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A theory of measuring fixed capital flows and stocks

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This paper presents a theoretical perpetual inventory model for measuring both the quantity and price sides of fixed capital flows and stocks. Because conventional models only measure the quantity side of fixed capital flows and stocks, it is difficult to fit the economic depreciation variable into the models. This paper provides a clear picture of how physical depreciation and economic depreciation are applied to the quantity side and price side of the model. The new model presented in this paper also generalizes the conventional model. In any case, the conventional model which uses the deflator of the gross fixed capital formation as the deflator of all fixed capital flows and stocks is questionable.

1. Introduction

Capital and labour are the two most important input factors of production whether one lives in the centrally controlled economy of a communist country, where the majority of capital is owned by the state, or the free market economy of a capitalist country, where the majority of capital is privately owned. An accurate measurement of capital is vital for the economic development of all countries. As an indicator of wealth, the measurement of capital provides an important information on how wealth is divided and shared by the sectors of economy, how rich a nation is, and the progresses of a nation in building up its wealth. Whether one uses quantities or prices, a new measurement of fixed capital flows and stocks will affect the empirical analysis of investment behaviour such as those by Jorgenson [12, 13] and Stevens [34]; capital stock theory such as those by Robinson [29], Christensen and Jorgenson [6] and Jorgenson [14]; producer behaviour such as those by Bernstein [2] and Jorgenson [15]; macro-economic models such as those by Robertson and McDougall [28], and Rose and Selody [30]; tax policy such as those by Coen [8] and Hulten [20]; productivity such as those by Kendrick [23], Solow [32], Brown [4], Nadiri [27] and Jorgenson [16]; and capacity utilization such as those by Hickman [19], and Klein and Preston [24].

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Fixed capital flows and stocks can be estimated either directly or indirectly. The direct method would consist of collecting and compiling the data directly from surveys or accounting records of administrative files. Tax files have been used extensively by the statistical agencies of many countries. The most commonly used method is the indirect perpetual inventory method which derives the gross capital stock for a given year by cumulating past investments up to that year and deducting the cumulated portion of investments that has been retired. Similarly, the net capital stock is derived by deducting the cumulated portion of investments that has been depreciated.

However, the conventional measurement of fixed capital flows and stocks (FCFAS) only measures the quantity side of FCFAS. The gross fixed capital formation deflator derived from capital expenditure price indexes is used as the deflator for the net fixed capital formation, the capital consumption allowance, the gross capital stock and the net capital stock. In other words, the same price index is used as the deflator for all the items of FCFAS. Those can be found in the publications of statistical agencies of most developed countries (for example, Statistics Canada [33] and the Bureau of Labor Statistic of the U.S. [5]). Measuring an economic variable by breaking it into price and quantity components is not new. For example, Christensen and Jorgenson [7][1] broke down all the economic variables in their work of measuring economic performance in the private sector of the U.S. economy. They measured the income as a flow, and the wealth as a stock. They made an assumption thai physical depreciation is geometric so that the rental price of capital has a simple form and capital income can easily be calculated. Also because of the duality characteristics of geometric depreciation, i.e., if physical depreciation is geometric, then economic depreciation is also geometric, and vice versa, the consistency between quantity and price in their measurements can easily be maintained. In this paper, a new theory of measuring FCFAS which measures both the quantity and price sides of FCFAS will be developed. On the quantity side, we will follow the conventional perpetual inventory method to derive the FCFAS as those studied by Goldsmith [11], Tice [36], Young and Musgrave [37], and Biorn et al. [3]. The discards will be derived by cumulating the past gross fixed capital formation (GFCF) time series weighted by a retirement function; the gross fixed capital stock (GFCS) will be derived by applying the survival function, which can be derived from the retirement function, to the same time series; the capital consumption allowance (CCA) will be derived by cumulating the past GFCF time series weighted by a physical depreciation function; and finally, the net fixed capital stock (NFCS) will be derived by applying the survival function derived from the physical depreciation function to the same time series.

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[1] My many thanks go to Prof. Jorgenson who indicated to me an approach similar to the one he used in a joint paper of 1973.

Both the form and rate of a retirement function will be used to derive the formulas for calculating the discards and the GFCS; and similarly, both the form and rate of a physical depreciation function will be used to derive the formulas for calculating the CCA and the NFCS. On the price side, the discard deflator will be calculated as the weighted average of those deflators of the GFCF time series which contribute to the discards, where the weights are calculated as the ratios of the fractions of the GFCF time series contributed to the discards on the quantity side. Similarly, the GFCS deflator will be calculated as the weighted average of those deflators of the GFCF time series which contribute to the GFCS with weights calculated as the ratios of the fractions of the GFCF time series contributed to the GFCS on the quantity side. Furthermore, the CCA deflator will be calculated as the weighted average of the economically depreciated deflators of those physically depreciated GFCF time series which contribute to the CCA on the quantity side. The weights are calculated as the ratios of the fractions of the physically depreciated GFCF time series contributed to the CCA on the quantity side. Finally, the NFCS deflator will be calculated as the weighted average of those economically depreciated deflators of the physically depreciated GFCF time series which contribute to the NFCS on the quantity side. The weights are calculated as the ratios of the fractions of the physically depreciated GFCF time series contributed to the NFCS on the quantity side. All of the formulas for calculating deflators will be derived from both the form and rate of economic and physical depreciation.

In summary, this paper provides a theory which gives a clear picture of how physical and economic depreciation are applied to the quantity and price sides of the perpetual inventory model. It turns out that the conventional model is a special case of the new model presented in this paper. It shows how different deflators are derived instead of one deflator for all fixed capital flows and stocks used by the conventional model. It also shows how the conventional model over estimates the quantities of the fixed capital flows and stocks. In Section 2, we discuss briefly the differences between physical and economic depreciation. In Section 3, we lay out a new perpetual inventory model that illustrates the principal concepts of this paper. Section 4 presents an example that illustrates the new method of estimation proposed in this paper. Some concluding remarks are presented in Section 5.

2. Physical versus economic depreciation

During a production process, two types of depreciation happen to a capital good: physical and economic depreciation. Physical depreciation, also called replacement, occurs so as to maintain the efficient service of the existing facilities. For example, the transmission of a car has to be replaced after running for a certain mileage so that the car can be operated as safely and efficiently as when the car is new.

Replacement also applies to the actual retirement of a capital good at the end of its economic life. The forms of physical depreciation are different for different capital goods. In general, the replacement cost is a fraction of the total cost of the whole capital good when it is new. However, in some case such as the light bulbs, there is no replacement cost, until the whole bulb is burnt out. On the other hand, economic depreciation is usually just called depreciation, and it is a price concept. The price of a used capital good decreases with its age, because the remaining useful economic life of that good is reduced.

The examples shown above are cases of one capital good or one type of homogenous capital goods. At an industry level, the measurement of replacement becomes more complicated as it involves a process of aggregating heterogenous capital goods. Economists usually use the dollar figures in some base year prices to measure the quantity of such aggregates. And in such case, the deflator becomes the unit price corresponding to the base year price dollar unit of measurement of quantity. Physical depreciation is an aggregate of all replacement costs in some base year price of all capital goods at a certain age within an industry. The age distribution of the aggregated replacement costs is the form of physical depreciation of the capital goods pertaining to that industry. On the other hand, economic depreciation is an aggregate loss of values of those capital goods pertaining to the industry associated with their aging. The aggregated price of the capital goods of an industry is usually calculated as the weighted average of all component prices of those capital goods pertaining to the industry. The age distribution of the aggregated prices is the form of economic depreciation of the capital goods pertaining to that industry. In the following, we will refer to the base year price values as the quantity, and the deflator as the price.

As discussed above, physical and economic depreciation are different, but they are also closely related to each other. Coen [8] derived a formula to convert physical depreciation into economic depreciation. On the other hand, a project is underway in Statistics Canada to convert economic depreciation into physical depreciation. Hence, physical and economic depreciation can be estimated from one another. From an empirical point of view, it is easier to measure economic depreciation than to measure physical depreciation. For example, Ackerman [1] studied used cars as depreciating assets, and found that their prices declined with age at a constant exponential rate. Hulten and Wykoff [22] applied the Box–Cox power transformation to a sample of used building prices to estimate the rate and form of economic depreciation, and found that the appropriate pattern was approximately geometric. Koumanakos and Hwang [26] applied Hulten and Wykoff's [22] model to the price-age profiles of certain types of assets derived from a sample of disposals and sales of fixed assets reported in the Capital and Repair Expenditures Survey at Statistics Canada, and derived the form and rate of economic depreciation by type of asset. They also derived aggregate forms and rates of economic depreciation for selected Canadian industries by calculating the weighted average of the forms and rates derived for the types of assets pertaining to the selected industries. Jorgenson [17][2] summarized recent studies of empirical research on depreciation based on an econometric model of asset prices. However, the conventional approach seems inadequate, because it uses a single equation to estimate the parameters of the asset price function, the best fit price function does not necessarily guarantee for the best behavior of the rates of economic depreciation and inflation. An other project is also underway in Statistics Canada, where a new approach is introduced. The simultaneous equations which consist of a price function, an economic depreciation rate function and an inflation rate function, are used to estimate the parameters of the price function. In other words, the parameters of the price function are estimated such that it has a minimum sum of square residual errors under the constraints on both the economic depreciation rate function and the inflation rate function such that they also have a minimum sum of square errors.

In summary, physical depreciation is a measure of the loss of efficiency or capacity of a fixed asset as it ages. Economic depreciation is a measure of the loss of value of a fixed asset with respect to age.[3]

3. A new perpetual inventory model

Using the perpetual inventory model to measure fixed capital flows and stocks (FCFAS) is well known. However, conventional models only measure the quantity side of FCFAS. In the conventional model, the deflator of gross fixed capital formation is used as the deflator for all flows and stocks. In other words, the same price index is used as the deflector for the gross fixed capital formation, net fixed capital formation, discards, capital consumption allowances, end year gross capital stock and end year net capital stock. Since economic depreciation is not included in deriving the price side of FCFAS, it has to be embodied in the measurement of the quantity side of FCFAS. The items of FCFAS have different contents, they represent different quantities, and they should have different prices. In the following, both the quantities and prices of FCFAS will be measured. Physical depreciation will be applied to depreciate only the quantities of FCFAS, and economic depreciation will be applied to depreciate the value of FCFAS. As a theoretical model, the form and rate of both physical and economic depreciation will be considered as given hypotheses in this paper.[4]

[2] Again, my many thanks go to Prof. Jorgenson who provided to me the paper which will appear in Economic Inquiry in the near future.

[3] Age is a principal independent variable of a depreciation function. There could be other independent variables in the depreciation function, for example, the time variable in Hulten and Wykoff's model [22] indicates that the depreciation function is not time invariant.

[4] In this paper, to simplify the model, we make the assumption that the retirement distribution and both physical and economic depreciation distributions and their corresponding survival distributions are

For an industry or component of an industry, here follows a list of input to be considered as given.

I_i, $i = 0, 1, 2, \ldots$	is a time series of gross fixed capital formation in constant dollars,
P_i, $i = 0, 1, 2, \ldots$	is a time series of deflators corresponding to I_i,
a_i, $I = 0, 1, 2, \ldots$	is a time series of used assets price index[5],
f_i, $i = 0, 1, 2, \ldots$	is a retirement distribution,
F_i, $i = 0, 1, 2, \ldots$	is the survival distribution corresponding to f_i,
d_i, $i = 0, 1, 2, \ldots$	is a physical depreciation distribution,
D_i, $i = 0, 1, 2, \ldots$	is the survival distribution corresponding to d_i,
e_i, $i = 0, 1, 2, \ldots$	is an economic depreciation distribution,
E_i, $i = 0, 1, 2, \ldots$	is the survival distribution corresponding to e_i,
L	is an average life,
$n = 2L - 1$	is the maximum life[6],
μ	is a rate of retirement,
δ	is a rate of physical depreciation,
α	is a rate of inflation/deflation of used assets,
r	is a rate of economic depreciation.

On the quantity side

An end-year gross capital stock G_t, can be calculated as follows:

$$G_t = \sum_{i=0}^{\infty} F_i I_{t-i}. \tag{1}$$

In the practical case, we let $F_i = 0$ for $i > n$, then equation (1) becomes

$$G_t = \sum_{i=0}^{n} F_i I_{t-i}. \tag{2}$$

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time invariant. In a more complicated model, those distributions could be a function of time and other variables. Even the average life could be a function of time and other variables.

[5]The time series of used assets price index is not the time series of the deflator of any one of the fixed capital flows and stocks. It may be considered as the time series of the deflator of the used gross fixed capital formation. To measure the deflators of the fixed capital flows and stocks, our model requires the price change of used assets over time and the price index of used assets will fulfill that requirement. The price index of used assets is usually not available. This many be the reason why the deflator of gross fixed capital formation is used as the deflator for all the fixed capital flows and stocks in the conventional model.

[6]This is for the range of summation, since the range starts from 0, it ends at $2L - 1$ so that there are exactly $2L$ investment figures contributed to all flows and stocks. In other words, the contribution of an investment figure at age $2L$ to all fixed capital flows and stocks will be zero. This will become clear in the case of a straight line distribution.

From equation (2)

$$G_t - G_{t-1} = \sum_{i=0}^{n} F_i I_{t-i} - \sum_{i=0}^{n} F_i I_{t-i-1} = F_0 I_t - \sum_{i=0}^{n} (F_i - F_{i+1}) I_{t-i-1}, \tag{3}$$

where $F_{n+1} = 0$ by definition $F_i = 0$ for $i > n$. Now let $F_0 = 1$, and

$$f_i = F_i - F_{i+1}, \tag{4}$$

then equation (3) becomes

$$G_t - G_{t-1} = I_t - \sum_{i=0}^{n} (F_i - F_{i+1}) I_{t-i-1} = I_t - \sum_{i=0}^{n} f_i I_{t-i-1}. \tag{5}$$

The second term at the right hand side of equation (5) can be defined as a discard, DC_t, i.e.,

$$DC_t = \sum_{i=0}^{n} f_i I_{t-i-1}. \tag{6}$$

There is a recurring relation between F_i and f_i which can be described as follows.

$$\begin{aligned} F_0 &= 1, \\ F_1 &= 1 - f_0, \\ &\dots \\ F_i &= 1 - f_0 - f_2 - \dots - f_{i-1}, \\ &\dots \\ F_n &= 1 - f_0 - f_1 - \dots - f_{n-1}. \end{aligned} \tag{7}$$

But $F_n = f_n$, since $F_{n+1} = 0$, therefore

$$\sum_{i=0}^{n} f_i = 1. \tag{8}$$

From equations (5) and (6), we have

$$G_t = G_{t-1} + I_t - DC_t. \tag{9}$$

Now assume that the retirement distribution f_i is geometric, i.e., it has a constant rate μ of retirement, the survival distribution F_i can be characterized by the following equation in the discrete time case,

$$F_i = (1-\mu)^i, \quad i = 0, 1, 2, \dots . \tag{10}$$

Substitute equation (10) into equation (2), we obtain

$$G_t^r = \sum_{i=0}^{n} (1-\mu)^i I_{t-i}, \tag{11}$$

which is the end year gross capital stock estimation formula derived from a constant rate μ geometric retirement distribution.

Now substitute equation (11) into equation (3), we obtain

$$\begin{aligned} G_t^r - G_{t-1}^r &= \sum_{i=0}^{n} (1-\mu)^i I_{t-i} - \sum_{i=0}^{n} (1-\mu)^i I_{t-i-1} \\ &= I_t - \mu \sum_{i=0}^{n} (1-\mu)^i I_{t-i-1} = I_t - \mu G_{t-1}^r, \end{aligned} \tag{12}$$

where

$$DC_t^r = \mu G_{t-1}^r \tag{13}$$

is a simple formula to calculate the discard by the rate μ, and

$$G_t^r = G_{t-1}^r + I_t - \mu G_{t-1}^r = G_{t-1}^r + I_t - DC_t^r, \tag{14}$$

which is similar to equation (9) derived from the form of retirement distribution. Note that the assumption $F_{n+1} = (1-\mu)^{n+1} = 0$ has been applied in deriving equation (12), although $(1-\mu)^{n+1}$ is actually not equal to zero.

By the same token, we can derive the equations corresponding to equations (1) to (14) for the end year net capital stock K_t and the replacement R_t simply by replacing F_i, f_i, μ, G_t and DC_t by D_i, d_i, δ, K_t and R_t, respectively. We will not repeat the whole procedure of deriving the equations instead we will list a few representative. Corresponding to equations (2), (4), (6), (9), (10), (11), (13) and (14) are the following equations,

$$K_t = \sum_{i=0}^{n} D_i I_{t-i}, \tag{15}$$

$$d_i = D_i - D_{i+1}, \tag{16}$$

$$R_t = \sum_{i=0}^{n} d_i I_{t-i-1}, \tag{17}$$

$$K_t = K_{t-1} + I_t - R_t, \tag{18}$$

$$D_i = (1-\delta)^i,\ i = 0, 1, 2, \ldots, \tag{19}$$

$$K_t^r = \sum_{i=0}^{n} (1-\delta)^i I_{t-i}, \tag{20}$$

$$R_t^r = \delta K_{t-1}^r, \tag{21}$$

$$K_t^r = K_{t-1}^r + I_t - \delta K_{t-1}^r = K_{t-1}^r + I_t - R_t^r. \tag{22}$$

When we estimate fixed capital flows and stocks, we can define a cut off point such that the summation of all estimation equations will terminate at that point, and in such case, the average life will have no role to play in the model. In other words, if $F_{m+1} < F_c < F_m$ where F_c is a cut off constraint constant number, then $F_i = 0$ for $i > m$, i.e., the summation of all the estimation equations will terminate at m instead of n. This condition can also be applied to the geometric distribution, i.e., if $(1-\mu)^{m+1} < F_c < (1-\mu)^m$ then $(1-\mu)^i = 0$ for $i > m$.

On the price side

At any particular time, the price of an item of fixed capital flows and stocks can be estimated as the weighted average of the prices of those investments which contribute to the item. Where the weight is calculated as the ratio of the fraction of investment contributed to the item on the quantity side. At the time t, a deflator for the end year gross capital stock PG_t can be estimated by the following equation

$$PG_t = \sum_{i=0}^{\infty} W_i A_{t-i} P_{t-i}, \tag{23}$$

where

$$W_i = \frac{F_i I_{t-i}}{\sum_{i=0}^{\infty} F_i I_{t-i}} = \frac{F_i I_{t-i}}{G_t} \tag{24}$$

is the weight of current or past investment contributed to the end year gross capital stock G_t on the quantity side, and

$$A_{t-i} = \frac{a_t}{a_{t-i}} \tag{25}$$

is the inflation price adjustment factor.

In the practical case, we let $F_i = 0$ for $i > n$, then equation (23) becomes

$$PG_t = \sum_{i=0}^{n} W_i A_{t-i} P_{t-i}. \tag{26}$$

Similarly, a deflator for the discard DC_t can be estimated by the following equation,

$$PDC_t = \sum_{i=0}^{n} w_i A_{t-i-1} P_{t-i-1}, \tag{27}$$

where

$$w_i = \frac{f_i I_{t-i-1}}{\sum_{i=0}^{n} f_i I_{t-i-1}} = \frac{f_i I_{t-i-1}}{DC_t} \tag{28}$$

is the weight of past investment contributed to the discard DC_t on the quantity side, and A_{t-i-1} is the inflation price adjustment factor calculated by replacing i by $i+1$ from equation (25).

Now assume a geometric form of retirement distribution, then equation (26) becomes

$$PG_t^r = \sum_{i=0}^{n} W_i^r A_{t-i}^r P_{t-i}, \tag{29}$$

where

$$W_i^r = \frac{(1-\mu)^i I_{t-i}}{\sum_{i=0}^{n} (1-\mu)^i I_{t-i}} = \frac{(1-\mu)^i I_{t-i}}{G_t^r} \tag{30}$$

is the weight of current or past investment contributed to the end year gross stock G_t^r on the quantity side, and

$$A_{t-i}^r = (1+\alpha)^i \tag{31}$$

is the constant rate inflation price adjustment factor.

Similarly, under the assumption of a geometric form of retirement distribution, equation (27) becomes

$$PDC_t^r = \sum_{i=0}^{n} w_i^r A_{t-i-1} P_{t-i-1}, \tag{32}$$

where

$$w_i^\tau = \frac{\mu(1-\mu)^i I_{t-i-1}}{\sum_{i=0}^{n} \mu(1-\mu)^i I_{t-i-1}} = \frac{\mu(1-\mu)^i I_{t-i-1}}{DC_t^\tau} \tag{33}$$

is the weight of past investment contributed to the discard DC_t^τ on the quantity side, and A_{t-i-1}^τ is the constant rate inflation price adjustment factor calculated by replacing i by $i+1$ from equation (31).

Let t' be the date of the base year, the deflator $PG_{t'}$ calculated by equation (26) for $t = t'$ is not equal to 1 in general and rebasing is required, therefore the final deflator of the end year gross capital stock G_t is obtained by dividing PG_t by $PG_{t'}$. Accordingly, the quantity G_t has to be adjusted by multiplying each G_t by $PG_{t'}$ to cancel out the price rebasing effect. Similarly, the deflator $PDC_{t'}$ calculated by equation (27) for $t = t'$ is not equal to 1 in general, and the final deflator of the discards DC_t is obtained by dividing each PDC_t by $PDC_{t'}$. Accordingly, the quantity DC_t has to be adjusted by multiplying each DC_t by $PDC_{t'}$.

In the special case, when the percentage change in the price index of used assets is in pace with the percentage change in the deflator of the gross fixed capital formation over time, i.e.,

$$A_{t-i} = \frac{a_t}{a_{t-i}} = \frac{P_t}{P_{t-i}}, \tag{34}$$

equations (26) and (27) can be simplified as follows,

$$PG_t = P_t,$$

and

$$PDC_t = P_t,$$

respectively. The above discussion can be summarized in the following proposition.

Proposition 3.1. *Under the assumption given at the beginning of this section, a deflator of the end year gross capital stock G_t and a deflator of the discard DC_t can be calculated by equations (26) and (27), and divided by their base year figures $PG_{t'}$ and $PDC_{t'}$, respectively. They are in general not equal to the deflator of the gross fixed capital formation, but in the particular case, if the percentage change in the price index of the used assets is in pace with the percentage change in the deflator of the gross fixed capital formation over time, then the deflator of the gross fixed capital formation can be used as the deflator for both G_t and DC_t. The quantities G_t and DC_t estimated from the conventional model are in general over estimated by the multiplicative factors $PG_{t'}$ and $PDC_{t'}$, respectively.*

By the same token, we can derive the equations to estimate the deflator for the end year net stock K_t and the replacement R_t, corresponding to equations (26) and (27), as follows. The deflator PK_t for the end year net stock K_t can be estimated by

$$PK_t = \sum_{i=0}^{n} V_i E_i A_{t-i} P_{t-i}, \tag{35}$$

where

$$V_i = \frac{D_i I_{t-i}}{\sum_{i=0}^{n} D_i I_{t-i}} = \frac{D_i I_{t-i}}{K_t} \tag{36}$$

is the weight of current or past investment contributed to the end year net stock K_t on the quantity side, and A_{t-i} is defined by equation (25).

Similarly, the deflator PR_t for the replacement R_t can be estimated by

$$PR_t = \sum_{i=0}^{n} v_i E_{i+1} A_{t-i-1} P_{t-i-1}, \tag{37}$$

where

$$v_i = \frac{d_i I_{t-i-1}}{\sum_{i=0}^{n} d_i I_{t-i-1}} = \frac{d_i I_{t-i-1}}{R_t} \tag{38}$$

is the weight of past investment contributed to the replacement R_t on the quantity side, and A_{t-i-1} is defined by equation (25) by replacing i by $i+1$.

When both physical and economic depreciation have a geometric form of depreciation, substituting

$$D_i = (1-\delta)^i, \qquad d_i = \delta(1-\delta)^i$$

and

$$E_i = (1-r)^i$$

into equations (35) to (38), we obtain

$$PK_t^r = \sum_{i=0}^{n} V_i^r (1-r)^i A_{t-i}^r P_{t-i}, \tag{39}$$

where

$$V_i = \frac{(1-\delta)^i I_{t-i}}{\sum_{i=0}^{n}(l-\delta)^i I_{t-i}} = \frac{(l-\delta)^i I_{t-i}}{K_t^r} \tag{40}$$

is the weight of current or past investment contributed to the end year net stock K_t^r on the quantity side, and A_{t-i}^r is defined by equation (31).

Similarly, the deflator PR_t^r for the replacement R_t^r can be estimated by

$$PR_t^r = \sum_{i=0}^{n} v_i^r (1-r)^{i-1} A_{t-i-1}^r P_{t-i-1}, \tag{41}$$

where

$$v_t^r = \frac{\delta(1-\delta)^i I_{t-i-1}}{\sum_{i=0}^{n}\delta(l-\delta)^i I_{t-i-1}} = \frac{\delta(1-\delta)^i I_{t-i-1}}{R_t^r} \tag{42}$$

is the weight of past investment contributed to the replacement R_t^r on the quantity side, and A_{t-i-1}^r is defined by equation (31) by replacing i by $i+1$.

Let t' be the date of the base year, the deflator $PK_{t'}$ calculated by equation (38) is not equal to 1 in general, and the final deflator of the end year net capital stock K_t can be obtained by dividing each PK_t by $PK_{t'}$. Accordingly, the quantity K_t has to be adjusted by multiplying each K_t by $PK_{t'}$. Similarly, the deflator $PR_{t'}$ calculated by equation (41) is not equal to 1 in general, and the final deflator of the replacement R_t is obtained by dividing each PR_t by $PR_{t'}$. Accordingly, the quantity R_t has to be adjusted by multiplying each R_t by $PR_{t'}$.

In the special case, if equation (34) holds, equations (35) and (37) can be simplified as follows,

$$PK_t = P_t \sum_{i=0}^{n} V_i E_i, \tag{43}$$

and

$$PR_t = P_t \sum_{i=0}^{n} v_i E_{i+1}. \tag{44}$$

At the base year t', equations (43) and (44) become

$$PK_{t'} = P_{t'} \sum_{i=0}^{n} V_i' E_i. \tag{45}$$

and

$$PR_{t'} = P_{t'} \sum_{i=0}^{n} v'_i E_{i+1}. \tag{46}$$

After rebasing, equations (43) and (44) become

$$PK_{t/t'} = (P_t/P_{t'}) \left(\sum_{i=0}^{n} V_i E_i \Big/ \sum_{i=0}^{n} V'_i E_i \right), \tag{47}$$

and

$$PR_{t/t'} = (P_t/P_{t'}) \left(\sum_{i=0}^{n} v_i E_{i+1} \Big/ \sum_{i=0}^{n} v'_i E_{i+1} \right). \tag{48}$$

Since $P_{t'} = 1$, the left hand side of equations (47) and (48) can be reduced to P_t, if the numerator and the denominator of the second bracket are equal. But the weighting factors V_i, V'_i, v_i and v'_i are a function of the gross fixed capital formation, and the equality can occur only if the gross fixed capital formation is constant over time. The above discussion can be summarized in the following proposition.

Proposition 3.2. *Under the assumptions given at the beginning of this section, a deflator of the end year net capital stock K_t and a deflator of the replacement R_t can be calculated by equations* (35) *and* (37), *and divided by their base year figures $PK_{t'}$ and $PR_{t'}$, respectively. They are in general not equal to the deflator of the gross fixed capital formation, but in the special case, if the percentage change in the price index of the used assets is in pace with the percentage change in the deflator of the gross fixed capital formation, and either the gross fixed capital formation is constant over time or there is no economic depreciation, then the deflator of the gross fixed capital formation can be used as the deflator for both K_t and R_t. The quantities K_t and R_t estimated by the conventional model are in general over estimated by the multiplicative factors $PK_{t'}$ and $PR_{t'}$, respectively.*

Some special cases

When there are insufficient investment time series available, i.e., $m < n$, and I_{t-m} and P_{t-m} are the earliest available investment quantity and price, respectively, then equation (2) becomes

$$G_t = \sum_{i=0}^{m} F_i I_{t-i}. \tag{49}$$

Similarly, replace n by m in equations (6), (11), (15), (17), (20), (26), (27), (29), (32), (35), (37), (39), and (41).

When there is an initial stock existing, say K_t' or G_t' with price PK_t' or PG_t', respectively, it should be added to the investment I_t to derive an equivalent investment I_t^e, i.e.,

$$I_t^e = I_t + K_t'. \tag{50}$$

The price should be adjusted too,

$$P_t^e = \frac{P_t I_t + PK_t' K_t'}{I_t + K_t'}. \tag{51}$$

Then proceed as usual to derive all quantities and prices of fixed capital flows and stocks.

4. An example

The input data used in this example are obtained from Investment and Capital Stock Division, Statistics Canada. The gross fixed capital formation time series shown in column 2 of Table 1 are 1970 SIC total manufacturing and nonmanufacturing figures in millions of constant 1981 dollars. The gross fixed capital formation deflator time series shown in column 2 of Table 2 are derived by dividing the 1970 SIC total manufacturing and nonmanufacturing current dollars figures by their corresponding constant dollars figures. Assume that the fixed assets have the following characteristics:

- the average life is $L = 20$,
- the maximum life is $n = 39$,
- the form of retirement follows a truncated normal distribution,
- the form of physical depreciation follows a straight line distribution,
- the form of economic depreciation follows a geometric distribution with a double declining rate of depreciation, i.e., $r = 2/L = 0.1$,
- the percentage change of the price index of the used assets is in pace with the percentage change of the deflator of the gross fixed capital formation,
- the rate of retirement is $\mu = 0.6/L = 0.03$,
- the rate of physical depreciation is $\delta = 1/L = 0.05$,
- the rate of economic depreciation is $r = 2/L = 0.1$,
- the rate of inflation of the used assets is $\alpha = 0.02$,

and there is no initial stock, then the end year gross capital stock and its deflator can be calculated by

$$G_t = \sum_{i=0}^{39} F_i I_{t-i}$$

and

$$PG_t = (1/G_t)\sum_{i=0}^{39} F_i I_{t-i} A_{t-i} P_{t-i} = P_t,$$

respectively, where

$$F_i = 1, \quad i = 0, 1, 2, \ldots, 8$$

$$F_i = \left(1.1133/20^5\right)\int_i^{31} x^2\left(x^2 - 160x + 6400\right)\mathrm{d}x, \quad i = 9, 10, \ldots, 31,$$

$$F_i = 0, \quad i = 32, 33, \ldots, 39.$$

The discard and its deflator can be calculated by

$$DC_t = \sum_{i=0}^{39} f_i I_{t-i-1}$$

and

$$PDC_t = (1/DC_t)\sum_{i=0}^{39} f_i I_{t-i-1} A_{t-i-1} P_{t-i-1} = P_t,$$

respectively, where

$$f_i = 0, \quad i = 0, 1, 2, \ldots, 8$$

$$f_i = \left(1.1133/20^5\right)\int_i^{i+1} x^2\left(x^2 - 160x + 6400\right)\mathrm{d}x, \quad i = 9, 10, \ldots, 31$$

$$f_i = 0, \quad i = 32, 33, \ldots, 39.$$

The end year net capital stock and its deflator can be calculated by

$$K_t = \sum_{i=0}^{39}(1 - i/40) I_{t-i}$$

and

$$PK_t = (1/K_t)\sum_{i=0}^{39}(1 - i/40) I_{t-i}(1 - 0.1)^i A_{t-i} P_{t-i}.$$

Table 1
Fixed capital flows and stocks calculated from the forms of retirement and depreciation

Year	Gross fixed capital formation	Discard	Capital consumption allowance	End-year gross capital stock	End-year net capital stock
		(in millions of constant 1986 dollars)			
1971	42,705.7	16,587.3	6,048.5	583,026.8	253,804.9
1972	43,711.1	17,523.8	6,367.6	607,944.8	265,120.1
1973	47,731.7	18,489.3	6,703.8	635,815.6	277,848.5
1974	50,925.8	19,560.5	7,067.7	665,682.6	291,544.2
1975	54,583.6	20,807.4	7,451.9	697,811.9	306,393.8
1976	54,024.9	22,225.5	7,862.3	727,974.8	320,391.9
1977	54,947.8	23,565.1	8,255.4	757,749.0	334,243.1
1978	55,428.2	24,781.3	8,659.8	786,761.2	347,747.9
1979	61,270.1	25,964.8	9,071.0	820,382.8	363,401.8
1980	67,641.6	27,172.7	9,516.1	859,115.4	381,380.0
1981	75,192.5	28,413.6	9,996.2	904,124.8	402,179.2
1982	67,858.2	29,629.3	10,535.4	940,967.2	418,777.4
1983	62,883.2	30,870.8	11,014.3	970,479.3	432,367.3
1984	63,997.9	32,197.0	11,465.8	1,000,148.5	445,854.0
1985	70,474.2	33,702.6	11,932.0	1,034,775.7	461,711.7
1986	72,500.0	35,186.2	12,433.5	1,069,900.3	477,776.1
1987	77,608.3	36,533.2	12,926.9	1,108,734.9	495,490.4
1988	87,589.2	37,836.4	13,447.3	1,156,050.8	517,107.9
1989	94,801.1	39,425.0	14,043.4	1,208,783.3	541,238.6
1990	93,968.5	41,233.2	14,693.7	1,258,623.5	564,046.1
1991	95,958.0	43,284.6	15,321.9	1,308,535.7	586,866.6
1992	91,706.6	45,209.9	15,942.8	1,352,390.6	606,781.3
1993	98,262.1	46,985.1	16,521.8	1,401,013.7	628,925.4

The capital consumption allowance and its deflator can be calculated by

$$R_t = \sum_{i=0}^{39} (1/40) I_{t-i-1}$$

and

$$PR_t = (1/R_t) \sum_{i=0}^{39} (1/40) I_{t-i-1} (1-0.1)^{i-1} A_{t-i-1} P_{t-i-1},$$

respectively.

The results of above calculations are shown in Table 1 for the quantity, and Table 2 for the price, where the deflators PK_t and PR_t have been divided by 0.467479 and 0.328033, respectively, to force PK_{1986} and PR_{1986} equal to 1, and the quantities K_t and R_t have been multiplied by the same two numbers, respectively. All figures are calculated from 1925 to 1993, but only figures from 1971 to 1993 are shown in the tables.

Table 2
Deflators of fixed capital flows and stocks calculated from the forms of retirement and depreciation

Year	Gross fixed capital formation deflator	Discard deflator	Capital consumption allowance deflator	End-year gross capital stock deflator	End-year net capital stock deflator
1971	0.378300	0.378300	0.402705	0.378300	0.428916
1972	0.397999	0.397999	0.419946	0.397999	0.445743
1973	0.432399	0.432399	0.454572	0.432399	0.477358
1974	0.508000	0.508000	0.533324	0.508000	0.555853
1975	0.569799	0.569799	0.598726	0.569799	0.619695
1976	0.606800	0.606800	0.634953	0.606800	0.657594
1977	0.646600	0.646600	0.672817	0.646600	0.695656
1978	0.701900	0.701900	0.725045	0.701900	0.747931
1979	0.762700	0.762700	0.787067	0.762700	0.802983
1980	0.815700	0.815700	0.845214	0.815700	0.854705
1981	0.883900	0.883900	0.923854	0.883900	0.927790
1982	0.952100	0.952100	0.989088	0.952100	1.005987
1983	0.960800	0.960800	0.984162	0.960800	1.006816
1984	0.982800	0.982800	0.993677	0.982800	1.011414
1985	0.989400	0.989400	0.994571	0.989400	0.999758
1986	1.000000	1.000000	1.000000	1.000000	1.000000
1987	1.009199	1.009199	1.007765	1.009199	1.000607
1988	1.026599	1.026599	1.031580	1.026599	1.013530
1989	1.039999	1.039999	1.054025	1.039999	1.030395
1990	1.057899	1.057899	1.074689	1.057899	1.054730
1991	0.997100	0.997100	1.013226	0.997100	0.995191
1992	0.957400	0.957400	0.966435	0.957400	0.955719
1993	0.928600	0.928600	0.934813	0.928600	0.920571

Similarly, we can use the rates of retirement, physical depreciation and economic depreciation to calculate all fixed capital flows and stocks, and their deflators. The end year gross capital stock and its deflator can be calculated by

$$G_t^r = G_{t-1}^r + I_t - 0.03G_{t-1}^r$$

and

$$PG_t^r = \left(1/G_t^r\right)\sum_{i=0}^{39}(1-0.03)^i I_{t-i}(1+0.02)^i P_{t-i},$$

respectively.

The discard and its deflator can be calculated by

$$DC_t^r = 0.03G_{t-1}^r$$

and

$$PDC_t^r = \left(1/DC_t^r\right)\sum_{i=0}^{39}0.03(1-0.03)^i I_{t-i-1}(1+0.02)^{i+1} P_{t-i-1},$$

Table 3
Fixed capital flows and stocks calculated from the rates of retirement and depreciation

Year	Gross fixed capital formation	Discard	Capital consumption allowance	End-year gross capital stock	End-year net capital stock
		(in millions of constant 1986 dollars)			
1971	42,705.7	11,340.0	8,496.2	419,181.8	210,148.9
1972	43,711.1	11,883.2	8,878.3	438,500.9	219,190.0
1973	47,731.7	12,430.9	9,260.2	460,174.1	229,577.1
1974	50,925.8	13,045.3	9,699.1	483,527.8	240,873.4
1975	54,583.6	13,707.3	10,176.3	508,849.9	253,240.6
1976	54,024.9	14,425.2	10,698.8	533,004.6	264,739.7
1977	54,947.8	15,109.9	11,184.6	557,108.2	276,076.5
1978	55,428.2	15,793.2	11,663.5	580,839.1	287,061.4
1979	61,270.1	16,466.0	12,127.6	608,120.7	300,109.6
1980	67,641.6	17,239.4	12,678.9	639,233.1	315,354.9
1981	75,192.5	18,121.4	13,323.0	674,921.7	333,214.8
1982	67,858.2	19,133.1	14,077.5	704,188.0	346,901.8
1983	62,883.2	19,962.7	14,655.8	728,946.2	357,679.4
1984	63,997.9	20,664.6	15,111.1	753,775.1	368,416.6
1985	70,474.2	21,368.5	15,564.7	782,584.7	381,513.4
1986	72,500.0	22,185.2	16,118.0	812,008.0	394,861.2
1987	77,608.3	23,019.3	16,681.9	844,276.2	409,826.3
1988	87,589.2	23,934.1	17,314.2	882,859.0	428,506.8
1989	94,801.1	25,027.8	18,103.4	925,546.6	449,478.5
1990	93,968.5	26,238.0	18,989.4	966,346.0	469,029.3
1991	95,958.0	27,394.6	19,815.4	1,007,373.2	488,492.3
1992	91,706.6	28,557.6	20,637.6	1,044,067.4	505,080.9
1993	98,262.1	29,597.9	21,338.5	1,084,444.2	523,771.8

respectively.

The end year net capital stock and its deflator can be calculated by

$$K_t^r = K_{t-1}^r + I_t - 0.05K_{t-1}^r$$

and

$$PK_t^r = \left(1/K_t^r\right)\sum_{i=0}^{39}(1-0.05)^i I_{t-i}(1-0.1)^i(1+0.02)^i P_{t-i},$$

respectively.

The capital consumption allowance and its deflator can be calculated by

$$R_t^r = 0.05K_{t-1}^r$$

and

$$PR_t^r = \left(1/R_t^r\right)\sum_{i=0}^{39}0.05(1-0.05)^i I_{t-i-1}(1-0.1)^{i+1}(1+0.02)^{i+1}P_{t-i-1},$$

Table 4
Deflators of fixed capital flows and stocks calculated from the rates of retirement and depreciation

Year	Gross fixed capital formation deflator	Discard deflator	Capital consumption allowance deflator	End-year gross capital stock deflator	End-year net capital stock deflator
1971	0.378300	0.443747	0.403803	0.438695	0.403388
1972	0.397999	0.458737	0.416339	0.454062	0.416647
1973	0.432399	0.475024	0.430026	0.472586	0.436814
1974	0.508000	0.494336	0.450840	0.498067	0.470476
1975	0.569799	0.520936	0.485583	0.528887	0.513073
1976	0.606800	0.553148	0.529547	0.560517	0.551836
1977	0.646600	0.586036	0.569553	0.593593	0.590177
1978	0.701900	0.620754	0.609127	0.629600	0.630517
1979	0.762700	0.658526	0.650764	0.671205	0.681670
1980	0.815700	0.701808	0.703556	0.717214	0.740696
1981	0.883900	0.749621	0.764473	0.769377	0.810781
1982	0.952100	0.804216	0.836809	0.820113	0.865410
1983	0.960800	0.857434	0.893195	0.865602	0.900885
1984	0.982800	0.905299	0.929812	0.910732	0.937474
1985	0.989400	0.952647	0.963449	0.956169	0.968889
1986	1.000000	1.000000	1.000000	1.000000	1.000000
1987	1.009199	1.045384	1.032101	1.042855	1.031345
1988	1.026599	1.089875	1.064448	1.086726	1.070044
1989	1.039999	1.135681	1.104388	1.130196	1.108968
1990	1.057899	1.181074	1.144562	1.171814	1.138736
1991	0.997100	1.224178	1.175279	1.204661	1.149358
1992	0.957400	1.258191	1.186236	1.231244	1.143229
1993	0.928600	1.285947	1.179909	1.254061	1.135602

respectively.

The results of above calculations are shown in Table 3 for the quantity, and Table 4 for the deflator, where the deflators PG_t^{τ}, PDC_t^{τ}, PK_t^{τ} and PR_t^{τ} have been divided by 0.729668, 0.689504, 0.447221 and 0.377881, respectively, to force PG_{1986}^{τ}, PDC_{1986}^{τ}, PK_{1986}^{τ} and PR_{1986}^{τ} equal to 1, and the quantities G_t^{τ}, DC_t^{τ}, K_t^{τ} and R_t^{τ} have been multiplied by the same set of numbers, respectively. All figures are calculated from 1925 to 1993, but only figures from 1971 to 1993 are shown in the tables.

The rates used to estimate the figures of fixed capital flows and stocks shown in Table 3, and their deflators shown in Table 4 are arbitrarily assigned. The differences in figures between those shown in Table 1 and Table 3 for the quantity comparison, and those shown in Table 2 and Table 4 for the price comparison are partially due to such arbitrary assignments. The deflators of capital consumption allowance and end-year net capital stock in columns 4 and 6, respectively, of Table 2, and the deflators of the discards, capital consumption allowance, end-year gross capital stock and end-year net capital stock in columns 3, 4, 5 and 6, respectively, of Table 4 are quite different from the deflators of the gross fixed capital formation shown in column 2 of the same tables, which is conventionally used as the deflator for all fixed capital flows and stocks. The figures of the end year net capital stock and the capital consumption allowance of the conventional approach are over estimated by the multiplicative factors 0.467479 and 0.328033, respectively, and resulted in columns 6 and 4, respectively, of Table 1. The figures of the end year gross capital stock, discards, end year net capital stock and capital consumption allowance of the conventional approach are over estimated by the multiplicative factors 0.729668, 0.689504, 0.447221 and 0.377881, respectively, and resulted in columns 5, 3, 6 and 4, respectively, of Table 3.

The end-year net capital stock in current dollars is commonly used as an indicator for the national wealth, although in this example, it is a national wealth of total manufacturing and nonmanufacturing. The example indicates that the national wealth measured by the end-year net capital stock in current dollars is reduced by more than 50% due to the effect of economic depreciation. It demonstrates the importance of economic depreciation in the measurement of fixed capital flows and stocks.

5. Concluding remarks

In this paper, we have proposed a new theory of measuring fixed capital flows and stocks (FCFAS). We have applied the physical depreciation to measure the quantity side of FCFAS, and the economic depreciation to measure the price side of FCFAS. Both the form and the rate of physical and economic depreciation have been used to derive formulas for measuring the quantity and price of FCFAS. Empirical studies for measuring the economic and physical depreciations are currently underway in Statistics Canada. Actual measurement of FCFAS in Canadian industries will be carried out in the near future.

The model presented in this paper is a simple one. There are many other variables which could affect the measurement of FCFAS, for example, the technical change, economic crisis, environment conditions and etc. Hopefully, those variables which are not explicitly included in the model, might some how be embodied in the economic and physical depreciation variables. Because economic depreciation will be derived from the market behaviour of new and used assets markets, and physical depreciation will be derived from the economic depreciation and the market interest rate.

While the conventional model only estimates the quantity side of FCFAS, the new model presented in this paper measures both the quantities and the prices of FCFAS. Physical depreciation and economic depreciation are clearly distinguished and applied to the measurement of the quantity and price sides of FCFAS. It turns out that the new model also generalizes the conventional one.

The conventional model usually over estimates the quantities of the fixed capital flows and stocks by some multiplicative factors which are sometimes even less than 0.5. Those multiplicative factors might be considered as the adjustment factors transfered from the price side to the quantity side of FCFAS due to economic depreciation. In any case, the conventional model which uses the deflator of the gross fixed capital formation as the deflator for all FCFAS is questionable.

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