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A SIMPLE TEST FOR STABLE SEASONALITIES

by

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Summary

The adjustment of economic and social time series for seasonal variation has been and continues to be the subject of much attention. As a first step towards seasonally adjusting a series, it is essential to test for the presence of seasonalities. This paper uses an ANOVA type model for seasonality in time series. Because autocorrelation invalidates usual ANOVA F-tests, we discuss an exact modified F-test for testing for stable seasonalities. The test procedures are illustrated using two sets of data.

KEY WORDS: Seasonalities; Two-way correlations; Dependent quadratic forms; Modified F-tests; Seasonal adjustments.

1. INTRODUCTION

Seasonal adjustment procedures are widely employed in the analysis of economic data. One of the main reasons for the adjustment of economic and social time series for seasonal variation is that seasonal components may represent the effects of non-economic factors that are exogenous to the economic system and hence are uncontrollable.

There are various methods available to deseasonalize a time series. The U.S. Bureau of the Census Method II-X-11 variant developed by Shiskin, Young, and Musgrave (1967) and the X-11-ARIMA version developed by Dagum (1975, 1980) are widely used by government agencies and statistical bureaus. These seasonal adjustment methods are based mainly on moving average techniques. Kenny and Durbin (1982) provided some methods of improving the performance of the X-11 seasonal adjustment procedures. Wallis (1982) suggested procedures for seasonal adjustment and revision of current data by linear filter methods. However, there are situations where the seasonality in a time series may not be significant. In such cases the adjustment for seasonality is unnecessary. This suggests testing for the presence of significant seasonality in a time series before making seasonal adjustments. But this problem of testing for seasonality has not been adequately addressed in the literature.

To identify seasonality in a time series, the X-11-ARIMA method uses the results of the usual F-test in a one-way ANOVA of between-season variation of the seasonal plus irregular terms (from an additive X-11 decomposition). Usually the residuals in this ANOVA will be found to be autocorrelated, so the nominal significance level of the F-test is not valid, especially when a strong autocorrelation effect is present. The three classical test procedures, likelihood ratio, Wald and Lagrange multiplier, all encounter similar difficulties {cf. Franzini and Harvey (1983, section 3), Sargan and Bhargava (1983), Sutradhar and Bartlett (1990)}.

Franzini and Harvey (1983) developed an exact test for testing for deterministic trend and seasonality in a time series. Under a model formulated in first differences, they have also discussed a test proposed mainly for testing for stability of a seasonal pattern. Since this test cannot be applied to decide whether the stable seasonality is significant, it leaves open an important question.

In the present paper we deal with a model for the detrended time series which is similar to Franzini and Harvey's (1983) model formulated in first differences. As a supplement to the Franzini and Harvey methodology we propose a test for the significance of stable seasonality for seasonal adjustment purposes. To do this, we follow Sutradhar, MacNeill and Sarhrmann (1987) {see also Sutradhar and Bartlett (1989)} and develop a modified F-test which accounts for autocorrelation in the detrended series. The proposed modified F-test is simple to compute. Distribution theory for the modified F-statistic and two examples illustrating the use of the test are discussed.

2. THE MODEL

We assume that an observable time series at time t, Y_t, can be represented as follows:

$$Y_t = T_t + S_t + I_t \quad , \qquad (t = 1, \ldots, T) \tag{2.1}$$

where T_t, S_t, and I_t are unobservable trend-cycle, seasonal and irregular components. If T contains only stochastic trend, then a detrended series, $\{Z_t\}$, may be obtained from the original series, $\{Y_t\}$, by taking appropriate differences. However, if T contains both deterministic and stochastic trends, differencing will result in a model similar to the partially deterministic model considered by Franzini and Harvey {1983, equation (4.1)}. We write this model as follows:

$$Z_t = \beta_t + \gamma_t + u_t \quad , \qquad (t = 1, \ldots, T_1 \leq T) \tag{2.2}$$

where β_t represents the putative deterministic trend, and γ_t and u_t are seasonal and irregular components.

With the assumptions that $\beta_t = \beta_{t-1} + \eta_t$ with $\eta_t \sim N(0, \sigma_\eta^2)$, and $\sum_{h=0}^{s-1} \gamma_{t-h} = \zeta_t - \zeta_{t-1}$, where $\zeta_t = \sum_{h=0}^{s-1} S_{t-h} \sim N(0, \sigma_\zeta^2)$, s being the seasonality period, Franzini and Harvey tested the null hypothesis, Ho:$\sigma_\eta^2 = 0, \sigma_\zeta^2 = 0$ for the presence of deterministic trend and for seasonality. Notice that $\sigma_\zeta^2 = 0$ indicates that the seasonal pattern is constant over the years, and $\sigma_\zeta^2 > 0$ indicates that the seasonal pattern is changing over the years. Since the test for $\sigma_\zeta^2 = 0$ does not test for significance of the deterministic seasonality, we formulate the hypothesis in the following way.

Let the detrended series $\{Z_t\}$, be obtained by applying the X-11 method due to Shiskin, Young and Musgrave (1967) or the X-11-ARIMA method due to Dagum (1980). These methods perform well in removing the deterministic and stochastic trends from the data without any distortion of the seasonal pattern. The detrended series will be referred to as the series of seasonal irregular differences. Suppose there are k seasons under each of n years in the series. If we write $Z(t)$ for Z_t, then the detrended series, $\{Z_t\}$, may be expressed as

$$Z\{(i-1)n+j\} = \gamma\{(i-1)n+j\} + u\{(i-1)n+j\} \quad , \tag{2.3}$$

$$i = 1, \dots, k \quad , \qquad j = 1, \dots, n$$

where $Z\{(i-1)n+j\}$ is the jth observation under the ith season. We re-express (2.3) as

$$Z_i(j) = \gamma_i(j) + u_i(j) \quad . \tag{2.4}$$

Since the seasonal pattern of the original series is assumed to be unaffected by detrending, $\sum_{i=1}^{k} \gamma_i(j)$ should vanish for all $j = 1, \dots, n$. Further, suppose that $\mathrm{Var}\{\gamma_i(j)\} = \sigma_j^2$ for all $i = 1, \dots, k$.

Then Franzini and Harvey's (1983) test for the presence of a constant seasonal pattern is equivalent to the test of $H_0 : \sigma_1^2 = \cdots = \sigma_n^2 = \sigma^2 > 0$ against the alternative hypothesis, H_1: Variances are unequal. It will be assumed in this paper that the variances are equal, that is, the seasonal pattern is constant over time. Then the test for the presence of significant stable seasonality is equivalent to the test $H_0 : \sigma_1^2 = \cdots = \sigma_n^2 = \sigma^2 = 0$ against $H_1 : \sigma^2 > 0$. When $\sigma_j^2 = \sigma^2$ for all $j = 1, \ldots, n$, without any loss of generality, one can express (2.4) as

$$Z_i(j) = \gamma_i + u_i(j) \quad . \tag{2.5}$$

Then, testing $\sigma^2 = 0$ is equivalent to testing $\gamma_i = 0$. The condition $\sum_{i=1}^{k} \gamma_i = 0$ must be satisfied whether all $\gamma_i = 0$ or $\gamma_i \neq 0$ for at least one i. The model in (2.5) is a one-way ANOVA model. The main difference between a traditional ANOVA model and the ANOVA model in (2.5) is that unlike the traditional ANOVA model, the residuals of the present model (2.5) are autocorrelated. Since $\{Z_i(j)\}$ in (2.5) is a detrended series, it is reasonable to assume that the residuals $\{u_i(j)\}$ follow a general SARMA $(p, q)(P, Q)_s$ process. In the error process, p and q are the autoregressive (AR) and moving average (MA) orders of the regular part of the series, and P and Q are the AR and MA orders of the seasonal part of the series, s being the seasonality period. Usually $s = k$.

3. TEST FOR SEASONALITY

Let $\boldsymbol{Z}_i = [Z\{(i-1)n+1\}, \ldots, Z\{(i-1)n+j\}, \ldots, Z\{in\}]'$ be the $n \times 1$ column vector containing the n seasonal irregular differences under the ith season, and $\boldsymbol{Z}^* = (\boldsymbol{Z}_1', \ldots, \boldsymbol{Z}_i', \ldots, \boldsymbol{Z}_k')'$. Also let $\boldsymbol{U}^*$ be the $kn \times 1$ stacked vector of residuals, which corresponds to $\boldsymbol{Z}^*$. Then for $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_{k-1})'$, and by exploiting the notation in (2.3) and (2.4), one may write the model (2.5) as

$$\boldsymbol{Z}^* = \boldsymbol{X}\boldsymbol{\gamma} + \boldsymbol{U}^* \quad , \tag{3.1}$$

where $\boldsymbol{X}$ is the $kn \times (k-1)$ design matrix given as follows:

$$\boldsymbol{X} = \begin{pmatrix} \mathbf{1}_n' & \mathbf{0}' & \mathbf{0}' & \ldots & \mathbf{0}' & \mathbf{0}' \\ \mathbf{0}' & \mathbf{1}_n' & \mathbf{0}' & \ldots & \mathbf{0}' & \mathbf{0}' \\ & & & & & \\ \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \ldots & \mathbf{1}_n' & \mathbf{0}' \\ \mathbf{0}' & \mathbf{0}' & \mathbf{0}' & \ldots & \mathbf{0}' & (-\mathbf{1}_n') \end{pmatrix}$$

where '$\mathbf{1}_n$' and '$\mathbf{0}$' are the $n \times 1$ column vectors of ones and zeroes respectively. In terms of the γ_k parameters in (3.1), the hypothesis $H_0 : \gamma_i = 0$ with $\sum_{i=1}^{k} \gamma_i = 0$, may be expressed in the general linear form

$$H_0 : \boldsymbol{C\gamma} = \mathbf{0} \ , \tag{3.2}$$

where $\boldsymbol{C}$ is the $(k-1) \times (k-1)$ matrix given by

$$\boldsymbol{C} = \begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 0 & 0 & \dots & -1 \\ 2 & 1 & 1 & \dots & 1 \end{pmatrix} .$$

Let $\boldsymbol{\Sigma}^*$ be the $kn \times kn$ covariance matrix of $\boldsymbol{U}^*$ given by

$$\boldsymbol{\Sigma}^* = \begin{pmatrix} \boldsymbol{\Sigma} & \boldsymbol{\Lambda}_1 & \boldsymbol{\Lambda}_2 & \dots & \boldsymbol{\Lambda}_{k-1} \\ \boldsymbol{\Lambda}_1' & \boldsymbol{\Sigma} & \boldsymbol{\Lambda}_1 & \dots & \boldsymbol{\Lambda}_{k-2} \\ \vdots & \vdots & \vdots & & \vdots \\ \boldsymbol{\Lambda}_{k-1}' & \boldsymbol{\Lambda}_{k-2}' & \boldsymbol{\Lambda}_{k-3}' & \dots & \boldsymbol{\Sigma} \end{pmatrix} \tag{3.3}$$

where $\boldsymbol{\Sigma}$ and $\boldsymbol{\Lambda}_h$'s $(h = 1, \dots, k-1)$ are $n \times n$ symmetric matrices. More specifically $\boldsymbol{\Sigma}$ and the $\boldsymbol{\Lambda}_h$'s are defined by

$$\boldsymbol{\Sigma} = \sigma_u^2 (\sigma_{jj'}) \ ,$$

and

$$\boldsymbol{\Lambda}_h = \sigma_u^2 (\lambda_{jj'}^{(h)}) \ ,$$

for $j, j' = 1, \dots, n$ and $h = |i - r| > 0$, $i, r = 1, \dots, k$. Notice that $\sigma_u^2 \sigma_{jj'}$ is the covariance between the residuals corresponding to the jth and j'th years for season i $(i = 1, \dots, k)$, and $\sigma_u^2 \lambda_{jj}^{(h)}$ is the covariance between the residuals corresponding to the jth and j'th years when the seasons are separated by $h = |i - r|$ for $i \neq r$, $i, r = 1, \dots, k$.

For moderately strong serial correlation among the residuals, the linear hypothesis $H_0 : \boldsymbol{C\gamma} = \mathbf{0}$ (3.2) may be tested in a generalized least squares (GLS) framework. However, the GLS approach runs into convergence problems when $\boldsymbol{\Sigma}^*$ is too weak to obtain its inverse; this occurs when the residuals are highly autocorrelated. In testing $\boldsymbol{C\gamma} = \mathbf{0}$, Pierce (1978, p.250) has discussed an F-test which is asymptotically valid. The test was constructed based on transformed residuals which are approximately white noise. We propose to test for the significance of stable seasonality, that is, $H_0 : \boldsymbol{C\gamma} = \mathbf{0}$, by using a modification to the standard F-test which takes the autocorrelations of the residuals into account. Unlike Pierce (1978), we do not require a transformation to construct the test. Moreover, as it is discussed in Section 4, one can exploit a finite sample approach to the computation of the p-values of the proposed modified statistic.

The standard F-test for testing the linear hypothesis $\boldsymbol{C\gamma} = \mathbf{0}$, is given by

$$F = \frac{\frac{(\boldsymbol{C\hat{\gamma}} - \boldsymbol{C\gamma})'(\boldsymbol{C}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{C}')^{-1})(\boldsymbol{C\hat{\gamma}} - \boldsymbol{C\gamma})}{(k-1)}}{\frac{(\boldsymbol{Z}^* - \boldsymbol{X\hat{\gamma}})'(\boldsymbol{Z}^* - \boldsymbol{X\hat{\gamma}})}{k(n-1)}} \tag{3.4}$$

where $\hat{\boldsymbol{\gamma}}$ is the ordinary least square estimate of $\boldsymbol{\gamma}$. Following Sutradhar, MacNeill and Sahrmann (1987), we now modify this standard F-statistic such that the denominator and the numerator of the modified statistic have the same mean under the null hypothesis, irrespective of the correlation structure of the data. The modified F-statistic F_M is given by

$$F_M = \frac{(k-1)\{\operatorname{trace}(\boldsymbol{I} - \boldsymbol{D}_2)\hat{\boldsymbol{\Sigma}}^*\}F}{k(n-1)\operatorname{trace}(\boldsymbol{D}_1\hat{\boldsymbol{\Sigma}}^*)} \quad , \tag{3.5}$$

where F is given in (3.4), $\hat{\boldsymbol{\Sigma}}^*$ is a consistent estimate of $\boldsymbol{\Sigma}^*$, and $\boldsymbol{D}_1 = \boldsymbol{R}'(\boldsymbol{RR}')^{-1}\boldsymbol{R}$, with $\boldsymbol{R} = \boldsymbol{C}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}$ and $\boldsymbol{D}_2 = \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$. Note that in the present setup, one seasonal time series is partitioned into k correlated groups, each group containing n observations due to n years.

For the cases when k groups are independent and when n observations in each group follow a SARMA $(0,q)(0,Q)_s$ process, Sutradhar and Bartlett (1989, Table

IV, p.1587) have shown by a simulation study that the distribution of F_M may be approximated by the usual F-distribution with $(k-1)$ and $k(n-1)$ degrees of freedom. In general, the distribution of F_M may not be approximated by the usual F-distribution, specially when k groups are correlated and n is small (for example in a series of 120 observations with 12 seasons, n is only 10). In the following section, we provide a finite sample approach to the calculation of quantiles of F_M.

4. p-VALUE COMPUTATION

For significance testing one requires the p-value of the statistic, namely, $\Pr(F_M \geq f_M)$, where f_M is the data based value of F_M. For known $\boldsymbol{\Sigma}^*$, the modified F-statistic (3.5) may be expressed under the null hypothesis as

$$F_M = \frac{\operatorname{trace}\{(\boldsymbol{I}-\boldsymbol{D}_2)\boldsymbol{\Sigma}^*\}\{\boldsymbol{U}^{*\prime}\boldsymbol{D}_1\boldsymbol{U}^*\}}{\operatorname{trace}(\boldsymbol{D}_1\boldsymbol{\Sigma}^*)\{\boldsymbol{U}^{*\prime}(\boldsymbol{I}-\boldsymbol{D}_2)\boldsymbol{U}^*\}} \quad . \tag{4.1}$$

Then computing $\Pr(F_M \geq f_M)$ is equivalent to computing f_M so that

$$\Pr[Q = m_1 Q_1 + m_2 Q_2 \geq 0] \quad , \tag{4.2}$$

where

$$m_1 = \operatorname{trace}(\boldsymbol{I}-\boldsymbol{D}_2)\boldsymbol{\Sigma}^* \quad , \qquad m_2 = -f_M \operatorname{trace} \boldsymbol{D}_1\boldsymbol{\Sigma}^* \quad ,$$

$$Q_1 = \boldsymbol{U}^{*\prime}\boldsymbol{A}_1\boldsymbol{U}^* \quad , \qquad \text{and} \qquad Q_2 = \boldsymbol{U}^{*\prime}\boldsymbol{A}_2\boldsymbol{U}^* \quad ,$$

with $\boldsymbol{A}_1 = \boldsymbol{D}_1$, and $\boldsymbol{A}_2 = \boldsymbol{I} - \boldsymbol{D}_2$, $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ being given in (3.5). Note that Q_1 and Q_2 are two quadratic forms in $\boldsymbol{U}^*$, where $\boldsymbol{U}^*$ has a kn-dimensional normal distribution with zero mean vector and covariance matrix $\boldsymbol{\Sigma}^*$. Consequently, for a given f_M, in a manner similar to Sutradhar (1990, p.7–8), one may exploit the mixed cumulants of Q_1 and Q_2 up to order four, to obtain the first four moments of Q, which is a linear combination of Q_1 and Q_2. Next the distribution of Q may be approximated using, for example, the algorithm AS99 due to Hill, Hill and Holder (1976) to fit the Johnson (1949) curve which has the same first four moments as Q. But this four moment approximation is computationally expensive. As an alternative method, we use the well-known Satterthwaite (1946) approximation to compute the significance level of the F_M test for testing the significance of stable seasonality.

4.1 SATTERTHWAITE APPROXIMATION

We re-write (4.1) in the form

$$\Pr(F_M \geq f_M) = \Pr\left[U^{*\prime}\{d^* D_1 - f_M(I - D_2)\}U^* \geq 0\right] \quad , \tag{4.3}$$

where $d^* = \operatorname{trace}\{(I - D_2)\Sigma^*\}/\operatorname{trace} D_1\Sigma^*$. The probability in (4.3) is equivalent to

$$\Pr\left[\delta^{*\prime}\Sigma^{*\frac{1}{2}}\left\{d^* D_1 - f_M(I - D_2)\right\}\Sigma^{*\frac{1}{2}}\delta^* \geq 0\right] \quad , \tag{4.4}$$

where $\delta^* \sim N(0, I_{kn})$. Further, this probability is equivalent to

$$\Pr\left[\left\{\frac{\sum_{j=1}^{r}\lambda_j\chi_j^2}{\sum_{j=m+1}^{N}|\lambda_j|\chi_j^2}\right\} \geq 1\right] \quad . \tag{4.5}$$

where $\lambda_1 \geq \cdots \geq \lambda_r > 0 = \lambda_{r+1} = \cdots = \lambda_m > \lambda_{m+1} \geq \lambda_{m+2} \geq \cdots \geq \lambda_N$ are the eigenvalues of $\Sigma^{*\frac{1}{2}}(d^* D_1 - f_M(I - D_2))\Sigma^{*\frac{1}{2}}$. Then, the Satterthwaite approximation yields

$$\Pr[F_M \geq f_M] = \Pr\left[F_{a,b} \geq \frac{bd}{ac}\right] \quad , \tag{4.6}$$

where $F_{a,b}$ denote the usual F-ratio with degrees of freedom a and b, with

$$a = \frac{\left(\sum_{j=1}^{r}\lambda_j\right)^2}{\sum_{j=1}^{r}\lambda_j^2} \quad \text{and} \quad b = \frac{\left(\sum_{j=m+1}^{N}\lambda_j\right)^2}{\sum_{j=m+1}^{N}\lambda_j^2} \quad .$$

In (4.6),

$$c = \frac{\sum_{j=1}^{r}\lambda_j^2}{\sum_{j=1}^{r}\lambda_j} \quad \text{and} \quad d = \frac{\sum_{j=m+1}^{N}\lambda_j^2}{\sum_{j=m+1}^{N}|\lambda_j|} \quad .$$

If a and b are fractions, the probability is computed by interpolation.

5. APPLICATIONS

CANADIAN EXPORT SERIES

The data file for this study contains export records of Canada from 1972 to 1981. We now test for the presence of seasonality in the export data, where seasonality represents the composite effect of climatic and institutional events which repeat more or less regularly. A plot of the data shows a noticeable trend {see Figure 1(a)} over the years. Hence, the data were detrended using the appropriate steps of the X-11-ARIMA procedure due to Dagum (1980). It was found that the detrended data are best modelled by a $(0,1)(0,1)_{12}$ process with

$$\theta = 0.2765 \ , \quad \Theta = 0.2995 \quad \text{and} \quad \sigma_u^2 = 37.1523 \ .$$

The detrended data are shown in Figure 1(b). The component matrices of the $kn \times kn$ covariance matrix $\boldsymbol{\Sigma}^*$ (3.3) of $\boldsymbol{Z}^*$ are:

$$\boldsymbol{\Sigma} = (\sigma_u^2) \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & \dots & 0 & 0 \\ \sigma_{21} & \sigma_{11} & \sigma_{12} & & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_{11} & \sigma_{12} \\ 0 & 0 & 0 & \dots & \sigma_{21} & \sigma_{11} \end{pmatrix}$$

$$\Lambda_1 = (\sigma_u^2) \begin{pmatrix} \lambda_{11}^{(1)} & \lambda_{12}^{(1)} & 0 & \dots & 0 & 0 \\ \lambda_{21}^{(1)} & \lambda_{11}^{(1)} & \lambda_{12}^{(1)} & & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \lambda_{11}^{(1)} & \lambda_{12}^{(1)} \\ 0 & 0 & 0 & \dots & \lambda_{21}^{(1)} & \lambda_{11}^{(1)} \end{pmatrix}$$

$$\Lambda_{11} = (\sigma_u^2) \begin{pmatrix} \lambda_{11}^{(11)} & 0 & 0 & \dots & 0 & 0 \\ \lambda_{21}^{(11)} & \lambda_{11}^{(11)} & 0 & \dots & 0 & 0 \\ \lambda_{31}^{(11)} & \lambda_{21}^{(11)} & \lambda_{11}^{(11)} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \lambda_{11}^{(11)} & 0 \\ 0 & 0 & 0 & \dots & \lambda_{21}^{(11)} & \lambda_{11}^{(11)} \end{pmatrix}$$

and $\Lambda_h = 0$ for $h = 2, \ldots, 10$; where

$$\sigma_{11} = (1+\theta^2)(1+\Theta^2) \ , \qquad \sigma_{12} = \sigma_{21} = \Theta(1+\theta^2) \ ,$$

$$\lambda_{11}^{(11)} = \lambda_{21}^{(11)} = -(1+\Theta^2) \qquad \text{and} \qquad \lambda_{12}^{(1)} = \lambda_{21}^{(1)} = \lambda_{11}^{(11)} = \lambda_{31}^{(11)} = \theta\Theta \ .$$

To test $H_0 : C\gamma = 0$ (3.2), we find $a = 10.15$, $b = 80.33$, $c = 12.66$, and $d = 12.41$. Then the Satterthwaite approximation (4.6) yields the p-value, $\Pr(F_{10,80} \geq 7.76) \simeq 0.00$. Therefore, we reject H_0 and conclude that the series contains significant stable seasonality.

This confirms the behaviour of the detrended series in Figure 1(b).

We remark that the original series was also detrended by taking appropriate differences. The first difference was enough to obtain a detrended series, which was found to follow a (0,1) process with $\theta = 0.808$, and $\sigma_u^2 = 105.52$. The modified F-value, f_M was found to be 2.2316. By computations similar to those performed above, the Satterthwaite approximation (4.6) can be shown to yield $\Pr(F_{8,72} \geq 2.24) = 0.034 < 0.05$. Thus, the decision based on the detrended series obtained by using differencing remains the same as in the case where the data were detrended by the X-11-ARIMA method.

CANADIAN HOMICIDE DATA (TOT SERIES)

The TOT data refer to all murders in Canada for the period from 1961 to 1980 {see Figure 2(a)}; this excludes manslaughter and infanticide as they are not classified as murder and are not available for the entire period. We do not use the X-11-ARIMA technique to detrend this series since Figure 2(a) shows an upward movement over the years suggesting straightforward first differencing. The series detrended by differencing is shown in Figure 2(b), and is modelled as a (0,1) process with $\theta = 0.8911$, and $\sigma_u^2 = 309.23$.

In testing for the presence of stable seasonality, we find $a = 7.90$, $b = 71.07$, and $bd/ac = 1.158$. Since by (4.6) $\Pr(F_{8,71} \geq 1.158) = 0.337$, we decided in favour of $H_0 : \gamma_i = 0$, i.e., there is no significant stable seasonality in the TOT series, which confirms the behaviour of the detrended series in Figure 2(b). This conclusion regarding the seasonal movement in the monthly murder series is also in agreement with the conclusion in Dagum, Huot and Morry (1988).

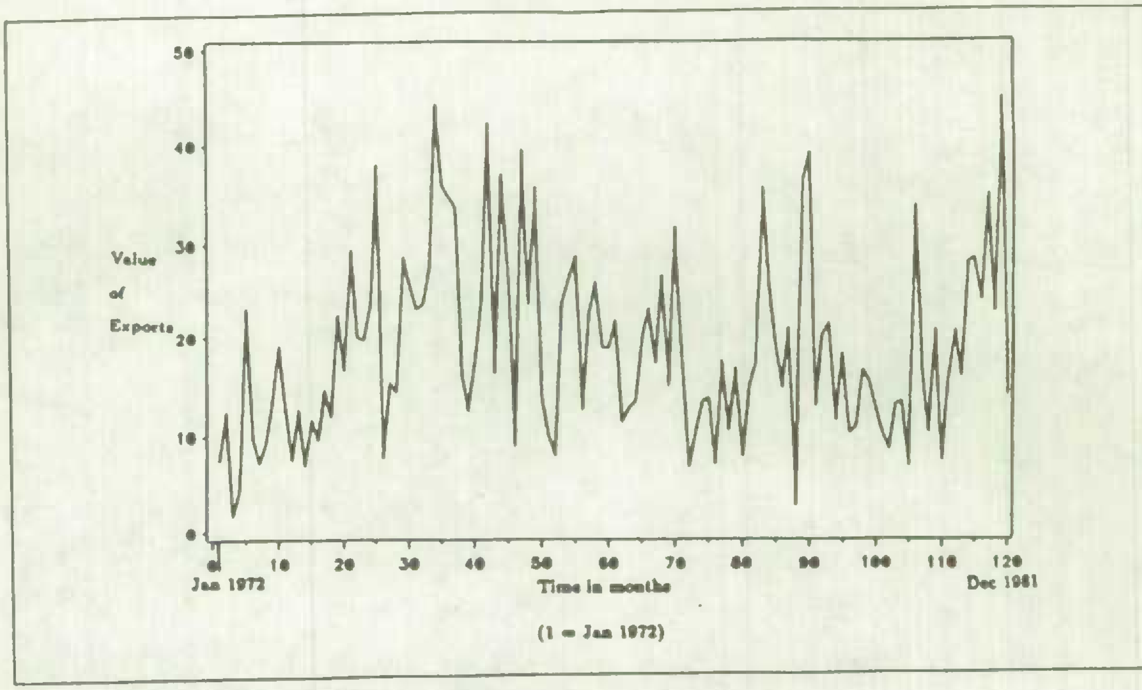

Figure 1a. Canadian monthly exports (1972-1981)

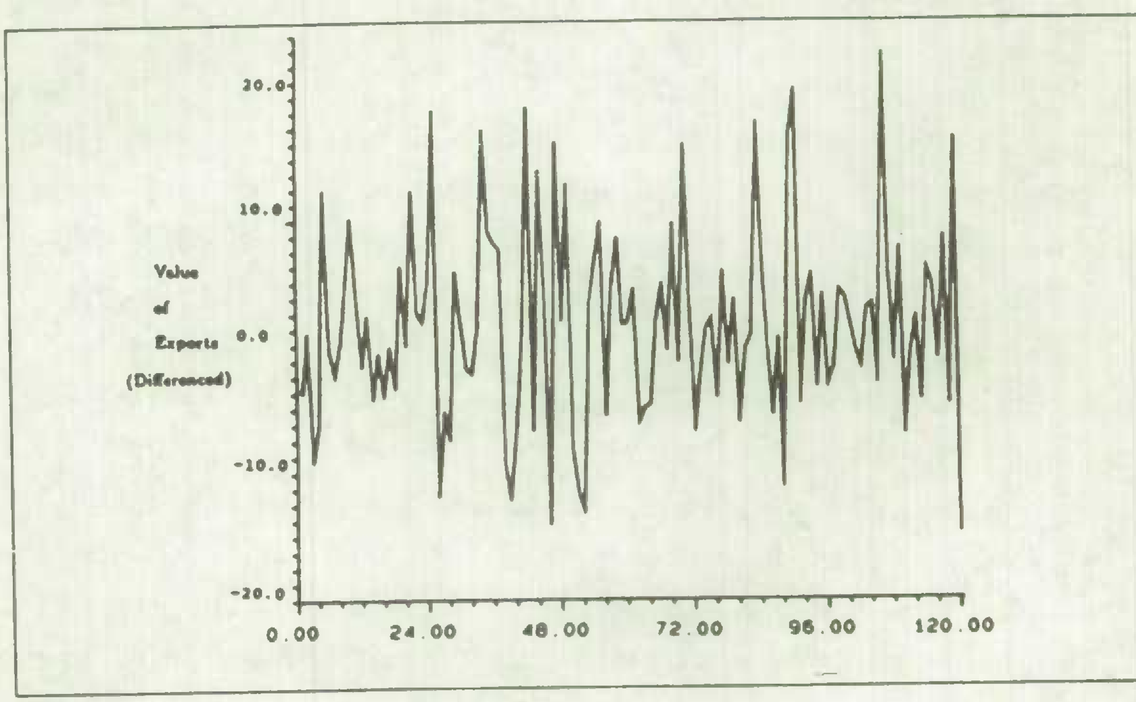

Figure 1b. Export series–detrended by X-11-ARIMA

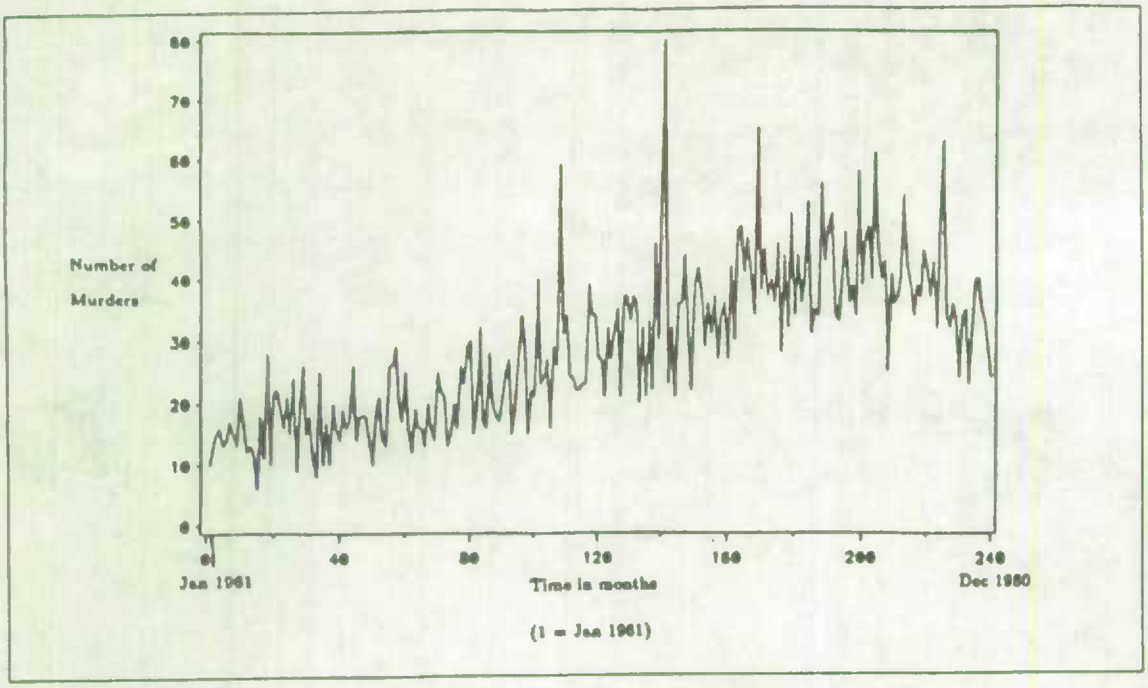

Figure 2a. Canadian monthly homicide data (TOT series) (1961-1980)

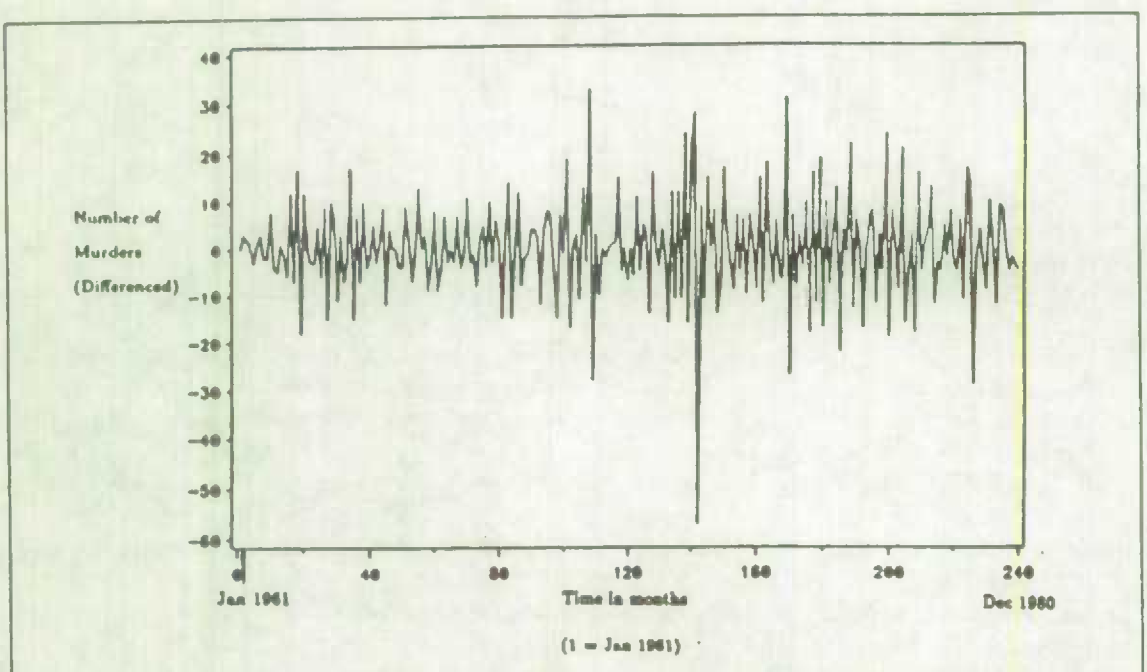

Figure 2b. Trend-free Canadian homicide data (TOT series)

6. CONCLUSION

Under the assumption that there is no non-stable seasonality in a detrended series, we have provided a simple modified F-test for testing the significance of stable seasonality. A small sample distribution of the modified F-statistic is discussed. The hypothesis that the seasonal pattern is constant over time against the alternative that the pattern changes over time, has been discussed by Franzini and Harvey (1983). In the present approach, this test is equivalent to testing for the homogeneity of variances of several correlated groups, a problem which is under investigation.

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