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ON A DYNAMIC PRODUCTIVITY INDEX NUMBER FORMULA

BY

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#27

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Introduction

This article presents a new dynamic index number formula for measuring productivity growth within a rectangular input-output framework. This index is derived by steps, building up successively on the traditional multifactor productivity index at the industry level presented in section 1, a new interindustry index derived in section 2 and a final end products index in section 3. For each index formulated, a distinct economic interpretation is suggested together with an outline of its relationship with the other indices[1].

There are two important advantages to analyzing productivity measures in an input-output framework extended to include estimates of capital and labour inputs. First, it allows to construct estimates of multifactor productivity from a consistent set of industry data on commodity inputs and outputs, in volume and in value terms, at a detailed level. Secondly, it advantageously provides information on interindustry relationships embedded in the input-output tables. Using these relationships, it is possible, as explained below, to express the outputs of an industry as a function of the primary inputs used directly and indirectly by that industry, that is by vertically integrating an industry with its intermediate inputs supplying industries.

The interindustry relationships are exploited one step further to derived indices of productivity growth on final end products. Direct and indirect primary input requirements associated with volume expenditure for each final end product are derived through the usual input-output impact matrix with their associated cost shares.

The dynamic index formula is introduced in section 4. It extends the interindustry and final end product indices across time periods. In that formulation, the capital stock used in all industries is considered as being composed of various goods produced by capital goods producing industries in earlier periods. Productivity gains are made on the production of these capital goods at the time they are produced. This results in the economy benefiting from a higher capital stock at the beginning of each period than it would have in the absence of productivity gains. The growth in the capital stock is therefore seen as being the result of both past savings and productivity growth. This productivity growth is hidden in the measured volume of the capital stock in the traditional, the interindustry and the end products multifactor productivity indices. The dynamic index reveals it by measuring capital goods in forgone consumption units of a base year. This section also establishes a parallel with the alternative non dynamic index suggested earlier by Rymes on the basis of the same intuition.

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[1]More detailed derivations and, in particular, the aggregation rules for each index, are available from the authors.

The last section provides further insight on the new productivity index and draws major conclusions among which the conclusion that productivity growth estimates which neglect the contribution of technical progress to capital accumulation systematically understate the contribution of technical progress to economic growth.

Appendix 1 details the notations and basic definitions used throughout the paper. Appendix 2 delineates a set of matrix differentiation operators used extensively in the paper and derives the vector of traditional productivity growth rates. Appendix 3 contains the mathematical derivations which support the final output productivity measure. Appendix 4 derives a fundamental result which throws light on a basic issue raised in this paper. Il shows the equivalence between the interindustry measure of productivity gains and the alternative of expressing productivity growth in terms of direct and indirect primary input requirements in producing a given output vector.

1 - The Traditional Productivity Measure

This Section introduces basic elements of the input-output accounting framework together with the theoretical assumptions which are required for the derivation of the industries' neo-classical formulation of productivity growth which is then presented.

The technological input-output framework, which will be referred to throughout the paper, includes a gross output or make matrix V, a commodity input or use matrix U, a vector of commodity prices p and two primary input matrices, K and L with service price vectors r and w. These variables obey the cost-sales or industry budget identity

$$V p = U^T p + K^T r + L^T w \quad p>0, r>0, w>0 \tag{1.1}$$

We represent each industry's production technology by a separable (in inputs and outputs) neoclassical production function

$$g_i(v_i) = f_i(u_i, k_i, l_i; t) \tag{1.2}$$

where the variables are the ith industry's vectors of outputs, commodity inputs, capital and labour services in physical units and t is the time variable. We maintain the usual assumptions, namely, that production functions are homogeneous of degree 1 in factor inputs and that inputs and outputs are supplied by competitive markets but also add that technical progress affects the output vector uniformly. The latter implies that productivity change results in a proportional shift of the elements of industries' output vectors. These assumptions permit the usual results of production theory: maximizing the profit function:

$$\pi_i = p_{v_i}^T v_i - p_{u_i}^T u_i - w_i^T L_i - r_i^T K_i \tag{1.3}$$

subject to condition (1.2) leads to the Divisia index of productivity growth, making use, in the derivations, of Euler's rule for homogenous functions. The productivity index for industry i can be written as:

$$\tau_i = c_i^T \dot{v} - b_i^T \dot{u} - \sigma_{Li}^T \dot{L} - \sigma_{Ki}^T \dot{K} \tag{1.4}$$

where the doted variables represent the time derivative of the logarithm of the variables (time percentage rates of growth) and where the c's the b's and the σ's represent respectively the value shares of commodities in industry's output, intermediate inputs, labour and capital inputs.

The conditions we have specified above allow us to represent productivity growth, or the derivative of the logarithm of the production function f_i with respect to time, by a Divisia index of aggregate output growth less a Divisia index of aggregate input growth. The aggregates are, in this case, revenue share and cost share weighted averages of commodity output growth rates and input growth rates respectively. This residual growth rate, denoted by τ, measures the gains in output unaccounted for by purchased factor inputs and is a measure of productivity gains resulting from technological influences other than those captured in the measured variables, that is, essentially from technical progress under the assumption of constant returns to scale.

In common with the atomistic view of traditional price theory, the traditional or neo-classical formulation of an industry's productivity growth measures technical changes which have occurred within a single stage of productive activity - the stage commencing with the industry's purchase of commodity and service inputs (both primary and intermediate) and ending with the sale of the transformed inputs. Using matrix notation, in Appendix 2, the traditional measure τ can also be derived in compact form by algebraic manipulations of industry budget equations when differentiated with respect to time. The vector of industries' traditional productivity growth rate, τ, is:

$$\begin{aligned}\tau &= C^{*}\,\dot{v}^{*} - B^{T*}\,\dot{u}^{*} - H^{T*}\,\dot{l}^{*} - J^{T*}\,\dot{k}^{*} \\ &= -(C\,\dot{p} - B^{T}\,\dot{p} - H^{T}\,\dot{w} - J^{T}\,\dot{r})\end{aligned} \tag{1.5}$$

Here C^{*} is the "row-diagonal" of the product mix matrix C made of current dollar value shares of output components in the value of industries' output, i.e.

$$C^{*} = \begin{bmatrix} C_{11}C_{12}\ldots C_{1n} & 0\ldots\ldots 0 & \ldots\ldots & 0\ldots\ldots 0 \\ 0\ldots\ldots 0 & C_{21}C_{22}\ldots C_{2n} & \ldots\ldots & 0\ldots\ldots 0 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ 0\ldots\ldots 0 & 0\ldots\ldots 0 & \ldots\ldots & C_{m1}C_{m2}\ldots C_{mn} \end{bmatrix} \tag{1.6}$$

B^{*} is a similar share matrix for intermediate inputs, and H^{*} and J^{*} are share matrices for the values of labour and capital services respectively. The other variables, $\dot{v}^{*}$, $\dot{u}^{*}$ and $\dot{k}^{*}$ are "vector-forms" of matrices of growth rates of output, intermediate inputs, labour and capital services, while their unit price vectors are p (for both v and u), w and r. For instance,

$$v^{*} = \begin{bmatrix} v_{11} \\ v_{12} \\ \cdot \\ v_{1n} \\ v_{21} \\ v_{22} \\ \cdot \\ v_{2n} \\ \cdot \\ \cdot \\ \cdot \\ v_{m1} \\ v_{m2} \\ \cdot \\ v_{mn} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{1} \\ \mathbf{v}_{2} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{v}_{m} \end{bmatrix} \quad \text{and} \quad C^{*}\,\dot{v}^{*} = \begin{bmatrix} c_{1}^{T}\,\dot{v}_{1}^{*} \\ c_{2}^{T}\,\dot{v}_{2}^{*} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ c_{m}^{T}\,\dot{v}_{m}^{*} \end{bmatrix} \tag{1.7}$$

The left-hand side of identity (1.5) is the Divisia index of factor inputs subtracted from the Divisia index of outputs and is the same productivity measure as given in (1.4) in vector notation or "stacked" form for all industries. The right-hand side of (1.5) is similarly the vector of traditional residual we would obtain from the dual unit cost functions of the same industries under the same assumptions.

In representing the history of a single economic unit, it is natural to enquire into the extent of technical change which accompanied its evolution. Measuring the sum of influences given by its productivity growth is an exercise in partial equilibrium, namely, we explicitly abstract from changes which are induced elsewhere in the system, and those which are required, to produce general economic equilibrium. Hence, the basic feature of the traditional productivity measure that has to be emphasized is that it focuses on each industry's productivity gain separately. In particular, inputs are considered as exogenously given to each industry. No distinction is therefore made between intermediate and primary inputs. But for the economy as a whole, intermediate inputs are not given. Only primary inputs are given and this has important implications which are explained in the next Section.

2. An Interindustry Measure of Productivity Growth

In an interindustry context, the outputs of productive units are either intermediate inputs for other industries, objects of consumption by final demand agents, or both. Resources which are "primary" to this system, namely capital and labour and natural resources - in the sense that they are not outputs of industries being considered which are re-utilized within the same period as intermediate inputs - are supplied by final demand agents in exchange for present and future command over consumption. From the point of view of the economy **as a whole**, primary inputs, are the only resources which ultimately support, directly or indirectly, an industry's level of productive activity. Replacing every industry's intermediate inputs growth rate vector with the rate of growth of the inputs required by the supplying industries to produce them is equivalent, in the end, to replacing all industries' required intermediate inputs with the primary inputs indirectly used to produce them. An alternative procedure for obtaining this result is to deflate the rate of growth of intermediate inputs by the productivity gain obtained in their respective producing industries (Appendix 4 deals with the equivalence of the two approaches).

Each industry is therefore viewed as an element of an integrated system, where their intermediate inputs are outputs originating from other industries. The appropriate measure in this context addresses the question: "what are the productivity gains realized by each industry in its use of the resources of the economy as a whole - that is, in its use of primary inputs?". These resources include not only those directly used by each industry but also those which are indirectly employed, through outputs of other industries, to generate the industry's own output. Following this procedure, we substitute for $\dot{u}^*$ in equation (1.5) its equivalent primary input requirements $\dot{u}^* - (i \otimes D^T)\tau^+$ where D is the current price market share matrix defined in Appendix 1. The *D* matrix transforms productivity rates from industry space into commodity space by assigning to commodities a market share weighted average of their producing industries' productivity growth rates. The resulting productivity growth rates, called here **interindustry** productivity growth rates τ^+ are

$$\tau^+ = C^*\dot{v}^* - B^{T*}[\dot{u}^* - (i \otimes D^T)\tau^+] - H^{T*}\dot{l}^* - J^{T*}\dot{k}^* \qquad (2.1)$$

$B^{T*} \beta \tau^{+}$

We can now show the relationship between the traditional τ and the interindustry measure τ^+ by simply substituting in (2.1) for τ from (1.5):

$$\tau^+ = \tau + B^T D^T \tau^+ \qquad (2.2)$$

$$\tau^+ = [I - B^T D^T]^{-1} \tau \qquad (2.3)$$

This equation finds τ^+ as a linear function of the traditional productivity measure τ, with the relationship depending on the intermediate input share matrix *B* and the market share matrix *D*. From this equation, it is clear that $|\tau^+| >> |\tau|$ if the productivity rates τ are all positive or all negative.

It should be noted that this analysis is consistent with an open economy framework, where commodities are imported for industries' intermediate use. By treating competitive imports as if they were domestically produced, the growth rate of such commodities are transformed into the growth rates of their primary domestic resource equivalents. These are precisely the resources which would have been required by the business sector to domestically produce the imported commodities[2]. Such primary resource

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[2] This involves the empirical approximation that the domestic technology equally applies to domestic and imported commodities. The alternative is to consider imports and other leakages as primary inputs used by the business sector.

equivalents can thus be treated as a claim against final demand uses. This is precisely what is accomplished when we net out imported commodities from the vector of final demand. Considering imports as intermediate inputs could lead to some overstatement of the productivity of the domestic primary inputs. This potential bias is also present in the aggregate business sector productivity estimate defined on real value added.

The interindustry productivity index computes the productivity of all production activities which a fully vertically integrated industry would carry. The vertically integrated industry has the same output bundle as the traditionally defined industry but a different set of inputs. The latter comprises direct and indirect use of primary inputs. This introduces an important qualitative difference between the traditional multifactor productivity index and the interindustry index. The traditional view looks at the productivity of an industry taken in isolation from the rest of the economy. The interindustry index looks at the productivity of the vertically integrated industry as a component of the business sector as a whole. Indeed, it computes the productivity gains made by that industry on the use of the primary resources which are the sole resources available to the business sector as a whole.

3 - Productivity Gains in Producing Final Demand Commodities

This Section introduces a measure for the productivity gains realized in the production of final commodities. In a rectangular framework, the input-output impact matrix (see Appendix 3) can be used to form a relationship between the quantity of each final demand commodity (noted here as components of the vector e) and the amount of direct and indirect primary inputs required by every industry to produce them. Using this relationship, we can express the proportional gain in the output of final commodities, ρ, as follows:

$$\rho = \dot{e} - \hat{p}^{-1}\hat{e}^{-1}L_t^{T*}(I\otimes\hat{w})\dot{l}^* - \hat{p}^{-1}\hat{e}^{-1}K_t^{T*}(I\otimes\hat{r})\dot{k}^* \qquad (3.1)$$

In this equation L and K are matrices of labour and capital requirements arranged by commodity (columns) and input type (rows), and l^* and k^* are their respective vector-forms (see Appendix 2).

4 - A Dynamic Index Number Formula of Productivity Growth

This section develops an alternative specification of technical change which incorporates capital both intertemporally and intersectorally into the framework of Section 2. To clarify the issue, we note that capital commodities used in any period are produced by means of capital services and other inputs in previous periods. Capital goods, like intermediate inputs, can therefore be seen as outputs of the productive system as well as inputs to this system. Similarly to the intermediate inputs, capital goods can consequently be considered as endogenous inputs rather than as exogenously given inputs. Their availability to the production process rests on their production through time with the primary inputs given in each period to the economy. The primary inputs included in the cumulated capital stock available for industries' use at the beginning of each period, therefore, could be seen as the cumulated direct and indirect primary input requirements used to produce the additions to the capital stock (investment) in previous periods after proper depreciation. Clearly, with increasing productivity, the rate of growth of the primary inputs used to produce investment goods should be smaller than the rate of growth of the investment goods produced.

To make a brief parallel, Rymes (1972, p.81) argued for, and Rymes and Cas (1985, 1990) estimated, what they termed "the Harrod-Robinson-Reed" formulation of productivity growth which endeavours to account for capital as a produced (and reproducible) productive factor in an interdependent system. Although their extensive work on this subject cannot be adequately discussed here, we will present their formulation for comparison and interpretation. Placing the productivity growth equation derived and used in these studies (Rymes, 1972, p. 85, equation III.5; Rymes and Cas, 1985, equation IV.7) in a rectangular input-output framework and treating depreciation implicitly, it becomes

$$\begin{aligned}\tau^* = C^* \dot{v}^* - B^{T*} [\dot{u}^* - (i \otimes D^T)\, \tau^*] \\ - H^{T*} \dot{l}^* - J^{T*} [\dot{k}^* - (i \otimes D^T)\, \tau^*]\end{aligned} \tag{4.1}$$

The dependence of this measure on interindustry relationships is best illustrated when we substitute for the traditional measure τ given by (1.5) into (4.1) to obtain

$$\tau^* = [I - B^T D^T - J^T D^T]^{-1}\, \tau \tag{4.2}$$

Rymes argues that in order to take account of "the fact that 'the capital input' is being produced with ever-increasing efficiency in an economy where technical progress is occurring" (p.81) the "capital input measured in neoclassical terms must be 'reduced' by technical change" (p.84). In performing this operation, however, Rymes reduces the growth rates of industries' capital stock by the productivity growth rates of the capital goods producing industries obtained during the **period of consumption** of capital services, not those obtained during the **production period** of capital goods.

Returning to our own formulation, the primary input requirements to produce final demand output, investment goods or consumption goods, may themselves be measured final good units. In order to do so, however, it must be taken into account that productivity gains change the amount of final goods which can be obtained from a given quantity of primary inputs through time as shown in the previous section. The transformation of the measure of primary inputs from their "natural" units such as hours for labour into final good units must therefore refer to a fixed base year period. Labour inputs can be measured in hours or equivalently in final goods of a base year using the base year wage rates for each type of labour. Capital, as a primary input, does not have a natural unit. Capital can only be measured in base year units of final goods.

In the light of the dynamic character of capital accumulation, isolating all of the productivity contributions entering into a given production process, delivered through the services of its capital stock, is equivalent to formulating a measure of capital stock net of all (intertemporal and interindustry) productivity gains.

As in traditional analysis, the measure of capital services can then be taken to be proportional to the stock of "productivity deflated" capital stock. Let us denote the matrix of "deflated" capital stock, arranged by commodity type and using industry, by $\mathbf{K}_*$ and its vector-form (see Appendix 2) by κ^*. When these magnitudes are known, the interindustry measure of productivity introduced in Section 2 can be extended to incorporate capital services by replacing its matrix of commodity capital stock with $\mathbf{K}$. We will then have

$$\mathrm{T}^* = C^* \dot{v}^* - B^{T*} [\dot{u}^* - (i \otimes D^T)\, \mathrm{T}^*] - H^{T*} \dot{l}^* - J^{T*} \dot{\kappa}^* \tag{4.3}$$

where again, it can be shown that, whenever the productivity rates τ^* are all positive or all negative, $|\mathrm{T}^*| >> |\tau^*|$, since the vector of industry growth rates will now include technical progress channelled through its use of capital commodities as well as through intermediate commodity inputs. It must be emphasized here that the measure of productivity growth given by $\mathbf{T}^*$ is a dynamic equation and, the authors' knowledge, is the first dynamic index number presented in the literature.

The dynamically dependent matrix of capital stock, $\mathbf{K}$ can,in principle, be directly constructed from information on investment expenditures by commodity, depreciation, the relevant price data and multifactor productivity growth rates of producing industries for the periods in which capital commodities were produced. Solving this dynamic, simultaneous equation system is, however, complex. The backward substitution of primary inputs for intermediate inputs across industries as shown in Appendix 4 is already quite involved. Nevertheless, we have seen that it was not actually necessary to operate the backward substitution to derive the required interindustry productivity growth index. Building on the same intuition, we shall instead derive the dynamic index by deflating the rate of growth of the conventionally measured capital stock by just what is required to compute the primary input growth necessary to sustain the growth of the capital stock.

We start first by noting that we do have a benchmark value of the productivity deflated capital stock for some arbitrarily chosen base year. Indeed, the nominal value of the productivity deflated capital stock is just the same as the nominal value of the conventionally measured capital stock. In the base year, by setting the price indices of these alternative measures of the capital stock equal to one, we therefore have their base year constant price measures. Taking this measure is equivalent to defining the productivity deflated capital stock in final good units of the base year.

In order to compute the impact of productivity gain on the growth of the capital stock, we must first make a basic stock-flow distinction. Capital is a stock and cannot just be deflated by the productivity growth rate of the capital good producing industries just as done for the flow of intermediate inputs. The growth of the capital stock is given by

$$\dot{K} = i/K - \delta \tag{4.4}$$

where i is the investment rate at time t and δ is the rate of depreciation. Using the final output measure of technical progress presented in the previous section, we may compute the rate of growth of primary inputs necessary to sustain the rate of growth of investment:

progress presented in the previous section, we may compute the rate of growth of primary inputs necessary to sustain the rate of growth of investment:

$$\dot{i} = \phi + \rho \tag{4.5}$$

where ϕ is the Divisia index of primary input growth and ρ the rate of technical progress made on final output. It may be noted that the estimate of ρ must initially be derived from the conventional measure of the capital stock. Next, we may compute the rate of change in investment which could be supported with the same rate of growth of primary inputs in the absence of technical progress:

$$\dot{I} = \phi \tag{4.6}$$

The investment level which is sustainable through time in both cases is thus given by

$$i = i_0 \, e^{\int_{t_0}^{t} (\phi + \rho)\, dt} \tag{4.7}$$

$$I = i_0 \, e^{\int_{t_0}^{t} \phi \, dt} \tag{4.8}$$

where the initial value of investment is the same in both (4.7) and (4.8) given the "base year" units chosen. The path variation in the growth of the capital stock is therefore given by

$$\begin{aligned} \delta K &= \dot{K} - \dot{\mathrm{K}} = i/K - I/\mathrm{K} \\ &= \frac{i}{K}\left[1 - \frac{K}{\mathrm{K}} \, e^{-\int_{t_0}^{t} \rho(t)\, dt}\right] \end{aligned} \tag{4.9}$$

conventional measure of the capital stock. This initial value can nevertheless be used to derive a second estimate of the final output productivity growth rate (which obviously will be larger) and the latter be used to re-estimate the productivity deflated capital stock K. This iterative procedure should converge rapidly toward final estimates of the productivity deflated capital stock[3].

It should also be clear from (4.9) that the path variation between the alternative measures of the capital stock depends on all productivity gains realised on the production of investment goods from the initial time t_0 up to time t. The two paths traced by the conventional and the dynamically deflated capital stock differ precisely in that the first one measures the capital stock produced in a world where there is technical progress and the second one measures the capital stock in the absence of technical progress, both measures being derived from the same amount of primary input uses in the production of capital goods. The difference in the two paths, therefore, measures solely the contribution of technical progress to capital accumulation as conventionally measured.

To emphasize again, the fact that capital goods are produced goods (transformed primary inputs) do not make them intermediate inputs. What distinguishes capital goods from intermediate inputs is rather that capital goods are commodities which are saved rather than consumed in order that their use in the production process in future periods will yield enlarged consumption possibilities. The capital stock may therefore be seen as an accumulation of waiting sacrifices in base year final good units rather than as an accumulation of current year commodities. In this perspective, the capital stock, in the form of waiting sacrifices, may be seen as an input supplied by households on the same footing as labour, the latter being a sacrifice of leisure time. On the other hand, capital, as an actual stock of commodities, may be seen as an output of the production process rather than as an input and measured in units equivalent to consumption units of the current year.

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[3]Convergence is insured by noting that deflated gross investments I are bounded from below by zero or, equivalently, the rate of growth of the productivity deflated capital stock is bounded from below by the depreciation rate δ (this lower bound is not violated even assuming ρ tends to infinity as can be seen from (4.9) using (4.4) in solving for $\dot{K}$). This set a lower bound to ϕ which is the average weighted rate of growth of the deflated capital stock and exogenously given rate of labour input growth with exogenously given capital and labour cost shares. Using (4.5) in which investment is also exogenous (therefore bounded) set an upper bound to ρ.

Since the nominal value of the capital stock is identical in both of the alternative formulations just described, this implies that their prices correspondingly differ. The price of the conventional measure of the capital stock is excluding all productivity gains. Indeed its price is an output deflator. In the simple one-good-one-sector growth model, for instance, the relative price of capital to consumption goods is identically equal to one at all time periods. This points out to the fact that the conventionally measured capital is in some sort of "efficiency units": technical progress is incorporated in the measure of the volume of the capital stock.

The price of the productivity deflated capital stock, on the other hand, is fully incorporating the effect of productivity growth. The real price of capital goods measured in units from which productivity gains have been removed should, therefore, increase through time as a consequence of technical progress as does the real price of labour hours, that is the price of labour in terms of consumption goods.

The important conclusion that comes out of the above derivation is, therefore, that the conventional measure of productivity growth defines labour in "natural" hour units which exclude productivity gains and capital in output units which include the productivity gains. This differential treatment of capital and labour certainly raises an issue for the interpretation of the conventional measure of the "residual" as technical progress. This residual excludes productivity gains implicitly included in the growth of the volume of the capital stock and overstates the contribution of past savings on output growth. The alternative measure proposed here is free of such an ambiguity.

Concluding Remarks

This paper presented an extension of the traditional concepts and formulation of technical change to the multi-input and multi-output case within the framework of a dynamic rectangular input-output system.

The dynamic formulation of productivity growth appropriately accounts for productivity gains transmitted from producers of capital goods to users of capital services across time and industries. The information content and interpretation of this measure differ somewhat from the interindustry measure discussed above, since capital goods now carry productivity gains across time periods. Clearly, past savings alone (in terms of a time homogeneous form of waiting sacrifice) cannot explain the large production capacity of modern economies. Through technical progress, the efficiency of waiting has been increasing through time and so the price of waiting units in terms of real consumption. The dynamic formula of productivity growth possesses the fundamental advantage over all other measures in that it conveys a clear interpretation of the productivity growth "residual". The latter incorporates unambiguously all productivity gains. The other measures attributes part of the growth in output resulting from technical progress to the growth of produced inputs. The dynamic formulation can be seen as a first extension of index number theory into the realm of dynamic economics.

APPENDIX 1.

Notations and basic relationships

Throughout this paper, upper case bold letters will be used to denote matrices and lower case bold letters will denote vectors. The symbol ^ implies diagonalization of a vector while * ,defined in Appendix 2, is used to row-diagonalize a matrix and obtain its vector-form, respectively. A unit vector of appropriate length will always be denoted by i. We also define the following:

$V = [v_{ic}]$ the make (output) matrix of industries ($i = 1,2,...n$) for all commodities ($c = 1, 2, ...C$) in physical units;

$U = [u_{ci}]$ the use (intermediate commodity input) matrix for all commodities by industries in physical units;

$F = [f_{fc}]$ the matrix of final demand for ($f=1,2,...F$) final demand categories for all commodities in physical units;

$p = [p_c]$ the vector of current period unit prices of produced commodities;

$K = [k_{ci}]$ the matrix of type c capital service inputs used by the ith industries;

$L = [l_{Li}]$ the matrix of L types of labour service inputs used by the ith industry;

$r = [r_c]$ the vector of imputed unit rental prices for capital services of type c;

$w = [w_L]$ the vector of compensation paid per unit of labour services for labour of type L;

$g = V\,p$ the vector of current dollar industry gross output;

$C = \hat{g}^{-1}V\hat{p}$ the matrix of current price commodity shares in the value of gross output of each industry;

$\hat{p}q = (V\hat{p})^{T}i$ the vector of current price commodity outputs;

$D = V\hat{q}-1$ the matrix of current (and constant) price market shares;

$\dot{g} = C\dot{v}^{*}$ the vector of the Divisia indices of industry gross output growth rates;

$B = \hat{p}U\hat{g}^{-1}$ the matrix of current dollar intermediate commodity shares in the value of gross output of the using industry;

$J = \hat{r}K\hat{q}^{-1}$ the matrix of current dollar capital services expenditure shares in the value of gross output of the using industry;

$H = \hat{w}L\hat{q}^{-1}$ the matrix of current dollar labour services expenditure shares in the value of gross output of the using industry;

$L_t = [L_{Lc}]$ the matrix of L types of labour input required for the production of c commodities;

$K_t = [K_{cc}]$ the matrix of c types of capital input required for the production of c commodities;

$l = Li$ the vector of inputs of labour services by type;

$k = Ki$ the vector of inputs of capital services by type;

$e = [e_c]$ the vector of final demand commodity in constant price, net of import and final demand supply (non business supply);

$l_t = L_t i$ the vector of inputs of labour services by type;

$k_t = K_t i$ the vector of inputs of capital services by type;

$\sigma_L = w^T \hat{l}_t / p^T e$ the vector of shares of labour types in the value of aggregate net output;

$\sigma_K = r^T \hat{k}_t / p^T e$ the vector of shares of types of capital services in the value of aggregate net output;

τ the vector of neoclassical productivity growth rates by industry;

τ^+ the vector of interindustry productivity growth rates by industry;

τ^* the Rymes-Cas vector of productivity growth rates by industry;

T^* the vector of dynamic and interindustry productivity growth rates by industry;

ρ the vector of productivity growth rates by final demand commodity;

i the investment rate at time t;

I	the investment rate at time t excluding technical progress;
δ	the depreciation rate of the capital stock;
ϕ	the rate of growth of primary input requirements to sustain the growth of investment.

APPENDIX 2.

The Traditional Formulation of Productivity Growth

In order to make effective use of matrix algebra, we need to define two operations which will be used extensively in the following Appendices. For any (m x n) matrix Z we define its row diagonal Z^* as follows

$$Z^* = \begin{bmatrix} z_{11}z_{12}\ldots z_{1n} & 0\ldots\ldots 0 & \ldots\ldots & 0\ldots\ldots 0 \\ 0\ldots\ldots 0 & z_{21}z_{22}\ldots z_{2n} & \ldots\ldots & 0\ldots\ldots 0 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ 0\ldots\ldots 0 & 0\ldots\ldots 0 & \ldots\ldots & z_{m1}z_{m2}\ldots z_{mn} \end{bmatrix}$$

The relationship between Z and Z^* is then:

$$Z = Z^* \, (i \otimes I) \tag{A2.1}$$

where $\otimes$ is the Kronecker product operator. For the same matrix, we define its vector-form z^* as:

$$z^* = \begin{bmatrix} z_{11} \\ z_{12} \\ . \\ z_{1n} \\ z_{21} \\ z_{22} \\ . \\ z_{2n} \\ . \\ . \\ . \\ z_{n1} \\ z_{n2} \\ . \\ z_{mn} \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ . \\ . \\ . \\ . \\ z_m \end{bmatrix} \quad \text{and} \quad Z^* \dot{z}^* = \begin{bmatrix} z_1^T \dot{z}_1^* \\ z_2^T \dot{z}_2^* \\ . \\ . \\ . \\ . \\ z_m^T \dot{z}_m^* \end{bmatrix}$$

Using the above definitions of Z^* and z^*, we can now represent the time derivative of the product of Z and a vector of variables x in terms of the growth rates of Z and x in the following alternative way

$$\begin{aligned} \frac{d(Zx)}{dt} &= \frac{dZ}{dt}\,x + Z\,\frac{dx}{dt} \\ &= \frac{d[Z^*(i \otimes I)]}{dt}\,x + Z\,\hat{x}\,\dot{x} \\ &= Z^*(I \otimes \hat{x})\,\dot{z}^* + Z\,\hat{x}\,\dot{x} \end{aligned} \tag{A2.2}$$

where the elements of the diagonal $Z^*(I \otimes \hat{x})$ are $[z_{ij}x_j]$ and the elements of $\dot{z}^*$ are $[(dz_{ij}/dt)/z_{ij}]$

The traditional productivity equation (1.5) is obtained from the budget identity as follows. The identity is first differentiated with respect to time:

$$\begin{aligned} [dV/dt]p + V[dp/dt] = {} & [dU^T/dt]p + U^T[dp/dt] \\ & + [dL^T/dt]w + L^T[dw/dt] \\ & + [dK^T/dt]r + K^T[dr/dt] \end{aligned} \tag{A2.3}$$

Rearranging the quantity-derivative terms on the left-hand side and price-derivative terms on the right, it becomes

$$\begin{aligned} & [\frac{dV}{dt}]p - [\frac{dU^T}{dt}]p - [\frac{dL^T}{dt}]w - [\frac{dK^T}{dt}]r = \\ & -\left\{V[\frac{dp}{dt}] - U^T[\frac{dp}{dt}] - L^T[\frac{dw}{dt}] - K^T[\frac{dr}{dt}]\right\} \end{aligned} \tag{A2.4}$$

Premultiplication of both sides of this identity by $\hat{g}^{-1}$ and using the alternative expressions for the time derivatives of the product of matrices and vectors, we find

$$\begin{aligned} & \hat{g}^{-1}V^*(I\otimes\hat{p})\,\dot{v}^* - \hat{g}^{-1}U^{T*}(I\otimes\hat{p})\,\dot{u}^* \\ & - \hat{g}^{-1}L^{T*}(I\otimes\hat{w})\,\dot{l}^* - \hat{g}^{-1}K^{T*}(I\otimes\hat{r})\,\dot{k}^* = \\ & -\{\hat{g}^{-1}V\hat{p}\dot{p} - \hat{g}^{-1}U^T\hat{p}\dot{p} - \hat{g}^{-1}L^T\hat{w}\dot{w} - \hat{g}^{-1}K^T\hat{r}\dot{r}\} \end{aligned} \tag{A2.5}$$

However, the row-diagonal of the output matrix V^* premultiplying the term in brackets is simply the row-diagonal of this matrix in current prices. Premultiplying this matrix by $\hat{g}^{-1}$ is, by definition, the row-diagonal of the product share matrix C^*. Other share matrices are found similarly so that

$$\begin{aligned} \tau = {} & C^*\dot{v}^* - B^{T*}\dot{u}^* - H^{T*}\dot{l}^* - J^{T*}\dot{k}^* \\ & -(C\dot{p} - B^T\dot{p} - H^T\dot{w} - J^T\dot{r}) \end{aligned} \qquad \text{(A2.6)}$$

where $C^*\dot{v}^*$ is the vector of the Divisia indices of the growth rate of aggregate output (in continuous time) $\dot{g}$, and $C\dot{p}$ is the vector of the Divisia index of the growth rate of industry prices.

APPENDIX 3.

End Products Productivity Indices

We begin from the standard input-output relationship

$$g - D\,B\,g = D\,\hat{p}\,e \tag{A3.1}$$

where e is the vector of final demand commodities. Rearranging this equation, we have another well-known relationship:

$$g = (I - D\,B)^{-1} D\,\hat{p}\,e \tag{A3.2}$$

which has the vector of (required) industry gross output (in current prices) as a function of the vector of final demand (net of imports and other non domestic business supply). We now introduce a new matrix, G, which expresses the vector of output requirements for each industry associated with the value of each final commodity. This is accomplished by diagonalizing e in equation (A3.2):

$$G = (I - D\,B)^{-1} D\,\hat{p}\hat{e} \tag{A3.3}$$

so that

$$G\,i = g$$

In G (dimensions N X C) each column gives the value of industry outputs associated with the corresponding column in the final demand matrix $\hat{p}\hat{e}$. To make use of this relationship, we now define L_t and K_t to be the input requirements matrices of labour and capital associated with each commodity (columns) by type of labour and capital (rows). With definitions of H and J as primary input-output value shares in mind, the input requirements matrices must satisfy

$$\hat{w}\,L_t = H\,G = H(I - D\,B)^{-1} D\,\hat{p}\,\hat{e} \tag{A3.4}$$

$$\hat{r}\,K_t = J\,G = J(I - D\,B)^{-1} D\,\hat{p}\,\hat{e} \tag{A3.5}$$

Premultiplying both equations by i^T, and summing them, we can express the economy's direct and indirect input requirements for the values of elements of e by

$$\begin{aligned} w^T L_t + r^T K_t &= (i^T H + i^T J)(I-DB)^{-1} D\hat{p}\hat{e} \\ &= (i^T H + i^T J)\ G \end{aligned} \tag{A3.6}$$

From the additivity property of the market share matrices we have

$$i^T B = i^T D B$$

and by the definitions of share matrices

$$i^T H + i^T J = i^T - i^T B$$

with unit vectors of appropriate length. We can now write (A3.6) as

$$\begin{aligned} w^T L_t + r^T K_t &= (i^T - i^T DB)(I-DB)^{-1} D\hat{p}\hat{e} \\ &= i^T D\,\hat{p}\,\hat{e} \\ &= p^T \hat{e} \end{aligned} \tag{A3.7}$$

which states an intuitively obvious result, namely, the equality of the values of net outputs and inputs of the economy. Totally differentiating this equation with respect to time and using the expressions (see Appendix 2) derived for the time derivative of products of matrices and vectors, we have

$$\begin{aligned} \hat{p}\,\hat{e}(\dot{e} + \dot{p}) &= L_t^{T*}(I \otimes \hat{w})\,\dot{l}_t^* + L_t^T\,\hat{w}\,\dot{w} \\ &\quad + K_t^{T*}(I \otimes \hat{r})\,\dot{k}_t^* + K_t^T\,\hat{r}\,\dot{r} \end{aligned} \tag{A3.8}$$

Premultiplying both sides by $\hat{p}^{-1}\hat{e}^{-1}$and grouping quantity terms on the left-hand side we obtain the vector of commodity productivity rates ρ:

$$\rho = \dot{e} - \hat{p}^{-1}\hat{e}^{-1}L_t^{T*}(I \otimes \hat{w})\dot{l}_t^* - \hat{p}^{-1}\hat{e}^{-1}K_t^{T*}(I \otimes \hat{r})\dot{k}_t^*$$
$$= -[\dot{p} - \hat{p}^{-1}\hat{e}^{-1}L_t^T\,\hat{w}\,\dot{w} - \hat{p}^{-1}\hat{e}^{-1}K_t^T\,\hat{r}\,\dot{r}] \qquad \text{(A3.9)}$$

The left-hand side of (A3.9) comprises a vector of the growth rates of final demand commodities less the growth rates of direct and indirect input requirements (by type) each premultiplied by the current dollar share of each type in total expenditure on labour and capital respectively. The right-hand side can be interpreted analogously. For instance, $\hat{p}^{-1}\hat{e}^{-1}L_t^T\hat{w}\dot{w}$ results in a matrix of expenditure shares (by commodity and labour type) premultiplying the vector of growth rates of wages by labour type. In terms of quantities, the left-hand side is a Divisia index of the efficiency with which direct and indirect inputs have been utilized in the production of the vector of final demand commodities **e**.

APPENDIX 4.

Substituting indirect primary inputs for intermediate inputs

It was suggested in Section 2 that industries' direct use of intermediate inputs can be transformed into their indirect use of the economy's primary inputs by deflating their growth rates by the productivity growth rates of their producing industries. This Appendix deals with the equivalence of these alternative methods. Restating the fundamental input-output relationships (A3.4) and (A3.5):

$$\hat{w}\, L_t = H\, G = H(I - D\, B)^{-1} D\, \hat{p}\, \hat{e} \qquad \text{(A3.4)}$$

$$\hat{r}\, K_t = J\, G = J(I - D\, B)^{-1} D\, \hat{p}\, \hat{e} \qquad \text{(A3.5)}$$

we can express the value of the primary input requirements associated with g by replacing the value of final demand in the above equation by the value of gross output:

$$\hat{\omega}^T\, \Omega = \begin{bmatrix} H \\ J \end{bmatrix} (I - D\, B)^{-1} \hat{g} \qquad \text{(A4.1)}$$

where we have grouped primary inputs (K, L) by type and by industry into the same matrix Ω and their prices into the combined vector $\omega^T = [w, r]$.

Summing over input types in the last equation, we find the value of gross output by industry equal to its direct and indirect primary input costs:

where we used equation (A3.7) from Appendix 3. Since we have

$$g = \Omega^T\, \omega = \Omega_U^T\, \omega + \Omega_0^T\, \omega \qquad \text{(A4.3)}$$

$$\omega^T \Omega = i^T \begin{bmatrix} H \\ J \end{bmatrix} (I - D\,B)^{-1}\hat{g} \tag{A4.2}$$

$$= i^T \hat{g} = g^T$$

$$g = U^T p + [L^T, K^T] \begin{bmatrix} w \\ r \end{bmatrix} \tag{A4.4}$$

where Ω_0 and Ω_U are respectively the direct and indirect primary input requirements associated with g and because, by definition

$$\Omega_0^T \omega = [L^T, K^T] \begin{bmatrix} w \\ r \end{bmatrix}$$

then

$$\Omega_U^T \omega = U^T p \tag{A4.5}$$

Differentiating this equation with respect to time, we have

$$\Omega_U^{T*}(I \otimes \hat{\omega})\, \dot{z}_U^* + \Omega_U^T \hat{\omega}\, \dot{\omega} = U^{T*}(I \otimes \hat{p})\, \dot{u}^* + U^T \hat{p}\, \dot{p} \tag{A4.6}$$

where $\dot{z}_U^*$ is the vector-form (see Appendix 2) of the growth rate of indirect primary inputs by input type. Grouping quantity terms on the left-hand side and transforming the right-hand side of (A4.6) leads to

$$\hat{g}\, B^{T*}\, \dot{u}^* - \Omega_U^{T*}(I \otimes \hat{\omega})\, \dot{z}_U^* = \Omega_U^T \hat{\omega}\, \dot{\omega} - U^T \hat{p}\, \dot{p} \tag{A4.7}$$

Using the relationship in (2.3) above

$$\tau^+ = [I - B^T D^T]^{-1}\tau \qquad (2.3)$$

and substituting for τ from output and input price relationships given in (A2.6), we have

$$\tau^+ = -\,[I - B^T D^T]^{-1}\{(C - B^T)\dot{p} - [H^T, J^T]\ \dot{\omega}\} \qquad (A4.8)$$

where $[H^T, J^T]$ is the partitioned matrix of primary input share coefficients. Premultiplying by $\hat{g}$, this equation becomes

$$\hat{g}\,\tau^+ = -\,\hat{g}[I - B^T D^T]^{-1}(C - B^T)\dot{p} + \hat{g}[I - B^T D^T]^{-1}[H^T, J^T]\,\dot{\omega} \qquad (A4.9)$$

$$= -\,\hat{g}[I - B^T D^T]^{-1}(C - B^T)\dot{p} + \Omega^T\,\hat{\omega}\ \dot{\omega} \qquad (A4.10)$$

By similar substitution, we also find

$$\Omega_0^T\,\hat{\omega}\ \dot{\omega} = \hat{g}[I - B^T D^T]\tau^+ + \hat{g}(C - B^T)\dot{p} \qquad (A4.11)$$

which implies that

$$\Omega_U^T\,\hat{\omega}\ \dot{\omega} = \Omega^T\,\hat{\omega}\ \dot{\omega} - \Omega_0^T\,\hat{\omega}\ \dot{\omega} \qquad (A4.12)$$

$$= B^T D^T \tau^+ + \hat{g}\{[I - B^T D^T]^{-1}(C - B^T) - (C - B^T)\}\dot{p} \qquad (A4.13)$$

and

$$\hat{g}^{-1}\Omega_U^T \hat{\omega}\ \dot{\omega} - B^T\dot{p} = B^T D^T \tau^+ + \hat{g}\{[I - B^T D^T]^{-1}(C - B^T) - C\}\dot{p} \tag{A4.14}$$

Let us first examine this equation in a square input-output framework where $C = D = I$ and the second term on the right-hand side will vanish. In this case, we can use the right-hand side of (A4.7) to show that

$$B^{T*}\dot{u}^* - \hat{g}^{-1}\Omega_U^{T*}(I \otimes \hat{\omega})\dot{z}_U^* = B^T D^T \tau^+ \tag{A4.15}$$

or, alternatively

$$B^{T*}\{\dot{u}^* - (I \otimes D^T)\tau^+\} - \hat{g}^{-1}\Omega_U^{T*}(I \otimes \hat{\omega})\dot{z}_U^* = 0 \tag{A4.16}$$

which shows the equivalence of recursive substitution of intermediate inputs with their respective input requirements (or the reduced form of the interindustry equations) and the alternative of deflating intermediate input growth rates with the productivity rates of their producing industries (or the structural form of the interindustry equations). To demonstrate their equivalence in a rectangular framework, we need to have

$$\{[I - B^T D^T]^{-1}(C - B^T) - C\}\dot{p} = 0 \tag{A4.17}$$

which requires that

$$B^T(I - D^T C)\dot{p} = 0 \tag{A4.18}$$

However, this equation will only hold if variations of commodity output prices are uniform for every industry and equal to the variation in the industry's aggregate price index, i.e.,

$$\dot{p} = D^T C \dot{p} \tag{A4.19}$$

This relationship between commodity prices and industry price indices is generally not satisfied in a rectangular framework where commodity price changes are unequal for many reasons, including differential impacts of technical change. It is satisfied, however, under our assumption about the separability of inputs and outputs. Since each commodity output cannot be identified with the factors used by the industry for its production (a distinct production technology for each commodity), commodity outputs are assumed to be separable from industry inputs.

An implication of this assumption is that commodity outputs have nearly identical isoquants and are subject to uniform price changes, including those caused by productivity growth. Given this relationship (A4.16), the two procedures are equivalent so that we may deflate input growth rates by their producing industries' productivity growth rates rather than the more cumbersome procedure of recursive substitution.

In the absence of this assumption, the two formulations differ slightly as they rest on a different commodity technology assumption. The structural form assumes that all commodities produced by an industry experience the same rate of productivity growth. The reduced form, obtained by substitution of primary inputs for intermediate inputs, assumes that each commodity produced by an industry has the same industry current price input-output coefficients. When the rate of growth of the prices of these commodities differ, this assumption amounts to attribute a distinct rate of productivity gain to each commodity. When the rates of growth of their prices are identical, it amounts to assume that their rate of productivity gain is the same as in the structural form.

Of course, if commodity technologies were known, the accounting framework would again be a square commodity by commodity framework and the results would hold equally as in the case of the square industry by industry accounting framework. In the absence of complete information on commodity technology, some additional assumption among alternative choices has to be made.

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