMARKING

This section presents a regression benchmarking model, which consists of the following equations:

$$s_t = a + \theta_t + e_t , \quad E(e_t)=0,\ E(e_t e_{t-k}) \neq 0, \quad t=1,...,T, \tag{2.1a}$$

$$y_m = \Sigma_{t \in m}\ \theta_t + w_m , \quad E(w_m)=0,\ E(w_m w_{m-k}) \neq 0, \quad m=1,...,M. \tag{2.1b}$$

In equation (2.1a), the s_t's denotes the T monthly measurements of a socio-economic variable; the θ_t's the "true" un-observed values of the variable; and a, a bias parameter. This parameter reflects the fact that most subannual measurements are subject to bias. Parameters θ_t and a must be estimated. The estimates of θ_t will be the benchmarked series. The e_t's denote the errors affecting the observations, e.g. sampling errors; they may have a general covariance structure. Equation (2.1a) therefore states that the observations of the "true" values of the variable are contaminated with sampling error and bias.

In equation (2.1b), the y_m's denote the M annual benchmark measurements of the variable. If a benchmark y_m is not subject to error, i.e. $w_m = 0$ $\sigma_w^2=0$, it is fully reliable and *binding*; in the alternative case, it is *non-binding*. The latter are not benchmark measurement in a strict sense, but simply less frequent measurements of the target variable. Equation (2.1b) states that the observations of the annual sums of the target variable are also contaminated with errors, which may have a general covariance structure. It is assumed that e_t and w_m are mutually independent.

The system of equation (2.1) can be written in matrix algebra in one equation

$$\begin{bmatrix} \mathbf{s} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{I} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \begin{bmatrix} a \\ \theta \end{bmatrix} + \begin{bmatrix} \mathbf{e} \\ \mathbf{w} \end{bmatrix} = \mathbf{X} \begin{bmatrix} a \\ \theta \end{bmatrix} + \begin{bmatrix} \mathbf{e} \\ \mathbf{w} \end{bmatrix}, \qquad (2.2)$$

$$E(\mathbf{e})=0,\ E(\mathbf{w})=0,\ E(\mathbf{e}\ \mathbf{e}')=\mathbf{V}_e,\ E(\mathbf{w}\ \mathbf{w}')=\mathbf{V}_w,\ E(\mathbf{e}\ \mathbf{w}')=0,$$

where $\mathbf{1}$ is a T by 1 vector of ones, and where $\mathbf{J}$ is a M by T design matrix with ones and zeroes such that, for any variable, say, $\mathbf{z}$, $\mathbf{Jz}$ yields the annual sums of $\mathbf{z}$. The covariance matrices $\mathbf{V}_e$ and $\mathbf{V}_w$ are assumed known and will be specified later.

In summary, equation (2.2) specifies that the desired benchmarked series θ fits both the subannual and the annual observations and is such that the residuals display some behaviour specified by $\mathbf{V}_e$ and $\mathbf{V}_w$, as explained in Section 4.

Model (2.2) can be written as

$$\mathbf{Y} = \mathbf{X}\ \underline{\beta} + \mathbf{u}, \quad E(\mathbf{u})=0, \quad E(\mathbf{u}\ \mathbf{u}')=\mathbf{V}, \qquad (2.3)$$

where $Y' = [\ s'\ y'\ ]$, $\underline{\beta}' = [a\ \theta']$, $u' = [\ e'\ w']$, V is a block diagonal matrix with blocks V_e and V_w and where X is a design matrix implicitly defined in (2.2).

The General Least Squares solution to (2.3) yields

$$\underline{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}Y. \tag{2.4}$$

If V is the *true* (known) covariance matrix of the disturbances u, the covariance matrix of the estimates $\underline{\beta}$ is given by

$$\text{cov } \underline{\beta} = (X'V^{-1}X)^{-1}. \tag{2.5}$$

When another covariance matrix V^* is used instead of V to obtain an estimate of β, say $\underline{\beta}^*$, then

$$\text{cov } \underline{\beta}^* = [(X'V^{*-1}X)^{-1}X'V^{*-1}]\ V\ [(X'V^{*-1}X)^{-1}X'V^{*-1}]'. \tag{2.6}$$

Assuming V is used, substituting the partitions of X, V and $\underline{\beta}$ in (2.4) and matrix transformations yield

$$\begin{bmatrix} \hat{a} \\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} 1'V_e^{-1}1 & 1'V_e^{-1} \\ V_e^{-1}1 & (V_e^{-1}+J'V_w^{-1}J) \end{bmatrix}^{-1} \begin{bmatrix} 1'V_e^{-1} & 0 \\ V_e^{-1} & J'V_w^{-1} \end{bmatrix} \begin{bmatrix} s \\ y \end{bmatrix}, \tag{2.7}$$

$$\begin{bmatrix} \hat{a} \\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} V_{aa} & V_{a\theta} \\ V_{\theta a} & V_{\theta\theta} \end{bmatrix} \begin{bmatrix} 1'V_e^{-1} & 0 \\ V_e^{-1} & J'V_w^{-1} \end{bmatrix} \begin{bmatrix} s \\ y \end{bmatrix}, \tag{2.8}$$

where V_{aa} and $V_{\theta\theta}$ are the estimated variance of $\hat{a}$ and covariance matrix of $\hat{\theta}$ respectively. As shown in Appendix A, V_{aa}, $V_{a\theta}$ and $V_{\theta\theta}$ may be written as

$$V_{aa} = 1 / [1'J'(JV_eJ'+V_w)^{-1} J\ 1] = h \tag{2.9a}$$

$$V_{a\theta} = V_{\theta a}' = h\ 1' + h\ 1'J'(JV_eJ'+V_w)^{-1} J\ V_e \tag{2.9b}$$

$$\begin{aligned} V_{\theta\theta} = {} & [V_e - V_e\ J'(JV_eJ'+V_w)^{-1} J'V_e] \\ & + [I - V_eJ'(JV_eJ'+V_w)^{-1} J]\ 1\ h\ 1'\ [I - V_eJ'(JV_eJ'+V_w)^{-1} J]' , \end{aligned} \tag{2.9c}$$

which implies

$$\hat{a} = -\ h\ 1'\ J'(JV_eJ'+V_w)^{-1}\ (y - J\ s) \tag{2.10a}$$

$$\hat{\theta} = s^* + V_eJ'(JV_eJ'+V_w)^{-1}(y - J\ s^*), \quad s^{*\prime} = [s_1-\hat{a}\ s_2-\hat{a}\ \dots\ s_T-\hat{a}]. \tag{2.10b}$$

When the model includes a bias parameter, the estimated benchmarked series is given by (2.10b); and its covariance matrix, by equation (2.9c). In the absence of bias, the benchmarked series is given by (2.10b), where $\mathbf{s}^*$ is replaced by $\mathbf{s}$ ($\hat{a}$=0); and its covariance matrix reduces to the first term, in brackets, of (2.9c). In this case, the first term of (2.9c) shows that benchmarking *always* reduces the variance $\mathbf{V}_e$ of the original series s_t by the positive semi-definite matrix $(\mathbf{V}_e\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}'+\mathbf{V}_w)^{-1}\mathbf{J}'\mathbf{V}_e)$. This result conforms to that given by Hillmer and Trabelsi (1987) for an ARIMA model-based benchmarking method.

3. RELATION TO OTHER BENCHMARKING METHODS

We will here show that the benchmarking methods of the Denton type are particular cases of the regression benchmarking model (2.1); and how the latter relates to the ARIMA model-based methods of Trabelsi and Hillmer.

3.1 The Benchmarking Methods of the Denton Type

The regression model (2.1) is equivalent to the additive variant of the benchmarking methods of the Denton type, under the following assumptions:

(1) the benchmarks are binding, which implies that $\mathbf{V}_w$ is the null matrix,

(2) there is no bias parameter,

(3) the covariance matrix $\mathbf{V}_e$ of e_t is equal to $\mathbf{V}^*_e=(\mathbf{D}'\mathbf{D})^{-1}\ \sigma_v^2$, where $\mathbf{D}$ is usually a first difference operator.

In the Denton approach, the e_t's are interpreted as corrections to be made to original observations, in order to obtain a benchmarked series consistent with the annual benchmarks. First differences specify that these corrections follow a random walk process, $e_t = e_{t-1} + v_t$ (in which the variance σ_v^2 of v_t is minimized). As a result, the corrections are as constant as possible, and the benchmarked series preserves the month-to-month changes of the original as much as possible. Under assumptions (1) to (3), the model (2.1) states that the benchmarked series fits both the subannual and the annual observations and maximizes a criterion of parallelism to the original series.

The first difference operator of the original Denton (1971) method is a T by T matrix with 1's in the diagonal and -1's in the first subdiagonal. The elements v^*_{ij} of the covariance matrix $\mathbf{V}^*_e=(\mathbf{D}'\mathbf{D})^{-1}\ \sigma_v^2$ are then known to be equal to $min(i,j)\ \sigma_v^2$. The benchmarked series and its covariance matrix are respectively given by

$$\hat{\theta}^* \quad = \quad (\mathbf{X}_2'\mathbf{V}^*_e\ \mathbf{X}_2)^{-1}\ \mathbf{X}_2'\mathbf{V}^*_e\ \mathbf{Y} \tag{3.1}$$

$$\text{cov}\ \hat{\theta}^* = \quad (\mathbf{X}_2'\mathbf{V}^*_e\ \mathbf{X}_2)^{-1} \tag{3.2}$$

where $\mathbf{X}_2$ consists of the T last columns of $\mathbf{X}$ in Section 2. In situations of sizable *discrepancies* between the benchmarks and the corresponding annual sums of the original series, this difference operator produces a large divergence between the movements of the original and the benchmarked series, because it minimizes the size of the first correction. This problem has been solved by Helfand, Monsour and Trager (1977) and by Cholette (1979).

One way to avoid movement distortions, is to use the quasi-first difference operator:

$$\underset{T \text{ by } T}{\mathbf{D}} = \begin{bmatrix} (1-\phi^2)^{1/2} & 0 & 0 & \cdots \\ -\phi & 1 & 0 & \cdots \\ 0 & -\phi & 1 & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \tag{3.3}$$

where the autoregressive parameter ϕ is lower but very close to 1.0 (e.g. 0.99999). Then $\mathbf{V}^*_e=(\mathbf{D}'\mathbf{D})^{-1}\ \sigma_v^2$ is known algebraically:

$$\mathbf{V}^*_e = \begin{bmatrix} 1 & \phi & \phi^2 & \cdots & \phi^{T-1} \\ \phi & 1 & \phi & \cdots & \phi^{T-2} \\ \phi^2 & \phi & 1 & \cdots & \phi^{T-3} \\ \vdots & \vdots & \vdots & & \vdots \\ \phi^{T-1} & \phi^{T-2} & \phi^{T-3} & \cdots & 1 \end{bmatrix} \sigma_v^2\ /\ (1-\phi^2). \tag{3.4}$$

With $\mathbf{D}$ defined by (3.3), the size of the first correction is still minimized but the term is given negligible weight, and the resulting benchmarked series is virtually the same as that obtained with the strict first difference operator (in the quadratic minimization framework). With covariance matrix (3.4), the Denton method will be referred in this paper as the modified Denton method. Solution (3.1)-(3.2) is applicable.

If the original or the modified Denton is applied to a series which follows (2.1), where there bias parameter is not zero and the covariance of the sampling error is V_e the covariance of the resulting benchmarked series becomes

$$\mathrm{cov}\ \hat{\theta}^* = [(X_2'V^{*-1}X_2)^{-1}X_2'V^{*-1}]\ V\ [(X_2'V^{*-1}X_2)^{-1}X_2'V^{*-1}]' \qquad (3.5)$$

instead of (3.2).

3.2 The Trabelsi and Hillmer (1990) Method

Trabelsi and Hillmer (1990) discuss a benchmarking method (TH), which allows for autocorrelated sampling errors. The regression model (2.1) is equivalent to the TH method, under the following assumptions:

(1) the benchmarks are binding which implies that V_w is the null matrix,

(2) there is no bias parameter,

(3) the sampling error e_t follows an ARMA model.

Although the authors cast their model in the signal extraction approach, the same estimates can be obtained in the regression approach, where the benchmarked series is given by (3.1) and its covariance matrix by (3.2). The authors point out that the additive Denton method is a particular case of their model if the ARMA model followed by e_t is a random walk. This TH model is a particular case of their previous model (Hillmer and Trabelsi, 1987), where the true benchmarked series is also assumed to follow an ARIMA model and where V_w is not necessarily zero.

4. BENCHMARKING ORIGINAL SERIES WITH AUTOCORRELATED SAMPLING ERRORS

Sampling errors e_t, which may be autocorrelated can be specified in the regression benchmarking model (2.1). This may be achieved in two different manners: one, by making the elements of $\mathbf{V}_e$ (and eventually $\mathbf{V}_w$) equal to values estimated as a by-product of the surveys; or two, by modelling the behaviour of e_t by means of ARMA models. The latter approach is now discussed.

We assume that w_m=0 and $\mathbf{V}_w$=0 and that e_t follows a stationary ARMA model of order (p,q):

$$e_t - \phi_1 e_{t-1} - \phi_2 e_{t-2} - \ldots - \phi_p e_{t-p} = v_t - \zeta_1 v_{t-1} - \ldots - \zeta_q v_{t-q}, \qquad (4.1)$$

where the ϕ_j's and ζ_j's are the known p autoregressive and q moving average parameters respectively. Model (4.1) may be written in the random shock form (Box and Jenkins, 1970)

$$e_t = v_t - \psi_1 v_{t-1} - \psi_2 v_{t-2} - \psi_3 v_{t-3} - \ldots ,$$

$$\psi_j = -\zeta_j + \Sigma^{p'}_{k=1} \phi_k \psi_{j-k}, \ p'=\min(p,j), \ \zeta_j=0 \text{ if } j>q, \ \psi_0=1. \qquad (4.2)$$

Equation (4.2) implies that the elements $v_{i,j}$ of $\mathbf{V}_e$ are given by

$$v_{1,1+k} = \mathrm{E}(e_1 e_{1+k}) = \sigma_v^2 \ \Sigma^{\infty}_{j=0} \ \psi_j \psi_{j+k},$$

$$v_{1+k,1} = v_{1,1+k} \ , \qquad k=0,1,2,\ldots,T-1, \qquad (4.3a)$$

$$v_{t,t+k} = v_{t-1,t-1+k} \ , \ v_{t+k,t} = v_{t,t+k} \ , \qquad k=0,1,2,\ldots,T-t; \ t=2,\ldots,T, \qquad (4.3b)$$

where the summation converge by virtue of the stationarily of the model.

Table 1 shows nine ARMA models for the sampling errors that we will use in the regression model (2.1), to assess the relative efficiency of the Denton model discussed in Section 3.1. The notation uses the backshift operator B, such that $B^k e_t \equiv e_{t-k}$. As explained in Section 3.1, model (1a) is the one implicitly assumed by the modified Denton benchmarking model. Model (1b) was used by Hillmer and Trabelsi (1987) to illustrate their ARIMA model-based benchmarking method. Models (2a) and (2b) were used by Trabelsi and Hillmer (1990), they are supposed to account for the effect of composite estimation and the sample rotation scheme of the U.S. Retail Trade survey. Model (4) was discussed by Bell and Wilcox (1990) for the same purpose. Model (3), proposed by Binder and Dick (1989), is supposed to account for sample rotation in the Canadian Labour Force Survey. Finally, the remaining models (5) and (6) have been included to investigate the effects due to autocorrelated errors which follow a purely moving average model or a simple ARMA (1,1) model.

The covariance matrices of the sampling error models (2a) to (6) are generated by means of (4.2) and (4.3). For the first order autoregressive models (1a) to (1c), V_e is obtained from (3.4), which provides the values directly, with no need of ψ weights.

Table 1: ARMA models used for modelling the sampling error

(1a)	(1,0)(0,0)	$(1 - 0.99999B)\ e_t = \nu_t$
(1b)	(1,0)(0,0)	$(1 - 0.80000B)\ e_t = \nu_t$
(1c)	(1,0)(0,0)	$(1 - 0.20000B)\ e_t = \nu_t$
(2a)	(3,1)(1,0)	$(1 - 0.75B)(1 - 0.60B^3)(1 - 0.60B^{12})\ e_t = (1 - 0.50B)\ \nu_t$
(2b)	(3,1)(1,0)	$(1 - 0.75B)(1 - 0.30B^3)(1 - 0.30B^{12})\ e_t = (1 - 0.50B)\ \nu_t$
(3)	(3,6)(0,0)	$(1 - 0.2575B + 0.3580B^2 + 0.6041B^3)\ e_t =$ $(1 + 0.1847B + 0.5873B^2 - 0.3496B^3 - 0.0647B^4 - 0.0982B^5 - 0.0347B^6)\ \nu_t$
(4)	(3,1)(1,0)	$(1 - 0.75B)(1 - 0.70B^3)(1 - 0.75B^{12})\ e_t = (1 + 0.10B)\ \nu_t$
(5)	(0,1)(0,0)	$e_t = (1 - 0.8B)\ \nu_t$
(6)	(1,1)(0,0)	$(1 - 0.95B)\ e_t = (1 + 0.80B)\ \nu_t$

The sampling errors e_t may be both autocorrelated and heteroscedastic; following Bell and Hillmer (1989), we may express the new structure of e_t by

$$e_t = k_t\ e^*_t \tag{4.4}$$

where the k_t's are weights representing changing variance over time and the $e\dagger_t$'s follow ARMA model (4.2) with covariance matrix $V_{e\dagger}$ given by (4.3). The covariance matrix of e_t is then

$$V_e = KV_{e\dagger}K \ , \qquad (4.5)$$

where K is a diagonal matrix containing the weights k_t. In this paper, all models considered are homoscedastic; it is well established that if the k_t's are available, their use will produce a more efficient estimator.

Given V_e, the bias is estimated by (2.10a) (with V_w=0); the benchmarked series, by (2.10b); and its covariance matrix is given by (2.9c).

5. CALCULATING THE RELATIVE EFFICIENCIES

To calculate the relative efficiency of the various estimators, covariance matrices V_e were generated for stationary models (1b) to (6) of Table 1; and, matrix V^*_e for stationary model (1a). V^*_e is the covariance matrix of the modified Denton model which implicitly assumes a random walk behaviour for the sampling error. Matrices V_e and V^*_e were standardized so that their diagonal elements be equal to 1.0 (instead of some other constant). The reason for this standardization is that in empirical applications the variance of the sampling error itself would be known instead of that of the noise generating the process. When the correct regression model is used for benchmarking (i.e. V_e is used in (2.2)), the variances of the benchmarked series are given by the diagonal values of (2.5). When the Denton model is applied (i.e. V^*_e is used), the variance of the benchmarked series is given by the diagonal values of covariance matrix (3.5). The relative efficiency is the ratio of the traces of the two covariance matrices.

We assume that the series s_t to be benchmarked contains 7 years and 7 months of observations; that annual benchmarks are available for years 1 to 5; that the benchmark of year 6 (t=61,...,72) is not available; and that year 7 (t=73,...,79) is incomplete. Missing benchmarks and incomplete years at the end of series are typical of real benchmarking situations. The regression model "preliminarily" benchmarks years 6 and 7, by *projecting* the estimated sampling error according to the error model selected. The benchmarking situation just described implies the following design matrix in (2.2)

$$\mathbf{J} = [\ \mathbf{I}_5 \otimes \mathbf{j} \quad \mathbf{0}\ ], \tag{5.1}$$

where **j** is a 1 by 12 vector of 1's and **0** is a 5 by 19 null matrix.

Some ARMA models considered for e_t imply complex subannual movements of the sampling error, and consequently, besides annual benchmarks, subannual benchmarks must be available. The role of these subannual benchmarks is to peg the subannual movement of the sampling error (in the same way as annual benchmarks can be described as pegging the annual sums of the sampling errors). We assume that 11 subannual benchmarks are available for the first 11 months of year 4. The design matrix in (2.2) is then

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_a \\ \mathbf{J}_s \end{bmatrix}, \quad \mathbf{J}_s = [\ \mathbf{0}_1\ \mathbf{I}_{11}\ \mathbf{0}_2\ ], \tag{5.2}$$

where $\mathbf{J}_a$ is given by (5.1), and $\mathbf{0}_1$ and $\mathbf{0}_2$ are 11 by 36 and 11 by 32 null matrices respectively.

6. ANALYSIS OF THE RESULTS

Table 2 and 3 display the relative efficiencies for "historical" benchmarking (years 1 to 5) and for "preliminary benchmarking" respectively, when: (a) only annual benchmarks are available and (b) annual and subannual benchmarks are available. Subannual benchmarks are required when the sampling models is of a seasonal type. For a given model, the relative efficiency is defined by the ratio of the trace of (3.5) over the trace of (2.5) and reflects the increase in variance due to the application of the Denton model instead of the optimal regression model.

Analysis for Historical Benchmarking - According to Table 2, when only annual benchmarks are available, the optimal model is only slightly more efficient than the Denton method, except for sampling error model (3). When both annual and subannual benchmarks are available, the gain in efficiency is higher (than with annual benchmarks only). It is highest for models which imply stronger subannual movements, namely, seasonal models (2a) and (4), and non-seasonal model (3), because the subannual benchmarks now specify the subannual movements. (Seasonal model (2b) is dominated by the regular autoregressive part.) For the remaining models, the gain in efficiency is only marginally higher (than with annual benchmarks only), because there is not much subannual movement to specify..

Table 2: Relative efficiencies of the Denton model versus "optimal" regression models for "historical" benchmarking

ARMA model (p,q)(P,Q) for the sampling errors	When only annual benchmarks are available	When annual and sub-annual benchmarks are available
(1b) (1,0)(0,0)	1.021	1.075
(1c) (1,0)(0,0)	1.012	1.014
(2a) (3,1)(1,0)	1.003	1.403
(2b) (3,1)(1,0)	1.006	1.064
(3) (3,6)(0,0)	1.129	2.011
(4) (3,1)(1,0)	1.010	1.770
(5) (0,1)(0,0)	1.050	1.057
(6) (1,1)(0,0)	1.005	1.006

Analysis for Preliminary Benchmarking - Table 3 displays the relative efficiencies for preliminary benchmarking, e.g. for years 6 and 7. For those years, the estimates of the sampling error do not have to satisfy any annual or subannual benchmarks. They are in effect ARMA forecasts, with initials values provided by the estimates of years 5 and 4, which eventually converge to the value given by the estimated bias parameter (except for the Denton model (1a)). When only annual benchmarks are available, Table 3 shows a gain in efficiency of the optimal model over the Denton model. The gain is higher for error models which imply less subannual movement, namely (1b), (1c) and (2b). The gain is lower for the models which imply strong subannual movement, namely models (2a), (3) and (4), because no subannual benchmark is there to specify a particular subannual movement. The gain is also lower for model (5), because MA models have short memory and are inherently harder to predict. When both annual and subannual benchmarks are available, the gain in efficiency remains high for the models which imply less subannual movement, because it does not hurt to have subannual benchmarks; it becomes high for models which imply stronger subannual movement, because the subannual benchmarks now specify such movement; it remains low for MA model (5).

Table 3: Relative efficiencies of the Denton model versus "optimal" regression models for "preliminary" benchmarking

ARMA model (p,q)(P,Q) for the sampling errors	When only annual benchmark are available	When annual and sub-annual benchmarks are available
(1b) (1,0)(0,0)	1.369	1.373
(1c) (1,0)(0,0)	1.147	1.149
(2a) (3,1)(1,0)	1.044	1.134
(2b) (3,1)(1,0)	1.141	1.146
(3) (3,6)(0,0)	1.050	1.439
(4) (3,1)(1,0)	1.065	1.174
(5) (0,1)(0,0)	1.028	1.031
(6) (1,1)(0,0)	1.090	1.090

7. SUMMARY AND CONCLUSION

The Denton method is widely applied by statistical agencies to perform "historical" and "preliminary" benchmarking of original values, without any explicit consideration of the sampling errors affecting the data.

In this paper, we have calculated the relative efficiency of the Denton method versus "optimal" regression models that incorporate various ARMA models for the sampling error. These ARMA models have been proposed by Hillmer and Trabelsi (1987), Trabelsi and Hillmer (1990), Binder and Dick 91989) and Bell and Wilcox (1990) in the context of model-based benchmarking and sampling.

The results discussed in Section 6 show that for "historical" benchmarking, the increase in efficiency of the optimal model versus the Denton model is very large if the sampling error follows seasonal ARMA or a complex MA process, *as long as annual and subannual benchmarks are available.* However, under the assumption that only annual benchmarks are available (which usually is the case), the improvement in the relative efficiencies is negligible except for the complex MA models.

The greater improvements brought about by the optimal method are observed for preliminary benchmarking, whether subannual benchmarks are available or not. However, the improvement is larger when sub-annual (and annual) benchmarks are available for those models which imply seasonal or complex sub-annual movements.

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APPENDIX A: Derivation of result (2.9) and (2.10)

Performing the inversion in (2.7) by blocks yields

$$V_{aa} = 1 / [\mathbf{1}'V_e^{-1} \mathbf{1} - \mathbf{1}'V_e^{-1} (V_e^{-1} + J'V_w^{-1}J)^{-1} V_e^{-1} \mathbf{1})] = h, \quad (A.1a)$$

$$V_{a\theta} = V_{\theta a}' = - h \, \mathbf{1}'V_e^{-1} (V_e^{-1} + J'V_w^{-1}J)^{-1}, \quad (A.1b)$$

$$V_{\theta\theta} = [(V_e^{-1} + J'V_w^{-1}J)^{-1}] \quad (A.1c)$$

$$+ [(V_e^{-1} + J'V_w^{-1}J)^{-1} V_e^{-1}\mathbf{1} \, h \, \mathbf{1}'V_e^{-1} (V_e^{-1} + J'V_w^{-1}J)^{-1}].$$

Substitution of (A.1) in (2.8) and some lengthy algebra yields

$$\hat{a} = h \, \mathbf{1}'V_e^{-1} \mathbf{s} - h \, \mathbf{1}'V_e^{-1} (V_e^{-1} + J'V_w^{-1}J)^{-1} (V_e^{-1} \mathbf{s} + J'V_w^{-1} \mathbf{y}), \quad (A.2a)$$

$$\hat{\theta} = (V_e^{-1} + J'V_w^{-1}J)^{-1} (V_e^{-1} \mathbf{s}^* + J'V_w^{-1} \mathbf{y}), \quad \mathbf{s}^{*\prime} = [s_1-\hat{a} \; s_2-\hat{a} \; \ldots \; s_T-\hat{a}]. \quad (A.2b)$$

The benchmarking methods of the Denton type were originally based on minimization of the quadratic form $(\theta-\mathbf{s})'V_e^{-1}(\theta-\mathbf{s})$. This process starts by specifying V_e^{-1} (and not V_e). In such cases, solution (A.1)-(A.2) is appropriate and requires one matrix inversion, that of $(V_e^{-1} + J'V_w^{-1}J)$. Note that $(V_e^{-1} + J'V_w^{-1}J)$ has to have full rank, but not necessarily V_e^{-1}.

If the covariances matrices V_e and V_w are given, solution (A.1)-(A.2) can be written in terms of V_e and V_w as (2.9)-(2.10) respectively, using matrix identities and lengthy algebra as in Hillmer and Trabelsi (1987). The matrix inversion of $(JV_eJ' + V_w)$ required by (2.9)-(2.10) is of smaller dimension (M by M) than that required in (A.1)-(A.2) (T by T). Furthermore, solution (2.9)-(2.10) admits the particular case where V_w is exactly equal to $\mathbf{0}$, contrary to (A.1)-(A.2) where V_w may only *tend* to zero.

c. 2