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PRODUCTIVITY ANALYSIS AND THE MEASUREMENT OF GROSS OUTPUT NET OF INTRAINDUSTRY SALES

BY
RENÉ DURAND

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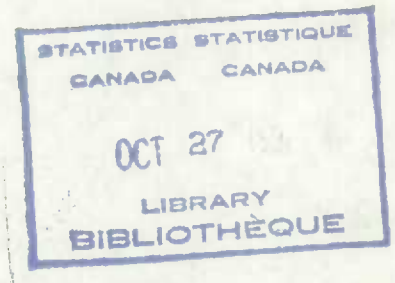

JANUARY 1991

In input-output tables, industries' gross output is defined as their total production of goods and services. It is measured as total sales corrected for inventory changes. Intermediate inputs include the purchase of all goods and services originating from all industries as well as imported goods and services and other leakages such as government supply of goods and services. The leakages are in fact preferably considered as primary inputs rather than intermediate inputs as they are purchased from outside the business sector.

Intermediate inputs net of leakages include the purchases of goods and services of all establishments of industries including intra-industry purchases, that is the purchases of establishments of an industry from other establishments in the same industry. As it is now considered to defined gross output of industries for productivity analysis net of intra-industry sales, this note discusses the methodology to estimate these intra-industry sales.

One simple answer to the question is to look at the commodity composition of industries' outputs and inputs and to match goods and services which appear in both commodity sets. Matching inputs would then be subtracted from the corresponding outputs to calculate gross output net of intra-industry sales. This is the correct answer for a closed economy in a square input-output framework. As our productivity measures apply to an open economy within a rectangular input-output framework, two modifications must be brought to the above simple methodology.

First, in a rectangular input-output framework, commodities may be produced by more than one industry. Consequently, an industry may use a commodity that it is partly producing but which is also partly originating from the other industries producing that same commodity. However, no statistics exist on the exact origin and destination of commodities so that some assumption must be made on their origin and destination. One such assumption correspond to taking the market shares of industries in the total supply of goods and services of the business sector. If, say, 80% of the production of a commodity which is used as an input by an industry is produced by that industry and 20% is produced by other industries, then it would be assumed that 80% of the use of that commodity would also be originating from that same industry and 20% from other industries. Under that assumption, the market share matrix D, would be applied to input uses of industries, net of leakages, to determine their origin:

$$U^O = D\,U^N \qquad (1)$$

where U^N is the matrix of intermediate inputs of industries net of all leakages and U^O is the matrix of intermediate inputs by industry of origin. Inputs net of leakages are computed by using the present specification of leakage parameters as follows:

$$U^N = (I - \hat{\alpha} - \hat{\beta} - \hat{\gamma})(I - \hat{\mu})\ U \qquad (2)$$

where the variables and leakage parameters are the same as in other previous notes. Equations (2) and (1) provide estimates of own use outputs along the diagonal of U^o . Indeed, the first column of U^o gives the vector of gross output (aggregated over commodities) of all industries associated with the net input uses of the first industry. The gross output of industry 1, that is the first element of that vector gives the gross output of the first industry associated with its own use of inputs. Similarly, the second element of the second column vector of U^o will give own use of outputs of industry 2, etc.. In other words, the diagonal of the matrix U^o gives the values of own use of outputs of industries which must be subtracted from their gross output g to give their gross net output g^N:

$$g^N = g - (I * U^o)\,\imath \qquad (3)$$

where the star product stands the element by element product of two matrices.

Equation (1) does not seem, at first glance to provide the required answer as U^o is giving the output by industry associated with intermediate input uses by industry rather than the output by industry and commodity associated with intermediate uses by industries. The matrix U^o is indeed a square industry by industry matrix. The commodity classification is lost in the transformation process while it may seem that it has to be maintained in order to be able to subtract, commodity-wise, own commodity output uses from commodity outputs of industries.

In fact, things are a little simpler as the passage from the actual gross output measure to gross output minus own use of inputs can be done by re-weighting the neoclassical productivity results by the ratio of these two output measures themselves[1]. To determine this ratio, it is not necessary to know the commodity breakdown of own use of outputs. The re-weighting of the productivity indices should occur before the computation of the Tornqvist indices to maintain the single step aggregation procedure.

Hence, current price outputs of own use must first be computed along the lines suggested above. In the above transformation, however, we have to take into account that inputs include taxes

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[1] For a demonstration, see Gullickson W. and M.J. Harper, "Multifactor Productivity Measurement for Two-Digit Manufacturing Industries", paper presented at the Western Economic Association in San Francisco, July 1-5, 1986.

minus subsidies while outputs are at producers' prices with, in some cases, correction for subsidies and royalties. This raises the issue of whether own use inputs must be transformed from input to output prices before being deducted from output. The answer is negative. At the aggregate level, that is when considering the deliveries of the business sector to final demand, the output measure not only shift from gross output to value added but from gross output at producers' prices to value added at factor cost. In the open economy model wherein leakages are treated as primary inputs, the latter are valued at prices including taxes minus subsidies. In general, whatever the level of aggregation, only the taxes paid by all producers on their inputs are considered in both costs and sales valuation. At the business sector level for the closed economy model, these taxes comprise only the direct income taxes on capital and labour plus the indirect taxes on capital services (other indirect taxes). This is how "factor cost" is defined in the productivity database. At the desegregated industry level, the taxes paid on intermediate inputs are included in the valuation of inputs and implicitly included in producers' prices on the output side. When, removing some intermediate inputs from the input list, it then seems only logical to remove their associated taxes altogether. The residual inputs will include all taxes on primary inputs plus the taxes on remaining intermediate inputs. As leakages are treated as primary inputs, their taxes should be included in their purchase prices.

Going from desegregated industry levels to gradually higher levels of aggregation is therefore done by removing gradually more and more commodity indirect taxes. These taxes, except those on the leakages in the open economy model, are completely removed at the business sector level. "Valuation" of transactions on the output side, therefore, gradually shifts from producers' prices to "factor cost" in a smooth fashion as the gross output similarly converges smoothly to final demand deliveries (open economy model) or value added (closed economy model).

The ratio of the actual gross output to the gross net output must be computed at all levels of aggregation. Own use of outputs for an industry group is not the simple sum of own use of outputs of the components industries. That is, when aggregating industries together, some outputs which were in the inter-industry category now fall into the intra-industry category. For instance, at the business sector level and for the closed economy model, the gross net output is identical to value added (all intermediate inputs become own use outputs), so that the results should be the same as before.

In order to compute the intra-industry flows at various levels of aggregation, one simply have to add up non-diagonal elements of U^o. These elements give the inter-industry flows. When aggregating various industries together, their corresponding inter-industry flows are transformed into intra-industry flows. Both directional flows have to be aggregated. If K_A is an industry aggregation

matrix at the A level, then the intra-industry flows at that aggregation level are given by the diagonal of the following matrix:

$$INTRA = Diagonal\ of\ K_A\ U^{\circ}\ K_A^T \qquad (4)$$

The above results imply that the aggregation weights of the neoclassical productivity indices must be redefined at all levels of aggregation accordingly[2]. At the P level of aggregation, the weights, previously given by the identity matrix are replaced by a diagonal matrix giving the ratio of the **nominal** gross output to the **nominal** gross net output. At a higher aggregation level, the weights become the value of each industry's gross output over the value of the group of industries gross net output.

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[2]See Durand R. "Aggregation formulas for multifactor productivity", Statistics Canada, Input-Output Division, June 1989.

Computer steps

1 - all data are in the productivity database.
2 - Intermediate inputs should be purified from leakages as already done so as to compute intermediate inputs net of leakages.
3 - Current price D matrix has to be done.
4 - Intermediate inputs at current input prices and net of leakages are pre-multiplied by the current price D matrix.
5 - The current price outputs of industries in the g vector are corrected for own use of outputs by subtracting own use inputs taken from the diagonal of the matrix computed in step 4.
6 - The vector g is divided by the vector g^N. This give the neo-classical aggregation A matrix at the P level of aggregation.
7 - Aggregated intra-industry flows are obtained from the diagonal of the aggregated matrix U^o. Aggregation is made on both sides of the matrix and the resulting matrix is square.

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