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BENCHMARKING TIME SERIES WITH AUTOCORRELATED SAMPLING ERRORS

by

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abstract

The Denton method is widely used by statistical agencies to benchmark time series (i.e. to adjust them to annual benchmarks). This method does not take into account the presence of autocorrelated sampling errors in the original data. This paper investigates to which extent this omission affects the efficiency of the method relative to a regression method that incorporate various types of ARMA process for autocorrelated sampling errors.

KEYWORDS: ARMA process; Denton method; Relative efficiency; Sampling error

résumé

La méthode de Denton est très utilisée par les instituts de statistique pour étalonner les chroniques (c.-à-d. pour les ajuster aux jalons annuels). Cette méthode ne tient pas compte de la présence d'autocorrélation dans les erreurs d'échantillonnage. Ce document examine l'effet de cette omission sur l'efficience relative de la méthode par rapport à une méthode de régression qui suppose que les erreurs d'échantillonnage se comportent comme divers processus ARMA.

Mots clés: Modèle de Denton; Erreur d'échantillonnage, Processus ARMA, Efficience relative

1. INTRODUCTION

Benchmarking is a procedure very widely used in statistical agencies. Benchmarking situations arise whenever two (or more) sources of data are available for the same *target variable* with different frequencies, e.g. monthly versus annually, monthly versus quarterly. Generally, the two sources of data do not agree; for example, the annual sums of monthly measurements of a variable are not equal to the corresponding annual measurements. Furthermore, one source of data, typically the less frequent, is more reliable than the other, because it originates from a census, exhaustive administrative records or a larger sample. The more reliable measurements are considered as *benchmarks*. Traditionally, benchmarking has consisted of adjusting the less reliable series to make it consistent with the benchmarks. Benchmarking, however, can be defined more broadly as the process of optimally combining two sources of measurements, in order to achieve improved estimates of the series under investigation. Under such a definition, benchmarks are treated as auxiliary observations (Cholette and Dagum 1989).

A typical example of benchmarking is the following. In Statistics Canada, the monthly estimates of Wages and Salaries originate from the Survey of Employment, Payrolls and Hours, whereas the annual benchmark measurements of the same variable originate from exhaustive administrative records, namely the Income Tax forms filed by Canadians and compiled by Revenue Canada. Benchmarking adjusts the monthly data so that they conform to the benchmarks and preserve the original month-to-month movement as much as possible.

Statistical agencies also use benchmarking to interpolate (and extrapolate) more frequent values from less frequent data. It is common, for instance, to benchmark a quarterly indicator, deemed to behave like a target variable, to annual data. The resulting benchmarked values are interpolations (and extrapolations), in the sense that no original quarterly measurements existed for the target variable. Similarly, monthly interpolations are obtained by benchmarking a monthly indicator to quarterly or annual data; and annual interpolations, by benchmarking an annual indicator to quinquennial data. In some cases, the indicator is in fact a mere pattern in percentages, possibly a seasonal-trading-day pattern.

It is also common to benchmark a daily pattern (of relative activity of days within the week) to data which cover four or five weeks; the resulting daily interpolations are then combined into monthly values by taking the monthly sums (Cholette and Chhab 1991). Similarly, calendar year values may be obtained, by benchmarking a subannual series to the fiscal year data and by taking the calendar year sums (Cholette and Baldwin 1989; Cholette 1990); calendar quarter values, by benchmarking a monthly indicator to fiscal quarter data (Cholette 1989). In many of these cases, the interpolations are of no interest per se, and the process is referred to as calendarization.

For simplicity, it is henceforth assumed that the original values are monthly and the benchmarks annual. The benchmarking methods most widely used by statistical agencies are of the Denton (1971) type. Under these methods, the benchmarked series fully conforms to the benchmarks, which are considered as *binding*, and the month-to-month movement of the original series is preserved as much as possible.

One current preoccupation among statisticians is that for repeated surveys, estimation procedures - and benchmarking procedures in particular - should reflect the fact that the sampling errors are autocorrelated. (Rotating panels, for instance would produce such kind of errors.) This was discussed by

Hillmer and Trabelsi (1987) and by Trabelsi and Hillmer (1990) in relation to their ARIMA model-based benchmarking method.

The main purpose of this paper is to estimate the relative efficiency of the Denton method, when the original series are contaminated with bias and autocorrelated sampling errors. According to the results presented in Section 4, taking into account bias and the behaviour of the sampling error reduces the variances of the estimates. The improvement varies with the type of ARMA model followed by the sampling error.

Section 2 presents a benchmarking method which is based on a regression model and allows for bias in the original series and for a general covariance structure of the sampling error. Section 3 discusses the relationship between this regression method and both the Denton method and the ARIMA model-based approach. Section 4 shows how the covariance structure of the sampling error in the regression method can reflect the ARMA behaviour of the error; describes how the relative efficiencies are calculated; and examines the results. Section 5 discusses a real example using the Canadian Retail Trade series. Section 6 gives the conclusions.

2. A BENCHMARKING METHOD BASED ON REGRESSION

This section presents a benchmarking method based on a regression model consisting of the following equations:

$$s_t = a + \theta_t + e_t, \quad E(e_t)=0,\ E(e_t e_{t-k}) \neq 0,\quad t=1,\ldots,T, \tag{2.1a}$$

$$y_m = \Sigma_{t\in m}\, \theta_t + w_m, \quad E(w_m)=0,\ E(w_m w_{m-k}) \neq 0,\quad m=1,\ldots,M. \tag{2.1b}$$

In equation (2.1a), the s_t's denotes the T monthly measurements of a socio-economic variable; the θ_t's the "true" un-observed values of the variable; and a, a bias parameter. This parameter reflects the fact that most subannual measurements are subject to bias. Parameters θ_t and a must be estimated. The estimates of θ_t will be the benchmarked series. Depending on the nature of the variable under question, θ_t can follow a deterministic or a stochastic model. The e_t's denote the errors affecting the observations, e.g. sampling errors; they may have a general covariance structure. Equation (2.1a) therefore states that the observations of the "true" values of the variable are contaminated with error and bias.

In equation (2.1b), the y_m's denote the M annual benchmark measurements of the variable. If a benchmark y_m is not subject to error, i.e. $w_m = 0$ $\sigma_w^2=0$, it is fully reliable and *binding*; in the alternative case, it is *non-binding*. The latter are not benchmark measurements in a strict sense, but simply less frequent measurements of the target variable. Equation (2.1b) states that the observations of the annual sums of the target variable are also contaminated with errors, which may have a general covariance structure. It is assumed that e_t and w_m are mutually independent.

The system of equation (2.1) can be written in matrix algebra in one equation

$$\begin{bmatrix} \mathbf{s} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{I} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \begin{bmatrix} a \\ \boldsymbol{\theta} \end{bmatrix} + \begin{bmatrix} \mathbf{e} \\ \mathbf{w} \end{bmatrix} = \mathbf{X} \begin{bmatrix} a \\ \boldsymbol{\theta} \end{bmatrix} + \begin{bmatrix} \mathbf{e} \\ \mathbf{w} \end{bmatrix}, \qquad (2.2)$$

$$E(\mathbf{e})=0,\ E(\mathbf{w})=0,\ E(\mathbf{e}\ \mathbf{e}')=\mathbf{V}_e,\ E(\mathbf{w}\ \mathbf{w}')=\mathbf{V}_w,\ E(\mathbf{e}\ \mathbf{w}')=0,$$

where $\mathbf{1}$ is a T by 1 vector of ones, and where $\mathbf{J}$ is a M by T design matrix with ones and zeroes such that, for any variable, say, $\mathbf{z}$, $\mathbf{Jz}$ yields the annual sums of $\mathbf{z}$.

In summary, equation (2.2) specifies that the desired benchmarked series θ fits both the subannual and the annual observations and is such that the residuals display some behaviour specified by known matrices V_e and V_w, as explained in Section 4.

Model (2.2) can be written as

$$Y = X\beta + u, \quad E(u)=0, \quad E(u\,u')=V, \tag{2.3}$$

where $Y' = [\,s'\ y'\,]$, $\beta' = [a\ \theta']$, $u' = [\,e'\ w'\,]$, V is a block diagonal matrix with blocks V_e and V_w and where X is a design matrix implicitly defined in (2.2).

The General Least Squares solution to (2.3) yields

$$\dot{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}Y. \tag{2.4}$$

If V is the *true* (known) covariance matrix of the disturbances u, the covariance matrix of the estimates $\dot{\beta}$ is given by

$$\text{cov}\ \dot{\beta} = (X'V^{-1}X)^{-1}. \tag{2.5}$$

When another covariance matrix V^* is used instead of V to obtain an estimate of β, say β^*, then

$$\text{cov}\ \beta^* = [(X'V^{*-1}X)^{-1}X'V^{*-1}]\ V\ [(X'V^{*-1}X)^{-1}X'V^{*-1}]'. \tag{2.6}$$

Assuming V is used, substituting the partitions of X, V and $\dot{\beta}$ in (2.4) and matrix transformations yield

$$\begin{bmatrix} \hat{a} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1'V_e^{-1}1 & 1'V_e^{-1} \\ V_e^{-1}1 & (V_e^{-1}+J'V_w^{-1}J) \end{bmatrix}^{-1} \begin{bmatrix} 1'V_e^{-1} & 0 \\ V_e^{-1} & J'V_w^{-1} \end{bmatrix} \begin{bmatrix} s \\ y \end{bmatrix}, \tag{2.7}$$

$$\begin{bmatrix} \hat{a} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} V_{aa} & V_{a\theta} \\ V_{\theta a} & V_{\theta\theta} \end{bmatrix} \begin{bmatrix} 1'V_e^{-1} & 0 \\ V_e^{-1} & J'V_w^{-1} \end{bmatrix} \begin{bmatrix} s \\ Y \end{bmatrix}, \tag{2.8}$$

where V_{aa} and $V_{\theta\theta}$ are the estimated variance of $\hat{a}$ and covariance matrix of $\dot{\theta}$ respectively. As shown in Appendix, V_{aa}, $V_{a\theta}$ and $V_{\theta\theta}$ may be written as

$$V_{aa} = 1 / [\mathbf{1}'\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}' + \mathbf{V}_w)^{-1} \mathbf{J}\,\mathbf{1}] = h \tag{2.9a}$$

$$\mathbf{V}_{a\theta} = \mathbf{V}_{\theta a}' = -h\,\mathbf{1}' + h\,\mathbf{1}'\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}' + \mathbf{V}_w)^{-1} \mathbf{J}\,\mathbf{V}_e \tag{2.9b}$$

$$\mathbf{V}_{\theta\theta} = [\mathbf{V}_e - \mathbf{V}_e\,\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}' + \mathbf{V}_w)^{-1} \mathbf{J}\,\mathbf{V}_e] + [\mathbf{I} - \mathbf{V}_e\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}' + \mathbf{V}_w)^{-1} \mathbf{J}]\,\mathbf{1}\,h\,\mathbf{1}'\,[\mathbf{I} - \mathbf{V}_e\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}' + \mathbf{V}_w)^{-1} \mathbf{J}]' , \tag{2.9c}$$

which implies

$$\hat{a} = -h\,\mathbf{1}'\,\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}' + \mathbf{V}_w)^{-1} (\mathbf{y} - \mathbf{J}\,\mathbf{s}) \tag{2.10a}$$

$$\hat{\theta} = \mathbf{s}^* + \mathbf{V}_e\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}' + \mathbf{V}_w)^{-1}(\mathbf{y} - \mathbf{J}\,\mathbf{s}^*), \quad \mathbf{s}^{*\prime} = [s_1-\hat{a}\;\; s_2-\hat{a}\; \ldots \; s_T-\hat{a}]. \tag{2.10b}$$

The estimated benchmarked series is given by (2.10b); and its covariance matrix, by equation (2.9c). In the absence of bias in the model, the benchmarked series is given by (2.10b), where $\mathbf{s}^*$ is replaced by $\mathbf{s}$ ($\hat{a}$=0); and its covariance matrix reduces to the first term, in brackets, of (2.9c). In this case, the first term of (2.9c) shows that benchmarking *always* reduces the variance $\mathbf{V}_e$ of the original series s_t by the positive semi-definite matrix $(\mathbf{V}_e\mathbf{J}'(\mathbf{J}\mathbf{V}_e\mathbf{J}'+\mathbf{V}_w)^{-1}\mathbf{J}'\mathbf{V}_e)$.

If the regression method is applied for the whole length of the series, the bias a is deterministic. If the method is applied on moving estimation intervals (of 5 years say), the bias is stochastic, because it evolves according to the innovations entering each interval.

3. RELATION TO OTHER BENCHMARKING METHODS

We here show how the regression benchmarking method of Section 2 relates to the benchmarking method of the Denton type and to the ARIMA model-based method of Trabelsi and Hillmer.

3.1 The Benchmarking Methods of the Denton Type

First of all, it should pointed out that, in the benchmarking methods of the Denton type (e.g. Helfand, Monsour and Trager, 1977), e_t stands for the aggregate of both the bias and the sampling error of model (2.1), and that this aggregate follows a random walk process, $e_t = e_{t-1} + v_t$, $t=2,\ldots,T$.

The regression method produces estimates close to the additive variant of the Denton method, under the following assumptions:

(1) the benchmarks are binding, which implies that $\mathbf{V}_w$ is the null matrix,

(2) there is no bias parameter,

(3) the covariance matrix $\mathbf{V}_e$ of e_t is equal to $\mathbf{V}^*_e=(\mathbf{D}'\mathbf{D})^{-1}\,\sigma_v^2$, where $\mathbf{D}$ is the quasi first difference operator.

$$\underset{T \text{ by } T}{\mathbf{D}} = \begin{bmatrix} (1-\phi^2)^{1/2} & 0 & 0 & \cdots \\ -\phi & 1 & 0 & \cdots \\ 0 & -\phi & 1 & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \tag{3.1}$$

where the autoregressive parameter ϕ is near 1.0 (e.g. 0.99). Then $\mathbf{V}^*_e=(\mathbf{D}'\mathbf{D})^{-1}\,\sigma_v^2$ is known algebraically:

$$\mathbf{V}^*_e = \begin{bmatrix} 1 & \phi & \phi^2 & \cdots & \phi^{T-1} \\ \phi & 1 & \phi & \cdots & \phi^{T-2} \\ \phi^2 & \phi & 1 & \cdots & \phi^{T-3} \\ \vdots & \vdots & \vdots & & \vdots \\ \phi^{T-1} & \phi^{T-2} & \phi^{T-3} & \cdots & 1 \end{bmatrix} \sigma_v^2 \,/\, (1-\phi^2). \tag{3.2}$$

Applying the results of section 2, the benchmarked series and its covariance matrix are respectively given by $\theta^* = \mathbf{s} + \mathbf{V}^*_e\mathbf{J}'(\mathbf{J}\mathbf{V}^*_e\mathbf{J}' + \mathbf{V}_w)^{-1}(\mathbf{y} - \mathbf{J}\,\mathbf{s})$, cov $\theta^* = [\mathbf{V}^*_e - \mathbf{V}^*_e\,\mathbf{J}'(\mathbf{J}\mathbf{V}^*_e\mathbf{J}' + \mathbf{V}_w)^{-1}\,\mathbf{J}\,\mathbf{V}^*_e]$.

The Denton method uses the first difference operator, consisting of the (T-1) last of (3.1) where ϕ=1.0. The appropriate solution is derived by means of quadratic minimization methods (e.g. Cholette, 1979)

$$\begin{bmatrix} \theta^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} D'D & J' \\ J & 0 \end{bmatrix}^{-1} \begin{bmatrix} D'D & 0 \\ 0 & I_M \end{bmatrix} Y = \begin{bmatrix} W_\theta \\ W_\lambda \end{bmatrix} \begin{bmatrix} s \\ y \end{bmatrix} \rightarrow \theta^* = W_\theta \begin{bmatrix} s \\ y \end{bmatrix}, \tag{3.3}$$

where λ contains the Lagrangian multipliers.

If the Denton method is applied to a pair of series which follows model (2.1), where the bias parameter is non zero and the covariance of the sampling error is $\mathbf{V}_e$, the covariance of the resulting benchmarked series is

$$cov\,\theta^* = W_\theta\ V\ W_\theta', \tag{3.4}$$

where V is block diagonal with blocks V_e and V_w. The Denton method produces the same estimates of θ whether a constant bias is present or not in the original series. In fact replacing **s** by (**s** + 1 a) in (3.3) yields

$$\begin{bmatrix}\theta^*\\ \lambda^*\end{bmatrix} = \begin{bmatrix}\boldsymbol{D}'\boldsymbol{D} & \boldsymbol{J}'\\ \boldsymbol{J} & 0\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{D}'\boldsymbol{D} & 0\\ 0 & \boldsymbol{I}_M\end{bmatrix}\begin{bmatrix}\boldsymbol{s}\\ \boldsymbol{y}\end{bmatrix} + \begin{bmatrix}\boldsymbol{D}'\boldsymbol{D} & \boldsymbol{J}'\\ \boldsymbol{J} & 0\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{D}'\boldsymbol{D} & 0\\ 0 & \boldsymbol{I}_M\end{bmatrix}\begin{bmatrix}\mathbf{1}a\\ 0\end{bmatrix}, \tag{3.5}$$

which is equal to (3.3) because $\boldsymbol{D}'\boldsymbol{D}\,\mathbf{1}\,a = \begin{bmatrix}-1 & 1 & 0 & 0 & \cdots\\ -1 & 2 & -1 & 0 & \cdots\\ 0 & -1 & 2 & -1 & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{bmatrix}\begin{bmatrix}1\\1\\1\\ \vdots\end{bmatrix} a = \begin{bmatrix}0\\0\\0\\ \vdots\end{bmatrix}$.

The proportional variant of the Denton methods can be achieved by replacing matrix $\boldsymbol{D}$ by $(diag(\boldsymbol{s}))^{-1}\boldsymbol{D}$ in (3.3), and it is easy to show that the estimates are unaffected by a multiplicative constant bias.

3.2 The Trabelsi and Hillmer (1990) Method

Trabelsi and Hillmer (1990) discuss a benchmarking method, in which the sampling errors follow an ARMA model, the true series θ_t (the signal) follows an ARIMA model and the benchmarks are binding. (This method was a particular case of their previous method (Hillmer and Trabelsi, 1987), which allowed for non-binding benchmarks.) The Trabelsi and Hillmer (1990) paper further focuses on a particular variant, where the inverse of the signal-to-noise ratio is low; the effect of this being that the ARIMA model of the benchmarked series becomes non-operative. The regression method is equivalent to this particular variant of the Trabelsi and Hillmer method, under the following assumptions:

(1) the benchmarks are binding which implies that V_w is the null matrix,

(2) there is no bias parameter,

(3) the sampling error e_t follows an ARMA model.

The benchmarked series is then given by (3.1) and its covariance matrix by (3.2). The authors point out that the additive Denton method is a particular case of this method, if the ARMA model followed by e_t is a random walk.

4. BENCHMARKING ORIGINAL SERIES WITH AUTOCORRELATED SAMPLING ERRORS

Sampling errors e_t, which may be autocorrelated can be specified in the regression benchmarking method of Section 2. This may be achieved in two different manners: one, by making the elements of $\mathbf{V}_e$ (and eventually $\mathbf{V}_w$) equal to values estimated as a by-product of the surveys; or two, by modelling the behaviour of e_t by means of ARMA models. The latter approach is now discussed.

We assume that w_m=0 and $\mathbf{V}_w$=**0** and that e_t follows a stationary ARMA model of order (p,q):

$$e_t - \phi_1 e_{t-1} - \phi_2 e_{t-2} - \ldots - \phi_p e_{t-p} = v_t - \zeta_1 v_{t-1} - \ldots - \zeta_q v_{t-q}, \qquad (4.1)$$

where the ϕ_j's and ζ_j's are the known p autoregressive and q moving average parameters respectively. The algorithm of McLeod (1975) is used to compute the covariance matrix $\mathbf{V}_e$ of $\mathbf{e}$ in terms of the $p+q$ ARMA parameters.

(place Table 1 about here)

Table 1 shows eight ARMA models for the sampling errors that we will use in the regression method, to assess the relative efficiency of the Denton method discussed in Section 3.1. The notation uses the backshift operator B, such that $B^k e_t \equiv e_{t-k}$. Model (1a) was applied by Hillmer and Trabelsi (1987) to illustrate their ARIMA model-based benchmarking method. Models (2a) and (2b) were used by Trabelsi and Hillmer (1990), they are designed to account for the effect of composite estimation and the sample rotation scheme of the U.S. Retail Trade survey. Model (4) was discussed by Bell and Wilcox (1990) for the same purpose. Model (3), proposed by Binder and Dick (1989), is supposed to account for sample rotation in the Canadian Labour Force Survey. Finally, the remaining models (5) and (6) have been included to investigate the effects due to autocorrelated errors which follow a pure moving average model or a simple ARMA (1,1) model.

The sampling errors e_t may be both autocorrelated and heteroscedastic; following Bell and Hillmer (1989), we may express the new structure of e_t by

$$e_t = k_t \, e^*_t \tag{4.4}$$

where the k_t's are weights representing changing variance over time and the e^*_t's follow ARMA model (4.2) with covariance matrix $\mathbf{V}_{e^*}$ given by (4.3). The covariance matrix of e_t is then

$$\mathbf{V}_e = \mathbf{K} \, \mathbf{V}_{e^*} \, \mathbf{K} \; , \tag{4.5}$$

where $\mathbf{K}$ is a diagonal matrix containing the weights k_t. If $\mathbf{V}_{e^*}$ is defined to contain the correlations (instead of the covariances), the k_t's are the standard deviations of the sampling errors.

Given $\mathbf{V}_e$, the bias is estimated by (2.10a) (with $\mathbf{V}_w$=0); the benchmarked series, by (2.10b); and its covariance matrix is given by (2.9c).

4.1 Calculating the Relative Efficiencies

Equation (3.5) shows that the Denton estimates are insensitive to the presence of a constant bias in the original series. It is therefore appropriate to calculate the relative efficiency of the Denton method with respect to the regression method. To calculate the relative efficiency of the various estimators, covariance matrices $\mathbf{V}_e$ were generated for stationary models (1a) to (6) of Table 1. Matrices $\mathbf{V}_e$ were standardized so that their diagonal elements be equal to 1.0 (instead of some other constant). The reason for this standardization is that in empirical applications the variance of the sampling error itself would be known instead of that of the noise generating the process. When the regression method is used for benchmarking, the variances of the benchmarked series are given by the diagonal values of (2.5). When the Denton method is applied to situation (2.1), the variance of the benchmarked series is given by the diagonal values of covariance matrix (3.4). The relative efficiency is the ratio of the traces of the two covariance matrices.

We assume that the series s_t to be benchmarked contains 7 years and 7 months of observations; that annual benchmarks are available for years 1 to 5; that the benchmark of year 6 (t=61,...,72) is not available; and that year 7 (t=73,...,79) is incomplete. Missing benchmarks and incomplete years at the end of series are typical of real benchmarking situations. The estimates of θ_t obtained for years 6 and 7 will be referred to as the *preliminary benchmarked values*. The benchmarking situation just described implies the following design matrix in (2.2), $\mathbf{J} = [\ \mathbf{I}_5 \otimes \mathbf{j}\ \mathbf{0}\ ]$, where **j** is a 1 by 12 vector of 1's and **0** is a 5 by 19 null matrix.

4.2 Analysis of the Results

Table 2 and 3 display the relative efficiencies for *historical* benchmarking (years 1 to 5) and for *preliminary* benchmarking respectively. The relative efficiency is defined by the ratio of the trace of (3.4) over the trace of (2.5) and reflects the increase in variance due to the application of the Denton method instead of the regression method.

According to Table 2, for historical benchmarking, the regression method is more efficient in the cases of sampling error models (3) and (5) and practically equivalent to the Denton method for the remainder. The gains realized with the regression method may be smaller, if the true ARMA models of the sampling errors are unknown and have to be estimated.

(Place Tables 2 and 3 about here)

Table 3 displays the relative efficiencies for preliminary benchmarking, i.e. for years 6 and 7. The gain in efficiency is higher for error models which imply less subannual movement, namely (1a), (1b), (2b) and (6). The gain is lower for the models which imply strong subannual movement, namely models (2a), (3) and (4). The gain is also lower for model (5), because MA models have short memory and are inherently harder to predict.

5. EXAMPLE: AN APPLICATION TO THE CANADIAN RETAIL TRADE SERIES

This section compares the benchmarked Canadian Retail Trade series obtained by three benchmarking methods: (1) the proportional Denton method, (2) the additive Denton method and (3) the regression method with sampling error ARMA model (2b). We use model (2b), developed for the U.S. Retail Trade series (Trabelsi and Hillmer, 1990), as a reasonable ARMA model for our series, since we do not have sufficient information on the autocovariance of the sampling errors. The regression method uses the coefficients of variations (CV's) available both for the monthly and the annual data; they vary between 0.8% to 1.4% for the monthly values and between 0.0% and 0.2% for the annual. (The CV's are converted into standard deviations, which are substituted in matrix K of (4.5) to generate V_e which is substituted in the regression method.) The proportional and the additive Denton methods, on the other hand, implicitly assume constant monthly CV's and constant monthly variances respectively; and, zero annual variances.

Figure 1 displays the original series and the annual benchmarks (divided by 12) and clearly indicates the need to raise the level of the original data. The monthly *corrections* to be made to the original series (to obtain the benchmarked series) under the three methods are very different.

Figure 2 (a) shows that, for the proportional Denton method, the corrections are as proportional to the original series (as made possible by the benchmarks) and therefore reflect the seasonal pattern of the original series. On the other hand, for the additive Denton method, the corrections are independent of the original values and are therefore as flat as possible.

Figure 2 (b) shows that the corrections made by the regression method has an additive part, given by its bias estimate (also displayed), and a much smaller proportional part due to the CV's. The larger correction for July 1987 is due to the larger CV for this observation; it is therefore corrected more than the other observations. For the last year (1989), the regression corrections converge towards the bias estimate. Consequently, the regression method has the good property of generating preliminary corrections that are closer to the average of the past *discrepancies* between the benchmarks and the corresponding sum of the original series. We would like to point out, however, that the convergence is faster if the sampling error follows a non-seasonal ARMA model. If the model is seasonal (as in figure 2 (b)), the convergence is slower, because it takes into account the seasonal effect.

On the other hand, both Denton methods simply repeat the last (additive or proportional) correction of the last year with a benchmark (December 1988 in figure 2 (a)).

Finally, Figure 3 displays both the original and the benchmarked series using the regression method with ARMA model (2b). It shows that, without preliminary benchmarking, a bigger drop would occur between December 1988 and January 1989 (distance AB) than with preliminary benchmarking (distance AC). Since the level of the benchmarked series is better than that of the original series, preliminary benchmarking is desirable; it avoids unwanted steps between the historically benchmarked series and the observations of the current year.

6. CONCLUSION

The Denton method is widely applied by statistical agencies to perform historical and preliminary benchmarking of original values, without any explicit consideration of the sampling errors affecting the data.

In this paper, we have calculated the relative efficiency of the Denton method versus a method based on regression, which incorporates various ARMA models for the sampling error. These ARMA models have been discussed by Hillmer and Trabelsi (1987), Trabelsi and Hillmer (1990), Binder and Dick (1989) and Bell and Wilcox (1990) in the context of model-based benchmarking and sampling.

The theoretical results presented in section 4 indicate that for historical benchmarking, the increase in efficiency of the regression method versus the Denton method is considerably large only when the errors follow a moving average model; otherwise the gain in efficiency is negligible. On the other hand, the increase in efficiency is large for preliminary benchmarking in all cases. This is of particular importance, because the preliminary corrections are applied to the most current observations which are critical for decision making.

These conclusions are illustrated with the Canadian Retail Trade series using both the additive and proportional Denton methods and a regression method with an ARMA model for the sampling error. This real case example clearly shows how the nature of the monthly corrections, made to the original series to produce a benchmarked series, differs depending on the benchmarking method used. The results show that the preliminary corrections generated from the regression method are more adequate than those from both Denton methods. In fact, whereas the Denton preliminary corrections consist of repeating the last correction estimated for the last year with a benchmark, the regression preliminary corrections take into account the average level of the corrections over the whole series.

We are currently investigating more general models for the bias (deterministic and stochastic). There is also a need for further research on the reliability of benchmarks, and on benchmarking methods for groups of series classified according to different attributes, as apposed to single time series.

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APPENDIX: DERIVATION OF RESULTS (2.9) and (2.10)

Performing the inversion in (2.7) by blocks yields

$$\mathbf{V}_{aa} = 1 / [\mathbf{1}'\mathbf{V}_e^{-1} - \mathbf{1}'\mathbf{V}_e^{-1} (\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})^{-1} \mathbf{V}_e^{-1})] = h, \qquad \text{(A.1a)}$$

$$\mathbf{V}_{a\theta} = \mathbf{V}_{\theta a}' = - h\, \mathbf{1}'\mathbf{V}_e^{-1} (\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})^{-1}, \qquad \text{(A.1b)}$$

$$\mathbf{V}_{\theta\theta} = [(\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})^{-1}] \qquad \text{(A.1c)}$$

$$+ [(\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})^{-1} \mathbf{V}_e^{-1}\mathbf{1}\, h\, \mathbf{1}'\mathbf{V}_e^{-1} (\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})^{-1}].$$

Substitution of (A.1) in (2.8) and some lengthy algebra yields

$$\hat{a} = h\, \mathbf{1}'\mathbf{V}_e^{-1} \mathbf{s} - h\, \mathbf{1}'\mathbf{V}_e^{-1} (\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})^{-1} (\mathbf{V}_e^{-1} \mathbf{s} + \mathbf{J}'\mathbf{V}_w^{-1} \mathbf{y}), \qquad \text{(A.2a)}$$

$$\dot{\theta} = (\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})^{-1} (\mathbf{V}_e^{-1} \mathbf{s}^* + \mathbf{J}'\mathbf{V}_w^{-1} \mathbf{y}), \quad \mathbf{s}^{*\prime} = [s_1-\hat{a}\ s_2-\hat{a}\ \ldots\ s_T-\hat{a}]. \qquad \text{(A.2b)}$$

The benchmarking methods of the Denton type were originally based on minimization of the quadratic form $(\theta-\mathbf{s})'\mathbf{V}_e^{-1}(\theta-\mathbf{s})$. This process starts by specifying $\mathbf{V}_e^{-1}$ (and not $\mathbf{V}_e$). In such cases, solution (A.1)-(A.2) is appropriate and requires one matrix inversion, that of $(\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})$. Note that $(\mathbf{V}_e^{-1} + \mathbf{J}'\mathbf{V}_w^{-1}\mathbf{J})$ has to have full rank, but not necessarily $\mathbf{V}_e^{-1}$.

If the covariances matrices $\mathbf{V}_e$ and $\mathbf{V}_w$ are given, solution (A.1)-(A.2) can be written in terms of $\mathbf{V}_e$ and $\mathbf{V}_w$ as (2.9)-(2.10) respectively, using matrix identities and lengthy algebra as in Hillmer and Trabelsi (1987). The matrix inversion of $(\mathbf{J}\mathbf{V}_e\mathbf{J}' + \mathbf{V}_w)$ required by (2.9)-(2.10) is of smaller dimension (M by M) than that required in (A.1)-(A.2) (T by T). Furthermore, solution (2.9)-(2.10) admits the particular case where $\mathbf{V}_w$ is exactly equal to $\mathbf{0}$, contrary to (A.1)-(A.2) where $\mathbf{V}_w$ may only *tend* to zero.

Table 1: ARMA models used for modelling the sampling error

(1a)	(1,0)(0,0)	$(1 - 0.80000B)\ e_t = \nu_t$
(1b)	(1,0)(0,0)	$(1 - 0.20000B)\ e_t = \nu_t$
(2a)	(2,1)(1,0)	$(1 - 0.75B)(1 - 0.60B^3)(1 - 0.60B^{12})\ e_t = (1 - 0.50B)\ \nu_t$
(2b)	(2,1)(1,0)	$(1 - 0.75B)(1 - 0.60B^3)(1 - 0.30B^{12})\ e_t = (1 - 0.50B)\ \nu_t$
(3)	(3,6)(0,0)	$(1 - 0.2575B + 0.3580B^2 + 0.6041B^3)\ e_t = (1 + 0.1847B + 0.5873B^2 - 0.3496B^3 - 0.0647B^4 - 0.0982B^5 - 0.0347B^6)\ \nu_t$
(4)	(2,1)(1,0)	$(1 - 0.75B)(1 - 0.70B^3)(1 - 0.75B^{12})\ e_t = (1 + 0.10B)\ \nu_t$
(5)	(0,1)(0,0)	$e_t = (1 - 0.8B)\ \nu_t$
(6)	(1,1)(0,0)	$(1 - 0.95B)\ e_t = (1 + 0.80B)\ \nu_t$

Table 2: Relative efficiencies of the Denton method versus the regression method for *historical* benchmarking

ARMA model (p,q)(P,Q) for the sampling errors		Relative Efficiency
(1a)	(1,0)(0,0)	1.017
(1b)	(1,0)(0,0)	1.014
(2a)	(2,1)(1,0)	1.001
(2b)	(2,1)(1,0)	1.002
(3)	(3,6)(0,0)	1.130
(4)	(2,1)(1,0)	1.002
(5)	(0,1)(0,0)	1.052
(6)	(1,1)(0,0)	1.006

Table 3: Relative efficiencies of the Denton method versus the regression method for *preliminary* benchmarking

ARMA model (p,q)(P,Q) for the sampling errors	Relative Efficiency
(1a) (1,0)(0,0)	1.424
(1b) (1,0)(0,0)	1.171
(2a) (2,1)(1,0)	1.045
(2b) (2,1)(1,0)	1.141
(3) (3,6)(0,0)	1.052
(4) (2,1)(1,0)	1.051
(5) (0,1)(0,0)	1.031
(6) (1,1)(0,0)	1.107

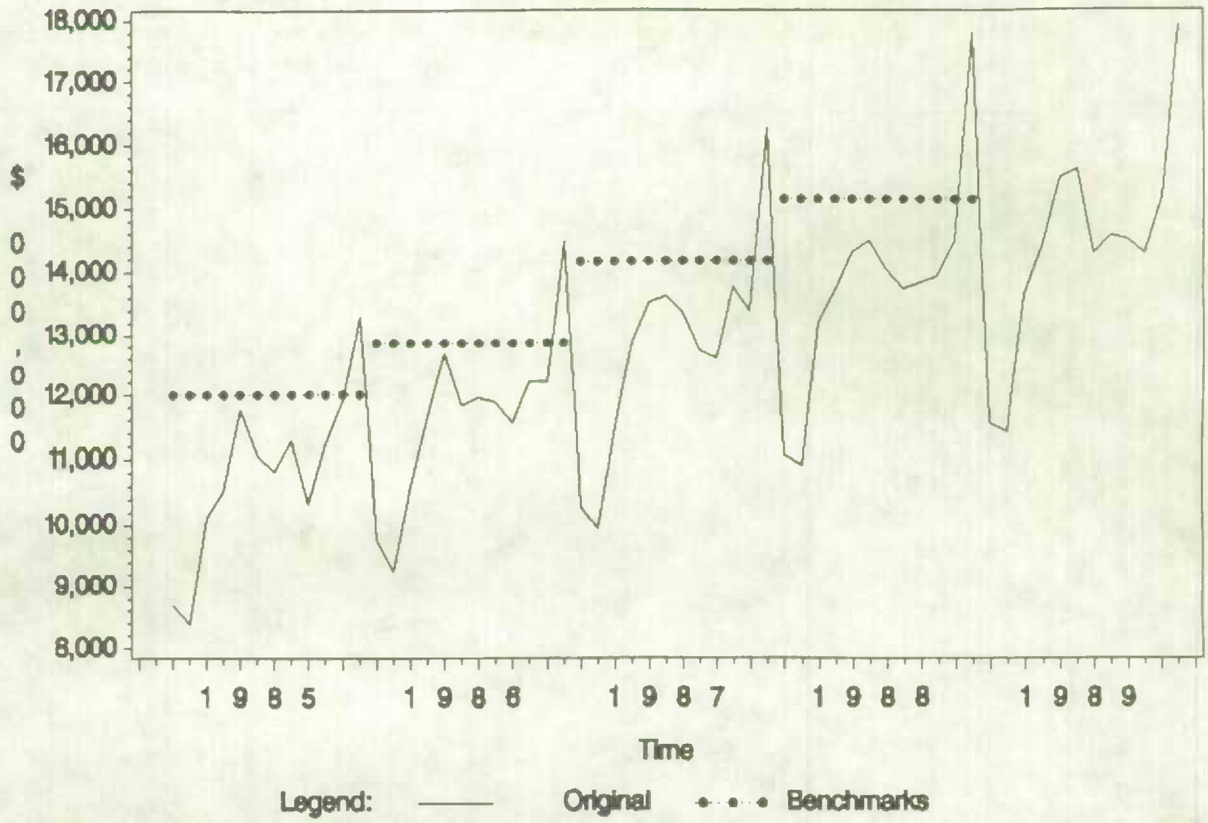
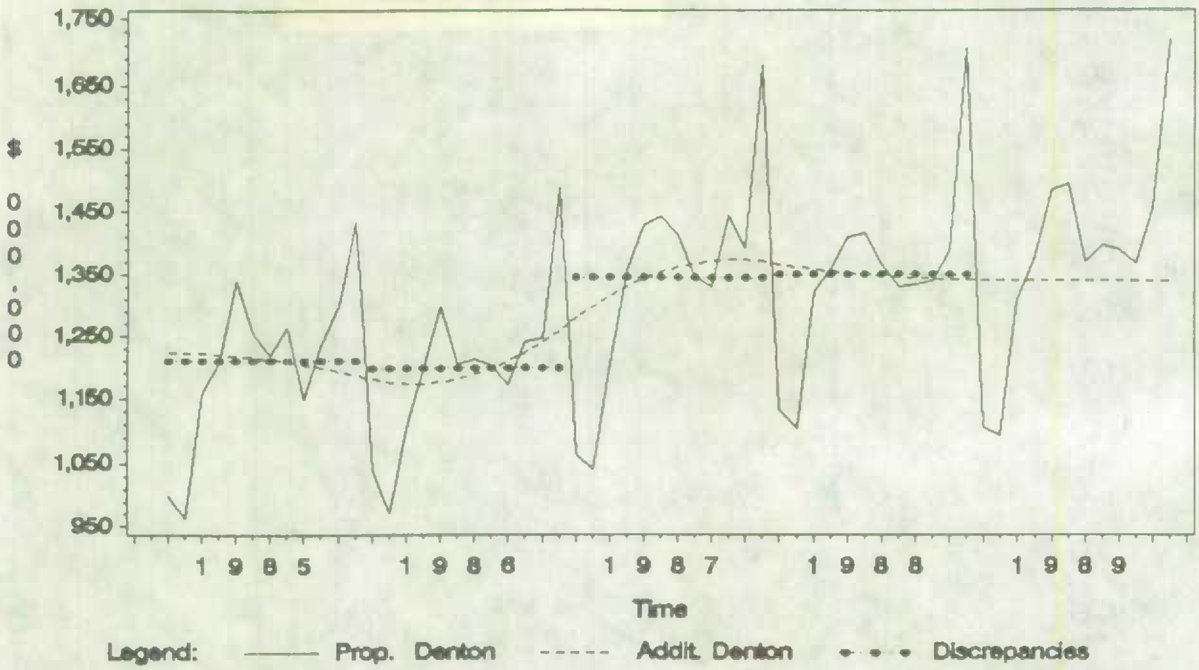
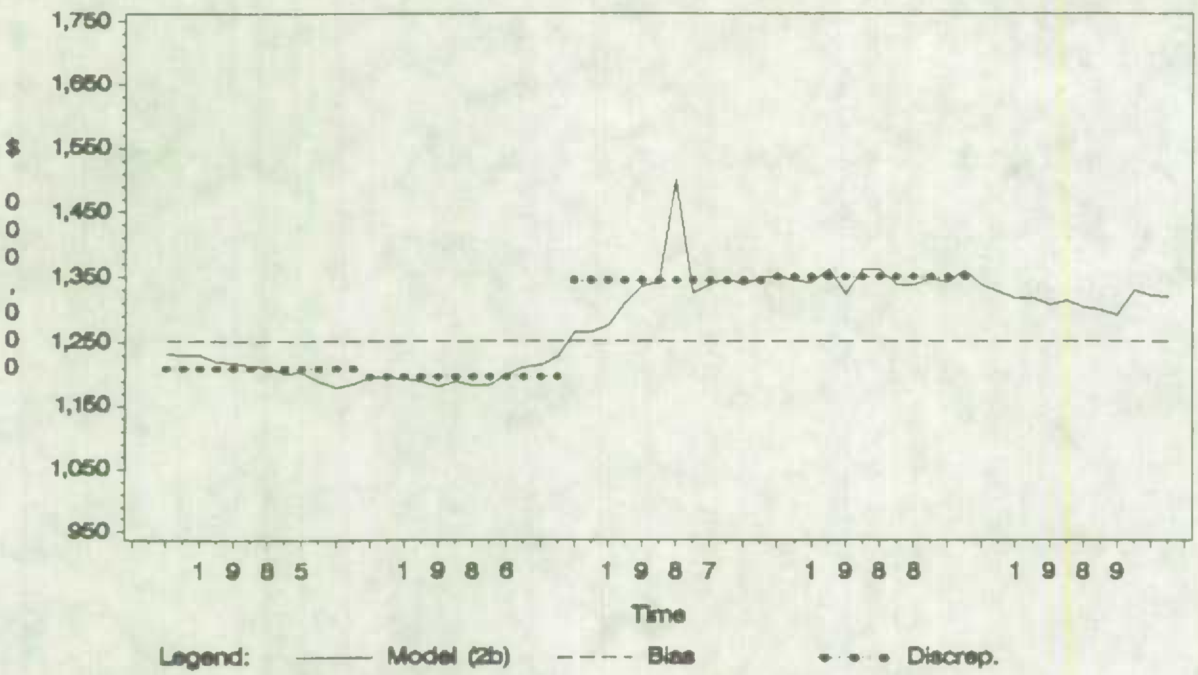


Figure 1: Original Canadian Retail Trade Series and its Annual Benchmarks



(a)



(b)

Figure 2: Monthly Corrections Made to the Original Series (a) under the Proportional and the Additive Denton Methods and (b) under the Regression Method with ARMA Model (2b)

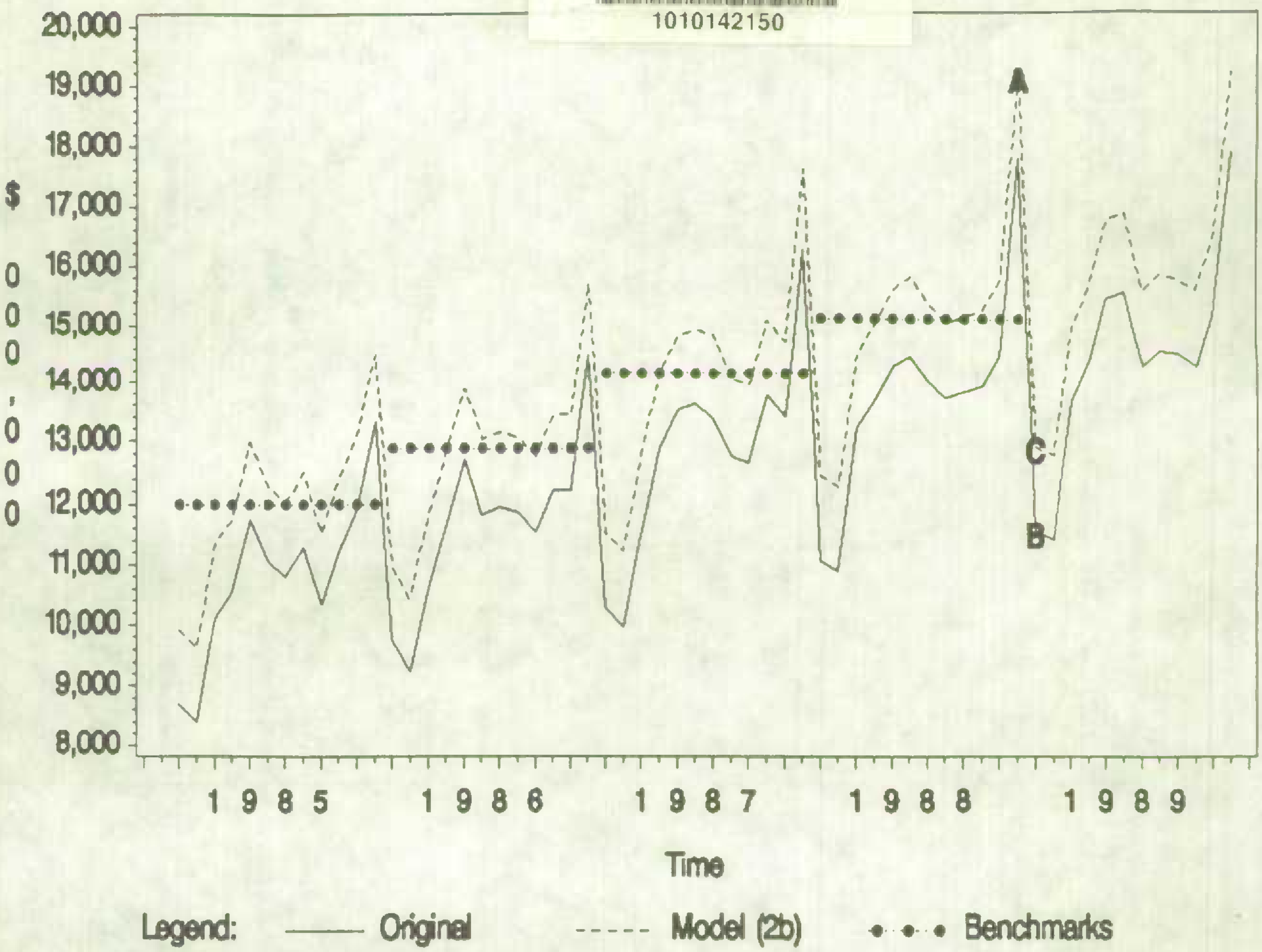


Figure 3: Original and Benchmarked Canadian Retail Trade Series under the Regression Method with ARMA Model (2b)