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Business Survey Methods Division

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VARIANCE ESTIMATION IN SAMPLE SURVEYS

J. Kovar, P. Ghangurde, M.-F. Germain, H. Lee, G. Gray

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1. Introduction

The variance estimation problem has traditionally received a great deal of attention in the sample survey literature. This note will attempt to guide the reader through the papers relating to variance estimation of nonlinear statistics in sample surveys; surveys with complex designs in particular. While variance estimators of statistics expressable as linear functions of the observations can be derived explicitly (Rao, 1975 and 1982), in the case of nonlinear functions of the observations various approximation methods must be used. The methods considered here include Taylor series linearization, balanced repeated replication (BRR), jackknife repeated replication (JRR) and the bootstrap. Most of these approaches were originally introduced for the independent, identically distributed (iid) case. With a few exceptions, investigations of their application to the complex sample survey have been only relatively recent, notably Frankel (1971), Bean (1975) and Woodruff and Causey (1976). Good overviews of these methods as they apply to complex surveys can be found in Rao (1985), Rust (1984) and Wolter (1979) among others.

In the next section, we will review these methods and indicate their properties and limitations. Topics of special interest to Statistics Canada, both current and upcoming, will be discussed in subsequent sections.

2. Variance Estimation

2.1 Taylor Series Linearization

The Taylor method (also called the delta method) is based on linearizing a complicated estimator using the Taylor expansion about the mean.

Let $\hat{\theta} = f(Y_1, \ldots, Y_m)$ be an estimator of interest expressed as a function of

m totals $(Y_1, \ldots, Y_m) = \underset{\sim}{Y}$. The first order Taylor expansion of f about the expected value of $\underset{\sim}{Y}$, $E(\underset{\sim}{Y})$, is given by

$$f(Y) \doteq f(E(Y)) + \sum_{i=1}^{m} \frac{\partial f(E(Y))}{\partial Y_i} (Y_i - E(Y_i)), \tag{2.1.1}$$

and thus the variance of $\hat{\theta}$ can be approximated by

$$V(\hat{\theta}) \doteq V\left(\sum_{i=1}^{m} \frac{\partial f}{\partial Y_i} Y_i\right), \tag{2.1.2}$$

where the partial derivatives, $\frac{\partial f}{\partial Y_i}$, are evaluated at the expected values of Y_i, $E(Y_i)$ (Tepping, 1968). Now, $Y_i = \sum_{j=1}^{n} y_{ij}$, where y_{ij} is the weighted observation of the j-th unit in the sample. Then

$$\begin{aligned} V(\hat{\theta}) &\doteq V\left(\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial f}{\partial Y_i} y_{ij}\right) \\ &= V\left(\sum_{j=1}^{n} d_j\right) \end{aligned} \tag{2.1.3}$$

where

$$d_j = \sum_{i=1}^{m} \frac{\partial f}{\partial Y_i} y_{ij} \,. \tag{2.1.4}$$

The variance estimate, $v(\hat{\theta})$, can be obtained by evaluating the derivatives at the estimated Y_i instead of $E(Y_i)$, and by applying the usual variance formula for a single variable to $V(\sum_{j=1}^{n} d_j)$. Reordering the summation simplifies the evaluation of the variance greatly by avoiding the computation of m variances of Y_i's and m(m-1)/2 covariances of Y_i and Y_j $(i \neq j)$ in (2.1.2). This simplication is due to Woodruff (1971).

The Taylor linearization method is general and is applicable to any sample design, provided the partial derivatives $\frac{\partial f}{\partial Y_i}$ exist and the variance formula for $V(\sum_{j=1}^{n} d_j)$ is available. Woodruff and Causey (1976) computerized the method for general use. An application of the method to the variance of ratio-adjusted estimators in household surveys is given in Ghangurde and Gray (1981).

2.2 Balanced Repeated Replication

The origins of replication can be found in the works of survey statisticians as early as the 1940s. Among others, Mahalanobis (1944) and Deming (1956) suggested the use of several interpenetrating independent samples (replicates), each selected with the same design, for the purposes of variance estimation. This idea of simple replication was later adapted to yield a number of procedures, three of them being presented in this section, namely the random groups, the balanced repeated replication (BRR) and the partially balanced repeated replication (PBRR) methods. Rust (1984) provides a good overview of these techniques.

For the random groups method, developed at the U.S. Bureau of the Census and described by Hansen, Hurwitz and Madow (1953), the original sample is divided into a number of groups which are formed so that each reflects the sample design. For instance, in stratified sampling, the sample is allocated to the groups within each stratum. Let $\hat{\theta}$ be the full sample estimator of the parameter of interest, θ, and let $\hat{\theta}_r$ be the estimator computed from the r'th random group ($r=1, \ldots, k$). The average $\hat{\bar{\theta}} = \sum_{r=1}^{k} \frac{\hat{\theta}_r}{k}$ is an estimate of $\hat{\theta}$ and its variance

$$v(\hat{\bar{\theta}}) = \frac{1}{k(k-1)} \sum_{r=1}^{k} (\hat{\theta}_r - \hat{\bar{\theta}})^2, \qquad (2.2.1)$$

provides an estimate of the variance of $\hat{\theta}$. This procedure, as all simple replication procedures, suffers from a dilemma with regard to the choice of the number of replicates (groups). The larger the number, the higher the bias of the variance estimator, whereas a fewer number of groups yield a lower precision variance estimate. As such the estimator performs well in large surveys, or surveys with a large number of primary sampling units (PSU) per stratum.

However, for surveys with a large number of strata but few PSU per stratum, the random groups method is unsatisfactory. For such designs, and in the case where two PSU per stratum are selected in particular, the method of balanced repeated replication has been developed by McCarthy (1966a). The BRR variance estimator is based on a set of "balanced" half-samples formed by deleting one unit from the sample in each stratum. The method is as follows. Let θ and $\hat{\theta}$ be the population parameter of interest and its estimator. For each stratum, denote at random one of the two sampled PSU as being first and the other as second. A set of S half-samples may be defined by an SxH matrix $\Delta = ((\delta_{jh}))$ of +1 or -1 (j=1, ..., S; h=1,..., H) depending on whether the first or the second PSU is in the h'th stratum in the j'th half-sample. The columns of Δ are balanced and Δ is orthogonal, that is

$$\sum_{j=1}^{S} \delta_{jh} = 0 \quad \text{and} \quad \sum_{j=1}^{S} \delta_{jh}\,\delta_{jh'} = 0 \quad (h \neq h').$$

A minimal set of S half-samples may be constructed using the method developed by Plackett and Burman (1946), with $H+1 \leq S \leq H+4$. From the half-samples, S estimates of θ denoted $\hat{\theta}^{(j)}$ are computed and the variance estimator of $\hat{\theta}$ is given by

$$v_{BRR-H}(\hat{\theta}) = \frac{1}{S} \sum_{j=1}^{S} (\hat{\theta}^{(j)} - \hat{\theta})^2 \quad . \tag{2.2.2}$$

This estimator does not have the shortcomings of the previous one, achieving good precision with only a limited number of replicates. One can also use the S complementary half-samples and estimate $v(\hat{\theta})$ with three other variations of the BRR variance estimator:

$$v_{BRR-S}(\hat{\theta}) = \frac{1}{2S} \sum_{j=1}^{S} \{(\hat{\theta}^{(j)}-\hat{\theta})^2 + (\hat{\theta}_c^{(j)}-\hat{\theta})^2\} \tag{2.2.3}$$

$$v_{BRR-C}(\hat{\theta}) = \frac{1}{S} \sum_{j=1}^{S} (\hat{\theta}_c^{(j)} - \hat{\theta})^2 \tag{2.2.4}$$

$$v_{BRR-D}(\hat{\theta}) = \frac{1}{4S} \sum_{j=1}^{S} (\hat{\theta}^{(j)} - \hat{\theta}_c^{(j)})^2 \tag{2.2.5}$$

where $\hat{\theta}_c^{(j)}$ is the estimator of θ computed from the complement of the j'th half-sample. (Note that $v_{BRR-H}(\hat{\theta})$ is equivalent to $v_{BRR-C}(\hat{\theta})$). When sampling with replacement is used, the BRR will have a low bias and be relatively precise for non-linear statistics. For designs in which sampling without replacement is used, the BRR variance estimator is positively biased.

For surveys where a large number of strata renders the balanced repeated replication method prohibitive, McCarthy (1966b) and Lee (1972 and 1973a) have proposed the method of partially balanced repeated replication. The H strata are now divided into g groups with H/g strata in each group. A set of k_{PB} partially balanced half-samples is constructed in the first group of strata and the same construction is then repeated in each group. The variance estimator has the same form as (2.2.2) but is based on a fewer number k_{PB} of replicates where $H/g + 1 \leq k_{PB} \leq H/g + 4$.

A number of authors have studied empirically the BRR as a method of estimating the variance of statistics like ratio estimators, regression coefficients and correlation coefficients (McCarthy, 1969; Kish and Frankel, 1970 and 1974; Frankel, 1971; Levy, 1971), post-stratified means (Bean, 1975) and even estimates of covariance matrices (Freeman, 1975). Gurney and Jewett (1975) also extended the use of BRR to designs where the number of sampled PSU per stratum is a prime.

Krewski (1978b) provides some insight as to the efficiency of the random groups, the BRR and PBRR methods in the linear case $\hat{\theta} = \bar{y}_{st}$. Also, a few studies (both empirical and theoretical) compare the BRR with the jackknife and the linearization methods. These papers are discussed in the following sections. We will only mention that Efron (1981) links the BRR with the bootstrap method and that Rao and Wu (1983b) study the asymptotic behavior of the four BRR variance estimators ((2.2.2) to (2.2.5)).

2.3 Jackknife Repeated Replication

Another sample reuse method, the jackknife, was originally introduced by Quenouille (1949) as a method of reducing the first order bias of a statistic. In a later paper Quenouille (1956) generalized the method and explored its bias reduction properties in the context of simple random sampling from infinite populations. The use of the jackknife was expanded when Tukey (1958) suggested that the individual subsample estimators may be regarded as iid random variables. In this way, a very simple estimate of variance and an approximate t statistic could be calculated and used for hypothesis testing as well as interval estimation.

The method, in its simplest form, is described as follows. Let $Y_1, \ldots, Y_n$ be iid random variables from a distribution F and $\theta(F)$ be the parameter of interest. An estimator $\hat{\theta}$ of θ is computed from the sample. Suppose we divide the sample into g groups of size $r = n/g$, and for each group, we compute $\hat{\theta}_{(j)}$, the estimator of θ of

the same functional form as $\hat{\theta}$ but computed from the sample after the omission of the j'th group. We define the "pseudo-values" $\hat{\theta}_j$ as

$$\hat{\theta}_j = g\hat{\theta} - (g-1)\, \hat{\theta}_{(j)}, \tag{2.3.1}$$

and the jackknife estimator of θ as

$$\hat{\theta}_J = \sum_{j=1}^{g} \frac{\hat{\theta}_j}{g}\,. \tag{2.3.2}$$

The jackknife estimator of the variance of $\hat{\theta}_J$ and of $\hat{\theta}$ (Hinkley, 1977a) is given by

$$v(\hat{\theta}_J) = \frac{1}{g(g-1)} \sum_{j=1}^{g} (\hat{\theta}_j - \hat{\theta}_J)^2 \quad , \tag{2.3.3}$$

the variance between the subsample estimates. The statistic

$$\hat{t} = \frac{\sqrt{g}\,(\hat{\theta}_J - \theta)}{\{\frac{1}{g-1} \sum_{j=1}^{g} (\hat{\theta}_j - \hat{\theta}_J)^2\}^{\frac{1}{2}}} \tag{2.3.4}$$

is assumed to follow an approximate t distribution with g-1 degrees of freedom. In order to maximize the degrees of freedom, g=n is by far the most common choice in applications.

Further research of the bias reduction and variance estimation properties was done in the context of simple random sampling with replacement or sampling from infinite populations, on statistics such as ratio estimators (Durbin, 1959; Rao, 1965; Rao and Webster, 1966), regression coefficients (Miller, 1974a; Hinkley, 1977b), maximum-likelihood estimators (Brillinger, 1964) or U-statistics (Arvesen, 1969).

Although the usefulness of the jackknife has been proven in many areas it fails to be consistent in the case of order statistics and quantiles (Miller, 1964). In fact, Miller, in his subsequent review (1974b) mentions that in order "for the jackknife to operate correctly, the estimator $\hat{\theta}$ has to have a locally linear quality".

The extension of the jackknife to finite population sampling and surveys with complex designs is not straightforward. The initial studies were empirical investigations concerned for the most part with stratified (without replacement) schemes (Folsom, Bayless and Shah, 1971; Frankel, 1971, Kish and Frankel, 1974; Sharot, 1976; Lemeshow and Levy, 1979). In their comparison of the linearization, the jackknife and the BRR variance estimators, Kish and Frankel (1974) found that the linearization estimator was more stable than the jackknife which in turn was more stable than the BRR estimator. However, for the purpose of interval estimation, their performance was in the reverse order, that is, the BRR method provided a t-statistic which followed the t distribution more closely than the other methods. At that time, several versions of the jackknife variance estimator in the stratified sampling context were proposed (Lee, 1973b; Jones, 1974; McCarthy, 1966a). Wolter (1979) provides a good overview of their findings.

Theoretical justification of the linearization, the BRR and the JRR methods has begun only recently with papers by Krewski (1978a) and Royall and Cumberland (1978) whose concern was with the asymptotic behaviour of the methods. Later, Krewski and Rao (1981) established the asymptotic normality and the consistency of all three variance estimators in stratified (with replacement) multistage sampling. The first order asymptotic results of that paper did not, however, separate the three methods. It was in a subsequent paper by Rao and Wu (1983b) where the second order asymptotic properties established an important result. That is, in the special case of two units per stratum, all of the jackknife variance estimators presented in earlier papers were found to be asymptotically equivalent to the linearization variance estimator but not to the BRR variance estimator.

2.4 The Bootstrap

The bootstrap was introduced recently by Efron (1979) in the context of simple random sampling or sampling from infinite populations. Briefly and freely adapted from therein, the method can be described as follows. Given a sample $\{y\}_n$ from a distribution function F and a parameter of interest $\theta(F)$, we estimate F by $\hat{F}$: {mass 1/n on each y_i}. A bootstrap sample of size m, $\{y^*\}_m$, is then obtained from $\hat{F}$ by sampling it with replacement. An estimate of θ can then be computed from this sample, say $\hat{\theta}^* = \hat{\theta}(\{y^*\})$. This process can be repeated a large number (B) of times, leading to $\hat{\theta}^*_1, \hat{\theta}^*_2, \ldots, \hat{\theta}^*_B$. A bootstrap variance estimator of $\hat{\theta}$ is then given by

$$v_B(\hat{\theta}) = \sum_{i=1}^{B} (\hat{\theta}^*_i - \hat{\theta}^*_.)^2 / B \tag{2.4.1}$$

where

$$\hat{\theta}^*_. = \sum_{i=1}^{B} \hat{\theta}^*_i / B \tag{2.4.2}$$

is a bootstrap estimator of θ. Efron (1979) links the jackknife and bootstrap estimators in the above set up by showing that the jackknife estimator is in fact the first order Taylor approximation of the bootstrap estimator.

Three adaptations of this method to data arising from complex sample surveys exist to date. First, there is the Bickel and Freedman (1984) approach, which consists of creating a superpopulation from the sample at hand, by replicating each of its units a number of times equal to its weight. The superpopulation is then sampled without replacement to obtain the individual bootstrap samples. Secondly,

McCarthy and Snowden (1983) resample the sample with replacement, choosing the bootstrap sample size (m) such that the variance of the bootstrap estimator of the mean is asymptotically correct. Subsequent studies suggest, however, that this approach fails to capture moments of higher order and can be misleading if applied to highly skewed populations. Both approaches can be applied to simple random samples or stratified simple random samples with or without replacement. Thirdly, an approach due to Rao and Wu (1983a) involves the use of adjusted values, somewhat analogous to Tukey's pseudo values for the jackknife. Their approach is applicable to virtually any survey design and any estimator that can be written as a function of means.

The theoretical justification of the bootstrap for finite samples is scarce. Babu and Singh (1983) prove some asymptotic properties of the bootstrap variance estimator by letting the stratum sample size go to infinity. Bickel and Freedman (1984) generalize these results by considering stratified samples selected with or without replacement and by letting the total sample size tend to infinity (that is, either the number of strata, or the stratum sample size or both tend to infinity). They show that the bootstrap variance estimator, in their superpopulation context, is asymptotically normal and consistent for the case of estimating linear combinations of stratum means. Rao and Wu (1983a) extend these first order asymptotic results to various other designs and to all estimators of functions of means.

Other empirical justifications of the boostrap in the case of finite populations are scarce and remain mostly unpublished. Clearly, this is an area that is in need of a lot more research.

3. Special Topics

3.1 Estimation of the Variance of the Median

The sample median is a robust alternative to the mean for estimation of location for skew and heavy tailed populations. Quantiles and percentiles are also of practical interest for distributions of variables such as personal income. Their variances can be obtained by methods developed for the median. Direct theoretical estimation of the median and its variance is possible for samples composed of iid observations. Various resampling methods such as the bootstrap and the jackknife based on Monte Carlo approximations are also available. The following paragraphs present some of these methods, including methods based on order statistics, resampling methods as well as Woodruff's method based on the inversion of the distribution function. The conspicuous lack of results pertaining to samples from complex surveys indicates the need for further research in this context.

Let $x_{(m)}$ be the sample median, where $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ are the order statistics based on a random sample of size n ($n = 2m-1$) from a distribution function F. It is known from standard asymptotic theory, that as n tends to infinity, $V(x_{(m)})$ approaches $1/(4nf^2(\theta))$, where $f(\theta)$ is the probability density at θ, the population median (Kendall and Stuart, 1950). An estimate of this variance, $v(x_{(m)})$, can be obtained by estimating $f(\theta)$ based on the observed order statistics. Maritz and Jarrett (1978) have obtained the values of $v(x_{(m)})$ for rectangular, exponential and standard normal distributions. However, even for simple random samples from finite populations, the substitution by sample order statistics does not work and an implicit assumption of an infinite super population may be necessary.

Alternatively, we can consider methods based on resampling the parent sample, including the bootstrap and the jackknife. Extensions of these methods to complex surveys and various estimators have been discussed above. However, these

extensions are generally not available for functionals such as the median. For independent, identically distributed observations from a distribution F with a continuous density f(x), Efron (1982) shows that in the case of the median, the bootstrap variance estimator approaches the correct asymptotic value of $1/(4nf^2(\theta))$. No comparable results exist in the context of sample surveys. In the case of the jackknife, on the other hand, it can be shown (Miller, 1964) that even in the case of iid variables, the jackknife variance estimator is inconsistent.

The above nonparametric methods necessarily emphasize the parent sample and with it, the uncertainty about the sampling distribution. If the form of the distribution is known, as can be argued in the case of income, it should be possible to improve the estimates. One alternative is to fit an appropriate distribution, say Pareto, to the data and obtain the appropriate parametric estimates. More generally, one can use a broad parametric family, with Pareto as a special case, and obtain, by maximum likelihood estimation, $\hat{\theta}$ and $v(\hat{\theta})$. These methods will better exploit different data sets collected over time from different areas. Steinberg and Davis (1981) compared various confidence interval estimates for quantiles including those based on order statistics and the bootstrap. They found that when the form of the distribution is known, the parametric confidence intervals are often considerably shorter than those of nonparametric methods. It is not clear, however, how extensions of the parametric methods to complex sample designs can be made.

Finally, Woodruff (1952) obtains a confidence interval for the median of the empirical distribution function, which can be used to obtain a confidence interval for the median itself. The method seems to work in the case of complex sample designs; however, the sampling distribution of the estimator of the standard error of the median has not been investigated. It is not appropriate to obtain the same by using normal approximation results, especially for skew populations, which would

make the sampling distribution of $\hat{\theta}$ skew in moderately large samples. The extension of the method to quantiles and percentiles is simple as far as confidence intervals are concerned. However, obtaining accurate estimates of standard errors for quantiles and percentiles for skew populations seems more difficult.

3.2 Seasonally Adjusted Data

Seasonally adjusted data receive considerable attention in data analysis pertaining to continuous surveys where trends over time, adjusted for seasonal factors, are frequently dealt with. While variances of unadjusted data in surveys have been estimated with regularity, there has only been a marginal amount of work undertaken to estimate the variances of seasonally adjusted data.

The variances of linear and nonlinear estimates are generally easily derived theoretically with first order Taylor series or various sample reuse approximations, as described in Section 2. However, the variance of seasonally adjusted data, whatever the method employed, has not, to our knowledge, been theoretically obtained. The closest to a theoretical development of that variance may be found in Wolter and Monsour (1980), where the problem of variance estimation for a deseasonalized series is discussed and a variance of the seasonally adjusted data by each of several procedures is obtained, assuming a time-series model for the seasonal adjustment algorithm.

From a practical point of view, for a continuous survey, the balanced repeated replication method can be used to derive the variance of seasonally adjusted data. An identical orthogonal BRR matrix is used for both current and previous surveys. The derived sets of half sample data are then seasonally adjusted and a variance between these adjusted figures is derived. See Gray (1976) for an application to the Canadian Labour Force Survey.

4. Current Uses at Statistics Canada

While estimates of variance are as yet not available for all characteristics produced by all surveys at Statistics Canada, numerous surveys do produce variances of their estimates by many varied methods. The following serve as examples only, they are not meant to be all exhaustive and perhaps not even representative of all the surveys at Statistics Canada.

Of the methods discussed above, with the exception of the bootstrap, all are in use at Statistics Canada. The National Farm Survey estimates, for example, are <u>all</u> accompanied by coefficients of variation estimated by independent replication (Davidson, 1984). The Taylor linearization method is in frequent use, notably to estimate the variances of the first iteration raking ratio estimates of the Canadian Labour Force Survey (LFS) characteristics (Chaudhry and Lee, 1984). Moreover, studies are underway to produce the variances of the seasonally adjusted data from the LFS using balanced repeated replication (Gray, 1976). Finally, the jackknife methodology is employed in estimating reliability measures for the Industry Selling Price Index (Sande,*et al*, 1983).

As of late, numerous demands are being made of Statistics Canada to produce reliability measures for more and more new characteristics; nonlinear estimates in particular. Variances of functionals such as the median and other percentiles are also being sought. It is our responsibility to produce these measures but first and foremost, it is our duty to ensure that our estimators are theoretically sound. To this end, several research endeavours have been undertaken at Statistics Canada; other initiatives need to follow.

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