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E.B. Dagum, N. Chhab and K. Chiu
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Estela Bee Dagum, Norma Chhab and Kim Chiu
Time Series Research and Analysis Centre
STATISTICS CANADA

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ABSTRACT

The linear properties of the seasonal adjustment and trend-cycle filters of X11ARIMA applied to central observations have been studied by several authors for a "standard" combination of seasonal and trend-cycle filters (see e.g. Young, 1968; Wallis, 1974 and 1982; Dagum, 1982 and 1983). For current observations, the corresponding combination of asymmetric filters have been discussed, among others, by Dagum, (1982 and 1983); Wallis, (1982); Burridge and Wallis (1984); Dagum and Laniel (1987).

This study extends previous analyses by including all the possible combinations of trend-cycle and seasonal filters available in the X11ARIMA program. The corresponding "cascade"f filters for historical and current observations are discussed, including the combined filters for the residuals.

It is shown that the variance amplification and phase shift of the standard concurrent filters fall between those corresponding to the convolution of the longest and shortest trend-cycle and seasonal moving averages. The variance amplification of the shortest filters is the largest whereas the corresponding phase shifts are near zero at low frequencies. The contrary is observed for the longest concurrent filters.

The use of ARIMA extrapolations produces gain functions closer to those of the symmetric filters if the ARIMA parameter values are consistent with the filters' implicit assumptions of trend-cycle and seasonality. The presence of inconsistency highly affects the linear properties of the trend-cycle filters.

Key Words: seasonality, trend-cycle, irregular variations, symmetric cascade linear filters, asymmetric cascade linear filters.

RÉSUMÉ

Les propriétés linéaires des filtres saisonniers et de tendance-cycle symétriques du X-11-ARMMI qui sont appliqués aux observations centrales ainsi que les propriétés de la combinaison standard de ces mêmes filtres ont été étudiées par plusieurs auteurs (par exemple Young, 1968; Wallis, 1974 and 1982; Dagum, 1982 and 1983). Les propriétés de la combinaison standard des filtres non symétriques appliqués aux observations courantes ont été examinées entre autre dans Dagum, (1982 et 1983); Wallis, (1982); Burridge and Wallis, (1984); Dagum et Laniel (1987).

Cette analyse va plus loin que toutes celles réalisées jusqu'à présent parce qu'elle inclue toutes les combinaisons possibles de filtres saisonniers et de tendance-cycle disponibles dans le programme X-11-ARMMI. L'analyse englobe les combinaisons de filtres pour les données historiques et courantes ainsi que celles servant à l'estimation des résidus.

Le déphasage et l'amplification de la variance correspondant à la combinaison standard des filtres non symétriques se situent entre le déphasage et l'amplification de la variance de la combinaison des filtres saisonnier et de tendance-cycle les plus longs et de la combinaison des plus courts. Ce sont les filtres les plus courts qui génèrent la plus grande amplification de la variance alors que leur déphasage est près de zéro dans le voisinage des basses fréquences. Il en va inversement pour les filtres les plus longs.

L'utilisation d'extrapolations ARMMI produit des fonctions de gain qui se rapprochent de celles des filtres symétriques à la condition les valeurs des paramètres ARMMI soient compatibles avec les hypothèses implicites de la tendance-cycle et de la saisonnalité. L'incompatibilité affecte grandement les propriétés linéaires des filtres de tendance-cycle.

Mots clés: saisonnalité, tendance-cycle, aléa, combinaison de filtres linéaires symétriques, combinaison de filtres linéaires non symétriques.

1. Introduction

The X11ARIMA seasonal adjustment method, with or without ARIMA extrapolations, is widely applied by statistical agencies. This method developed by Dagum (1980 and 1988) is an extension of the Census Method II-X11 variant developed by Shiskin, Young and Musgrave (1967). Both methods apply moving averages or linear something filters to estimate the trend-cycle and the seasonal variations of a time series.

It is inherent in any moving average procedure that the first and last points of a time series cannot be smoothed with the same symmetric filters applied to middle (central) values. The current and most recent observations are smoothed by asymmetric filters which have different properties concerning the type of functions they reproduce or eliminate.

The linear properties of the seasonal adjustment and trend-cycle symmetric filters applied to middle observations have been already studied for a fixed combination of moving averages which is the standard (default) option of the X11 variant and the 1980 version of X11ARIMA (see e.g. Young, 1968; Wallis, 1974 and 1982; Dagum 1982 and 1983). The same combination of moving averages has been analysed for the asymmetric filters applied to the last and most recent observations, by Dagum (1982 and 1983); Wallis (1982); Burridge and Wallis (1984) and Dagum and Laniel (1987).

Besides the standard (default) moving averages option the X11ARIMA and Census X11 variant have a broad range of seasonal and trend-cycle filters the properties of which have not yet been studied. These non-standard filters are also used by statistical agencies to seasonally adjust real data.

Furthermore, the default option of the last version X11ARIMA/88 no longer applies a fixed filter combination but a variable combination of seasonal and trend-cycle moving averages based on given values of the signal to noise ratio for each component.

This study extends previous works by discussing the linear properties of all possible combinations of seasonal and trend-cycle moving averages available in the X11ARIMA/88 method. The resulting "cascade" filters for the estimation of the seasonal component, the trend-cycle and the irregulars are analysed by means of their gain and phase-shift functions.

Section 2 introduces the definitions of symmetric and asymmetric smoothing linear filters. Section 3 provides the formulas for the central and end-weights trend-cycle moving averages available in the X11ARIMA/88. Section 4 gives the various formulas for the calculations of the central and end-weights of the seasonal moving averages. Section 5 introduces the "cascade" filters resulting from the convolutions of the different trend-cycle and seasonal linear filters. Section 6 discusses the linear properties of the "cascade" filters for seasonal adjustment, trend-cycle and irregulars. It analyses several combinations of symmetric and asymmetric filters, with and without ARIMA extrapolations. Finally, section 7 gives the conclusions of this study.

2. Smoothing Linear Filters

Given an input series x_t, $t = 1, \ldots, T$, for t sufficiently far removed from the end of the series ($m+1 \le t \le T-m$) the output value y_t by X11ARIMA is obtained by application of a symmetric filter $h_m(B)$.

$$y_t = h_m(B)\ x_t = \sum_{j=-m}^{m} h_{m,j}\ x_{t-j} \tag{2.1}$$

where B is the backshift operator defined as $B^m\ x_t = x_{t-m} (B^0=1)$ and $h_{m,j} = h_{m,-j}$. The length of the filter is $2m+1$.

For current and recent data ($T-m < t \leq T$) a symmetric filter cannot be applied and truncated asymmetric filters $h_j(B)$ are used. For example,

$$
\begin{aligned}
y_T^{(0)} &= h_0(B)\,x_T &&= \sum_{j=0}^{m} h_{0,j}\,x_{T-j} \\
y_{T-k}^{(k)} &= h_k(B)\,x_{T-k} &&= \sum_{j=-k}^{m} h_{k,j}\,x_{T-k-j} \\
y_{T-m}^{(m)} &= h_m(B)\,x_{T-m} &&= \sum_{j=-m}^{m} h_{m,j}\,x_{T-m-j}
\end{aligned}
\tag{2.2}
$$

For the filter $h_k(B)$; $k = 0, 1, \ldots, m$ the subscript k indicates the number of values of x entering the filter after the observation x_T or the negative of the lower limit of summation in the expression $\Sigma\, h_{k,j}\, x_{t-j}$.

At different points in time different outputs corresponding to a given input can be calculated and the superscript on y keeps track of this. Thus

$$
y_t^{(k)} = h_k(B)\,x_t = \sum_{j=-k}^{m} h_{k,j}\,x_{t-j} \tag{2.3}
$$

is the output value of x_t calculated from observations $t-m,\ t-m+1, \ldots,\ t, \ldots,\ t+k$.

The filters are time varying in the sense that on running X11ARIMA at time t with original data $x_t, \ldots, x_T$ each of the first and last $m+1$ adjusted values results from $m+1$ **different** filters applied to the input.

When ARIMA extrapolations are used, the asymmetric filters result from the combination of the X11 smoothing filters with the ARIMA extrapolation filters (see Dagum 1983).

The ARIMA forecasts, x^*_{T+k}, can be written as a linear combination of past values that is,

$$x^*_{T+k} = \sum_{j=0}^{p} \pi_{k,j}\, x_{T-j} \qquad k = 1,\ 2,\ \ldots,\ n \tag{2.4}$$

where $\pi_{k,j}$ denotes the forecast coefficients to be applied to past values of x_T to generate the k-th step ahead forecast and n denotes the number of step-ahead forecasts, usually equal to 4 or 12 for quarterly and monthly series, respectively. Therefore, the asymmetric combined filter applied to the last observation of a series which has been extended with 12 forecasts is,

$$h_0^*(B)\ x_T = \sum_{j=0}^{\max(m,p)} \left(h_{0,j} + \sum_{k=1}^{12} \pi_{k,j}\, h_{0,-k}\right) x_{T-j} \tag{2.5}$$

The outputs from the X11ARIMA method result from the sequential application of several linear filters to estimate the trend-cycle and seasonal components. These linear filters are discussed in the next two sections with reference to monthly data only. The extension to quarterly series is straightforward.

3. Trend-cycle Filters

The estimation of the trend-cycle component by the X11ARIMA method is made by the application of two different linear filters, namely, the centred 12-term moving average and the Henderson trend-cycle filters.

The centred 12-term m.a. is applied in the first iteration for a preliminary estimation of the trend. This filter reproduces exactly the central point of a linear trend and annihilates a stable seasonality over a 12-month period if the decomposition models is additive. If the relationship among the components is multiplicative, then only a constant trend multiplied by a stable seasonality will be perfectly reproduced.

The centred-term moving average D is defined by,

$$D(B) = (1/24)\ B^{-6}(1+B)(1+B+B^2+...+B^{11}) \tag{3.1}$$

The X11ARIMA method generates the six missing estimates of the trend-cycle at either end of the series by repeating the first (last) available estimate six times.

The final estimate of the trend-cycle is made by the application of one of the three different Henderson filters available in the computer package, namely the 9-, 13- and 23-term. These filters were developed by Henderson (1916) based on summation formulae mainly used by actuaries. The basic principle for the summation formulae is the combination of operations of differencing and summation in such a manner that when differencing above a certain order is ignored, they will reproduce the functions operated on. The merit of this procedure is that the smoothed values thus obtained are functions of a large number of observed values whose errors, to a considerable extent, cancel out. These filters have the properties that, when fitted to second or third degree parabolas, their output will fall exactly on those parabolas and, when fitted to stochastic data, they will give smoother results than can be obtained from the weights which give the middle point of a second degree parabola fitted by least squares. Recognition of the fact that the smoothness of the resulting filtering depends on the smoothness of the weight diagram led Robert Henderson (1916) to develop a formula which makes the sum of squares of the third differences of the smoothed series a minimum for any number of terms. In other words, the $\Sigma(\Delta^3 y_t)^2$ is minimized (where $\Delta = 1-B$ is the difference operator, and y_t is the output or smoothed series) if and only if $\Sigma(\Delta^3 h_k)^2$ (where the h_k's are the weights) is minimized (Dagum 1978 and 1985).

If the span of the average is $2m-3$, Henderson showed that the general expression for the n-th term of the filter that minimizes $\sum(\Delta^3 h_k)^2$ is:

$$\frac{315\{(m-1)^2-n^2\}\{m^2-n^2\}\{(m+1)^2-n^2\}\{(3m^2-16)-11n^2\}}{8m(m^2-1)(4m^2-1)(4m^2-9)(4m^2-25)} \qquad (3.2)$$

To derive a set of 13 weights from this formula, 7 is substituted for m and the values are obtained for each n from -6 to 6.

2x7-3=11
So it should be 2m-1 which gives 13.

The Henderson 13-term trend-cycle filter follows,

$$\begin{aligned} H_{13}(B) = &- .019B^{-6} - .028B^{-5} + .00B^{-4} + .065B^{-3} + .147B^{-2} \\ &+ .214B^{-1} + .24B^{0} + .214B + .147B^{2} + .065B^{3} \\ &+ .00B^{4} - .028B^{5} - .019B^{6} \end{aligned} \qquad (3.3)$$

The calculation of the weights of the asymmetric Henderson filter in the X11ARIMA method is based on the minimization of the mean squared revision (MSR) between the final estimates (obtained by the application of the symmetric filter) and the preliminary estimate (obtained by the application of an asymmetric filter) subject to the constraint that the sum of the weights is equal to one (Laniel 1985, Dagum 1988). The assumption made is that at the end of the series the seasonally adjusted values are equal to a linear trend-cycle plus a purely random irregular $NID \sim (0,\sigma_a^2)$. The equation used in X11ARIMA is,

$$E[r_t^{(i,m)}]^2 = c_1^2(t-\sum_{j=-i}^{m} h_{ij}(t-j)^2 + \sigma_a^2\sum_{j=-m}^{m}(h_{mj}-h_{ij})^2) \qquad (3.4)$$

where h_{mj} and h_{ij} are the weights of the symmetric (central) filter and the asymmetric filters, respectively; $h_{ij} = 0$ for $j = -m, \ldots, -i-1$, c_1 is the slope of the line and σ_a^2 denotes the noise variance.

ormula (3.4)

Under the assumption:

$$X_t = c_0 + c_1 t + I_t, \quad I_t \sim (0, \sigma_a^2).$$

Asymmetric filter results in (see (2.2))

$$y_t^{(i)} = \sum_{j=-i}^{m} h_{ij} X_{T-i-j} = \sum_{j=-m}^{m} h_{ij} \{(T-i-j)c_1 + c_0 + I_t\}, \quad \left\{ \begin{array}{l} h_{ij}=0 \\ j=-i+1, \cdots -m \end{array} \right.$$

the symmetric filter results in (after $X_{T+1}, \ldots X_{T-i+m}$ are observed)

$$y_t^{(m)} = \sum_{j=-m}^{m} h_{mj} X_{T-i-j} = \sum_{j=-m}^{m} h_{mj} \{(T-i-j)c_1 + c_0 + I_{T-i-j}\}$$

($t = T-i$)

$$r_t^{(i,m)} = y_t^{(m)} - y_t^{(i)} = c_1 \sum_{j=-m}^{m} (h_{mj} - h_{ij})(\underbrace{T-i}_{t}-j) + c_0 \sum_{j=-m}^{m} (h_{mj} - h_{ij})$$

$$+ \sum_{j=-m}^{m} (h_{mj} - h_{ij}) I_{\underbrace{T-i}_{t}-j}$$

The second term is 0 due to $\sum_j h_{ij} = 1$, $\quad \sum_{j=-m}^{m} h_{mj}(t-j) = t$,

So

$$E\left(r_t^{(i,m)}\right)^2 = E\left\{c_1\left(t - \sum_{j=-i}^{m} h_{ij}(t-j)\right) + \sum_{j=-m}^{m} (h_{mj} - h_{ij}) I_{t-j}\right\}^2$$

$$= c_1^2 \left(t - \sum_{j=-m}^{m} h_{ij}(t-j)\right)^2 + \sigma_a^2 \sum_{j=-m}^{m} (h_{mj} - h_{ij})^2, \quad \left\{ \begin{array}{l} h_{ij}=0 \\ j=-i+1, \cdots -m \end{array} \right.$$

There is a relation between c_1 and σ_a^2 such that the noise to signal ratio, I/C is given by,

$$I/C = (4\sigma_a^2/\pi)^{1/2} \;/\; |c_1| \tag{3.5}$$

The I/C noise to signal ratio (3.5) determines the length of the Henderson trend-cycle filter to be applied. Thus, setting $t = 0$ and $m = 6$ for the end weights of the 13-term Henderson, we have,

$$\frac{E[r_o^{(i,6)}]^2}{\sigma_a^2} = \frac{4}{\pi(I/C)^2}\left(\sum_{j=-i}^{6} h_{ij}\right)^2 + \sum_{j=-6}^{6}(h_{6j}-h_{ij})^2 \tag{3.6}$$

Making $I/C = 3.5$ (the most noisy situation where the 13-term Henderson is applied), equation (3.6) gives the same set of end weights of Census X-11 variant (Shiskin, Young and Musgrave, 1967). The end weights for the remaining monthly Henderson filters are calculated using, $I/C = .99$ for the 9-term filter and $I/C = 7$ for the 23-term filter.

4. Seasonal Filters

The seasonal filters are applied to the seasonal-irregular ratios (differences) of each month separately over a period ranging from 3 to 11 years in order to estimate the seasonal component. The weights are all positive and, consequently, they reproduce exactly the middle value of a straight line within their spans. This property enables the X11ARIMA program to estimate a linearly moving seasonality within three and eleven years spans. Therefore, these filters can approximate quite adequately gradual seasonal changes that follow non-linear patterns over the whole range of the series.

The seasonal filters of X11ARIMA enable the estimation of different seasonal patterns, the shorter the filter the higher the variance of the seasonal component is assumed to be. If the seasonal component appears very stable, then it can be best estimated by an unweighted average of the seasonal-irregular ratios over the whole span of the series.

The seasonal filters available are: (1) a weighted 3-term m.a. (3x1); (2) a weighted 5-term m.a. (3x3); (3) a weighted 7-term m.a. (3x5); (4) a weighted 11-term m.a. (3x9); and (5) Stable.

The seasonal linear filters applied to central values are symmetric and calculated as follows:

$$S_3(B) = (1/3)\, B^{-12}\, (1+B^{12}+B^{24}) \qquad \text{(3x1 m.a.)} \qquad (4.1)$$

$$S_5(B) = (1/9)\, B^{-24}\, (1+B^{12}+B^{24})\, (1+B^{12}+B^{24}) \qquad \text{(3x3 m.a.)} \qquad (4.2)$$

$$S_7(B) = (1/15)\, B^{-36}\, (1+B^{12}+B^{24})\, (1+B^{12}+B^{24} + B^{36}+B^{48}) \qquad \text{(3x5 m.a.)} \qquad (4.3)$$

$$S_{11}(B) = (1/27)\, B^{-60}\, (1+B^{12}+B^{24}) \times (1+B^{12}+B^{24} + B^{36}+B^{48}+B^{60}+B^{72}+B^{84}+B^{96}) \qquad \text{(3x9 m.a.)} \qquad (4.4)$$

$$S_N(B) = (1/N)\, (1+B^{12}+B^{24}+B^{36}+ \ldots +B^{12N}) \qquad (4.5)$$

(unweighted average; N = number of years)

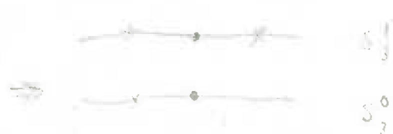
The X11ARIMA also enables the application of **different** seasonal filters for each month, e.g., a 3x3 m.a. for the months of June, July, August and a 3x5 m.a. for the remaining months.

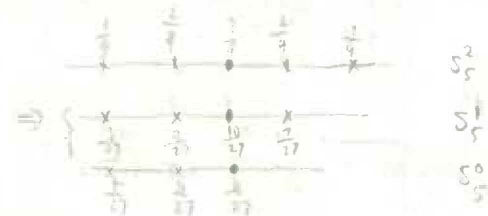
The asymmetric filters applied to non-central values are obtained as follows,

(by the similar criterion as in Section 3 ?)

Asymmetric Filter of the 3x1 m.a.

$$S_3^0(B) = (.61+.39\, B^{12}) \qquad (4.6)$$





Asymmetric Filters of the 3x3 m.a.

$$S_5^0(B) = [(11/27)(1+B^{12}) + (5/27)B^{24}]$$
$$S_5^1(B) = [(7/27)(B^{-12}+B^{12}) + (10/27) + (3/27)B^{24}] \qquad (4.7)$$

Asymmetric Filters of the 3x5 m.a.

$$S_7^0(B) = [(17/60)(1+B^{12}+B^{24}) + (9/60)B^{36}]$$
$$S_7^1(B) = [(15/60)(B^{-12}+1+B^{12}) + (11/60)B^{24} + (4/60)B^{36}] \qquad (4.8)$$
$$S_7^2(B) = [(9/60)B^{-24} + (13/60)(B^{-12}+1+B^{12}) + (8/60)B^{24} + (4/60)B^{36}]$$

Asymmetric Filters of the 3x9 m.a.

$$S_{11}^0(B) = .246+.221B^{12}+.197B^{24}+.173B^{36}+.112B^{48}+.051B^{60}$$
$$S_{11}^1(B) = .208+.192B^{12}+.176B^{24}+.160B^{36}+.144B^{48}+.092B^{60}+.028B^{72}$$
$$S_{11}^2(B) = .173+.163B^{12}+.154B^{24}+.143B^{36}+.133B^{48}+.123B^{60}+.079B^{72}+.032B^{84} \qquad (4.9)$$
$$S_{11}^3(B) = .141+.137B^{12}+.132B^{24}+.128B^{36}+.123B^{48}+.117B^{60}+.113B^{72}+.075B^{84}+.034B^{96}$$
$$S_{11}^4(B) = .084+.120B^{12}+.118B^{24}+.117B^{36}+.116B^{48}+.114B^{60}+.113B^{72}+.111B^{84}+.073B^{96}+.034B^{108}$$

5. Cascade Linear Filters

The estimated trend-cycle and seasonal components from X11ARIMA are obtained by cascade filtering that results from the convolution of the various linear filters discussed in previous sections. In fact, if the output from the filtering operation H is the input to the filtering operation Q, the coefficients of the cascade filter C result from the convolution of $H * Q$. For symmetric filters $H * Q = Q * H$ but this is not valid for asymmetric filters.

Assuming an input series $x(t)$, $t=1,2,\ldots,T$, we can define a matrix $H(k,j)$, $k=1,\ldots,m_h$; $j=1,2,\ldots,2m_h+1$ where each row is a filter and m_h is the half length of the symmetric filter. $H(1,.)$ denotes an asymmetric filter where the first m_h coefficients are zeroes and $H(m_h+1,.)$ denotes the symmetric filter.

Given data up to time T, the m_h+1 most recent values of the output (filtered series) are given by

$$y_h(T+1-k) = \sum_{j=m_h-k+2}^{2m_h+1} H(k,j)\ x(T-k+m_h+2-j) \qquad (5.1)$$

$$k=1,2,\ldots,m_h+1$$

For example, the 13-term Henderson filter can be put in matrix form as follows,

0	0	0	0	0	0	-.092	-.058	.012	.120	.244	.353	.421
0	0	0	0	0	-.043	-.038	.002	.080	.174	.254	.292	.279
0	0	0	0	-.016	-.025	.003	.068	.149	.216	.241	.216	.148
0	0	0	-.009	-.022	.004	.066	.145	.208	.230	.201	.131	.046
0	0	-.011	-.022	.003	.067	.145	.210	.235	.205	.136	.050	-.018
0	-.017	-.025	.001	.066	.147	.213	.238	.212	.144	.061	-.006	-.034
-.019	-.028	0	.066	.147	.214	.240	.214	.147	.066	0	-.028	-.019

To calculate the coefficients of the cascade filter C in

$$y_c(T+1-k) = \sum_{j=m_c-k+2}^{2m_c+1} C(k,j)\ x(T-k+m_c+2-j) \qquad (5.2)$$

$$k=1,\ldots,m_c+1$$

where $C = Q * H$,

$$y_c(T+1-k) = \sum_j Q(k,j)\, y_h(T-k+m_Q+2-j) \quad k=1,\ldots,m_Q+1 \qquad (5.3)$$

and $m_c = m_h + m_q$. Since the asymmetric filters are represented by a complete row of the relevant matrix, with initial entries being zero, the lower limit in the above summation can always be taken as $j=1$.

The X11ARIMA **Seasonal Cascade Filter** most often applied results from the convolution of: (i) 12-term centred m.a.; (ii) 3x3 m.a.; (iii) 3x5 m.a.; and (iv) 13-term Henderson m.a.
In symbols,

for D, see (3.1) (centred-term MA)

$$S(B) = D^c(B) * S_7(B) * [H_{13}(B) * (D^c(B) * S_5(B) * D^c(B))^c]^c \qquad (5.4)$$

For central observations, D, H_{13}, S_5 and S_7 are defined by equations (3.1), (3.3), (4.2), (4.3) respectively and by 'c' we denote the complement of the corresponding filter. For **end-values**, D, H_{13}, S_5 and S_7 are defined by equations (3.1), (3.6), (4.7) and (4.8) respectively.

The complement of (5.4) defines the corresponding **Seasonal Adjustment Cascade Filter**.

The **Trend-Cycle Cascade Filter** most often applied is,

$$TC(B) = H_{13}(B)\{I - D^c(B) * S_7(B) * [H_{13}(B) * (D^c(B) * S_5(B) * D^c(B))^c]^c\} \qquad (5.5)$$

Where I denotes the identity filter and D, H_{13}, S_5 and S_7 are defined by equations (3.1), (3.3), (4.2), (4.3) respectively. For **end-values**, D, H_{13}, S_5 and S_7 are defined by equations (3.1), (3.6), (4.7) and (4.8), respectively.

Finally, the **Irregular Component Cascade Filter** is given by the complement of the Trend-Cycle Cascade Filter. + Seasonal

$I = 1 - (S + TC)$

$SA = TC + I$

6. Properties of the X11ARIMA Cascade Linear Filters

The properties of the cascade filters can be studied by analyzing their corresponding frequency response functions.

The frequency response function is defined by

$$H(\omega) = \sum_{j=-m}^{m} \alpha_j e^{-i\omega j}; \quad 0 \le \omega \le \tfrac{1}{2} \tag{6.1}$$

where α_j are the weights of the filter and ω is the frequency in cycles per unit of time.

In general, the frequency response functions can be expressed in polar form as follow,

$$H(\omega) = A(\omega) + iB(\omega) = G(\omega)\ e^{i\phi(\omega)} \tag{6.2}$$

where $G(\omega) = \{A^2(\omega) + B^2(\omega)\}^{1/2}$ is called the **gain** of the filter and $\phi(\omega) = \arctan\{B(\omega)/A(\omega)\}$ is called the **phase shift** of the filter and is usually expressed in radians. The expression (6.2) shows that if the input function is a sinusoidal variation of unit amplitude and constant phase shift $\psi(\omega)$, the output function will also be sinusoidal but of amplitude $G(\omega)$ and phase shift $\psi(\omega) + \phi(\omega)$. The gain and phase shift vary with ω. For symmetric filters the phase shift is 0 or $\pm\pi$, and for asymmetric filters take values between $\pm\pi$ at those frequencies where the gain function is zero. For a better interpretation the phase shifts will be here given in months instead of radians (the phase shift in months is given by $\phi(\omega)/2\pi\omega$ for $\omega \ne 0$).

The gain function shown should be interpreted as relating the spectrum of the original series to the spectrum of the output obtained with a linear time-invariant filter. For example, let $Y_t^{(0)}$ be the estimated seasonally adjusted observation for the current period based on data x_t, $t=1,2,\ldots,T$, then the time series $\{Y_t^{(o)}\}$ is obtained from $\{X_t\}$ by application of the concurrent linear time invariant filter $h^{(o)}(B)$. The gain functions discussed below relate the spectrum of $\{X_t\}$ to the spectrum of $\{Y_t^{(o)}\}$ not to the spectrum of the complete seasonally adjusted series produced at time t (which includes $Y_t^{(o)}$, a first revision of time $t-1$, a second revision of time $t-2$, and so on).

6.1 Properties of Symmetric (Central) Filters

The gain functions of three different seasonal adjustment cascade filters are shown in Figure 1. The gain function of the cascade filter resulting from the convolution of the shortest moving averages, ie. (3x3)(3x3) [H-9] has broader dips around the fundamental seasonal frequency $\omega = 0.083$ and its five harmonics 0.167, 0.250, 0.330, 0.417 and 0.50. Therefore, this combination is more appropriate for series affected by rapidly changing seasonality.

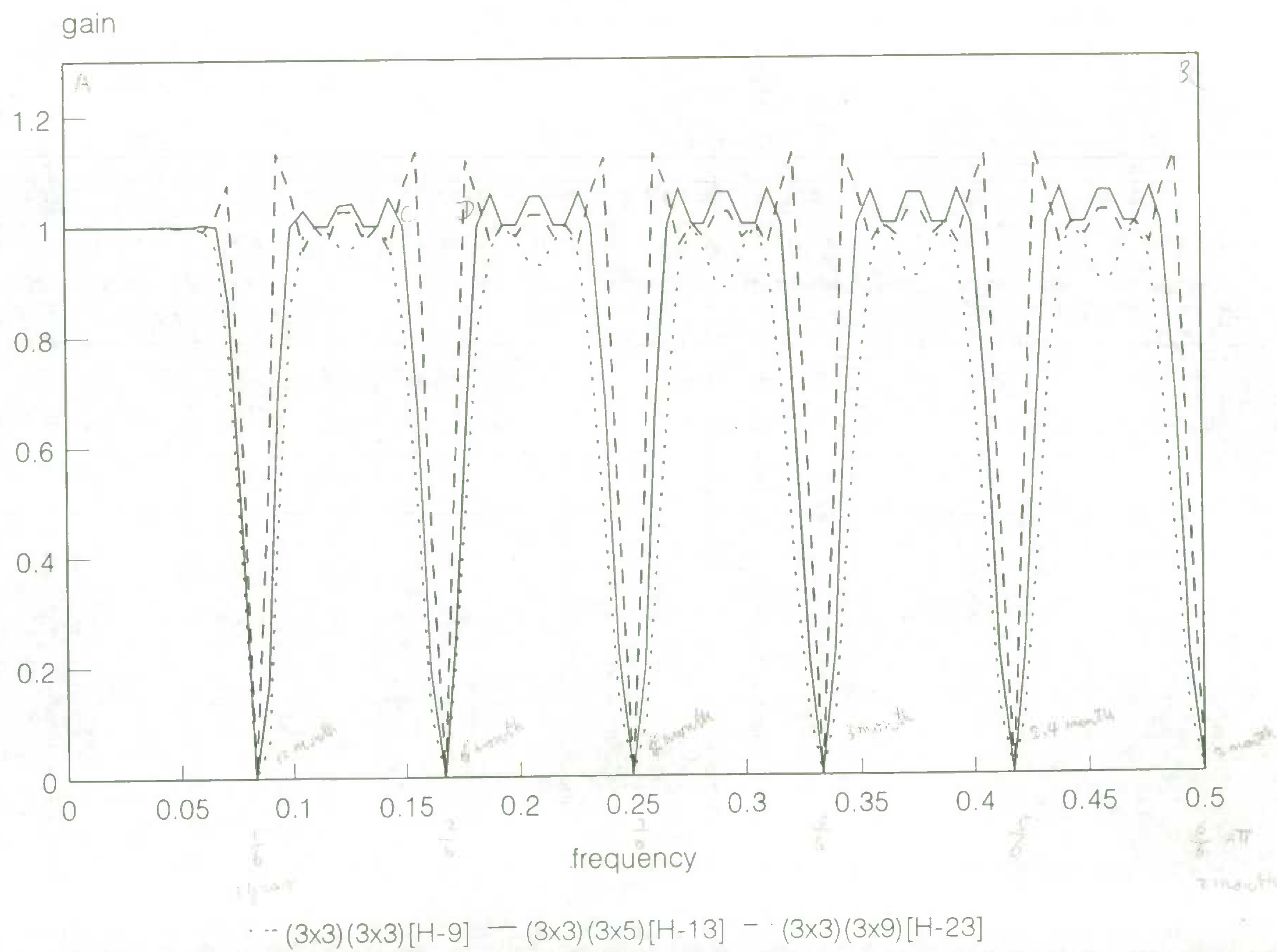
(Place Figure 1 here)

On the other hand, the gain function of the cascade filter corresponding to the convolution of the longest filters, i.e., (3x3)(3x9)[H-23] shows narrower seasonal dips, therefore, being more appropriate for series with an underlying stable or regular seasonal pattern. The gain of the cascade filter for the standard combination (3x3)(3x5)[H-13] falls between that of the shortest and longest filters. It will be seen that this middle position for the standard cascade filter is always present irrespectively of the filter convolutions.

Figure 2 shows the gains of the trend-cycle cascade filters for the same combinations discussed above. The shortest and standard cascade filters modify slightly the variances of the low frequency components; i.e., $0 < \omega \leq 0.055$ which correspond to cycles of periodicities equal to and longer than

Fig. 1

SEASONAL ADJUSTMENT SYMMETRIC CASCADE FILTERS



18 months.

On the other hand, the longest cascade filter reflects the pattern of the long Henderson filter [H-23] decreasing fast to zero at the fundamental seasonal frequency and hovering around zero for the remaining higher frequencies

The total variance of the trend-cycle cascade filter as shown by the area under the gain function clearly indicates that it is smallest for the longest combination which makes this combination suitable for series with an underlying stable (more rigid) trend-cycle.

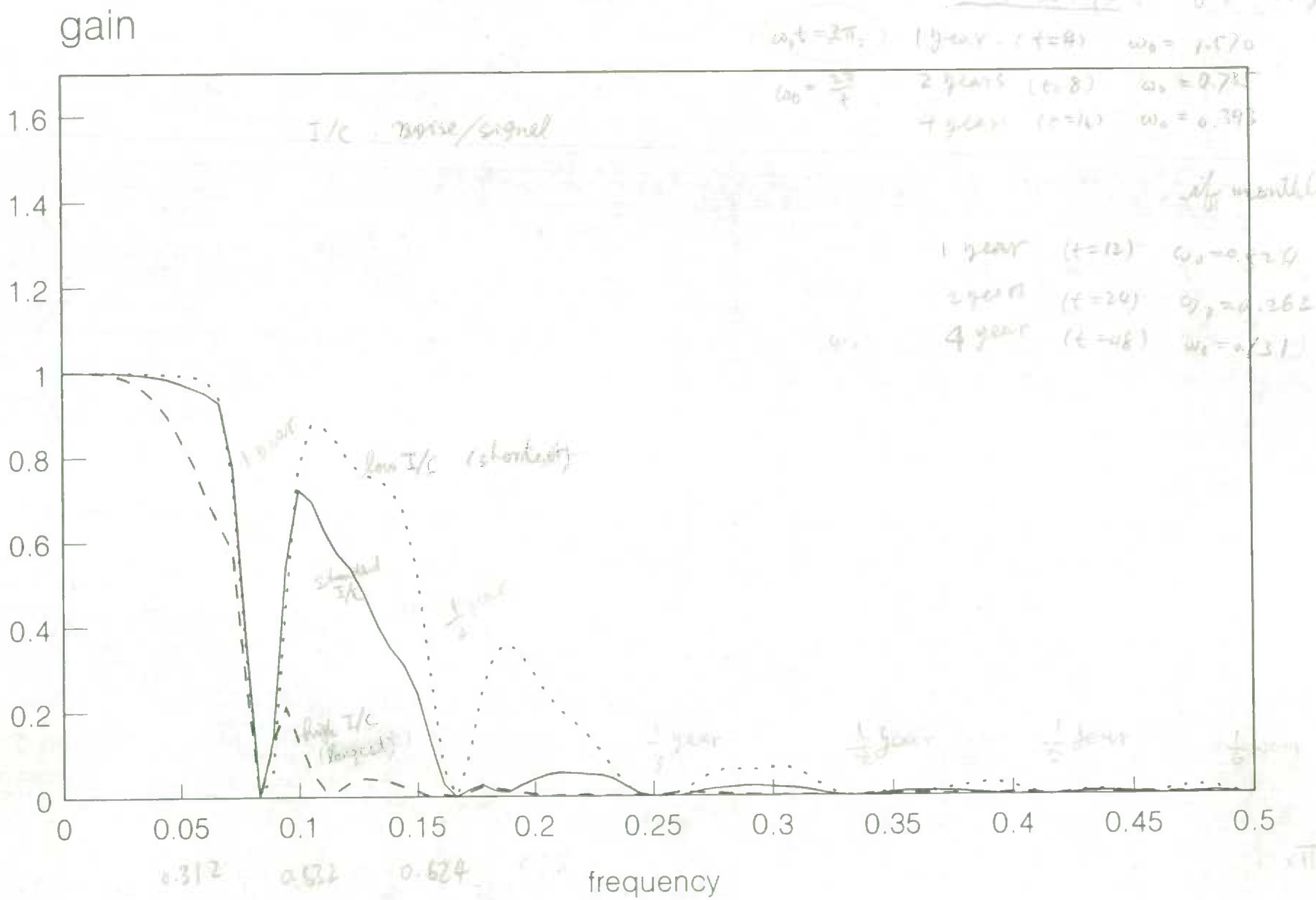
On the other hand, the shortest cascade filter passes about 75% of the variance associated with the frequency band $0.08 < \omega < 0.16$ and 25% for $0.16 < \omega < 0.25$, which makes it more appropriate for series affected by rapidly changing trend-cycle variations.

(Place Figure 2 about here)

Finally, Figure 3 gives the gain of the cascade filters for the irregular component. The variance of the irregulars is largest for the longest combination and smallest for the shortest cascade filter. The gain of the standard combination falls between those two.

Fig. 2

TREND-CYCLE SYMMETRIC CASCADE FILTERS



··· (3x3)(3x3)[H-9] — (3x3)(3x5)[H-13] - - (3x3)(3x9)[H-23]

It is also apparent the negative autocorrelation of the irregulars for lags 1, 2 and 12. In fact for the standard option, the autocorrelations of the residuals (assuming the irregulars affecting the series are white noise) are as follows:

$\rho_1 = -0.34$	$\rho_8 = -0.03$
$\rho_2 = -0.21$	$\rho_9 = 0.02$
$\rho_3 = -0.06$	$\rho_{10} = 0.07$
$\rho_4 = 0.05$	$\rho_{11} = 0.11$
$\rho_5 = 0.08$	$\rho_{12} = -0.32$
$\rho_6 = -0.03$	$\rho_{13} = 0.11$
$\rho_7 = -0.05$	

and the variance $\sigma_I^2 = 0.55$ (Dagum, Chhab and Solomon, 1991).

For the shortest cascade filter the autocorrelations of the residuals show a similar pattern but being higher at lags 1 and 12.

$\rho_1 = -0.47$	$\rho_8 = -0.04$
$\rho_2 = -0.17$	$\rho_9 = -0.04$
$\rho_3 = 0.08$	$\rho_{10} = 0.07$
$\rho_4 = 0.10$	$\rho_{11} = 0.20$
$\rho_5 = -0.03$	$\rho_{12} = -0.43$
$\rho_6 = -0.01$	$\rho_{13} = 0.21$
$\rho_7 = 0.01$	

and the variance $\sigma_I^2 = 0.36$.

For the longest cascade filter, the autocorrelations of the residuals are smaller at lags 1 and 12 than those of the previous combinations. The autocorrelations follow,

$\rho_1 = -0.19$	$\rho_8 = 0.05$
$\rho_2 = -0.17$	$\rho_9 = 0.05$
$\rho_3 = -0.13$	$\rho_{10} = 0.04$
$\rho_4 = -0.08$	$\rho_{11} = 0.03$
$\rho_5 = -0.04$	$\rho_{12} = -0.15$
$\rho_6 = 0.00$	$\rho_{13} = 0.02$
$\rho_7 = 0.03$	

and $\sigma_I^2 = 0.73$.

(Place Figure 3 about here)

6.2 Properties of the Asymmetric Cascade Filters

6.2.a Concurrent Cascade Filters without ARIMA Extrapolations

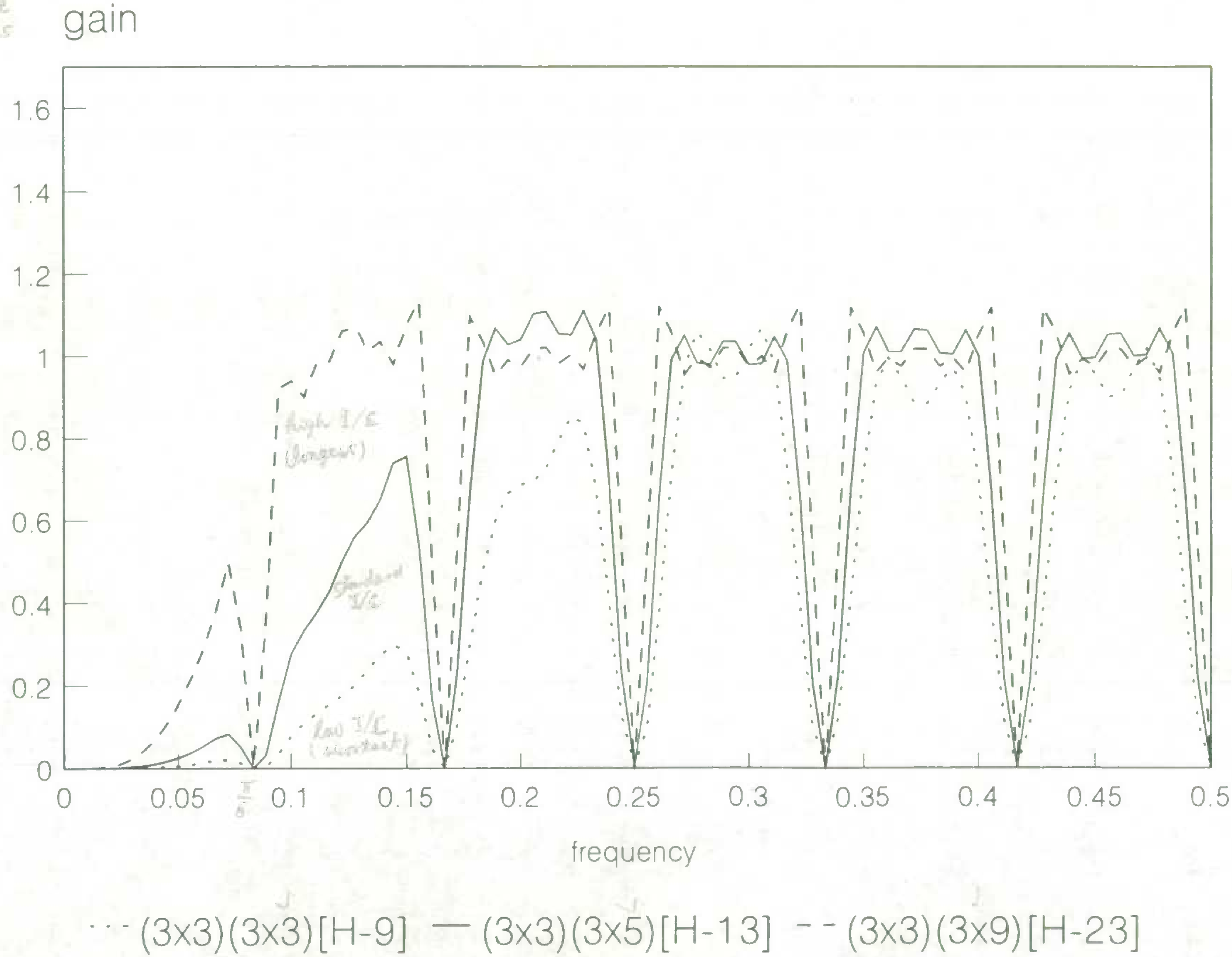
To analyse the effects of various asymmetric seasonal and Henderson filters on the concurrent cascade filter for seasonal adjustment (the concurrent filter is applied to last available observation) we selected three combinations as follows:

(1.a) Short seasonal moving averages combined with each one of the three Henderson filters; that is, (3x3)(3x3)[H-9], (3x3)(3x3)[H-13] and (3x3)(3x3)[H-23]

(2.a) Standard seasonal moving averages combined with each one of the three Henderson filters, that is, (3x3)(3x5)[H-9]; (3x3)(3x5)[H-13] and (3x3)(3x5)[H-23];

Fig. 3

IRREGULAR SYMMETRIC CASCADE FILTERS



and

(3.a) Long seasonal moving averages combined with each one of the three Henderson filters, that is, (3x3)(3x9)[H-9], (3x3)(3x9)[H-13] and (3x3)(3x9)[H-23].

Figures 4, 5 and 6 give the corresponding gain and phase shift functions for each case respectively.

We can see that if the seasonal filters are short (Figure 4), the short Henderson filter amplifies very much the variance at frequencies near the fundamental seasonal as well as those frequencies between the fundamental seasonal and its first harmonic. On the other hand, short seasonal filters combined with the long Henderson do not amplify the gain but increase the phase shift.

As we enlarge the seasonal filters from (3x3) to (3x5) to (3x9), the influence of the Henderson filters diminishes. In fact, as plotted in Figure 6 the combinations (3x3)(3x9)[H-9] has reduced very much the amplification observed in the shortest cascade filter. Furthermore, it has also improved the results from the other two combinations, i.e., using [H-13] and [H-23] by reducing the variance amplification and also the phase shift for the combination with [H-23].

All of the above seems to indicate that if ARIMA extrapolations are not used, the longer seasonal moving averages are to be preferred for concurrent seasonal adjustment, at least, from the view point of the linear properties of the X11ARIMA method.

(Place Figures 4, 5 and 6 about here)

Combinations (1.a), (2.a) and (3.a) are applied to obtain the gain and phase shifts of the trend-cycle concurrent cascade filters plotted in Figure 7, 8 and 9 respectively. The Henderson filters significantly increase the variance and phase shift at the frequencies near the fundamental seasonal and between

Fig. 4

SEASONAL ADJUSTMENT CONCURRENT CASCADE FILTERS
Short Seasonal m.a. Combined with Three Henderson Filters

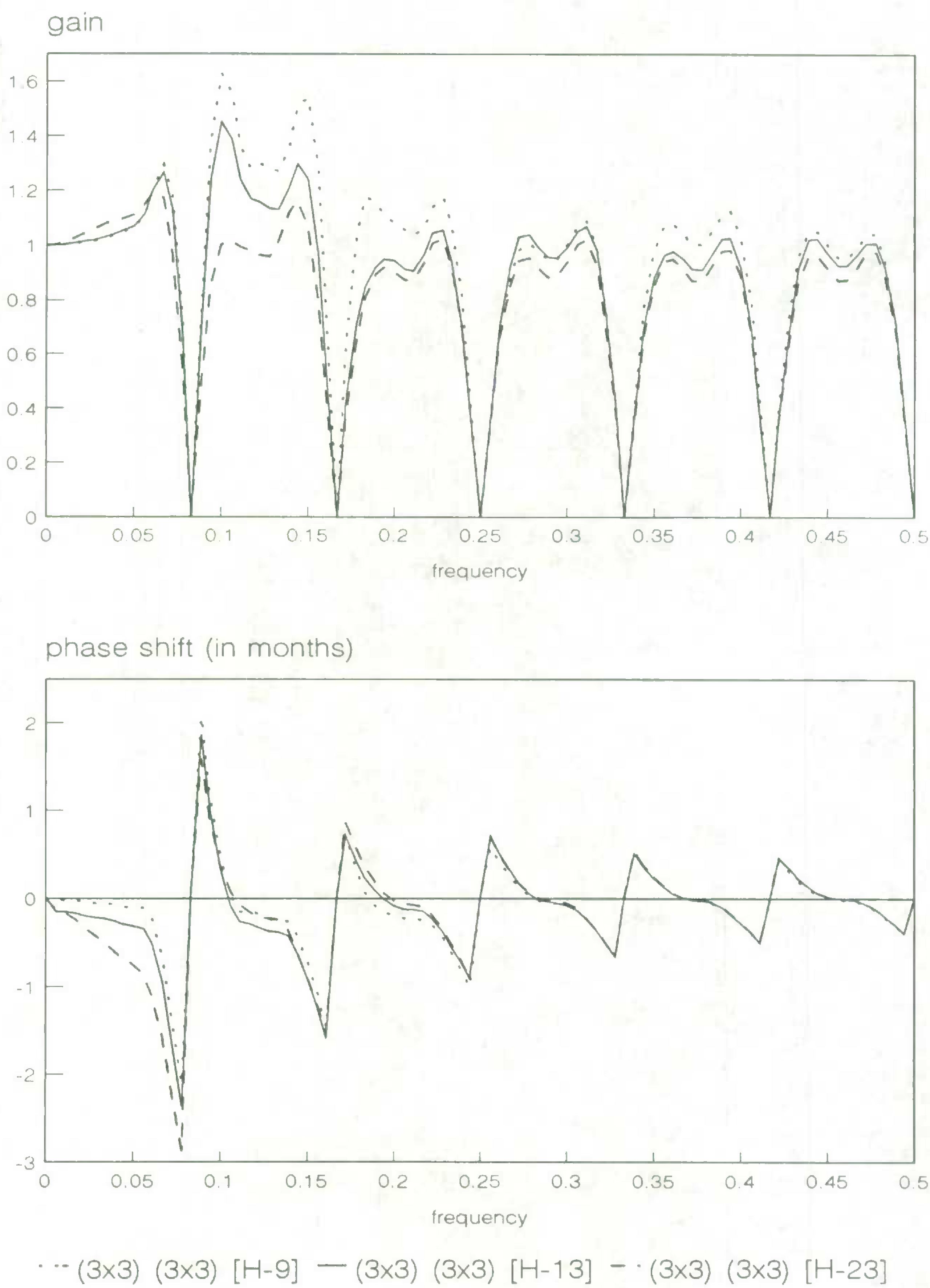


Fig. 5

SEASONAL ADJUSTMENT CONCURRENT CASCADE FILTERS
Standard Seasonal m.a. Combined with Three Henderson Filters

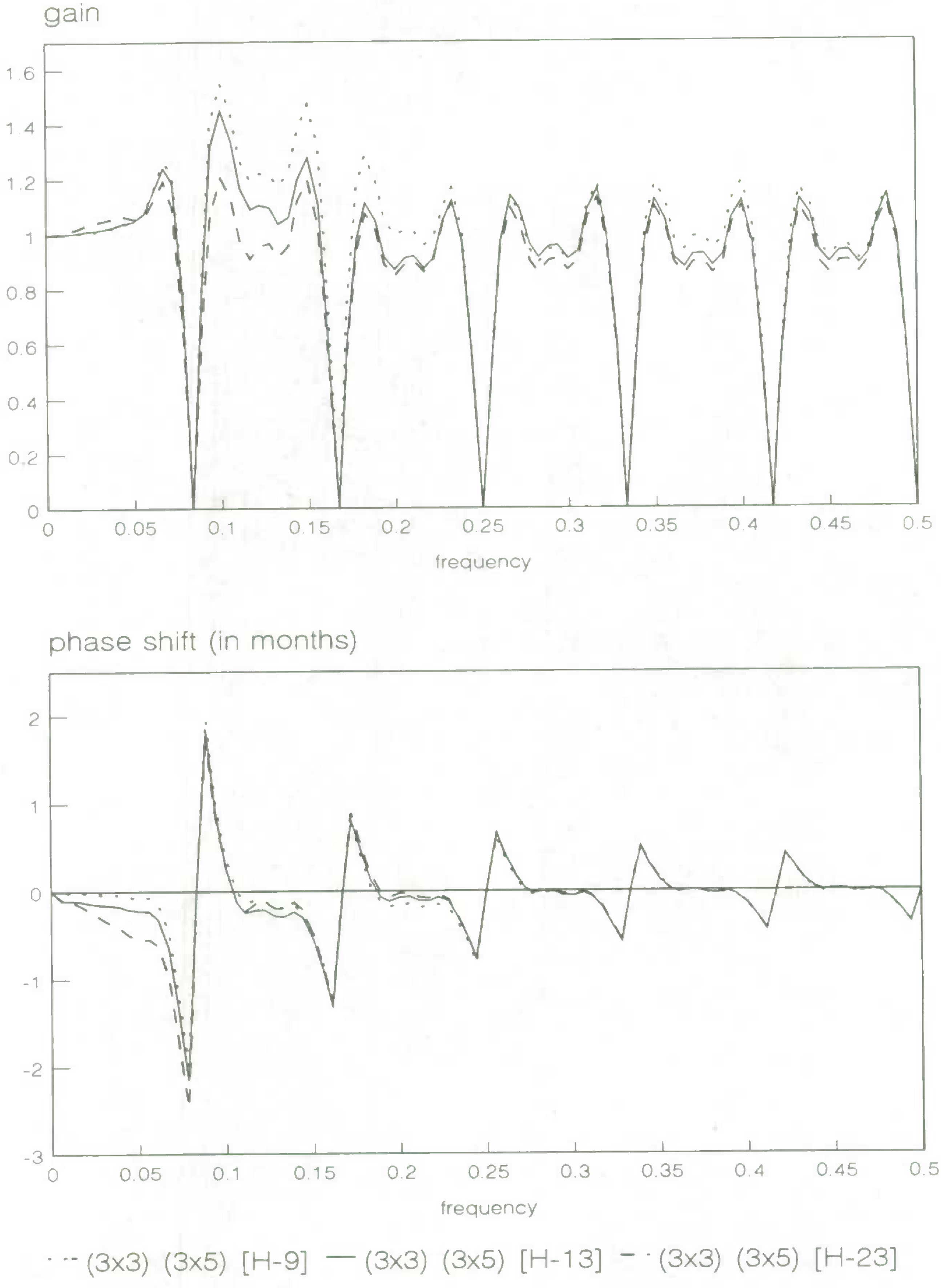
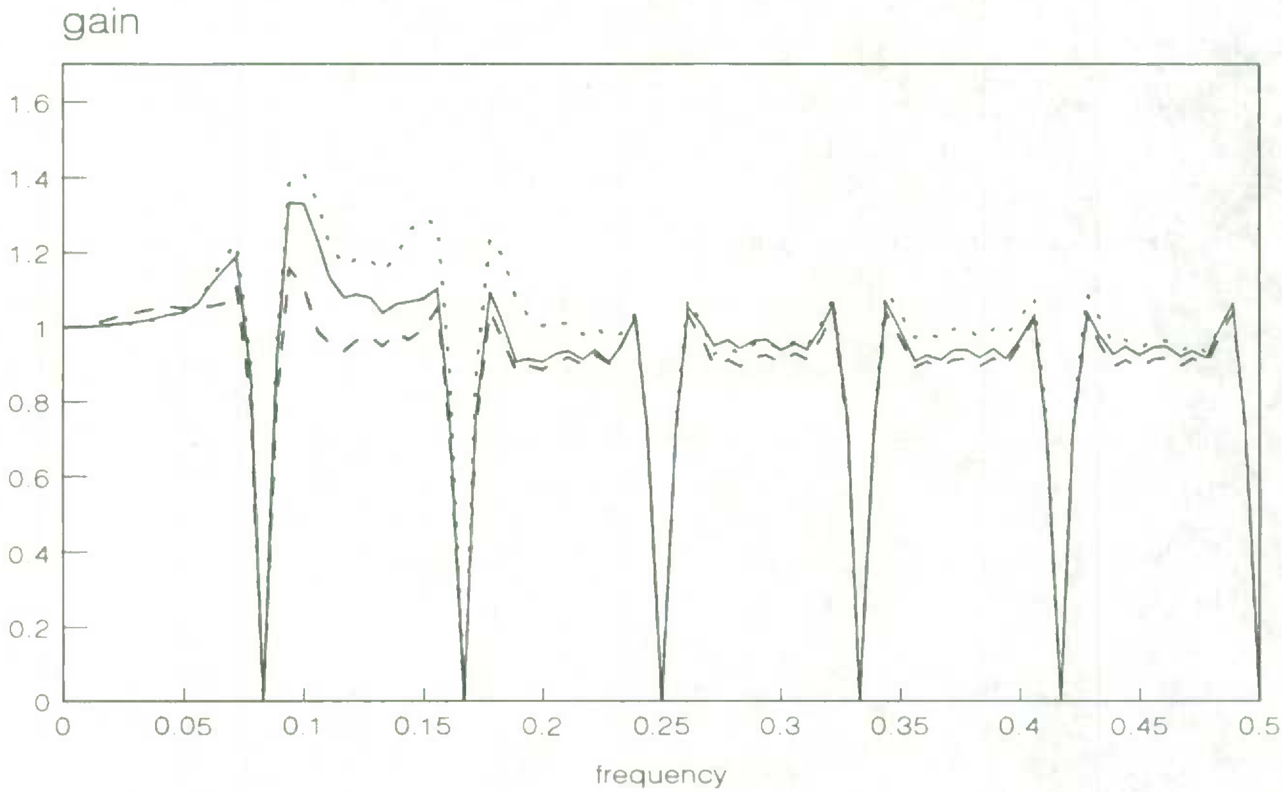


Fig. 6

SEASONAL ADJUSTMENT CONCURRENT CASCADE FILTERS
Long Seasonal m.a. Combined with Three Henderson Filters



··· (3x3) (3x9) [H-9] — (3x3) (3x9) [H-13] – · (3x3) (3x9) [H-23] the fundamental seasonal and its first harmonic. This clearly supports the common practise among statistical agencies of not publishing the trend estimates of the last available observation. It is also obvious that the trend-cycle concurrent cascade filter passes a significant amount of noise at high frequencies.

The relationship between the Henderson filter's length and the trend-cycle cascade filter's length is similar to that observed for the seasonal adjustment filters although less pronounced. Long seasonal filters seem to reduce the variance amplification introduced by the short Henderson filter while producing similar phase shifts (see Figures 7 and 9).

(Place Figures 7, 8 and 9 around here)

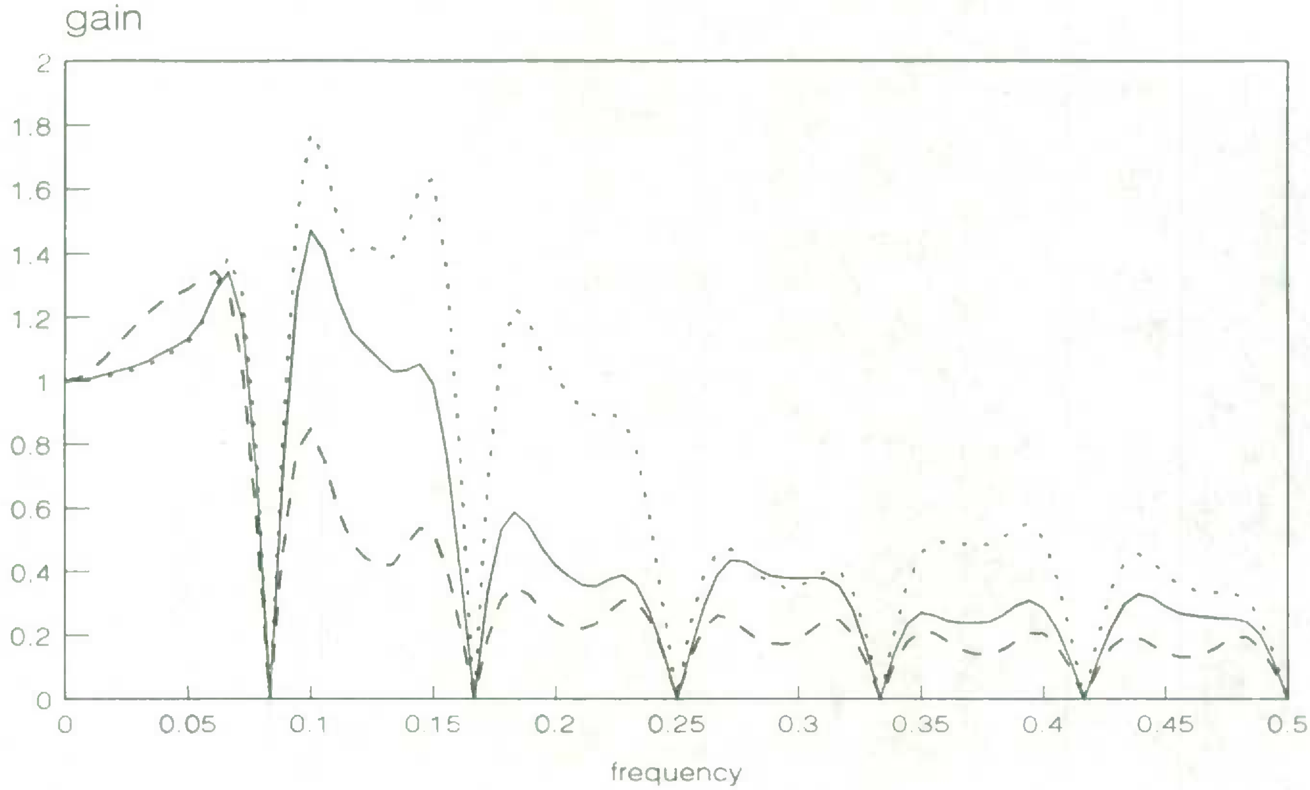
Figures 10-12 give the gain and the phase shifts of the concurrent irregular cascade filter for the same combinations (1.a), (2.a) and (3.a).

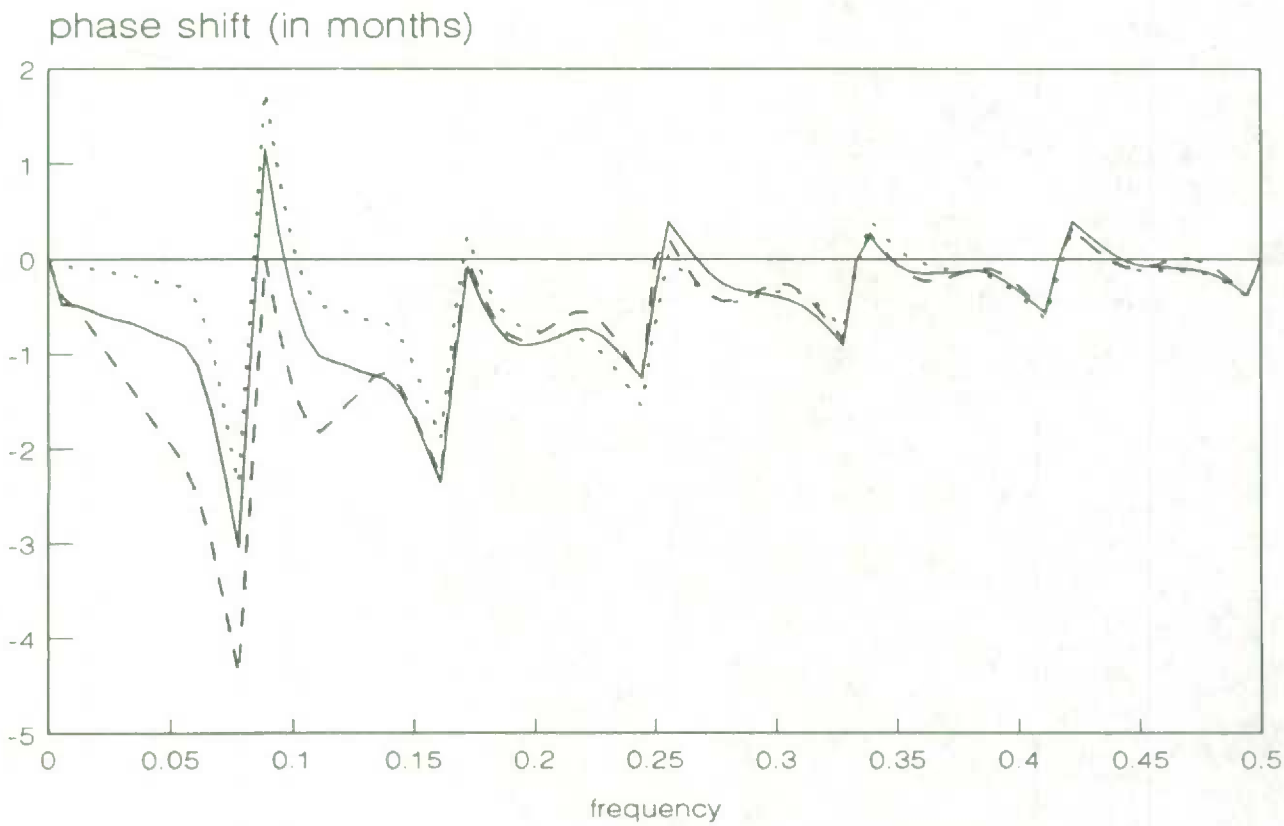
The variance of the concurrent irregular cascade filters are systematically larger than the corresponding symmetric filters at low frequencies and between the fundamental seasonal and its first harmonic but smaller at the remaining frequencies. Among the various irregular concurrent filters the variance is largest for the longest which agrees with the fact that its complement (the trend-cycle concurrent filter) smooths more than any other combination. The opposite can be observed for the shortest irregular filter.

(Place figures 10, 11 and 12 about here)

Fig. 7

TREND-CYCLE CONCURRENT CASCADE FILTERS
Short Seasonal m.a. Combined with Three Henderson Filters





(3x3)(3x3)[H-9]   (3x3)(3x3)[H-13]   (3x3)(3x3)[H-23]

Fig. 8

TREND-CYCLE CONCURRENT CASCADE FILTERS
Standard Seasonal m.a. Combined with Three Henderson Filters

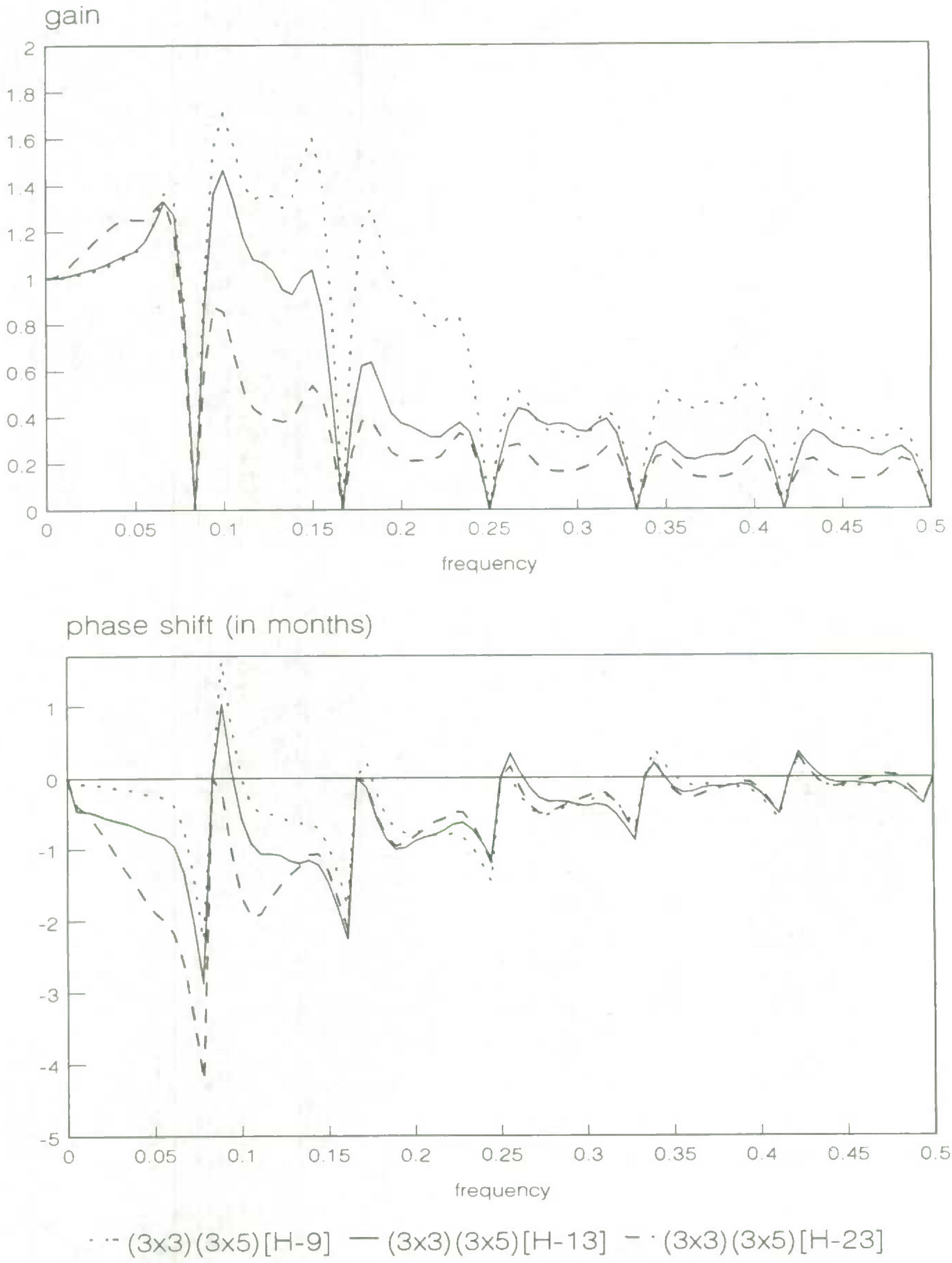


Fig. 9

TREND-CYCLE CONCURRENT CASCADE FILTERS
Long Seasonal m.a. Combined with Three Henderson Filters

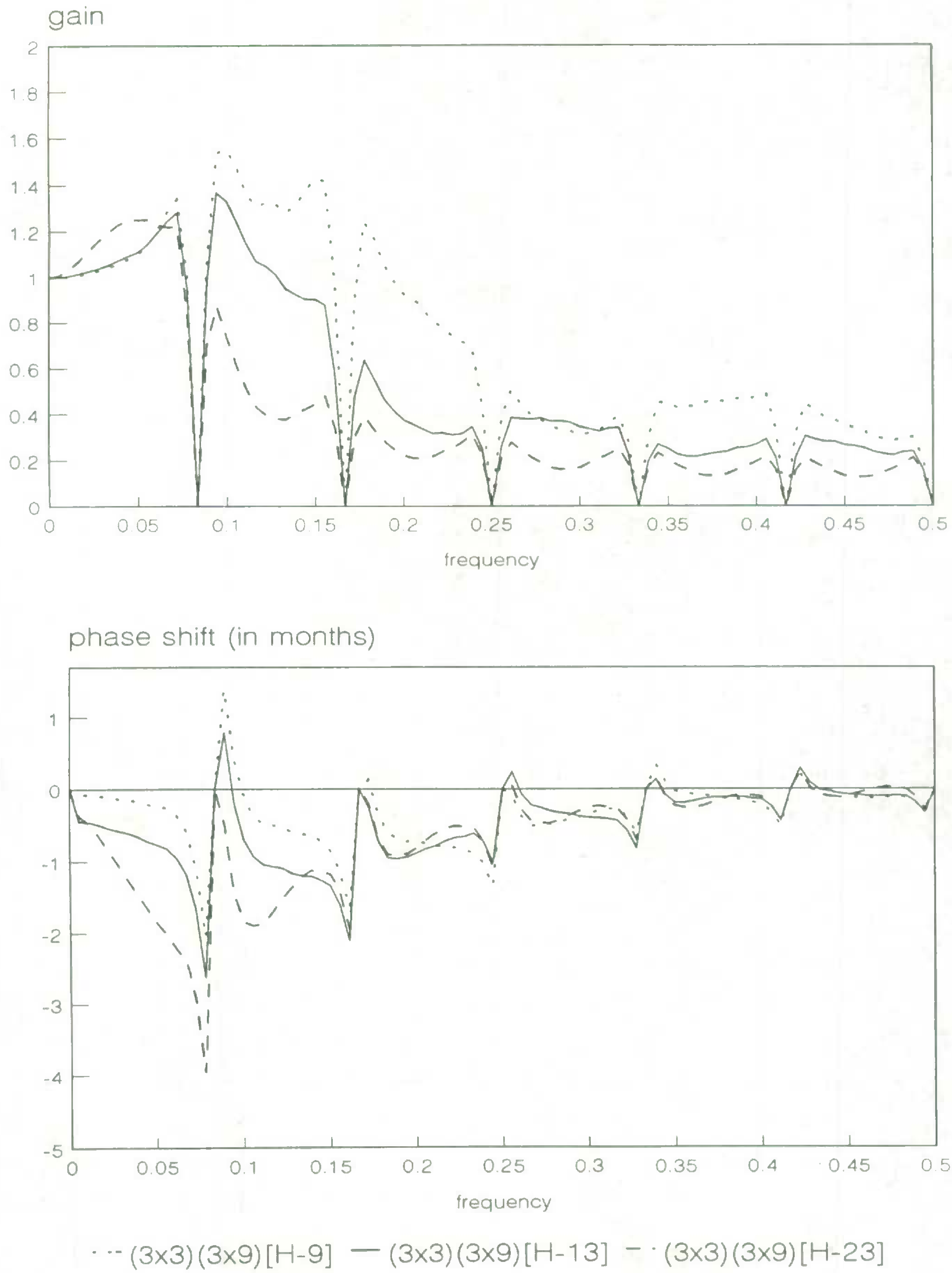


Fig. 10

IRREGULAR CONCURRENT CASCADE FILTERS
Short Seasonal m.a. Combined with three Henderson Filters

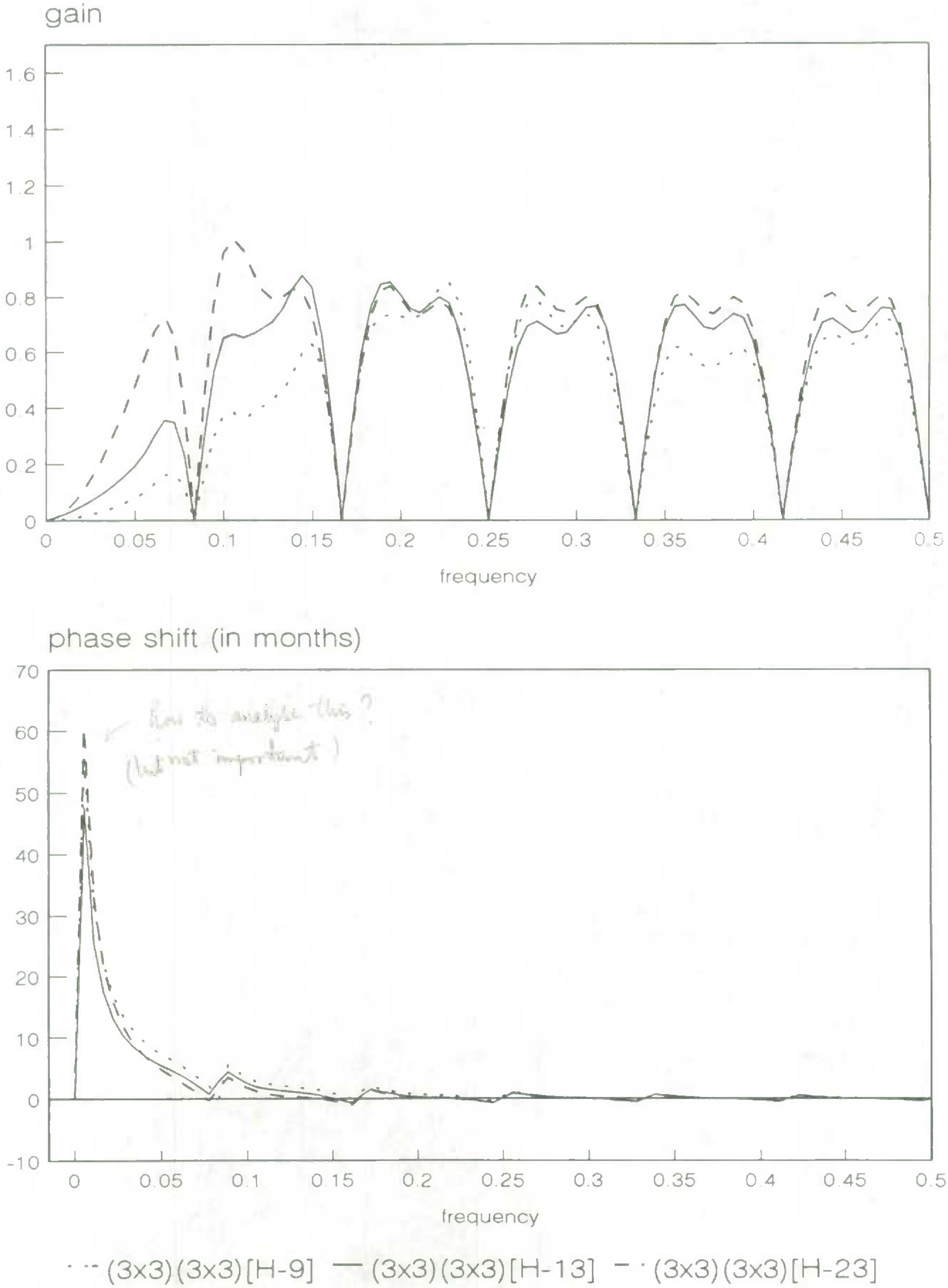


Fig. 11

IRREGULAR CONCURRENT CASCADE FILTERS
Standard Seasonal M.A. combined with Three Henderson Filters

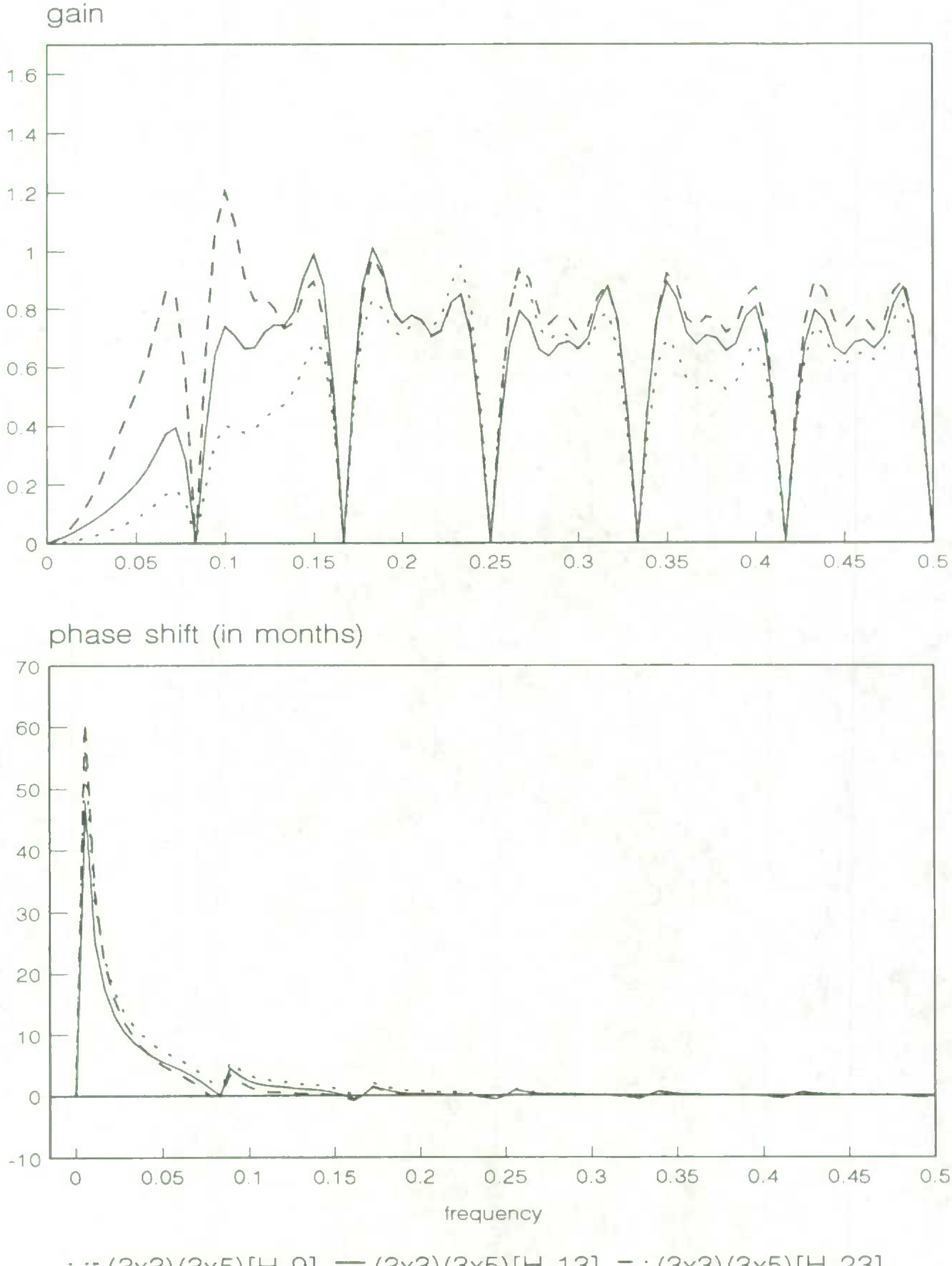
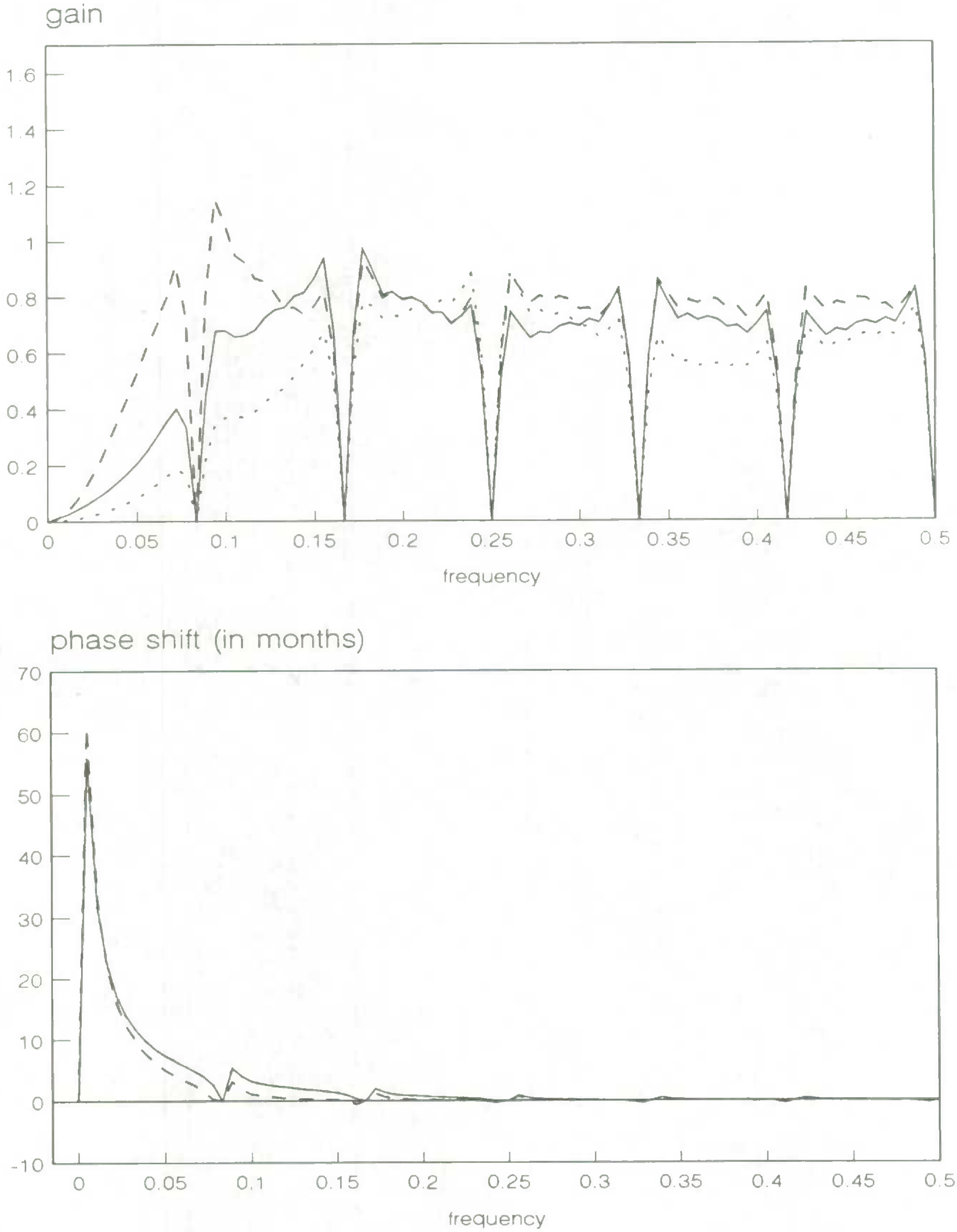


Fig. 12

IRREGULAR CONCURRENT CASCADE FILTERS
Long Seasonal m.a. combined with Three Henderson Filter



- -- (3x3)(3x9)[H-9] — (3x3)(3x9)[H-13] – · (3x3)(3x9)[H-23]

6.2.b Concurrent Cascade Filters with ARIMA Extrapolations

To discuss the impact of the ARIMA extrapolations on the concurrent cascade filters analysed in the previous section we selected the following combinations:

(1.b) Shortest cascade filter (3x3)(3x3)[H-9] extended with ARIMA forecasts from a (0,1,1)(0,1,1) model with parameter values $\theta = .30$ $\Theta = .30$.

(2.b) Shortest cascade filter (3x3)(3x3)[H-9] extended with same ARIMA model as in (1) but $\theta = .80$ $\Theta = .80$.

(3.b) Standard cascade filter (3x3)(3x5)[H-13] extended with same ARIMA model as in (1) but $\theta = .40$ $\Theta = .60$.

(4.b) Long cascade filter (3x3)(3x5)[H-23] extended with same ARIMA model as in (1) but $\theta = .30$ $\Theta = .30$.

The $(0,1,1)(0,1,1)_{12}$ ARIMA model may be expressed by,

$$(1-B)(1-B^{12})\ Z_t = (1-\theta B)(1-\Theta B^{12})\ a_t \qquad (6.2.1)$$

with invertibility conditions $|\theta| < 1$ and $|\Theta| < 1$ (Box and Jenkins, 1970).

The extrapolation filter of the (0,1,1) (0,1,1) ARIMA model implies an instantaneously straight-line trend and an instantaneously constant zero-sum seasonal pattern, both changing their level and slope in a stochastic fashion, proportional to the value of innovation a.

The parameters $\lambda = 1-\theta$ and $\Lambda = 1-\Theta$ can be interpreted as representing the extent to which the trend level and the seasonal pattern respond to new innovations. Thus a low value of θ corresponds to a fast-changing trend, and a high value to an underlying stable trend. Similarly, a low value of Θ corresponds to a rapidly changing seasonality, and a high value to a stable underlying seasonal pattern.

The sets of parameter values chosen are those discussed by Dagum (1983) which have been observed in empirical cases. The parameter values for combination (1.b) are often encountered in some retail trade series, those of combination (2.b) have been found in industrial employment data and those of combination (3.b) correspond to the classical international airline passengers series discussed by Box and Jenkins (1970).

For the combinations (1.b) and (3.b) the ARIMA parameter values conform to the assumptions implied by the corresponding symmetric cascade filters. For example, the shortest combination will be appropriate for a fast changing trend-cycle and fast changing seasonality which agrees with the low parameter values of the ARIMA model. On the other hand, combinations (2.b) are (4.b) are selected to assess the impact of using extrapolations from an ARIMA model not consistent with the assumptions of the corresponding symmetric cascade filter.

How do you get the weight?
(How do you get ARIMA forecasting weight?)

Figures 13-15 exhibit for combination (1.b) the gains and phase shifts of the seasonal adjustment, trend-cycle and irregulars with and without extrapolations, respectively.

It can be seen that the effects of the ARIMA extrapolations on the gain functions are the following:

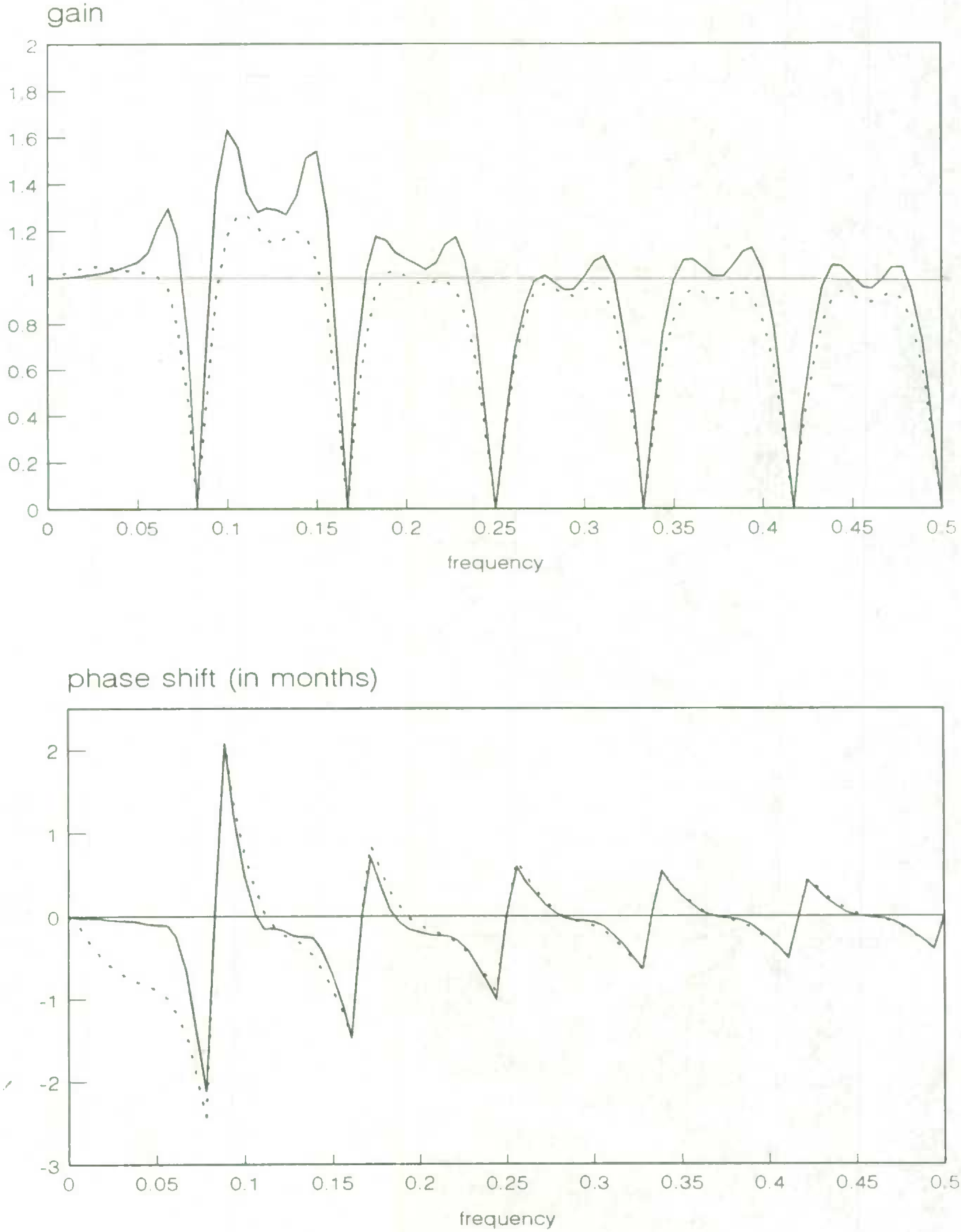
(1) a significant reduction of variance amplification at low frequencies and between the fundamental seasonal and its first harmonic; and

(2) broader seasonal dips.

You can see that ARIMA improves seasonal adjustment results very much in lower frequencies but worsen the T-C filter in higher frequencies (larger)

Fig. 13 SEASONAL ADJUSTMENT CONCURRENT CASCADE FILTERS (3x3)(3x3)[H-9]

Short, ARIMA

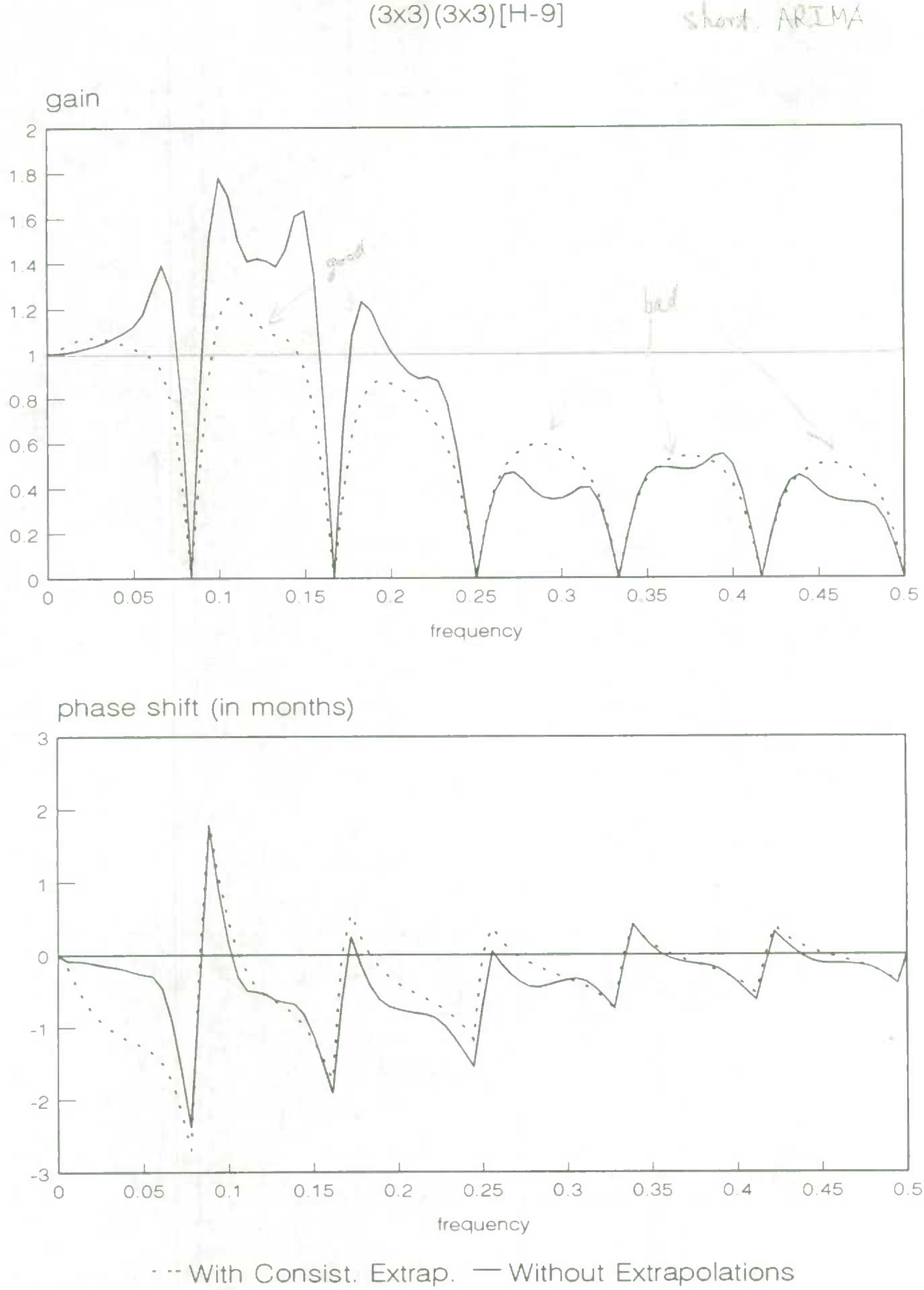


--- With Consist. Extrap. — Without Extrapolations

Model (0,1,1)(0,1,1) $\theta = .30$ $\Theta = .30$

Fig. 14

TREND-CYCLE CONCURRENT CASCADE FILTERS
(3x3)(3x3)[H-9]

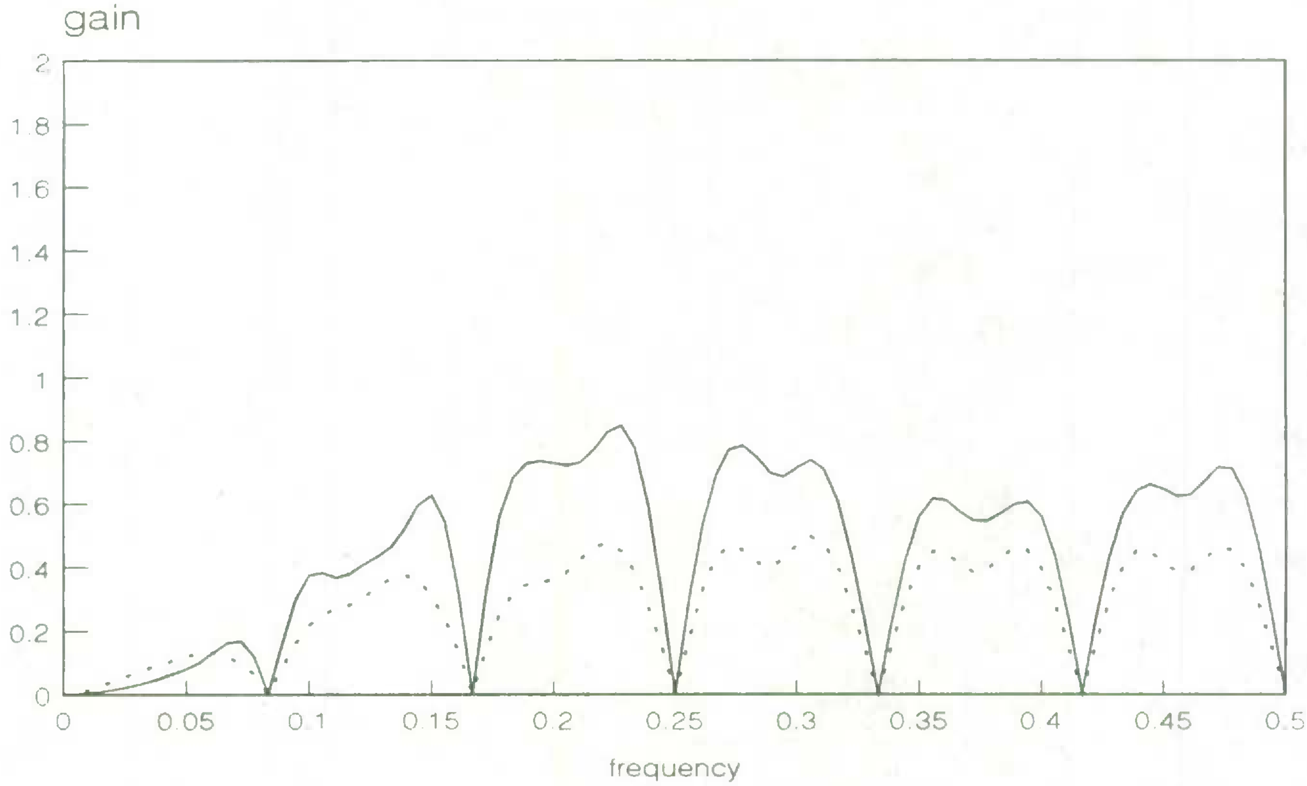


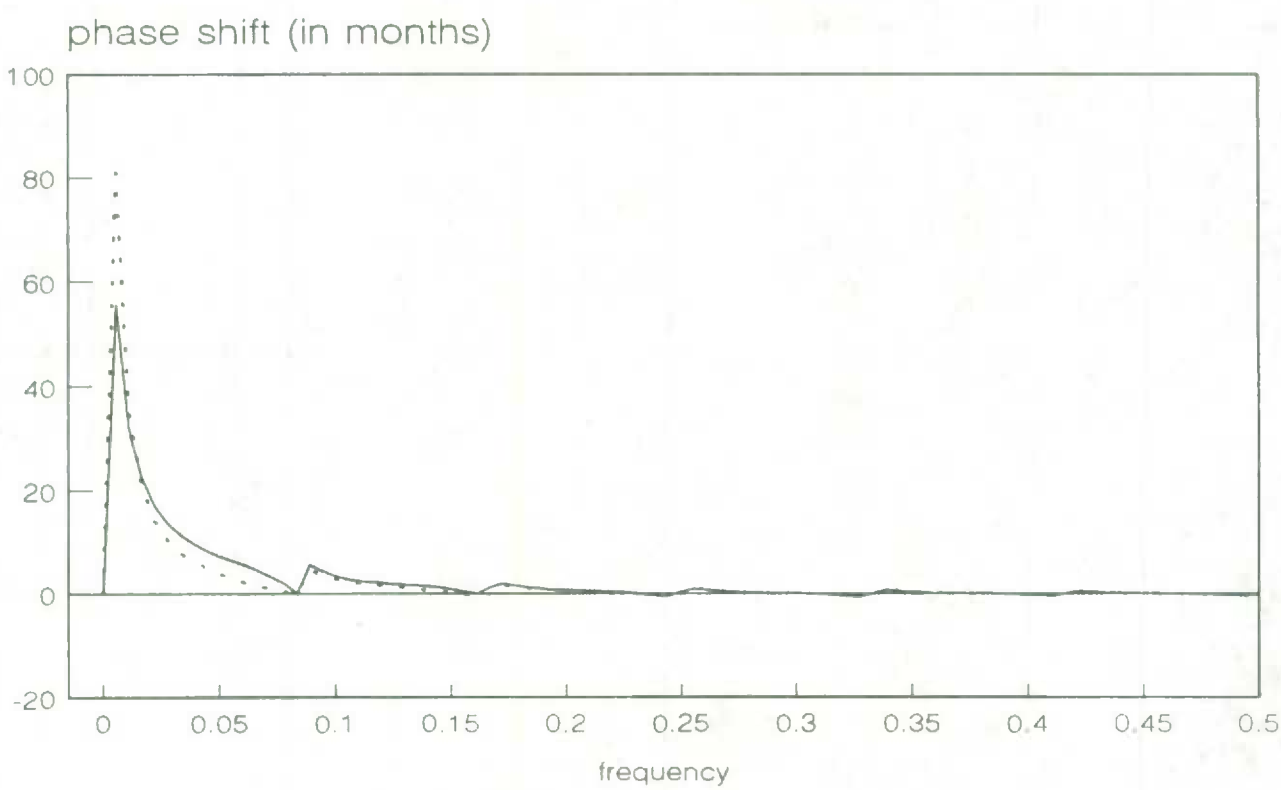
Model (0,1,1)(0,1,1) θ=.30 Θ=.30

Fig. 15

IRREGULAR CONCURRENT CASCADE FILTERS
(3x3)(3x3)[H-9]

Short ARIMA

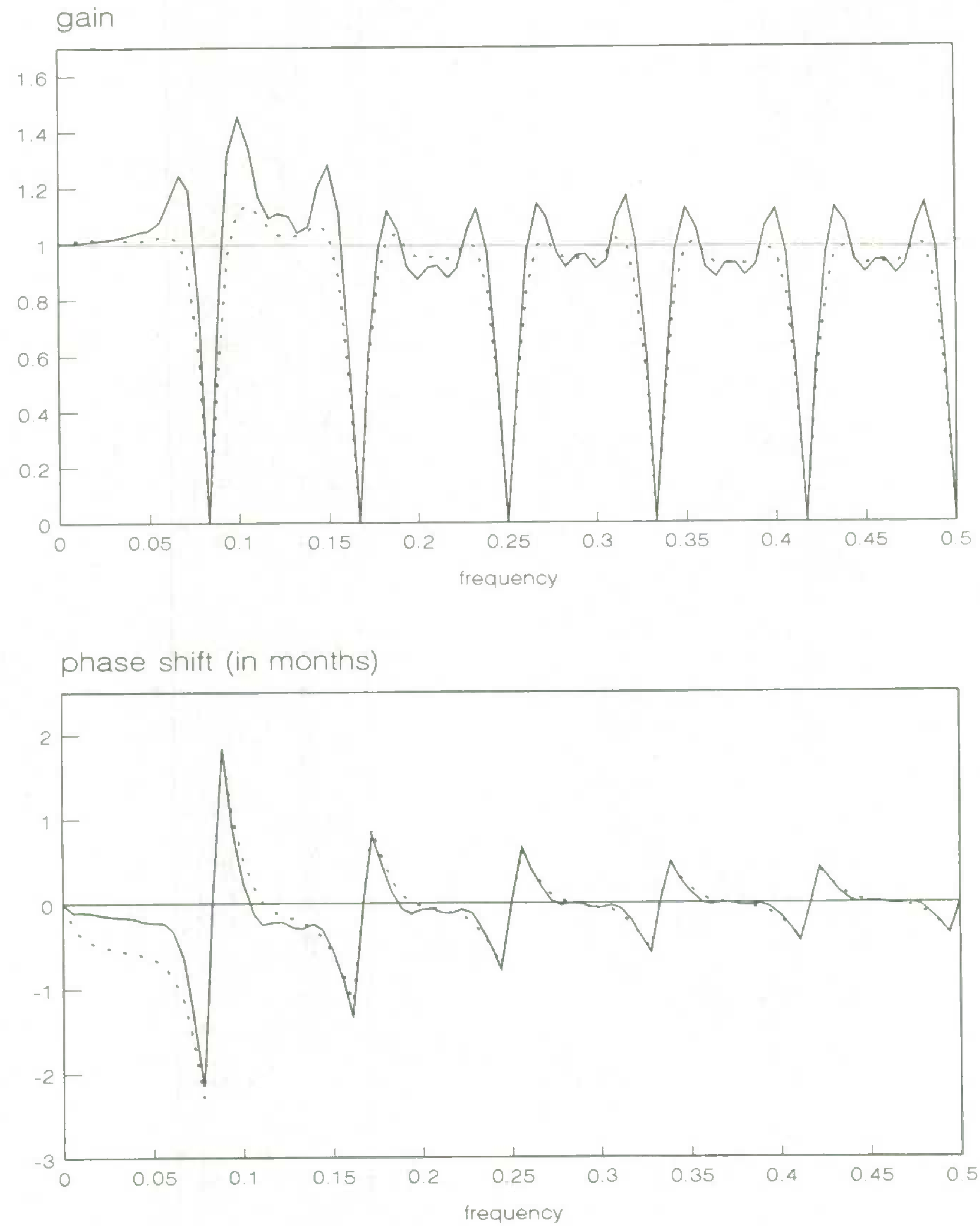




··· With Consist. Extrap. — Without Extrapolations

Model (0,1,1)(0,1,1) $\theta=.30$ $\Theta=.30$

Fig. 16 SEASONAL ADJUSTMENT CONCURRENT CASCADE FILTERS
(3x3)(3x5)[H-13] Standard ARIMA



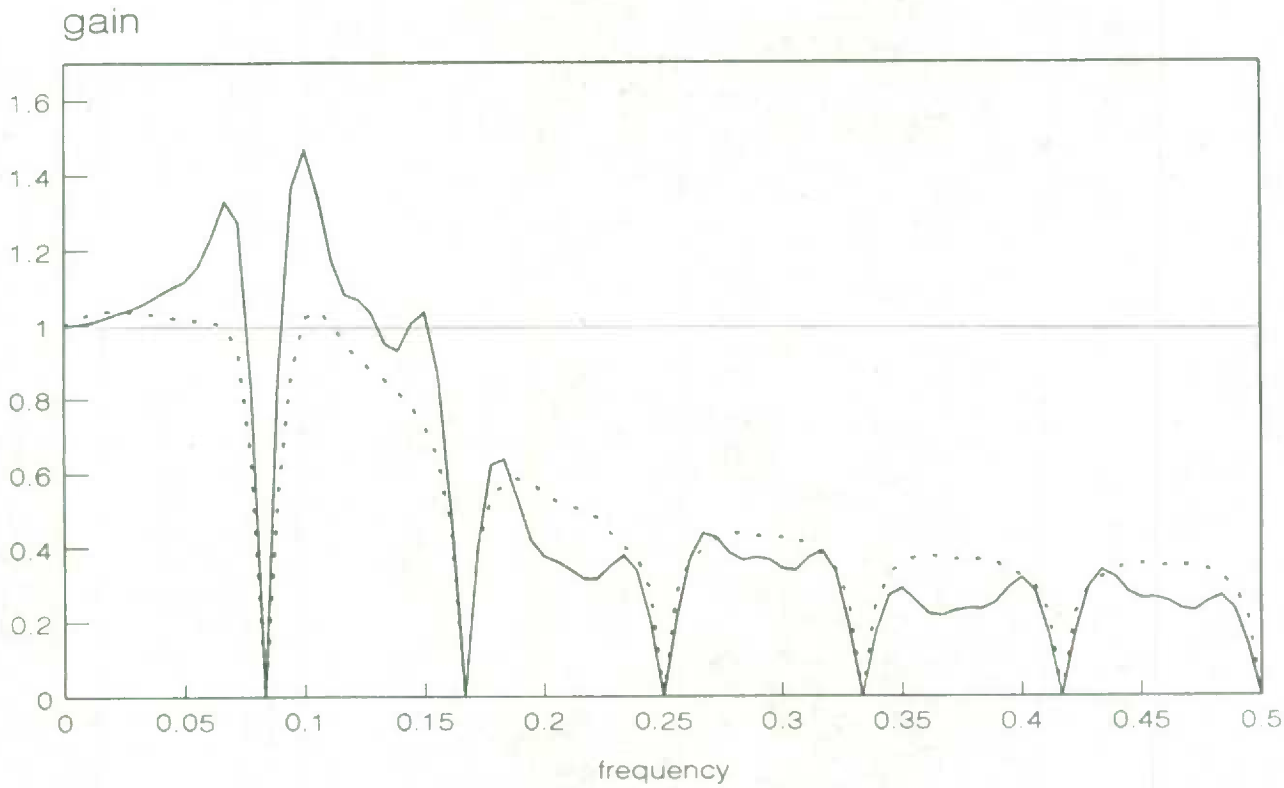
··· With Consist. Extrap. — Without Extrapolations

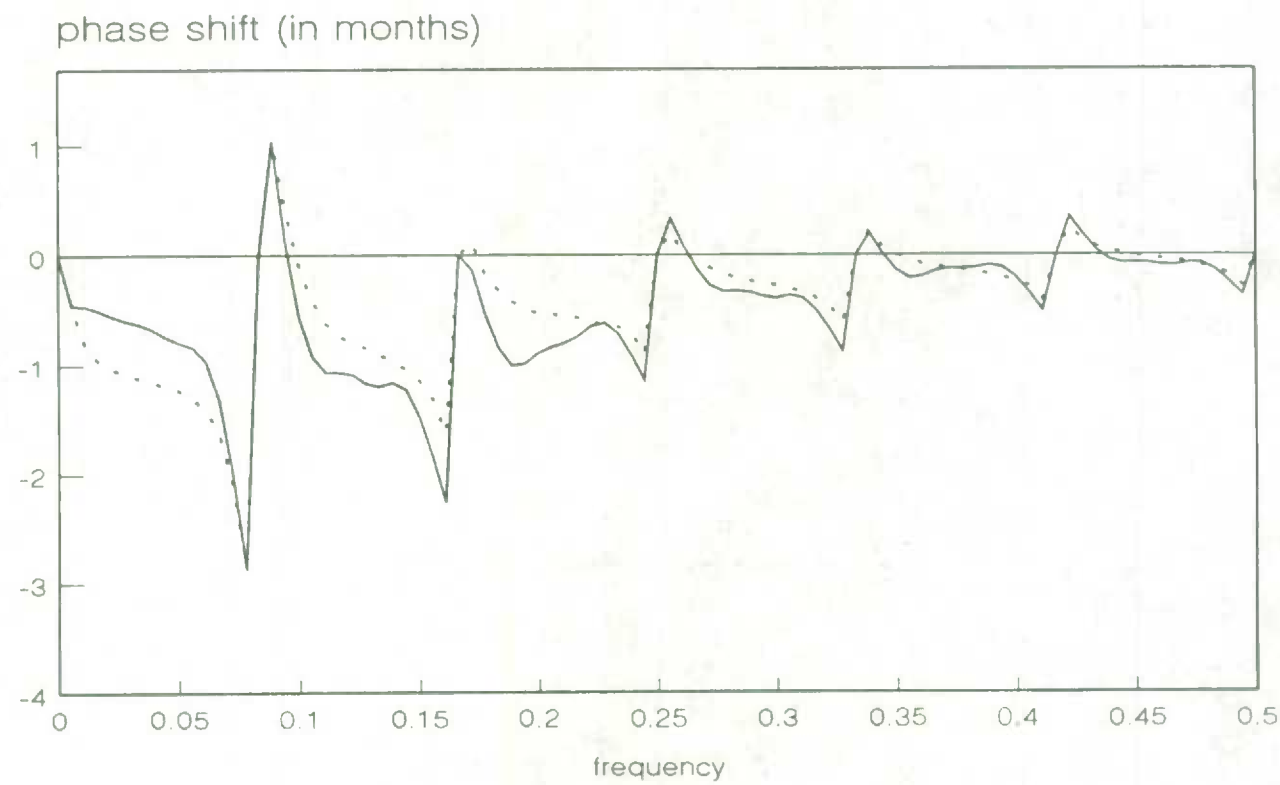
Model (0,1,1)(0,1,1) $\theta=.40$ $\Theta=.60$

Fig. 17

TREND-CYCLE CONCURRENT CASCADE FILTERS
(3x3)(3x5)[H-13]

standard. ARIMA





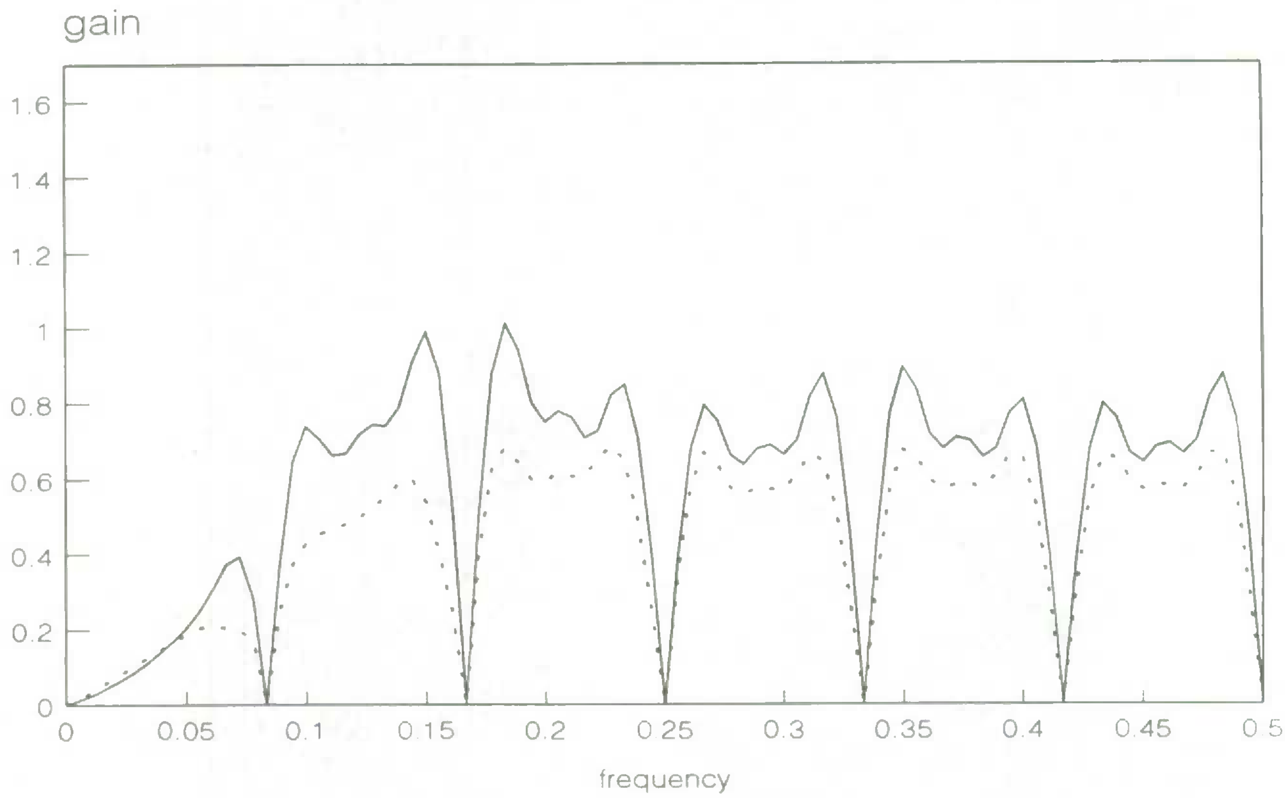
--- With Consist. Extrap. — Without Extrapolations

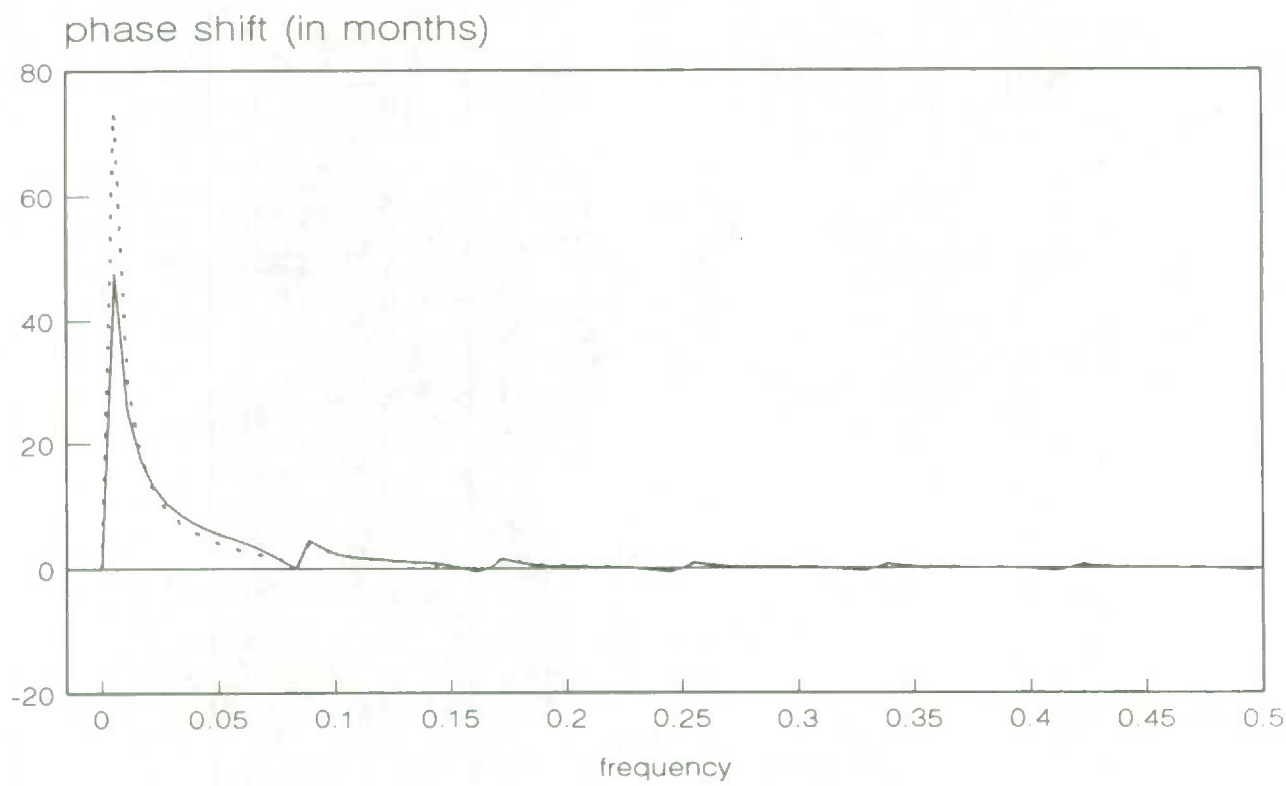
Model (0,1,1)(0,1,1) $\theta = .40$ $\Theta = .60$

Fig. 18

IRREGULAR CONCURRENT CASCADE FILTERS
(3x3)(3x5)[H-13]

Standard, ARIMA





··· With Consist. Extrap. — Without Extrapolations

Model (0,1,1)(0,1,1) $\theta=.40$ $\Theta=.60$

As a consequence, the gain functions of the seasonal adjustment and trend-cycle filters resemble more their corresponding symmetric cascade filters. Since the concurrent trend-cycle filter still passes a significant amount of noise, the gain of the irregular concurrent filter has smaller variance compared to the corresponding symmetric filter. As per the phase shift, there is a systematic increase of about one month at the low frequencies.

(Place figures 13, 14, 15 here)

All the above observations are valid for combination (3.b) and, in general, whenever the ARIMA parameter values are consistent with the implicit assumptions of the corresponding symmetric cascade filters.

(Place figures 16, 17 and 18 about here)

The gain and phase shift functions of the shortest cascade filters extended with inconsistent ARIMA extrapolations in the sense that assume underlying stable trend-cycle and seasonality (combination 2.b) are plotted in Figures 19-21 for the various components.

Figures 19 and 20, show a major reduction of variance over all frequency. This is attributed to the high value of the trend-cycle parameter θ which plays a crucial role in determining the shape of the gain function. On the other hand, a high value of Θ has not affected the seasonal dips which continue to be broader as in the previous combinations (1.b) and (3.b).

We can also observe a substantial increase in phase shift at low frequencies, being largest for the trend-cycle cascade filter than the seasonal adjustment one.

Finally Figure 21, shows the distortion introduced in the gain function of the irregular, mainly at low frequencies.

(Place Figures 19-21 about here)

Figures 22-24 exhibit the gain and phase shift functions of combination (4.b).

The effect of the a $\theta = 0.30$ is clearly seen in Figure 23 where with the exception of the low frequencies, the variance and phase shift have been greatly increased at all the remaining ones.

The irregular concurrent cascade filter (Figure 24) shows a decrease of variance over all the frequencies together with a high increase in phase shift.

Finally, the long seasonal adjustment concurrent filter (Figure 22) seems not to be affected by the extrapolation model parameters except by a small increase in phase shift.

(Place Figures 22-24 about here.)

7. Conclusions

We have introduced and analysed the cascade filters for seasonal adjustment, trend-cycle estimation and the estimated irregulars (residuals) resulting from the convolution of; (a) very short seasonal and trend-cycle moving averages; (3x3)(3x3)[H-9]; (b) standard (most frequently applied) seasonal and trend-cycle moving averages; (3x3)(3x5)[H-13] and (c) long seasonal and trend-cycle moving averages (3x3)(3x9)[H-23].

The shortest symmetric seasonal adjustment filter is characterized by broad seasonal dips which make it more appropriate for series affected by rapidly changing seasonality. Its corresponding trend-cycle cascade filter also seems more adequate for fast changing trend-cycle since the gain function passes all the variance at low frequencies and about 75% of the power at frequencies between the fundamental seasonal and its first harmonic. Consequently, the gain function of the corresponding irregular cascade filter exhibits small variance at all frequency and its shape indicates the presence of negative autocorrelations at low and seasonal lags.

The opposite is concluded by looking at the gains function of the long symmetric cascade filters. These filter convolutions seem to fit better series with underlying regular trend and stable seasonality. The gain of the corresponding irregular cascade filter shows higher variance and lower autocorrelations at low and seasonal lags. Finally, the gain of the standard symmetric cascade filters fall between the above two cases.

We also analysed the asymmetric cascade filters applied to the last available point (also known as the concurrent filter), with and without ARIMA extrapolations. If there are no ARIMA extrapolations, the phase-shift for the short concurrent cascade filter is nearly zero at low frequencies but the gain function is highly amplified. On the contrary, the convolution of the long filters shows phase-shifts of about one month at low frequencies and practically no amplification of variance at these frequencies. Finally, the phase shift and variance amplification produced by the concurrent standard cascade filter fall between the above two cases.

To analyse the impact of the ARIMA extrapolations on the concurrent cascade filters we discussed four cases as follows; (1) shortest cascade filter (3x3)(3x3)[H-9] extended with ARIMA extrapolations from a $(0,1,1)(0,1,1)_{12}$ model where $\theta = 0.30$ and $\Theta = 0.30$; (2) same as (1) with $\theta = 0.80$, $\Theta = 0.80$; (3) standard cascade filter (3x3)(3x5)[H-13] extended with (0,1,1)(0,1,1) ARIMA extrapolations where $\theta = 0.40$, $\Theta = 0.60$; and (4) long cascade filter (3x3)(3x5)[H-23] with

$\theta = 0.30$, $\Theta = 0.30$.

Combinations (1) and (3) have ARIMA parameter values that agree with the assumptions implied by the corresponding short and standard symmetric cascade filters concerning the behaviour of the trend-cycle and seasonal variations.

On the other hand, combinations (2) and (4) have ARIMA parameter values not consistent with the implicit assumptions of the corresponding symmetric cascade filters. In fact, $\theta = 0.80$, $\Theta = 0.80$ assume a series with underlying regular trend-cycle and stable seasonality whereas (3x3)(3x3)[H-9] implies rapidly moving seasonality and trend-cycle. The opposite occurs for combination 4.

The results show that the use of extrapolated values highly improved the gain functions of the concurrent cascade filters if the parameter values of the ARIMA extrapolation model are consistent with the assumptions of the corresponding symmetric cascade filters.

On the other hand, extrapolations from ARIMA models where the parameter values are inconsistent with the implicit assumptions of the symmetric filters produce contradictory results summarized as follows:

(a) the gains of the short cascade filters are mainly affected by high values of θ and practically nothing by Θ. Furthermore, the phase shifts are increased up to 2 months for the concurrent trend-cycle filter;

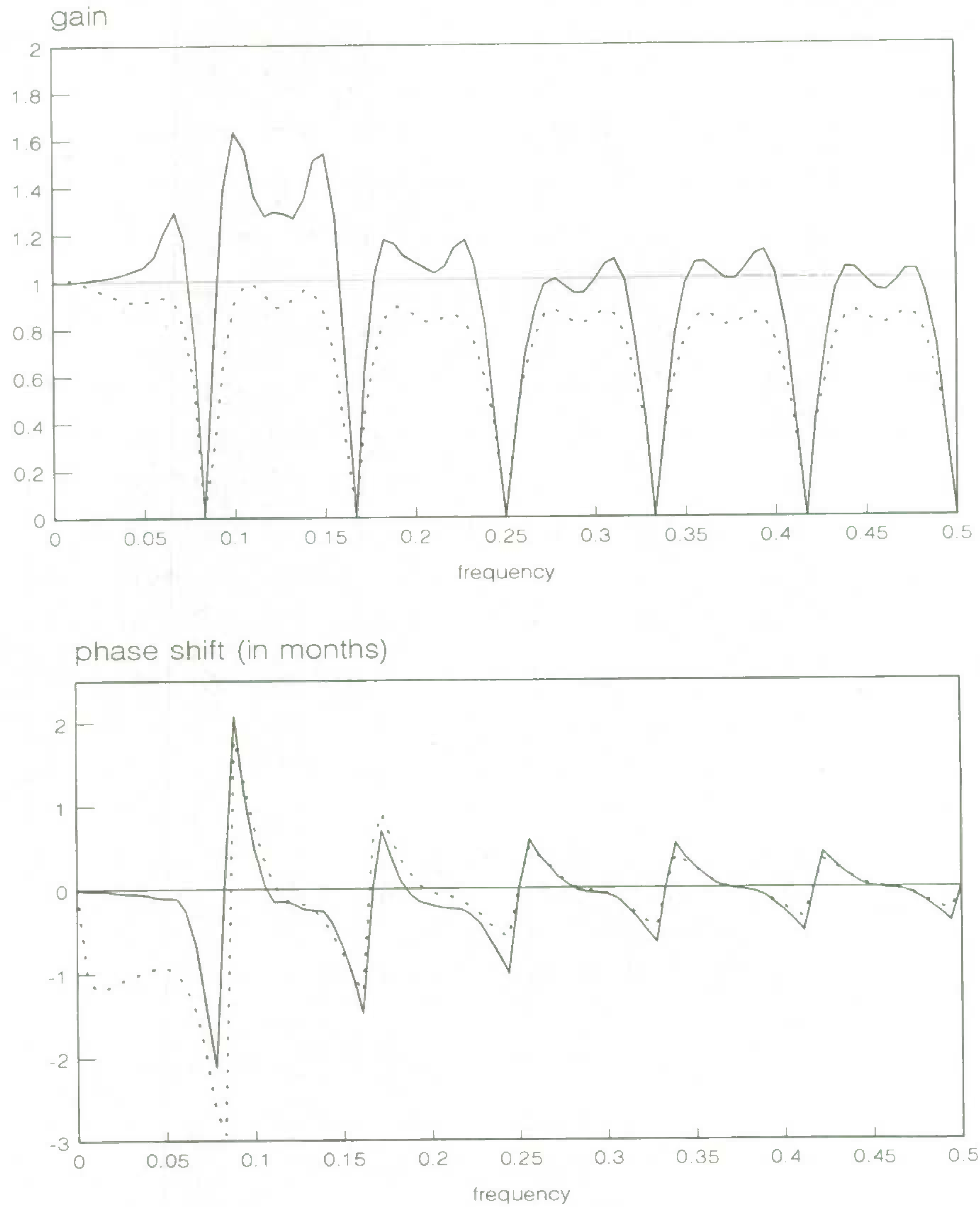
(b) the gain and phase shift of the seasonal adjustment long filter with low values of θ and Θ are slightly affected but those for the trend-cycle are completely distorted, being dominated by the low parameter values of the ARIMA model.

In general, ARIMA parameter values not consistent with the symmetric cascade filters mainly affect the trend-cycle gain and phase shift functions, an indication that the value of θ is more dominant than that of Θ. The inconsistency slightly affects the gain function of the seasonal adjustment filter but increases, in general, the phase shift at low frequencies.

Although not discussed in this study, the discrepancies between the asymmetric and symmetric cascade filters is related to the problem of revision of seasonally adjusted data. Longer asymmetric filters have gain functions that change little as they approach to the symmetric ones, an indication that the seasonally adjusted values will be slightly modified when some observations are added to the series. On the other hand the convergence of the preliminary estimates to the final takes a long number of years. For shorter filters the opposite can be concluded.

Fig. 19 SEASONAL ADJUSTMENT CONCURRENT CASCADE FILTERS
(3x3)(3x3)[H-9]

Short, bad ARIMA

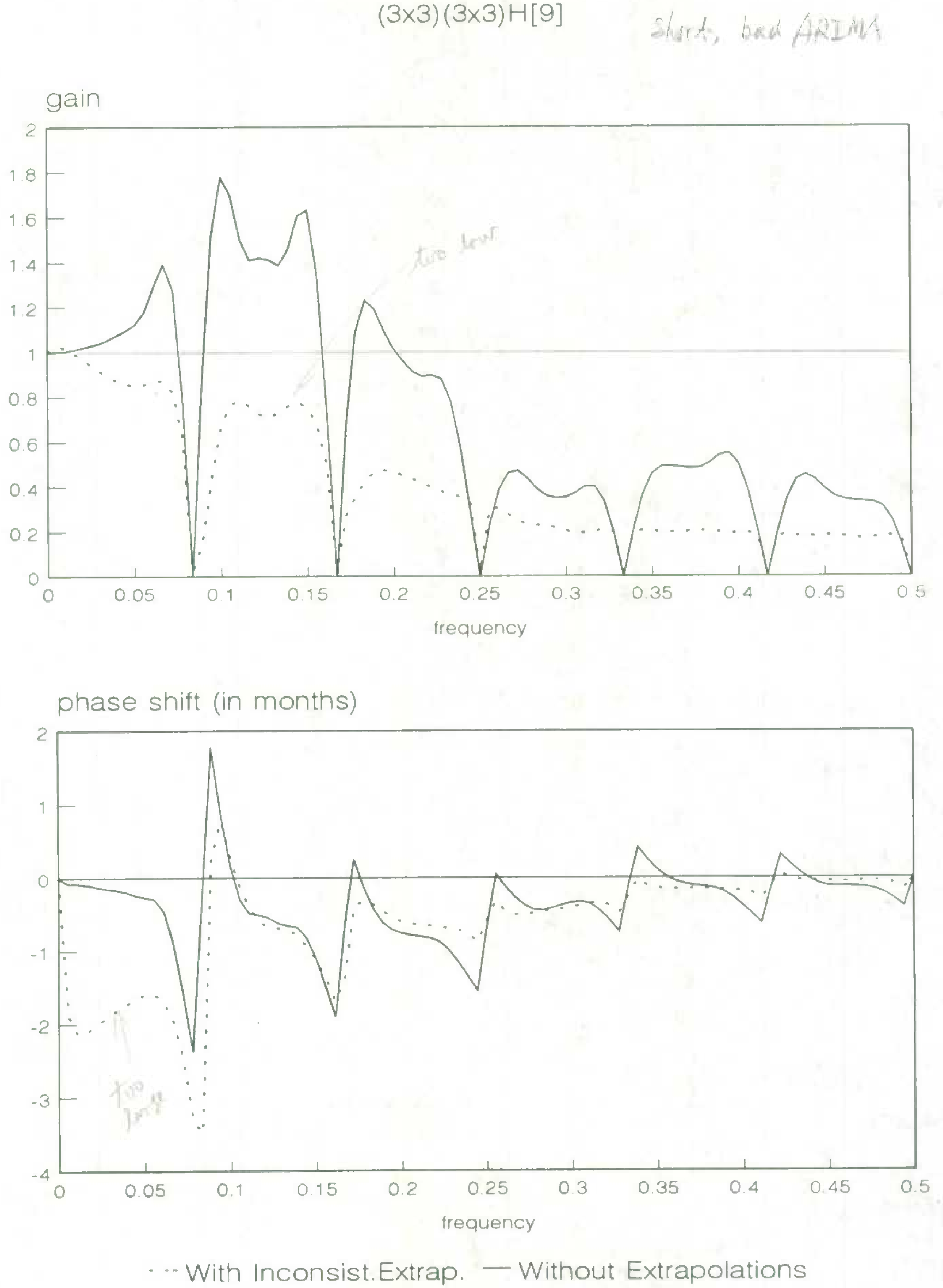


··· With Inconsist.Extrap. — Without Extrapolations

Model (0,1,1)(0,1,1) Θ=.80 Θ=.80

Fig. 20

TRENDS-CYCLE CONCURRENT CASCADE FILTERS
(3x3)(3x3)H[9]

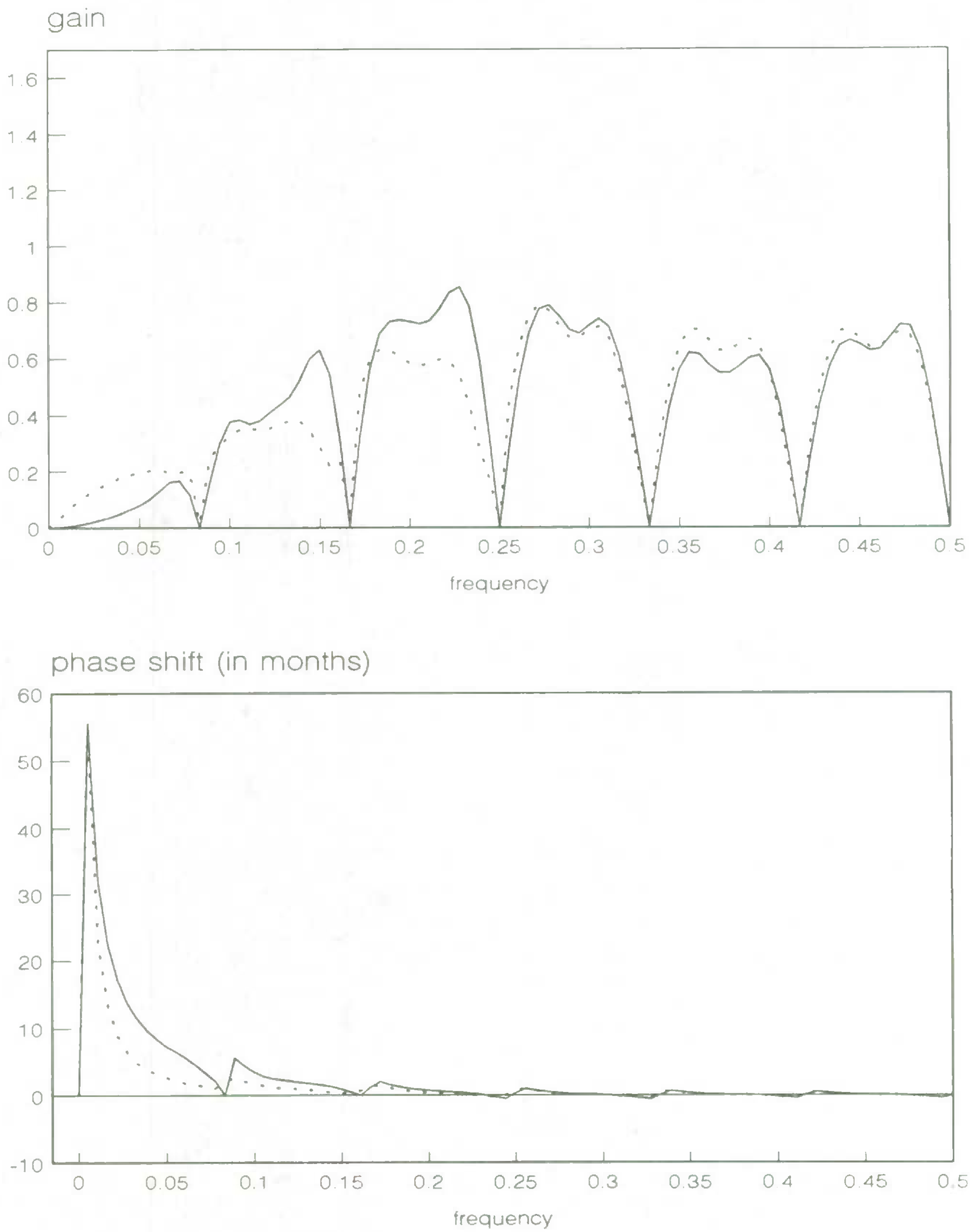


Model (0,1,1)(0,1,1) Θ=.80 Ⓗ=.80

Fig. 21

IRREGULAR CONCURRENT CASCADE FILTERS
(3x3)(3x3)[H-9]

Short, bad ARIMA



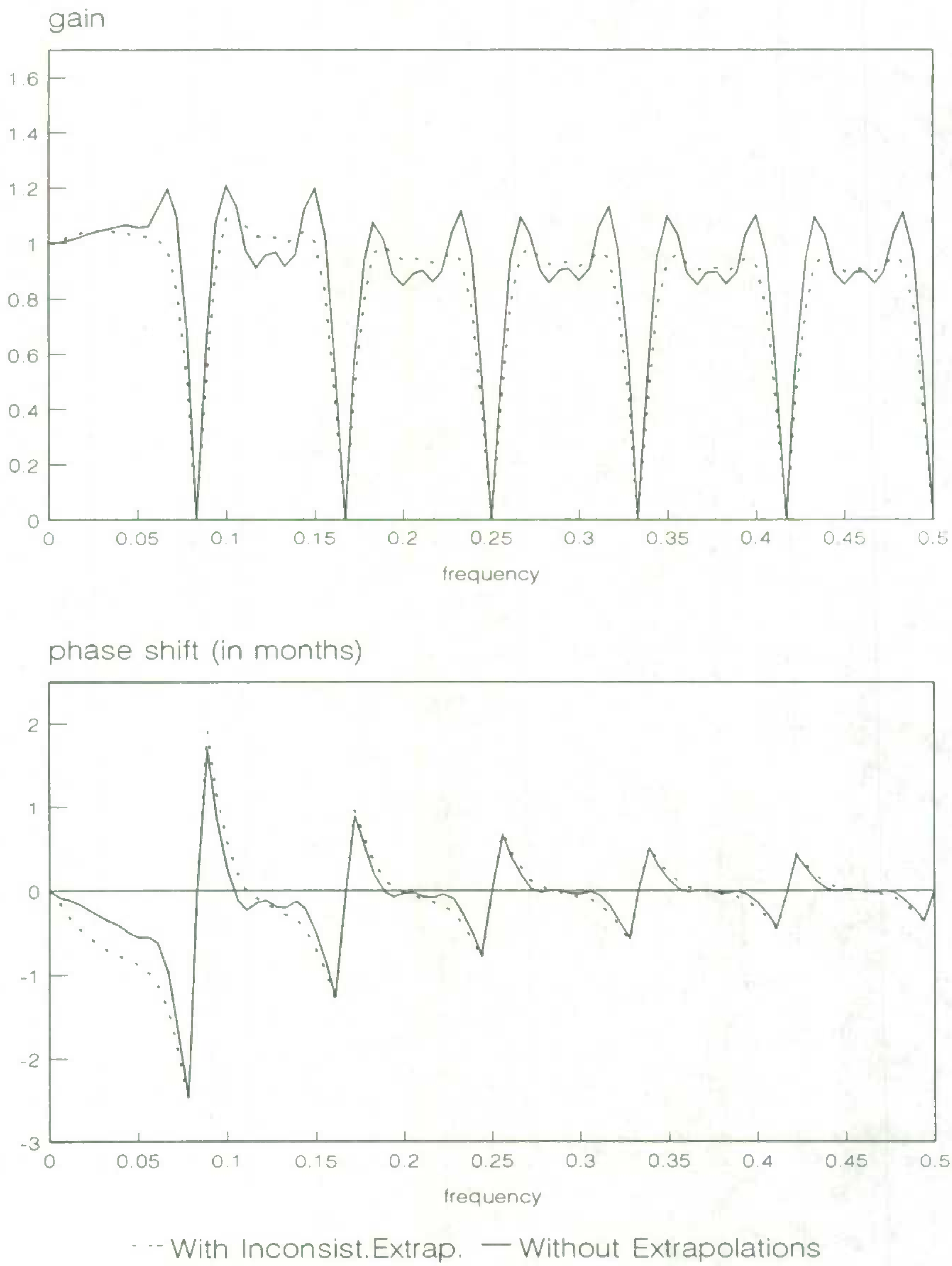
··· With Inconsist.Extrap. — Without Extrapolations

Model (0,1,1)(0,1,1) θ=.80 Θ=.80

Fig. 22 SEASONAL ADJUSTMENT CONCURRENT CASCADE FILTERS

(3x3)(3x5)[H-23]

long. bad ARIMA

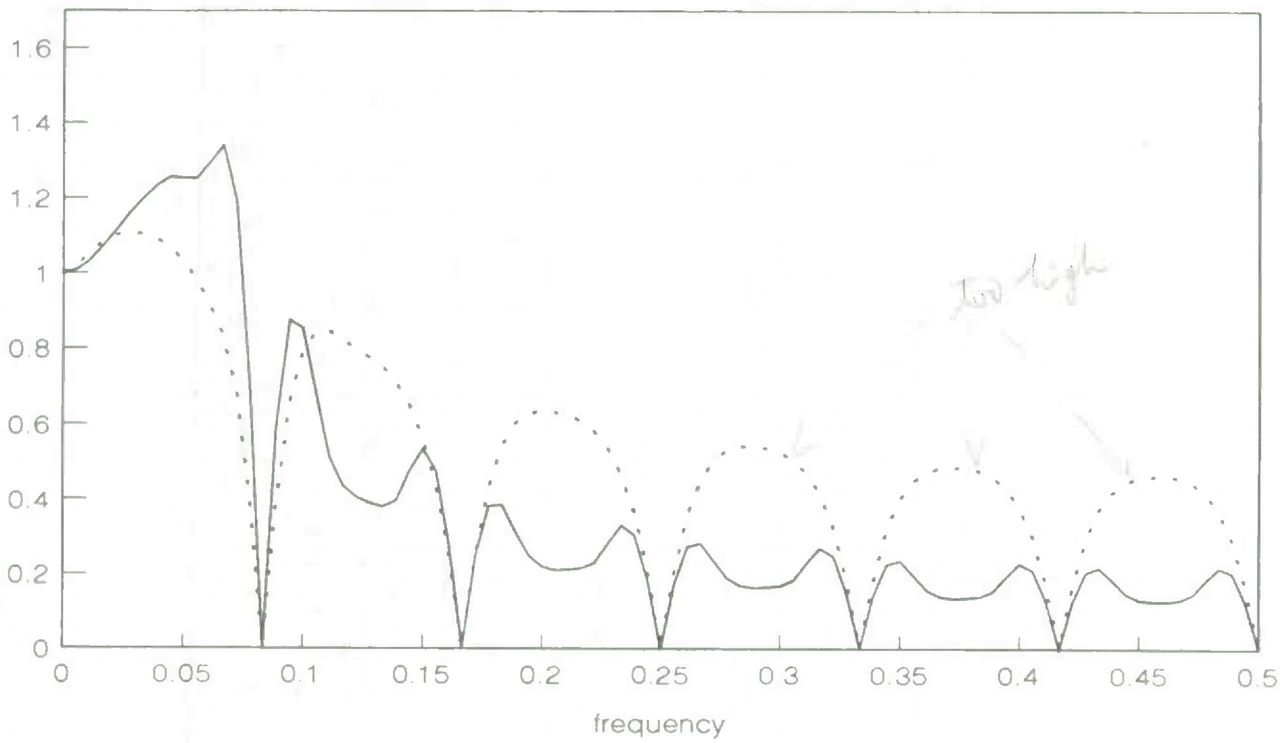


Model (0,1,1)(0,1,1) $\theta = .30$ $\Theta = .30$

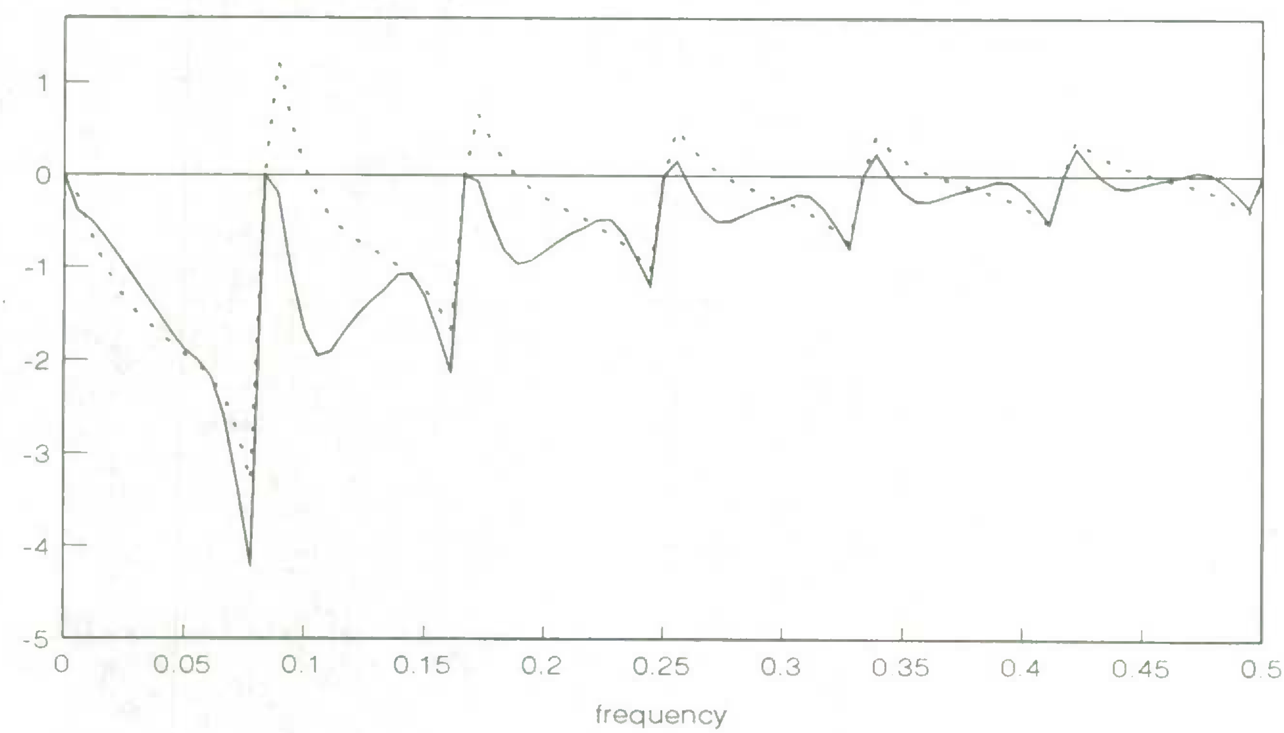
Fig. 23

TREND-CYCLE CONCURRENT CASCADE FILTERS
(3x3)(3x5)[H-23]

gain



phase shift (in months)



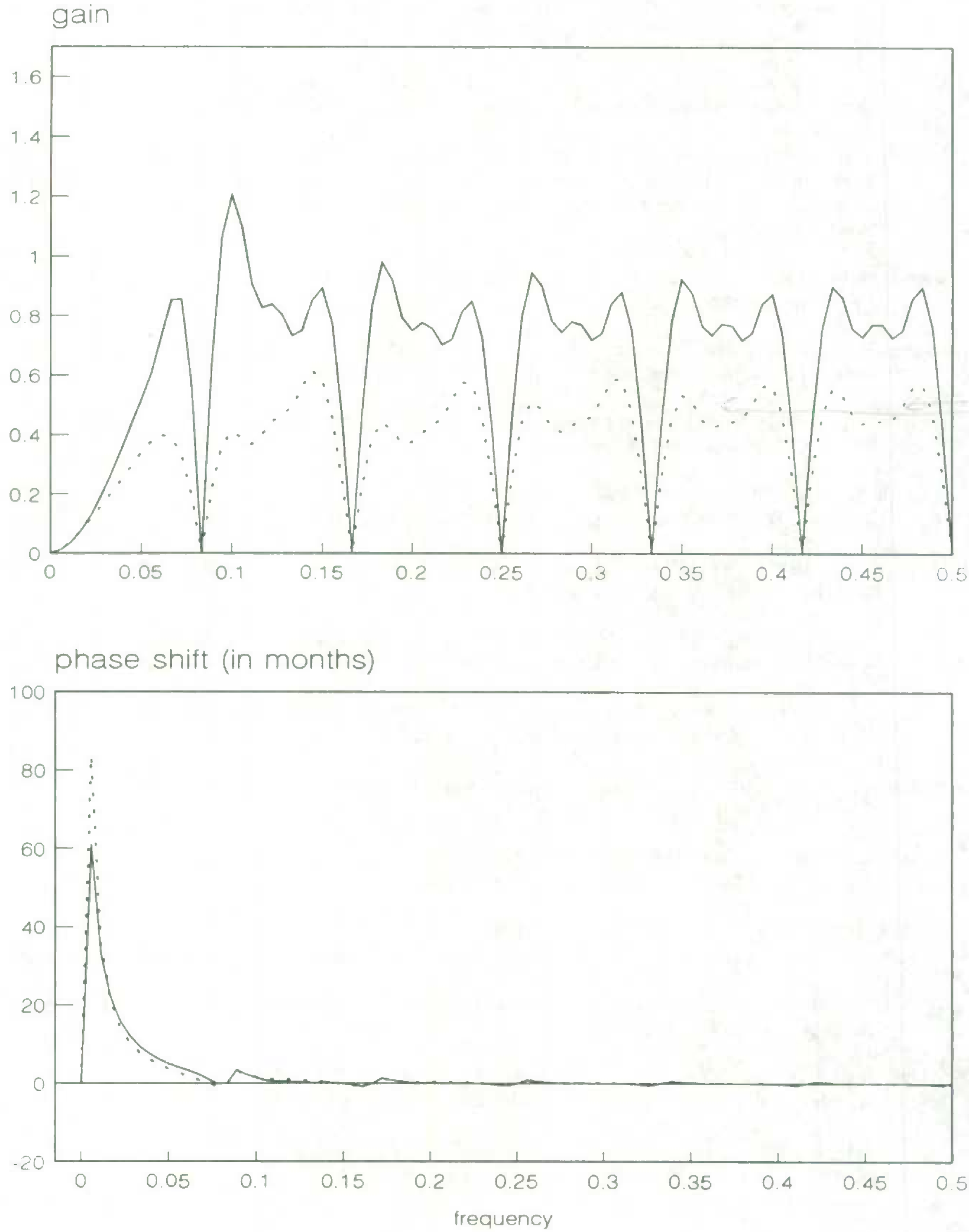
--- With Inconsist.Extrap. — Without Extrapolations

Model (0,1,1)(0,1,1) Θ=.30 Θ=.30

Fig. 24

IRREGULAR CONCURRENT CASCADE FILTERS
(3x3)(3x5)[H-23]

long. bad ARIMA



··· With Inconsist.Extrap. — Without Extrapolations

Model (0,1,1)(0,1,1) $\theta=.30$ $\Theta=.30$

References

Box, G.E.P. and Jenkins, G.M. (1970): *Time Series Analysis Forecasting and Control*, San Francisco; Holden Day.

Burridge, P. and Wallis, K.F. (1984) "Unobserved Components Models for Seasonal Adjustment Filters", *Journal of Business and Economic Statistics*, 2, 350-359.

Dagum, E.B. (1978): "Comparison and Assessment of Seasonal Adjustment Methods for Labour Force Series", Background Paper No. 5, *U.S. National Commission on Employment and Unemployment Statistics*, Superintendent of Documents, U.S. Government Printing Office, Washington, D.C..

Dagum, E.B. (1980): *The X11ARIMA Seasonal Adjustment Method*, Ottawa: Statistics Canada, Catalogue No. 12-564E.

Dagum, E.B. (1982): "The Effects of Asymmetric Filters on Seasonal Factor Revisions", *Journal of the American Statistical Association*, Vol. 77, No. 380, pp. 732-738.

Dagum, E.B. (1983): "Spectral Properties of the Concurrent and Forecasting Seasonal Linear Filters of the X11ARIMA Method", *The Canadian Journal of Statistics*, 11, 73-90.

Dagum, E.B. (1985): "Moving Averages" in *Encyclopedia of Statistical Sciences* (Kotz, S. and Johnson, N., Editors), John Wiley and Sons Inc., Vol. 5, pp. 630-634.

Dagum, E.B. (1988): *The X11ARIMA/88 Seasonal Adjustment Methods - Foundations and User's Manual*, Ottawa: Statistics Canada.

Dagum, E.B. and Laniel, N. (1987): "Revisions of Trend-Cycle Estimators of Moving Average Seasonal Adjustment Method", *Journal of Business and Economic Statistics*, Vol. 5, No. 2, pp. 177-189.

Dagum, E.B. and Chhab, N. and Solomon, B. (1991): "Autocorrelations of Residuals from X11ARIMA Method", *Journal of Official Statistics*, Vol. 7, No. 2, pp. 181-194.

Henderson, R. (1916): "Note on Graduation by Adjusted Average", *Transactions of the Actuarial Society of America*, Vol. XVII, pp. 43-48.

Laniel, N. (1985): "Design Criteria for the 13-Term Henderson End-Weights", *Working Paper, Methodology Branch*, Ottawa: Statistics Canada.

Shiskin, J., Young, A.H. and Musgrave, J.C. (1967): "The X-11 Variant of Census Method II Seasonal Adjustment", *Technical Paper No. 15*, Bureau of the Census, U.S. Department of Commerce.

Wallis, K.F. (1974): "Seasonal Adjustment and Relations between Variables", *Journal of the American Statistical Association*, 69, 18-31.

Wallis, K.F. (1982): "Seasonal Adjustment and Revision of Current Data: Linear Filter for the X11-Method", *Journal of the Royal Statistical Society*, Ser. A, 145, 74-85.

Young, A.H. (1968): "Linear Approximations to the Census and BLS Seasonal Adjustment Methods", *Journal of the American Statistical Association*, 63, pp. 445-471.