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New Alternative Estimate of Real Industry Value-Added for Canada

By

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Abstract

This paper presents a few of the estimates of the real value-added of the Canadian industries that were obtained through the application of a new deflation methodology presented in Durand (1994). These estimates, which bear over the last three decades, are compared to the standard estimates based on the double deflation method. The results show that the estimates based on these alternative methodologies differ substantially so that the choice between them is not a question of theoretical nicety. It does really matter. New theoretical developments are also reported that add further interpretation to the new deflation methodology.

1 - Introduction

Simple ideas are as difficult to explain as axioms are in pure mathematics since they call upon the immediate intuition. This is the case for the new methodology presented in Durand (1994) for the deflation of the value-added of industries. We nevertheless begin this second paper on the subject with new brief intuitive explanations. As it is almost always easier to grasp the significance of a mathematical theory by its properties rather than its axioms, we will, in a similar fashion, also derive the properties of the new deflation method in comparison to the properties of the traditional double deflation method. The comparison of the two methodologies will be illustrated with a series of figures exhibiting some of the empirical results.

We have to add that concepts and methods cannot be proved either right or wrong. Concepts have to be assessed with respect to their usefulness for analysis. This entails some subjective judgmental analysis based on intuition. The associated measurement methodology should be assessed on the basis of its congruity with the underlying concept it intends to capture. Congruity is established by stating statistical or theoretical properties that the ideal measure must satisfy and showing that the proposed measure does satisfy these requirements. We begin by formulating the concept of value-added as clearly as possible in order to avoid the ambiguities that may have prevailed in past discussions on the subject.

2 - An Intuitive Appraisal of the Concept of Real Value-added

The nominal value-added of industries, as conceived by national accountants, represents the value which industries add to the commodities they deliver to final demand. That value is equal to the industries' primary input costs. Two intuitive ideas follow that are discussed in this section. The first is that the production processes of the commodities delivered to final demand are distributed across the many industries of the economy rather than being delimited by industries' boundaries. The other is that the distribution of income of the primary inputs in the economy must itself be related, through the economy's input-output industrial network, to their specific contribution to final output.

The first idea relates the nominal notion of value-added in terms of costs and sales with the idea of a real production process. Industries may be seen are contributing *jointly*, by using their capital and labour resources, to the production of the commodities delivered to final demand. In that production framework, each industry is viewed as contributing only partly to the production of some final demand commodity (ies). The contributions of each industry to all final demand commodities may consequently be seen as its contributions to the net final output of the economy or as its real value-added. Hence, the notion of real value-added being primarily analyzed here is tied to the notion of a production function specified on commodities rather than on industries[1].

Therefore, the economy may be viewed as being made of industries, more or less vertically integrated, each of which operating a subset of the transformation processes that are required to deliver goods and services to final demand. The sets of transformation processes are considered as the production functions for these goods and services, one function for each of them. The inputs of these transformation processes comprise only the primary inputs of all contributing industries. Strictly speaking, however, separability between the commodity production processes is not required by what follows. One could specify a more general production function that relates the vector of final demand commodity outputs of the economy to its vector of primary inputs[2].

This opposes to the more recent and widely spread practice which is to specify the production process on industries' gross output, intermediate inputs and primary inputs. According to the latter view, the production function is entirely delimited within the boundaries of the industries, while according to the previous view, it extends across the boundaries of the industries. Otherwise, both views conform entirely to the basic concepts and results of modern production theory of which they constitute alternative applications. Furthermore, nothing in production theory gives precedence to either one of these models. In fact, these models, rather than opposing themselves, will be shown below to provide complementary views of the production process of the economy in which the industry gross output model can be viewed as an incomplete structural specification form while the final output model can be viewed as the complete reduced form.

The new methodology is consistent with standard results of general equilibrium theory. It assumes that firms base their decisions on the current relative prices of inputs with respect to output prices in order to minimize costs or to maximize profits in competitive mar-

1. That it is was indeed also the notion that national accountants had in mind from its inception in national accounting can be found in Fabricant (1940): "The ideal index of the net physical output of an industry would measure the changes in the aggregate value of net output attributable exclusively to changes in the physical quantities of the final products and to changes in the quantities of the materials and other commodities consumed in the fabrication of the final products..." (p.25).

2. Though we used the input-output relationships to relate specific input uses to each final demand commodity separately.

kets. If industries were perfectly integrated vertically, they would deliver their whole output to final markets and they would not purchase any intermediate inputs. Hence, their output prices would be the final demand prices and their input prices would be their primary input prices. But the market leads to the same choices as if the industries were perfectly integrated. Decision makers just split their decisions into many steps through the market rather than through transactions internal to the firm. It follows that the degree of vertical integration of industries should not change the valuation of their real value-added or the real income of their primary inputs. The new deflation method estimates the real value-added of industries as if they were perfectly vertically integrated and therefore satisfies the above requirement. Double deflation does not.

The primary input costs of industries are to be contrasted with their intermediate input costs which are the costs of goods and services they purchase from *upstream* supplying industries.Through a process of successive substitution of the input costs of upstream industries to their sales, intermediate input costs may, in turn, be seen as the direct and indirect value-added of upstream industries or their primary input costs. Hence, the second idea developed in this section is that the value of the gross output of any industry may conceivably be split into its value-added and the value-added generated in all of its direct and indirect upstream suppliers, that is, as income accruing to its primary inputs as opposed to the income accruing to the primary inputs of the upstream industries.

On intuitive grounds, therefore, the issue of deflating value-added may also be seen as the one of estimating the real income of the primary inputs of the industries or, of computing the disaggregated functional income distribution of the economy. At the macroeconomic level, the functional income distribution is estimated by applying the *nominal* income shares of capital and labour to the deflated value of total output. Income shares are not deflated. Only output is deflated with the output price itself. It is a single deflation process.

Similar to the aggregate level, a single deflation method, applied at a disaggregated level, has also been proposed in the past. It consists in deflating the industries value-added by their gross output prices. Single deflation has generally been rejected as an alternative to double deflation but, as will be shown below, it makes much sense when used to measure the real primary input costs of industries in the framework of an industry production model in which output is taken to be the gross output of industries and the inputs comprise intermediate inputs in addition to primary inputs.

Single deflation is also at the root of the new deflation method: This method consists in applying the single deflation method to the alternative production model in which output is taken to be the direct and indirect real final demand deliveries of industries. Consequently, inputs comprise only the primary inputs of industries and output prices are final demand prices. Lets call it the *indirect* single deflation method to oppose it to the direct single deflation method based on the gross output price and which consists in deflating symmetrically all intermediate and primary inputs of industries.

Similarly to the computation of the functional distribution of income at the macroeconomic level, it follows that the real income of the primary inputs of any industry and the real income of the primary inputs of its upstream supplying industries should be proportional to their *nominal* share of output, that is, they should be derived from the nominal income of these inputs deflated by some specific output price. Therefore, as explained with more details in the previous article, the application of the new deflation method is extremely simple. It consists in three steps. First, using the standard current price input-output model (its impact matrix), the nominal value-added of industries made on each final demand delivery (a matrix) are computed. This gives also, equivalently, the associated nominal income of their primary inputs. Second, these value-added are deflated by the corresponding commodity prices. Third, these deflated value-added are aggregated over commodities to give the real value-added of industries.

This, by itself, eliminates double deflation as a possible candidate to estimate the real value-added of industries. Indeed, according to the double deflation methodology, intermediate inputs are deflated by their own price indices. Their total value is subtracted from the deflated value of industries' output to obtain the measure of their real value-added. The price deflator of value-added is obtained implicitly by dividing the nominal value-added into the real value-added. That price could only by chance, or under very restrictive conditions, be equal to the intermediate input average price. Hence, double deflation does not generally distribute the real income of the primary inputs in the economy according to their nominal income shares.

Again, the issue consists mainly in distributing the real net output of the economy as measured by its real final demand to the originating industries. This involves a two-step operation. Both the double deflation and the new methodology consist, first, in *distributing* the real value-added of each final demand commodity by industry and, secondly, in *aggregating* back the value-added over commodities to obtain real value-added by industry.

As will be made clear by the empirical results shown below, the index number issue of aggregation is of very minor importance. Indeed, the empirical results have been compiled using the fixed base year Laspeyres index aggregation formula for both the new and the double deflation methodologies. This has been done primarily in order to isolate the fact that the divergences between the results of the two methods originate essentially from differences in the way final demand value-added by commodity is first distributed to the industries.

The question, therefore, is primarily a *distributional* issue contrary to what is sometimes perceived as an index number issue, that is as an *aggregation* issue. It follows that the double-deflation method cannot be salvaged by switching from the fixed base year

Laspeyres index number formula generally applied by national accountants to some more sophisticated chained index number formula.

To summarize, the new methodology attempts to determine the functional income distribution of capital and labour at the disaggregated level of industries, taking into consideration that final output consists in many goods and services and that these goods and services are related in a specific manner, through the input-output network, to the use of the primary inputs of the various industries in the economy. As stated above, the primary income shares are determined by the input-output impact matrix, the so-called Leontief inverse. One may either use the deflated income shares, which correspond to the double deflation method, or the nominal income shares, which correspond to the new methodology and to the standard way of calculating the functional income distribution at the macroeconomic level.

In the case of the double deflation method, the deflated income shares are a function of the base year relative prices, while in the case of the new method, the deflated income shares are based on current year relative prices. This is essentially why the results of the two methods differ. This is also, perhaps, the limit of the comparisons which can be established between these alternative methodologies on intuitive grounds. We therefore switch to the comparative analysis of their properties in the next section, starting first with the statistical properties and continuing with the economic properties.

3 - Properties of the Alternative Methodologies[3]

Statistical properties

Since the double deflation measure of industries' net output consists in *subtracting* an estimate of the real value of their intermediate input uses from an estimate of their real gross output, it is closely linked to the Laspeyres index number formula. Consequently, in the application, double deflation is generally based on a fixed base year Laspeyres index number formula. The use of other aggregation formula for the computation of the real value of output and intermediate inputs would therefore, despite that they have been proposed at times as the *solution* to the problem of the double deflation method, involves some theoretical inconsistency. With the new methodology, no such constraint is imposed on the choice of the index number formula for aggregating inputs and outputs.

In current practice, the base year applies to a delimited number of years before being moved forward. Then either the whole historical series of real value-added are entirely based on that new base year (historical series are re-based on the new set of relative pric-

3. Some of the theoretical and statistical properties have already been reported in Durand (1994) and will be only briefly discussed here.

es) or historical series are statistically linked to the new estimates while still based on the set of the past relative prices of their previous base year as done in Canada[4]. The estimates to be presented below are based on the Canadian method of linking estimates across base years for both the new and the double deflation methodologies.

As stated in the previous article, the new deflation method does not have the major defects of double deflation, including the potential generation of a negative real value-added when nominal value-added is positive (or a negative implicit price deflator) or yielding real value-added estimates greater than gross output when it cannot be explained by subsidies. Indeed, the new implicit deflator of value-added is a regular weighted average of final demand positive commodity prices and it generally exhibits a very smooth behavior contrary to the double deflation implicit deflators which often exhibit erratic behavior. As will be illustrated below, the new deflators are also generally highly correlated to the corresponding gross output deflators and more so than is the case for the double deflation deflators. This result follows from the theoretical link which will be established between the new deflators and the gross output deflators.

In any case, the new implicit real value-added deflators cannot take negative values, which is a condition which should be met by any proposed estimator to be acceptable from the point of view of pure statistical theory. Indeed, in statistical theory, an estimator is defined as a statistics which takes its values *within* the parameter space. This is not the case for the double deflation estimator of the implicit value-added deflators. That estimator, therefore, is not acceptable.

As mentioned above, the new deflation methodology distributes the final output of the economy on the basis of the impact matrix of the economy in current prices and that distribution is *uniquely* defined. The double deflation method distributes the same final output on the basis of the constant price impact matrix. That distribution is not uniquely defined as it depends arbitrarily on the choice of the base year. Still, the constant price impact matrix involves the specification of sets of input-output coefficients in constant prices which depend arbitrarily on units of measurement as shown in Durand and Markle (1994). These authors have shown that the input structure of an industry defined as a set of coefficients adding up to one can only be meaningfully defined in current prices. Hence the *distribution* of final output to industries based on double deflation depends on arbitrary units of measurement of inputs and outputs and, if the latter are measured in constant prices, on the arbitrary choice of the base year[5].

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4. The historical rates of growth of industries' real value-added are preserved and the series are projected backward from the new base year on that basis while the following years are established on the basis of the relative prices of the new base year.

5. One must note carefully here that we are not referring to the fact that final demand aggregate output, in real terms, may be dependent on the choice of the base year but its distribution across industries itself.

Economic theoretical properties

On the economic theoretical side, besides its clear conceptual meaning, the validity of the new methodology does not rest on the stringent assumptions of double deflation. In particular and similarly to direct single deflation, it does not rest on the separability condition between intermediate and primary inputs, of either the weak or strong category. This is because real value-added is not obtained by subtracting some estimate of real intermediate inputs from the estimate of real gross output. In general, real value-added plus real intermediate inputs does not equal real gross output. Nor does the validity of the new methodology depends on other specific assumptions about the production function[6].

Furthermore, as explained in Durand (1994) and further detailed below, the new methodology allows for the measurement of multifactor productivity gains of industries without bias, contrary to double deflation. Finally, interpreting changes in the relative intermediate to output prices as changes in the terms of trade of industries, we show below that only the new methodology generates meaningful results.

The direct single deflation method, in the gross output production framework of industries, provides estimates of the real primary and intermediate input costs of industries. Dividing these costs by the corresponding quantities of the inputs gives their real prices. Under market equilibrium, these real input prices measure their real marginal products. It is easy to show that the changes in the real primary and intermediate input prices through time, properly weighted by their nominal cost shares provide estimates of industries' multifactor productivity gains on their gross output. This is indeed the standard neoclassical "dual" multifactor productivity index number formula: Productivity growth is given by the weighted changes in input prices minus the change in the gross output price.

Similarly, deflating the value-added of industries by (a weighted average of) the prices of the commodities that they have directly and indirectly delivered to final demand provides an estimate of their real primary input costs in the alternative production framework in which final demand deliveries are taken to be the output of the production processes. Dividing again the real primary input costs by the quantity of the primary inputs used gives their real prices. Weighting these price changes by the cost share of each input in the total value of final demand deliveries provide an estimate of productivity gains on final output. Allocating these productivity gains according to the participation shares of each industry in final output gives the estimates of their productivity gains on value-added. Alternatively, this may be seen as weighting these real input price changes by the cost share of each input in the total value-added of the industry.

6. Bruno for instance, referring to the double deflation bias when estimating the marginal products of primary inputs and the assumptions needed for that bias to vanish states that "...single deflation value-added (SVA) functions would be free of the above bias even without any special assumptions" (Bruno, 1978, p.10).

Growth in the real value-added of industries, consequently, is provided by adding their productivity gains to their primary input growth. The estimates of real value-added are equivalently obtained by adding up the real primary input costs of the industries. This is similar to adding all primary and intermediate input costs in the industry gross output production framework. By construction, these costs sum to real gross output. In the alternative production framework, the real primary input costs of the industries also sum, by construction, to their real value-added or their real direct and indirect final demand deliveries.

As reported in the previous article, double deflation will give results identical to the new method only when either it satisfies the strong separability condition between intermediate and primary inputs, when relative intermediate to gross output prices remain constant or when the input-output technical coefficients are fixed. Note that these conditions are the only three cases in which double deflation properly deflate real value-added, that is, according to the criterion fixed by Bruno (1978), when it provides an unbiased estimate of multifactor productivity growth on gross output up to a scaling factor. The new method satisfies that criterion without imposing those restrictions as shown below.

4 - Examples of Similar Empirical Results[7]

As a simple example, the new method would consist in deflating the value-added of the manufacturers of loves of bread, if all loves of bread were delivered to final demand, by the price of bread. But the value-added of the retailers of bread realized on the sales of bread would also be deflated by the price of bread as well as the contribution of all other industries to the value-added of bread.

Deflating value-added in the trade industry by the price of the commodities sold to final demand is quite reasonable as it amounts to set a real percentage trading mark-up on sales. If all sales prices were to double without any change in real sales and if the trade industry would charge a fixed percentage mark-up on nominal sales, as is usually the case, then the nominal trade margins would also double. This means that the real margins and the real value-added of the trade industry would remain unchanged as it should for an equal volume of trading services offered. The real value-added of the trade industry would increase only if the nominal percentage trade margin would increase.

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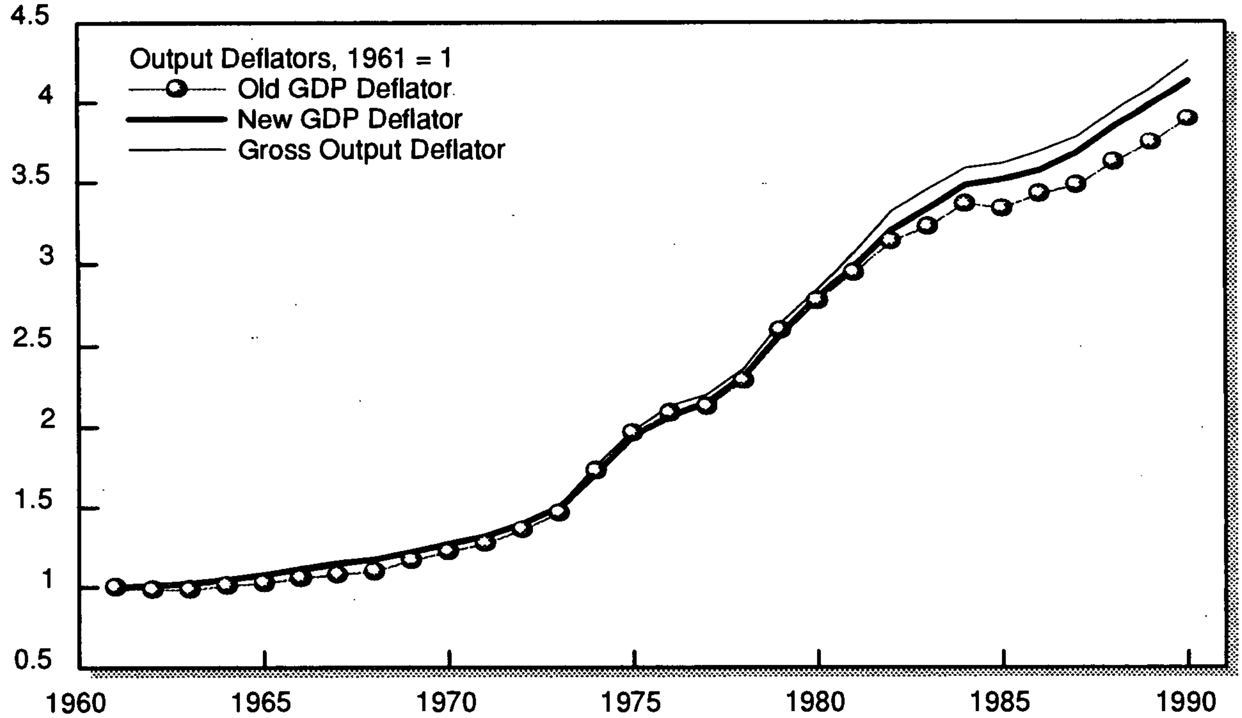
7. Some adjustments have to be done to the equations presented in Durand (1994) in the open economy case to properly account for the use of imported inputs. Similarly, necessary valuation adjustments need to be made to final output, usually expressed in either producers' (or purchasers') prices, to express it at factor costs, taking into account the indirect taxes paid and subsidies received on intermediate inputs (and final output for the purchasers' price base). These adjustments have been incorporated in the estimates. For further details, see Durand, R. and R. Rioux (1994).

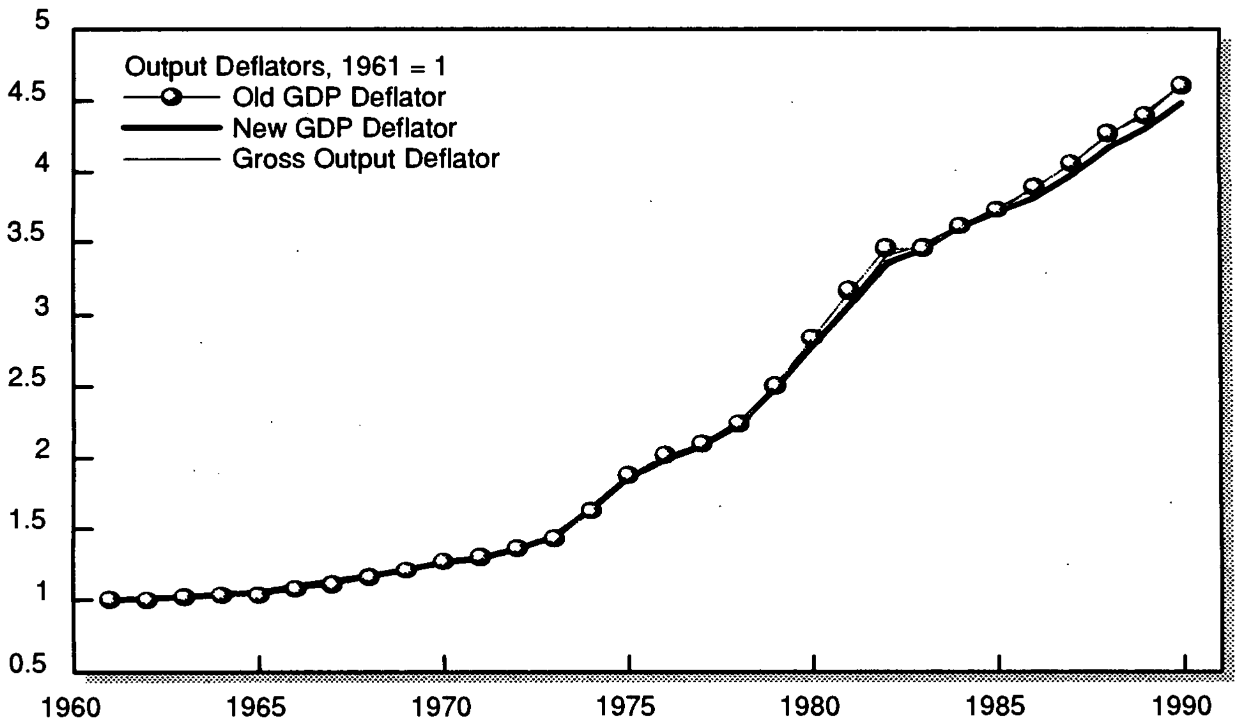
Figure 1 - Alternative Output Deflators for the Canadian Wholesale Trade Industry, 1961 to 1990, 1961 = 1.0.



The reader may look at the empirical results for the Wholesale and Retail Trade industries on the accompanying figures 1 and 2 (industries 135 and 136 at the L level of aggregation of the Canadian input-output tables)[8]. The new value-added deflators for these industries are almost identical to the ones actually used for the production of the official Canadian estimates and which are based on this idea of a percentage margin. The indirect single deflation and the double deflation methods give almost identical results for these industries as well as for other industries not reported here. Results are similar when the relative intermediate input to the gross output prices are fairly stable through time as will be shown more formally below. In those cases, the implicit value-added deflators of either types are close to the gross output deflator. Direct single deflation would, therefore, provide similar results in those cases as well.

8. In the figures, the new deflator of value-added is referred to as the new GDP deflator and the double deflation deflator as the Old GDP deflator. Both are compared to the gross output deflator. For ease of comparison, 1961 was selected as the base year.

Figure 2 - Alternative Output Deflators for the Canadian Retail Trade Industry, 1961 to 1990, 1961 = 1.0.



5 - Some further theoretical considerations

Productivity growth estimates based on the new real value-added measures and the estimates based on the gross outputs of industries form a perfectly consistent set of estimates of productivity gains. Productivity growth on value-added (vertically integrated production processes or reduced form of the production model) takes into account the productivity gains made in the *production* of the intermediate inputs (considered as endogenous or produced inputs) while the computation of productivity gains on gross output (non-integrated production processes or structural form of the production model) does not take the productivity gains made on the production of intermediate inputs (considered as exogenous or non-produced) into account.

To account for the productivity gains made on intermediate inputs, the latter have to be taken as endogenous and equations have to be added to account for their production. The reduced form of the complete set of structural equations gives the alternative production framework in which final demand commodities are a function of the primary inputs of the various industries that have contributed directly or indirectly to their production. This is the basis for the interindustry productivity growth equation to be introduced below.

Equivalently, the measure of productivity gains on gross output does not take into account the productivity gains made in the indirect *use* of upstream industries' primary inputs, a fact which is expressed by the difference in the rate of growth of intermediate input uses and the rate of growth of the primary inputs used in their production[9]. The industry productivity gains attached to the use of primary inputs should be the same whatever the source of the primary inputs. Accounting for all primary input uses, therefore, amounts to add the missing component into the productivity equation as follows:

If τ_y is the productivity gains on primary inputs, productivity gains on gross output, τ_g are usually measured as:

$$\tau_g = \lambda_y \tau_y + (1 - \lambda_y)\, 0 \tag{1}$$

where λ_y is the share of the primary input costs of the industry. Allocating the same productivity estimates to the primary inputs of the upstream industries in (1) would inflate the productivity gains from τ_g to τ_y. This can be equivalently done by solving the previous equation for τ_y, which explains why the inflation factor is $1/\lambda_y$.

It will be shown shortly that if real intermediate input prices were constant, real primary input prices would be growing at the same rate in both direct and indirect single deflation models and productivity growth on value-added would differ from its value on gross output by the ratio of nominal gross output to value-added (Domar's (1961) integration weight or Bruno's (1978) *scaling* factor $1/\lambda_y$). But it is more generally the case. Productivity growth on value-added is *always* larger than productivity growth on gross output by that factor, not because it is biased but because it covers more production processes as revealed by the discussion leading to equation (1).

Here, a little of mathematics might help clarify the point further. Let p_g and p_f be price indices for respectively gross output, g, and the final demand deliveries, f, of an industry (both direct and indirect), that is, its implicit value-added deflator according to the indirect single deflation method. Let w and r be respectively the wage rate of labour L and the rental price of capital services K. Finally, let $\boldsymbol{u}$ and $\boldsymbol{p}_u$ be respectively the vectors of intermediate input uses and corresponding prices. Then real value-added according to the direct single deflation (dsd) and the indirect single method (isd) are given by:

$$\frac{wL + rK}{p_g} = \text{dsd} = g - \frac{\boldsymbol{p}_u^T u}{p_g} \qquad\qquad \frac{wL + rK}{p_f} = \text{isd} = \frac{p_g g - \boldsymbol{p}_u^T u}{p_f} \tag{2}$$

Now, it can be seen immediately from the left hand sides of these two equations that these two estimates of real value-added will be equal when and only when p_g is equal to p_f. That

9. This provides a clue as to why there are two alternative estimates of the real income of the primary inputs of industries and of how the benefits of productivity gains are transmitted and shared in the economy.

is, the two methods give identical results in that particular case. Otherwise, the estimates of real value-added obtained from the two methods differ.

The productivity measures associated with gross output and real value-added are:

$$\tau_g = \omega_{gu}^T \dot{p}_u + \omega_{gL} \dot{w} + \omega_{gK} \dot{r} - \dot{p}_g \qquad \tau_y = \omega_{yL} \dot{w} + \omega_{yK} \dot{r} - \dot{p}_f \tag{3}$$

where, as above, we have used the subscript "g" to refer to gross output and "y" to refer to value-added. The cost shares of inputs are given by ω with the appropriate subscript, while growth rates of prices are given by dotted symbols. Therefore, the first equation in (3) reads: Multifactor productivity growth on gross output is given by the cost share weighted rates of growth of input prices minus the growth rate of the gross output price. This is the well known neoclassical dual definition of multifactor productivity on gross output which we referred to above.

The second equation in (3) reads: Multifactor productivity growth on real value-added is given by the cost share weighted rates of growth of primary input prices minus the rate of growth of the new value-added price (or more generally, of any price used to deflate nominal value-added). Note that the cost shares in the first equation are taken with respect to the value of gross output while, in the second equation, the cost shares are taken with respect to nominal value-added.

Let us assume, as a particular case, that gross output, real value-added and intermediate input prices remain constant[10]. In such a case, the two productivity measures differ by the value of nominal gross output over nominal value-added, exactly as required by Bruno's no bias criterion. This result follows by comparing the weights which applies to the rate of growth of the nominal primary input prices in the two formula. In one case, the values of the inputs are divided by the value of gross output and in the other case, they are divided by nominal value-added. In that particular case, single deflation and the new method (which will be shown to satisfy Bruno's criterion below) will give the correct measure of productivity growth as well as double deflation.

The weight differences in the two equations of (3) always apply whatever happens to prices. Consider another particular case in which the relative intermediate input to output prices remain constant. Then, one of the three alternative conditions that need to be satisfied by double deflation to yield the correct measure of productivity growth is met according to Bruno (1978). It is also equal to the new measure in that case. In such a case, productivity

10. Note that the implicit value-added price index is endogenous and that such condition may not be satisfied by all alternative definitions of that price. For instance, it will be satisfied by the direct single deflation value-added price deflator when the gross output price is constant, whatever happens to intermediate input prices. This single condition is not sufficient for the double deflation price index to be constant as can be seen from equation (7) below.

on value-added will be equal to productivity on gross output multiplied by the same inflation factor as above (namely, the value of gross output over value-added) only if the rate of growth of the final demand weighted prices (the new value-added implicit deflator) is identical to the rate of growth of the gross output price. That it is indeed the case will be shown below. Direct single deflation, therefore, also gives the correct answer in that case. But for the time being, it can be noted that the above equation provides quite a strong relationship between these prices. To conclude, whether firms look at the gross output prices or the final demand prices provides them with exactly the same signals. Stated differently, the behavior of the market does not depends on the degree of vertical integration of firms or industries.

To help in the understanding of the price relationship, let us transform the productivity growth equations in (3) above in two ways. First, let us multiply the productivity gains on gross output by the nominal gross output to value-added ratio $1/\lambda$, where $\lambda = p_f y/p_g g$. This allows us to equate the two productivity measures according to Bruno's economic criterion or equation (1) (and only provisionally in the case of the new deflator until we prove that it satisfies that condition in all cases). Second, let us transform all weights in the first productivity equation by taking them out of value-added rather than out of gross output. That is, let us multiply all weights by λ and change their subscript (i.e. $\omega_g = \lambda\omega_y$):

$$\frac{\tau_g}{\lambda} = \frac{1}{\lambda}[\lambda\omega_{yu}^T(\dot{\boldsymbol{p}}_u - \boldsymbol{i}\dot{p}_g) + \lambda\omega_{yL}(\dot{w} - \dot{p}_g) + \lambda\omega_{yK}(\dot{r} - \dot{p}_g)] = \tau_y \tag{4}$$

where we have used the fact that the sum of the weights on the gross output price changes is equal to one to insert the rate of growth of the output price as a deduction to the rate of growth of the input prices. Now clearly, λ cancels out everywhere in the second expression of (4) and replacing τ_y by its value from (3), we get:

$$[\omega_{yu}^T(\dot{\boldsymbol{p}}_u - \boldsymbol{i}\dot{p}_g) + \omega_{yL}(\dot{w} - \dot{p}_g) + \omega_{yK}(\dot{r} - \dot{p}_g)] = \omega_{yL}(\dot{w} - \dot{p}_f) + \omega_{yK}(\dot{r} - \dot{p}_f) \tag{5}$$

Now, it should be pretty clear that, for each industry, there is a close relationship between the intermediate input prices, the gross output prices and the optimal (bias free) value-added implicit deflator. It can also be seen from (5) that the alternative estimates of the real primary input prices much depends on what happens to the relative intermediate input to output prices. If intermediate input prices grow faster than the gross output price, then the real primary input prices from the industry perspective increase at a lower rate than for the vertically integrated set of industries. Eliminating the primary input prices from the last relationship gives:

$$\dot{p}_g + \omega_{yu}^T(\boldsymbol{i}\dot{p}_g - \dot{\boldsymbol{p}}_u) = \dot{p}_f \tag{6}$$

The two special cases just discussed above are more easily interpreted from (6). Case one (no price changes) is trivial and case two is given when the second term on the left hand side of equation (6) is equal to zero. In that case, the equality is maintained only if the gross output and the final output prices (implicit value-added deflator) grow at the same rate, i.e. their indices are identical. This shows why the three deflation methods (single, double and new) are identical in that case[11].

Now, when the intermediate input prices grow faster than the gross output price, the residual of these two, on the left hand side of the price relationship is negative. Solving that inequality, therefore, shows that the optimal value-added price is growing less rapidly than the gross output price in such a case. But that is quite consistent with the idea that the *terms of trade* of the industries are deteriorating, i.e., upstream suppliers are gaining greater real value-added out of the joint production of the final demand deliveries. Indeed, if intermediate input prices rise, the output price will rise to maintain the equality between costs and sales. With the value-added deflator rising less rapidly than the intermediate input prices, the share of nominal value-added in the value of the gross output of the industry will fall while the share of intermediate input costs, which represent the nominal value-added of upstream industries, will increase. The converse applies when intermediate input prices grow less rapidly than the gross output price.

Direct single deflation would not properly take that phenomena into consideration and would overstate the downward adjustment made to the real value-added of the industry. Aggregating over the real value-added of the industries will not yield a value equivalent to the real final demand. Direct single deflation does not generally satisfy the aggregate balance equation except under one of the two polar cases under which double deflation also gives the correct answer. Hence, for the purpose of allocating the *total* real value-added of the economy to originating industries, double deflation is always, under that criterion, at least as good or better than direct single deflation.

This also explains why the directly deflated primary input prices do not necessarily grow at the same rate as multifactor productivity measured on either gross output or value-added. Their rates of growth also depend on the changing relative prices of intermediate inputs with respect to gross output prices. With fixed relative prices, the real primary input prices grow at the same rate under both the gross output and the final demand output production framework. Therefore, weighting their rates of change using value-added cost shares gives an unbiased measure of total factor productivity growth in all cases only in the latter production framework.

To clarify things further, following similar derivations (see Appendix 2), we can show that:

$$\dot{\pi} - \dot{p}_{\mathrm{g}} = \omega_{\mathrm{yu}}^{\mathrm{T}} (i\dot{p}_{\mathrm{g}} - \dot{\boldsymbol{p}}_{\mathrm{u}}) + \left(\omega_{\mathrm{yu}}^{\mathrm{T}} - \frac{u^{\mathrm{T}}}{y}\right)(i\dot{g} - \dot{\boldsymbol{u}}) \tag{7}$$

11. Note that the second term of the left hand side of equation (6) is a scalar. The changes in the gross output price are compared to cost share weighted average of the changes in intermediate input prices.

where π is the implicit value-added deflator obtained from double deflation. The first term of that relationship on the right hand side is similar to the one of equation (6). The second term on the right hand side measures the bias associated with double deflation. It depends on the relative growth of the quantities g and $\boldsymbol{u}$ and on the difference between current and constant price input-output coefficients. If the current price coefficients are equal to the constant price coefficients (no change in relative input-output prices), then double deflation satisfies condition (6). As well, double deflation meets condition (6) when intermediate input-output coefficients are constant. In general, however, the second term, on the right hand side of the last equation, will be different from zero. Current price input coefficients tend to diverge from constant price coefficients when relative input to output prices change. The sign of the second term, however, cannot be determined without knowing the elasticity of demand for the intermediate inputs following a change in their real prices[12].

In many cases, intermediate input prices grow more or less at the same rate as gross output prices since the intermediate inputs of industries are themselves the output of other industries except for imported inputs and inputs supplied from other sources. Hence, one would expect that the second term, on the left hand side of the equation (7), be close to zero. In other words, one would expect that the gross output prices of industries be closely correlated with their new value-added implicit deflators generated from final demand prices. That this is the case has been generally observed over the Canadian historical record of the last thirty years.

6 - Divergent empirical results

Noticeable exceptions are provided by many industries. In those cases, the indirect single deflation and the double deflation methods give widely divergent views of the history of the last thirty years. For instance, large discrepancies were observed for the Refined Petroleum & Coal Products industry starting around the first energy shock of 1973 when relative input/output price changes have been important. This is a case in which the gross output price grew less rapidly than the intermediate input prices up to the mid-eighties and, therefore, according to the new methodology, when the new implicit value-added deflator grew less rapidly than the gross output deflator (see figure 3).

With the deterioration in the terms of trade, the industry's share of total value-added fell; it also happened that the real value-added of the industry grew much less rapidly than real gross output as shown on figure 4 (which is not necessary implied by the above results). The share of the real value-added accruing to the industry declined as the nominal share

12. Hence, an improvement in the terms of trade of an industry may lead to a reduction of its real value-added and conversely, a deterioration in its terms of trade lead to an improvement in its real value-added! That this is sometimes the case with double deflation was observed from the numerous charts of the price indices which were drawn.

Figure 3 - Alternative Output Deflators for the Canadian Refined Petroleum & Coal Products Industries, 1961 to 1990, 1961 = 1.0.

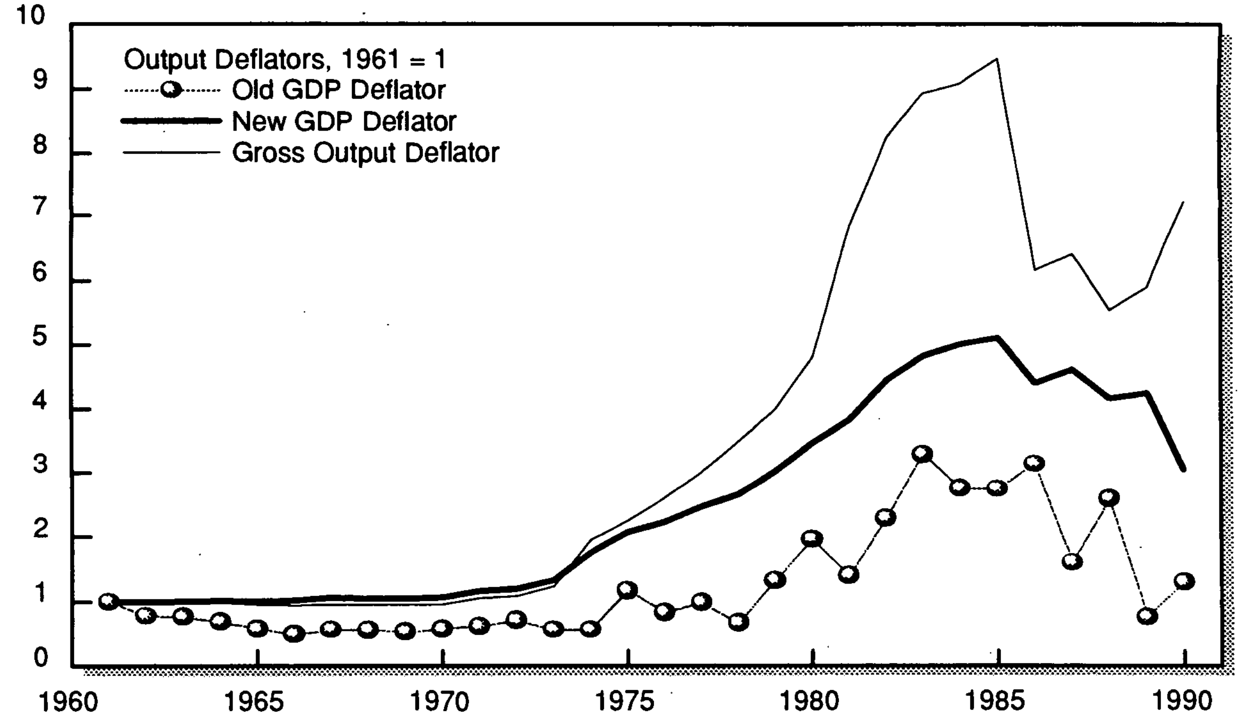


Figure 4 - Alternative Measures of Output for the Canadian Refined Petroleum & Coal Products Industries, 1961 to 1990, 1961 = 1.0.

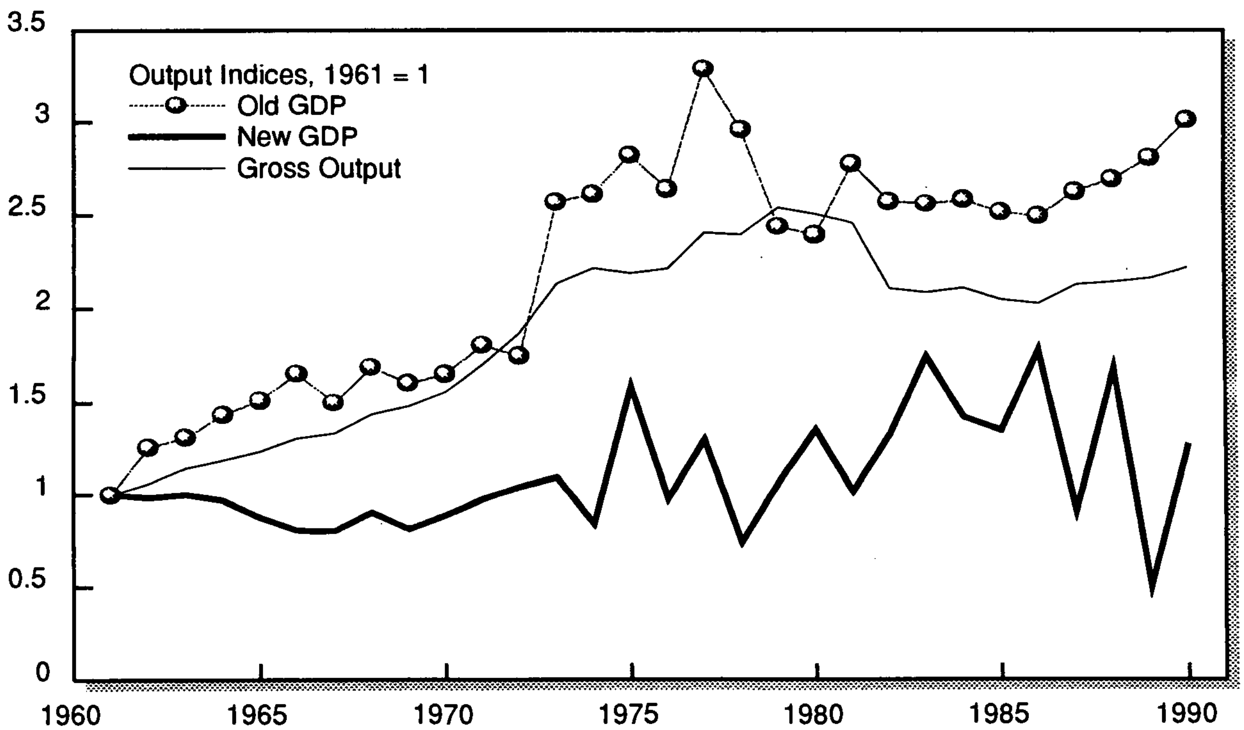


Figure 5 - Alternative Output Deflators for the Canadian Crude Petroleum & Gas Industries, 1961 to 1990, 1961 = 1.0.

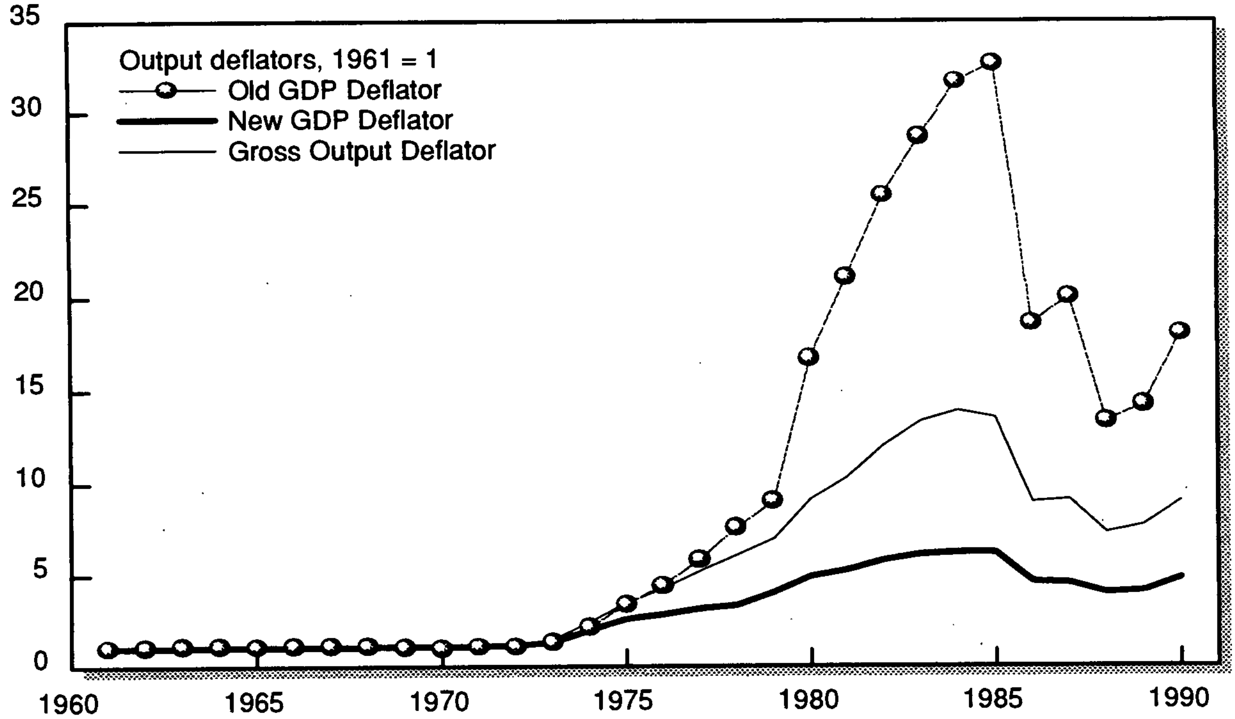


Figure 6 - Alternative Measures of Output for the Canadian Crude Petroleum & gas Industries, 1961 to 1990, 1961 = 1.0.

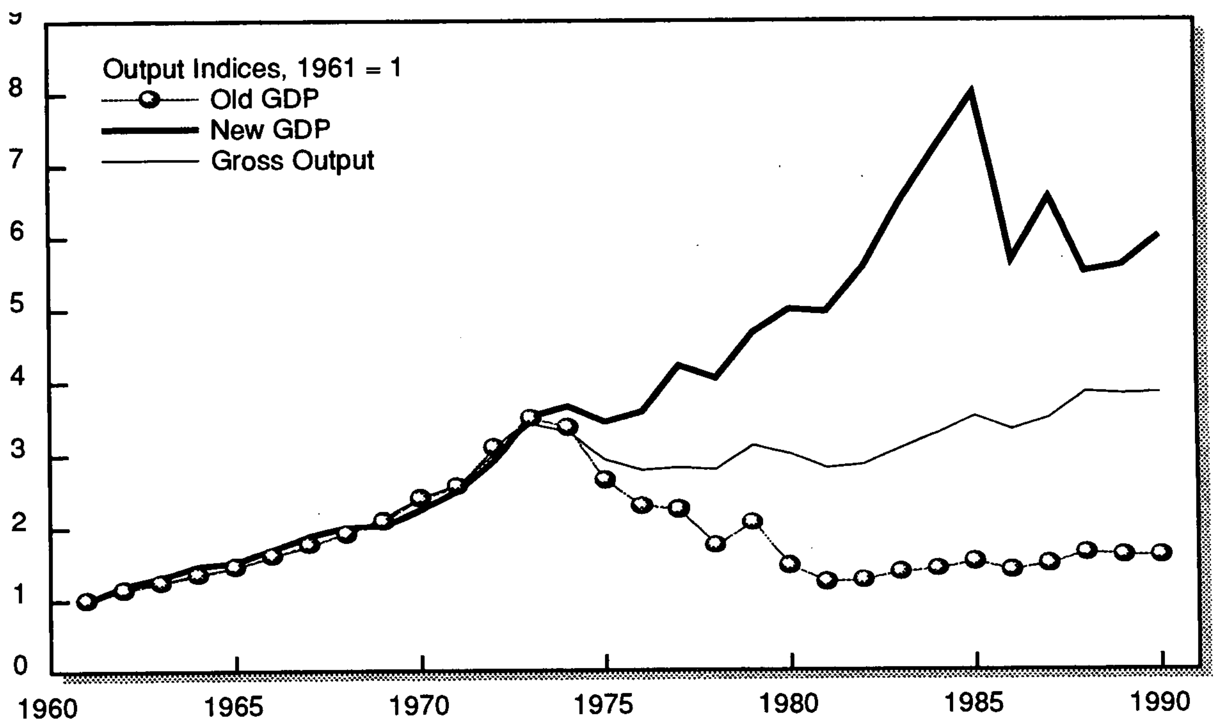
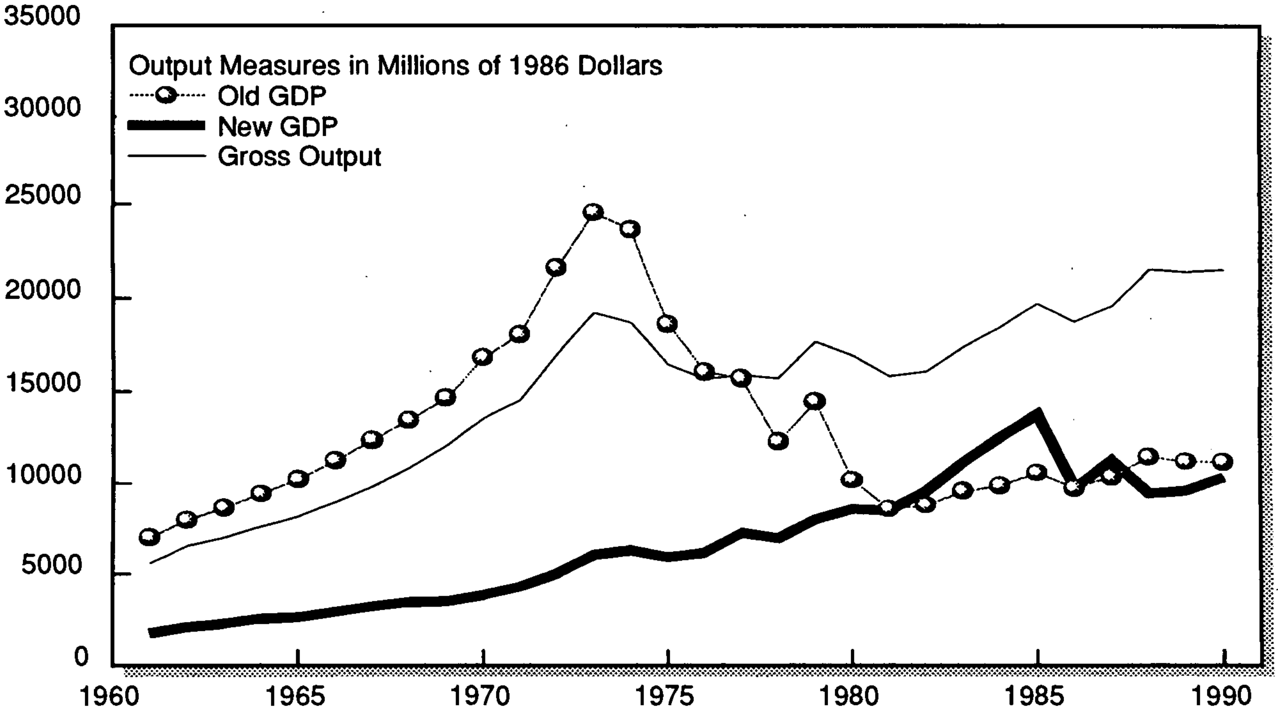


Figure 7 - Alternative Measures of Output for the Canadian Crude Petroleum & Gas Industry, 1961-1990, millions of 1986 dollars



of intermediate inputs increased (that is the nominal and real shares of total value-added of upstream industries)[13].

As depicted on figure 6, the new estimate of real value-added of the Crude Petroleum & Gas industry, the major upstream supplier of the Refined Petroleum & Coal Products industry, increased as expected and more so than its gross output. But as shown on figure 5, in that case, the new value-added deflator also increased by less than the gross output deflator suggesting that intermediate input prices grew by still more as well as the share of value-added of upstream suppliers. The data, in effect, show that this is the case. The same upward trend in the intermediate input nominal share was observed for the Refined Petroleum & Coal Products industry: Intermediate input prices grew more on average than

13. According to equation (A1.3) of Appendix 1 and the optimality condition (6), one has the primal optimality condition $\dot{y}-\dot{g} = \omega_{yu}^{T}(i\dot{g}-\dot{u})$. Hence, the ratio of value-added to gross output increases when the intermediate input-output coefficients decrease. This is to be expected when the relative intermediate input to output prices increase. In the present case, a large portion of the "intermediate" inputs is comprised of imported crude oil so that a larger share of the real value-added of final goods goes to non-residents rather than to upstream industries.

the gross output price for these two industries up to the middle of the eighties[14]. This seems reasonable for the Refined Petroleum & Coal Products industry which uses crude petroleum as its major input. The reason why this happened in the Crude Petroleum & Gas industry is not that obvious and would need further investigation. In any case, we have an example, with this industry, where the value-added price increased by less than the gross output price and yet, real value-added nevertheless increased more than real gross output, pushed upward by what is happening in the major downstream industry using crude petroleum.

Double deflation gives a substantially different message for these industries. The real value-added of the refined Petroleum & Coal Products industry increased and the one of the Crude Petroleum & Gas industry declined. This is inconsistent with the increased profitability of the Crude Petroleum & Gas industry and the higher costs faced by the refined Petroleum & Coal Products industry. Double deflation does not tell the correct story in those cases. In addition, re-basing all prices to 1986 = 100 resulted in a double deflation estimate of the level of real value-added in the Crude Petroleum & Gas industry being larger than real gross output over a large portion of the historical record, namely from 1961 to 1976 (see figure 7).

The subsidies paid to the industry over that period do not justify such a result. Indeed, nominal value-added is consistently and importantly lower than nominal gross output over that period. On the contrary, subsidies became important after the first energy shock only. The consequence of all of this is that the average intermediate input implicit deflator turned negative from 1961 to 1976 when it suddenly jumped to a very high positive value of 23 (from the 1986 base year value = 1)! Therefore, the estimates of the average intermediate input prices that we have computed residually are unreliable. They result from the linking of the highly distorted double deflation real value-added growth rates between the periods when shifting the base year[15].

7 - Further interpretation of the dynamic price equations

Turning back to the previous discussion, the case of separability of intermediate inputs from primary inputs is more difficult to read from equation (7). However, in that case, double deflation provides the correct answer as we know in the limiting continuous Divisia case. But weak separability is not sufficient as shown by Bruno when fixed base weights double deflation is applied. Strong or additive separability is required:

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14. The Chained Paasche price indices indicate effectively that intermediate input prices grew more rapidly than output prices over that period and less thereafter in both industries.

15. This is another issue which is not discussed here.

"When base weight DVA functions are used to measure growth in total productivity, functional separability is no longer sufficient to eliminate the resultant bias. Also, in this case, the direction of the bias is unknown unless the price of intermediate goods relative to the price of output is non-increasing." (Bruno (1978), p.16)[16].

If follows that, for each industry, strong separability must imply[17]:

$$\left(\omega_{yu}^{T} - \frac{u^{T}}{y}\right)(i\dot{g} - \dot{u}) = 0 \tag{8}$$

Note that the last expression is a scalar, that is, a weighted sum of changes in intermediate input-output coefficients. It is the sum which is required to be equal to zero rather than each element of the products as in the previous two polar cases of composite goods. This is a somewhat less stringent condition than the ones of the previous two polar cases.

Hence, double deflation would provide unbiased estimates of productivity growth (up to a scaling factor) only when it provides the same answer as the new method. In all other cases it would provide biased estimates.

Now it was asserted above that the new method satisfies the dynamic price equation (6) so that it provides unbiased estimates of productivity growth in all cases. Still, this has to be shown as we have equated the two productivity growth formula above without showing that the two expressions were always equal in the case where the value-added deflator was the indirect single deflation one. We only know so far that these expressions must be equal for productivity growth estimates on real value-added to be unbiased up to a scaling factor according to Bruno's criterion, whatever method is used to deflate value-added.

To show that the indirect single deflation method satisfies Bruno's condition requires a little more tedious mathematical exercise that we now undertake. But that proof should settle the issue of choosing between the alternative deflation methods. First, it means that in all cases the new method is at least as good or better than double deflation. Second, that the new method provides a measure of real value-added that could be used to estimate multifactor productivity growth without bias. Third, that it provides a clear economic interpretation of the scaling factor converting productivity growth on gross output to productivity growth on real value-added: that scaling factor allows to take into account the

16. See also Denny and May (1977,1978).

17. What the implication of the strong additive separability condition implies for the double deflation price change relationship (7) is not derived here. But this is implied if double deflation is to provide unbiased estimates of productivity growth. But precisely, double deflation has been shown by Bruno to provide unbiased estimates of productivity growth in that case. The second term of (7) does not need to be zero for each component but only in total when summing. Indeed, the shares are row vectors while the growth rates are column vectors so that the expression is a sum of the products. That sum may be equal to zero even if none of its elements is.

productivity gains made in the production of intermediate inputs contrary to productivity measures on gross output which exclude such gains.

First, from Durand (1994), the vector of industries' productivity growth on value-added, τ_y^n , is defined by

$$\tau_y^n = (\hat{\boldsymbol{p}}_f\hat{\boldsymbol{y}})^{-1}(\boldsymbol{Y} \bullet \Gamma)\,\boldsymbol{i} \tag{9}$$

where the superscript *n* (for new) has been added to identify the new method, and where Γ is a matrix of rates of growth of real value-added by industry and by commodity minus the rate of growth of the primary inputs needed to generate that value-added (a matrix of "productivity" measures associated with the cells of the real value-added matrix $\boldsymbol{Y}^k$)[18]. In that notation, $\boldsymbol{Y}$ is the matrix of nominal value-added by industry (rows) and commodities (columns) obtained with the input-output model and $\boldsymbol{Y}^k$ its constant price value obtained by deflating its columns by the final demand commodity prices, $\boldsymbol{p}$. Productivity growth on industry value-added is a weighted average of the productivity gains made on commodities in each industry (cells of Γ) weighted by the share of each commodity in the value-added of the industry. Now according to Bruno, productivity growth on the value-added of *all* industries must satisfy the condition:

$$\tau_y = \hat{\lambda}^{-1}\tau_g \tag{10}$$

where λ ($=1/\omega_y$ in each industry) is now a vector of the nominal value-added to nominal gross output ratios of all industries. Hence, it remains to be shown that this relationship is satisfied also for τ_y^n .

First we note that productivity gains on final demand commodities are weighted averages of interindustry productivity gains on gross output, the weights being the market shares of industries. In other words, the productivity gains on a given commodity, in the reduced form of the production model, is by construction a weighted average of the productivity gains of its industry of origin including the productivity gains of upstream suppliers[19]:

$$\tau_e = \boldsymbol{D}^T\tau_{gi} \tag{11}$$

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18. The same current price impact matrix referred to above properly adjusted for primary inputs can be used to estimate separately the primary input costs associated with the final demand deliveries of industries. The quantities of these primary inputs and their costs shares can therefore be identified and used with the corresponding net output estimates to generate the matrix Γ.

19. This can be shown more formally by using the linear space transformations given in Appendix 2. In a square input-output framework, the matrix $\boldsymbol{D}$ is replaced by the identity matrix and the two productivity measures are more easily seen as being equal.

where τ_{gi} is the interindustry productivity index providing the productivity gains made on industries' commodity bundles (gross output) including the gains made by upstream suppliers in the production of intermediate inputs and where $\boldsymbol{D}$ is the market share matrix of industries which contains the value shares of industries in the production of each commodity. Replacing the interindustry index by the neoclassical multifactor productivity index on gross output τ_g using the relationship (Cas and Rymes (1991)) between these two indices gives:

$$\tau_e = \boldsymbol{D}^T(\boldsymbol{I}-\boldsymbol{B}^T\boldsymbol{D}^T)^{-1}\tau_g \tag{12}$$

where $\boldsymbol{B}$ is the matrix of nominal intermediate input coefficients of industries of the rectangular input-output framework ($\boldsymbol{DB}$ is the square Leontief technical coefficient matrix $\boldsymbol{A}$ and the whole expression is the transpose of the impact matrix for gross output as a function of final demand expenditure). It is shown in Appendix 2 that productivity gains on final demand and productivity gains on value-added according to indirect single deflation are related by:

$$\tau_e = \boldsymbol{D}_y^T\tau_y^n \quad \text{and} \quad \tau_y^n = \boldsymbol{C}_y\tau_e \tag{13}$$

where

$$\boldsymbol{D}_y^T = (\hat{\boldsymbol{p}}\hat{\boldsymbol{e}})^{-1}\boldsymbol{Y}^T \text{ and } \boldsymbol{C}_y = \hat{\boldsymbol{y}}^{-1}\boldsymbol{Y} \tag{14}$$

Productivity gains on industry value-added have already been defined as the weighted average of the productivity gains made on the cells of $\boldsymbol{Y}^k$ above, the average being taken over the commodities in equation (9) using the weights of $\boldsymbol{C}_y$ which give the nominal share of commodities in industries' value-added. Productivity gains by commodity is also obtained by averaging the productivity gains made on the cells of $\boldsymbol{Y}^k$ but, this time, over industries rather than commodities, using the weights of $\boldsymbol{D}_y$ which gives the nominal value-added market shares of industries in the final value of commodities (similarly to $\boldsymbol{D}$ which gives the industries' nominal shares of the gross output of commodities)[20]. Substituting in (12) for the value of final demand productivity gains using (13) gives:

$$\tau_y^n = \boldsymbol{C}_y\boldsymbol{D}^T(\boldsymbol{I}-\boldsymbol{B}^T\boldsymbol{D}^T)^{-1}\tau_g \tag{15}$$

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20. Intuitively speaking, productivity gains on value-added as discussed above when deriving equation (1) refer to the productivity gains made in the production of the commodities included in the gross output of the industry, including the contribution of the primary inputs of upstream industries. Part of that output is sold to downstream industries and eventually ends up in final demand sales. Accounting for productivity gains on final sales is, therefore, shifting downstream the productivity gains made by industries as measured on their value-added. Hence, accounting for productivity gains on value-added or on final sales are just different ways of cumulating productivity gains associated with the use of the primary inputs in the production of specific final sales commodities, using industry "product-mix" or market shares weights.

$$\tau_y^n = C_y(\hat{p}\hat{e})^{-1}(\hat{p}\hat{e})D^T(I-B^TD^T)^{-1}\hat{\lambda}\hat{\lambda}^{-1}\tau_g = C_y(\hat{p}\hat{e})^{-1}Y^T\hat{\lambda}^{-1}\tau_g = C_yD_y^T\hat{\lambda}^{-1}\tau_g \quad (16)$$

where $\hat{pe}$ is the current price vector of final demand. In virtue of the results of Appendix 2, it finally gives:

$$\tau_y^n = \hat{\lambda}^{-1}\tau_g \quad (17)$$

Hence the new value-added productivity measure satisfies the optimality criterion of Bruno in all cases.

8 - Conclusion

This paper has provided first time comparative empirical estimates of real industry value-added for some Canadian industries over the last three decades based on the double deflation method and the alternative method proposed in Durand (1994). Because of the similitude of the new method with the single deflation method based on gross output prices, we suggested to name it the indirect single deflation method.

This second paper has also formulated new statistical and economic theoretical properties of the new methodology or reinforced the derivations presented in the previous paper. On the statistical properties, the new estimators of real value-added and of its implicit price deflator were found to satisfy the defining criteria of a *bona fide* estimator in pure statistical theory while the double deflation estimators were not. In addition, the new estimators of real value-added and of its price deflators are generally more correlated to the corresponding gross output quantity and price estimators than is the case for the double deflation estimators.

Both set of estimators are derived by allocating real final demand expenditure to industries on the basis of income shares. In the case of the double deflation income shares, we have shown that these shares, which are in constant prices, were, at best, dependent on the choice of the base year and, consequently, not uniquely defined, contrary to the case of the shares based on the new deflation methodology.

The new methodology is also more consistent with the manner in which the functional income distribution of the primary factors is determined at the macroeconomic level. That income distribution is based on nominal income shares rather than on deflated income shares as is the case for double deflation.

The new methodology generates an estimate of real value-added of industries which may be used with their primary inputs and their cost shares to estimate multifactor productivity gains without bias. In addition, the new income distribution estimates are consistent with changes in the terms of trade of the industries. This is not generally the case for the double deflation methodology. This was exemplified by the estimates given for the Crude Petroleum & Gas industry.

Finally, we have shown that the underlying difficulty of the double deflation method is not an aggregation issue but rather, a distributional issue. That methodology cannot possibly be improved by using an index number formula other than the fixed base year Laspeyres index number formula actually in use. Hence, on both statistical and economic grounds, the new deflation methodology was shown to fare better than double deflation.

Appendix 1: Dynamic Link Between Prices

The implicit value-added price obtained from double deflation, π, is related to the gross output price p_g (we assume for simplicity but without loss of generality one commodity output) and the intermediate input prices $\boldsymbol{p}_u$ according to the following identity:

$$p_g g = \boldsymbol{p}_u^T \boldsymbol{u} + \pi y \tag{A1.1}$$

This identify actually applies to all implicit deflators π of real value added y given that all such deflators have to satisfy the nominal value-added identity. Any method used to deflate value-added must indeed be such that real value-added times the associated deflator gives a predetermined nominal value-added. Differentiating totally that identity with respect to time, one has:

$$p_g g\dot{g} + p_g g\dot{p}_g = \boldsymbol{p}_u^T \hat{\boldsymbol{u}}\dot{\boldsymbol{u}} + \boldsymbol{p}_u^T \hat{\boldsymbol{u}}\dot{\boldsymbol{p}}_u + y\pi\dot{y} + y\pi\dot{\pi} \tag{A1.2}$$

where the dot symbol represents the percentage time derivative. Solving for $\dot{\pi} - \dot{p}_g$, one has:

$$\dot{\pi} - \dot{p}_g = \omega_{yu}^T (\boldsymbol{i}\dot{p}_g - \dot{\boldsymbol{p}}_u) + \dot{g} - \dot{y} + \omega_{yu}^T (\boldsymbol{i}\dot{g} - \dot{\boldsymbol{u}}) \tag{A1.3}$$

Now, in the special case of double deflation, we have:

$$y = g - \boldsymbol{i}^T \boldsymbol{u} \tag{A1.4}$$

differentiating with respect to time and rearranging gives:

$$\dot{g} - \dot{y} = -\frac{\boldsymbol{u}^T}{y} (\boldsymbol{i}\dot{g} - \dot{\boldsymbol{u}}) \tag{A1.5}$$

Substituting that result into the dynamic price relationship gives:

$$\dot{\pi} - \dot{p}_g = \omega_{yu}^T (\boldsymbol{i}\dot{p}_g - \dot{\boldsymbol{p}}_u) - \frac{\boldsymbol{u}^T}{y} (\boldsymbol{i}\dot{g} - \dot{\boldsymbol{u}}) + \omega_{yu}^T (\boldsymbol{i}\dot{g} - \dot{\boldsymbol{u}}) \tag{A1.6}$$

Grouping the last two terms, one finally gets:

$$\dot{\pi} - \dot{p}_g = \omega_{yu}^T (\boldsymbol{i}\dot{p}_g - \dot{\boldsymbol{p}}_u) + \left(\omega_{yu}^T - \frac{\boldsymbol{u}^T}{y}\right) (\boldsymbol{i}\dot{g} - \dot{\boldsymbol{u}}) \tag{A1.7}$$

Now condition (6) in the text is satisfied when, from (A1.3), we have:

$$0 = \dot{g} - \dot{y} + \omega_{yu}^T (\boldsymbol{i}\dot{g} - \dot{\boldsymbol{u}}) \tag{A1.8}$$

Transforming the weights again to expressed cost shares as fractions of the value of gross output, one has that:

$$0 = \dot{g} - \omega_{gy}\dot{y} + \omega_{gu}^{T}(i\dot{g} - \dot{\boldsymbol{u}}) \tag{A1.9}$$

This is the primal productivity growth equation applied to gross output, intermediate inputs and real value-added. It means that if real value-added is substituted to the primary inputs in the productivity equation specified on gross output, there is no residual productivity gains to account for.

Appendix 2. - Relationships between matrix margins.

Let $\boldsymbol{V}$ be any rectangular matrix and let $\boldsymbol{g}$ and $\boldsymbol{v}$ be its margin vectors such that:

$$\boldsymbol{g} = \boldsymbol{V}\boldsymbol{i} \quad \text{and} \quad \boldsymbol{V}^T\boldsymbol{i} = \boldsymbol{v} \tag{A2.1}$$

Consider the share matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ defined as:

$$\boldsymbol{C} = \hat{\boldsymbol{g}}^{-1}\boldsymbol{V} \quad \text{and} \quad \boldsymbol{D} = \boldsymbol{V}\hat{\boldsymbol{v}}^{-1} \tag{A2.2}$$

The product mix shares of $\boldsymbol{C}$ sum to one by construction when post-multiplied by the summation vector while the market shares of $\boldsymbol{D}$ also sum to one when pre-multiplied by the summation vector. Then it follows, by construction from (A2.2), that:

$$\boldsymbol{V} = \hat{\boldsymbol{g}}\boldsymbol{C} = \boldsymbol{D}\hat{\boldsymbol{v}} \tag{A2.3}$$

It also follows that the margins of the matrix $\boldsymbol{V}$, by summing respectively over the columns and the rows are related by the following two relationships:

$$\boldsymbol{g} = \boldsymbol{D}\boldsymbol{v} \quad \text{and} \quad \boldsymbol{v} = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{g} \tag{A2.4}$$

Or, using these last relationships again:

$$\boldsymbol{g} = \boldsymbol{D}\boldsymbol{C}^T\boldsymbol{g} \quad \text{or} \quad \boldsymbol{v} = \boldsymbol{C}^T\boldsymbol{D}\boldsymbol{v} \quad \text{etc.} \tag{A2.5}$$

These relationships are often used in input-output modelling. One has to be careful in that if $\boldsymbol{D}$ is exogenous, then $\boldsymbol{C}$ is endogenous and vice-versa. Next, consider the matrix $\boldsymbol{V}^*$ given by

$$\boldsymbol{V}^* = (\boldsymbol{V} \bullet \boldsymbol{S}) \tag{A2.6}$$

where $\boldsymbol{S}$ is any matrix having the same dimension as $\boldsymbol{V}$ and where the product in (A2.6) is element by element. The margins of $\boldsymbol{V}^*$ are related by relationships similar to (A2.4) above:

$$\boldsymbol{g}^* = \boldsymbol{D}^*\boldsymbol{v}^* \quad \text{and} \quad \boldsymbol{v}^* = \boldsymbol{C}^{\mathrm{T}*}\boldsymbol{g}^* \tag{A2.7}$$

If there exists diagonal matrices $\hat{\gamma}$ and $\hat{\delta}$ such that:

$$\boldsymbol{g}^* = \hat{\gamma}\boldsymbol{g} \quad \text{and} \quad \boldsymbol{v}^* = \hat{\delta}\boldsymbol{v} \tag{A2.8}$$

Then, from (A2.7) and (A2.8), it follows that:

$$\hat{\gamma} g = \overset{*}{D}\hat{\delta} v \qquad \text{(A2.9)}$$

But $g = Dv$ as we have written above. Replacing in the previous equation gives:

$$\hat{\gamma} D v = \overset{*}{D}\hat{\delta} v \qquad \text{(A2.10)}$$

This last relationship is true for any v such that simplifying, one has:

$$\hat{\gamma} D = \overset{*}{D}\delta \qquad \text{(A2.11)}$$

Summing finally over the rows, given that the shares of D^* (and D) sum to one in each column by construction, one has:

$$\delta = D^T \gamma \qquad \text{(A2.12)}$$

A similar reasoning leads to

$$\gamma = C\delta \qquad \text{(A2.13)}$$

As an application of the theorem, one can identify V^* as the current price make matrix, V as the constant price make matrix, S as the price matrix of commodities produced by industry (admitting, in the most general case, distinct prices for the same commodity when produced in different industries) g and v as respectively the constant price gross output and commodity vectors and γ and δ as the corresponding price vectors. The nominal gross output by industry vector is therefore identified with g^* and the nominal commodity vector is identified with v^*. The commodity price vector has to be interpreted as comprising averages of commodity prices. One then obtains the well known dual price relationships of the input-output model: The industries' gross output prices are weighted averages of the commodity prices, the weights being the product mix shares C. And, conversely, average commodity prices are obtained by weighting the unit costs of the industries from where they originate according to the market share matrix D.

Identifying V^* as the nominal value-added matrix Y and S as the matrix Γ, v and g respectively as the commodity vector $\hat{p}e$ and the nominal value added vector $p_f y$ and γ and δ as the productivity growth vectors on value-added τ_y and final demand commodity sales τ_e, shows the relationships presented in the text between productivity gains on value-added by industry and productivity gains by final demand commodity. The vectors v^* and g^*, if divided by total value-added, can be identified as the weighted productivity gains by commodity and by industry. The sum of their elements gives the aggregate business sector multifactor productivity gains.

Productivity gains by commodity are given by weighting the productivity gains by industries according to the market shares of industries (i.e. according to the origin of the commodities) and conversely, the productivity gains by industry are given by weighting the productivity gains of the commodities according to the product mix shares, that is according to the composition of the output of industries.

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