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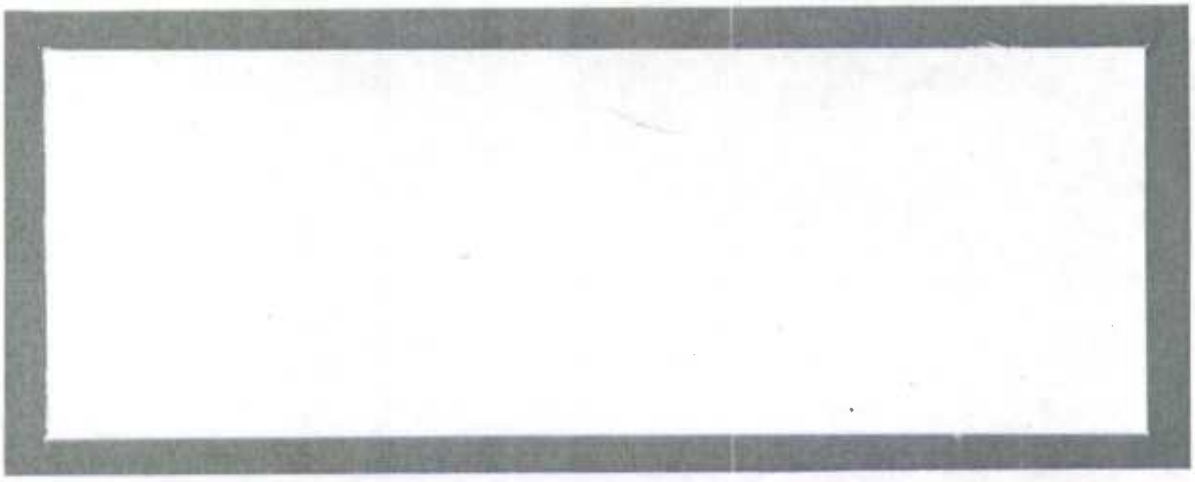

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VARIANCE ESTIMATION OF NONLINEAR STATISTICS IN STRATIFIED SAMPLES

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1. Introduction

Variances of nonlinear statistics are generally approximated by either Taylor linearization or by various sample reuse methods. In this note we compare, on an empirical basis, four such methods, including Taylor linearization, balanced repeated replication (BRR), the jackknife and the bootstrap. The study is essentially an extension of Hansen and Tepping's (1985) study where the BRR and the jackknife variance estimators of the ratio are compared. In this paper we confirm their results and use the same hypothetical populations in order to study the behaviour of the above four variance estimators in the case of the ratio, the regression coefficient and the correlation coefficient.

The results indicate that in the case of the ratio estimator, all of the methods perform well for populations that exhibit a low coefficient of variation of the estimate of the mean of the independent variable, CV ($\bar{x}$). This is not the case when the coefficient of variation increases, or in the case of more nonlinear estimators like the regression or correlation coefficients, especially in non-normal populations.

2. The Populations and the Parameters

All three populations considered in this study are hypothetical in nature, however they are intended to resemble real populations encountered by Hansen and Tepping (1985) in the National Assessment of Educational Progress study. All consist of 32 strata with two variables, X and Y. In stratum h, X_h and Y_h are assumed to have a joint distribution with

the respective means μ_{xh} and μ_{yh} and standard deviations σ_{xh} and σ_{yh}. The correlation ρ of X_h and Y_h is assumed to be constant across strata. The actual values of these parameters, as well as the weights W_h, can be found in Tables 1 to 3. Other populations have been derived from the basic populations by increasing the standard deviations σ_{xh} and σ_{yh} by various choices of the factors f_x and f_y respectively, and by modifying the value of ρ. In all, thirty populations were considered.

In order to further diversify the populations, two different underlying distributions were considered, thus effectively doubling the number of populations studied. First, (X_h, Y_h) were assumed to have a bivariate normal distribution with the prespecified parameters in each stratum. Secondly, X_h was assumed to follow the gamma (α,β) distribution with a shape parameter $\alpha = \mu^2_{xh}/\sigma^2_{xh}$ and a scale parameter $\beta = \sigma^2_{xh}/\mu_{xh}$ so that X_h have mean μ_{xh} and standard deviation σ_{xh}. Then, under the linear regression model, the Y_h variable was derived from the X_h variable by multiplying X_h by $\sigma_{yh}\,\rho/\sigma_{xh}$ and by adding on a normal deviate with mean $\mu_{yh} - \sigma_{yh}\,\mu_{xh}\,\rho/\sigma_{xh}$ and variance $\sigma^2_{yh}\,(1-\rho)$ in order that (X_h, Y_h) have a joint distribution with the prespecified parameters.

From each population, a stratified simple random sample of size 64 ($n_h = 2$ for all h) was selected in order to estimate the ratio R, the regression coefficient B and the correlation coefficient C. The true values of R, B and C can be obtained for each population from equations (2.1), (2.2) and (2.3) respectively, given below.

$$R = \frac{\mu_y}{\mu_x} = \frac{\sum_{h=1}^{32} W_h\,\mu_{yh}}{\sum_{h=1}^{32} W_h\,\mu_{xh}}\,, \qquad (2.1)$$

$$B = \frac{\sum_{h=1}^{32} W_h(\rho\sigma_{xh}\sigma_{yh} + (\mu_{xh}-\mu_x)(\mu_{yh}-\mu_y))}{\sum_{h=1}^{32} W_h(\sigma_{xh}^2 + (\mu_{xh} - \mu_x)^2)} \quad , \tag{2.2}$$

$$C = \frac{\sum_{h=1}^{32} W_h(\rho\sigma_{xh}\sigma_{yh} + (\mu_{xh}-\mu_x)(\mu_{yh}-\mu_y))}{\{\sum_{h=1}^{32} W_h(\sigma_{xh}^2+(\mu_{xh}-\mu_x)^2) \cdot \sum_{h=1}^{32} W_h(\sigma_{yh}^2+(\mu_{yh}-\mu_y)^2)\}^{1/2}} \quad . \tag{2.3}$$

In order to calculate the corresponding sample estimates, consider a function t of the individual observations, say $t = t(x_{hi}, y_{hi})$. The stratified sample mean of t, is given by

$$\bar{t} = \sum_{h=1}^{32} W_h \sum_{i=1}^{2} t(x_{hi}, y_{hi}) / 2 \quad . \tag{2.4}$$

The sample estimates of R, B and C are then given, respectively by

$$r = \bar{t}_2 / \bar{t}_1 \quad , \tag{2.5}$$

$$b = (\bar{t}_3 - \bar{t}_1\bar{t}_2) / (\bar{t}_4 - \bar{t}_1^2) \quad , \tag{2.6}$$

$$c = (\bar{t}_3 - \bar{t}_1\bar{t}_2) / \{(\bar{t}_4 - \bar{t}_1^2)(\bar{t}_5 - \bar{t}_2^2)\}^{1/2} \quad , \tag{2.7}$$

where $t_1(x_{hi},y_{hi}) = x_{hi}$, $t_2(x_{hi},y_{hi}) = y_{hi}$, $t_3(x_{hi},y_{hi}) = x_{hi}y_{hi}$, $t_4(x_{hi},y_{hi}) = x_{hi}^2$, and $t_5(x_{hi},y_{hi}) = y_{hi}^2$. While at this point the above notation may seem somewhat cumbersome, as compared to the usual $r = \bar{y}/\bar{x}$ for example, it will become more convenient in the following sections.

In order to obtain the "true" mean square errors (MSE) of the estimators in (2.5)-(2.7), for each population 1,000 independent samples were selected according to the criteria set out above, and the estimates r, b and c were consructed. The MSE of r, for example, was then computed by

$$MSE(r) = \sum_{i=1}^{1000} (r_i - R)^2/1000. \tag{2.8}$$

The MSE(b) and MSE(c) were obtained in an analogous manner. It was found that 1000 replications were sufficient to obtain reasonable estimates of the MSE's in the three basic populations. The MSE's were therefore taken as reference points to which all alternative variance estimators were compared. The following four sections describe these variance estimators in more detail.

3. Taylor Linearization

Suppose the estimate of interest can be written as a function of k means, $f(\bar{t}_1, \bar{t}_2, \ldots, \bar{t}_k)$ say. Then the Taylor approximation of the population MSE is given by

$$V_T(f(\underline{\bar{t}})) = \nabla'_{f(\underline{\bar{T}})} \cdot D(\underline{\bar{t}}) \cdot \nabla_{f(\underline{\bar{T}})} \tag{3.1}$$

where D is the variance-covariance matrix of the vector $\underline{\bar{t}} = (\bar{t}_1, \ldots, \bar{t}_k)$ and $\nabla_{f(\underline{\bar{T}})}$ is the gradient vector of f with respect to $\underline{\bar{t}}$ evaluated at the expected value of $\underline{\bar{t}}$, $\underline{\bar{T}}$.

In the case of estimating the ratio r from a stratified sample, for example, we have

$$r = f(\bar{t}_1, \bar{t}_2) = \bar{t}_2/\bar{t}_1 \ , \tag{3.2}$$

$$\nabla_{r|\underline{\bar{t}} = \underline{\bar{T}}} = \mu_x^{-1} \ (-R, \ 1)' \ , \tag{3.3}$$

$$D(\bar{t}_1, \bar{t}_2) = \begin{bmatrix} \Sigma \ W_h^2 \ \sigma_{xh}^2/n_h & \Sigma \ W_h^2 \ \sigma_{xh} \ \sigma_{yh} \ \rho/n_h \\ & \Sigma \ W_h^2 \ \sigma_{yh}^2/n_h \end{bmatrix} \tag{3.4}$$

Applying (3.1), we obtain the standard formula

$$V_T(r) = \mu_x^{-2} \sum_{h=1}^{32} W_h^2(\sigma_{yh}^2 - 2R \ \sigma_{xh} \ \sigma_{yh} \ \rho + R^2 \ \sigma_{xh}^2)/n_h \ . \tag{3.5}$$

Similar expressions have been derived in the case of the regression and correlation coefficients. They are given in the appendix for the case of the bivariate normal distribution. However, no comparable derivations were made in the case of the gamma distribution. In what follows, the Taylor approximation to the MSE will be referred to as the TAYLOR-P method.

The corresponding sample estimates can be obtained by first evaluating the gradient at the sample estimate $\underline{\bar{t}}$ instead of its expected value $\underline{\bar{T}}$, and secondly by switching the variance operation with the dot product (Woodruff, 1971), to obtain

$$v_T(f(\underline{\bar{t}})) = \sum_{h=1}^{32} W_h^2 \ s_{eh}^2/n_h \ , \tag{3.6}$$

with

$$s_{eh}^2 = \sum_{i=1}^{n_h} (e_{hi} - \bar{e}_h)^2/(n_h - 1), \tag{3.7}$$

$$\bar{e}_h = \sum_{i=1}^{n_h} e_{hi}/n_h \tag{3.8}$$

and

$$e_{hi} = \nabla'_{f(\underset{\sim}{\bar{t}})} \cdot \underset{\sim}{t}(x_{hi}, y_{hi}) \quad . \tag{3.9}$$

In the case of the ratio, regression coefficient and correlation coefficient the elements in (3.9) are given respectively by (3.10), (3.11) and (3.12) below.

$$e_{hi}(r) = (y_{hi} - rx_{hi}) \; / \; \bar{t}_1 \quad , \tag{3.10}$$

$$e_{hi}(b) = ((2\bar{t}_1 b - \bar{t}_2)\, x_{hi} - \bar{t}_1 y_{hi} + x_{hi}\, y_{hi} - b x_{hi}^2)/s_x^2 \quad , \tag{3.11}$$

$$\begin{aligned} e_{hi}(c) = {} & ((c\bar{t}_1\, s_y/s_x - \bar{t}_2)\, x_{hi} + (c\bar{t}_2\, s_x/s_y - \bar{t}_1)\, y_{hi} \\ & + x_{hi} y_{hi} - (cs_y/2s_x)\, x_{hi}^2 - (cs_x/2s_y)\, y_{hi}^2) \; / \; s_x s_y \quad , \end{aligned} \tag{3.12}$$

where $s_x^2 = \bar{t}_4 - \bar{t}_1^2$ and $s_y^2 = \bar{t}_5 - \bar{t}_2^2$. In what follows these sample estimators of variance will be refered to as the TAYLOR-S estimators.

4. Balanced Repeated Replication

The balanced repeated replication method of variance estimation relies on reusing the sample to recompute the estimate of interest a number of times from carefully chosen half samples. The variation between these subsample estimates is then assumed to be the sampling variance of the estimate at hand (McCarthy, 1966). The half samples are constructed so that exactly one unit is chosen from each stratum and so that the half samples are mutually orthogonal. (That is to say, the matrix of pointers is orthogonal). The matrix, M (r, h), used in this study is given in Table 4 where the (r, h) entry m indicates that (x_{hm}, y_{hm}) in stratum h is to be included in the r'th replicate. The half sample estimates, $f(\bar{t}_r)$, are obtained from (2.5)-(2.7) by substituting for each $\bar{t}$ its half sample estimate, given for the r'th replicate by

$$\bar{t}_r = \sum_{h=1}^{32} W_h \cdot t(x_{hm_{rh}}, y_{hm_{rh}}) , \tag{4.1}$$

where m_{rh} is the (r, h) entry of M. Note that here and in subsequent sections the subscripts 1 to 5 of t, which determine its particular functional form, have been omitted, as in (2.4), in order not to be confused with other subscripts.

The BRR variance estimator of $f(\bar{t})$ based on these 32 half samples is given by

$$v_{BRR-H}(f(\bar{t})) = \sum_{r=1}^{32} (f(\bar{t}_r) - f(\bar{t}))^2 / 32 , \tag{4.2}$$

and denoted by BRR-H. Interchanging the ones and twos in the matrix M, we obtain the complementary half sample estimates, denoted by $f(\bar{t}_r)$, and the corresponding variance estimate, denoted by BRR-C,

$$v_{BRR-C}(f(\bar{\underline{t}})) = \sum_{r=1}^{32} (f(\bar{\underline{t}}_r) - f(\bar{\underline{t}}))^2/32 \quad . \tag{4.3}$$

The complementary estimates can be derived from the half sample estimates by noting that $\bar{t}_{\bar{r}} = 2\bar{t} - \bar{t}_r$ thus saving unnecessary recomputations.

A difference variance estimator (BRR-D) can also be derived from the half sample estimates and their complements by taking the corresponding differences, concretely

$$v_{BRR-D}(f(\bar{\underline{t}})) = \sum_{r=1}^{32} (f(\bar{\underline{t}}_r) - f(\bar{\underline{t}}_{\bar{r}}))^2 \; / \; 128 \; . \tag{4.4}$$

Finally, the full variance estimator, BRR-F, is defined as the average of the half sample variance estimate and its complement,

$$v_{BRR-F}(f(\bar{\underline{t}})) = \{v_{BRR-H}(f(\bar{\underline{t}})) + v_{BRR-C}(f(\bar{\underline{t}}))\} \; / \; 2. \tag{4.5}$$

While all four variance estimators were computed for all populations, in all cases the differences in their performance were only minor. Hence, for the sake of brevity, only the BRR-F estimator is reported in the summaries that follow.

5. Jackknife repeated replication

Analogous to BRR, the jackknife repeated replication (JRR) method relies on re-estimating the parameter of interest a number of times from the original sample. The jackknife subsample estimate, $f(\bar{\underline{t}}_{.j})$, based on the j'th replicate, is obtained from (2.5)-(2.7)

by deleting the second unit from stratum j in computing the averages $\bar{t}$. That is, by letting

$$\bar{t}_j = \sum_{h \neq j} W_h \sum_{i=1}^{2} t(x_{hi}, y_{hi})/2 + W_j \cdot t(x_{j1}, y_{j1})$$

$$= \bar{t} + W_j(t(x_{j1}, y_{j1}) - t(x_{j2}, y_{j2})) / 2 \quad . \tag{5.1}$$

The complementary estimates, $\bar{t}_j^{\sim}$, are derived from

$$\bar{t}_j^{\sim} = \sum_{h \neq j} W_h \sum_{i=1}^{2} t(x_{hi}, y_{hi})/2 + W_j \cdot t(x_{j2}, y_{j2})$$

$$= \bar{t} - W_j(t(x_{j1}, y_{j1}) - t(x_{j2}, y_{j2}))/2. \tag{5.2}$$

As in the case of BRR, four variance estimators can be defined (McCarthy, 1966; Kish and Frankel, 1974).

$$v_{JRR\text{-}H}(f(\bar{\underline{t}})) = \sum_{j=1}^{32} (f(\bar{\underline{t}}_j) - f(\bar{\underline{t}}))^2 \quad , \tag{5.3}$$

$$v_{JRR\text{-}C}(f(\bar{\underline{t}})) = \sum_{j=1}^{32} (f(\bar{\underline{t}}_j^{\sim}) - f(\bar{\underline{t}}))^2 \quad , \tag{5.4}$$

$$v_{JRR\text{-}D}(f(\bar{\underline{t}})) = \sum_{j=1}^{32} (f(\bar{\underline{t}}_j) - f(\bar{\underline{t}}_j^{\sim}))^2/4 \quad , \tag{5.5}$$

$$v_{JRR\text{-}F}(f(\bar{\underline{t}})) = \{v_{JRR\text{-}H}(f(\bar{\underline{t}})) + v_{JRR\text{-}C}(f(\bar{\underline{t}}))\}/2 \quad . \tag{5.6}$$

As in the preceeding section, here also the performance of the four estimators is indistinquishable, and hence only the JRR-F estimator will be reported in what follows. Unlike the case of BRR, however, in this instance the equivalence is theoretically justified (Rao and Wu (1985)).

6. The Bootstrap

The bootstrap is yet another sample reuse method and as such relies on recomputing the estimate of interest a large number (B) of times; in this case, by resampling the original sample. In this study we follow the approach due to Rao and Wu(1983), that is, in stratum h, a with replacement sample of size m_h is selected from the corresponding stratum of the original sample.

Just as in the preceeding two sections, the estimator $f(\bar{t}_b)$ for the b'th replicate is derived from (2.5)-(2.7) by substituting $\bar{t}_b$ for $\bar{t}$, where, in the case of $n_h = 2$, $\bar{t}_b$ is given by

$$\bar{t}_b = \bar{t} + \sum_{h=1}^{32} W_h \, m_h(\bar{t}_{b(h)} - \bar{t}_{(h)}). \tag{6.1}$$

In (6.1), $\bar{t}_{(h)}$ is the stratum mean computed from the original sample and $\bar{t}_{b(h)}$ is its corresponding estimate obtained from the b'th bootstrap sample (of size m_h), from stratum h, that is

$$\bar{t}_{(h)} = \sum_{i=1}^{2} t(x_{hi}, y_{hi}) \; / \; 2 \;, \quad \text{and} \tag{6.2}$$

$$\bar{t}_{b(h)} = \sum_{i \in I_{bh}} t(x_{hi}, y_{hi}) \; / \; m_h \;. \tag{6.3}$$

Here I_{bh} denotes the set of indices representing the b'th bootstrap sample in the h'th stratum.

The bootstrap variance estimator of $f(\underline{\bar{t}})$ is then given simply by

$$v_{BOOT}(f(\underline{\bar{t}})) = \sum_{b=1}^{B} (f(\underline{\bar{t}}_b) - f(\underline{\bar{t}}))^2 / B \quad . \tag{6.4}$$

While in principle the number of replications (B) should be infinite, we found the choice of B = 100 to be both sufficient and computationally affordable. On the other hand, by selecting various values of m_h (e.g. $m_h = 2$ or 3, for all h), we obtain various bootstrap estimators of the variance of $f(\underline{\bar{t}})$. In the following sections, they are denoted by BOOT-MH2 and BOOT-MH3.

It should be pointed out at this stage, that even if $t(x_{hi}, y_{hi})$ is a nonnegative function, it is mathematically possible for $\bar{t}_b$ in (6.1) to take on negative values. While such situations are rare and can occur only if $m_h \geq 2$, their incidence increases with m_h and the underlying variance of the characteristic of interest. In fact, for the gamma populations, estimation of the denominators of the regression and correlation coefficients caused problems in all but the basic populations ($f_x = f_y = 1$). As such, the bootstrap variance estimates for the regression and correlation coefficients are not reported in what follows in the case of the non-basic gamma populations.

7. The Simulation Study

For all the populations described in Section 2, the MSE's for the three estimators of interest (r, b, c) were computed based on 1,000 independent samples according to the cirteria set out above, using the appropriate parameters from Tables 1-3. The simulation study consisted of selecting, for each population, 100 further samples and computing the estimates r, b and c and their sampling variance estimates using the methods of Sections 3-6. Note that since all the variance estimates use the same 100 samples, they are not independent. For each sample s, therefore, a variance estimate v_s was computed corresponding to the appropriate point estimator and the particular method at hand.

For each point estimator and method of variance estimation, the average relative variance was computed by

$$\text{rel. var.} = \{ \sum_{s=1}^{100} v_s/100 \} \; / \; \text{MSE} \; , \tag{7.1}$$

and the relative stability of the variance estimator by

$$\text{stab.} = \{ \sum_{s=1}^{100} (v_s - \text{MSE})^2 \; / \; 100 \}^{1/2} \; / \; \text{MSE} \; . \tag{7.2}$$

In the case of the Taylor-P method, which does not depend on the selected sample, comparable measures were obtained by:

$$\text{rel. var.} = v_{\text{TAYLOR-P}} \; / \; \text{MSE} \tag{7.3}$$

$$\text{stab.} = (v_{\text{TAYLOR-P}} - \text{MSE}) \; / \; \text{MSE} \tag{7.4}$$

The above relative variances and stabilities are tabulated for the thirty bivariate normal populations and 6 methods of variance estimation (TAYLOR-P, TAYLOR-S, JRR-F, BRR-F, BOOT-MH2 and BOOT-MH3) in Tables 5-7 for the ratio, the regression coefficient and the correlation coefficient respectively. Similar summaries of the results for the thirty populations with the underlying gamma distribution can be found in Tables 8-10. The relative variances are also presented for both underlying populations in graphical form in Figures 1-6, in order to illustrate how their performance depends on the coefficient of variation of $\bar{x}$. We note here, that all results obtained by Hansen and Tepping (1985) for the BRR, JRR and Taylor-P variance estimators for the ratio in normal populations have been confirmed by this study.

8. Summary

In the case of the ratio estimator, all the methods considered in this study tend to perform well when the $CV(\bar{x})$ is low (less than 10%). However, as the $CV(\bar{x})$ increases, the differences become more transparent: the BRR and the bootstrap estimators tend to overstate the sampling variance while the Taylor-P aproximation leads to severe understatement of the true MSE. The same conclusions are even more pronounced as the nonlinearity of the point estimators increase (r → b → c) and as the underlying distribution becomes skewed (normal→gamma). In fact, in the case of the ratio, the positive bias of the BRR and bootstrap variance estimators, as well as the slight negative bias of the Taylor-S and jackknife estimators are only evident for populations with high $CV(\bar{x})$ (Figure 1). However, in the case of the regression and correlation coefficients these biases are clearly evident in all cases (Figures 2 and 3).

Finally, we also note that the Taylor-S and the jackknife methods perform equivalently, confirming the second order asymptotic equivalence established by Rao and Wu

(1985). From a computational point of view, the savings can be substantial. For a design with 32 strata, in the case of the ratio, for example, the number of mathematical operations needed to compute the jackknife variance estimator is approximately 22 times larger than that for the Taylor-S estimator (Hidiroglou and Paton, 1985). The bootstrap estimators resemble the BRR estimators as would be expected in the case of $m_h = 1$. Increasing m_h serves only to increase the noise in the estimator as is evident from the tabulated stabilities. The positive bias of the BRR estimators is more pronounced in the two estimators that use only half the replicates (BRR-H, BRR-C), while the BRR-D tends to perform the best. However, the differences between the BRR estimators are minor compared to the differences between any of them and the jackknife or Taylor-S estimator. In this study, for the purpose of variance estimation, the jackknife and Taylor-S estimators tend to perform the best.

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Appendix

The first order Taylor approximation of the variance of the ratio, r, the regression coefficient, b, and the correlation coefficient, c, in case of sampling from a stratified bivariate normal distribution is given, in general for the estimate f, by:

$$V(f(\underline{\bar{t}})) = \sum_{h=1}^{32} W_h^2 (\nabla'_{f(\underline{\bar{T}})} \cdot D(\underline{\bar{t}}) \cdot \nabla_{f(\underline{\bar{T}})}) / n_h \cdot \tag{A.1}$$

In the case of the ratio, ∇_r is given by

$$\nabla_{r|\underline{\bar{t}} = \underline{\bar{T}}} = \mu_x^{-1} \cdot (-R, 1)' \tag{A.2}$$

while in the case of the regression coeefficient we have:

$$\nabla_{b|\underline{\bar{t}} = \underline{\bar{T}}} = S_x^{-2} \cdot (2\mu_x B - \mu_y, \ -\mu_x, \ 1, \ -B)' \tag{A.3}$$

and in the case of the correlation coefficient,

$$\nabla_{c|\underline{\bar{t}} = \underline{\bar{T}}} = S_x^{-1} S_y^{-1} \cdot (C\mu_x S_y S_x^{-1} - \mu_y, \ C\mu_y S_x S_y^{-1} - \mu_x ,$$

$$1, \ - C S_y S_x^{-1}/2, \ - C S_x S_y^{-1}/2)' \ , \tag{A.4}$$

where

$$S_x^2 = \sum_{h=1}^{32} W_h (\sigma_{xh}^2 + (\mu_{xh} - \mu_x)^2)$$

and

$$S_y^2 = \sum_{h=1}^{32} W_h(\sigma_{yh}^2 + (\mu_{yh} - \mu_y)^2) \qquad \text{(A.5)}$$

The elements of the (symmetric) matrix D, under the assumption of normality, are given for each stratum in (A.6). In the case of the regression coefficient only the leading 4x4 submatrix is needed while for the ratio only the leading 2x2 submatrix is of interest.

$$\begin{aligned}
d_{11} &= \sigma_{xh}^2 \\
d_{12} &= \sigma_{xh}\sigma_{yh}\rho \\
d_{13} &= \mu_{yh}\sigma_{xh}^2 + \mu_{xh}\sigma_{xh}\sigma_{yh}\rho \\
d_{14} &= 2\mu_{xh}\,\sigma_{xh}^2 \\
d_{15} &= 2\mu_{yh}\,\sigma_{xh}\sigma_{yh}\rho \\
d_{22} &= \sigma_{yh}^2 \\
d_{23} &= \mu_{xh}\,\sigma_{yh}^2 + \mu_{yh}\sigma_{xh}\sigma_{yh}\rho \\
d_{24} &= 2\mu_{xh}\,\sigma_{xh}\,\sigma_{yh}\rho \\
d_{25} &= 2\mu_{yh}\,\sigma_{yh}^2 \\
d_{33} &= \mu_{yh}^2\,\sigma_{xh}^2 + \mu_{xh}^2\,\sigma_{yh}^2 + \sigma_{xh}^2\,\sigma_{yh}^2 + \sigma_{xh}^2\,\sigma_{yh}^2\rho^2 + 2\mu_{xh}\,\mu_{yh}\,\sigma_{xh}\,\sigma_{yh}\,\rho \\
d_{34} &= 2(\mu_{xh}\,\mu_{yh}\,\sigma_{xh}^2 + \mu_{xh}^2\,\sigma_{xh}\,\sigma_{yh}\,\rho + \sigma_{xh}^3\,\sigma_{yh}\,\rho) \\
d_{35} &= 2(\mu_{yh}\,\mu_{xh}\,\sigma_{yh}^2 + \mu_{yh}^2\,\sigma_{yh}\,\sigma_{xh}\,\rho + \sigma_{yh}^3\,\sigma_{xh}\,\rho)
\end{aligned} \qquad \text{(A.6)}$$

$$d_{44} = 4\,\mu_{xh}^2\,\sigma_{xh} + 2\sigma_{xh}^{*}$$

$$d_{45} = 4\,\mu_{xh}\mu_{yh}\,\sigma_{xh}\,\sigma_{yh}\,\rho + 2\,\sigma_{xh}^2\,\sigma_{yh}^2\,\rho^2$$

$$d_{55} = 4\,\mu_{yh}^2\,\sigma_{yh}^2 + 2\sigma_{yh}^{*}$$

Since the assumption of normality is needed only for moments of order three or higher, the variance of the ratio as expressed here is valid for any bivariate distribution with the specified first and second moments.

Table 1: Parameters for population 1

Stratum (h)	W_h	μ_{xh}	μ_{yh}	σ_{xh}	σ_{yh}
1	0.042	100.000	90.000	10.000	25.000
2	0.042	95.000	75.000	9.500	24.000
3	0.042	90.000	70.000	9.000	22.000
4	0.039	98.000	75.000	9.800	22.000
5	0.039	93.000	70.000	9.300	20.000
6	0.037	98.000	75.000	9.800	24.000
7	0.037	96.000	75.000	9.600	23.000
8	0.037	94.000	75.000	9.400	22.000
9	0.037	92.000	70.000	9.200	24.000
10	0.034	96.000	75.000	9.600	23.000
11	0.034	94.000	70.000	9.400	20.000
12	0.034	92.000	70.000	9.200	22.000
13	0.034	90.000	70.000	9.000	22.000
14	0.031	96.000	75.000	9.600	25.000
15	0.031	94.000	70.000	9.400	20.000
16	0.031	92.000	70.000	9.200	18.000
17	0.031	90.000	70.000	9.000	19.000
18	0.031	88.000	70.000	8.800	20.000
19	0.031	86.000	65.000	8.600	20.000
20	0.031	84.000	60.000	8.400	18.000
21	0.031	82.000	60.000	8.200	16.000
22	0.031	80.000	60.000	8.000	20.000
23	0.028	90.000	70.000	9.000	22.000
24	0.028	85.000	65.000	8.500	18.000
25	0.028	80.000	60.000	8.000	20.000
26	0.025	90.000	70.000	9.000	20.000
27	0.025	85.000	60.000	8.500	18.000
28	0.025	80.000	50.000	8.000	15.000
29	0.025	75.000	50.000	7.500	14.000
30	0.020	75.000	50.000	7.500	16.000
31	0.016	75.000	45.000	7.500	14.000
32	0.013	75.000	45.000	7.500	12.000

Table 2: **Parameters for population 2**

Stratum (h)	W_h	μ_{xh}	μ_{yh}	σ_{xh}	σ_{yh}
1	0.042	100.000	90.000	10.000	25.000
2	0.042	95.000	75.000	9.500	24.000
3	0.042	90.000	73.000	9.000	22.000
4	0.039	95.000	77.000	9.800	22.000
5	0.039	90.000	71.000	9.300	20.000
6	0.037	95.000	74.000	9.800	24.000
7	0.037	90.000	74.000	9.600	23.000
8	0.037	85.000	71.000	9.400	22.000
9	0.037	80.000	68.000	9.200	24.000
10	0.034	90.000	75.000	9.600	23.000
11	0.034	85.000	70.000	9.400	20.000
12	0.034	80.000	69.000	9.200	22.000
13	0.034	75.000	66.000	9.000	22.000
14	0.031	95.000	73.000	9.600	25.000
15	0.031	90.000	72.000	9.400	20.000
16	0.031	85.000	69.000	9.200	18.000
17	0.031	80.000	67.000	9.000	19.000
18	0.031	75.000	65.000	8.800	20.000
19	0.031	70.000	64.000	8.600	20.000
20	0.031	65.000	61.000	8.400	18.000
21	0.031	60.000	60.000	8.200	16.000
22	0.031	55.000	57.000	8.000	20.000
23	0.028	90.000	70.000	9.000	22.000
24	0.028	80.000	70.000	8.500	18.000
25	0.028	70.000	63.000	8.000	20.000
26	0.025	90.000	69.000	9.000	20.000
27	0.025	80.000	66.000	8.500	18.000
28	0.025	70.000	65.000	8.000	15.000
29	0.025	60.000	59.000	7.500	14.000
30	0.020	80.000	71.000	7.500	16.000
31	0.016	70.000	62.000	7.500	14.000
32	0.013	60.000	58.000	7.500	12.000

Table 3: Parameters for population 3

Stratum (h)	W_h	μ_{xh}	μ_{yh}	σ_{xh}	σ_{yh}
1	0.013	100.000	90.000	10.000	25.000
2	0.016	95.000	75.000	9.500	24.000
3	0.020	90.000	70.000	9.000	22.000
4	0.025	98.000	75.000	9.800	22.000
5	0.025	93.000	70.000	9.300	20.000
6	0.025	98.000	75.000	9.800	24.000
7	0.025	96.000	75.000	9.600	23.000
8	0.028	94.000	75.000	9.400	22.000
9	0.028	92.000	70.000	9.200	24.000
10	0.028	96.000	75.000	9.600	23.000
11	0.031	94.000	70.000	9.400	20.000
12	0.031	92.000	70.000	9.200	22.000
13	0.031	90.000	70.000	9.000	22.000
14	0.031	96.000	75.000	9.600	25.000
15	0.031	94.000	70.000	9.400	20.000
16	0.031	92.000	70.000	9.200	18.000
17	0.031	90.000	70.000	9.000	19.000
18	0.031	88.000	70.000	8.800	20.000
19	0.031	86.000	65.000	8.600	20.000
20	0.034	84.000	60.000	8.400	18.000
21	0.034	82.000	60.000	8.200	16.000
22	0.034	80.000	60.000	8.000	20.000
23	0.034	90.000	70.000	9.000	22.000
24	0.037	85.000	65.000	8.500	18.000
25	0.037	80.000	60.000	8.000	20.000
26	0.037	90.000	70.000	9.000	20.000
27	0.037	85.000	60.000	8.500	18.000
28	0.039	80.000	50.000	8.000	15.000
29	0.039	75.000	50.000	7.500	14.000
30	0.042	75.000	50.000	7.500	16.000
31	0.042	75.000	45.000	7.500	14.000
32	0.042	75.000	45.000	7.500	12.000

Table 4: The BRR specification matrix

Replicate	Stratum 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1
3	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1
4	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1
5	1	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2
6	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1
7	1	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2
8	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1
9	1	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2
10	1	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2
11	1	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2
12	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1
13	1	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2
14	1	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2
15	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1
16	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1
17	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1
18	1	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2
19	1	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2	2
20	1	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2	2
21	1	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2	2
22	1	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1	2
23	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1	1
24	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2	1
25	1	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2	2
26	1	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1	2
27	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2	1
28	1	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1	2
29	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	2	1	1
30	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1	1	2
31	1	1	1	2	1	2	2	1	1	2	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	2
32	1	2	1	1	2	1	2	2	1	1	2	2	2	2	2	1	1	1	2	2	1	2	2	2	1	2	1	2	1	1	1	1

Table 5: Relative variances (and their relative stabilities) computed by various methods in the case of the ratio for all normal populations

Pop'n	ρ	f_x	f_y	CV($\bar{x}$)	R	r	MSE	TAYLOR-P	TAYLOR-S	JJR-F	BBR-F	BOOT-MH2	BOOT-MH3
1	0.8	1	1	.013	.760	.760	5.5-4	0.97 (.03)	0.95 (.25)	0.95 (.25)	0.95 (.25)	0.93 (.29)	0.96 (.28)
1	0.8	5	5	.065	.760	.754	1.3-2	1.00 (.01)	1.00 (.36)	1.00 (.36)	1.02 (.37)	1.02 (.40)	1.03 (.44)
1	0.8	10	1	.130	.760	.755	6.7-3	0.88 (.12)	0.93 (.52)	0.93 (.53)	1.09 (.67)	1.12 (.74)	1.13 (.81)
1	0.5	1	1	.013	.760	.755	7.1-4	1.00 (.01)	1.01 (.28)	1.01 (.28)	1.01 (.28)	1.00 (.29)	1.02 (.32)
1	0.5	5	5	.065	.760	.758	1.8-2	1.01 (.01)	1.01 (.31)	1.01 (.31)	1.02 (.31)	1.00 (.33)	1.01 (.35)
1	0.5	10	1	.130	.760	.755	8.4-3	0.91 (.09)	0.95 (.61)	0.96 (.62)	1.14 (.87)	1.20 (1.0)	1.14 (.90)
1	0.2	1	1	.013	.760	.770	9.4-4	0.95 (.05)	1.00 (.30)	1.00 (.30)	1.01 (.30)	0.99 (.30)	0.98 (.32)
1	0.2	5	5	.065	.760	.739	2.4-2	0.93 (.07)	0.92 (.27)	0.92 (.27)	0.93 (.28)	0.95 (.31)	0.95 (.32)
1	0.2	10	1	.130	.760	.766	1.2-2	0.80 (.20)	0.98 (.98)	0.98 (1.0)	1.44 (2.8)	1.44 (2.6)	1.21 (1.4)
2	0.8	1	1	.014	.847	.847	8.2-4	0.97 (.03)	0.98 (.27)	0.96 (.27)	0.98 (.27)	1.00 (.33)	0.96 (.28)
2	0.8	4	1	.057	.847	.852	9.1-4	0.96 (.04)	0.97 (.33)	0.97 (.33)	0.99 (.34)	0.99 (.36)	0.99 (.40)
2	0.8	4	4	.057	.847	.843	1.0-2	0.92 (.08)	0.97 (.38)	0.97 (.38)	0.99 (.39)	0.99 (.41)	1.00 (.45)
2	0.8	10	1	.142	.847	.868	1.2-2	0.79 (.21)	0.94 (.54)	0.95 (.54)	1.18 (.79)	1.22 (1.2)	1.17 (.87)
2	0.5	1	1	.014	.847	.848	8.0-4	1.05 (.05)	1.05 (.32)	1.05 (.32)	1.05 (.32)	1.05 (.37)	1.04 (.33)
2	0.5	5	5	.071	.847	.848	2.2-2	0.97 (.03)	0.98 (.32)	0.98 (.32)	0.99 (.33)	0.98 (.35)	0.99 (.36)
2	0.5	10	1	.142	.847	.866	1.5-2	0.78 (.22)	0.92 (.53)	0.93 (.53)	1.17 (.78)	1.14 (.78)	1.20 (.91)
2	0.2	1	1	.014	.847	.846	9.7-4	1.11 (.11)	1.15 (.38)	1.15 (.38)	1.15 (.38)	1.16 (.43)	1.17 (.43)
2	0.2	4	1	.057	.847	.855	2.8-3	1.01 (.01)	1.05 (.38)	1.05 (.38)	1.08 (.40)	1.08 (.47)	1.04 (.42)
2	0.2	4	4	.057	.847	.848	1.7-2	1.03 (.03)	1.04 (.33)	1.04 (.33)	1.05 (.34)	1.06 (.39)	1.07 (.37)
2	0.2	10	1	.142	.847	.858	1.8-2	0.76 (.24)	0.86 (.50)	0.87 (.50)	1.08 (.70)	1.06 (.71)	1.08 (.78)
3	0.8	1	1	.013	.741	.742	4.9-4	0.94 (.06)	0.95 (.24)	0.95 (.24)	0.95 (.24)	0.95 (.26)	0.96 (.26)
3	0.8	4	1	.051	.741	.743	5.3-4	0.98 (.02)	1.05 (.34)	1.05 (.34)	1.07 (.36)	1.06 (.38)	1.04 (.38)
3	0.8	4	4	.051	.741	.737	7.5-3	0.98 (.02)	1.00 (.27)	1.00 (.27)	1.01 (.28)	1.04 (.34)	1.03 (.34)
3	0.8	10	1	.127	.741	.753	5.7-3	0.95 (.05)	1.10 (.73)	1.11 (.74)	1.32 (1.0)	1.29 (.94)	1.35 (1.2)
3	0.5	1	1	.013	.741	.741	5.9-4	1.04 (.04)	1.07 (.28)	1.07 (.28)	1.07 (.28)	1.10 (.33)	1.08 (.29)
3	0.5	4	1	.051	.741	.737	1.3-3	0.88 (.12)	0.84 (.30)	0.84 (.30)	0.85 (.30)	0.85 (.34)	0.81 (.34)
3	0.5	4	4	.051	.741	.749	9.7-3	1.02 (.02)	1.03 (.23)	1.03 (.23)	1.04 (.23)	1.08 (.30)	1.03 (.26)
3	0.5	10	1	.127	.741	.759	8.3-2	0.84 (.16)	1.06 (.72)	1.07 (.73)	1.29 (1.1)	1.30 (1.2)	1.38 (1.8)
3	0.2	1	1	.013	.741	.742	7.0-4	1.12 (.12)	1.15 (.34)	1.15 (.34)	1.16 (.34)	1.17 (.41)	1.15 (.36)
3	0.2	10	1	.127	.741	.746	1.0-2	0.83 (.17)	0.95 (.60)	0.96 (.60)	1.13 (.83)	1.12 (.89)	1.21 (1.2)

Note: 1) Relative stabilities less than .01 are reported as .01.
2) The MSE's are reported in a condensed scientific notation (e.g. 5.5-4 = .00055)

Table 6: Relative variances (and their relative stabilities) computed by various methods in the case of the regression coefficient for all normal populations

Pop'n	ρ	f_x	f_y	CV($\bar{x}$)	B	b	MSE	TAYLOR-P	TAYLOR-S	JJR-F	BBR-F	BOOT-MH2	BOOT-MH3
1	0.8	1	1	.013	1.63	1.63	2.2-2	0.99 (.01)	0.95 (.37)	0.96 (.38)	1.07 (.41)	1.11 (.53)	1.10 (.50)
1	0.8	5	5	.065	1.82	1.81	3.4-2	1.00 (.01)	0.96 (.39)	0.98 (.41)	1.12 (.45)	1.15 (.53)	1.16 (.55)
1	0.8	10	1	.130	.190	.191	4.8-4	0.95 (.05)	0.94 (.45)	0.95 (.46)	1.09 (.53)	1.10 (.53)	1.10 (.53)
1	0.5	1	1	.013	1.19	1.19	4.4-2	0.98 (.02)	0.93 (.40)	0.94 (.41)	1.08 (.46)	1.08 (.50)	1.09 (.51)
1	0.5	5	5	.065	1.15	1.08	6.6-2	1.05 (.05)	1.02 (.42)	1.03 (.43)	1.18 (.52)	1.20 (.57)	1.20 (.58)
1	0.5	10	1	.130	.121	.118	9.0-4	0.92 (.08)	0.99 (.43)	1.01 (.45)	1.15 (.52)	1.16 (.54)	1.22 (.65)
1	0.2	1	1	.013	.749	.755	5.7-2	0.97 (.03)	0.99 (.47)	1.00 (.47)	1.12 (.53)	1.16 (.64)	1.09 (.52)
1	0.2	5	5	.065	.477	.494	9.6-2	0.93 (.07)	0.88 (.40)	0.90 (.40)	1.05 (.46)	1.05 (.48)	1.07 (.51)
1	0.2	10	1	.130	.053	.054	1.1-3	0.97 (.03)	0.97 (.44)	0.99 (.45)	1.14 (.53)	1.17 (.59)	1.20 (.62)
2	0.8	1	1	.014	1.00	.981	1.8-2	0.93 (.07)	0.93 (.43)	0.94 (.44)	1.04 (.49)	1.04 (.52)	1.05 (.57)
2	0.8	4	1	.057	.464	.470	2.2-3	0.94 (.06)	0.95 (.39)	0.96 (.40)	1.09 (.44)	1.10 (.48)	1.12 (.50)
2	0.8	4	4	.057	1.71	1.74	3.3-2	1.00 (.01)	0.97 (.40)	0.99 (.41)	1.13 (.52)	1.16 (.55)	1.20 (.65)
2	0.8	10	1	.142	.189	.188	4.2-4	0.96 (.04)	1.03 (.49)	1.05 (.51)	1.20 (.58)	1.23 (.66)	1.26 (.69)
2	0.5	1	1	.014	.748	.762	2.7-2	0.94 (.06)	0.96 (.39)	0.97 (.39)	1.04 (.42)	1.06 (.48)	1.04 (.45)
2	0.5	5	5	.071	1.11	1.09	7.1-2	0.95 (.05)	0.93 (.45)	0.94 (.46)	1.08 (.54)	1.12 (.61)	1.10 (.61)
2	0.5	10	1	.142	.121	.122	8.5-4	0.91 (.09)	0.84 (.37)	0.86 (.37)	1.00 (.38)	1.03 (.43)	1.02 (.42)
2	0.2	1	1	.014	.493	.496	3.3-2	0.96 (.04)	1.01 (.53)	1.01 (.53)	1.09 (.57)	1.11 (.62)	1.11 (.59)
2	0.2	4	1	.057	.153	.157	5.6-3	0.97 (.04)	0.89 (.40)	0.90 (.40)	1.04 (.44)	1.03 (.46)	1.09 (.52)
2	0.2	4	4	.057	.464	.445	8.5-2	0.96 (.04)	0.96 (.43)	0.97 (.44)	1.11 (.50)	1.15 (.52)	1.17 (.63)
2	0.2	10	1	.142	.054	.050	1.1-3	0.93 (.07)	0.91 (.42)	0.93 (.43)	1.07 (.48)	1.08 (.52)	1.09 (.49)
3	0.8	1	1	.013	1.57	1.60	1.9-2	0.92 (.08)	0.93 (.40)	0.94 (.41)	1.04 (.46)	1.03 (.42)	1.09 (.61)
3	0.8	4	1	.051	.483	.482	2.3-3	0.97 (.03)	0.93 (.37)	0.94 (.38)	1.08 (.44)	1.09 (.52)	1.07 (.47)
3	0.8	4	4	.051	1.77	1.79	3.1-2	0.93 (.07)	0.97 (.44)	0.99 (.45)	1.12 (.52)	1.15 (.58)	1.15 (.56)
3	0.8	10	1	.127	.187	.188	4.6-4	0.98 (.04)	0.93 (.41)	0.94 (.41)	1.07 (.45)	1.07 (.48)	1.1[illegible]
3	0.5	1	1	.013	1.19	1.19	3.3-2	1.02 (.02)	1.08 (.46)	1.09 (.46)	1.21 (.54)	1.21 (.56)	1.2[illegible]
3	0.5	4	1	.051	.323	.320	5.0-3	0.86 (.14)	0.89 (.37)	0.90 (.37)	1.02 (.40)	1.06 (.42)	1.03 (.42)
3	0.5	4	4	.051	1.12	1.13	5.7-2	1.02 (.02)	0.99 (.39)	1.01 (.40)	1.15 (.46)	1.15 (.48)	1.19 (.56)
3	0.5	10	1	.127	.120	.123	8.3-4	0.92 (.08)	0.88 (.36)	0.89 (.36)	1.02 (.38)	1.02 (.43)	1.08 (.46)
3	0.2	1	1	.013	.798	.787	4.7-2	0.95 (.05)	1.00 (.37)	1.01 (.38)	1.12 (.44)	1.10 (.44)	1.16 (.53)
3	0.2	10	1	.127	.054	.053	1.0-3	0.91 (.09)	0.90 (.38)	0.91 (.39)	1.05 (.44)	1.09 (.52)	1.08 (.46)

Note: 1) Relative stabilities less than .01 are reported as .01.
2) The MSE's are reported in a condensed scientific notation (e.g. 5.5-4 = .00055)

Table 7: Relative variances (and their relative stabilities) computed by various methods in the case of the correlation coefficient for all normal populations

Pop'n	ρ	f_x	f_y	CV($\bar{x}$)	C	c	MSE	TAYLOR-P	TAYLOR-S	JJR-F	BBR-F	BOOT-MH2	BOOT-MH3
1	0.8	1	1	.013	.810	.804	2.0-3	0.93 (.07)	0.93 (.48)	0.94 (.48)	1.08 (.57)	1.13 (.63)	1.10 (.56)
1	0.8	5	5	.065	.797	.789	2.4-3	0.98 (.04)	0.97 (.53)	0.99 (.54)	1.16 (.63)	1.16 (.66)	1.19 (.68)
1	0.8	10	1	.130	.756	.751	3.4-3	0.90 (.10)	0.98 (.63)	1.00 (.64)	1.17 (.77)	1.21 (.85)	1.17 (.76)
1	0.5	1	1	.013	.591	.601	7.1-3	0.96 (.04)	0.84 (.42)	0.86 (.43)	0.98 (.48)	0.96 (.47)	1.00 (.62)
1	0.5	5	5	.065	.503	.481	8.8-3	1.12 (.12)	1.11 (.48)	1.13 (.50)	1.28 (.60)	1.31 (.64)	1.29 (.68)
1	0.5	10	1	.130	.483	.483	1.0-2	0.98 (.02)	1.03 (.41)	1.05 (.42)	1.18 (.50)	1.19 (.56)	1.25 (.65)
1	0.2	1	1	.013	.373	.373	1.2-2	0.96 (.04)	0.88 (.34)	0.89 (.34)	0.98 (.35)	1.00 (.39)	0.98 (.37)
1	0.2	5	5	.065	.208	.213	1.7-2	0.96 (.04)	0.86 (.32)	0.87 (.32)	0.98 (.32)	0.99 (.35)	0.99 (.35)
1	0.2	10	1	.130	.210	.211	1.8-2	1.00 (.01)	0.94 (.33)	0.95 (.33)	1.06 (.36)	1.09 (.42)	1.09 (.43)
2	0.8	1	1	.014	.681	.670	4.7-3	0.95 (.05)	1.04 (.56)	1.05 (.58)	1.19 (.70)	1.22 (.74)	1.22 (.80)
2	0.8	4	1	.057	.807	.807	2.2-3	0.98 (.02)	0.93 (.54)	0.95 (.55)	1.10 (.63)	1.13 (.65)	1.15 (.69)
2	0.8	4	4	.057	.777	.784	2.7-3	1.00 (.01)	0.95 (.43)	0.96 (.43)	1.14 (.55)	1.17 (.59)	1.20 (.68)
2	0.8	10	1	.142	.788	.779	2.5-3	1.01 (.01)	1.14 (.73)	1.17 (.75)	1.36 (.91)	1.36 (.93)	1.40 (1.0)
2	0.5	1	1	.014	.508	.513	9.1-3	0.97 (.03)	0.95 (.39)	0.96 (.39)	1.06 (.44)	1.09 (.58)	1.06 (.42)
2	0.5	5	5	.071	.495	.487	1.0-2	1.00 (.01)	0.98 (.50)	0.99 (.52)	1.12 (.59)	1.18 (.72)	1.10 (.62)
2	0.5	10	1	.142	.506	.505	1.1-2	0.90 (.10)	0.86 (.39)	0.88 (.39)	1.00 (.41)	1.01 (.46)	1.03 (.44)
2	0.2	1	1	.014	.335	.339	1.3-2	0.95 (.05)	0.95 (.44)	0.96 (.45)	1.03 (.48)	1.05 (.54)	1.06 (.52)
2	0.2	4	1	.057	.266	.274	1.6-2	0.98 (.02)	0.88 (.37)	0.89 (.37)	1.00 (.39)	1.00 (.43)	1.05 (.46)
2	0.2	4	4	.057	.211	.204	1.8-2	1.00 (.01)	0.95 (.34)	0.97 (.35)	1.07 (.39)	1.11 (.41)	1.11 (.48)
2	0.2	10	1	.142	.224	.209	1.7-2	0.97 (.03)	0.92 (.35)	0.94 (.35)	1.04 (.37)	1.05 (.42)	1.08 (.44)
3	0.8	1	1	.013	.821	.826	1.8-3	0.96 (.04)	0.94 (.54)	0.95 (.56)	1.09 (.67)	1.07 (.63)	1.16 (1.1)
3	0.8	4	1	.051	.784	.783	2.5-3	0.94 (.06)	0.99 (.50)	1.01 (.52)	1.17 (.67)	1.19 (.71)	1.15 (.68)
3	0.8	4	4	.051	.797	.798	2.3-3	0.90 (.10)	0.90 (.54)	0.92 (.55)	1.07 (.65)	1.11 (.72)	1.10 (.68)
3	0.8	10	1	.127	.743	.739	3.2-3	0.98 (.02)	0.99 (.42)	1.00 (.42)	1.15 (.53)	1.17 (.58)	1.21 (.62)
3	0.5	1	1	.013	.818	.609	5.6-3	1.00 (.01)	1.05 (.44)	1.06 (.44)	1.18 (.51)	1.18 (.55)	1.19 (.55)
3	0.5	4	1	.051	.524	.518	9.2-3	0.89 (.11)	0.85 (.36)	0.86 (.36)	0.96 (.38)	0.99 (.35)	0.97 (.38)
3	0.5	4	4	.051	.507	.500	8.1-3	1.07 (.07)	1.05 (.41)	1.06 (.42)	1.19 (.49)	1.21 (.55)	1.24 (.59)
3	0.5	10	1	.127	.478	.486	1.0-2	0.94 (.06)	0.86 (.34)	0.86 (.34)	0.97 (.35)	0.99 (.40)	0.99 (.37)
3	0.2	1	1	.013	.416	.404	1.0-2	0.97 (.03)	0.98 (.36)	0.99 (.37)	1.08 (.42)	1.08 (.42)	1.11 (.48)
3	0.2	10	1	.127	.213	.207	1.5-2	0.95 (.05)	0.88 (.29)	0.89 (.29)	0.98 (.29)	1.01 (.36)	1.01 (.31)

Note: 1) Relative stabilities less than .01 are reported as .01.
2) The MSE's are reported in a condensed scientific notation (e.g. 5.5-4 = .00055)

Table 8: Relative variances (and their relative stabilities) computed by various methods in the case of the ratio for all gamma populations

Pop'n	ρ	f_x	f_y	CV($\bar{x}$)	R	r	MSE	TAYLOR-P	TAYLOR-S	JJR-F	BBR-F	BOOT-MH2	BOOT-MH3
1	0.8	1	1	.013	.760	.762	3.8-4	1.40 (.40)	1.03 (.30)	1.03 (.30)	1.03 (.30)	1.03 (.31)	1.02 (.29)
1	0.8	5	5	.065	.760	.746	1.0-2	1.30 (.30)	0.95 (.32)	0.95 (.32)	0.96 (.33)	0.99 (.34)	0.95 (.34)
1	0.8	10	1	.130	.760	.775	6.2-3	0.95 (.05)	0.97 (.40)	0.98 (.41)	1.13 (.54)	1.13 (.52)	1.11 (.51)
1	0.5	1	1	.013	.760	.757	4.6-4	1.53 (.53)	1.04 (.26)	1.04 (.26)	1.04 (.26)	1.03 (.29)	1.05 (.31)
1	0.5	5	5	.065	.760	.761	1.3-2	1.34 (.34)	0.91 (.30)	0.91 (.30)	0.92 (.30)	0.93 (.33)	0.93 (.31)
1	0.5	10	1	.130	.760	.761	8.6-3	0.87 (.12)	0.83 (.43)	0.84 (.43)	0.96 (.49)	0.96 (.48)	0.97 (.52)
1	0.2	1	1	.013	.760	.752	7.7-4	1.16 (.16)	0.97 (.26)	0.97 (.26)	0.97 (.26)	0.97 (.30)	1.01 (.31)
1	0.2	5	5	.065	.760	.758	1.9-2	1.14 (.14)	0.97 (.29)	0.97 (.29)	0.98 (.29)	0.97 (.33)	0.98 (.32)
1	0.2	10	1	.130	.760	.767	1.1-2	0.87 (.13)	0.92 (.38)	0.93 (.39)	1.05 (.46)	1.09 (.51)	1.12 (.56)
2	0.8	1	1	.014	.847	.846	4.2-4	1.42 (.42)	1.02 (.26)	1.02 (.26)	1.02 (.27)	1.01 (.28)	1.00 (.29)
2	0.8	4	1	.057	.847	.853	7.2-4	1.22 (.22)	0.95 (.26)	0.95 (.26)	0.97 (.28)	0.98 (.35)	0.99 (.30)
2	0.8	4	4	.057	.847	.849	6.4-3	1.49 (.49)	1.02 (.35)	1.03 (.35)	1.04 (.37)	1.03 (.36)	1.04 (.38)
2	0.8	10	1	.142	.847	.845	9.6-3	0.96 (.04)	0.88 (.39)	0.89 (.40)	1.04 (.48)	1.08 (.55)	1.09 (.58)
2	0.5	1	1	.014	.847	.847	5.5-4	1.52 (.52)	1.06 (.34)	1.06 (.34)	1.06 (.34)	1.07 (.37)	1.08 (.37)
2	0.5	5	5	.071	.847	.868	1.6-2	1.35 (.35)	0.93 (.32)	0.93 (.32)	0.95 (.32)	0.94 (.35)	0.94 (.34)
2	0.5	10	1	.142	.847	.859	1.3-2	0.90 (.10)	0.91 (.50)	0.92 (.53)	1.08 (.65)	1.06 (.65)	1.10 (.82)
2	0.2	1	1	.014	.847	.847	9.8-4	1.10 (.10)	0.90 (.27)	0.90 (.27)	0.90 (.27)	0.87 (.30)	0.89 (.30)
2	0.2	4	1	.057	.847	.850	2.6-3	1.06 (.06)	1.03 (.29)	1.03 (.29)	1.05 (.30)	1.07 (.35)	1.06 (.33)
2	0.2	4	4	.057	.847	.856	1.4-2	1.21 (.21)	1.06 (.28)	1.06 (.28)	1.07 (.28)	1.04 (.30)	1.09 (.33)
2	0.2	10	1	.142	.847	.861	1.6-2	0.86 (.14)	0.91 (.37)	0.92 (.38)	1.10 (.51)	1.12 (.57)	1.12 (.57)
3	0.8	1	1	.013	.741	.742	3.6-4	1.27 (.27)	0.91 (.27)	0.91 (.27)	0.91 (.27)	0.90 (.30)	0.90 (.29)
3	0.8	4	1	.051	.741	.741	3.9-4	1.32 (.32)	1.01 (.28)	1.01 (.28)	1.02 (.29)	1.04 (.33)	1.01 (.30)
3	0.8	4	4	.051	.741	.738	5.8-3	1.31 (.31)	0.93 (.30)	0.93 (.30)	0.94 (.30)	0.93 (.34)	0.93 (.36)
3	0.8	10	1	.127	.741	.758	5.7-3	0.95 (.05)	0.95 (.40)	0.95 (.40)	1.09 (.51)	1.09 (.53)	1.[illegible]
3	0.5	1	1	.013	.741	.743	4.1-4	1.50 (.50)	1.01 (.25)	1.01 (.25)	1.01 (.25)	1.02 (.27)	1.0[illegible]
3	0.5	4	1	.051	.741	.741	8.9-4	1.30 (.30)	1.10 (.35)	1.10 (.35)	1.12 (.37)	1.13 (.41)	1.12 (.42)
3	0.5	4	4	.051	.741	.741	7.0-3	1.42 (.42)	1.01 (.26)	1.01 (.26)	1.02 (.28)	1.01 (.29)	1.00 (.32)
3	0.5	10	1	.127	.741	.743	7.0-3	1.00 (.01)	1.00 (.53)	1.01 (.54)	1.15 (.67)	1.17 (.68)	1.14 (.61)
3	0.2	1	1	.013	.741	.739	5.8-4	1.32 (.32)	1.13 (.31)	1.13 (.31)	1.13 (.31)	1.16 (.39)	1.13 (.35)
3	0.2	10	1	.127	.741	.762	8.5-3	1.00 (.01)	1.06 (.43)	1.07 (.44)	1.22 (.58)	1.22 (.64)	1.21 (.65)

Note: 1) Relative stabilities less than .01 are reported as .01.
2) The MSE's are reported in a condensed scientific notation (e.g. 5.5-4 = .00055)

Table 9: Relative variances (and their relative stabilities) computed by various methods in the case of the regression coefficient for all gamma populations

Pop'n	ρ	f_x	f_y	CV($\bar{x}$)	B	b	MSE	TAYLOR-S	JJR-F	BBR-F	BOOT-MH2	BOOT-MH3
1	0.8	1	1	.013	1.63	1.62	1.3-2	1.00 (.51)	1.02 (.53)	1.15 (.60)	1.21 (.73)	1.23 (.78)
1	0.8	5	5	.065	1.82	1.81	2.2-2	0.94 (.48)	0.98 (.50)	1.19 (.62)	-	-
1	0.8	10	1	.130	.190	.188	4.0-4	0.92 (.59)	1.00 (.64)	1.28 (.77)	-	-
1	0.5	1	1	.013	1.19	1.19	3.0-2	0.99 (.42)	1.00 (.43)	1.13 (.49)	1.16 (.56)	1.16 (.54)
1	0.5	5	5	.065	1.15	1.18	5.1-2	0.84 (.45)	0.86 (.46)	1.04 (.50)	-	-
1	0.5	10	1	.130	.121	.125	6.3-4	0.95 (.61)	1.02 (.66)	1.36 (.86)	-	-
1	0.2	1	1	.013	.749	.728	4.9-2	0.91 (.38)	0.93 (.38)	1.03 (.41)	1.06 (.47)	1.07 (.51)
1	0.2	5	5	.065	.477	.423	8.6-2	0.85 (.50)	0.88 (.51)	1.05 (.55)	-	-
1	0.2	10	1	.130	.053	.053	9.6-4	0.86 (.55)	0.93 (.58)	1.23 (.71)	-	-
2	0.8	1	1	.014	1.00	1.00	1.2-2	0.85 (.40)	0.86 (.41)	0.96 (.43)	0.96 (.49)	0.97 (.46)
2	0.8	4	1	.057	.464	.467	1.3-3	0.90 (.40)	0.93 (.42)	1.09 (.47)	-	-
2	0.8	4	4	.057	1.71	1.69	1.9-2	1.00 (.45)	1.04 (.52)	1.23 (.65)	-	-
2	0.8	10	1	.142	.189	.190	3.0-4	0.86 (.66)	0.92 (.70)	1.21 (.79)	-	-
2	0.5	1	1	.014	.748	.765	1.9-2	0.98 (.42)	0.99 (.42)	1.06 (.45)	1.07 (.48)	1.08 (.54)
2	0.5	5	5	.071	1.11	1.12	4.7-2	0.95 (.51)	0.99 (.54)	1.19 (.61)	-	-
2	0.5	10	1	.142	.121	.117	6.5-4	0.92 (.74)	1.01 (.86)	1.34 (1.0)	-	-
2	0.2	1	1	.014	.493	.517	3.0-2	0.89 (.39)	0.90 (.40)	0.96 (.40)	0.96 (.42)	0.98 (.41)
2	0.2	4	1	.057	.153	.164	5.2-3	0.86 (.45)	0.89 (.48)	1.04 (.53)	-	-
2	0.2	4	4	.057	.464	.462	6.9-2	1.00 (.47)	1.03 (.48)	1.21 (.56)	-	-
2	0.2	10	1	.142	.054	.048	9.5-4	0.89 (.58)	0.97 (.63)	1.31 (.83)	-	-
3	0.8	1	1	.013	1.57	1.57	1.1-2	0.99 (.42)	1.00 (.43)	1.11 (.48)	1.08 (.48)	1.09 (.50)
3	0.8	4	1	.051	.483	.489	1.8-3	0.92 (.48)	0.94 (.52)	1.08 (.58)	-	-
3	0.8	4	4	.051	1.77	1.75	1.8-2	0.91 (.41)	0.93 (.42)	1.09 (.49)	-	-
3	0.8	10	1	.127	.187	.186	4.5-4	1.05 (.74)	1.16 (1.1)	1.44 (1.2)	-	-
3	0.5	1	1	.013	1.19	1.18	2.2-2	1.05 (.43)	1.07 (.44)	1.18 (.51)	1.20 (.59)	1.19 (.56)
3	0.5	4	1	.051	.323	.319	3.2-3	0.95 (.41)	0.97 (.43)	1.14 (.51)	-	-
3	0.5	4	4	.051	1.12	1.11	4.2-2	0.94 (.41)	0.96 (.43)	1.12 (.49)	-	-
3	0.5	10	1	.127	.120	.126	6.9-4	0.90 (.52)	0.97 (.56)	1.30 (.75)	-	-
3	0.2	1	1	.013	.798	.804	3.6-2	1.00 (.40)	1.01 (.40)	1.13 (.47)	1.13 (.53)	1.17 (.54)
3	0.2	10	1	.127	.054	.058	1.0-3	0.86 (.53)	0.93 (.57)	1.22 (.72)	-	-

Note: 1) Relative stabilities less than .01 are reported as .01.
2) The MSE's are reported in a condensed scientific notation (e.g. 5.5-4 = .00055)

Table 10: Relative variances (and their relative stabilities) computed by various methods in the case of the correlation coefficient for all gamma populations

Pop'n	ρ	f_x	f_y	CV($\hat{x}$)	C	e	MSE	TAYLOR-S	JJR-F	BBR-F	BOOT-MH2	BOOT-MH3
1	0.8	1	1	.013	.810	.868	4.6-3	0.21 (.79)	0.22 (.79)	0.26 (.76)	0.27 (.75)	0.28 (.74)
1	0.8	5	5	.065	.797	.860	6.2-3	0.24 (.78)	0.25 (.76)	0.31 (.71)	-	-
1	0.8	10	1	.130	.756	.792	6.4-3	0.56 (.55)	0.64 (.54)	0.89 (.55)	-	-
1	0.5	1	1	.013	.591	.662	1.0-2	0.51 (.55)	0.51 (.55)	0.57 (.51)	0.58 (.50)	0.59 (.51)
1	0.5	5	5	.065	.502	.582	1.5-2	0.54 (.54)	0.56 (.54)	0.66 (.50)	-	-
1	0.5	10	1	.130	.483	.548	1.5-2	0.69 (.48)	0.75 (.55)	0.91 (.55)	-	-
1	0.2	1	1	.013	.373	.393	1.2-2	0.87 (.32)	0.89 (.33)	0.97 (.34)	0.99 (.38)	0.99 (.37)
1	0.2	5	5	.065	.206	.203	1.8-2	0.87 (.37)	0.90 (.38)	1.02 (.40)	-	-
1	0.2	10	1	.130	.210	.232	1.6-2	0.97 (.42)	1.02 (.49)	1.20 (.62)	-	-
2	0.6	1	1	.014	.681	.729	6.2-3	0.53 (.54)	0.54 (.54)	0.61 (.54)	0.62 (.53)	0.62 (.52)
2	0.8	4	1	.057	.807	.867	5.1-3	0.22 (.79)	0.23 (.79)	0.29 (.74)	-	-
2	0.6	4	4	.057	.777	.845	6.4-3	0.25 (.77)	0.27 (.76)	0.33 (.71)	-	-
2	0.8	10	1	.142	.788	.844	5.9-3	0.40 (.66)	0.46 (.63)	0.65 (.58)	-	-
2	0.5	1	1	.014	.508	.581	1.2-2	0.59 (.50)	0.60 (.50)	0.67 (.47)	0.67 (.48)	0.69 (.54)
2	0.5	5	5	.071	.495	.570	1.4-2	0.60 (.51)	0.63 (.52)	0.75 (.51)	-	-
2	0.5	10	1	.142	.506	.548	1.5-2	0.67 (.51)	0.72 (.55)	0.89 (.60)	-	-
2	0.2	1	1	.014	.335	.380	1.4-2	0.80 (.42)	0.81 (.42)	0.89 (.42)	0.87 (.45)	0.90 (.47)
2	0.2	4	1	.057	.266	.302	1.8-2	0.83 (.36)	0.85 (.36)	0.96 (.36)	-	-
2	0.2	4	4	.057	.211	.224	1.7-2	0.90 (.33)	0.92 (.33)	1.03 (.35)	-	-
2	0.2	10	1	.142	.224	.208	1.7-2	0.84 (.44)	0.88 (.48)	1.04 (.54)	-	-
3	0.8	1	1	.013	.821	.878	4.2-3	0.19 (.82)	0.19 (.81)	0.22 (.79)	0.23 (.78)	0.23 (.78)
3	0.8	4	1	.051	.784	.836	4.3-3	0.40 (.65)	0.42 (.64)	0.51 (.61)	-	-
3	0.8	4	4	.051	.797	.862	5.9-3	0.19 (.82)	0.20 (.81)	0.24 (.77)	-	-
3	0.8	10	1	.127	.743	.765	5.6-3	0.61 (.51)	0.67 (.54)	0.88 (.58)	-	-
3	0.5	1	1	.013	.818	.688	9.6-3	0.42 (.62)	0.42 (.61)	0.47 (.57)	0.47 (.58)	0.47 (.58)
3	0.5	4	1	.051	.524	.581	3.2-3	0.63 (.47)	0.64 (.47)	0.73 (.43)	-	-
3	0.5	4	4	.051	.507	.571	1.3-2	0.57 (.50)	0.58 (.50)	0.67 (.46)	-	-
3	0.5	10	1	.127	.478	.544	1.4-2	0.66 (.48)	0.72 (.55)	0.87 (.56)	-	-
3	0.2	1	1	.013	.416	.449	9.9-3	0.85 (.33)	0.86 (.33)	0.94 (.33)	0.93 (.35)	0.94 (.35)
3	0.2	10	1	.127	.213	.242	1.6-2	0.91 (.41)	0.94 (.44)	1.08 (.49)	-	-

Note: 1) Relative stabilities less than .01 are reported as .01.
2) The MSE's are reported in a condensed scientific notation (e.g. 5.5-4 = .00055)

Figure 1: Relative variances by various methods summarized for all normal populations in the case of the ratio

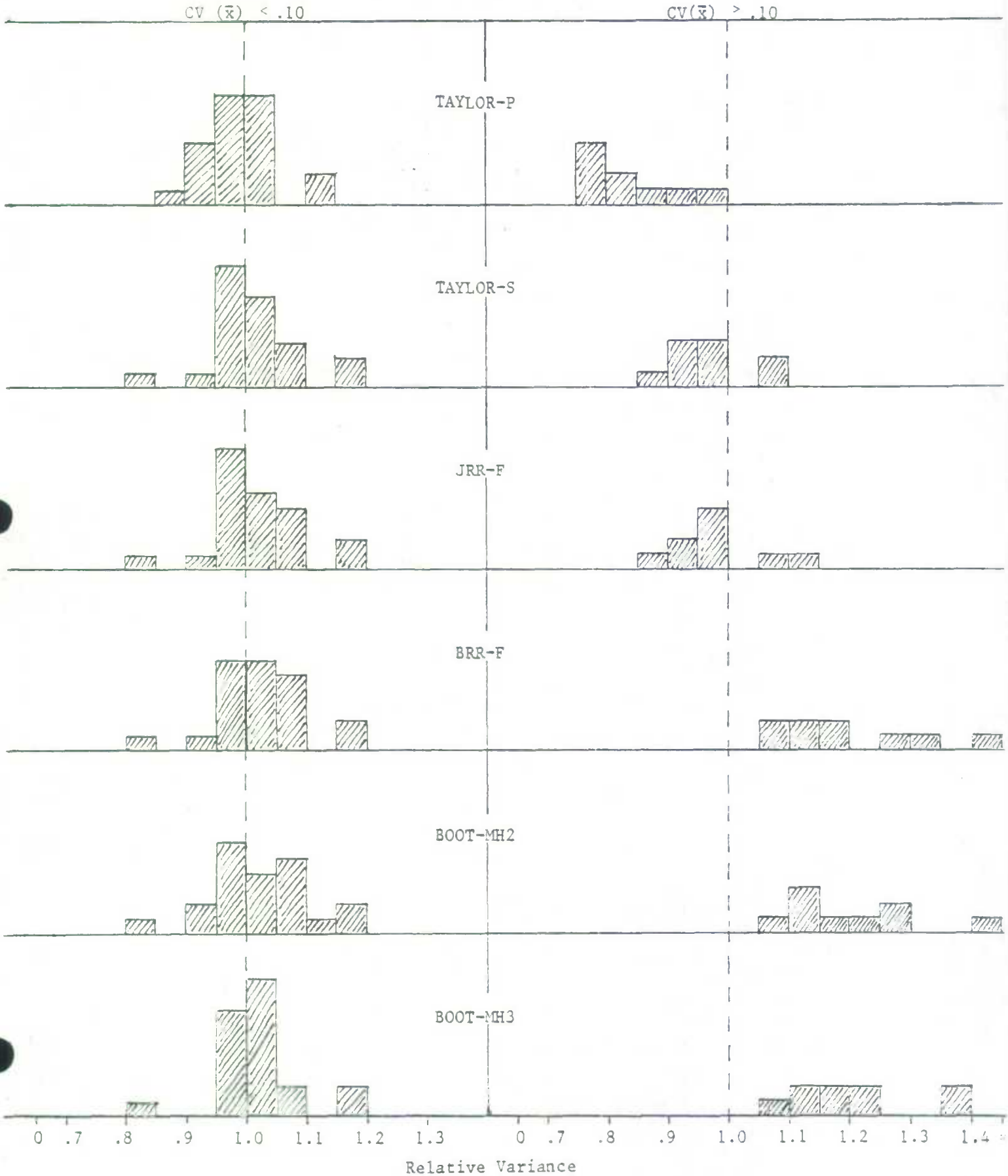


Figure 2: Relative variances by various methods summarized for all normal populations in the case of the regression coefficent

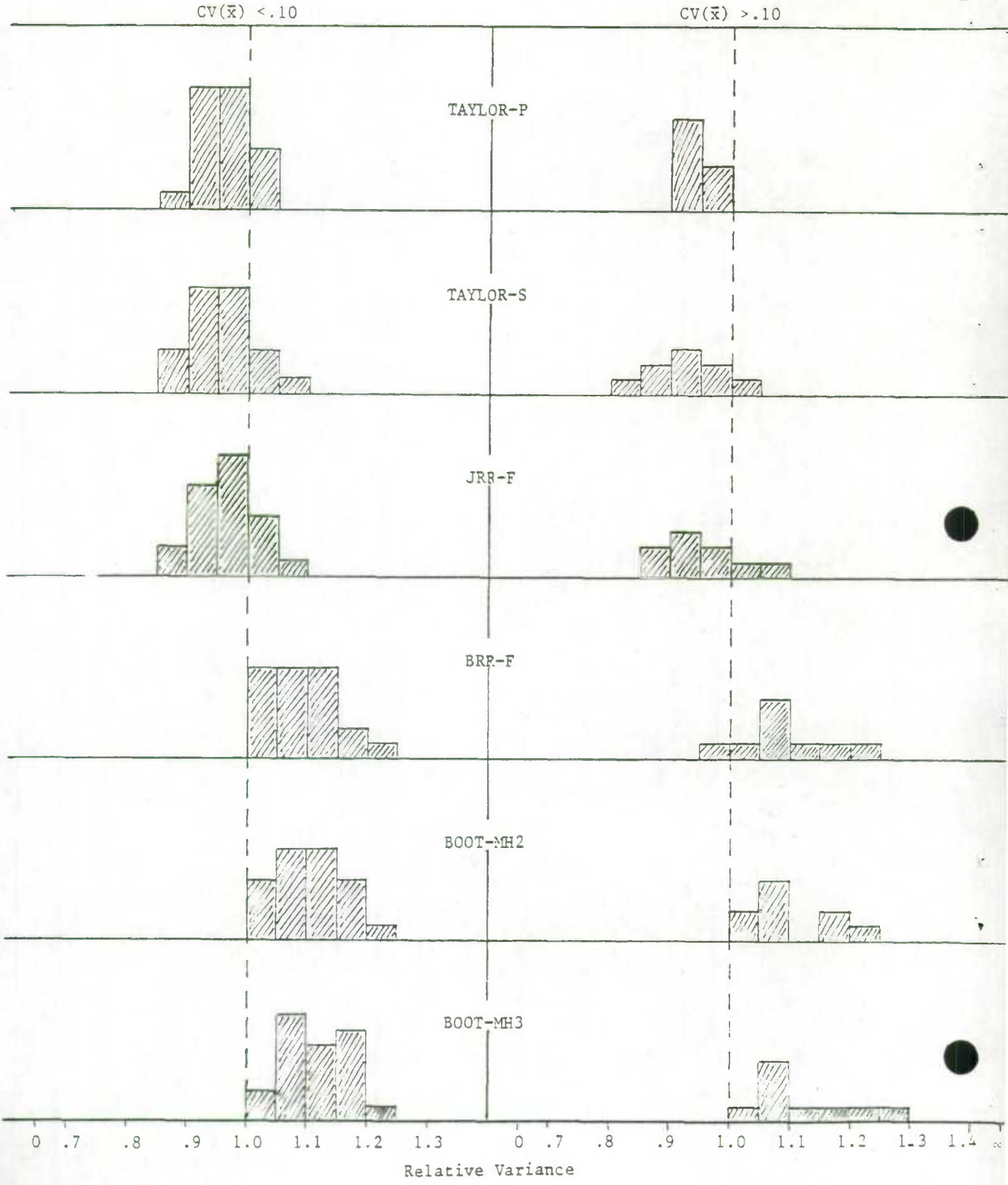


Figure 3: Relative variances by various methods summarized for all normal populations in the case of the correlation coefficent

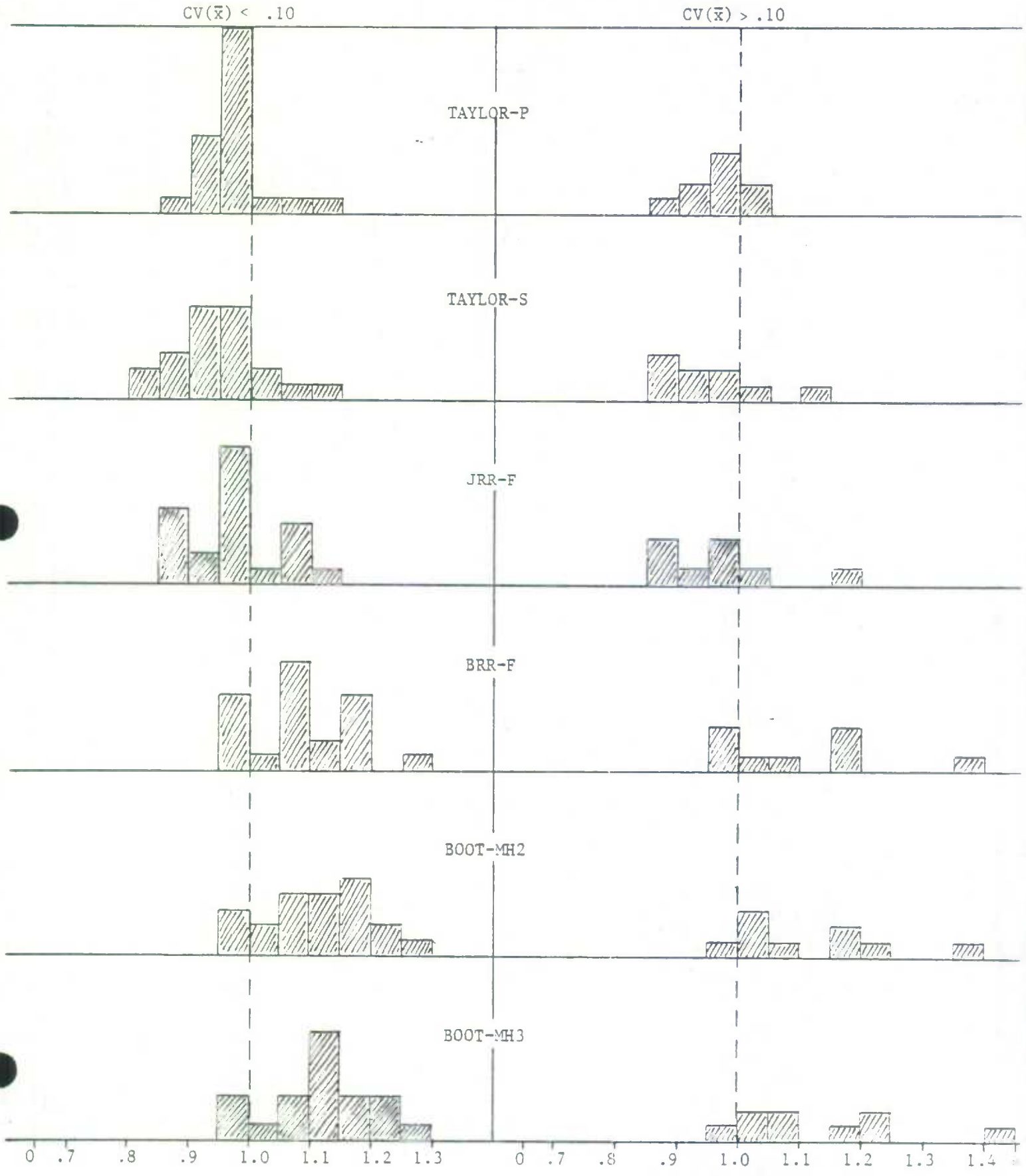


Figure 4: Relative variances by various methods summarized for all gamma populations in the case of the ratio

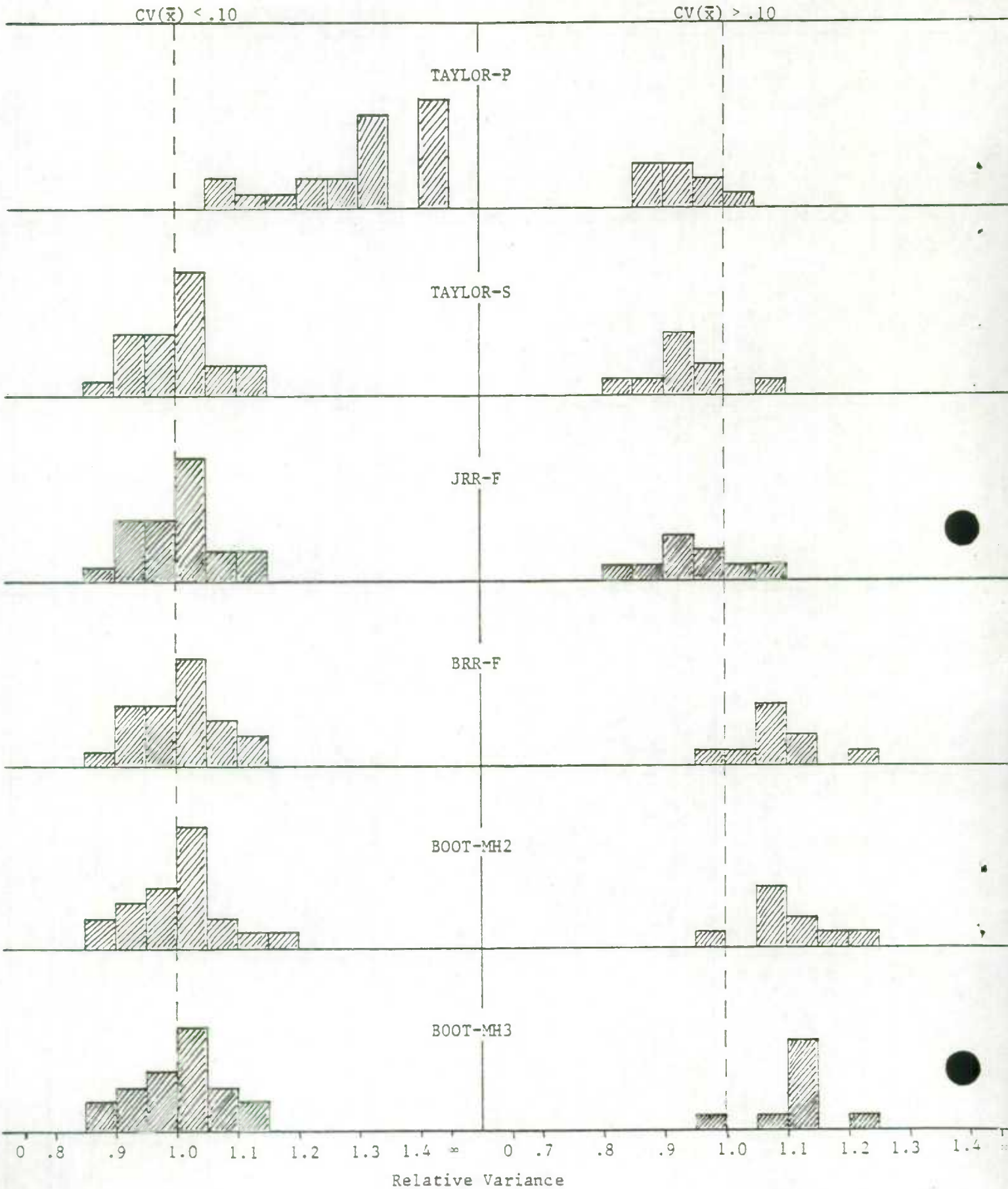
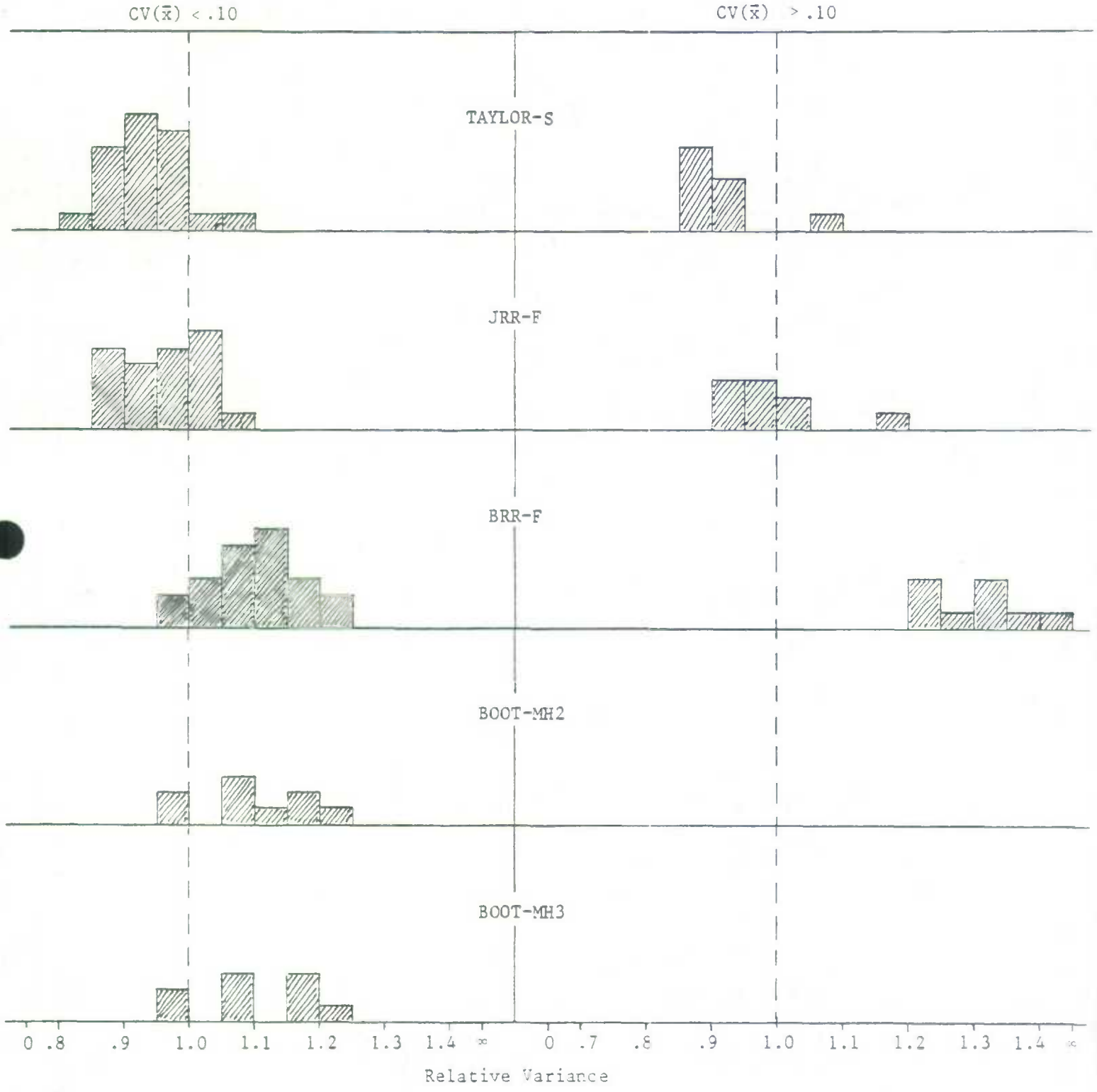


Figure 5: Relative variances by various methods summarized for all gamma populations in the case of the regression coefficient



Note: In the case of the bootstrap, only the basic populations ($f_x = f_y = 1$) were considered.

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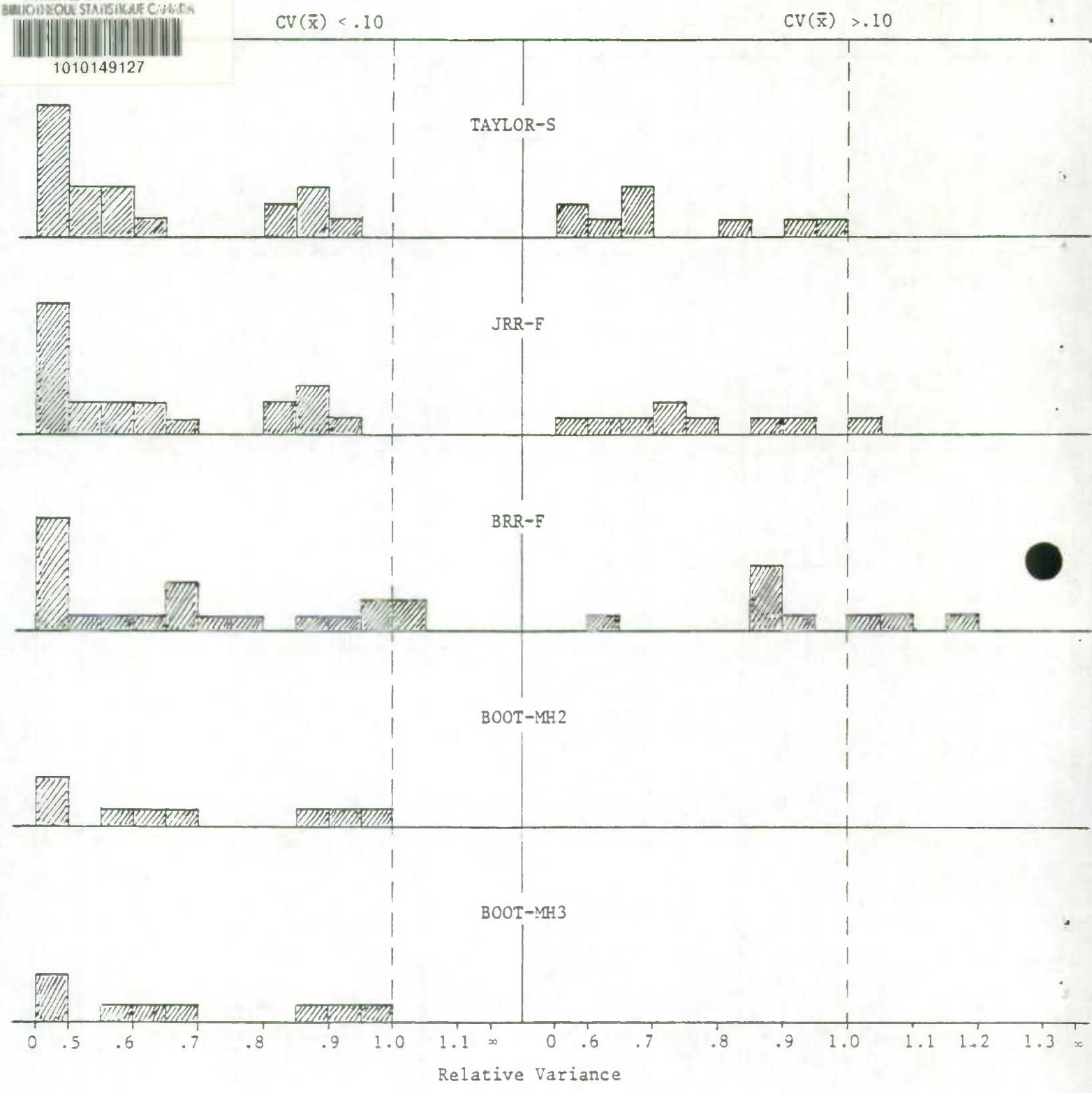
Figure 6: Relative variances by various methods summarized for all gamma populations in the case of the correlation coefficent



Note: In the case of the bootstrap, only the basic populations ($f_x = f_y = 1$) were conside