Analytical Paper

Methods for Constructing Life Tables for Canada, Provinces and Territories

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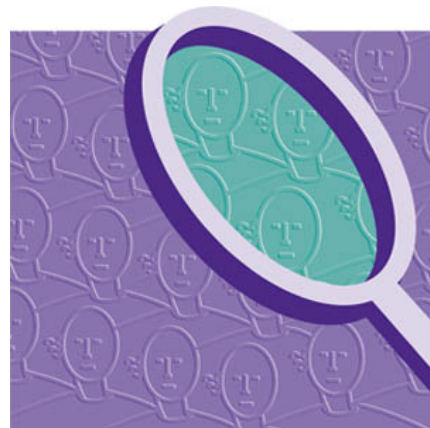
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Standard table symbols

The following symbols are used in Statistics Canada publications:

.	not available for any reference period
..	not available for a specific reference period
...	not applicable
0	true zero or a value rounded to zero
0^s	value rounded to 0 (zero) where there is a meaningful distinction between true zero and the value that was rounded
p	preliminary
r	revised
x	suppressed to meet the confidentiality requirements of the *Statistics Act*
E	use with caution
F	too unreliable to be published
*	significantly different from reference category (p < 0.05)

METHODS FOR CONSTRUCTING LIFE TABLES FOR CANADA, PROVINCES AND TERRITORIES

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Introduction

Life tables are used to compute life expectancy at birth and at different ages, but they are also used to compute many other indicators: death probabilities, probabilities of survival between two ages, years of life lived and the number of survivors at different ages. The construction of a life table makes it possible to summarize mortality within a population at a given time (period life table) or within a cohort (cohort life table). Accordingly, life tables meet many statistical needs, particularly in the fields of health, epidemiology, and actuarial science, and facilitate comparisons between regions or cohorts.

Statistics Canada has disseminated life tables, both complete (by single years of age) and abridged (by five-year age groups) for Canada, the provinces and territories, since 1939 and at irregular intervals, most often centered on census years. The methodology on which these period life tables are based has also been revised in some occasions, following advances in the field of mortality studies. An important revision was made for the construction of the life tables for the period 1995 to 1997 released in August 2002, with the introduction of a model for estimating old-age mortality.

A major revision was undertaken in 2013 of the methodology described in this document to take into account current methods for constructing life tables. This revision was also an opportunity to take into account the work conducted in the context of the development of the Human Mortality Database (HMD),[1] a database aiming to facilitate comparisons of mortality data between a large number of countries and regions, including Canada.[2] While differences remain between the two methods, the increased consistency between the methodology used by Statistics Canada and that of the HMD offers many advantages, especially for demographic projections and analysis.

Two methodology changes were then introduced in the elaboration of the life tables. The first change relates to the method of estimating old-age mortality. A logistic model based on the work of Kannisto (1992) replaced the quadratic model of Coale-Kisker (1990) first used in the construction of the tables for the period 1995 to 1997. This change was made because studies showed that in countries with high-quality data, the increase, from one age to the next, in mortality rates tended to follow a logistic curve (Kannisto et al. 1994; Thatcher et al. 1998). This logistic model is also used in the construction of the HMD life tables.

The second change applies to the method of smoothing age-specific death probabilities. The previous method, used since the 1970s and relying, among other factors, on pivotal ages, was replaced by a method based on splines. This method is more flexible and does not rely on arbitrary pivotal ages.

Statistics Canada has implemented another change related to the dissemination of the life tables, making them an annual product to better meet the needs of its many clients. As soon as new data on deaths from Vital Statistics is disseminated by Statistics Canada, the life tables are produced using the methodology presented in this document and made available on the Statistics Canada website in CANSIM and in Excel format. These tables cover three reference years, starting with the period 1980 to 1982, then 1981 to 1983, and so on.

This methodology document is divided into three parts. The first part presents data sources and steps for the construction of complete life tables. These tables are calculated for Canada and all of the provinces except Prince Edward Island. Complete life tables have the advantage of being more detailed than abridged tables, especially for old ages, where most of the decline in mortality has been concentrated for some years now.

The second part presents data sources and steps for the construction of the abridged life tables used for Prince Edward Island, Yukon, the Northwest Territories and Nunavut. Abridged life tables must be used when population sizes and death counts are too small to calculate complete life tables accurately.

Finally, the last part presents a short analysis of the impact of the changes in the methodology on the comparability of life expectancy data over time.

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1. See www.mortality.org.
2. See www.bdlc.umontreal.ca.

Methodology for complete life tables

Using the revised methodology of Statistics Canada's life tables, the construction of such tables is a seven-step process:

- **Step 1**: Calculation of observed mortality rates for ages 5 to 109 and for the open age group of 110 years and over;
- **Step 2**: Modeling of observed mortality rates for ages 95 to 109 and for the open age group of 110 years and over;
- **Step 3**: Calculation of death probabilities for ages 5 to the open age group of 110 years and over;
- **Step 4**: Calculation of death probabilities for ages 0 to 4;
- **Step 5**: Smoothing of death probabilities for ages 1 to 94;
- **Step 6**: Calculation of the elements of the life table;
- **Step 7**: Calculation of the margins of error for death probabilities and life expectancy.

Input data

Two data sources are used to construct complete life tables: Statistics Canada's Vital Statistics and Population Estimates Program.

More specifically, for a given sex and region, the following four data sets are required to calculate a complete life table for a period extending from calendar year $a-1$ to year $a+1$:

- Observed number of deaths by year of age (from 5 to 109 years and for the open age group of 110 years and over) for calendar years (January 1 to December 31) $a-1$, a and $a+1$ (source: Vital Statistics);
- Observed number of deaths by year of age (from 0 to 4 years) and year of birth for calendar years $a-1$, a and $a+1$ (source: Vital Statistics);
- Population estimates on July 1 by age (from 5 to 110 years) for calendar years $a-1$, a and $a+1$ (source: Population Estimates Program);[3]
- Population estimates on January 1 by age (from 0 to 4 years) for calendar years $a-1$, a, $a+1$ and $a+2$ (source: Population Estimates Program).

In general, the input data on deaths by age and sex from Canadian Vital Statistics are considered to be of very good quality (Bourbeau and Lebel 2000), even between 80 and 100 years of age (Beaudry-Godin 2010). Each year, there are very few deaths with unknown age or sex; any that exist are redistributed according to the known structure of observed deaths by age and sex. There are also few late registrations of deaths in Canada.

Similarly, Statistics Canada's population estimates are considered to be of very good quality. These estimates, used in the context of the Federal-Provincial Fiscal Arrangements Act, are based on the last available census, are adjusted for census net undercoverage and take into account demographic events since the last available census, by age and sex. Most often, postcensal population estimates are used to compute the life tables in a timely fashion, and these life tables are revised once the new population estimates are updated.

Estimating mortality at 100 years and over presents a challenge, since population sizes and observed death counts are lower and records are more subject to reporting errors. However, the use of a logistic model makes it possible to obtain a consistent series of mortality rates in old age, since this series is modeled.

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3. Populations of the open-aged age group 90 and over in population estimates between 1980 and 2000 were broken down by years of age using internal estimates produced in the Demographic Estimates Program (DEP) of Statistics Canada with the extinct cohort method (Vincent 1951) and survival ratios method (Thatcher 1992; Thatcher et al., 2002; Andreev 2004) based on deaths data from civil registers. The reader is referred to the demographic estimates methodological guide for more details. These methods have been used to distribute populations up to 99 years of age and for populations over 100 years of age (Atlantic, Quebec, Ontario, Prairies, British Columbia) with the exception of Prince Edward Island and territories for which only abbreviated tables with an open age group of 90 years have been produced. Subsequently, centenarian populations between 1980 and 2013, by year of age between 100 and 109 years and for those aged 110 and over, were derived using national distributions that are much more stable than the provincial distributions.

Step 1: Calculation of observed mortality rates for ages 5 to 109 and for the open age group of 110 years and over

For each year of age x included between 5 and the open age group of 110 years and over, the observed mortality rate is computed by taking the ratio of the sum of deaths during the three-year period to the sum of the populations in the middle of each of the three years of the same period. By doing so, it allows differentials that can be observed in the population growth rate of the three calendar years used in the calculation of the mortality rates to be taken into account.

More specifically,

$$_nM_x = \frac{\sum_{t=a-1}^{a+1} D_{x,t}}{\sum_{t=a-1}^{a+1} P_{x,t}}$$

where

$_nM_x$ is the observed mortality rate between ages x and x+n (in the case of complete tables, n = 1);

$\sum_{t=a-1}^{a+1} D_{x,t}$ is the sum of deaths between ages x and x+n for calendar years $a-1$, a and $a+1$ of the reference period, and;

$\sum_{t=a-1}^{a+1} P_{x,t}$ is the sum of the population counts estimated on July 1 between ages x and x+n for calendar years $a-1$, a and $a+1$ of the reference period.

In the case where, for a given age, no deaths are observed during the reference period, a mortality rate is imputed based on a geographic approach according to Table 1.

Table 1
Regions associated to the different provinces, for imputation

Province	Region associated for imputation[1]
Newfoundland and Labrador	
Nova Scotia	Atlantic provinces[2]
New Brunswick	
Quebec	Canada
Ontario	Canada
Manitoba	
Saskatchewan	Prairie provinces
Alberta	
British Columbia	Canada

1. If, for the Atlantic and Prairie provinces, no deaths were observed (a rare situation when three calendar years of data are used), the rate computed for Canada as a whole is used.
2. Including Prince Edward Island.

Step 2: Modeling of observed mortality rates for ages 95 to 109 and for the open age group of 110 years and over

From 95 years up to the open age group of 110 years and over, the mortality rates calculated in Step 1 can exhibit important variations due to the small number of deaths and persons submitted to the risk of dying. It can even happen that at some very old ages, often beyond age 105, it is impossible to calculate rates because of lack of deaths or at-risk population, a situation that arises frequently where there are small populations.

In these circumstances, it is better to use a model for estimating old-age mortality rates, which can yield both a more accurate representation of the mortality pattern, and a complete series of rates up to the open age group of 110 years and over. Therefore, a simplified logistic model based on the work of Kannisto (1992) was used. This model was adjusted by the maximum likelihood method, and it features an upper asymptote equal to 1.

The model takes the following form:

$$\mu_x = \frac{\alpha\, e^{\beta x}}{1 + \alpha\, e^{\beta x}}$$

where

μ_x is the mortality force (mortality hazard in continuous time) at age x, and;

α and β are the parameters to be estimated.

The parameter α corresponds roughly to the baseline mortality at age 0 and the parameter β corresponds to the rate of increase (logistic) of mortality from one age to the next. Parameters α and β are assumed to be positive, meaning that this condition is enforced as the model is estimated. The model is estimated by SAS software's NLIN procedure,[4] and for optimization, the Newton method is used (SAS Institute Inc. 2008A). The modeled mortality rate between age x and x+n corresponds to :

$$_n\widehat{M}_x = \widehat{\mu}_{(x+n)/2}$$

To ensure that the logistic model properly adjusts the observed old-age data, a minimum of 15 observed mortality rates between ages 80 up to the open age group of 110 years and over, calculated in Step 1, must be computable (and not imputed) for estimating the model. If this threshold were not reached, the model would not be estimated and an abridged life table, rather than a complete life table, would be calculated.

Step 3: Calculation of death probabilities for ages 5 to the open age group of 110 years and over

Steps 1 and 2 yield a series of mortality rates for ages 5 to 110. These rates are then converted to death probabilities by the so-called actuarial method:

$$_n q_x = \frac{2 \times n \times {}_n M_x^*}{2 + (n \times {}_n M_x^*)}$$

where

$_n q_x$ is the death probability between ages x and x+n, that is the probability that an individual aged x die before reaching age x+n;

n is the age interval (in the case of complete life tables, n = 1), and;

$_n M_x^*$ is either the observed mortality rate ($_n M_x$) between ages x and x+n for x comprised between 5 and 94 years or the modeled mortality rate ($_n\widehat{M}_x$) for x comprised between 95 and 110 years.

Within the open age group of 110 years and over, the death probability takes the value of 1.

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4.   NLIN stands for non-linear regression.

Step 4: Calculation of death probabilities for ages 0 to 4

Between ages 0 and 4, death probabilities are estimated directly, since mortality at these ages has its own particular pattern. For example, between 0 and 1 year of age, deaths are not distributed uniformly over the year, but are instead concentrated in the first days of life. The method used is the same as the one used in the most recent editions of the complete life tables produced by Statistics Canada (Statistics Canada 2006).

Schematically, and for a given year a, the calculation of the death probability at age 0 ($_1q_0$), also called the infant mortality rate, is based on a complement to 1 of a product of two ratios, the first representing the probability that a person of exact age x will survive to the end of the calendar year in which that person reaches age x, and the second representing the probability that a person living at the end of the calendar year in which he or she reaches age x will survive until the exact age of x+1.

Thus, according to the Lexis diagram (Figure 1), the value $E_{0,a}$ is obtained by adding to the population aged 0 on January 1 of year $a+1$ (value $P^*_{0,a+1}$ in Figure 1) the number of deaths at age 0 during year a, among children born during that same year a ($D'_{0,a}$). The value $E_{1,a}$ is obtained by subtracting from the population aged 0 on January 1 of year a ($P^*_{0,a}$) the number of deaths at age 0 during year a, among children born during the previous year ($D''_{0,a}$).

Figure 1
Lexis diagram
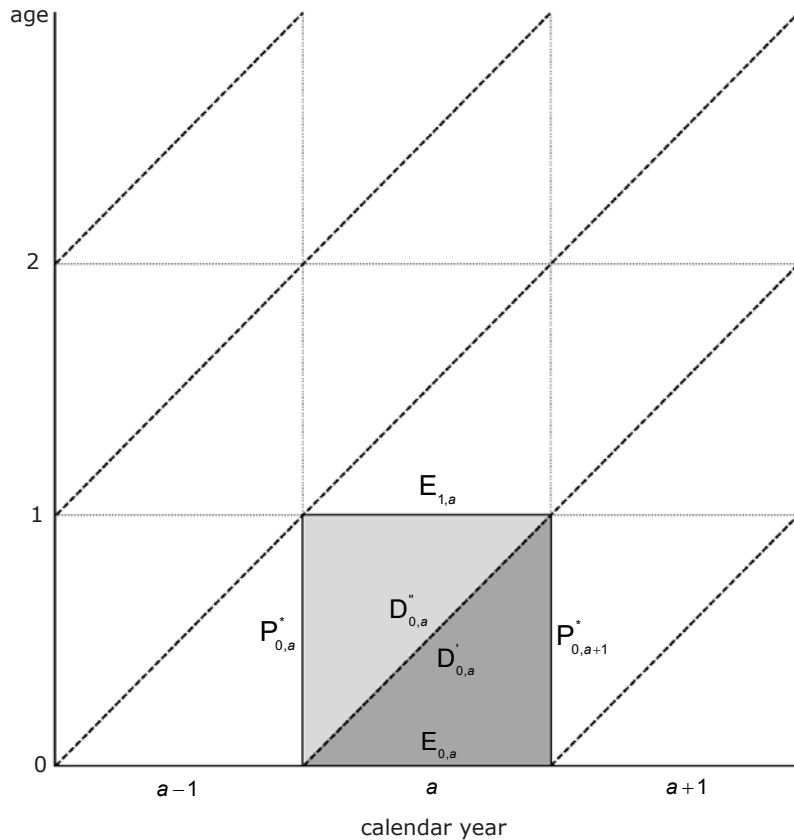
$$_1q_0 = 1 - \left(\frac{P^*_{0,a+1}}{E_{0,a}} \times \frac{E_{1,a}}{P^*_{0,a}} \right)$$

By transposition and using three years for the calculations:

$$_1q_0 = 1 - \left(\frac{\sum_{t=a-1}^{a+1} P_{0,t+1}}{\sum_{t=a-1}^{a+1} P_{0,t+1} + \sum_{t=a-1}^{a+1} D'_{0,t}} \times \frac{\sum_{t=a-1}^{a+1} P_{0,t} - \sum_{t=a-1}^{a+1} D''_{0,t}}{\sum_{t=a-1}^{a+1} P_{0,t}} \right)$$

where

$_1q_0$ is the death probability at age 0;

$\sum_{t=a-1}^{a+1} P_{0,t+1}$ is the sum of the population counts of age 0 estimated on January 1 of years a, $a+1$ and $a+2$, that is the last two years of the reference period and the year that follows;

$\sum_{t=a-1}^{a+1} D'_{0,t}$ is the sum of deaths at age 0 during the reference period, for children born in the same year as the year of their death;

$\sum_{t=a-1}^{a+1} P_{0,t}$ is the sum of the population counts of age 0 estimated on January 1 during the reference period, and;

$\sum_{t=a-1}^{a+1} D''_{0,t}$ is the sum of deaths at age 0 during the reference period, for children born during the year preceding the year of their death.

From ages 1 to 4 years, an equation following the same principle was used:

$$_nq_x = 1 - \left(\frac{\sum_{t=a-1}^{a+1} P_{x,t+1}}{\sum_{t=a-1}^{a+1} P_{x,t+1} + \sum_{t=a-1}^{a+1} D'_{x,t}} \times \frac{\sum_{t=a-1}^{a+1} P_{x,t} - \sum_{t=a-1}^{a+1} D''_{x,t}}{\sum_{t=a-1}^{a+1} P_{x,t}} \right)$$

In the case where, for each age between 1 and 4 years, no death is observed during the reference period, a death probability will be interpolated for this age in Step 5, where death probabilities are smoothed.

Step 5: Smoothing of death probabilities for ages 1 to 94

In steps 3 and 4, a complete series of death probabilities was computed for ages 0 to 110 years. However, between ages 1 and 94, these probabilities can exhibit an irregular pattern over time, especially for regions with small populations. To ensure that death probabilities evolve consistently from one age to another and to estimate, if needed, missing probabilities between ages 1 and 4, smoothing was applied using B-splines. This smoothing is applied to death probabilities from 0 to 109 years of age, to ensure a harmonious link between smoothing by means of B-splines and the model for estimating old-age mortality (Step 2).[5]

The B-splines smoothing technique has the advantage of being flexible, that is, to provide users many options to fit the data in the best way possible. Since B-splines, just like any splines, are constructed using piecewise polynomial functions joined together, it is appropriate to choose various positions on the horizontal axis–or knots–where these junctions occur. The greater the number of knots, the better the smoothed curve fits the original curve of death probabilities by age; conversely, a small number of knots provide more power to the smoothing. As a result, the fluctuations between ages are eliminated and a curve with a more regular appearance is obtained.

There are algorithms that can be used to find both the optimal number of knots to use and their position on the age distribution in the context of constructing life tables (Kaishev et al. 2009). However, these algorithms are complex to use. For these complete tables, the number and the position of the knots were instead determined empirically, a series of tests were performed to evaluate both the neutrality and the adjustment of the smoothing method chosen.[6] The smoothing method used must have the smallest effect possible on the age-specific life expectancy. Also, each series of smoothed probabilities is compared to the observed non smoother series of probabilities, so as to ensure goodness of fit.

Two series of knots are used, depending on the size of the population for which a complete mortality table is produced. The first series includes 11 knots, set at the following ages: 0, 1, 9, 15, 18, 24, 30, 35, 40, 50 and 90,[7] instead of 9 knots in the previous release of life tables (2016) to take into account recent evolution of mortality between age 30 and 50 with two new knots, at ages 35 and 40. Between ages 50 and 94, the number and position of the knots is of lesser importance than for younger ages, as the mortality pattern is mostly linear. Before age 50, chosen knots correspond often to inflexion points on the classic death probabilities curve. By choosing these knots, it was possible to enforce a similar pattern in death probabilities from one region to the next, while allowing enough flexibility to allow period and regional variations in mortality to be taken into account. This 11-knot series is always used for Canada, Quebec, Ontario, Alberta and British Columbia. It is also used most of the time for the other provinces except Newfoundland and Labrador, although an analysis is performed each time to select the most appropriate series of knots.

The second series consists of 7 knots, set at the following ages: 0, 9, 18, 24, 30, 50 and 90. Using this series, smoothing is "stronger" than for the other series and is used for provinces with smaller populations. This 7-knots series is used for Newfoundland and Labrador and, at times, Nova Scotia, New Brunswick, Manitoba and Saskatchewan.

The B-splines smoothing of the death probabilities between 1 and 94 years in the current life tables has been fitted using the TRANSREG[8] procedure in the SAS statistical software (SAS Institute Inc., 2008B).

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5. In the complete life table published, the death probabilities between 95 and 109 years remain those calculated in Step 3.
6. An evaluation of the method proposed by the Kaishev et al. (2009) was made for two regions, Canada as a whole and Newfoundland and Labrador. The number and the position of the knots given by the Kaishev et al. model were very close to those established empirically.
7. A knot is set at age 0 but the life table is always computed using the observed value at that age.
8. TRANSREG stands for transformation regression.

Step 6: Calculation of the elements of the life table

By creating a series of smoothed death probabilities between ages 0 and 110, it is possible to calculate all the life table elements based on a fictitious cohort of 100,000 newborns, according to the following equations:

- Number of survivors at exact age 0, also called the root of the life table (l_0):

$$l_0 = 100{,}000 \text{ newborns}$$

- Number of deaths between ages x and x+n ($_n d_x$):

$$_n d_x = l_x \times {_n q_x}$$

- Number of survivors at exact age x (l_x):

$$l_x = l_{x-n} - {_n d_{x-n}}$$

- Probability of survival between ages x and x+n ($_n p_x$):

$$_n p_x = 1 - {_n q_x}$$

- Number of years lived between ages x and x+n (stationary population) ($_n L_x$):

$$_n L_x = n \times l_{x+n} + {_n d_x} \times {_n f_x} \text{ for x comprised between 0 to 109 years}$$

   where, for complete life tables,

$$_1 f_x \text{ (separation factor)} = \begin{cases} 1 - \left(\dfrac{\displaystyle\sum_{t=a-1}^{a+1} D'_{x,t}}{\displaystyle\sum_{t=a-1}^{a+1} \left(D'_{x,t} + D''_{x,t} \right)} \right) & \text{for x=0 to 4 years if the numerator and denominator are greater than 0} \\[4ex] \text{and} \\ 0.5 \text{ for x} \geq 5 \text{ years.} \end{cases}$$

   If either the numerator or the denominator equals to zero in the calculation of $_1 f_x$ for x comprised between 0 to 4 years, a value of $_1 f_x$ is imputed based on a geographic approach like the one used in Step 1.

$$L_{110+} = l_{110} \times e_{110} \quad \text{for the open age group 110 years and over}$$

$$\text{where} \quad e_{110} = \frac{1}{\widehat{M}_{110}}$$

- Total number of cumulative years of life lived starting at age x (cumulative stationary population) (T_x):

$$T_x = \sum_{i=x}^{110} {_n L_i}$$

- Life expectancy between ages 0 and 109 (e_x):

$$e_x = \frac{T_x}{l_x}$$

Step 7: Calculation of the margins of error for death probabilities and life expectancy

Since publishing the complete life tables for the period 1995 to 1997, Statistics Canada has disseminated the coefficients of variation associated with death probabilities and life expectancies from life tables. This quality indicator gives the reader an idea of the variability of the estimate, which largely depends on the number of deaths on which the estimate is based.

The quality indicator used this time is the margin of error, which is used to directly calculate a 95% confidence interval of an estimate. The margin of error (m.e.) of death probabilities at age x is calculated as follows:

$$\text{m.e.}\left(_n q_x\right) = 1.96 \times \text{s.e.}\left(_n q_x\right)$$

where the standard error (s.e.) of $_n q_x$ is given by:

$$\text{s.e.}\left(_n q_x\right) = \sqrt{V\left(_n q_x\right)}$$

and the variance of $_n q_x$ is given by the Chiang formula (1984):

$$V\left(_n q_x\right) = \frac{_n q_x^2 \times \left(1 - _n q_x\right)}{D_x^*}$$

where $_n D_x^*$ are estimated deaths between age x and x+n in the population based on smoothed mortality rates, the latter being computed from smoothed death probabilities.

The margin of error and the standard deviation of life expectancies at age x are calculated using the same equations, with the exception of the equation used for the variance, which according to Chiang (1984), is:

$$V\left(e_x\right) = \frac{\sum_{i=x}^{110} l_i^2 \times \left[\left(1 - _n f_i\right) + e_{i+n}\right]^2 \times V\left(_n q_i\right)}{l_x^2}$$

For example, a margin of error of 0.00020 for a death probability at age 0 of 0.00556 enables to build a 95% confidence interval with lower and upper limits of 0.00536 and 0.00576. In other words, the death probability is precise within a range of 0.00020, 19 times out of 20. In rare cases, subtracting the margin of error to the associated probability of dying may yield a negative result. If so, the lower limit can be considered to be exactly zero.

Similarly, a margin of error of 0.2 on a life expectancy at birth of 81.9 years enables to build a 95% confidence interval with lower and upper limits of 81.7 and 82.1 years.

Methodology for abridged life tables

An abridged life table is used when the size of a population of a given region is too small to accurately compute a complete life table (by single years of age) since, often, no deaths are recorded for a number of ages, a common situation between the ages of 1 and 15 years. An abridged life table is produced for Prince Edward Island and for Yukon, the Northwest Territories and Nunavut, separately.

These abridged life tables are constructed using the methodology described in this section. It is mostly based on the methodology used in the complete life tables, to ensure maximum consistency between the two series of tables. In some cases, as in the calculation of the death probability at age 0, the two methods are identical. However, it is not necessary to use the model to estimate old-age mortality, since the abridged life table ends at the open age group of 90 years and over. Also, no method of smoothing death probabilities is used for the abridged life tables, as there are less random fluctuations. The methodology used is moreover very close to the one used in previous editions of the abridged life tables produced by Statistics Canada (2006), with the exception of the imputation of mortality rates from an aggregated region in the case where population sizes or death counts are too low.

Constructing abridged life tables is a five-step process:

Step 1: Calculation of observed mortality rates for age group 1 to 4 to the open age group 90 years and over;
Step 2: Calculation of death probabilities for age group 1 to 4 to the open age group 90 years and over;
Step 3: Calculation of death probabilities at age 0;
Step 4: Calculation of the elements of the life table;
Step 5: Calculation of the margins of error for death probabilities and life expectancy.

Input data

Calculating abridged tables requires the same data series as for complete tables.

Age groups

Abridged tables are produced using 20 age groups, for which the notation is of the form x to x+(n-1) as indicated in Table 2.

Table 2
Age interval by age group

Age group	n (age interval)
0 year	1
1 to 4 years	4
5 to 9 years to 85 to 89 years	5
90 years and over	--

Step 1:   Calculation of observed mortality rates for age group 1 to 4 to the open age group 90 years and over

For the age group 1 to 4, for each 5-year age group x to x+(n-1) from 5 to 89 years and the open age group 90 years and over, the observed mortality rate is computed by taking the ratio of the sum of deaths during the three-year (calendar year) period to the sum of the populations on July 1 for the same age group and during the same period. The formula used is the same as that of the complete life tables, adjusted to take into account age groups (see Step 1 of the section on complete life tables).

In a case where, for a given age group and sex, no deaths are observed during the reference period, an observed mortality rate is imputed on the basis of a geographic approach. First, the region is considered; for Prince Edward Island, the observed mortality rate imputed will be that of the region comprised of all of the Atlantic provinces. For Yukon, the Northwest Territories and Nunavut, the reference region consists of the three territories pooled together. This imputation procedure is also used for mortality rates above age 49 when, for a given age group, population counts are lower than 50 or death counts are lower than 10.

If no deaths were observed in these two major regions, a very rare situation, the observed mortality rate for Canada as a whole is used.

Step 2:   Calculation of death probabilities for age group 1 to 4 to the open age group 90 years and over

The observed mortality rates obtained in Step 1 are converted into death probabilities by the Greville method (1943). This method yields very similar results to the actuarial method used for complete tables (Ng and Gentleman 1995), while ensuring that the probabilities obtained are never greater than 1.

According to Greville,

$$_nq_x = \frac{_nm_x}{\frac{1}{n} + {_nm_x} \times \left[0.5 + \frac{n}{12} \times \left(_nm_x - \ln C\right)\right]}$$

where

$_nq_x$ is the death probability between ages x and x+n;

$_nm_x$ is the observed mortality rate between ages x and x+n;

n   is the size of the age group interval, which is 4 years in the case of age group 1 to 4 and 5 years after except for the last age group, and;

C   is equal to:

$$C = (1/45)\ln(\frac{_5m_{85}}{_5m_{40}})$$

Within the open age group of 90 years and over, the death probability takes the value of 1.

Step 3: Calculation of death probabilities at age 0

The method of calculating death probabilities at age 0 is identical to the method used for complete tables.

Step 4: Calculation of the elements of the life table

The various elements of the life table are computed in the same way as for the complete tables, adjusting for age groups (see Step 6 of the section on complete life tables).

Step 5: Calculation of the margins of error for death probabilities and life expectancy

The same equations are used to calculate margins of error as for the complete tables, adjusting for age groups (see Step 7 of the section on complete life tables).

Impact of methodology changes

Numerous authors have shown that life expectancy at birth calculated using a life table is a robust statistical indicator, relatively insensitive to changes in methodology (e.g., Ng and Gentleman 1995). However, it is appropriate to examine the impact of the changes made to the construction of complete and abridged life tables in comparison with Statistics Canada's previous edition (2006).
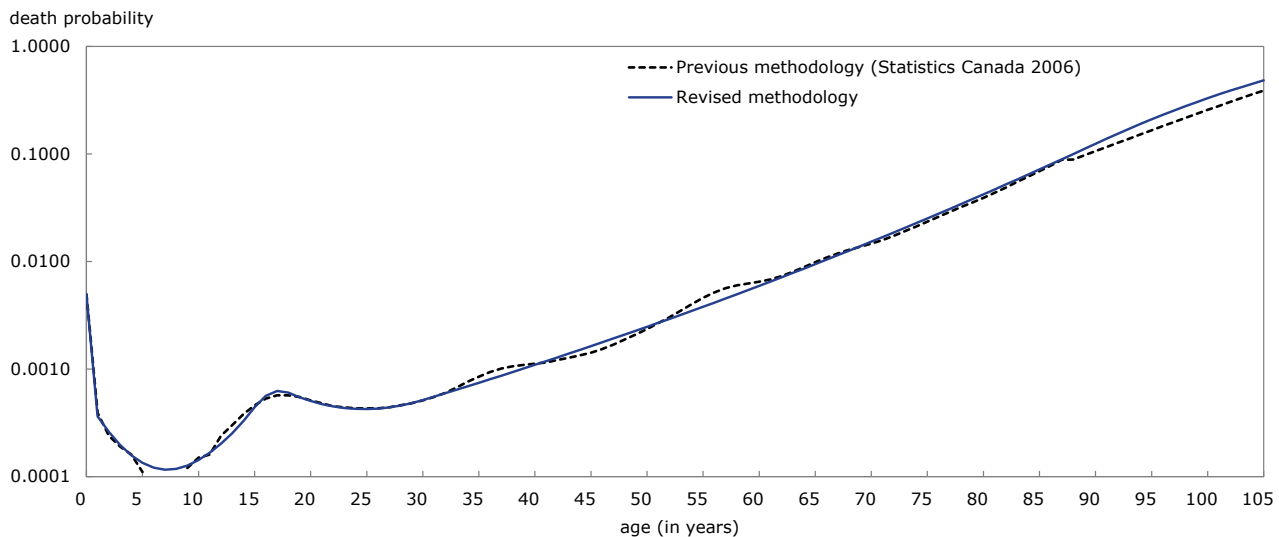
Complete life tables

Previously, two major changes have been made to the construction of the complete life tables: The Coale-Kishker model for estimating mortality at advanced ages has been replaced by a logistic model, and the method for smoothing the mortality rates has been replaced by a smoothing technique based on B-splines applied to the death probabilities.

These changes have very little impact on the comparability of life expectancy data through time. A comparison made between the life expectancy at birth drawn from the complete life table already published by Statistics Canada for the period 2000 to 2002 and that based on the life tables reproduced for the same period using the present methodology show a difference of at most 0.1 year for Canada as a whole.

For the nine provinces for which a complete life table is computed (males and females separately), this difference in life expectancy at birth between the two series is lower than 0.1 year in 13 of the 18 cases. For the 5 additional cases, this difference in life expectancy at birth do not exceed 0.4 year. In these 5 cases, the difference can be attributed to an improvement in the junction between the series of smoothed death probabilities and modeled death probabilities for old-ages, junction which occurred between 87 and 88 years in the life tables for the period 2000 to 2002 already released (Statistics Canada 2006) and between 94 and 95 years with the revised methodology. Figure 2 provides an overview of the differences in age-specific death probabilities computed using the previous and revised methodology for the life table of females in Saskatchewan for the period 2000 to 2002.

Figure 2

Age-specific death probabilities for females in Saskatchewan for the period 2000 to 2002 according to the previous and revised methodology

death probability

Abridged life tables

Concerning the abridged life tables, only very minor changes were made to the methodology used in previous life tables produced by Statistics Canada. Like the complete tables, the death probability at age 0 was calculated directly, to maintain consistency. Where possible, the separation factor (f_x) used in the equation on years of life lived (stationary population) is calculated for the province or territory of interest. These changes have very little impact on the computed life expectancies.

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