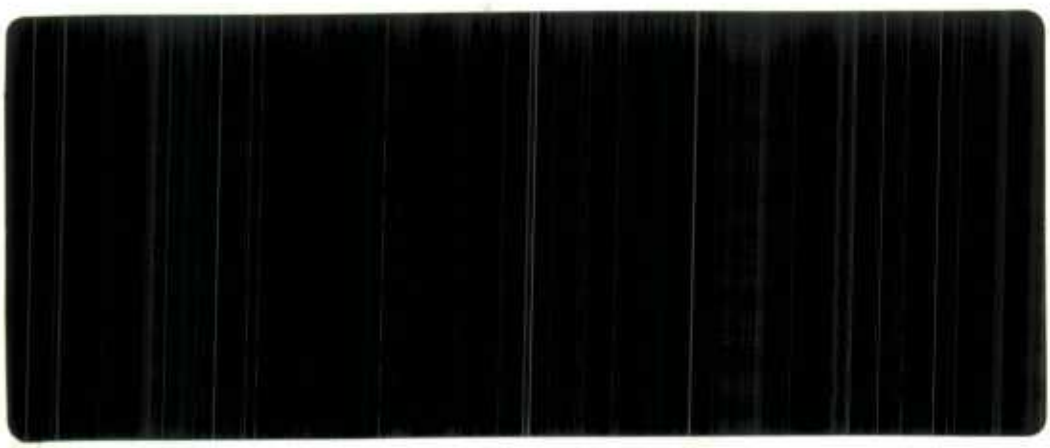




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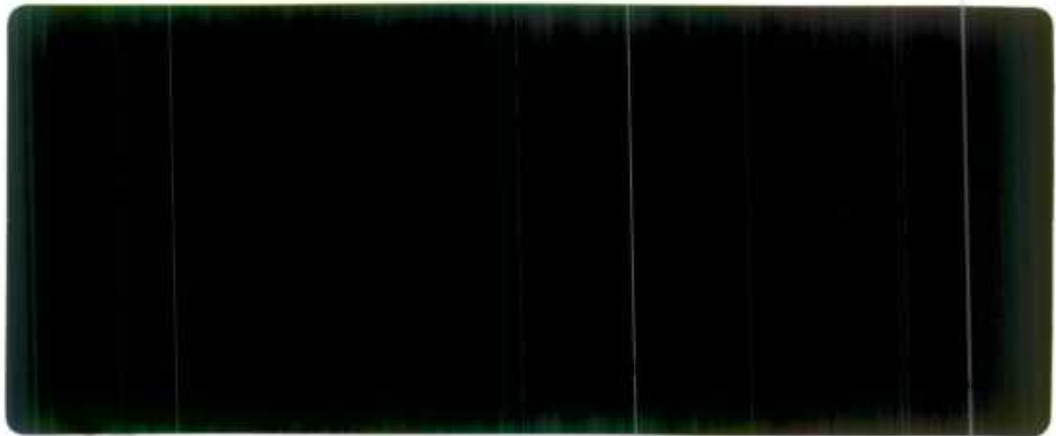


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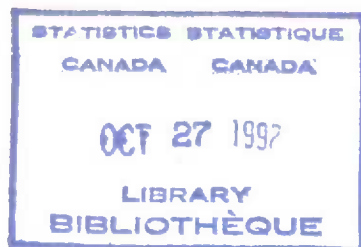
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Estimating Final Demand Expenditure at Factor Cost
and Net of Tax Indices
in the Canadian Input-Output Tables

By

René Durand
Ronald Rioux

3



Presented at an international Round Table
on Taxes and the CPI
March 3, 1987

1 - Introduction

This article presents the method used to compute final demand expenditure at factor cost in Canadian Input-Output Tables with the associated derivations of price indices net of all indirect taxes and subsidies. Indirect taxes are comprised of Commodity Indirect Taxes and Other (mostly property) Indirect Taxes in the Canadian System of National Accounts. Subsidies are considered as negative indirect taxes.

The price indices have come to be called *net price indices* (NPI) in the literature. They can also be seen as price indices at factor cost applied to final demand commodities as indirect taxes and subsidies account fully for the difference between market prices and factor cost in National Accounts. For analytical purposes, net price indices correspond to *before-tax* prices and must not be confused with *pre-tax* equilibrium prices. Prices, in the absence of taxes and subsidies, would establish somewhere between the market equilibrium prices and the before-tax prices depending on the distribution of the tax burden between producers and consumers.

Net price indices exclude all indirect taxes that are raised at both the intermediate (production) and final (distribution) demand stages. Taxes raised on intermediate inputs are reallocated to final demand commodities with an input-output model. The Canadian Input-Output Accounts usually present transactions between economic units at either *producers' prices* or *purchasers' prices*. Purchasers' prices are inclusive of all taxes and (negative) subsidies and other margins such as retail and wholesale margins which account for the difference in valuation between purchasers' and producers' prices. Purchasers' prices correspond in fact to market prices. Producers' prices are the prices received by producers from selling their commodities on the market. However, these prices are inclusive of the taxes and subsidies paid by the producers on the purchase of their inputs. Net price indices are computed by reallocating these taxes and subsidies to final demand expenditure in the Accounts. Final demand expenditure at market prices are transformed into factor cost prices by subtracting taxes and adding subsidies. Final demand expenditure at factor cost include (1) the producers' costs net of taxes paid and subsidies received on inputs and (2) the real margins such as the wholesale and retail margins. Net price indices could also be computed from producers' prices by subtracting input taxes net of subsidies from final demand expenditure at producers' prices. In the latter case, however, real margins would appear as distinct commodities.

The article presents the specification of the input-output transformations applied to the input-output tables which yield the appropriate valuation base for the computation of net price indices from market price indices. The following section explains, in simple terms, the basic principles which lie behind the computation of the NPI. This is translated in the Appendix into a mathematical language.

2 - The Method of Computation

The input-output model allows the accounting decomposition of the prices of final demand commodities into their various cost components. Among these cost components, there are the indirect taxes levied at both the final demand level (of which a large component is made of the

provincial sales tax) and the taxes levied on the purchases of inputs at the production stage which include, as a major component, the federal sales tax. The accounting input-output model does not rest on any behavioral assumption nor does it contain any behavioral equation which could permit to assess the impact of tax changes on price behaviour. The model simply accounts, ex-post, for the various costs making up the prices of commodities. The model cannot assess, for instance if, after a tax increase, the producers will lower their profit margin, thereby sharing the tax burden with consumers, in order to sustain market competition.

All taxes levied on intermediate inputs at the production stage are allocated to the final demand commodities along the following lines. First, using the Input-Output model, the value of the inputs purchased for the production of each final demand commodity in the various industries directly and indirectly involved is computed. Taxes paid (and subsidies received) on these purchases by type of tax are then compiled and associated with the corresponding final demand expenditure of that commodity.

Secondly, the value of final demand expenditure net of taxes and subsidies, levied on both the inputs at the intermediate production stage and on final demand sales, are compiled and compared to the value of expenditure at market prices. An overall effective tax rate is calculated by dividing the total taxes and subsidies on each final demand expenditure by the tax free expenditure. The market prices were then equated to one plus the effective tax rate times the before-tax prices, i.e.:

$$\text{Market Price} = (1 + \text{effective tax rate}) \text{ Net Price}$$

Market price *indices* are obtained by dividing market prices in any year by the value of the market prices in some arbitrarily selected base year. Dividing both sides of the previous equation by the base year equation gives the relationship between market and net price indices:

$$\frac{\text{Market Price}}{\text{index}} = \frac{(1 + \text{current year rate})}{(1 + \text{base year rate})} \frac{\text{Net Price}}{\text{index}}$$

It should be noted here that, from the last equation, both the market price indices and the net price indices are set equal to one in the base year. It should also be noted that the relationship between these indices depends on both the current year and the base year effective tax rates. Hence, in the years following the base year, the net price indices will be lower than the market price indices only if the effective tax rate in these years is higher than in the base year. Otherwise, the net price indices will be higher than the market price indices. Therefore, the comparison of the net price indices to the market price indices in any given year is indicative of how effective tax rates have changed since the base year.

APPENDIX

Mathematical Derivations of the NPI

We start the derivations with a few notational definitions¹. Let us first denote the current price rectangular (industry by commodity) Make matrix of the input-output tables by V . Then the vector of industries' gross output g and the vector of commodity output q will respectively be given by:

$$g = V i \quad (1)$$

$$q = V^T i \quad (2)$$

where i is taken in each case to be a summation vector of appropriate dimension and where the superscript T is used to indicate the transpose of a matrix. If we use the symbol " $\hat{}$ " to denote a diagonal matrix, then the market share matrix D which allocates the commodity demands to the industries will be defined by:

$$D = V \hat{q}^{-1} \quad (3)$$

Let respectively U and Y be the intermediate commodity by industry and primary input by industry use matrices. We may define the corresponding technical input coefficient matrices by:

$$B = U \hat{g}^{-1} \quad (4)$$

$$H = Y \hat{g}^{-1}$$

Given these notations and definitions, the basic output determination equation of the Canadian interindustry model is:

¹ We use and extend here the notation of the Open Output Determination Model as described in *The Input-Output Structure of the Canadian Economy, 1961-1981*, (Statistics Canada), cat. no.: 15-510, pp. 73-76.

$$g = \{[I - D(I - \hat{\mu} - \hat{\alpha} - \hat{\beta})B]^{-1} D(I - \hat{\mu} - \hat{\alpha} - \hat{\beta})\} e \quad (5)$$

where e is some final demand vector at producers' prices² and where the diagonals of μ , α and β contain the leakage parameters respectively for imports, government output of goods and services (included as a negative entry in final sales) and inventory withdrawals. According to (4), the vector of primary inputs (summation of the columns of Y over industries) is

$$y = H g \quad (6)$$

The reduced form for equations (5) and (6) may then be written as

$$y = J e \quad (7)$$

where the impact matrix J is the product of H by the expression within the braces in equation (5). The vector y includes indirect taxes paid at the intermediate production stages, labour income and other primary inputs. Since we are interested only in taxes let us define P as a matrix containing the rows of the identity matrix corresponding to the rows of the tax components of H . Indirect taxes on intermediate inputs will then be given by:

$$t_{p*} = P J e \quad (8)$$

The vector t_{p*} contains as many elements as there are types p of indirect taxes. The next step is to compute the appropriate final demand vector for consumer goods. This is done by summing over the forty categories of consumer expenditures in order to get the vector e_c of consumer expenditures by commodity at producers prices:

$$e_c = E_c i \quad (9)$$

where E_c is the matrix of consumer expenditures by commodity and category (excluding any negative leakage), a sub-matrix of the final demand sales matrix E . Intermediate level indirect taxes t_{p*} have to be computed separately for each component of e_c . This is accomplished by writing:

² Again, the distinction between producers' prices and consumers' (market) prices lies in the various margins included in the latter and, in particular, the tax margins.

$$T_* = P J \hat{e}_c \quad (10)$$

where T_* is now a matrix whose columns give the taxes on intermediate inputs by type corresponding to each element of \hat{e}_c . In the Canadian Input-Output accounting framework import taxes are included in the producers' prices value of the intermediate commodities imported. To compute these, let M be the matrix of intermediate imports by commodity and by industry given as:

$$M_* = \hat{\mu} U \quad (11)$$

where $\hat{\mu}$ is the diagonal matrix of import coefficients. Import taxes on intermediate inputs T_m are then given by:

$$T_{m*} = \hat{\tau}_m (I + \hat{\tau}_m)^{-1} M_* \quad (12)$$

where $\hat{\tau}_m$ is the diagonal matrix of import tax rates which apply to the before-tax prices. Import duties by industry are obtained by summing them over commodities in the previous equation:

$$t_{m*}^T = \tau_m^T (I + \hat{\tau}_m)^{-1} \hat{\mu} B \hat{g} \quad (13)$$

From (13), it can be seen that the import average tax rates by industry (by dividing by industries' gross output) are given by:

$$h_{m*}^T = \tau_m^T (I + \hat{\tau}_m)^{-1} \hat{\mu} B \quad (14)$$

These rates are fixed for fixed commodity import tax rates and fixed B and import propensities. We may therefore extend the definition of the matrix H to include the import tax rates h_{m*} . Indeed, from (5), (13) and (14) we have:

$$t_{m*}^T i = h_{m*}^T [I - D(I - \hat{\mu} - \hat{\alpha} - \hat{\beta})B]^{-1} D(I - \hat{\mu} - \hat{\alpha} - \hat{\beta})e \quad (15)$$

Equation (15) gives total import duties (a scalar) associated with the final demand vector e . Replacing, as before, the vector e by the diagonal matrix \hat{e}_c will yield again the import duties associated with each separate commodity component of consumers expenditure. The import duties by final demand commodity is, therefore, a vector having the same dimension as the other

row vectors of T . We may thus consider, in what follows that the matrix T has been extended to include the row of import duties associated with consumers expenditure but levied at the intermediate stage level and H as the correspondingly extended matrix of coefficients. Summing T over tax types yields the vector of total taxes by commodity raised on intermediate transactions t (which differs from t_p in (8) above which expresses taxes by type rather than by commodity).

To these intermediate taxes, we still have to add the final demand sales taxes t_c . These are given by commodity and category of final demand consumption in a tax matrix T_c (or set of tax matrices by type of commodity tax which may be added)³. Adding over the categories of consumption, the final consumption taxes by commodity, t_c are given by:

$$t_c = T_c i + \hat{\tau}_m(I + \hat{\tau}_m)^{-1} \hat{\mu} e_c \quad (16)$$

where i is a summation vector and where we have incorporated the import taxes to t_c . Total taxes t are therefore given by:

$$t = t_p + t_c \quad (17)$$

Consumer sales at market prices, z_c , can be broken down into their tax free components ϵ and taxes t :

$$z_c = \epsilon + t \quad (18)$$

The computations leading to (18) can be repeated for each year on current price flows. If we now let a bar over a symbol denote current price flows and symbols without a bar denote constant price flows, then the overall commodity market prices \bar{p}_z are given by:

³ Some Other Indirect taxes are also part of consumers expenditure like property taxes of universities but these account for a very minor share of total indirect taxes. These taxes have been prorated over the commodity expenditure in each final demand category where they appear. There are no subsidies on consumers expenditure.

$$\begin{aligned}
p_z &= \hat{z}^{-1} \bar{z}_c \\
&= \hat{z}_c^{-1} \bar{\epsilon} + \hat{z}_c^{-1} \bar{t} \\
&= (\hat{z}_c^{-1} \hat{\epsilon})(\hat{\epsilon}^{-1} \bar{\epsilon}) + (\hat{z}_c^{-1} \hat{t})(\hat{t}^{-1} \bar{t}) \\
&= \hat{\gamma} \phi + (I - \hat{\gamma}) \theta
\end{aligned} \tag{19}$$

The all inclusive market prices are thus equal to net price indices ϕ plus the tax indices θ with their respective weights $\hat{\gamma}$ and $(I - \hat{\gamma})$. The weights are the constant price shares of the tax free expenditure and the tax expenditure. These shares, in the case of taxes, are computed by taking base year average tax rates τ_0 given by:

$$\tau_0 = \hat{\epsilon}_0^{-1} \bar{t}_0 \tag{20}$$

In the base year, we therefore have:

$$z_{c0} = (I + \hat{\tau}_0) \epsilon_0 \tag{21}$$

Indeed we must have:

$$\phi_t = \hat{z}_{ct}^{-1} \hat{\delta} \bar{\epsilon}_t \tag{22}$$

where δ is a vector of normalization factors such that all net price indices are equal to one in the base year, that is:

$$\phi_0 = \hat{z}_{c0}^{-1} \hat{\delta} \epsilon_0 = I \tag{23}$$

where I is the summation vector. Solving (23) for δ using (21) gives:

$$\hat{\delta} = I + \hat{\tau}_0 \quad (24)$$

Substituting back this last result into (22) gives:

$$\phi_t = \hat{z}_{ct}^{-1} (I + \hat{\tau}_0) \bar{\epsilon}_t \quad (25)$$

From (25), we may derive how to compute ϵ_t as:

$$\epsilon_t = \hat{\phi}_t^{-1} \bar{\epsilon}_t = (I + \hat{\tau}_0)^{-1} z_{ct} \quad (26)$$

It follows that the weights γ of the price equation (19) are given by:

$$\hat{\gamma} = \hat{z}_{ct}^{-1} \hat{\epsilon}_t = (I + \hat{\tau}_0)^{-1} \quad (27)$$

Substituting back this result into the price equation (19), one gets:

$$p_{zt} = (I + \hat{\tau}_0)^{-1} \phi_t + \hat{\tau}_0 (I + \hat{\tau}_0)^{-1} \theta_t \quad (28)$$

The tax indices θ_t are given by:

$$\theta_t = \hat{t}_t^{-1} \bar{t}_t = \hat{\tau}_0^{-1} \hat{\epsilon}_t^{-1} \hat{\tau}_t \bar{\epsilon}_t = \hat{\tau}_0^{-1} \hat{\tau}_t \phi_t \quad (29)$$

Substituting the last result into (28) suggests the following reduced form relating market prices to net prices:

$$p_{zt} = (I + \hat{\tau}_t)(I + \hat{\tau}_0)^{-1} \phi_t \quad (30)$$

Looking at (30), an important conclusion may be derived. Indeed, net price indices ϕ_t can be derived from the market prices p_{zt} and current and base year tax rates τ_0 and τ_t . Therefore, constant price input-output tables are not needed in the computation of net (of transfer) price indices except trivially in the base year.

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