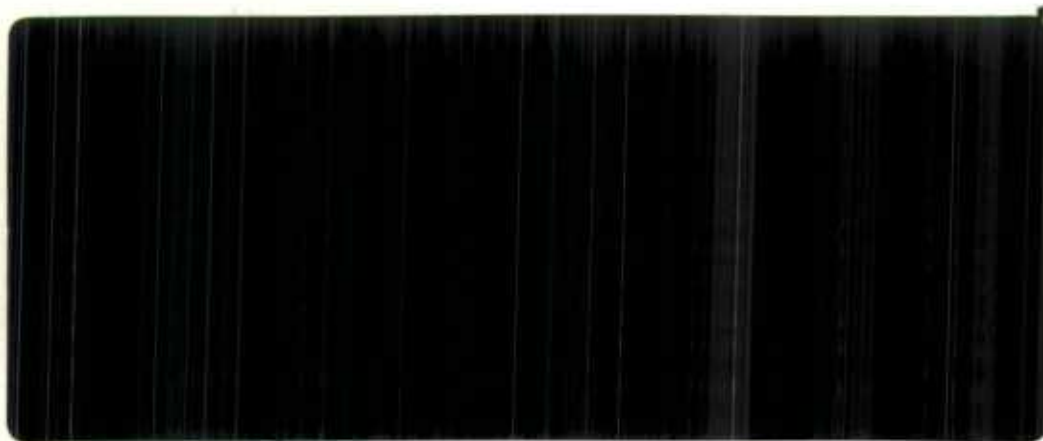




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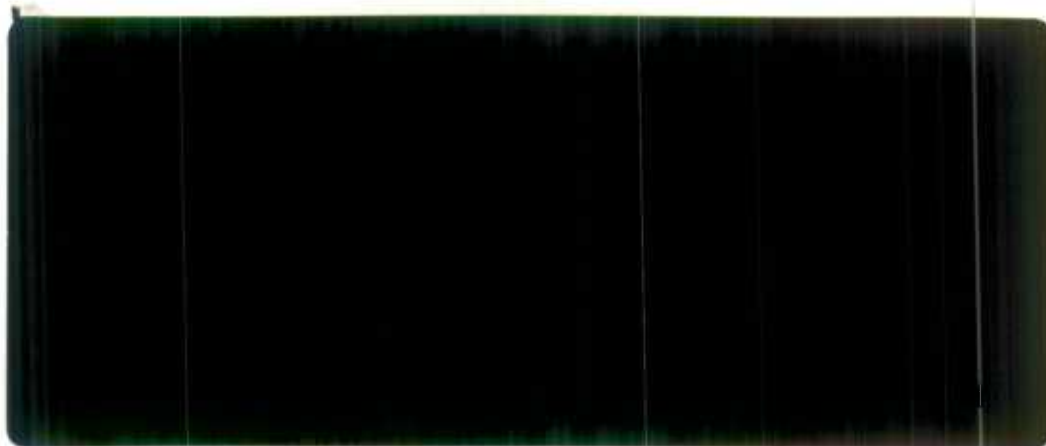
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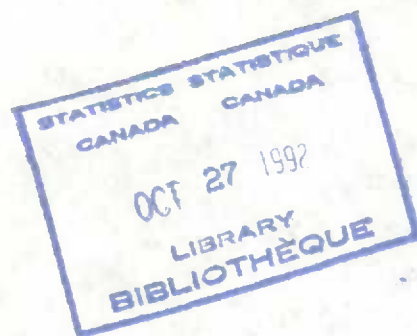
DOCUMENTATION OF THE PRODUCTIVITY DATABASE

Aggregation Formulas for
Multifactor Productivity

By

René Durand

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1 - Introduction

In the technical paper by Durand and Salem(1989)¹ alternative indices of productivity growth applying to the Canadian rectangular input-output tables were presented. These alternative measures differ in that they apply to different sets of inputs and outputs but are otherwise based on the same index number formula known as the Divisia index. Under constant returns to scale and exogenous output and input prices (perfect market competition), productivity growth can be formulated as the difference between the weighted rate of growth of outputs and the weighted rate of growth of inputs with weights equal to the value shares of outputs and inputs in the total value of output.

Three static and one dynamic measures of productivity were developed. The static measures are the traditional neo-classical measure at the industry level which, in this note, is called the **industry index**, a new **interindustry index** and an index defined on **final end products**. The industry, the final end products, the interindustry and the dynamic measures were all aggregated to the business sector of the economy and the aggregation rules for these alternative measures were related to one another. However, aggregation was performed with the Divisia index which is a continuous time index number formula. This formula has to be approximated, in empirical work, by a discrete time index. The discrete time approximation which was chosen is the Tornqvist index. This note deals with aggregation with Tornqvist productivity indices in discrete time for the three static formula.

More precisely, it is concerned with the fact that, contrary to the Divisia indices, Tornqvist indices are inconsistent in aggregation. It is meant by that that aggregation of Tornqvist indices on the basis of the Tornqvist index number formula does not yield an aggregate Tornqvist index. This opens two routes to estimate aggregate productivity indices. Aggregation may be performed on disaggregated Divisia indices in continuous time and the aggregate Divisia index then approximated with the Tornqvist formula. This is what is called in this note the one step aggregation. The second route is to compute Tornqvist indices at the disaggregated level and to aggregate these indices with the Tornqvist formula. This is called the two steps aggregation. The note favours the one step aggregation and shows how it was applied to estimate the published aggregate productivity indices.

¹Durand R. and M. Salem, "Alternative Measures of Productivity Growth in a Rectangular Framework", Statistics Canada, Input-Output Division, November 1987, revised May 1989.

2 - One Step Discrete Time Aggregation Rule The End Product and Industry Indices

In principle, that is using the Divisia index number formula, both of the industry and interindustry productivity indices aggregate to the same business sector productivity index. It has also been shown, in the referred to above paper, that this applies as well to the end product indices. The aggregate final demand productivity index is a Divisia index of productivity applied to the whole vector of final demand end products. Indeed, its formula is

$$\gamma = \alpha^T \rho = \alpha^T \theta = \sigma_L^T \dot{I}_a = \sigma_K^T \dot{k}_a \quad (2.1)$$

The aggregate productivity index γ is the weighted sum of the end product productivity indices given by the vector ρ , the weights, α , being the value shares of these end products in total business sector output. It can easily be seen that the aggregate productivity index which is obtained from the Divisia aggregation of the Divisia productivity indices of individual end products (denoted by the vector θ) is itself a Divisia index. Indeed, each end product is given a weight equal to its value share in the total value of the business sector output. Similarly, each of the inputs (denoted by the aggregate business sector vectors of labour \dot{I}_a and capital \dot{k}_a by type) is given a weight equal to its share of total business sector costs.

The discrete approximation to the aggregate productivity index can therefore be performed as a one step aggregation on the aggregate Divisia index itself rather than, in the first step of a two steps aggregation, on the individual commodity indices and, in the second step, by aggregating the commodity indices to the business sector level. Of the one or two steps aggregation rule, which is used is indifferent provided that the discrete time approximation index chosen is consistent in aggregation. The continuous time Divisia index is as just mentioned. However, this is not the case for the Tornqvist discrete time approximation to the Divisia index which was chosen in the derivation of the empirical estimates. The Tornqvist index of the individual commodity productivity Tornqvist indices is not identical to the aggregate Tornqvist index defined on the vector of final end products.

Which aggregation rule must be followed then comes as the next question. The answer to this question does not appear different than to answer the similar question raised when measuring an aggregate input index. Separate Tornqvist indices, say for capital by type and labour by type, can be constructed from which the Tornqvist input index of the capital and labour Tornqvist indices can be constructed. Alternatively, all capital and labour inputs can be combined, in a single step, into a Tornqvist input index. This last index will be different from the preceding input index. It

has been recommended to use a single step aggregation rule whenever possible in this case. This suggests that, to compute the aggregate productivity index for the business sector of the economy, a single step aggregation procedure also be used. This means that the Tornqvist discrete approximation to the business sector productivity growth from the end product side will be given by:

$$\bar{\gamma} = \bar{\alpha}^T \dot{e} = \bar{\sigma}_L^T \dot{I}_s = \bar{\sigma}_K^T \dot{K}_s \quad (2.2)$$

where the discrete time Tornqvist weights are given by their year t and $t-1$ average:

$$\bar{\omega}_t = \left(\frac{\omega_t + \omega_{t-1}}{2} \right) \quad (2.3)$$

$$\omega = \alpha, \sigma_L, \sigma_K$$

The time derivatives of the logarithm of the variables are likewise approximated in the Tornqvist formula by the logarithmic first difference, e.g., by

$$\dot{x} = \log(x_t/x_{t-1}) \quad (2.4)$$

As noted above, summing the Divisia indices of each Divisia productivity index of individual final demand commodity with the α 's weights gives exactly the same results than taking the Divisia index of productivity of the final demand vector e . However, in discrete time, this consistency in aggregation property breaks, that is:

$$\bar{\alpha}^T \bar{\rho} \neq \bar{\gamma} \quad (2.5)$$

Turning to the neo-classical productivity measure, its aggregate is given by the same γ value which can be shown to be equal to (see Durand and Salem(1989 p.33-34)):

$$\gamma = \beta^T (\bar{C} \dot{v} - \bar{B}^T \dot{u} - \bar{H}^T \dot{I} - \bar{J}^T \dot{K}) \quad (2.6)$$

where

v = the vector of all industries commodity outputs;
 u = the vector of all industries commodity inputs;
 l = the vector of labour inputs of all industries
 by types;
 k = the vector of capital inputs of all industries
 by types

where

$$\beta^T (\bar{C} \dot{v} - \bar{B}^T \dot{u}) - \alpha^T \dot{e} \quad (2.7)$$

and where

$$\begin{aligned} \beta^T \bar{H}^T l &= \sigma_L^T l_a \\ \beta^T \bar{J}^T k &= \sigma_K^T k_a \end{aligned} \quad (2.8)$$

However, in discrete time and similarly to the final demand productivity, the Tornqvist indices of individual industries cannot be aggregated to get the Tornqvist index of the aggregate economy. This is true whether aggregate output is defined as real value added or as the sum of the gross output of all industries. In the latter case, we must take the Tornqvist index of the aggregated economy wide industry by first summing all outputs and inputs commodity by commodity. This Tornqvist index defined on aggregate gross outputs and inputs, it is true, has no clear interpretation and would be of no use. However, for subset of industries, such as total manufacturing industries, this is how the computations should be done rather than aggregating the Tornqvist indices of the component industries. The same applies when computing the aggregate business sector productivity on real value added. Individual industries Tornqvist indices cannot be aggregated into an overall aggregate Tornqvist index.

The aggregate industry productivity index can be taken as being identical to the end products aggregate productivity index. The latter, which must obviously exclude all negative components of final demand accounting for non business supply can easily be compiled from the existing productivity data base. Indeed, performing the aggregation of gross outputs and inputs across industries as indicated above, one obtains the vector v_a of aggregate output by commodity, the vector u_a of aggregate intermediate inputs by commodity and the vectors k_a and l_a of aggregate capital and labour inputs by types. Their cost shares can be similarly established from the current price flows of the data base. One then computes

$$\begin{aligned}
e &= v_a - u_a \\
k_a &= (I^T \otimes I) k \\
l_a &= (I^T \otimes I) l
\end{aligned}
\tag{2.9}$$

where the subscript a indicates that the variables are aggregated over industries. Similar equations are defined for current price flows. It must be noted here that k_a and l_a might turn out to be identical to their disaggregated counterparts if the prices of the primary inputs by "type" in fact differs from one industry to another. Indeed, labour and capital types must not be defined according to their "label" but rather according to their prices. White collars in two industries, for instance, are likely to be of a different type.

From (2.8), it can immediately be seen that

$$\sigma_L^T \dot{l}_a = \omega_L^T \dot{l} \tag{2.10}$$

and

$$\sigma_K^T \dot{k}_a = \omega_K^T \dot{k} \tag{2.11}$$

where the weights ω_L and ω_K are the value shares of labour and capital by types and industries in the total business value added. Taken together, they sum to one. These shares can easily be calculated from the database. Their discrete counterparts, according to the Tornqvist formula, are then simply calculated as:

$$\bar{\omega}_{Lt} = (\omega_{Lt} + \omega_{Lt-1})/2 \tag{2.13}$$

and

$$\bar{\omega}_{Kt} = (\omega_{Kt} + \omega_{Kt-1})/2 \tag{2.14}$$

To conclude on the previous results:

(1) The discrete approximation to the neoclassical aggregate business sector productivity measure has been formulated as the Tornqvist index of the aggregate final demand Divisia index of productivity growth.

(2) The discrete approximation to the neoclassical productivity index on some aggregated subset of industries as been formulated as the Tornqvist index applied to the aggregate industry formed by summing commodity outputs and inputs for these industries.

In the above derivations, because competing imports are part of intermediate inputs, it may (and in fact, it does) happen that some components of the vector e turn out to be negative. In such a case, the negative components are considered as inputs and e is redefined to include only the positive components. This gives a measure of "gross" final demand output. The Divisia index may be computed on that output measure, with the correspondingly redefined set of inputs. The result can be transformed to a net output basis (similarly to the transformation of the neoclassical industrial formula) by multiply the productivity estimated growth rates by the ratio of the final demand "gross" output to net final demand output in current prices. It is on this last result that the Tornqvist approximation may be applied.

Given what precedes, the computation can equally be done on the business sector gross output and inputs obtained by summing over all industries. The result can then be transformed from a gross output basis to a net output basis by multiplying it by the aggregate β weight, that is by the ratio of gross output to net output in current prices. Again the Tornqvist approximation may be applied to that result.

It can be shown that the aggregate business sector productivity index can alternatively be computed by subtracting final demand "inputs" from final demand "gross" outputs by commodity to define a measure of net output which is then related to corresponding primary inputs used. The same result applies to industries' gross outputs from which intermediate inputs can be subtracted. The aggregate Divisia index can be applied to that measure of net output and corresponding primary inputs. In fact, any input, including some primary input such as non competing imports, government supply of goods and services, etc., can first be subtracted from gross output to obtain any desired measure of net output. This is this last alternative which has been used. Intermediate inputs and some primary inputs were subtracted from the aggregate business sector gross output so as to define net output as value added plus "Other Indirect Taxes"; the latter, being composed mostly of property taxes, were considered as part of gross capital income. Hence, we subtracted from the output vector v the vector of intermediate inputs u and all other primary inputs which are not part of the value added as just defined above. The corresponding Divisia index formula was then approximated.

3 - Aggregation for the Interindustry Indices

Discrete approximation indices for the interindustry productivity indices also have to be formulated. For the business sector as a whole, the above aggregate discrete approximation can be used as the aggregate interindustry index is identical to the aggregate neoclassical industry index. For subset of industries, such as manufacturing industries, we have to start with formula (A4.3) of Durand and Salem(1989):

$$\gamma = \alpha^T D^T \tau^+ \quad (3.1)$$

where τ^+ is the interindustry productivity index and D the market share matrix. Let us write, to simplify the notation

$$\phi^T = \alpha^T D^T \quad (3.2)$$

The ϕ weights are the shares of the deliveries of each industry to final demand while the α weights were the value share of each commodity in total final demand. Whatever the level of aggregation chosen, relation (3.2) has to be satisfied which implies that we must have:

$$\phi_a^T \tau_a^+ = \phi^T \tau^+ \quad (3.3)$$

where the subscript a indicates some level of aggregation of industries from the most disaggregated level which is not indexed in (3.3). For any subgroup k of industries which are aggregated together, it follows that we will have

$$\phi_{ak}^T \tau_{ak}^+ = \sum_{i \in k} \phi_i^T \tau_i^+ \quad (3.4)$$

But the aggregate share ϕ_{ak} also represent the deliveries of the industrial group to final demand. These deliveries are just but the sum of the deliveries of the component industries, that is

$$\phi_{ak} = \sum_{i \in k} \phi_i \quad (3.5)$$

Combining (3.4) and (3.5) one gets:

$$\tau_{ak}^+ = \frac{\sum_{i \in k} \phi_i \tau_i^+}{\sum_{i \in k} \phi_i} \quad (3.6)$$

Equation (3.6) is the interindustry index aggregation rule for industrial subgroups. The subgroup index is a weighted sum of the indices of the component industries, the weights being the shares of each industry's deliveries to final demand in the total share of the group's deliveries to final demand.

The interindustry indices at the disaggregated level can themselves be computed from the industry indices from the relationship (see again Durand and Salem 1989, equation 2.3):

$$\tau^+ = [I - B^T D^T]^{-1} \tau \quad (3.7)$$

where the vector τ of the industry neo-classical productivity index is given by the expression which is multiplied by β in equation (2.6). Substituting from (3.7) into (3.6) gives the final expression which has to be approximated by the Tornqvist index.

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