Input-output Division

AN ALTERNATIVE TO DOUBLE DEFLATION FOR MEASURING REAL INDUSTRY VALUE-ADDED

BY

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1 - Introduction

The double deflation method[2] is at the centre of the deflation of the gross domestic product of industries in Canada. This method is also widely used in other countries as its application follows a recommendation from the United Nations[3]. This note proposes an alternative measure of real value-added and discusses its properties for production analysis. It also criticizes the double deflation method as providing an inappropriate measure of industries' output except in the very special case in which all relative prices remain constant through time. In such a case, the double deflation method gives the same measure of real value added as the alternative method proposed here.

Numerous criticisms of the double deflation technique of national accountants can be found in the economic literature. However, these criticisms do not bear on the concept of real value-added of industries. They rather address the issue of how the presently used double deflation method should be amended to produce a valid measure of net output of industries. For instance, it has been suggested by Bruno (1978) that replacing the fixed base year Laspeyres index of real value-added by its Divisia index (or more precisely by some close chained approximation index) would yield valid results in that the resulting measure of net output would not entail any measurement bias of the industries' productivity growth estimates. However, the justification for such an alternative measure of real value added (and associated productivity measure) does not rest on a fundamental analysis of the notion of real value-added itself. The criticism which is brought here is more fundamental in that it addresses the meaning of real value added as such.

1 The author wishes to thank Terry Gigantes for valuable comments on an earlier draft of this paper.

2 The method, as is well known to national accountants, consists in deflating the outputs and intermediate inputs of industries in some base year prices, and in computing their real value-added by subtracting the deflated intermediate inputs from the deflated outputs.

3 United Nations, *A System of National Accounts*, Studies in Methods, Series F, No. 2, Rev. 3, New York, 1968

The note explains the alternative concept of real value added of industries and the method for its computation. Surprisingly, the alternative concept is not a new one. It has long been used by national accountants when referring to the *nominal* value-added of industries. That nominal value-added of an industry is the value which is added by an industry to the raw commodities which it processes and transforms into new commodities. This concept, when translated into real terms according to the simple method exposed below, leads to a measure of industry output which is completely different from the real value-added obtained with double deflation. That alternative concept has interesting properties for productivity analysis.

Briefly, the computation method which corresponds to that notion of real value-added consists of two steps. In the first step, nominal value-added is broken down by industry into commodity "slices". These slices correspond to the direct and indirect contributions of each industry to the total nominal value-added associated with each commodity delivered to final demand. In the second step, value-added by industry is deflated, slice by slice, by the corresponding commodity prices. The alternative method, therefore, is more closely connected to the true concept of value-added of industries of national accountants as will be explained further in the next section. The double-deflation method changes the interpretation which should be given to real value-added. It is not clear, however, which interpretation can be attached to the doubled-deflated real value-added, except in a very special case.

Section 2 describes, in more detail and in non technical terms, the proposed real value-added measure by industry and how it can be computed. Appendix 1 gives the corresponding mathematical derivations. For expository purposes, the economy is assumed to be closed (no leakages). The concept of value-added associated with this measure has a more immediate and meaningful economic interpretation as the *net output*[4] of industries. It follows that the proposed measure should be more suitable for production analysis than the double deflated real value-added measure of net output. This new interpretation rests both on the basic notion of "value-added" in national accounting and on production theoretic foundations presented in Section 3 which are further supported by mathematical derivations presented in Appendix 2. Appendix 2 presents an alternative derivation of the same real value-added

[4] The concept of net output or real value added of industries is, by itself, a central theme of this paper as "The concept of real value-added is not well-defined in the literature. There are ambiguities about the relationship of real value-added to the production function inclusive of all inputs. We are asserting that for our purposes real value-added is a function of the primary inputs and perhaps gross output." (Michael Denny and J. Douglas May (1978), footnote 2, p.54).

measure with its associated productivity measure. Links are established with various measures of productivity growth, namely productivity growth by final demand commodity, productivity growth for vertically integrated industries and the neoclassical productivity growth measure on gross output of industries. A valuation issue is dealt with in Appendix 4 as Canadian input-output tables express final demand at producers' prices and value-added at factor cost.

2 - The Alternative Real Value-Added Measure

2.1 Introducing the concept of value-added in national accounting

The value-added approach to national accounting, as an alternative to the income and the expenditure approach, consists in computing a slice of value-added attached to each commodity as it goes through the many transformation processes of industries. The value-added associated with bread delivered to final demand, for example, is computed as the value-added of growing wheat in agriculture, plus the value-added of transporting wheat to flour mills, plus the value-added of the flour mills plus the value-added of the bakeries and plus, finally, the value-added of the distributors of bread. The value-added of the contributing industries in the production of bread form a "layer" of value-added across industries. Summing value-added across industries for the bread layer yields a total value-added which is equal to the value of the bread delivered to final demand. This is the very essence of the value-added approach. Generally, an industry contributes directly or indirectly to more than one commodity. Its contribution to each commodity forms a "slice" of value-added. The contribution of an industry to all commodities is simply its total value-added[5].

[5] Final demand nominal values of expenditure can be used to compute the direct and indirect value-added by industry associated with each commodity. This can be performed by applying the current price impact matrix to the current price value of expenditure on each commodity. This yields layers of value-added in almost all industries associated with the commodities delivered to final demand. Adding over these commodity layers gives the total nominal value-added of the industries. Note that by construction of the impact matrix, this value-added is simply equal to the total value of the primary inputs or, equivalently, it is equal to the nominal values of gross output of industries minus the nominal values of their intermediate inputs. The purpose of using an impact matrix, therefore, is not to compute total nominal value-added by industry, which is readily given from a reading of the input-output tables, but to further desegregate nominal value-added by industry into commodity layers.

2.2 Outline of the proposed deflation method

The deflation method proposed in this note consists in deflating each commodity nominal value-added slice within an industry by the associated commodity price. Total real value-added by industry is computed (admitting a Laspeyres rule for computing real aggregates) by summing over the deflated slices[6]. Since the total nominal value of a commodity layer across industries is equal to the final demand value of that commodity, deflating the commodity layer across all industries gives a constant price value for the total business sector (using a Laspeyres rule) which is just equal to the real value of that commodity in final demand. Repeating the calculation for all commodities leads to distinct layers of real value-added for each industry; one for every commodity that the industry produces. For each commodity layer, there is a balance between the total real value-added of all industries and the corresponding real value of the commodity delivered to final demand.

Summing over commodities in the final demand, therefore, gives a total (fixed base year Laspeyres) value of final demand expenditure in real terms. This value is identical to the total real business value-added of industries found by summing over both industries and commodity layers. Hence, the proposed measure of real value-added of industries adds up to the same total real value of final demand as the real value-added obtained from the double deflation method. However, the resulting industrial distribution is quite different from the usual distribution obtained using the double deflation method on an industry by industry basis. This is the case whether double deflation is operated with a chained or an unchained index number formula.

[6] To open a brief parenthesis, note that the nominal flows of commodities can only be deflated by their own prices to compute their constant price flows. Hence, the constant price value of a commodity delivered to final demand users is equal to the constant price gross output of that commodity minus its constant price intermediate use. It corresponds to the real value-added for that commodity. Aggregating real value-added over all commodities yields an estimate of real value-added for the total business sector of the economy. This is the only unique logical way of computing and reconciling the quantities of the commodities produced and consumed at the intermediate stage and final stage level. However, a choice between many alternative aggregation formula (index number formula) has to be made when computing aggregate real value-added for all commodities. Aggregation of constant price commodity flows, in Canadian Input-Output Tables, is based on a fixed base year Laspeyres index formula. This method of computing final demand by commodity at constant prices and aggregate real value-added is not questioned here.

2.3 A numerical example

A simple example could perhaps be introduced at this stage to highlight the differences between the fixed base year double deflation method and the alternative deflation method proposed in this note. As such an example, one can take an industry producing a single commodity which is completely delivered to final demand. In that case, the alternative deflation method which is proposed consists in deflating both gross output and value-added of the industry by the commodity price. Assume first that current price gross output is fixed at 300 dollars (see table 1) and that the initial commodity price is 3.0. Assume further that current price intermediate inputs are worth 70 dollars so that nominal value-added is at 230 dollars. Lastly, assume that the intermediate input average price is 1.4 so that real intermediate inputs are worth 50 dollars. Real gross output is then worth 100 dollars and the double deflated real value-added is worth 50 dollars. According to the alternative deflation method proposed, real value-added would be given by deflating nominal value-added (230 dollars) by the commodity price (3.0) and would equal 76.66 dollars.

Table 1. Comparison of double deflation with an alternative deflation method: a simple example

	Gross output	Inter-mediate inputs	Value-Added double deflation	Value-Added alternative deflation
Current prices	300.00	70.00	230.00	230.00
Initial price	3.0	1.4	4.6	3.0
Initial constant prices	100.00	50.00	50.00	76.66
New Price	1.5	1.4	1.53	1.5
New constant prices	200.00	50.00	150.00	153.33

Now assume that the commodity price is cut by half to 1.5. Then gross output jumps to 200 dollars and, since real intermediate inputs remain the same, double deflation gives a real value-added of 150 dollars. That is, with a doubling of gross output, real value-added is tripled. With the alternative deflation method, nominal value-added, which is fixed at 230 dollars, is now deflated by 1.5 which gives 153.33 dollars. That is, real value-added, just like gross output, is also doubled.

To interpret these results, note that nominal prices and quantities of both intermediate and primary inputs were held constant so that the real factor prices were doubled when the output price was reduced by 50%. This should imply, under equilibrium, that the marginal product of both primary and intermediate inputs doubled and that increased factor productivity is what explains the increased output. The output share (real income) attributable to primary inputs as opposed to intermediate inputs should remain the same as the relative price (and quantity) of primary inputs to intermediate inputs remained the same. This is what happens with the alternative deflation method. With double deflation, the real share of primary inputs in total output is increased (if it is considered that real value-added is attributable to primary inputs) while the share of intermediate inputs is correspondingly decreased. Output shares are inconsistent with factor income shares. There is simply no explanation for such a result.

2.4 Empirical properties

The new real value-added measure has some interesting empirical properties which makes it more desirable than its alternative obtained from the double deflation method. In particular, the corresponding price index (using a fixed base year Paasche approximation) is a weighted average of the prices of the commodities, to which each industry has contributed directly and indirectly. The value-added price index is therefore well bounded from below by the smallest commodity price index and, bounded from above, by the highest commodity price index.

Sensible bounds cannot so easily be established in the case of the implicit value-added price index derived from the double deflation method. This price index tends to behave erratically and sometimes turns out to be negative (which, obviously, has no economic meaning) as relative input to output prices change through time. The sensitivity of the implicit value-added price index derived from the double deflation method tends to be higher, the smaller is the share of nominal value-added into the gross output of industries (see Lal (1987)). National accountants recommend, in such a case, abandoning the double deflation method for some alternative methods. These alternative methods could include using the gross output deflator to deflate value-added or, aggregating the problematic industries with other industries before deflating. The further away the current year is from the base year, the more likely such situations tend to occur. Problem cases also tend to increase with the desegregation of real value-added by industry. This is one of the major reasons that national accountants recommend updating the base year periodically. The alternative price index proposed here, being a weighted average of final demand

commodity prices, is, by construction, always positive and it is completely insensitive to the share of nominal value-added into gross output. Therefore, a selected fixed base years can be maintained for longer time spans, although they may still be revised for other reasons[7].

A last but not least important property of the alternative deflation method[8] is that it requires only final demand commodity prices. Intermediate input prices, particularly prices of service inputs do not need to be known. This property is important in light of the difficulties encountered in deflating many services and, in particular, business services.

It must be stressed clearly again here that the problem presently discussed is not an index number formula issue. On the contrary, the proposed deflation method breaks down final demand value-added by commodity into value-added by industry and by commodity. Total value-added by industry can be computed by adding over commodities, if desired, to obtain a fixed base year Laspeyres volume of net output. But other aggregation rules could be used. Aggregation or index number issues occur at that stage but these issues are independent of the problem of the double deflation method on which this note focuses. It has, therefore, been assumed throughout this section and in Appendix 1, for simplicity and without loss of generality, that aggregation was done with a Laspeyres index. But the conclusions reached in this paper are valid regardless of which aggregation rule is used. Consequently, when deriving productivity indices within a continuous time framework in Appendix 2, a Divisia aggregation rule was used.

3 - Interpretation of the New Real Value-Added Measure

The main focus of this note is to define a meaningful concept and associated measure of real value added for industries which could be used for economic analysis and, in particular, for production analysis. It starts from the usual notion of real value-added as a measure of the *real net output* of industries to be contrasted with their *real gross output*. Real value-added is clearly distinguished from the deflated value of industries' primary inputs. It is also distinguished from the double deflated concept which is shown to be ambiguous. It is rather considered, in

[7] Many economists would indeed prefer chained indices of output on the ground that they yield "quantities" and corresponding prices which are actually those economic units have in mind when making buying or selling decisions.

[8] This important property was noticed and communicated to the author by T. Gigantes.

parallel to the concept of nominal value added in national accounting described above, to be a measure of the direct and indirect contributions of an industry to final demand deliveries in real terms.

On the first point, the real value-added of industries, at least at the aggregate business sector level, has always been considered as the proper measure of their real net output. Real value-added represents the value of the production delivered by the business sector to other sectors of the economy. However, from the restricted point of view of any particular industry, real value-added has no such immediate meaning. The output delivered by an industry is usually equated with its gross output. The widely accepted notion of value-added at the industry level is meaningful only in reference to the total business sector. Value-added must correspond to the idea that the total value-added of the business sector can somehow be *distributed* among the various industries. It must be realized, hence, that the concept of value-added has a meaning only from the perspective of the business sector as a whole as *it relates to the deliveries of the business sector to other sectors of the economy*. The real value-added of an industry is just one output component of that fully *integrated* business sector in which all industries are interrelated together through exchanges of intermediate goods and services.

Without that interdependence, industries' value-added would coincide with their gross output. Industries would contribute to final demand output directly. Given their interdependence, industries actually contribute both directly and indirectly to final demand output. These contributions can be computed by *integrating vertically* all industries with the downstream industries purchasing their goods and services. That vertical integration is identical to the integration which is performed at the aggregate business sector when real value-added is substituted for gross output as a measure of output for the sector. In the process, intermediate inputs are "left" out of the output measure as well as of the input measure. On the input side, they are left out by substituting, to the intermediate inputs, the primary inputs used directly and indirectly to produced them. On the output side, similarly, the intermediate inputs are removed as if all production was carried within a single large and fully vertically integrated establishment encompassing all industries.

Ambiguities in the measurement of real value-added arise from the fact that nominal value added can be equivalently measured in three different ways. Nominal value-added can be seen as the value of gross output minus the value of intermediate inputs consumed in the business sector. It is also equal to the value of primary inputs used, which fully justifies the distinction made between intermediate and primary inputs in national accounting. Finally, nominal value added can be computed as the cumulative addition of

value to commodities as they are processed in the various industries of the business sector. The same identity hold at the industry level and is at the source of the conceptual difficulties. Indeed, this equality between the various measures of value-added breaks down in constant prices as will now be seen. The double deflation method simply extends the application the first of these alternative methods to constant price flows. But this is precisely where double deflation is inappropriate.

The real value-added of an industry has always been considered as distinct from the constant price value of its primary inputs. The constant price flows correspond to real input uses associated with real net output through a production process. If both of these magnitudes were identical, using the Laspeyres aggregation rule for inputs and outputs, this would mean that output growth would be equal to input growth (again, using the Laspeyres approximation to the Divisia index). In other words, there would be no productivity gains made through time on the use of primary inputs. Conceptually, therefore, real value-added as an output concept cannot be equated with the deflated flows of primary inputs.

Constant price intermediate goods and services have an ambiguous status since they can be seen simultaneously as outputs and inputs of the business sector. For the whole business sector, subtracting intermediate "outputs" from gross outputs gives its net output. The issue raised in this note is whether or not it is justifiable to subtract intermediate "inputs" from industries' gross outputs (double deflation method) to arrive at their net output. At the industry level, indeed, intermediate goods and services can only be considered as inputs as they are purchased from other industries[9]. Subtracting real intermediate inputs from an industry's real gross output gives its real primary inputs and a productivity residual associated with *all* of its inputs not only with its primary inputs[10]. This is what makes the double deflated real value-added an ambiguous concept. If this last point is accepted, then it follows that, in general, real value-added cannot be obtained by subtracting real intermediate inputs from real gross output at the industry level. Simply stated, constant price intermediate inputs and real value added generally do not add up to gross output.

To stress further the distinction between gross output and value-added, the former is linked to the notion of an industry considered in isolation from the rest of the economy with its own technology relating gross outputs of commodities to all primary and

[9] Except perhaps for intra-industry sales.

[10] In a non-Laspeyres context, this translate by: subtracting from the rate of growth of gross output the rate of growth of intermediate inputs gives the rate of growth of primary inputs and the associated multifactor productivity gain.

intermediate inputs. The concept of real value-added is linked to the notion that industries are integrated components of the business sector. Their technology relates net final demand outputs to the use of their primary inputs and the primary inputs of downstream industries to which they are integrated.

To summarize, in value terms, final demand value-added (total net sales) is equal to total primary factor costs (total costs) and the latter are shared among the various industries of the business sector. Also, at the aggregate business sector level, total real value-added is a meaningful and well defined output concept. For an industry considered in isolation from the rest of the business sector, however, the concept of real output corresponds to gross output. The concept of net output of any industry, to have any meaning, must be defined in reference to the output of the whole business sector for which total output and real value-added coincide. Real output (or more precisely the vector of real output by commodity) for the business sector can be related to the quantity of primary inputs used (real inputs) through a production function, not through a "sales equal cost" identity as is the case for nominal value flows. The aggregate production function shifts through time as productivity increases so that the rate of growth of outputs tends to exceed the rate of growth of inputs. This implies that real input growth plus a productivity residual sum to output growth or that real inputs do not need to sum to real outputs. At the industry level, net output or real value-added, must be defined by allocating the business sector productivity gains to each industry. The imputed productivity gains must be such that they aggregate "nicely" (that is, with weights which have an immediate interpretation) to the total business sector productivity gain.

A second major issue of this paper, therefore, is choosing the appropriate productivity growth measure to be added to primary input growth to yield the most meaningful measure of real value-added growth at the industry level. It seems, to follow the value-added approach, that all productivity gains made by an industry on the use of its primary inputs associated with each final demand commodity to which production it participated in must be taken into account. The vector of real net output of an industry would hence be defined, as above, as the deflated commodity slices of value-added. Indeed, the real value-added measure which is looked for has to be consistent with aggregate business sector real value-added into which all productivity gains are included. The aggregate business sector Divisia index of productivity growth would thereby follows (and does as shown in Appendix 2) by aggregating the productivity growth rates of industries on real value-added using industries' value-added shares as weights.

It is shown in Appendix 2 that the industries' multifactor productivity growths obtained from such a production function, when based on the proposed measure of real value-added, aggregate nicely to the total business sector multifactor productivity growth obtained by either aggregating the neoclassical industries' productivity gains associated with their gross output or aggregating the productivity gains associated with final demand output by commodity. These two alternative formulations of multifactor productivity growth are well established in the economic literature and provide a major theoretical justification for the real value-added measure which is proposed in this note.

As shown in Appendix 2, the aggregate productivity index computed from the industries' real value-added measure obtained through fixed base year double deflation differs from the aggregate productivity index obtained from well established principles and consequently, invalidates that measure of net output of industries for production analysis. When real value-added is obtained from the Divisia index of industries' outputs minus the Divisia index of their intermediate inputs, the corresponding residual productivity gains is identical to the one computed from gross output and all intermediate and primary inputs (see Appendix 2). However, this concept of real value-added is inappropriate for three reasons. First, it does not correspond to the idea standing behind the current price concept of value-added of industries which is associated with the direct and indirect contributions of industries to final demand deliveries. Secondly, the resulting measure is not useful as it can only be computed if gross outputs and intermediate inputs are known in which case productivity growth could be equivalently computed on gross outputs and all inputs. Furthermore, the aggregation weights of such productivity measures to the business sector level are identical to the aggregation weights of the productivity measures associated with gross outputs. No economic interpretation of these weights could be given when associated with net outputs of industries while these weights have a clear interpretation when associated with their gross output.

Many attempts at estimating the parameters of double-deflated real value-added production functions appeared in the earlier literature prior to criticisms that these estimates implied that the underlying "true" production function of the industries, (which, as was asserted, should be defined on gross output and all intermediate and primary inputs) was separable on their primary and intermediate input sets. Separability between primary and intermediate inputs was shown to be too restrictive an assumption on empirical grounds. It has also been shown that separability implies that the relative prices of intermediate inputs with respect to output prices should be constant through time (see for instance, Denny and May (1977), and see also Bruno(1978)). The

fixed base year double deflated real value-added measure of output has since then been dismissed in production analysis.

The alternative real value-added measure of net output proposed in this note is equal to the double deflated measure of real value-added only when relative intermediate to gross output prices are constant through time as shown in Appendix 1. Is it, nevertheless, subject to some similar restrictive implicit assumptions which would restrict its use in production analysis? It can be argued that it is not. First, it is not derived, as the double deflated value-added, by subtracting real intermediate inputs from real gross output of industries under any separability assumptions. As we have seen, the proposed real value-added measure and the constant price intermediate inputs generally do not add up to the constant price gross output of industries. Secondly, the new real value-added measure cannot be considered as an argument of a more general production function which would be specified on gross outputs, intermediate inputs, and real value-added considered as some kind of aggregate index of primary inputs. It is an independent concept of output at the industry level to be contrasted to the concept of gross output. The latter draws its meaning from considering each industry as an independent economic entity while the former draws its meaning from considering each industry as an integrated component of the business sector as a whole. Both production functions corresponding to these alternative concepts have their own (exhaustive) sets of outputs and inputs and are not connected otherwise than by the fact that they share the same set of primary inputs. The "separability" issue, as far as we can judge, was raised in the literature in an attempt to relate the real value-added to the gross output of an industry. Bruno (1978) for instance asks the question: "Under what alternative set of assumptions could one infer, from the observed estimates in terms of value-added, the "true" measure of total productivity (in terms of the underlying gross output function)?" The measure of real value-added proposed in this paper will not yield the same productivity residual than the one associated with gross output and it is not meant for that purpose. In other words, it is not developed to answer Bruno's question. Therefore, separability is not an issue.

Appendix 1: The Real Value-Added Equation

Let $\mathbf{e}$ be the vector of real final demand commodities delivered by the business sector of the economy, that is the vector of gross output by commodity, $\mathbf{v}$, minus the vector of intermediate inputs by commodity, $\mathbf{u}$ [11]. Let $\mathbf{p}$ be the price vector of the commodities (we assume, without loss of generality, a unique price for each commodity in all uses). Then, the vector of gross output by industry in current price, $\mathbf{g}$, is given by the usual "impact" equation:

$$\mathbf{g} = [\mathbf{I}-\mathbf{DB}]^{-1}\mathbf{D}\hat{\mathbf{p}}\mathbf{e} \tag{1.1}$$

where the "hat" symbol is used to indicate a diagonal matrix formed from a vector. In equation (1.1), $\mathbf{D}$ is the current price market share matrix and $\mathbf{B}$ the current price technology coefficient matrix for intermediate inputs, i.e., $\mathbf{D}\,\mathbf{B} = \mathbf{A}$, where $\mathbf{A}$ is the Leontief square technology matrix. It is assumed that the economy is closed and that all commodity supply comes from the business sector of the economy, i.e., that there is no leakages associated with imports, government supply of goods and services, inventory depletion, etc.. The vector $\mathbf{g}$ can be multiplied by a diagonal matrix $\boldsymbol{\lambda}$ of nominal value-added coefficients of industries such that value-added, $\mathbf{y}$, be given by

$$\mathbf{y} = \hat{\boldsymbol{\lambda}}\mathbf{g} \tag{1.2}$$

$$= \hat{\boldsymbol{\lambda}}[\mathbf{I}-\mathbf{DB}]^{-1}\mathbf{D}\hat{\mathbf{p}}\mathbf{e} \tag{1.3}$$

The vector $\mathbf{y}$ gives total value-added in current prices by industry. However, to get value-added decomposed also by commodity, it suffices to replace the vector $\mathbf{e}$ by its diagonal in equation (1.3). The matrix of value-added $\mathbf{Y}$ by industries (rows) and commodities (columns) is given by

$$\mathbf{Y} = \hat{\boldsymbol{\lambda}}[\mathbf{I}-\mathbf{DB}]^{-1}\mathbf{D}\hat{\mathbf{p}}\hat{\mathbf{e}} \tag{1.4}$$

[11] Bold faces types are used throughout for vectors and matrices.

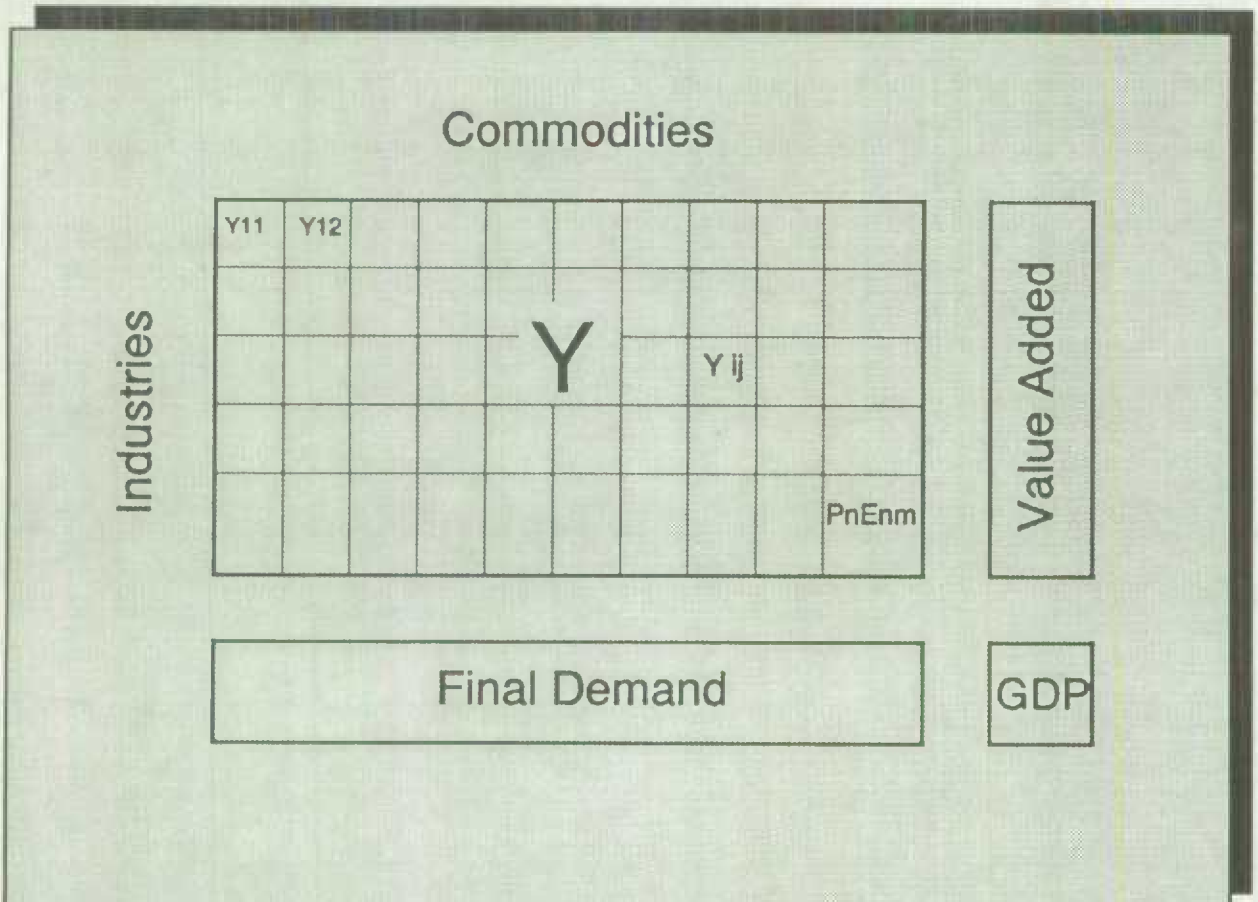
The matrix Y is such that summing over its rows gives the value of the final demand vector $\hat{p}\, e$ and summing over its columns, gives the value-added by industry vector, y:

$$i^T Y = p^T \hat{e} \tag{1.5}$$

$$Y\, i = y \tag{1.6}$$

The first property follows from the fact that the coefficients of the impact matrix yielding primary inputs add up to one. Hence, the Y matrix has the form depicted on Figure 2.

Figure 1: The Real Value Added Matrix



The **n** commodities form the **n** columns of the matrix and the **m** industries form the **m** rows. Any given row contains the direct and indirect contributions of an industry to all final demand commodities. The sum of an industry's contributions over all commodities, therefore, gives its total value-added. Similarly, the sum over all industries' contributions, for a given commodity, gives the total value of that commodity delivered to final demand. It is clear that adding the total value-added of industries over all industries will give the business sector total value-added which, from the construction of the table, will also give the sum of the total value-added of commodities over all commodities.

Each cell of the matrix Y represents the contribution of a specific industry to the value-added of a given commodity. It then seems only natural to estimate the contribution of an industry to the real value-added of a commodity by deflating its nominal contribution by that particular commodity's price. The nominal contributions of all other industries to the real value-added of that commodity may similarly be deflated by the same commodity price. In other words, the proposed deflation method consists in deflating each column of the Y matrix by the corresponding commodity price. This gives a constant price matrix Y^k. In mathematical terms, this amounts to post multiplying both sides of equation (1.4) by the inverse of the commodity prices $\hat{p}^{-1}$:

$$Y^k = Y\hat{p}^{-1} \tag{1.7}$$

$$Y^k = \hat{\lambda}[I-DB]^{-1}D\hat{e} \tag{1.8}$$

Equation (1.8) would be identical to the constant price impact equation if the current price matrices D B and $\hat{\lambda}$ were identical to their constant price counterparts. The market share matrix, D, in our simplified framework, in which commodity prices are identical in all uses, is identical to its constant price counterpart. The matrices B and $\hat{\lambda}$ are identical to their constant price counterparts when, and only when, all relative prices of inputs to outputs in any year t are the same, as in some arbitrarily chosen base year. In such a case, equation (1.8) yields, by construction, industries' constant price estimates of value-added which are identical to those obtained from the usual application of the double deflation method. Otherwise, the distribution of real value-added estimates by industry obtained from the two alternative methods differ even though the totals of their real value-added over all industries are equal.

Further inspection of equation (1.8) leads to an interesting interpretation. This equation applies current prices and current

year weights to real final demand expenditure by commodities. This means that each industry's share in real final output is directly associated with its nominal value share of that output (according to the relative prices prevailing in any period). Contrastingly, with the double deflated real value-added measure, each industry's share in real final output is given by what it would have been, given the prices prevailing in the base year. This seems to be a somewhat irrelevant measure for economic analysis as it answers the question "what would the real value-added have been of a given industry had relative prices have been the same as in the base year?".

The Laspeyres real value-added by industry, y^k, is obtained from (1.7) by summing over commodities, i.e., from:

$$y^k = Y^k i \tag{1.9}$$

$$y^k = \hat{\lambda}\ [I-DB]^{-1} D\ e \tag{1.10}$$

As mentioned above, other aggregation formulas could be used to aggregate real value-added over commodities for each industry. In the next appendix, the aggregation is carried out with the Divisia index.

Appendix 2: Productivity Indices on Real Value-Added

In order to define productivity indices on real value-added, use will be made of the results achieved on productivity indices on final demand commodities in Durand and Salem (1987, revised 1990). The equation for the vector of productivity indices, ρ, on the final demand commodity vector **e** is given by

$$\rho = \dot{e} - \hat{p}^{-1}\hat{e}^{-1}\overline{L}^{T}_{[1]}(I\otimes\hat{w})\dot{l}_{[1]} - \hat{p}^{-1}\hat{e}^{-1}\overline{K}^{T}_{[1]}(I\otimes\hat{r})\dot{k}_{[1]} \tag{2.1}$$

where $\dot{e}$ is the vector of rates of growth of final demand commodities, w and r are the vectors respectively of labour and capital prices by type , $\overline{L}^{T}_{[1]}(I\otimes\hat{w})$, is the matrix of direct and indirect labour cost by type of labour (columns) associated with each commodity (rows), $\overline{K}^{T}_{[1]}(I\otimes\hat{r})$, is similarly the matrix of direct and indirect capital costs, and where $\otimes$ is the Kronecker product. The dot over the symbols represents their continuous time rate of growth so that formula (2.1) represents, by definition, the vector of Divisia indices of productivity growth associated with final demand commodities. Indeed, it equates productivity gains on each commodity to the difference between the rate of growth of that commodity and the weighted rate of growth of the primary inputs used in its production. The weights are the direct and indirect cost shares of each input by type in total cost. Values of direct and indirect primary input requirements are obtained by the application of the usual current price impact matrix of the input-output model to the current price diagonal matrix formed with the vector of final demand. Real input requirements are obtained by deflating nominal values by input prices. The columns of the factor cost matrices have been extended along the commodity dimension; as well, the vectors of rates of growth of primary inputs by type have been extended over commodities so as to separate each commodity productivity equation. For instance:

$$\overline{L}^{T}_{[1]} = \begin{bmatrix} l_1^T, & 0^T, & 0^T, & \dots & 0^T \\ 0^T, & l_2^T, & \dots & \dots & 0^T \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0^T, & 0^T, & \dots & \dots & l_n^T \end{bmatrix} \tag{2.2}$$

and

$$l^T_{[l]} = (l_1^T, l_2^T, \ldots, l_n^T) \tag{2.3}$$

where l_i is the i^{th} component vector of labour input by types of labour and applies to commodity i. The notation of the extended form of a matrix includes a bar over the matrix symbol and, the dimension over which it is extended a subscript within square brackets. Extended vectors are similarly subscripted.

It is to be noted here that primary inputs are broken down by commodity and by type. To transpose the final demand productivity results into the industry space, it is necessary to reclassify the primary inputs so as to express them also by industry i.e.,to build a three dimensional array breaking down primary inputs by commodity, by industry and by type. This array can be constructed by first breaking down gross output by industry and by commodity as follows:

$$G = [I-DB]^{-1}D\hat{p}\hat{e} \tag{2.4}$$

Each column of G gives the gross output vector of industries associated with the production of a given final demand commodity. Applying the primary input requirement coefficient matrices to any column of G gives:

$$\begin{aligned} \hat{w}\, L_i &= H_L \hat{g}_i \\ \hat{r}\, K_i &= H_K \hat{g}_i \end{aligned} \tag{2.5}$$

Equation (2.5) provides a generalization of the equation used in Durand and Salem to compute primary input requirements. The labour and capital costs in (2.5) are now by type of labour and capital (rows), by industry (columns), and relate only to commodity i. Repeating the calculations over all commodities give the elements required to build the three dimensional arrays mentioned above. Real input requirements can be estimated by deflating the nominal flows in (2.5) by input prices. Actually, it is not necessary to construct such an array but simply to realise that the three dimensional breakdown of inputs and their associated cost is feasible. This breakdown of inputs corresponds exactly to the cells of the proposed real value-added matrix Y^k. Indeed, in value terms, the coefficients of the primary input requirement coefficient matrices add up exactly to the value-added coefficients and the impact matrices used to compute value-added and primary input costs are otherwise identical. To each cell of the real

value-added matrix correspond capital and labour inputs by type pertaining to a given commodity-industry breakdown of net output. It is, therefore, possible to define a Divisia index of productivity growth for each cell of Y^{*}. Letting γ_{ij} denote the Divisia productivity growth rate associated with each industry-commodity real value-added and Γ the corresponding matrix of Divisia indices of productivity growth associated with Y^{*}, then industries' productivity indices (denoted by the vector μ) are by definition given by weighting the row elements of Γ by the value shares of each commodity in the value-added of each industry:

$$\begin{aligned} \mu &= (\hat{y}^{-1} Y \cdot \Gamma)\, i \\ &= \hat{y}^{-1} (Y \cdot \Gamma)\, i \end{aligned} \tag{2.6}$$

where the dot product in (2.6) is the element by element (Schurr) matrix product. That is, (2.6) follows from the consistency in aggregation property of Divisia indices, which implies that Divisia indices of Divisia indices are themselves Divisia indices[12]. Similarly, the vector of productivity growth by commodity can be derived by adding the columns elements of Γ weighted by their value share in the total value of each commodity:

$$\begin{aligned} \rho &= (Y \hat{p}^{-1} \hat{e}^{-1} \cdot \Gamma)^{T} i \\ &= \hat{p}^{-1} \hat{e}^{-1} (Y^{T} \cdot \Gamma^{T})\, i \end{aligned} \tag{2.7}$$

By definition, the Divisia aggregate over industries of the productivity growth rates, γ, is, again from the consistency in aggregation property, given by the weighted sum of industries' productivity growth rates:

This is, of course, looking at the third equality in (2.8), also given by the weighted average of the productivity growth rates of each cell of Γ. Aggregating over commodities yields the same result:

The aggregate business sector productivity growth rate in (2.9) is the same as in (2.8) as long as aggregating value-added over commodities gives the same result as aggregating value-added over

12 This property is also shared by the more usual Laspeyres index which is computed by adding constant price values. Adding two Laspeyres indices, that is taking the Laspeyres' index of two Laspeyres indices, gives an aggregate Laspeyres index.

$$\gamma = \frac{y^T}{y^T i}\,\mu$$

$$= \frac{y^T}{y^T i}\hat{y}^{-1}(Y \cdot \Gamma)\,i \tag{2.8}$$

$$= \frac{i^T}{y^T i}(Y \cdot \Gamma)\,i$$

$$\gamma = \frac{p^T\hat{e}}{p^T e}\,\rho$$

$$= \frac{p^T\hat{e}}{p^T e}\hat{p}^{-1}\hat{e}^{1-}(Y^T \cdot \Gamma^T)\,i \tag{2.9}$$

$$= \frac{i^T}{p^T e}(Y^T \cdot \Gamma^T)\,i$$

industries. Therefore, the real value-added measure Y^k , derived in section 2, fulfils the requirements set out above. First, it provides a real value-added measure by industry which is derived by desegregating (and in reference to) total real business value-added. Secondly, real value-added growth by industry can be equated to primary input growth plus a productivity residual. Thirdly, the productivity residuals of industries add up as required to the aggregate business sector productivity gain with a set of weights which fulfil the condition of the Divisia aggregation in continuous time. This aggregate productivity gain is identical to the one which results from aggregating productivity gains over industries that have underlying production functions defined on gross outputs and intermediate and primary inputs. It is also equal to the aggregate business sector productivity gain obtained by aggregating productivity gains associated with final demand output by commodity. Finally, various discrete time approximations of the Divisia indices of real value-added by industry can be computed, including the widely used Laspeyres fixed base year quantity indices.

The alternative fixed base year double deflated value-added measure does not share the same properties. Value-added could still be distributed by industry and by commodity on the basis of the constant price impact matrix but the constant price cells of, say the matrix Z^k corresponding to Y^k, would not have prices corresponding to the commodity prices from which the same Divisia aggregate could be built. Consequently, the Divisia indices of productivity which could be built by using the nominal value shares

of Y (or the implicit prices obtained by dividing each cell of Y by the corresponding cell of Z^k), would not aggregate to the total factor productivity of the business sector. In other words, these prices and quantities form an inconsistent set for production analysis.

More precisely this can be seen as follows. First, it can be noted that if X is a matrix of real industry by commodity flows to which is associated a commodity price vector p and x is the sum of X over its rows or industries (an aggregate commodity vector), then, the Divisia index of the matrix X is equal to the Divisia index of the vector x. The proof of that proposition follows from inspection. Hence the Divisia index of Y^k is equal to the Divisia index of the commodity vector e which follows from summing over the rows of Y^k. Since inputs are the same for both Y^k and e with same prices, this shows more formally that the aggregate productivity index associated with Y^k is the same as the one associated with e. But even though the rows of Z^k also add up to e, the prices associated with Z^k vary from row to row and differ from the prices associated with e. The corresponding Divisia indices of output, therefore, differ and, consequently, so do their aggregate productivity index.

It is straightforward to show Bruno's proposition alluded to in section 2 that if real value added is defined as the Divisia index of industries' gross output minus the Divisia index of their intermediate inputs, the productivity gains associated with value added and primary inputs are identical to the productivity gains associated with gross outputs and all intermediate and primary inputs. Indeed, let the real value-added of some industry be defined as:

$$\dot{y}^k = c^T \dot{v} - b^T \dot{u} \qquad (2.10)$$

where v is the vector of the industry's commodity outputs whose value shares are in the vector c and u is the vector of intermediate inputs with shares (in the value of the industry's output) b. Total factor productivity of the industry is defined by:

$$\tau = c^T \dot{v} - b^T \dot{u} - \omega^T \dot{x} \qquad (2.11)$$

where x is the vector of primary inputs with associated cost shares ω. Then substituting (2.10) into (2.11) gives:

$$\tau = \dot{y}^k - \omega^T \dot{x} \qquad (2.12)$$

However, in defining such a measure of real value-added, Bruno gave no justification of the fact that the intermediate input cost shares were defined with respect to total industries costs rather than to intermediate input costs. In addition, the aggregation weights of the resulting productivity measures to the total business sector are identical to the aggregation weights associated with gross output productivity gains. If these weights can be given an economic interpretation in the latter case, they cannot be interpreted when associated with the net measure of industries' output. But Bruno's suggestion is worth pursuing a little more as, with some modifications, it leads to interesting conclusions.

In fact, there are two problems with Bruno's definition of real value-added. First, his formula does not, strictly speaking, correspond to the double deflation technique. To correspond to the double deflation technique, the intermediate input cost shares should add up to one. Correspondingly, the primary input cost shares in total value-added should add up to one. Secondly, if $\dot{y}^k$ were to be a measure of real value-added, it would have to apply also to all industries aggregated into a single industry at the business sector level. But at that level, productivity is given by

$$\gamma = \beta\tau \tag{2.13}$$

where β is Domar's productivity aggregation weight given by the value of total gross output divided by total value-added. This suggests to replace Bruno's measure of real value-added, at the aggregate business sector level by

$$\dot{y}^k = \beta(\boldsymbol{c}^T\dot{\boldsymbol{v}} - \boldsymbol{b}^T\dot{\mathbf{u}}) \tag{2.14}$$

from which follows:

$$\gamma = \beta\ \tau = \dot{y}^k - \beta\boldsymbol{\omega}^T\dot{\boldsymbol{x}} \tag{2.15}$$

In the latter formula, it is to be noted that primary input shares, $\beta\boldsymbol{\omega}^T$, are now defined with respect to value-added rather than gross output and furthermore, they add up to one. Hence, $\dot{y}^k$ must, by definition, be the index of the real value-added growth rate. From a different angle, since the net final demand vector of commodities $\boldsymbol{e}$ is given by $\boldsymbol{v} - \boldsymbol{u}$, it can be seen, taking the Divisia index of $\boldsymbol{e}$ that $\dot{y}^k$ is again given by (2.15)[13]. The interesting question which follows is to see what happens when such

[13] Note that the Divisia index of $\boldsymbol{v} - \boldsymbol{u}$ is different from the Divisia index of $\boldsymbol{v}$ minus the Divisia index of $\boldsymbol{u}$.

transformation of Bruno's equation is made at the desegregated industry level. For each industry, we could write:

$$\dot{y}_i^k = \beta_i (c_i^T \dot{v}_i - b_i^T \dot{u}_i) \\ = \beta_i \tau_i + \beta_i \omega_i^T \dot{x}_i \tag{2.16}$$

were again the β's are Domar's aggregation weights for each industry given by the value of the industries' gross output on the total business sector value-added. However, equation (2.17) needs further transformations as the primary factor cost shares in value-added do not add up to one. For that purpose, (2.17) has to be multiplied by the ratio of the business sector value-added y over the industry's value added y_i. Hence, the index of the rate of growth of industry *i* output can be defined as:

$$\dot{y}_i^k = \frac{y}{y_i} \beta_i \tau_i + \frac{y}{y_i} \beta i \omega_i^T \dot{x}_i \tag{2.17}$$

or

$$\dot{y}_i^k = \frac{g_i}{y_i} \tau_i + \frac{g_i}{y_i} \omega_i^T \dot{x}_i \tag{2.18}$$

where input cost shares now add up to one. The industry's productivity growth rate defined on gross output is now multiplied by the ratio of the industry's gross output over the industry's value-added. Therefore, the productivity residuals across all industries, that is

$$\mathbf{v} = \hat{y}^{-1} \hat{g} \tau \tag{2.19}$$

add up to the business sector productivity residual when weighted by the industries' value-added shares in the business sector value-added, similarly to the productivity residuals just derived above. Similarly, the Divisia index of industries' real value-added aggregate nicely, as above, to the Divisia index of the business sector real value-added with the same value-added shares as weights. This does not suffice, however, to show that this third alternative measure of real value-added just derived is identical to the one derived above, except at the aggregate business sector level. That this is the case will be shown in the next Appendix into which all productivity indices will be interrelated.

Appendix 3: Link between various productivity indices

The most desegregated level at which productivity indices can be computed within the framework of the rectangular input-output tables is the one provided by the breakdown of production activities by commodity and industry. Hence, the matrix Γ is the most desegregated matrix of productivity indices for the economy associated with the input-output tables. It assumes that technology is both commodity and industry specific. However, in computing the input requirements associated with each commodity, the square industry by industry impact matrix is used as the commodity technology is unknown. If commodity technologies were known, the square commodity by commodity impact matrix would be used instead to estimate the input requirements and there would be a perfect match between the industry productivity indices and the commodity productivity indices. In the rectangular framework, there is an implicit assumption that technology is industry specific and that outputs and inputs of industries are separable. Therefore, the productivity indices previously derived in Durand and Salem were based on the assumption that productivity growth is industry specific for the industry indices and, in principle, that it is commodity specific for the final demand commodity indices. However, for the latter indices, an implicit industry technology assumption was used to calculate the input requirements associated with each final demand commodity. Hence, commodity productivity growth rates are fundamentally function of industries' productivity growth rates. In any case, if it is assumed that the technology is commodity specific, then, from (2.6) it follows that the productivity indices on industries' real value added reduces to:

$$\mu = \hat{y}^{-1} Y \hat{\rho} i = \hat{y}^{-1} Y \rho \qquad (3.1)$$

That is, industries productivity gains appear as a weighted average of commodity productivity gains, the weights being the industries contribution shares to the value of the net output of commodities. Conversely, if it is assumed that technology is industry specific, the productivity indices on commodities can be expressed as:

$$\rho = \hat{p}^{-1} \hat{e}^{-1} Y^{T} \hat{\mu}\ i = \hat{p}^{-1} \hat{e}^{-1} Y^{T} \mu \qquad (3.2)$$

These indices correspond to the weighted average of industries productivity indices, the weights being the value share of each commodity accounted for by the industries. In the general case

where neither one or the other of these assumptions is made, no relationship can be established between the industry and the commodity productivity indices. Keeping the industry technology assumption, it is possible to relate the commodity productivity indices ρ to the industries' value added productivity indices μ and υ derived in Appendix 2.

The commodity productivity indices are simply a weighted average of the productivity growth rates of the vertically integrated industries from which they originate:

$$\rho = D^T \tau^+ \tag{3.3}$$

where τ^+ is Rymes's vector of interindustry productivity indices defined as:

$$\begin{aligned}\tau^+ = \overline{C}_{[j]}\dot{v}_{[j]} - \overline{B}^T_{[j]}[\dot{u}_{[j]} - (i \otimes D^T)\tau^+] \\ - \overline{H}^T_{L[j]}\dot{l}_{[j]} - \overline{H}_{K[j]}\dot{k}_{[j]}\end{aligned} \tag{3.4}$$

That is as the weighted (matrix C) rate of growth of outputs by commodity (vector v) minus the weighted rate of growth of inputs (intermediate u ,labour l and capital k with matrices of value shares B, H_L, and H_K). This equation is similar to the neoclassical productivity equation

$$\tau = \overline{C}_{[j]}\dot{v}_{[j]} - \overline{B}^T_{[j]}\ \dot{u}_{[j]} - \overline{H}^T_{L[j]}\dot{l}_{[j]} - \overline{H}_{K[j]}\dot{k}_{[j]} \tag{3.5}$$

except that intermediate input growth rates are deflated by the rate of growth of productivity of their originating industries. This is equivalent to replacing the intermediate inputs by the inputs of the supplying industries. Doing that substitution for all industries simultaneously amounts to replace the direct use of intermediate inputs by the indirect use of the primary inputs of all upstream suppliers, that is to integrate all industries vertically. The productivity associated with vertically integrated industries is similar to the productivity associated with final demand commodities as the latter was expressed above as a function of the rate of growth of their direct and indirect primary input requirements. Comparing equation (3.4) and (3.5) leads to

$$\tau^+ = (I - B^T D^T)^{-1}\ \tau \tag{3.6}$$

Using equation (3.3), therefore, final demand commodity productivity growth rates can be expressed as a function of industries' productivity growth rates

$$\rho = D^T [I - B^T D^T]^{-1} \tau \tag{3.7}$$

But, as the productivity growth rates υ just defined on value added can themselves be expressed as a function of industries' productivity growth rates τ by

$$\nu = \hat{g}\, \hat{y}^{-1}\, \tau = \hat{\lambda}^{-1} \tau \tag{3.8}$$

we have:

$$\rho = D^T [I - B^T D^T]^{-1} \hat{\lambda}\, \nu \tag{3.9}$$

In the last expression, the impact matrix for computing the value added matrix Y may be recognized so that we finally have:

$$\rho = \hat{p}^{-1} \hat{e}^{-1}\, Y^T \nu \tag{3.10}$$

Expression (3.10) is identical to expression (3.2) except that it expresses final[14] demand commodity productivity growth rates as a function of υ instead of μ. These expressions may be equated and solved. In a square input-output framework, each industry is producing an exclusive commodity so that the market share matrix is equal to the identity matrix. It follows that υ is equal to μ. In a rectangular framework, it suffices, to solve, that each industry produces one exclusive commodity, i.e., that the market share matrix has full industry rank. This is normally the case for a properly defined industrial classification.

A corollary can be establish from the above results which may have important empirical implications. Since the interindustry productivity growth rates τ^* can be computed from the industry (gross output) productivity growth rates τ which can, themselves, be computed from the industry value added productivity growth rates υ, it is possible to calculate the industries' Divisia index of gross output from current prices input-output flows, final demand commodity prices and primary input prices. Indeed, the interindustry productivity index is obtained by vertically integrating industries such that their production function expresses gross output as a function of the industries' direct and

[14] It consists in fact of the productivity of the commodity output vector **v** as well.

indirect uses of primary inputs. The latter inputs can be computed in both current and constant prices as well as their Divisia index. The Divisia index of industries' gross output can then be equated to the Divisia index of their primary (direct and indirect) inputs plus their interindustry productivity indexes. Given current prices data on gross output, this result means that the gross output deflator can be computed along the above lines.

This is an important result in so far as it is often difficult to deflate the gross output of many service industries. This is particularly the case for business services such as accounting and management services and the growing computer services. No obvious physical measure of output exists for these types of services. National accountants often use last resort prices such as wage rates and other input prices. These prices clearly yield unsatisfactory measure of real output for productivity analysis. Indeed, deflating gross output by some average of input prices has the effect of eliminating productivity gains from the industry. This does not affect aggregate productivity which depends on real final demand but reallocate the productivity gains from the service industries to the good industries. Part of the popular belief that productivity gains are larger for good industries than service industries depend on these measurement biases.

Appendix 4: Tackling the Price Base Valuation Problem

Canadian Input-Output Tables are estimated at either producers' prices or purchasers' prices. This means that all commodity transactions in a given set of tables are expressed in either producers' prices or purchasers' prices[15]. However, value-added is at factor cost[16]. It means that, using, say the producers' prices input-output tables, total final demand at producers' prices will differ from value-added at factor cost. In current prices, it is easy to deal with this problem by fixing λ in the impact matrix so as to exclude all taxes and subsidies for the computed Y value-added matrix which is then at factor cost. Valuation of final demand transactions on commodities does not need to be modified for that purpose. However, the matrix Y must be deflated by commodity price indices at factor cost. These prices are not available from the producers' prices tables. But these prices can be computed along the lines suggested in Durand and Rioux (1989).

Briefly, the methodology proposed by Durand and Rioux consists in computing the intermediate input requirements associated with each final demand commodity. Taxes minus subsidies are computed on these inputs and associated with the corresponding final demand commodity. As producers' prices include these taxes minus subsidies, it suffices to subtract them from the producers' value of the transactions to get final demand at factor cost. An effective overall tax rate per commodity is then computed by dividing the taxes by the factor cost demand on each commodity. The net of tax and subsidy prices are given by equating producers prices to one plus the effective tax rate times the before-tax prices, i.e.:

$$\textit{Producers Price} = (1 + \textit{effective tax rate})\ \textit{Net Price}$$

Price *indices* are obtained by dividing prices in any year by the value of the prices in some arbitrarily selected base year.

15 Producers' prices are the prices of the commodities at the gate of the selling establishment and include all margins such as retail and wholesale margins and taxes paid by producers but exclude margins paid by final users. Purchasers' prices include all margins and correspond to market prices.

16 More precisely, for productivity analysis, it is preferable that the valuation price base of primary inputs includes all direct and indirect taxes. This means that property taxes and other non-commodity indirect taxes should normally be included in the price of capital services.

Dividing prices in equation (4) by their value in the base year gives the relationship between producers and net price indices:

$$\text{Producers Price Index} = \frac{(1 + \text{current year rate})}{(1 + \text{base year rate})} \text{Net Price Index}$$

where the chosen base year is actually 1981. It should be observed here that, from the last equation, both the producers' price indices and the net price indices are set equal to one in the base year. It should also be observed that the relationship between these indices depend on both the current year and the base year effective tax rates. Hence, in the years following the base year, the net price indices will be lower than the producers' price indices only if the effective tax rate in these years is higher than in the base year. Otherwise, the net price indices will be higher than the producers' price indices.

The matrix of value-added by commodity and by industry **Y** has simply to be deflated by the factor cost prices to yield real value-added at factor cost[17]. Hence, this technical issue can easily be resolved. Similarly, final demand at factor cost prices can be deflated by factor cost prices[18].

[17] Factor cost prices for final demand commodities were computed on the basis of the Canadian Input-Output Tables over the 1981-1986 period and could easily be computed for previous years.

[18] Actually, this is how the final demand vector **e** must be expressed to compute productivity indices by end products, except, as noted above, that all indirect taxes paid on the purchase of primary inputs should be included in the price valuation base.

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