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ADJUSTMENT OF ECONOMIC PRODUCTION SUB-ANNUAL SERIES TO CORRESPONDING ANNUAL SERIES

by

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* This is a preliminary version.
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AJUSTEMENT AUX SÉRIES ANNUELLES DES SÉRIES SOUS ANNUELLES DE PRODUCTION ÉCONOMIQUE

SOMMAIRE

Statistique Canada mène présentement un projet de remaniement des enquêtes auprès des entreprises, font partie de ce projet les enquêtes annuelles et infra-annuelles de production économique. Des différences entre les valeurs estimées par les enquêtes annuelles et par les enquêtes infra-annuelles vont exister, ceci dû à l'utilisation de versions différentes de la base de sondage. On étudie ici le problème d'ajuster les séries infra-annuelles au jalons annuels dans le contexte d'un environnement de production de données. On fait la revue de méthodes traitant ce problème. On suggère des procédures, utilisant une méthode de minimisation sous contraintes, qui permettent de raccorder les années de données ajustées sans causer de bris dans la série. Ensuite, on compare ces dernières méthodes avec d'autres en utilisant des données des enquêtes annuelles et mensuelles sur le commerce de gros. Les procédures suggérées donnent une performance satisfaisante.

ADJUSTMENT OF ECONOMIC PRODUCTION SUB-ANNUAL SERIES TO CORRESPONDING ANNUAL SERIES

ABSTRACT

Statistics Canada is conducting a Business Survey Redesign Project, with part of it being the redesign of annual and sub-annual economic production surveys. Differences in terms of estimates produced will exist between the sub-annual and annual surveys figures due to different versions of the frame. The problem of adjusting sub-annual series to annual benchmarks is studied here in the context of a production environment. Some methods to solve the problem are reviewed. Suggested procedures, based on constrained minimization, will allow the years of adjusted data to be smoothly linked. The latter and other methods are compared using data from the Wholesale Trade Monthly and Annual Surveys. The suggested procedures appear to perform satisfactorily.

I INTRODUCTION

The problem of adjusting a sub-annual series of estimates to figures produced by an annual survey is well known by Statistical Agencies. For example, it may consist of adjusting monthly sample estimates of sales in an industry to their corresponding annual totals produced by a census. It may also be desired to adjust a quarterly series of estimated inventories to the fourth quarter values given by a census.

This study has been motivated by Statistics Canada's Business Survey Redesign Project. This project comprises, amongst other objectives, the design of a Central Frame Data Base (C.F.D.B.) for the storage, maintenance an access to frame data relevant to annual and sub-annual economic surveys (Infrastructure Project Team, 1984). The objectives of the design are to achieve, at the lowest cost possible, the development and provision of facilities for data integration; to ensure comprehensive and non-duplicative coverage within and between economic surveys; and to foster the replacement of business units reporting through direct survey by comparable information gathered from administrative sources.

One major implication of Statistics Canada's project is the redesign and integration of annual and sub-annual economic production surveys (i.e. the Wholesale and Retail Trade surveys). In order to realize the above objectives, these annual surveys will have a common frame, standardized sampling and estimation procedures so that they may be collectively viewed as one survey (Colledge, 1985). For the sub-annual surveys standardized procedures will be used and, for practical reasons, a different version of the frame used by annual surveys.

The aim of annual surveys of economic production will be to collect and publish statistics on economic activities broken down by industries, by province/major urban area and, possibly, by size. These annual surveys will provide counts and totals for use by the corresponding sub-annual surveys. There may be two or more sets of counts and totals (e.g. preliminary, revised and final) due to more than one mailout. The preliminary figures for a given year Y will be produced in the first months of year Y+2. The revised and final may be available later in year Y+2.

Sub-annual surveys of economic production will produce current estimates of key variables in economic activities at various levels of industrial and geographical aggregation. Differences between sub-annual and annual survey, results which are due to different versions of the frame, will be removed in the final estimates of all sub-annual surveys that can be benchmarked to annual surveys. For a given period P of a given year Y, a preliminary estimate will first be produced. Then, one month later, a revised version will be available. Finally, at the beginning of Y+1, the final estimate will be produced.

The structure of this report is as follows. In section 2, a review of the adjustment methods is presented. In section 3, the suggested procedure for benchmarking on a regular basis in a production environment are described. Some adjustment procedures are compared in the fourth section. Finally, the last section concludes as with the summary remarks.

II REVIEW OF ADJUSTMENT METHODS AND APPLICATIONS

2.1 Methods

One of the first methods proposed to adjust a sub-annual flow series (i.e. sales) to its annual benchmarks is Bassie's method (Bassie, 1939). There are two versions of the method, an additive and a multiplicative version, which are used depending on the type of correction applied. The multiplicative version does not yield an exact correction, a small amount of prorating is needed after application of the method, but the additive yields an exact solution. This method has been developped to handle two years at once, the first being previously adjusted but not the second. It assumes that the correction to be applied on the sub-annual series is an explicit function of time. This function must fulfill the following four conditions:

i) In the first year the average correction must be zero.

ii) For the second year, the total discrepancy between the benchmark and the sub-annual series is distributed among all the sub-annual periods.

iii) In order to avoid breaks in the steps from one year to the next, the correction applied at the beginning of the first year will be null.

iv) The correction funtion will tend to be horizontal at the end of the second year. This is done in order not to distort the original trend at the end of the second year.

The simplest function which satisfies the above conditions is a cubic curve.

Bassie's method has two weaknesses. Firstly, it may produce discontinuities between the last sub-annual period of one year and the first period of the next. Secondly, seasonality may be induced in the adjusted series (see Dagum, 1977).

Denton (1971) proposed an approach to the problem of adjusting sub-annual time series to make them accord with annual totals or averages without introducing artificial discontinuities. This method involves the constrained minimization of a quadratic function of the differences between the adjusted and the non-adjusted (original) series. With this approach one has to choose the appropriate quadratic function which will depend on the series to be adjusted. Also, as stated by Cholette (1983), care must be taken in the choice of constraint when the series to be adjusted has to be linked with historical data.

Smith (1977) suggested a method, called GSTEP, which is identical to the method proposed by Denton except in the quadratic function minimized. This function is the squared differences in the trend ratios between the adjusted and the unadjusted series. It's minimization leads to the solution of a non-linear equations system. Denton's functions all lead to solve systems of linear equations.

Baldwin (1980) derived a formula which can be used to benchmark stock series. The formula is the solution of a quadratic minimization problem where the loss function is the squared difference of the logarithms of the original and adjusted series. Trager (1980) derived also that formula.

Some authors, such as Chow and Lin (1971), Somermyer, Jansen and Lauter (1976) and Litterman (1983), dealt with a similar problem. It consists of interpolating between annual estimates of a variable in using measures of other variables, produced sub-annually, related to the former with an econometric model. The problem studied in this paper is a different matter.

2.2 Applications

In 1977, the U.S. Bureau of the Census introduced new samples and new definitions for the estimated variables in their Retail, Wholesale and Selected Services monthly surveys of the Business Division (see Monsour and Trager, 1979). It was then decided to reconcile and link the historical series to the new series. In order to achieve that the constrained minimization approach of Denton was used at the kind of business level. The August 77 sales estimate (link point) the 1967 and 1972 sales census estimates were the constraints, and the sum of squared differences between the relative revision of one month and the next was the quadratic function to be minimized. The revision of the historical series was quite satisfactory in that the month-to-month trend ratios were well reproduced.

In the Monthly Survey of Canadian Manufacturer's Shipments, Inventories and Orders, a benchmarking method, called Step Adjustment, is used (see Wightman, 1983). This method is applied on a regular basis (whenever an annual benchmark becomes available) to inventories and orders series (which are stock series). It consists essentially of computing the ratios of the non-adjusted series to a straight line between subsequent year-end values and of multiplying these ratios by the straight line computed between the two corresponding benchmarks (derived from the census of manufactures) to obtain the adjusted series. It will be observed in section IV that this method performs similarly to the constrained minimization approach.

III SUGGESTED ADJUSTMENT PROCEDURES

3.1 General Considerations and Assumptions

The adjustment procedures, which will be described here, are designed to solve the problem of benchmarking sub-annual series annually in the context of a production environment such as the one presented in section I. A necessary assumption that is made, when designing such procedures, is that figures produced by annual surveys are more reliable than the sub-annual level estimates. Otherwise, there will be no need to benchmark sub-annual series.

As mentioned in the introduction, it is expected that, for a given year Y, preliminary, revised and final benchmarks will be successively available in year Y+2. It is generally true that the revision and final figures are very close. But, it may happen, for some annual surveys, that the preliminary and final figures are as far apart as are the sub-annual estimates to the final figures. In such cases the preliminary benchmarks should not be used to adjust the sub-annual values since there is no gain in doing so. It will be better to wait for the revised figures before benchmarking sub-annual data.

The subannual surveys will produce preliminary, revised and final estimates. The revised and final values are generally very close to each other. But, the preliminary and final values are more than often sensibly different. Thus the preliminary sub-annual estimates should not be used when computing adjusted values.

In a given year Y a benchmark for year Y-2 will become available, but none for year Y-1 and on. If nothing is done when adjusting year Y-2, a discontinuity in the sub-annual series will appear between the last period of year Y-2 and the first period of year Y-1. One way to solve the problem is to project adjustment factors for year Y-1, Y and Y+1 (that year the next benchmark will be produced). To project the adjustment factors, the unadjusted sub-annual series will be extrapolated to the end of year Y+1 and benchmarks will be projected for year Y-1, Y and Y+1. How exactly this is to be done is described in the description of the procedures below. The benchmarks will be projected in assuming that the relative difference between the benchmark of year Y-2 and the sub-annual estimates of that year is the most likely difference to be reproduced in the future (e.g. years Y-1, Y and Y+1).

According to what has been written above, one may expect three types of sub-annual adjustment factors for a given year Y. Namely, the projected factors produced before year Y+2, the revised factors produced in year Y+2 when the revised benchmark becomes available, and the final factors produced when the final benchmark becomes available.

Generally, for surveys of economic production, figures are published for various levels of industrial and geographical aggregation. This implies that, in order to get the sum of the industrial aggregates to add up to the sum of the geographical aggregates, the adjustment of the sub-annual series should be done at a cross-classified level (industry by geographic area).

3.2 Specific Considerations

Since the approach of constrained minimization of a quadratic function permits the exact adjustment of a sub-annual series to benchmarks without introducing artificial discontinuities, that method will be used in the procedures described below.

Now, to choose the quadratic function to be minimized, consider the following points. In general, sub-annual surveys produce reliable estimates of the month-to-month (or quarter-to-quarter) changes in economic activities. Thus the quadratic function to be chosen should minimize the differences in the month-to-month (or quarter-to-quarter) trend ratios between the adjusted sub-annual series and the unadjusted (original) series. Thre are two quadratic functions which lead to this minimization while tending to equalize the relative differences in the trend ratios. These are the following:

$$\text{(i)} \quad \sum_t \left(\frac{x_t/x_{t-1} - y_t/y_{t-1}}{y_t/y_{t-1}} \right)^2 \qquad \text{(ii)} \sum_t \left(\frac{x_t}{y_t} - \frac{x_{t-1}}{y_{t-1}} \right)^2$$

where x_t and y_t denote respectively the series resulting from the adjustment and the original series.

In minimizing the above two functions, subject to annual benchmarks, it can be shown that (i) leads to the solution of a system of non-linear equations while (ii) leads to a system of linear equations. If the series to be adjusted is a flow series (i.e. sales) then the non-linear system of (i) has to be solved with an iterative method, but if it is a stock series (i.e. inventories) then the non-linear system can be solved directly (see Appendix A). This results from the type of constraints associated with the two types of series. For a flow series the constraints are sums of the sub-annual values while for a stock series they are year-end values.

For the practical reasons enumerated above, in the case of a flow series function(ii) will be used for benchmarking and in the case of a stock series either (i) or (ii).

3.3 Description of the Procedures

Two procedures will be described here, one for the flow series and the other for the stock series. The adjustment procedure for stock series will be simpler than the one for flow series, even if both are based on the same assumptions. This is a result of the simpler form of the constraints for stock series (year-end values).

. First adjustment of a sub-annual flow series

In the third year of the sub-annual survey, when a revised (or a good preliminary) benchmark becomes available for the first year of the series, the adjustment factors for the first year will all be equal to the ratio of the benchmark to the sum of the first year sub-annual estimates. The projected adjustment factors for the second, third and fouth years will all be equal to the first year factors. (It can be shown that this procedure minimize the appropriate quadratic function.)

. Subsequent adjustment of a sub-annual flow series

In year Y of the sub-annual survey, when a revised (or a good preliminary) benchmark for year Y-2 becomes available, adjustment factors will be calculated the following way.

Firstly, the sub-annual unadjusted series will be extrapolated till the end of year Y+1 using the data from year Y-3, Y-2, Y-1 and Y (only revised values of that current year) using a simple autoregression model. This model will be for a series z_t displaying seasonality

$$z_t = a_1 z_{t-1} + a_2 z_{t-p} + a_3 z_{t-p-1} + e_t$$

Where p will be equal to 4 for a quarterly series and 12 for a monthly series. The model for a series with no seasonality will be

$$z_t = a z_{t-1} + e_t$$

(These models have been chosen because experience has shown that they can explain the major portion of the variance of an economic time series.)

Secondly, benchmarks for year Y-1, Y and Y+1 are projected.

These are calculated as the product of the extrapolated series values sum for a given year with no benchmark times the ratio of year Y-2 benchmark to the sum of the year Y-2 sub-annual unadjusted estimates.

Thirdly, the appropriate quadratic function will be minimized using the extrapolated unadjusted series from Y-3 to Y+1 and, as constraints, year Y-3 final benchmark, year Y-2 revised benchmark and years Y-1, Y and Y+1 projected benchmarks. Also, if the current year Y is not the fourth year of the sub-annual survey, another constraint will be the year-end adjustment factor of year Y-4. (This last constraint will allow the adjusted values of year Y-4 and Y-3 to be smoothly linked, e.g. without discontinuity.) The results of the constrained minimization will be the final adjustment factors for year Y-3, revised factors for year Y-2 and projected factors for years Y-1, Y and Y+1.

The adjustment procedures for a stock series will be as follows.

- First adjustment of a sub-annual stock series

This precedure is exactly the same as the first adjustment of a sub-annual flow series, except that the adjustment factors will be equal to the ratio of the benchmark to the year-end estimated value of the series' first year.

- Subsequent adjustment of a sub-annual stock series

In year Y of the sub-annual survey, when a revised (or a good preliminary) benchmark for year Y-2 becomes available, adjustment factors will be calculated the following way. An appropriate quadratic function will be minimized using years Y-3 and Y-2 of the unadjusted series and, as constraints, year Y-3 final benchmark and year Y-2 revised benchmark. Also, if the current year Y is not the fourth year of the sub-annual survey, another constraint will be the year-end adjustment factor of year Y-4. The minimization will be the final adjustment factors for year Y-3 and revised adjustment factors for year Y-2. The projected factors for year Y-1, Y and Y+1 will all be equal to the year-end factor of year Y-2. (It can be shown that, for a stock series, this procedure leads to the same results as a procedure using extrapolated series data and projected benchmarks. This is not the case for a flow series, as it will be seen in the next section.)

IV COMPARISON OF THE PROCEDURES

In this section, the performance of three methods for adjusting flow series and four methods for adjusting stock series will be examined. To compare the methods, data on sales and inventories of Wholesale Merchants produced by the Wholesale Trade Monthly and Annual Surveys will be used. Only two industries and provinces will be considered. These are:

Industries
- Motor vehicles and accessories,
- Farm machinery, equipment and supplies,

Provinces
- Prince Edward Island,
- and Quebec.

The monthly estimates used are covering the period of January 1981 to July 1985 and the annual figures (benchmarks) are available for 1981, 1982 and 1983. In applying the procedures described in section III, adjustment factors (or adjusted figures) will be obtained as illustrated below.

Year of data / Year of adjustment	1981	1982	1983	1984	1985
1983	revised	projected	projected	-	-
1984	final	revised	projected	projected	-
1985	-	final	revised	projected	projected

Illustration of the type of adjustment factors obtained per year of adjustment.

In order to measure the performance of the different methods the following measures of error will be used:

i) Month-to-mongh trend ratio difference between the adjusted and the original series expressed as a percentage of the original trend ratio. Algebraically this gives

$$100 \times \left(\frac{x_t/x_{t-1} - y_t/y_{t-1}}{y_t/y_{t-1}} \right) \underset{=}{\Delta} 100 \left(\frac{c_t}{c_{t-1}} - 1 \right)$$

where x_t is the adjusted series, y_t is the original series and $c_t = x_t/y_t$ is the adjustment factor.

ii) Yearly average of the absolute value of level revisions from one adjustment year to the next. The level revisions will be expressed as a percentage of the level estimates of the original series. Algebraically, this measure can be written as

$$\sum_{t\ \varepsilon\ \text{given year}} 100 \left(\frac{x_t^{(Y+1)} - x_t^{(Y)}}{y_t} \right) = \sum_{t\ \varepsilon\ \text{given year}} 100 \left(c_t^{(Y+1)} - c_t^{(Y)} \right)$$

where $x_t^{(Y)}$ and $c_t^{(Y)}$ are respectively, the adjusted value and the adjustment factor of month "t" calculated in adjustment year Y.

The methods used in the comparison as well as the results will be presented in two parts. One part is for the sales series and the other part is for the inventories series.

Sales Series

The three methods used to adjust the four Wholesale Merchants sales series are:

- S-A, this method produces adjustment factors for a given year that are all equal to the ratio of the annual value for that year to the sum of the corresponding twelve monthly unadjusted estimates. This method has no provision for steps between years and does not give projected adjustment factors.

- S-B, this method is very similar to the one for flow series described in section III. The difference is that the projected factors are not calculated in extrapolating the original series and in projecting the benchmarks, but simply in repeating the December factor of the most recent year with a benchmark.

- S-C, this method is the one suggested for flow series in section III.

The results for the comparison of the above three methods are presented in tables 1 to 5 and figures 1 and 2 (see Appendix B).

Table 1 shows that with methods S-B and S-C the difference between the original January to December trend ratios and the corresponding adjusted trend ratios are much smaller than with method S-A. Result which was expected. Also methods S-B and S-C perform equally well in that respect. Figure 1 (with series adjusted by S-A) and figure 2 (with series adjusted by S-C) are showing graphically the same results. (Note the differences in adjusted and original January to December trend ratios.)

Table 2 demonstrates that method S-C leads to smaller level revisions in the data than S-B when a first set of projected factors for a given year is replaced by a second set of projected factors. This phenomenon is more accentuated for 1984. This advantage of method S-C over S-B is of a non-negligeable nature, especially when it is desired that published preliminary, revised and final estimates be as close as possible to each other.

Table 3 shows that method S-C has a slight advantage over S-B for level revisions induced by the replacement of projected factors by revised factors.

Table 4 does not show much difference between methods S-B and S-C concerning level revisions due to replacing revised factors by final factors. The reason for this is that, for the purpose of this analysis, the revised and final benchmarks are identical. This will not necessarily be the case in real practice.

Table 5 shows that methods S-B and S-C performs equally well vis-à-vis yearly average trend ratio differences.

Inventories Series

The four methods used to adjust the four Wholesale Merchants inventories series are:

- I-A, this method produces adjustment factors for a given year that are all equal to the ratio of the annual value for that year to the December estimate of that year. There is no provision for steps between years.

- I-B, this method is similar to the one for stock series described in section III. The difference is that it does not use a constrained minimization approach to calculate the factors, but the method used in the Monthly Survey of Canadian Manufacturer's Shipments, Inventories and Orders.

- I-C, this method is the one for stock series described in section III, with the sum of squared differences between the relative adjustment of one month and the next as the function to be minimized.

- I-D, this method is the one for stock series described in section III, with the sum of squared relative differences between the adjusted trend ratio and the original trend ratio as the function to be minimized.

The results for the comparison of the above four methods are presented in tables 6 to 9 and figures 3 and 4 of Appendix B.

Table 6 shows that the differences between the original and adjusted January to December trend ratios are much smaller for methods I-B, I-C and I-D than for method I-A. This is an expected result. Methods I-B, I-C and I-D perform equally well. Figures 3 and 4, which respectively show the graph of a series adjusted with method I-A and I-C, reflect the above observation.

Table 7 and 8 demonstrate that methods I-B, I-C and I-D perform equally well concerning level revisions due to the revision of adjustment factors.

Tabel 9 shows that methods I-B, I-C and I-D give the same average trend ratio differences between adjusted and original series. Method I-A also has the same values, which simply means that the three above methods distribute, over a year, the step produced by method I-A between years.

V CONCLUSION AND DISCUSSION

The results of section IV lead to the choice of method S-C for the adjustment of flow/sales series to annual benchmarks. This method allows years of adjusted data to be smoothly linked and minimizes level revisions as new benchmarks become available.

The choice of an adjustment method for stocks/inventories series is more difficult. Methods I-B, I-C and I-D allow, equally well, adjusted years of data to be linked smoothly. But, one may choose method I-C since it requires less algebraic calculations, which means less computer time.

When a redesigned survey will have been run for a while, it will be advisable to link the new data produced with the historical data. An approach based on similar methods as the ones described in this document may then be used.

As a last remark, it should be noted that the methods presented in this document are only remedies. The sources of the differences between sub-annual and annual surveys should be investigated in detail. This sort of study may eventually lead to a reconciliation of sub-annual and annual surveys at a micro-data level. Such a result would improve the quality of the published data.

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APPENDIX A

Solutions to constrained quadratic minimization problems for benchmarking.

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Solutions to constrained quadratic minimization problems for benchmarking.

1. Let first consider the problem of minimizing the sum of squared differences between the relative revision of one month (or quarter) and the next, with annual benchmarks and an initial adjustment factor as constraints.

   a) General Solution

   Let us formalize the problem as a non-linear program (NLP). It can be expressed as follows:

$$\min_{x} \sum_{t=1}^{N} \left(\frac{x_t}{y_t} - \frac{x_{t-1}}{y_{t-1}} \right)^2 \quad \text{with } \frac{x_o}{y_o} \text{ known}$$

subject to $\sum_{t=(k-1)p+1}^{kp} x_t = b_k$ for a flow series

or $x_{kp} = b_k$ for a stock series

with $k = 1, \ldots, K$.

Where y_t denotes the original series of estimates; x_t denotes the adjusted series; $N=Kp$ is the number of sub-annual values to be adjusted, with K the number of years and p the number of sub-annual periods per year; and b_k is the benchmark produced by the annual survey the k^{th} year.

Using $c_t = x_t/y_t$ the adjustment factors and the matrix notation, the problem can be rewritten in a simpler form. This gives

$$\min_{C} \; (D\,C - C_o)' \; (D\,C - C_o)$$

subject to $Y\,C = B$

where

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & & 0 & 0 \\ -1 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & -1 & 1 & 0 & & 0 & 0 \\ & \vdots & & & & \vdots & \\ 0 & 0 & 0 & 0 & \ldots & -1 & 1 \end{bmatrix}_{N \times N} ;$$

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}_{N \times 1} ; \quad C_o = \begin{bmatrix} c_o \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{N \times 1} ; \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}_{K \times 1} ;$$

$$Y = \begin{bmatrix} y_1 & y_2 & \ldots & y_p & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & y_{p+1} & \ldots & y_{2p} & \ldots & 0 & \ldots & 0 \\ \vdots & & & \vdots & & & \vdots & & \vdots & & \vdots \\ 0 & 0 & & 0 & 0 & & 0 & & y_{(K-1)p+1} & & Y_{KP} \end{bmatrix}_{K \times N}$$

for a flow series

and

$$Y = \begin{bmatrix} 0 & \ldots & y_p & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 \\ 0 & \ldots & 0 & 0 & \ldots & y_{2p} & 0 & \ldots & 0 & 0 \\ \vdots & & \vdots & & & \vdots & & & \vdots & \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 & Y_{KP} \end{bmatrix}_{K \times N}$$

for a stock series.

In order to solve the NLP, the Lagrangian is formed

$$f\,(C;L) = (DC-C_o)'(DC-C_o) - 2L'\,(B-YC)$$

where $L = \begin{bmatrix} l_1 \\ \vdots \\ l_k \end{bmatrix}$ is the vector of Lagrangian multipliers.

Then the first derivative of f to C is taken

$$\frac{\partial f}{\partial C} = 2D'DC - 2D'C_o + 2\,Y'L$$

a solution for C is obtained in letting $\frac{\partial f}{\partial C} = 0$ and in using YC = B. This results in the following linear system of equations.

$$\begin{bmatrix} D'D & Y' \\ Y & O \end{bmatrix} \begin{bmatrix} C \\ L \end{bmatrix} = \begin{bmatrix} D'C_o \\ B \end{bmatrix} . \qquad \text{(A.1)}$$

The solution for C, given by this system of equations, will always be a minimum since

for all $H = \begin{bmatrix} h_1, \ldots, h_N \end{bmatrix}'$ such that $Y\,H = O$

we have $H'D'D\,H \geq 0$.

If there is no initial adjustment factor as constraint (a reason for this would be that the values to be adjusted by minimization do not have to be linked with past data) then the problem can be formalized in matrix notation as follows.

$$\min_{C} \; C'D'DC \quad \text{subject to } YC = B$$

where

$$D = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ & & \vdots & & \ddots & & \vdots & \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{bmatrix}_{(N-1)XN}$$

The optimal solution to this problem is given by

$$\begin{bmatrix} D'D & Y' \\ Y & O \end{bmatrix} \begin{bmatrix} C \\ L \end{bmatrix} = \begin{bmatrix} O \\ B \end{bmatrix} . \qquad \text{(A.1)}$$

Once the c'_ts are obtained the adjusted series can be calculated as

$$x_t = c_t\, y_t \qquad t = 1, \ldots, N.$$

b) <u>Solution for stock series</u>

It can be shown that for a stock series the solution of system (A.1) leads to

$$c_t = c_{(k-1)p} + \frac{(c_{kp} - c_{(k-1)p})t}{p} \qquad \text{for } (k-1)p + 1 \leq t \leq kp \text{ with } k = 1, \ldots, K.$$

and the solution of system (A.2) leads to

$$c_t = \begin{cases} c_p & \text{if } 1 \leq t \leq kp \\ c_{(k-1)p} + \dfrac{(c_{kp} - c_{(k-1)p})t}{p} & \text{if } (k-1)p + 1 \leq t \leq kp \text{ with } k = 2, \ldots, k \end{cases}$$

where $c_{kp} = \dfrac{x_{kp}}{y_{kp}}$ with $k=1, \ldots, K$.

2. Now let us consider, for stock series only, the problem of minimizing the sum of squared relative differences between the adjusted trend ratio and the original trend ratios, with annual benchmarks and an initial adjustment factor as constraints.

This problem can again be formalized as a non-linear program. Using the same notation as the first problem one gets

$$\underset{x}{\text{Min}} \sum_{t=1}^{N} \left(\frac{x_t/x_{t-1} - y_t/y_{t-1}}{y_t/y_{t-1}} \right)^2 \qquad \text{with } \frac{x_o}{y_o} \text{ known}$$

subject to $x_t = b_k$ for $t=k_p$ and $k=1, ..., K$.

Substituting $x_t = c_t\, y_t$ in the above expressions and simplifying, the problem can be rewritten in the following simple form

$$\underset{c}{\text{Min}} \sum_{t=1}^{N} \left(\frac{c_t}{c_{t-1}} - 1 \right)^2 \qquad \text{with } c_o \text{ known}$$

subject to $c_t\, y_t = b_k$ for $t = kp$ and $k = 1, ..., K$

The problem can now be solved in forming the Lagrangian and in putting its gradient equal to zero. The Lagrangian is

$$f\ (c_t; l_k) = \sum_{t=1}^{N} \left(\frac{c_t}{c_{t-1}} - 1 \right)^2 + 2 \sum_{k=1}^{N} l_k\ (b_k - c_{kp}\ y_{kp}).$$

Taking the first order derivatives to the $c_{t's}$ equal to zero leads to a system of N non-linear equations. Among these, there are K independent sets of p-1 equations with p-1 unknowns (since the c_{kp}'s are known by the constraints) of the form

$$\left(\frac{c_{t+1}}{c_t} - 1 \right) \left(\frac{c_{t+1}}{c_t} \right) = \left(\frac{c_t}{c_{t-1}} - 1 \right) \left(\frac{c_t}{c_{t-1}} \right)$$

with $(k-1)p + 1 \leq t \leq kp-1$.

These equalities are verified in two cases, namely

$$c_{t+1}/c_t = \begin{cases} 1 - c_t/c_{t-1} \\ \text{or} \\ c_t/c_{t-1} \end{cases}$$

It can be shown that only the second case leads to a realisable solution which is minimum. This implies that

$$\frac{c_{(k-1)p+1}}{c_{(k-1)p}} = \frac{c_{(k-1)p+2}}{c_{(k-1)p+1}} = \dots = \frac{c_{kp}}{c_{kp-1}} = R_k \text{ a constant.}$$

Taking the product of the ratios above, one finds

$$\prod_{t=(k-1)p+1}^{kp} \left(\frac{c_t}{c_{t-1}}\right) = \frac{c_{kp}}{c_{(k-1)p}} = R_k^p$$

which implies that $R_k = \left(\dfrac{c_{kp}}{c_{(k-1)p}}\right)^{1/p}$

and that

$$c_t = c_{(k-1)p}\left(\frac{c_{kp}}{c_{(k-1)p}}\right)^{(t-(k-1)p)/p} \qquad \text{for } (k-1)p+1 \le t \le kp \text{ with } k=1, \dots, K$$

If $c_o = x_o/y_o$ does not exist then

$$c_t = c_p \quad \text{for} \quad t=1, \dots, p.$$

In the above formula if one substitutes (x_j/y_j) for c_j then it can be observed that it corresponds to the formula derived by Baldwin (1980) and Trager (1980).

APPENDIX B

TABLES AND FIGURES

Trend ratio dates	January 82/December 81			January 83/December 82			January 84/December 83		
Method / Series	S-A	S-B	S-C	S-A	S-B	S-C	S-A	S-B	S-C
Motor vehicles and accessories	8.69	1.42	1.39	-5.12	-0.93	-0.98	20.56	0	0.27
Farm Machinery equipment and supplies	6.51	1.34	1.33	-9.27	-1.41	-1.48	11.07	0	0.36
Prince Edward Island	0.17	0.29	0.31	-4.69	-0.65	-0.70	13.87	0	0.19
Quebec	-1.85	-0.40	-0.40	2.61	0.41	0.43	9.31	0	-0.10

Table 1: Monthly trend ratio differences in percentage for three link points of Sales Series.

Year: Y	1983			1984		
Series / Method	S-A	S-B	S-C	S-A	S-B	S-C
Motor vehicles and accessories	N.A.	8.63	7.03	N.A.	7.67	4.48
Farm machinery equipment and supplies	N.A.	7.44	6.10	N.A.	13.0	9.24
Prince Edward Island	N.A.	.196	.160	N.A.	5.52	4.34
Quebec	N.A.	2.08	1.69	N.A.	3.42	2.34

Table 2: Average absolute level revisions in percentage due to the difference in adjustment projected factors produced in year Y and Y+1.

Year: Y	1982			1983		
Series / Method	S-A	S-B	S-C	S-A	S-B	S-C
Motor vehicles and accessories	N.A.	6.95	6.95	N.A.	5.95	4.37
Farm machinery equipment and supplies	N.A.	6.00	5.98	N.A.	10.34	8.97
Prince Edward Island	N.A.	.157	.157	N.A.	4.23	4.22
Quebec	N.A.	1.67	1.67	N.A.	2.67	2.28

Table 3: Average absolute level revisions in percentage due to the difference between projected factors produced in year Y+1 and revised factors produced in year Y+2.

Year: Y	1981			1982		
Series / Method	S-A	S-B	S-C	S-A	S-B	S-C
Motor vehicles and accessories	N.A.	1.31	1.39	N.A.	0.78	0.61
Farm machinery equipment and supplies	N.A.	1.09	1.15	N.A.	1.36	1.25
Prince Edward Island	N.A.	.030	.032	N.A.	.54	.57
Quebec	N.A.	.31	.33	N.A.	.35	.32

Table 4: Average absolute level revisions in percentage due to the difference between revised factors produced in year Y+2.

Year	1981			1982			1983			1984		
Series / Method	S-A	S-B	S-C	S-A	S-B	S-C	S-A	S-B	S-C	S-A	S-B	S-C
Motor vehicles and accessories	0	.53	.56	.72	.63	.63	.43	.54	.49	1.71	0	.13
Farm Machinery equipment and supplies	0	.38	.40	.54	.73	.75	.77	.81	.75	.92	0	.17
Prince Edward Island	0	.01	.01	.01	.24	.26	.39	.38	.35	1.16	0	.09
Quebec	0	.11	.12	.15	.21	.21	.22	.22	.21	.78	0	.05

Table 5: Average trend ratio differences in percentage between the adjusted series and the original series.

Trend ratio dates	January 82/December 81				January 83/December 82				January 84/December 83			
Series / Method	I-A	I-B	I-C	I-D	I-A	I-B	I-C	I-D	I-A	I-B	I-C	I-D
Motor vehicles and accessories	-2.87	-.225	-.239	-.242	1.85	.153	.154	.153	14.44	0	0	
Farm Machinery equipment and supplies	13.81	1.06	1.15	1.08	-5.72	-.50	-.48	-.49	13.88	0	0	0
Prince Edward Island	-23.81	-2.11	-1.98	-2.24	-4.10	-.36	-.34	-.35	39.60	0	0	0
Quebec	-6.42	-.49	-.54	-.55	2.85	.25	.24	.23	13.88	0	0	0

Table 6: Monthly trend ratio differences in percentage for three link points of Inventories Series.

Year: Y	1983			1984		
Series / Method	I-B	I-C	I-D	I-B	I-C	I-D
Motor vehicles and accessories	2.53	2.53	2.53	1.58	1.58	1.58
Farm machinery equipment and supplies	11.30	11.30	11.30	5.33	5.33	5.33
Prince Edward Island	23.35	23.35	23.35	3.06	3.06	3.06
Quebec	5.86	5.86	5.86	2.43	2.43	2.43

Table 7: Average absolute level revisions in percentage due to the difference in adjustment projected factors produced in year Y and Y+1.

Year: Y	1982			1983		
Series / Method	I-B	I-C	I-D	I-B	I-C	I-D
Motor vehicles and accessories	1.34	1.37	1.38	.86	.86	.86
Farm machinery equipment and supplies	5.95	6.12	6.00	2.94	2.89	2.91
Prince Edward Island	12.91	12.65	13.17	1.69	1.66	1.67
Quebec	3.08	3.18	3.21	1.34	1.32	1.31

Table 8: Average absolute level revisions in percentage due to the difference between projected factors produced in year Y+1 and revised/final factors produced in year Y+2.

Year	1981				1982				1983				1984			
Series / Method	I-A	I-B	I-C	I-D	I-A	I-B	I-C	I-D	I-A	I-B	I-C	I-D	I-A	I-B	I-C	I-D
Motor vehicles and accessories	0	0	0	0	.24	.24	.24	.24	.15	.15	.15	.15	1.20	0	0	0
Farm Machinery equipment and supplies	0	0	0	0	1.15	1.08	1.08	1.08	.48	.49	.49	.49	1.16	0	0	0
Prince Edward Island	0	0	0	0	1.98	2.24	2.24	2.24	.34	.35	.35	.35	3.30	0	0	0
Quebec	0	0	0	0	.54	.55	.55	.55	.24	.23	.23	.23	1.16	0	0	0

Table 9: Average trend ratio differences in percentage between the adjusted and the original series.

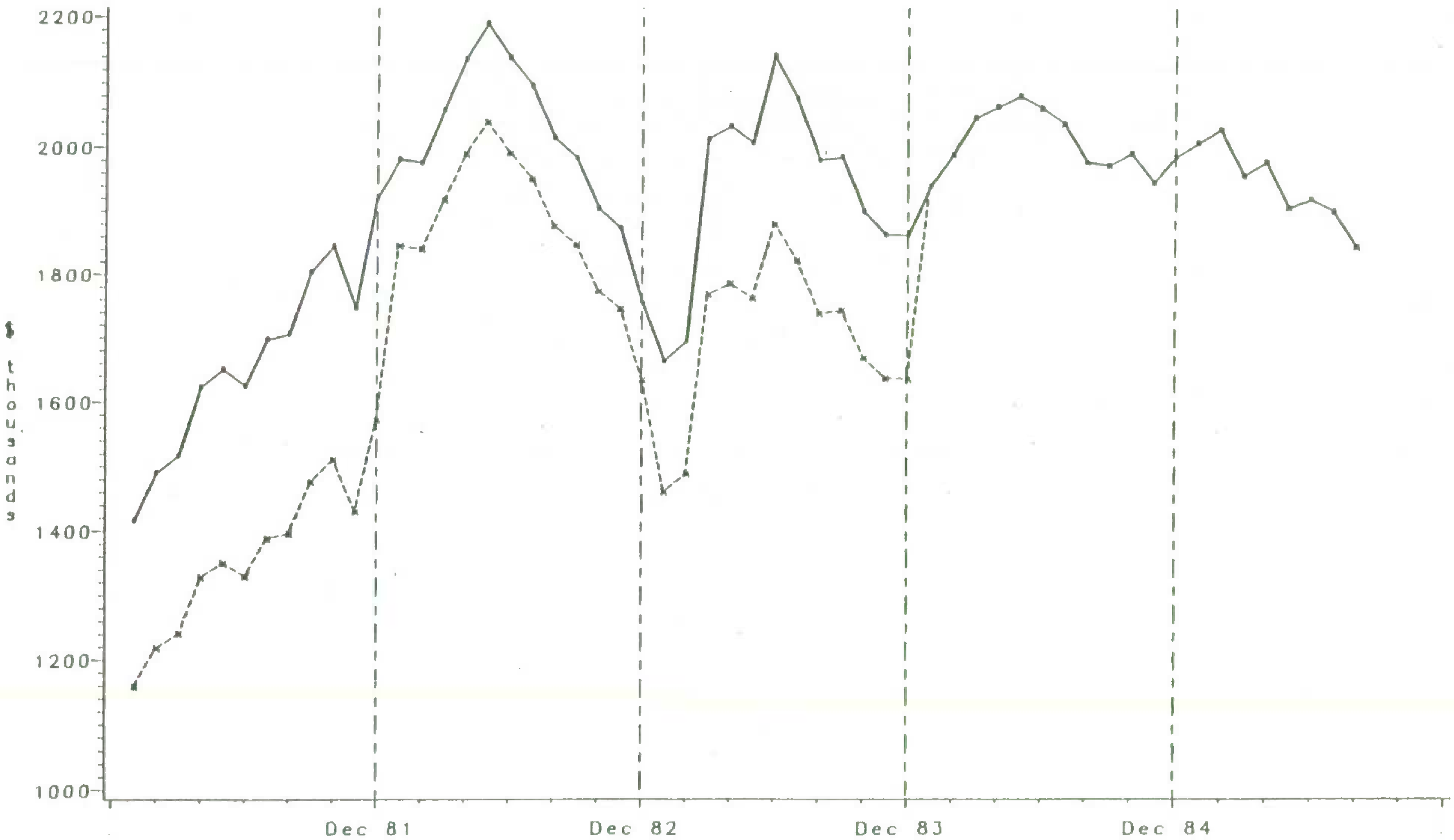
Figure 1
Motor Vehicles and Accessories Sales, Canada
Original and Adjusted Series with Method S-A

O = Original data (full line)
A = Adjusted data (dotted line)

Figure 2
Motor Vehicles and Accessories Sales, Canada
Original and Adjusted Series with Method S-C

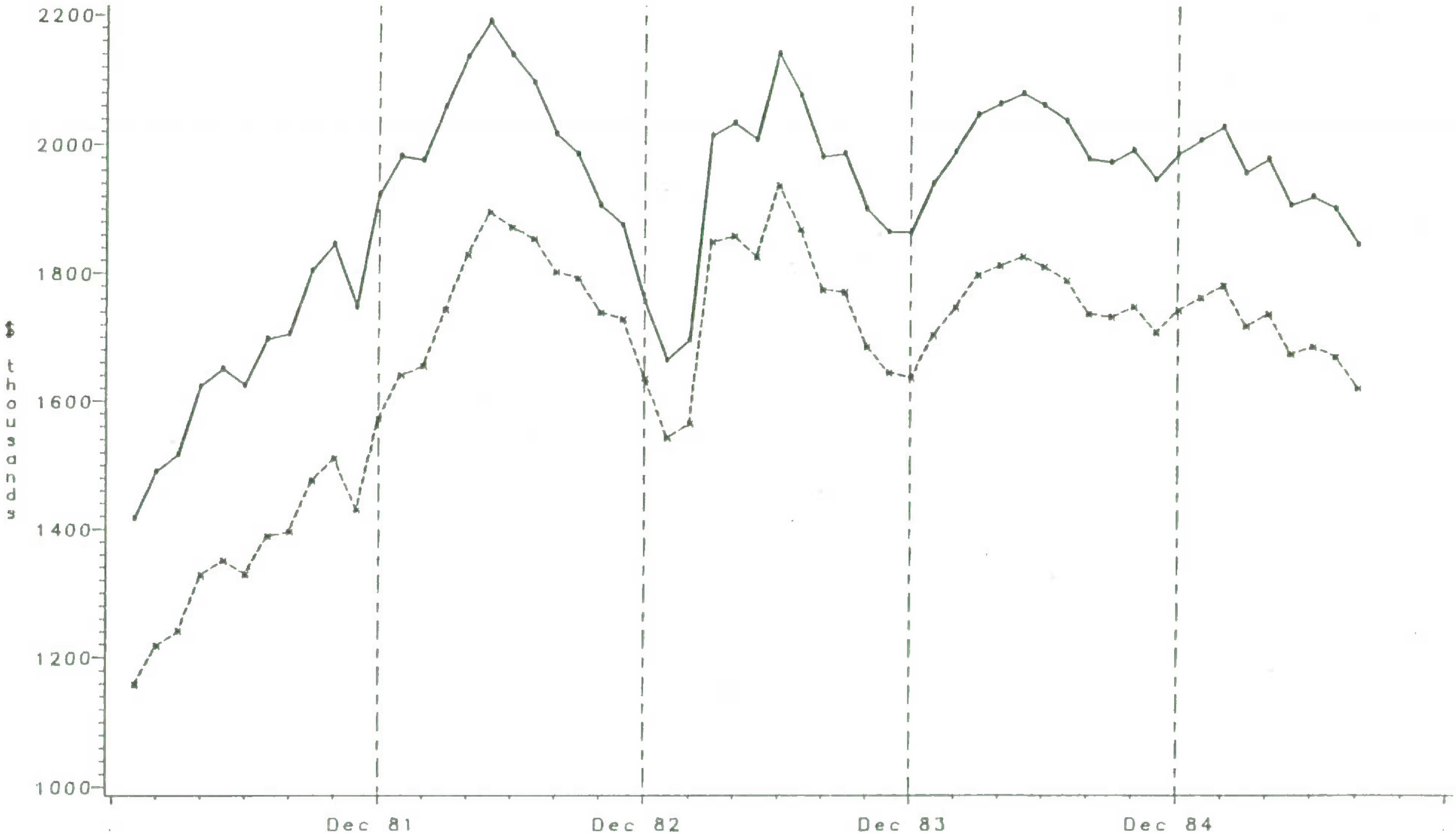
0 = Original data (full line)

Figure 3
Farm Machinery, Equipment and Supplies Inventories, Canada
Original and Adjusted Series with Method I-A



0 = Original data (full line)
A = Adjusted data (dotted line)

Figure 4
Farm Machinery, Equipment and Supplies Inventories, Canada
Original and Adjusted Series with Method I-C



0 = Original data (full line)
A = Adjusted data (dotted line)