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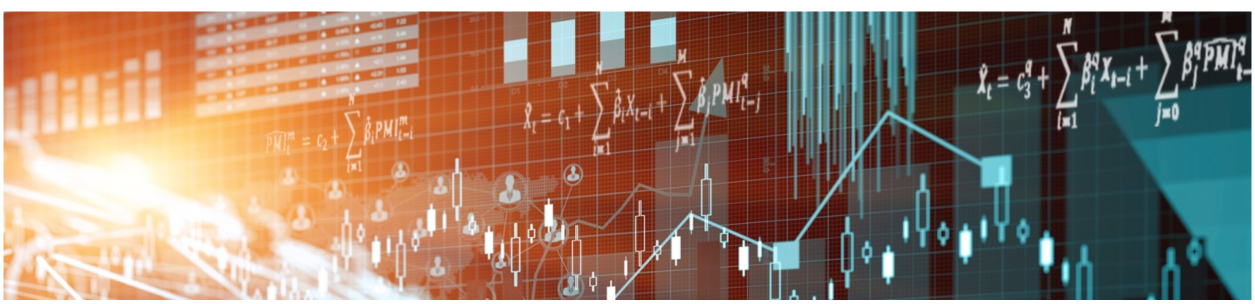

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by

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Abstract

We document that, across households, the money consumption ratio increases with age and decreases with consumption, and that there has been a large increase in the money consumption ratio during the recent era of very low interest rates. We construct an overlapping generations (OLG) model of money holdings for transaction purposes subject to age (older households use more money), cohort (younger generations are exposed to better transaction technology), and time effects (nominal interest rates affect money holdings). We use the model to measure the role of these different mechanisms in shaping money holdings in recent times. We use our measurements to assess the interest rate elasticity of money demand and to revisit the question of what the welfare cost of inflation is (which depends on how the government uses the windfall gains from the inflation tax). We find that cohort effects are quite important, accounting for half of the increase in money holdings with age. This in turn implies that our measure of the interest rate elasticity of money is -0.6, on the high end of those in the literature. The cost of inflation is lower by one-third in the model and, as a result, lower than previously estimated in the literature that does not account for the secular financial innovation.

Bank topics: Inflation: costs and benefits
JEL codes: E21, E41

Résumé

Notre analyse démontre que le ratio monnaie-consommation augmente avec l’âge des ménages et décroit avec la consommation, et que ce ratio a fortement augmenté dans le contexte de très bas taux d’intérêt des dernières années. Nous élaborons un modèle à générations imbriquées de la détention de monnaie pour des motifs de transaction, dans lequel nous introduisons des effets d’âge (l’utilisation de la monnaie augmente avec l’âge des ménages), des effets de cohorte (les jeunes générations font appel à des technologies financières plus évoluées) et des effets temporels (les taux d’intérêt nominaux influent sur la détention de monnaie). À l’aide du modèle, nous quantifions le rôle de ces différents mécanismes dans l’évolution de la détention de monnaie au cours de la période récente. Nous nous servons de ces mesures pour déterminer l’élasticité de la demande de monnaie par rapport aux taux d’intérêt et revenir sur la question du coût de l’inflation en termes de bien-être (qui dépend, en définitive, de ce que l’État fait des gains inattendus de la taxe d’inflation). Il ressort de notre analyse que les effets de cohorte sont plutôt importants, puisqu’ils expliquent plus de la moitié de l’accroissement de la détention de monnaie avec l’âge. Il en découle qu’à -0,6, notre mesure de l’élasticité-intérêt de la monnaie compte parmi les plus élevées recensées dans la littérature. Le coût de l’inflation est, dans le modèle, inférieur du tiers aux estimations antérieurement citées dans les études qui ne prennent pas en compte l’innovation financière de long terme.

Sujets : Inflation : coûts et avantages
Codes JEL : E21, E41

Non-Technical Summary

Holding money is costly as it does not accrue interests, i.e., inflation tax. Thus, this is one type of welfare costs associated with higher inflation that monetary policy should pay attention to. Measuring this cost is important today for two reasons. First, money holdings are at their highest level in the last four decades despite financial innovation that ought to reduce the need for money. Second, there have been discussions to raise the inflation target, which would lead to a higher long-run inflation rate, increasing the welfare cost of inflation on money holding. In addition, data show that money holding patterns across households are quite different, indicating their welfare costs likely differ accordingly among them. For instance, in the cross-section, old and poor households hold 10 times more money per unit of consumption than their young and rich counterparts. Across age groups, money holdings per unit of consumption differ by a factor of 3.

This paper calculates the welfare cost of inflation by focusing on distortions and costs that inflation poses on households who use money for transactions to avoid alternative means of payment and on the losses inflicted on the holders of public debt arising from unexpected inflation. In doing so, we take into account of the role of financial innovation that reduces over time the dependence of households on money to make transactions. We find a sizable aggregate welfare cost of money holdings, but lower than those found in previous studies. The difference mainly comes from the incorporation of financial innovation in our studies. Across households, the old and middle-aged and the poor face higher costs from inflation much more than the young and the rich. Costs are expected to be lower for future generations due to financial innovation.

1 Introduction

Money holdings are at their highest level in the last four decades. Hence, raising the level of long-term inflation, as it has been proposed recently in a variety of contexts,[1] can inflict more pain on money holders now than before. At the same time, financial innovation has altered and continues to alter how households manage their financial portfolios and pay for their consumption. Such financial innovation would likely reduce the need for money, raising a question as to the role of financial innovation in the observed high level of money holding in the economy. Motivated by these recent developments, we revisit the welfare cost of inflation, one of the classical questions in economics. We concentrate on the distortions and costs that inflation poses on households who use money for transactions to avoid alternative means of payment and on the losses inflicted on the holders of public debt arising from unexpected inflation, i.e., the inflation tax. These costs are likely to be unequal across household groups because money holdings are extremely unequal across groups. For instance, in the cross-section, old and poor households hold 10 times more money per unit of consumption than their young and rich counterparts. Across age groups, money holdings per unit of consumption differ by a factor of 3. These differences across household groups should be taken into account to assess the aggregate cost of inflation. In addition, if financial innovation were to play an important role and reduce the dependence of households on money to make transactions over time, the current estimates of the aggregate welfare cost of inflation in the literature without financial innovation would be overestimated. In this paper we take all these issues into account when providing a measure of the cost of inflation.

There is no standard model that incorporates financial innovation and the dimensions of household heterogeneities that we study. Thus, we start by building a model where households are heterogeneous in income, age, and cohort of birth that is capable of incorporating all these mechanisms. By confronting money holdings across time in the model and in the data in the presence of varying nominal interest rates we provide a decomposition of the higher money holdings of older people into those due to age (what we can think of as an intrinsic feature of the aging process) and those due to the cohort to which people belong (which we can associate with financial innovations, or to the adaptation of households to newly available money saving transacting technologies).

[1] Blanchard, Dell'Ariccia, and Mauro (2010) support raising the inflation target from around 2%, a commonly adopted target in most advanced countries. This issue was also debated at the 2015 Jackson Hole Economic Policy Symposium by Aruoba and Schorfheide (2016). In addition, the Bank of Canada officially reviewed a question of whether or not to raise the inflation target from 2% to 3% under its 2016 Renewal of the Inflation-Control Target. See Bank of Canada (2016).

The model builds on the work of Erosa and Ventura (2002) who analyzed the money demand of households that were heterogeneous in income and wealth. Our model accommodates differences in money holdings per unit of consumption by consumption levels of agents but also households that differ in the date of birth (and hence their age at any point in time). It is an overlapping generations model with age and cohort effects, both mechanisms contributing to generate larger money holdings among the elderly. The model is then asked to replicate aggregate money holdings over time. The large oscillations in nominal interest rates of the years help identify age versus cohorts effects by ensuring that the model replicates jointly the cross-sectional holdings with the variation of holdings over time. The model accommodates details of the windfall for the government of higher inflation (reductions in the real value of its liabilities and increases in seigniorage) and permits us to measure the costs of inflation under alternative scenarios for the allocation of these windfall gains.

Specifically, the paper makes four contributions.

1. Using a cross-sectional Canadian household survey data set and Canadian macroeconomic time-series data, we document the following three facts on the ratio of money holdings to consumption (which we label succinctly as money demand):

    - Across households, *money demand increases with age*: The money demand of households aged between 76 and 85 is on average more than 3 times higher than that of households aged 35 or younger.
    - *Money demand decreases with household's consumption level*: Households in the upper quintile of consumption have a money to consumption ratio that is more than 3 times lower than that of the lowest quintile.
    - *Aggregate money demand increased by 34% between 2000 and 2010.*

2. We develop an overlapping generations model where households choose to purchase consumption with money and credit. Cash is costly because its nominal rate of return is zero, while credit is subject to transaction costs that vary with both the age and the cohort of the household. Non-stationary versions of this model with varying interest rates, inflation and government policy generate *time effects* on the allocation of assets by households. Replicating the properties of money holdings in the data both over time and across ages and consumption groups provides the crucial identification of the relative roles of age and cohort effects in shaping how different household groups choose their money holdings. The

important insight is that the cross-sectional data allows us to identify the age-effect and cohort-effect parameters jointly while the changes in macroeconomic conditions over time help identify the cohort-effect parameter.

We find that the age effect accounts for 53% of the observed difference in money demand across age groups, whereas the cohort effect accounts for the other 47%. This decomposition has important implications for the welfare cost of inflation.

3. We provide estimates for the interest rate elasticity of money demand by individual household type. Aggregating over types and time yields an estimate of the economy-wide elasticity of -0.62, implying that a 1% increase in the nominal interest rate decreases the transaction demand for money (i.e., money-consumption ratio) by 0.62%. This estimate is just at the high end of the range found in the literature, between -0.25 and -0.6, which uses very different methods and data to derive the estimates. This implied elasticity is high enough to allow for a substantial role of financial innovation while accounting for the increased money holdings of the last few decades induced by the low nominal interest rates. Hence the increased money holdings coexisting with substantial financial innovation is not puzzling (see Teles and Zhou (2005)). Our findings about group-specific elasticities are that they are mostly increasing with age and social or income class.

4. Finally, we obtain measures of the welfare cost of inflation. We proceed by introducing an unexpected permanent change in inflation in 2010 with associated fiscal policies that accommodate the additional revenue generated by the higher inflation. Common among all fiscal policies is the existence of very large differences in the welfare costs of inflation among age, income and cohort groups. The old and middle-aged and the poor suffer much more than the young and the rich, about 3 times more.

   The aggregate costs of inflation depend crucially on the government's use of the windfall. An increase in inflation from 2% to 5% can cost as much as 25% of one year of consumption if the government squanders the resources or as low as 4% if the government returns to the households their gains. If we were to ignore the role of financial innovation we would obtain measures of the inflation cost that can be 47% higher in aggregate. In other words, there would be an over-estimation of the cost by about one-third for studies without considering financial innovation. When compared to the previous studies in the literature without financial innovation, our measure is 14% lower than that of Lucas (2000) and 32% lower than that of Erosa and Ventura (2002).

Accurately capturing these welfare costs of inflation in the presence of money holdings has important implications for the debate in the aftermath of the financial crisis of whether monetary policy should adopt a higher inflation target. On one side, Blanchard, Dell'Ariccia, and Mauro (2010) support the view that the inflation target should be higher to reduce the likelihood of reaching the zero lower bound for nominal interest rates that limits the ability of central banks to respond to adverse shocks. On the opposite side of the debate are Coibion, Gorodnichenko, and Wieland (2012) and Aruoba and Schorfheide (2016). They use a standard New Keynesian DSGE models and ask whether central banks should raise the inflation targets when the cost associated with the zero lower bound are present. Coibion, Gorodnichenko, and Wieland (2012) find that the optimal inflation rate is less than 2% based on the utility-based welfare analysis, providing little evidence for higher inflation. Aruoba and Schorfheide (2016) find the expected real effects, such as in GDP, of the higher inflation target to be essentially zero. However, these papers do not account for money holdings, which have almost tripled in the last three decades. We show that accounting for this observed elevated level of money holdings leads to very large welfare costs of higher inflation targets.

There is a long list of previous studies measuring the welfare cost of inflation with respect to money holdings. Lucas (2000) finds that when the annual inflation rate is reduced from 10% to 0%, the gain is slightly less than 1% of real income. However, Lucas points out that "...Using aggregate evidence only, it may not be possible to estimate reliably the gains from reducing inflation further, ...," recognizing the important role of distributional considerations to obtain a proper assessment. Mulligan and Sala-i-Martin (2000) document that almost 60% of U.S. households hold cash and chequing accounts, and do not hold interest-bearing financial assets, which they interpret as the existence of a fixed cost for financial transactions. Ignoring these fixed (adoption) costs will underestimate the welfare cost of inflation, pointing also to the role of the cross-sectional differences in the inflation costs. Attanasio, Guiso, and Jappelli (2002) use a unique Italian household-level data set with much richer information on cash holding, cash transactions, and usage of ATM cards. They estimate the demand for cash and for interest-bearing assets, and find that the welfare cost of inflation varies considerably within the population but is small (0.1% of consumption or less) and that the interest rate elasticity is between -0.3 (for non-ATM users) and -0.6 (for ATM users). Alvarez and Lippi (2009) make another important contribution to the analysis of money demand using micro data. They extend the cash-inventory management models of Baumol (1952) and Tobin (1956) to incorporate precautionary cash holdings due to uncertainty with respect to a chance to withdraw cash without cost. Using the same Italian household-level data as Attanasio, Guiso, and Jappelli (2002) do, they estimate structural parameters of the model. They use the estimated model to analyze quantitatively the elasticity of money demand as well as the welfare cost of inflation. They find that the interest rate elasticity is about -0.5 and that the welfare cost of inflation is about half of the cost implied by a pure Baumol-Tobin's model economy.

Our estimate of the welfare cost of inflation is generally larger than those of Attanasio, Guiso, and Jappelli (2002) and Alvarez and Lippi (2009). This is mostly due to the definition of money. These papers use a narrower definition of money (e.g., currency) while as in Lucas we are using a relatively broader definition of money (e.g., M1). Since welfare costs should increase with the stock of money, broader definitions lead to larger welfare results.

Erosa and Ventura (2002) have made an important contribution to accommodate differences in cross-sectional money holdings when assessing the welfare cost of inflation. They extend the Aiyagari model to include the cash-in-advance constraint and study the welfare distribution of changing inflation rates. They find that the distributional effects of inflation are large where low-income households are disproportionately hurt by inflation. However, their study ignores the cohort and age effects as well as the inflation-induced windfall gains/losses associated with nominal bond holdings of households and the government.[2]

How does our welfare cost of inflation compare to Lucas (2000) and Erosa and Ventura (2002) since they are also using a broader definition of money? Our welfare cost of inflation is lower than theirs, primarily due to the cohort effects. Cohort effects reduce the demand for money of future generations, and taking them into account dampens the aggregate welfare cost. When comparing our cost to those of Lucas (2000) and Erosa and Ventura (2002) under the model assumptions in our paper that are the closest to their studies, we find that our aggregate welfare cost estimate is 14% and 32% lower, respectively.

With respect to age, Heer, Maussner, and McNelis (2011) document that the money-age profile is hump-shaped in the U.S., and that money is only weakly correlated with income and wealth, but they do not explore its relation to the transaction demand for money. Klee (2008) uses grocery store transaction data and estimates the probability of using cash. She finds that the eldest group in the data, aged 65-74, has the highest probability of using cash and the lowest

[2] Chiu and Molico (2010) calibrate a search-theoretical model of demand for money, and find that the welfare cost of inflation is 40% smaller than that in complete market representative-agent models, such as Lucas (2000).

probability of using credit among all the age groups. Also related to our study is Ragot (2010), who finds that the distribution of money across households is more similar to the distribution of financial assets than to that of consumption. He concludes that transaction frictions in the financial market account for 85% of total money demand.

The rest of the paper is organized as follows. In Section 2, we document how money holding varies with age and consumption and the time series of aggregate money holdings using Canadian data. A life-cycle model of demand for money is developed in Section 3, and Section 4 maps it to the data. Section 5 discusses the implications of our model for nominal interest rate elasticities. In Section 6, we analyze the relative importance of the age and cohort effects in accounting for the cross-sectional age profile of money holdings. Section 7 evaluates welfare from changing inflation. Section 8 concludes.

2 Money Demand of Canadian Households

We use the ratio of money holdings to consumption as the centerpiece of our analysis. A better data set would be a panel of households with money holdings and consumption, which would allow us to identify age and cohort effects with fewer restrictive assumptions. No such data are readily available. However, there is a cross-sectional household survey in Canada with information both on money holdings and on household consumption. The Canadian Financial Monitor (CFM) by Ipsos Reid is an annually repeated cross-sectional household survey data set, containing information on household income, expenditure and balance sheets that is available with consistent methodology for the period after 2009Q3.[3] In order to observe money-consumption ratios across households, we group households by the age of the household head into six categories: less than or equal to 35 years old, 36-45 years old, 46-55 years old, 56-65 years old, 66-75 years old, and 76-85 years old. Given an age group, we further make five sub-groups based on their consumption quintile. Money-consumption ratios are calculated by taking the ratio of the average money holdings and the average consumption of a given group. Given our objective to analyze the welfare cost of inflation, we define money to include cash and low-interest bank account balances, i.e., chequing, chequing/savings and business accounts. This is a measure of the liquid

[3]Although the CFM contains information on bank account balances starting in 1999, it lacks information on cash, in addition to the fact that variables definitions of bank accounts differ between earlier and later years of the survey. Hence, it is difficult to have a reliable time series of the aggregate money-consumption ratios from the CFM. As a result, we only use data from the 2009-2012 period and we rely on macroeconomic data sources for the dynamics of aggregate money and consumption over time. We do not use the latest data from 2013, as sample weights can occasionally be revised and updated for more recent data. See Appendix A for a more detailed description of the CFM and a detailed table of the money holdings.

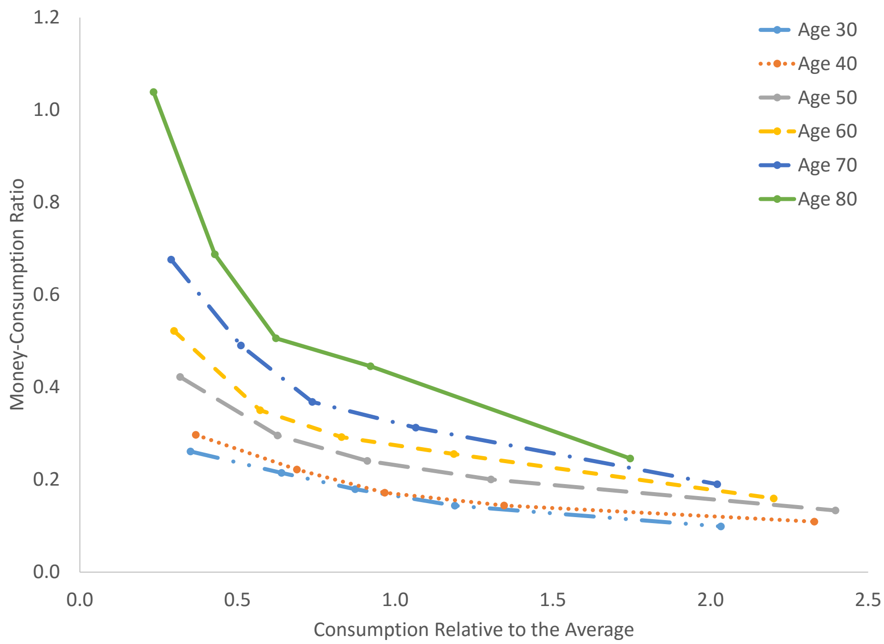


Figure 1: Money-Consumption Ratio by Age (Data source: CFM 2009Q3-2012Q2)

assets that are used for transactions and whose real values are sensitive to inflation, as their nominal rates of return are typically lower than the nominal interest rates and are not adjusted for inflation. When inflation is high, the cost of holding these assets will also be high due to the inflation tax. This definition of money is close to the definition of aggregate money held by households, which includes currency circulated outside banks and personal chequable deposits at banks. Furthermore, we define consumption in the survey data as the household's sum of gross (annualized) monthly spending on non-durable goods, services and durable goods but we exclude expenses on housing services. We complement these data with aggregate data on money and consumption from Statistics Canada; consequently, we apply a similar definition to construct aggregate consumption and use non-housing consumption by excluding actual and imputed rental fees for housing services from final consumption expenditures.

Figure 1 displays the cross-sectional relation for the various age groups of consumption and money-consumption ratios. The two main facts cited above can be readily seen: first, money holdings per dollar of consumption (i.e., money-consumption ratio) increases with age conditional on consumption and, second, the money-consumption ratio decreases with consumption. The second fact has been documented in previous studies (see Erosa and Ventura (2002) for a summary

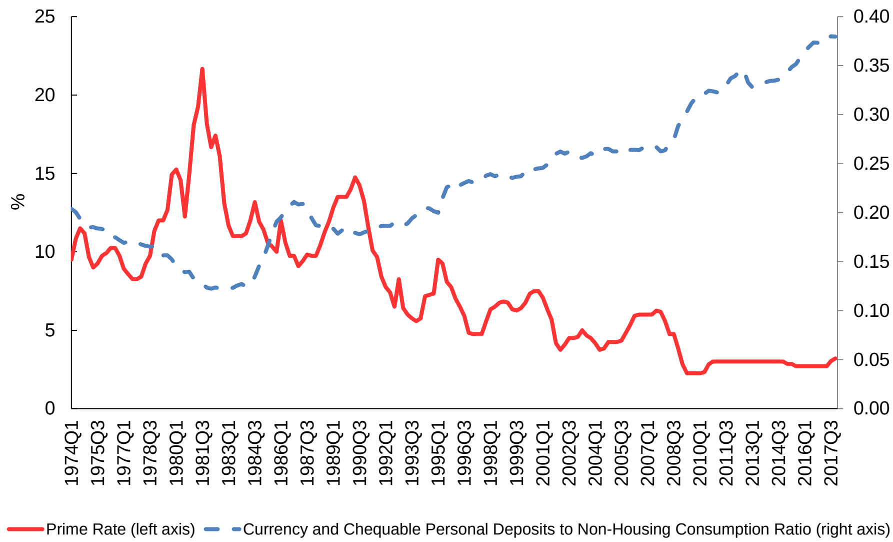


Figure 2: Aggregate Money-Consumption Ratio and Prime Rate

of previously documented stylized facts). It suggests that as household consumption increases, the fraction of consumption purchased with money becomes smaller, implying that non-cash payment methods become more important. What has not been extensively studied is the age aspect of money demand, i.e., the first fact.[4]

Why do older households hold a higher money to consumption ratio than younger households? Is it just because they are older (age effect),[5] or is it because they were born earlier in a world with fewer financial instruments and this shaped their ability to save in non-money financial instruments (cohort effect)? The answer matters. If it is an age effect we would predict that the current young will increase money holdings as they age and no persistent change in aggregate money holdings will occur over time, while if it is a cohort effect, we would not predict such an increase with age; instead we would expect a persistent decrease in aggregate money holdings over time as long as the cohort effects are active (which we assume is the case at least for all the living generations).

Tracking aggregate money holdings over time can shed some light on this question: if it is mostly an age effect, money holdings would not have come down over time, while if it is mostly a cohort effect, then money holdings would have shrunk. The notions of aggregate money and consumption

[4]There are studies that show positive correlations between age and money holding. One example is the study by Daniels and Murphy (1994). They use the U.S. survey data on currency and transaction account usage in the 1980s and find that currency inventory increases with age. Our focus is different from these studies in that the measure of transaction demand for money is the money-consumption ratio.

[5]It could be that old households face more difficulty processing complex financial information associated with the use of the credit technology. In the cash-inventory management literature, there is some empirical evidence that shows that older households pay fewer visits to bank branches than younger households. See Mulligan and Sala-i-Martin (2000).

from the macroeconomic data have to be consistent with those from the CFM. Our definition of aggregate money consists of “Currency outside banks” and “Personal deposits chequable.” These are the sub-categories of the narrowest definition (i.e., called M1+) of aggregate money supply measures available in Canada. Consumption is the annual non-housing consumption.[6]

Figure 2 displays the ratio of money to consumption over the period of 1974Q1 to 2017Q3 as well as the prime rate,[7] which displays the third fact that this paper highlights. That is, the money-consumption ratio is going up over time while the nominal interest rate, i.e., the opportunity cost of holding money, has been going down, indicating that, even in the presence of cohort effects or financial innovation, the interest rate elasticity of money demand has played a central role in shaping the aggregate quantity of money. Our objective of separating age and cohort effects in the determination of the higher money holdings of older households is compounded by the substitution elasticity. Fortunately, to disentangle these effects, we can use a structural model capable of making predictions simultaneously about the age distribution of money holdings and its evolution over time when nominal interest rates as well as inflation and fiscal variables are changing over time. In the next section, we describe such a model.

3 The Model

We pose a model with equal-sized overlapping generations of agents that use either money or credit to purchase goods. Within each cohort there are types (income groups or social classes) that determine both the endowments and the age profile of the desired timing of consumption. Households can purchase consumption using some combination of a cash-in-advance constraint and a credit-transaction technology that depends on the age and cohort of the household. We now turn to the details.

3.1 Model description and equilibrium

Each household, of which there is a continuum, is indexed by its age $i \in \{0, \cdots, I\}$ and by its type $j \in \{1, \cdots, J\}$. Types are permanent, and we think of them as equal-sized social classes. Households are also indexed by their cohort h, or period of birth. Households supply labour exogenously and have a fixed labour endowment. There is no uncertainty. In each period,

[6]See http://www.statcan.gc.ca for this information. Specifically for money, "Currency outside banks" corresponds to the series v37173 and "Personal deposits chequable" to v41552802. Annual non-housing consumption is total consumption minus housing consumption: V62700456-V62700469-V62700470. All series are deflated to the 2010 value using the core Consumer Price Index. For more information on measures of money supply in Canada, see http://www.bankofcanada.ca/wp-content/uploads/2010/11/canada_money_supply.pdf.

[7]The prime rate is the interest rate charged to the most creditworthy borrowers by chartered banks in Canada.

households use money and credit to purchase a consumption good.

The fraction of consumption purchased with money is subject to a cash-in-advance constraint. Purchasing by credit involves transaction costs. This generates a trade off between using money and using credit for purchases because holding money precludes gaining interest, which is particularly taxing in periods of high inflation or high interest rates. The credit transaction technology is a function of the fraction of consumption purchased with credit, s, and is given by[8]

$$\xi_{hi}(s) = \int_0^s \gamma_i \cdot \eta^h \cdot \left(\frac{x}{1-x}\right)^{\theta_i} dx, \tag{1}$$

where $\gamma_i > 0$, $\theta_i \geq 1$ and $\eta > 0$.[9] This function is convex and strictly increasing in s for all $s \in [0, 1)$. It is also independent of the level of consumption. Thus, the credit technology exhibits increasing returns to scale: the credit transaction cost per unit of consumption decreases with consumption given s. This assumption helps generate the second fact: the money-consumption ratio decreases with consumption. Both γ_i and θ_i vary with age to replicate both the different levels and the different slopes displayed in Figure 1 and are responsible for the age effects. Cohort effects are captured by η in Equation (1). A value of $\eta < 1$ would imply ceteris paribus that credit becomes less costly over time (as h increases) and that hence the demand for money declines. Proposition 1 makes this statement formally. This assumption implies that cohort effects take a form of secular changes over time, e.g., a secular financial innovation. The crucial difference between age and cohort effects is that the former predicts that the currently young households will use more money and less credit as they age, while the latter will have no direct impact on the money-credit choice within the life cycle of a given household. Similarly, cohort effects impact the money-credit choice between cohorts but there is no direct impact from age effects in this margin. Hence, the important property of the transaction cost in Equation (1) is the independence of age and cohort parameters. This assumption is crucial in the identification of age and cohort effects discussed in the next section.

Given this cost function, each household makes money-credit payment decisions, consumption-savings decisions, and decisions on money and non-money asset portfolios in savings. Let c_{hij} be consumption, a_{hij} real assets, and m_{hij} real money holdings of agents of cohort h, age i and class j. The sum of cohort and age indices, $h+i$, defines the time index t such that $t = h+i+constant$.[10]

[8] This specification is an extension of that used in Dotsey and Ireland (1996) and Erosa and Ventura (2002) with age-specific parameters and a parameter capturing cohort effects.

[9] These assumptions guarantee that money holdings increase with consumption, i.e., fact (ii) discussed in Section 2. See a discussion below with Equation (8) below.

[10] This linear dependency of these indices is the source of difficulty in identifying the three effects from age, cohort

The cohort h and class j household solves

$$\max_{\substack{\{c_{hij}, s_{hij},\\ a_{h,i+1,j}, m_{h,i+1,j}\}}} \sum_{i=0}^{I} \beta_{ij} \frac{c_{hij}^{1-\sigma} - 1}{1-\sigma} \qquad \text{s.t.} \tag{2}$$

$$c_{hij}(1 - s_{hij}) \leq m_{hij}, \tag{3}$$

$$\begin{aligned} c_{hij} + q_t \xi(s_{hij}) + a_{h,i+1,j} + (1 + \pi_{t+1}) m_{h,i+1,j} & \\ \leq [1 + r_t(1 - \tau_t)] a_{hij} + m_{hij} + (1 - \tau_t) w_t z_{ij} & \qquad \forall\, i < I, \end{aligned} \tag{4}$$

$$c_{hIj} + q_t \xi(s_{hIj}) \leq [1 + r_t(1 - \tau_t)] a_{hIj} + m_{hIj} + (1 - \tau_t) w_t z_{Ij}, \tag{5}$$

$$m_{h,0,j} = \underline{m}, \quad m_{hij} \geq 0 \quad \text{and} \quad t = h + i + constant, \tag{6}$$

where q_t is the price per unit of credit-transaction service, π_{t+1} the inflation rate from time t to $t+1$, w_t the wage rate, r_t the interest rate, τ_t the tax rate on non-money asset and wage income, all at time t.[11] The labour endowment, z_{ij}, is assumed to be independent of cohorts or time. To have an interior solution for money holdings even for the youngest agents, we assume that newborns are endowed with a small amount of initial money holdings, $\underline{m}$ (less than .3% of their average consumption).

Condition (3) is the cash-in-advance constraint. Given its current money holdings, a household chooses total consumption, which, given no uncertainty and hence that the cash-in-advance constraint holds with equality, also implies a certain amount of credit. Conditions (4) and (5) are the budget constraints for households aged $i < I$ and $i = I$, respectively. We pose age and class specific discount factors or utility weights to capture the consumption age profile of each class of households without attempting to understand the origins of such consumption patterns.

Regarding real assets, a_{hij}, we assume that they are a composite of government bonds b_{hij}, and other non-money assets a^n_{hij}, so that $a_{hij} = b_{hij} + a^n_{hij}$. We further assume that their markets are frictionless and that inflation is exogenous and deterministic. These assumptions imply that both assets have the same real rate of return. Hence, the composition of nominally denominated assets in a_{hij} is indeterminate. Households' holdings of government bonds will matter in the counterfactual analysis of inflation shock for the welfare calculations. As will be discussed later, we will use data to pin down the fraction of government bonds in a_{hij}.

In addition to households, the model features a government that spends, has assets and debts,

and time.

[11] Following Erosa and Ventura (2002), we assume $q_t = w_t$, implying that credit-transaction costs are specified in terms of time costs.

supplies money, and collects tax and seigniorage revenues. The government faces the following budget constraint every period:

$$G_t = -A^G_{t+1} + (1+r_t)A^G_t + B_{t+1} - (1+r_t)B_t + (1+\pi_{t+1})M_{t+1} - M_t + \tau_t(r_t K_t + w_t Z), \quad (7)$$

where G_t is government spending at time t, A^G_t is government assets, B_t government debt, M_t aggregate money supply, K_t aggregate productive capital, Z aggregate labour endowments, r_t the real interest rate, π_t the inflation rate, w_t the wage rate, and τ_t the income tax rate. Government debt B_t is held by domestic households and by the foreign sector.[12] All variables are real.

The term $(1+\pi_{t+1})M_{t+1} - M_t$ in the government budget constraint Equation (7) represents seigniorage. This specification is motivated by our assumptions that all money is held by households and that the central bank perfectly controls inflation.

We assume that there is no growth, an irrelevant assumption because we pose the demand for money relative to consumption units. To close the model we make the small open economy with a Cobb-Douglas production technology, $F(K_t, Z) = K^\alpha_t Z^{1-\alpha}$, that uses capital and labour with a capital depreciation rate of δ. The real interest rate, r_t, is exogenously determined in the global capital market and is time varying. The labour input is assumed to be non-tradable and only domestically supplied. In equilibrium, these assumptions imply time-varying wage rates that are consistent with the rest of the economy. The quantity of money and the tax rate are also determined in equilibrium. Specifically, the government is supplying whatever quantity of money households demand, and the level of taxation is determined by the government budget constraint. The nominal interest rate is implied by the Fisher equation. The exogenous macroeconomic variables in this economy are G_t, A^G_t, B_t, π_t and r_t. Then, the following formal definition describes the equilibrium.

Definition 1. *An equilibrium is defined by a set of variables, $\{\tau_t, K_t, M_t, Z, c_{hij}, s_{hij}, a_{h,i,j+1}, m_{h,i,j+1}, w_t\}$ for all h, i, j, t, such that*

- *Each cohort of households, h, optimally choose c_{hij}, s_{hij}, $a_{h,i,j+1}$ and $m_{h,i,j+1}$ by solving the household problem defined by (2);*
- *The government balances its budget every period (Equation (7));*
- *K_t is implied by $r_t = \alpha\left(\frac{Z}{K_t}\right)^{1-\alpha} - \delta$;*

[12] The exact composition of the debt matters because of the dilution of the real value of nominal assets in the inflation counterfactuals.

- *Wage rates are determined by* $w_t = (1-\alpha)\left(\frac{K_t}{Z}\right)^{\alpha}$*; and*
- *Aggregate consistency is satisfied by* $M_t = \sum_{ij}^{IJ} m_{hij}$, $Z = \sum_{ij}^{IJ} z_{ij}$ *and* $B_t = \sum_{ij}^{IJ} b_{hij} + B_t^F$*, where the cohort index* h *is consistent with* $t = h + i + constant$ *and* i *capturing all active cohorts in* t*,* b_{hij} *is government bonds held by household type* h, i, j*, and* B_t^F *is the amount of government bonds held by foreigners.*

3.2 Characterization

The household's optimization problem, (2), yields an interior solution defined by a set of first-order conditions (see Appendix B for details). The consumption Euler equation is

$$\frac{u'(c_{hij})}{1+(1-s_{hij})\widetilde{R}_t} = \frac{\beta_{i+1,j}}{\beta_{ij}}[1+r_{t+1}(1-\tau_{a,t+1})]\frac{u'(c_{h,i+1,j})}{1+(1-s_{h,i+1,j})\widetilde{R}_{t+1}},$$

where $\widetilde{R}_t \equiv (1+\pi_t)[1+r_t(1-\tau_t)]-1$ denotes the after-tax nominal interest rate. The left-hand side of this equation describes the marginal gain of increasing one unit of consumption today. While a fraction s_{hij} of consumption is purchased by credit and costs 1 unit of the numeraire, a fraction $1-s_{hij}$ is purchased by money that costs more because it incurs a loss in nominal interest income by carrying money forward from $t-1$ to t, $1+(1-s_{hij})\widetilde{R}_t$. Similarly, the right-hand side is discounted of marginal utility taking into account the cost of carrying money from t to $t+1$. Consumption dynamics in the model balance these elements.

Another important margin specific to the present model is that of the money-consumption ratio, which satisfies

$$\frac{m_{hij}}{c_{hij}} = 1 - s_{hij} = \frac{1}{1+\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i}}. \tag{8}$$

The following proposition summarizes the behaviour of money demand with respect to γ_i, θ_i, η, h and c_{hij}.

Proposition 1. *Assume* $\gamma_i > 0$, $\theta_i \geq 1$, $\eta \in (0,1)$, $c_{hij} > 0$ *for all* h, i *and* j*. All else equal, the money-consumption ratio increases with* γ_i *(i.e., higher credit-transaction cost). It decreases (increases) with* θ_i *when money is used less (more) than credit for consumption. It also increases with* η *(i.e., lower financial innovation). Furthermore, the money-consumption ratio decreases with* h *(i.e., more recent cohort) and with* c_{hij} *(i.e., richer). That is,* *(i)* $\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial\gamma_i > 0$,

(ii) $\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial\theta_i < 0$ *if* $\frac{m_{hij}}{c_{hij}} \in (0, \frac{1}{2})$ *and* $\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial\theta_i \geq 0$ *if* $\frac{m_{hij}}{c_{hij}} \geq \frac{1}{2}$,

(iii) $\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial\eta > 0$,

(iv) $\frac{m_{h+1,i,j}}{c_{h+1,i,j}} - \frac{m_{hij}}{c_{hij}} < 0$, *and*

(v) $\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial c_{hij} < 0$, *respectively.*

See Appendix C for the proof.

Statement *(i)* states that money demand increases with age if γ_i also increases with age, while *(ii)* states that how age impacts money holdings depends on the importance of money and credit in purchasing consumption. If money is less important, money holdings go up with θ_i. In addition, *(iii)* implies that money holdings are lower with higher financial innovation (e.g., lower η) or with higher degrees of cohort effects. Furthermore, *(iv)* says that money demand goes down over time, i.e., as new cohorts come into the economy. Hence, the model qualitatively allows both the age and the cohort effects to account for the increase in money holdings with age, i.e., the first fact discussed in Section 2. We discuss the important issue of how we identify and quantify these parameters in the next section. Note also that because of *(iv)*, cohort effects will push down aggregate money demand over time as younger cohorts replace their older peers if $\eta \in (0, 1)$. The model calibration results in the next section verify this. Statements *(iii)* and *(iv)* become relevant when we conduct the hypothetical welfare analysis by taking out the cohort effects that we identify. Finally, *(v)* implies the positive wealth effect on the use of credit rather than money, i.e., the second fact discussed in Section 2.

4 Mapping the Model to Data

The inherent difficulty in identifying age, cohort and time effects can be seen in the linear dependency of their indices, i.e., $t = h + i + \textit{constant}$ from Condition (6).[13] We can disentangle these three effects using the non-linear model and data, including the cross-sectional variation of

[13]For example, a simple linear regression model of money demand on i, h and t is not identifiable due to this dependency. The beginning of the social science literature on the age-period-cohort issue dates back to the 1970s. See Fienberg and Mason (1979). More recently, Shulhofer-Wohl (2013) deals with the identification of age, time and cohort effects in life-cycle models although he does not analyze money holdings. He adopts a semi-structural approach that models only age effects but not time and cohort effects and then proposes an estimation method of how to remove the time and cohort effects from the data before confronting the model. In contrast, we adopt a full structural approach where we enrich the model to include not only age effects but also time and cohort effects.

households' money-consumption ratios from Figure 1 and the change in aggregate money demand across time from Figure 2.

Given a set of values for the model parameters, Equation (8) allows us to simulate the dynamics of each household's money-consumption ratio and hence the equilibrium of the economy over time. Then, our identification strategy of the three effects based on the simulated model outcome can be described heuristically as follows:

1. Varying the term $\gamma_i \eta^h$ jointly, as well as θ_i, can be used to match the cross-sectional data on money-consumption ratios by household type in Figure 1 to their respective counterparts from the model in 2010. As seen from Equation (8), only the product of the age- and cohort-specific parameters, $\gamma_i \eta^h$, is identifiable in any given period, but matching the age and consumption profile gives us partial information of the elasticities of substitution between money and credit within any given group, and by aggregation for the population as a whole.

2. The other identification margin is the change in the aggregate money demand (i.e., the aggregate money-consumption ratio) over time. The key parameter here is η. Varying this parameter, and conditional on estimates of $\gamma_i \eta^h$ and θ_i, we can target the change in aggregate money demand from 2000 to 2010 in Figure 2.

3. Clearly this is a fixed point that we solve with standard iterative methods.

Once we have the parameter estimates we define the time effects to be the changes in money demand driven by the changes in the macroeconomic environment, including the after-tax nominal interest rates ($\widetilde{R}_t$) and the wage rates (w_t), which are equilibrium objects and do not have free parameters directly controlling them.

4.1 Calibration details

A model period is 10 years. There are five equal-sized income groups (social classes), $J = 5$. Households live seven periods, indexed by $i = 0, 1, ..., 6$, which corresponds to households aged 25 or younger, 26-35, 36-45, 46-55, 56-65, 66-75 and 76 or older, respectively. Our analysis will focus on the six oldest age groups ($i = 1, ..., 6$) as the youngest's money holdings are imposed exogenously to $\underline{m}$, which we set to 0.01% of the average consumption of all households or 0.024% of the average consumption of a newborn. The household's (inverse of the) inter-temporal elasticity of substitution, σ, is 2. Labour endowments of households are determined to replicate

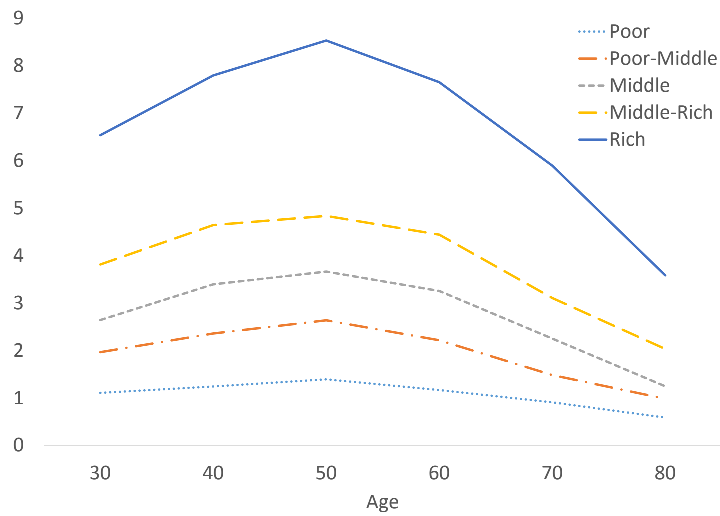


Figure 3: Labour Endowments by Household Type

the age profile in labour earnings from the 2009Q3-2012Q2 CFM with a normalization that their present value of life-cycle endowments is the same as that of consumption. This ensures that the model will be able to capture the observed consumption dispersion in the data. The resulting endowment profiles are shown in Figure 3.[14] The annual capital depreciation rate, δ, is 0.07 and the labour share in the production is 0.65, both of which are needed to obtain the wage rate given the rate of return on capital.

We now describe the rest of the parameters that are jointly determined in equilibrium by solving the model and matching a set of moments. In total, we have 42 parameters and 42 moments that we calibrate by solving the equilibrium of the model: 29 β_{ij}'s, 6 γ_i's, 6 θ_i's and η. Still, we find it useful for the discussion to associate a specific parameter with a particular target. This helps us better understand the potential link between parameters and data for the identification of the three effects. The job of replicating the hump-shaped consumption profiles over the life cycle of the various income groups is handled by households' type-discount factors, β_{ij}, perhaps better thought of as consumption-age weights. Such consumption profiles can be attributable to changes in household size and in costs of participating in the labour force. There are 30 β_{ij}'s in total. We normalize $\beta_{i=3,j=3}$ to 0.8, or 0.978 in annual terms, leaving 29 β_{ij}'s.

The credit transaction technology has three sets of parameters, γ_i's, θ_i's and η. The first two capture age effects, including the steepness at which money and credit substitute each other

[14]Since the groupings are based on consumption and there is no mobility, the resulting labour endowments profiles are flatter and more parallel than those that would result from panel data. From the point of view of this paper this is not a concern: the actual inequality in consumption profiles is not as relevant as is the money to consumption ratios by consumption quintiles by age group, which is what the model replicates.

h	1940	1950	1960	1970	1980	1990	2000	2010	2020	2030	2040	2050	2060	2070	2080	2090	2100	2110	2120
0	20	30	40	50	60	70	80												
1		20	30	40	50	60	70	80											
2			20	30	40	50	60	70	80										
3		Cohorts		20	30	40	50	60	70	80									
4					20	30	40	50	60	70	80								
5						20	30	40	50	60	70	80							
6							20	30	40	50	60	70	80						
7								20	30	40	50	60	70	80					
8						Age			20	30	40	50	60	70	80				
9										20	30	40	50	60	70	80			
10											20	30	40	50	60	70	80		
11												20	30	40	50	60	70	80	
12													20	30	40	50	60	70	80

Figure 4: Cohort and Age Structure of Households

over consumption, and η captures the cohort effects. Average money-consumption ratios for each age group i (i.e., $\frac{1}{5}\sum_{j=1}^{5}\frac{m_{ij}}{c_{ij}}$) in 2010, are mainly responsible for the corresponding γ_i. In addition, the θ_i's are mostly responsible for the slopes of the money-consumption ratio curve over consumption for each age group i (i.e., $\frac{1}{4}\sum_{j=2}^{5}\frac{m_{ij}/c_{ij}-m_{ij-1}/c_{ij-1}}{c_{ij}-c_{ij-1}}$) from Figure 1. Finally, the role of making aggregate changes in the money-consumption ratio of all households between 2000 and 2010, is what helps identify η^h separately from γ_i. The aggregate change in money demand between 2000 and 2010 is also determined by the changing macroeconomic conditions, which is what we call the time effects. Consequently, our model spans the experience of Canada over the last 80 years. That is, we start our model from a steady state in 1940 and we make it face the realized interest rates, inflation and taxes since then. Post-2010 we maintain the 2010 inflation and nominal interest rates with taxes, balancing the budget period by period all the way up to 2170, way past the time when all the living generations in 2010 have disappeared. The model is closed by assuming it was in a stationary equilibrium prior to 1940 and goes to another after 2170, without any cohort effects after 2070, when the youngest cohort alive in 2010 dies out (this is not quantitatively important given its distance into the future). Solving the model for such a long period is necessary to capture the potential impact of cohort and time effects on household decisions over the life cycle and the resulting implications for the equilibrium of the economy. For example, an 80-year-old household in 2010 was 20 years of age in 1950, and hence, its decisions since 1950 are reflected in its asset portfolio in 2010. Figure 4 visualizes the core part of the demographic structure of the model with 2010 highlighted as the period when the cross-sectional data are available.[15]

The values of the exogenous macroeconomic variables are in the upper panel of Table 1. Inflation

[15] There are more cohorts than shown in the figure for calibration and simulations.

Table 1: Macroeconomic Variables

	1940	1950	1960	1970	1980	1990	2000	2010
Exogenous variables:								
Annual inflation (%)	1.55	4.64	2.22	2.74	8.07	5.99	1.99	2.02
Annual nom. int. rate (%)	5.12	4.69	4.98	6.36	9.57	12.72	7.02	4.48
Government variables (% of annual GDP)								
Expenditure	9.13	23.91	15.74	15.28	18.53	21.85	19.85	15.52
Revenue	7.32	17.86	15.98	15.24	16.31	16.81	17.92	15.47
Debt	76.34	102.65	62.02	50.33	41.82	66.39	70.69	55.17
Asset	27.36	59.78	51.61	44.53	24.30	14.18	14.88	21.81
Endogenous variables:								
Tax rate (%)	6.51	26.38	18.54	16.77	21.42	15.87	19.21	19.22
Aggregate M-C ratio (%)	27.98	40.20	34.51	26.60	20.78	11.47	17.79	23.98
Wage rate	0.384	0.525	0.411	0.384	0.461	0.312	0.345	0.421

rates are the 10-year average of annual changes in CPI in Canada. Nominal interest rates are the prime rates. Government variables are expressed as a percent of annual GDP and obtained from the national accounts. Government assets correspond to A_t^G in the model and debts to B_t. The revenue and expenditure are, respectively,[16]

$$\begin{aligned} \text{Revenue} &= (1+\pi_{t+1})M_{t+1} - M_t + \tau_t(r_t K_t + w_t Z) + r_t^G A_t^G \quad \text{and} \\ \text{Expenditure} &= G_t + r_t B_t. \end{aligned}$$

4.2 Calibration results

The lower panel of Table 1 presents the main two endogenous macroeconomic variables, the income tax rates that balance the government budget each period and the aggregate money-consumption ratios, obtained from solving the household problems and aggregating. Note the negative relation between the money-consumption ratios and the nominal interest rates. The table also shows the wage rate consistent with the exogenous real interest rates.

[16] For the change in net worth of the government, we also calibrate directly from the data as: Change in Net worth $= A_{t+1}^G - A_t^G - B_{t+1} + (1+r_t)B_t + Adjustments_t$. The term $Adjustments_t$ represents the market value adjustment of government assets over time, e.g., the price of land held by the government, based on the Government Finance Statistics accounting framework developed by the International Monetary Fund.

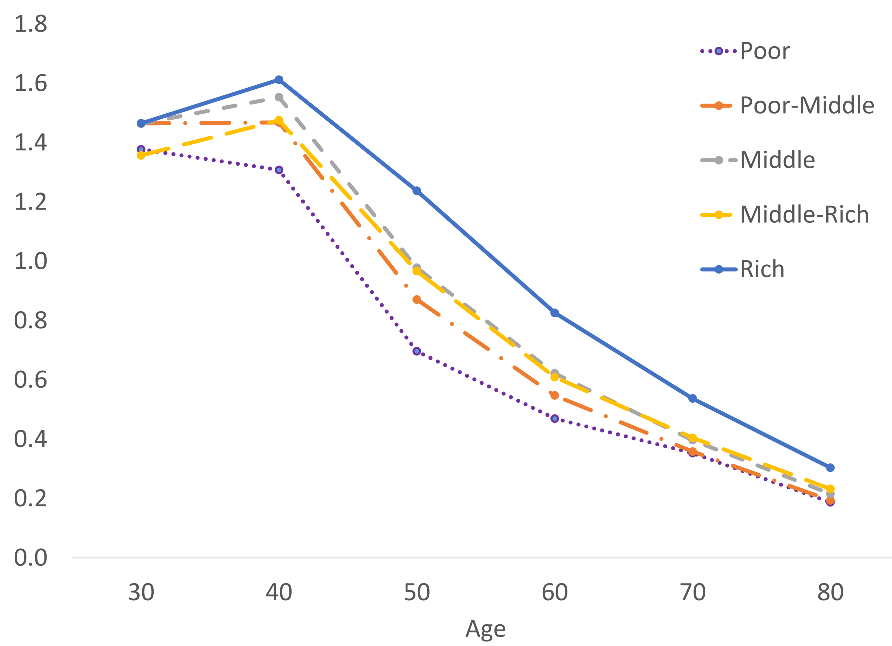


Figure 5: Calibration: Discount Factors

Given our specification of age-class-specific weight parameters for consumption shown in Figure 5, we are able to replicate exactly the consumption patterns in the data. To show how well the model matches the key moments from the data, Figure 6 separately displays money-consumption ratios by age in 2010 in the model and in the data. For each age group, the model-generated money-consumption ratios can replicate those from the data well. Notice that this is done with only one level-specific and one slope-specific parameter for each age group, yet the overall patterns of the money-consumption ratio over consumption decline are very well captured, indicating that our specification is a good one.[17]

Table 2 shows the parameter values for the γ_i's, the θ_i's and η. The value of main target associated with the cohort effect that we obtained is $\eta = .7922$, which suggests that the cost of credit transactions declines by about 21% for each new cohort from that of the previous cohort every 10 years, or about 2% per year (1.92%). The age-level-effect parameters, γ_i, monotonically increases with age, as should be expected, while the other parameter, θ_i, almost monotonically declines, meaning that the dependence of the money-consumption ratio to consumption is increasing with

[17] We could have used all these extra moments to estimate the parameters without any significant changes in their value.

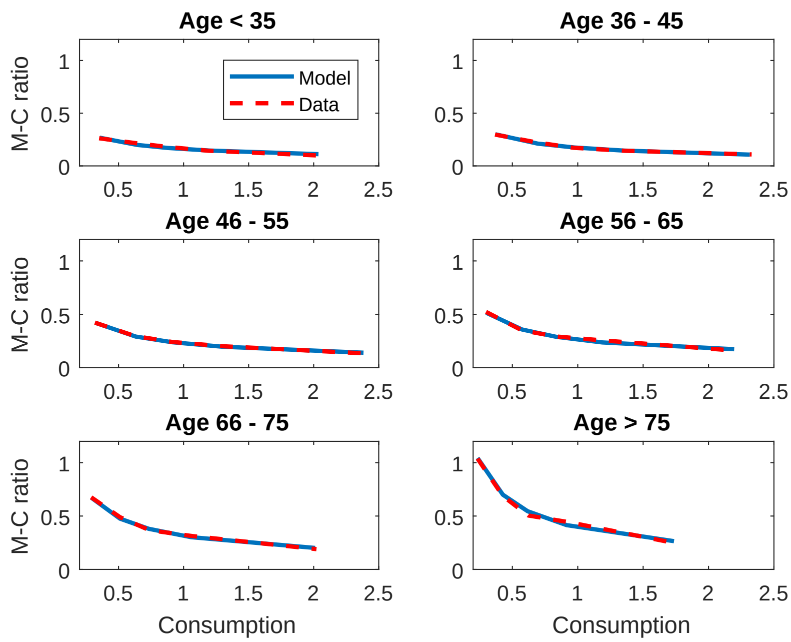
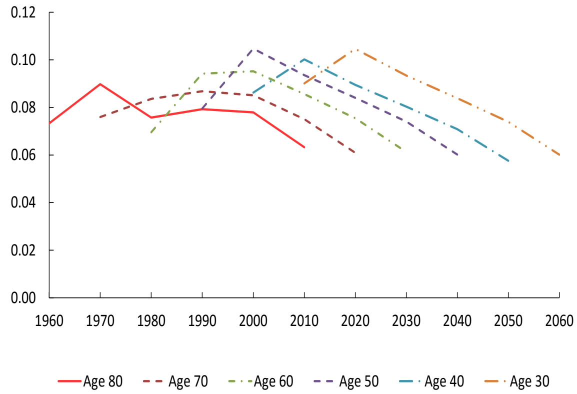


Figure 6: Comparison of Data and Calibrated Money-Consumption Ratios

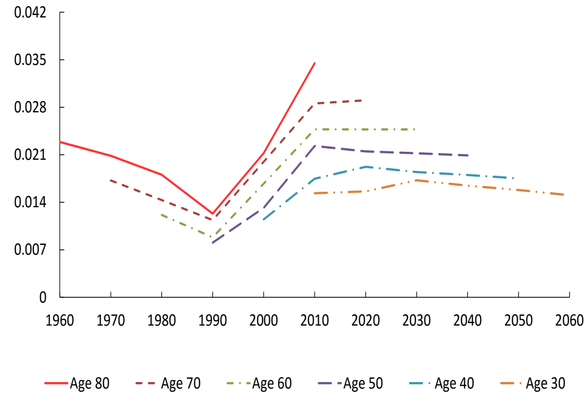
age. The table also shows the average money demands and average slopes in both model and data, confirming what Figure 6 showed.

4.3 Household decisions in equilibrium

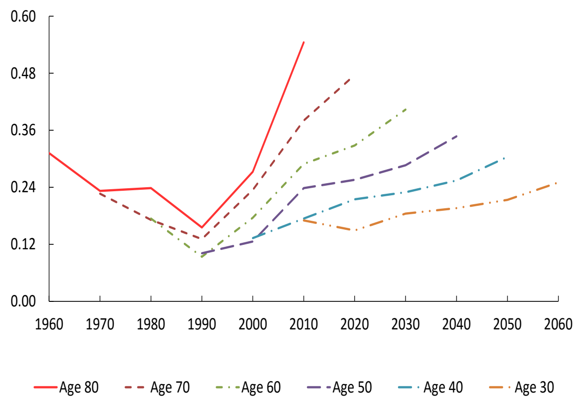
Figure 7 shows the main variables over the life cycle of the middle-class households for various cohorts who are active and identified by their age in 2010. We look at their consumption, money holdings, money-consumption ratios, the cost of credit and non-money assets. These figures give insight into how the cohort and the time effects generated by the changing the macroeconomic environment impact households born in different periods. Consumption varies little (Figure 7a), with all cohorts displaying the typical life-cycle consumption hump. There is, however, a lot more variation in money holdings (Figure 7b) and money-consumption ratios (Figure 7c) across cohorts. We can observe both the cohort effect reducing the money holdings of later cohorts and the low nominal interest rates increasing cross-sectional money demand since 1990. Cohort effects also show up as a decline in the cost of credit across generations (Figure 7d). Finally, Figure 7e shows the life-cycle non-money asset dynamics with distinct shapes by cohort, implying that the time effects are important.



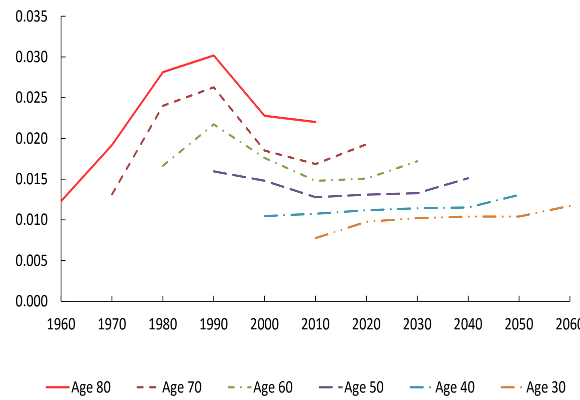
(a) Life-Cycle Consumption



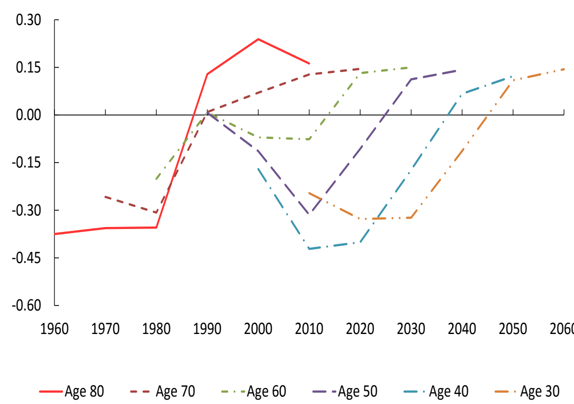
(b) Life-Cycle Money Holdings



(c) Life-Cycle Money-Consumption Ratio



(d) Life-Cycle Cost of Credit



(e) Life-Cycle Non-Money Assets

Figure 7: Evolution of Variables for Middle-Class Group: Various Cohorts

Table 2: Calibration Results

Parameter	Value	Target	Data	Model
γ_1	0.00167	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{1,j}$	0.1796	0.1788
γ_2	0.00366	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{2,j}$	0.1889	0.1875
γ_3	0.00419	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{3,j}$	0.2586	0.2573
γ_4	0.00562	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{4,j}$	0.3160	0.3140
γ_5	0.00952	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{5,j}$	0.4076	0.4063
γ_6	0.01821	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{6,j}$	0.5849	0.5937
θ_1	1.9619	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{1,j}/\Delta c_{1,j}$	-0.1195	-0.1193
θ_2	1.7475	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{2,j}/\Delta c_{2,j}$	-0.1308	-0.1302
θ_3	1.7800	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{3,j}/\Delta c_{3,j}$	-0.1917	-0.1913
θ_4	1.7055	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{4,j}/\Delta c_{4,j}$	-0.2626	-0.2616
θ_5	1.5416	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{5,j}/\Delta c_{5,j}$	-0.4193	-0.4170
θ_6	1.3762	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{6,j}/\Delta c_{6,j}$	-0.7968	-0.8040
η	0.7922	$\frac{\overline{m_{2009}}}{\overline{c_{2009}}} \frac{\overline{m_{1999}}}{\overline{c_{1999}}}$	1.35	1.34

5 Interest Rate Elasticity of Money Demand

What do the estimates imply for the nominal interest rate elasticity of money demand, a standard object of interest in monetary economics? Our calculation is based on a completely different estimation strategy, data set, and theory than those found in the literature. Moreover, our estimate of the interest rate elasticity of money demand results from the aggregation of the elasticities of individual household types based on an analytical expression for the elasticity for each household type by age, cohort, class and time, providing a much finer look at individual household elasticity than in the existing literature. Our main finding in this respect is that the nominal interest rate elasticity of money demand increases mostly with age and clearly with social class. That is, the older or the richer the household is, the higher his or her sensitivity to money demand is to the changes in nominal interest rates. More specifically for each $\{h, i, j\}$ type, the analytical expression of the elasticity is[18]

$$\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}} = -\frac{1}{\theta_i}\cdot\left(\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}\right)^{1/\theta_i}\cdot\left[1+\left(\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}\right)^{1/\theta_i}\right]^{-1}\cdot\frac{1+\widetilde{R}_t}{\widetilde{R}_t}\cdot\frac{R_t}{1+R_t}, \tag{9}$$

where R_t is the nominal interest rate, i.e., $R_t = (1+\pi_t)(1+r_t)-1$. All three sets of parameters from the credit-transaction technology, γ_i's, θ_i's and η, are present in Equation (9). We first note that (as is universally thought of) the elasticity is negative given the parameter restrictions discussed with Equation (1). The following proposition summarizes how the elasticity changes with γ_i, θ_i, η, h and c_{hij}.

Proposition 2. *Assume $\gamma_i > 0$, $\theta_i \geq 1$, $\eta \in (0,1)$, $c_{hij} > 0$ and $\widetilde{R}_t > 0$ for all h, i, j and t. The nominal interest rate elasticity of money demand decreases (or becomes less negative) with γ_i (i.e., higher credit-transaction cost). It also decreases (or becomes less negative) with θ_i if money is more important than credit in transactions. In addition, it decreases (or becomes less negative) with η (i.e., lower financial innovation per period). Finally, the elasticity increases (or becomes more negative) with h (i.e., more recent cohort) and c_{hij} (i.e., richer). That is,*

(i) $\partial\left(\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right)/\partial\gamma_i > 0$,

(ii) $\partial\left(\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right)/\partial\theta_i > 0$ *if* $\frac{m_{hij}}{c_{hij}} \in (0,\frac{1}{2})$,

(iii) $\partial\left(\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right)/\partial\eta > 0$,

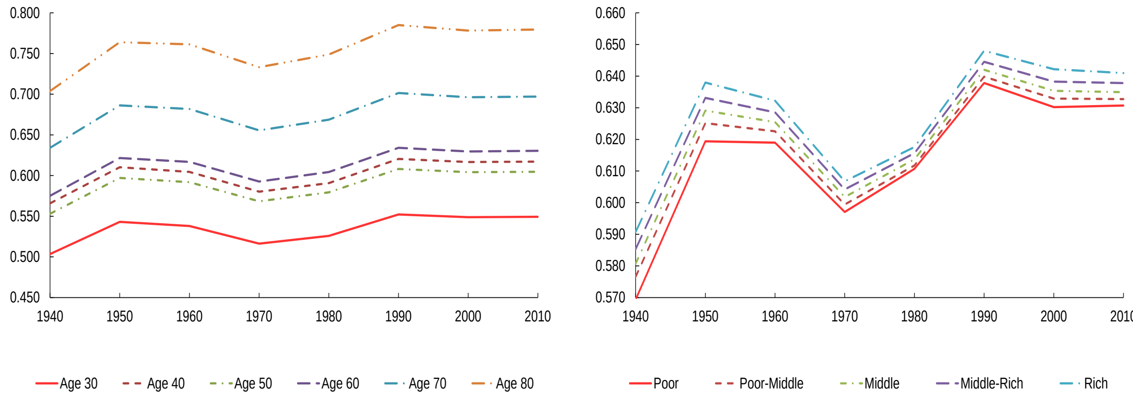
(iv) $\frac{\partial(m_{h+1,i,j}/c_{h+1,i,j})}{\partial R_t}\frac{R_t}{m_{h+1,i,j}/c_{h+1,i,j}} - \frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}} < 0$, *and*

(v) $\partial\left(\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right)/\partial c_{hij} < 0$.

See Appendix E for the proof.

Proposition 2 shows that the relation between age and the elasticity depends in subtle ways on the estimated values of the parameters and on the allocations. Our estimates of γ_i increase with age and, according to *(i)*, this dampens the elasticity. However, because θ_i mostly decreases with age and the money-consumption ratios are predominantly less than one-half, then *(ii)* tells us that the elasticity increases with age. Moreover, according to *(iii)*, larger cohort effects (i.e., lower η) lead money demand to be relatively more elastic, making younger cohorts (i.e., higher h) have a higher elasticity than their older peers as stated in *(iv)*. Also, higher consumption leads

[18] See Appendix D for the derivation.



(a) By Household Age

(b) By Household Class

Figure 8: Interest Rate Elasticity of Money Demand by Household Type

Table 3: Nominal Interest Rate Elasticity of Money Demand over Time

	1940	1950	1960	1970	1980	1990	2000	2010	Average
Time	0.585	0.633	0.628	0.604	0.615	0.645	0.638	0.638	0.623

to more elastic money demand from *(v)*. Our equilibrium measure of elasticity also depends on w_t and $\widetilde{R}_t$, implying that it is time specific in addition to being age, cohort and class specific.

Figure 8a shows the implied elasticities by age groups over time. We see clearly how among the different forces that affect the age profile of elasticities, those that increase with age prevail, except for those aged 40 and 50, which is likely driven by the non-monotonicity of θ_i. We also see a small secular increase over time of the elasticities consistent with Proposition 2.

Figure 8b displays the elasticities over time by household class. Richer households have almost proportionally higher elasticities than their poorer counterparts. Since there is no difference in the credit-cost parameter values across household classes, Proposition 2 tells us that it is the level of consumption that drives the observed differences: higher consumption corresponds to higher elasticity. We also observe a mild increase over time. Appendix G shows a detailed set of values for the elasticities by age, class, and time.

Table 3 shows the aggregate nominal interest rate elasticities of money demand in each period (they result from averaging all household types weighted by their consumption). It also shows the average elasticity of the cohort born in the period indicated. These average elasticities do not vary much over time. Across time, it was lowest in the 1940s at 0.585 and peaks in the 1990s at

0.645 with the simple average over time of 0.623. Time effects, including nominal interest rate changes, drive the fluctuations over time while cohort effects secularly push elasticities up over time as stated in Proposition 2.

These elasticity measures are at the higher end of the range found in the literature. The implied elasticity from the theoretical literature on the transactions demand for cash in wallet (e.g., Baumol (1952) and Tobin (1956)) is 0.5. Empirical studies that use cross-sectional survey data vary in their estimates, ranging from less than 0.25 (Lippi and Secchi (2009)) to 0.6 (Attanasio, Guiso, and Jappelli (2002)). Mulligan and Sala-i-Martin (2000) incorporate the analysis of the extensive margin in owning interest-bearing assets, and they estimate the elasticity between 0.2 and 0.5 for the median financial asset owner when the nominal interest rate is around 5%; Alvarez and Lippi (2009) and Attanasio, Guiso, and Jappelli (2002) each estimate two values of elasticities, one with ATM cards and another without, with Alvarez and Lippi (2009) obtaining 0.43 and 0.48, respectively, and Attanasio, Guiso, and Jappelli (2002) obtaining 0.59 and 0.27, respectively.

Mulligan and Sala-i-Martin (2000), Alvarez and Lippi (2009) and Attanasio, Guiso, and Jappelli (2002) also stress the importance of financial innovation/technology in estimating the elasticity, although the specific technology considered differs among them. Mulligan and Sala-i-Martin (2000) analyze the financial technology in the extensive margin between money and other interest-earning assets, arguing that the elasticity becomes higher with the interest rate as people have more incentive to access non-money assets earning interest at higher interest rates as long as there is enough of a pool of people without access to these assets. Alvarez and Lippi (2009) and Attanasio, Guiso, and Jappelli (2002) focus on financial innovation in withdrawing money, such as the availability of ATMs, hence the technology that eases access to money as opposed to non-money assets.

The cohort effects in our study reflect any secular financial innovation as our estimate of $\eta = 0.79 < 1$ implies a declining cost of using credit over time with each new generation entering the economy. This leads the elasticity to be higher on average over time, as implied by Proposition 2. In addition, financial innovation in our model is an important factor determining the portfolio choice of households over money and non-money assets/debt and makes it cheaper to hold non-money assets/debt, thus similar to that by Mulligan and Sala-i-Martin (2000) with the difference being that our model incorporates household decisions over time with respect to the dynamics of financial innovation and other changing macroeconomic factors, including inflation. In addition, other studies with financial innovations besides Mulligan and Sala-i-Martin (2000) typically analyze demand for "cash in wallet," whereas our study uses both cash and low-interest deposit accounts, a definition closer to the aggregate money supply, as our concern is the welfare cost of inflation due to the change in the value of liquid nominal assets.

6 Income versus Age versus Cohort Effects

The very large differences in the money-consumption ratio among different groups (the oldest poor have a 10.5 times larger ratio than the youngest rich) are due to a combination of the income effect already documented by Erosa and Ventura (2002) (high consumption households save on the credit transaction technology, so they use relatively less money to implement their consumption), the age effect (people use more money as they get older), and the cohort effect (young people become acquainted with the state-of-the-art transaction technologies). We want to measure the contribution of each of these effects.

The income effect can be readily defined as the ratio between the average money consumption ratio of the poorest relative to that of the richest, and this is 3.43. Note that this calculation can be done either from the data or from the model since our model is calibrated to replicate money-consumption ratios by age and income class in 2010.

The average money holdings of the oldest group in 2010 relative to the youngest is 3.26. The partition of those differences between age and cohort effects is more subtle and it requires the explicit use of theory. We proceed by solving the model under the assumption of no cohort effects or technological change beyond that experienced by the oldest cohort alive in 2010 and we ask what would have been the money holdings by age groups in that case. Money holdings of the oldest in 2010 is the same as in the baseline economy, but younger groups would be holding more money due to the lack of a cohort effect that makes younger cohorts more adept at using credit for transactions. More precisely, in the data the money-consumption ratio held by the oldest is 3.26 times that of the youngest while in the economy without cohort effects it is 1.74 times. We then say that age effects account for 53.4% of the differences in money holdings of the old relative to those of the young, and the rest, 46.6%, is accounted for by cohort effects. This finding implies that cohort effects are as important as those specific to aging in explaining the money holding patterns across age groups in a given period.

One more relevant comment is that if we use the ratio of average money consumption ratios between the old and the young (3.26) holdings by age as a measure of the joint age and cohort effect and we use the ratio of the poorest to the richest (3.43) as a measure of the income effect we see that they do not add up to the 10.5 ratio between the oldest poorest and the youngest richest. The reason is that these effects are not additive, and in fact, they compound each other: the income effect is largest for the oldest, or the age effect is largest for the poorest. For this reason we do not want to decompose the differences into three disjoint mechanisms. However, this is not the case for the decomposition between age and cohort effects, as we define the share that the age effect has in shaping differences in money holdings across age groups as the ratio of the average money consumption ratio by age that would exist if there were no cohort effects to that in the data.

7 Welfare Implications of a Permanent Switch to 5% Inflation

What are the welfare implications of a surprise permanent increase in inflation from 2% to a moderate inflation of 5%? To answer this question we have to determine three things. First, what is the gain (the windfall) to the government of the higher inflation due to the capital loss associated with the reduced value of the holdings of government debt that households have (Section 7.1)?[19,20] Second, what is the use that the government gives to the windfall generated by the surprise inflation? Here we consider various alternatives (Section 7.2). Third, what is the response of households to the increase in the costs of transacting for consumption (and the associated change in the relative costs of the available transaction technologies) (Section 7.3)? The response of households involves not only changes in the transactions costs that they pay but also induces changes in the amount of seignoirage that the government raises, which implies that for each type of use the government gives to the windfall, its amount has to be determined simultaneously with the effects of that policy on households.

7.1 Changes in the real value of bond holdings

There are myriad government bonds of different types, and there is insufficient detail in the data to ascertain how many bonds of each type are held by each group of households. To circumvent this difficulty, we proceed by first calculating the inflation-shock-induced change in the aggregate value of government bonds, then allocating the change into the different sectors by their share of bond holdings and, finally, assigning the change across household groups according to their bond ownership shares. We discuss these in reverse order.

Table 4 shows how the bonds held by households are distributed among the various groups

[19]See Doepke and Schneider (2006) and Meh and Terajima (2011).

[20]We assume that nominally denominated privately issued assets cancel since issuers' gains are holders' losses.

Table 4: Share of Bonds Held by Households (%), 2009Q3-2012Q2 CFM and 2011 NHS

	Age					
Class	30	40	50	60	70	80
Poor	0.18	0.64	1.70	1.99	1.20	2.27
Poor-Middle	0.31	1.15	5.11	3.18	2.41	2.62
Middle	0.66	0.97	3.67	3.35	3.90	2.00
Middle-Rich	1.15	2.38	5.30	6.43	4.98	3.86
Rich	2.28	3.33	9.00	11.61	5.94	6.45

Table 5: Canadian Government Bond Position by Sector in 2010Q1

	Household	Government	Business	Foreign
	Direct Position			
Value in $Mil	27,975	-556,157	430,987	97,195
Share in %	5.03	-100.00	77.49	17.48
	Direct and Indirect Positions Combined			
Value in $Mil	234,392	-556,157	-	321,765
Share in %	42.14	-100.00	-	57.86

using CFM and the 2011 National Household Survey data.[21] The numbers add up to 100%. As it should be expected, rich and old households tend to hold a larger share of government bonds.

We now document the government bond holdings at the sectoral level in Canada. The upper panel of Table 5 is from the National Balance Sheet Accounts.[22] Negative numbers represent liabilities. The market value of outstanding government bonds totalled $556 billion in 2010Q1, where 5.03%, 77.49% and 17.48% are held by the household, business and foreign sector, respectively. We further allocate the holdings of the business sector between the household and foreign sectors and display them in the lower panel of the table. The value of equity claims against businesses held by other sectors represents the value of business assets net of its liabilities. Hence, the owners of businesses (i.e., households and foreigners) will ultimately bear the impact of the change in the value of assets held by businesses. Households own 47.9% of the equity claim against businesses, and the rest is held by foreigners.[23] The household sector ultimately

[21] Appendix F discusses how we derived the numbers in the table, and Appendix A.2 describes the National Household Survey, which is a part of the Census program in Canada.

[22] See CANSIM Table 378-0121 from Statistics Canada.

[23] See Meh and Terajima (2011) for more discussion.

Table 6: Annual Aggregated Payment Schedule of Government Bonds, $Mil

Year	Amount Due	Year	Amount Due
2010	132,813	2026	9,102
2011	62,445	2027	10,071
2012	44,595	2028	3,114
2013	37,101	2029	15,919
2014	50,249	2030	2,378
2015	21,108	2031	8,178
2016	17,083	2032	2,146
2017	16,862	2033	15,556
2018	16,729	2034	1,374
2019	23,304	2035	1,374
2020	12,092	2036	7,224
2021	10,927	2037	15,198
2022	4,690	2038	499
2023	8,464	2039	499
2024	4,141	2040	499
2025	7,346	2041	16,049

holds 42.14% of all government bonds, while the foreign sector has 57.86%. We then impute the loss for each household type using their share of the government bonds held directly or indirectly within the household sector. The change in the real value of government bonds depends on their maturity, since longer bonds change more than shorter bonds.

Assuming efficient secondary bond markets, the changes in the future values of these payments will be reflected in the price of bonds instantaneously at the realization of the shock. Accordingly, to obtain the new value of these bonds, we should sum up the present values of changes in the real values of all future payments based on the schedule reported in Table 6 for all government bonds and T-bills.[24] The reduction in the aggregate value of government bonds from the permanent change in inflation comes to 4.72% of 2010 GDP. This loss is allocated to the household and foreign sectors, and then further into household groups according to the fractions discussed in this section.

7.2 Government policy

The effects of inflation crucially depend on what the government does with this windfall (and with that induced by the additional seigniorage). It is not the same if the government spends it on things that are not substitutes for private consumption compared to if it returns it to households

[24] Data are from the Bank of Canada and accessible from https://www.bankofcanada.ca/stats/goc/results/en-goc_tbill_bond_os_2009_12_31.html.

in the form of lower taxation or if it returns it to the original bondholders.

We look at five possible uses of the windfall. In all cases we look at it from a present value perspective rather than imposing some form of period by period balancing rule. Specifically, we define the present value government budget constraint to be

$$\begin{aligned}\sum_{t=t_{2010}}^{\infty} \frac{G_t}{(1+r_t)^{t-t_{2010}}} &= \sum_{t=t_{2010}}^{\infty} \frac{r_{t+1}A^G_{t+1} + r_{t+1}B_{t+1} + \pi_{t+1}M_{t+1} + \tau_t\left(r_tK_t + w_tZ\right)}{(1+r_t)^{t-t_{2010}}} \\ &- (1+r_{t_{2010}})\left(A^G_{t_{2010}} + B_{t_{2010}}\right) - M_{t_{2010}} + \Delta_{t_{2010}}, \end{aligned} \tag{10}$$

where t_{2010} is the time index for the model period 2010, and $\Delta_{t_{2010}}$ is the part of the windfall gain by the government calculated in the previous section and due only to lower real debt value, i.e., 4.72% of 2010 GDP. Using this government budget constraint, we consider the following five policies:

(a) The government transfers all gains from reductions in the real value of debt back to the bondholders so they do not experience any losses and spends all seigniorage proceedings as useless government expenditures.

(b) The governments spends all its windfall on government expenditures.

(c) The government lowers taxes forever at a constant rate.

(d) The government gives to household bondholders all the gains from the reduced value of bonds (which implies that these bondholders have net gains). It does so not on an equal basis, but proportionally to each bondholder's initial debt holdings. The increased seigniorage goes to lower taxation.

(e) The government gives the windfall of the debt reduction back to the bondholders while the increased seigniorage lowers taxation.

Policy (a) is designed to leave households with only the effects of inflation on money holding, without any wealth effect implied by the government windfalls and their management. Policy (b) adds to the inflation cost the wealth confiscation that results from the reduction on the value of government debt without any alleviation. In Policy (c) households receive on an equal basis all the gains through lower taxes. Policy (d) adjusts the tax rate for the change in seigniorage while redistributing the windfall gain from the bond revaluation back to households proportionally to their holding of the bonds. In Policy (d), the government targets the different sets of households to compensate for the seigniorage and the bond-revaluation channels. The inflation tax on money holdings induces a long-term impact covering both current and all future cohorts, and thus the government reduces tax permanently, benefiting both current and future cohorts. In contrast, the losses in bond values impact only the active cohorts in 2010, and hence the transfer of the windfall from this channel back to them. The amount of this transfer to households equals that of the entire windfall from bonds including the loss incurred by foreigners. Thus, all households receive a net gain in bonds. Finally, Policy (e) isolates the impact of inflation tax only on money, and the government lowers taxes without the wealth effects from the revaluation of government bonds.

7.3 Welfare measures of inflation

Changes in welfare are measured as the consumption equivalent variation. Let λ_{hj} be the consumption equivalent variation for cohort h and social class j expressed as a constant proportion to the life-cycle consumption of the household, where a positive value indicates a cost and a negative value a benefit. Then, define V_{hj}^{0} to be the lifetime utility of the households of type $h \geq 1$ and $\forall j$, i.e.,

$$V_{hj}^{0} = \sum_{i=1}^{6} \beta_{ij} u\left(c_{hij}^{0}\right),$$

where c_{hij}^{0} is the consumption obtained in the baseline calibration of the model. Let us define the value for an alternative economy with policy ℓ to be

$$V_{hj}^{\ell} = \sum_{i=1}^{6} \beta_{ij} \;\; u\left(c_{hij}^{\ell}\right),$$

where c_{hij}^{ℓ} is the household's optimal consumption decision under policy $\ell \in \{a, b, c, d, e\}$ as described in Section 7.2. Then, λ_{hj}^{ℓ} is implicitly defined by[25]

$$\sum_{i=1}^{6} \beta_{ij} \;\; u\left[\left(1+\lambda_{hj}^{\ell}\right) \; c_{hij}^{\ell}\right] = V_{hj}^{0}.$$

[25] For households who are alive in 2010, $1+\lambda_{hj}^{\ell}$ is multiplied only by their consumption from 2010, since pre-2010 consumption is not affected in this exercise. Hence, for households who are alive in both 2010 and earlier periods, λ_{hj}^{ℓ} is set to zero in the pre-2010 periods and accordingly their welfare measure reflects the consumption equivalence since 2010. We do not use the "i" subscript for λ to avoid the implication that λ can change for each and every age.

In addition to λ^{ℓ}_{hj}'s, which are measures of the welfare cost specific to each household type, we calculate aggregated welfare measures. Such aggregation is not straightforward since the λ's are expressed relative to each group's specific consumption. Hence, we aggregate the changes in the consumption of individual households implied by the respective λ_{hj} and discount them using real interest rates to their 2010 values, and we report total aggregate welfare losses (present and future) in terms relative to 2010 consumption (a one-year loss, not a recurrent one). We do this by household type and aggregate them.[26] To see the details, we define the changes in units of 2010 consumption to be

$$\Delta c^{\ell}_{hij} \equiv \lambda^{\ell}_{hj} \cdot c^{\ell}_{hij}/(1+r_{2010})^{h+i-1-t_{2010}} \ \ \textit{for } h \in \{1, ..., 12\}, i \in \{1, ..., 6\}, \ \textit{and } h+i-1 \geq t_{2010},$$

where the last inequality restricts the relevant consumption to be that in 2010 or later and $t_{2010} = 6$. r_{2010} is the 10-year real interest rate in 2010 and constant for all future periods. We now define three aggregate welfare measures by aggregating households into different groups. Specifically, for $h \in \{1, ..., 12\}$, $i \in \{1, ..., 6\}$, and $h+i-1 \geq t_{2010}$, we define

$$W_h \equiv \left(\sum_{ij} \Delta c^{\ell}_{hij}\right) / C^{h}_{2010}, \tag{11}$$

$$W_j \equiv \left(\sum_{hi} \Delta c^{\ell}_{hij}\right) / C^{j}_{2010}, \tag{12}$$

$$W \equiv \left(\sum_{hij} \Delta c^{\ell}_{hij}\right) / C_{2010}, \text{ where} \tag{13}$$

$$C^{h}_{2010} \equiv \sum_{ij} \left[c^{0}_{hij}/(1+r_{2010})^{h+i-1-t_{2010}}\right],$$

$$C^{j}_{2010} \equiv \sum_{hi} c^{0}_{hij} \ \ \forall\, h \in \{1, ..., 6\} \text{ and } i \in \{1, ..., 6\} \text{ such that } h+i-1 = t_{2010}, \text{ and}$$

$$C_{2010} \equiv \sum_{hij} c^{0}_{hij} \ \ \forall\, h \in \{1, ..., 6\}, \ i \in \{1, ..., 6\} \text{ and } j \in \{1, ..., 5\} \text{ such that } h+i-1 = t_{2010}.$$

C^{h}_{2010} is the sum of the life-cycle consumption of cohort h discounted to 2010. C^{j}_{2010} and C_{2010} are the 2010 aggregate consumption of class j and of all groups, respectively. These consumption numbers are annualized from their 10-year model period versions. W_h, W_j, and W are the measures of aggregate welfare for cohort h, class j and all households, respectively.

[26] For the purposes of welfare calculation, we track future households up to and including the 12th cohort who is born in year 2060. See Figure 4.

7.3.1 Results of individual welfare: λ_{hj}^{ℓ}

Table 7 displays the resulting λ_{hj}^{ℓ}, the welfare costs of the increase in permanent inflation, expressed as the constant proportion of per-period consumption for each household type active in 2010 under the five government policies. Under Policy (a) the only costs are the increased transaction costs of using money, which in the model do not affect the oldest households. The losses are clearly ranked by income with the poor losing a lot more (three to four times) than the rich: they use more money than the rich. Across cohorts, costs peak for the households in their 60s and then monotonically decline towards the younger cohorts. Younger cohorts are victims of the higher inflation for a longer time but are protected by the cohort effects that reduce their vulnerability.

Policy (b) is more costly (higher waste), with the additional costs concentrated in richer and older households who are the ones that hold the most bonds. When the government returns its windfall in the form of lower taxation (Policy (c)), the tax rate permanently decreases to 18.91% from 19.22% in 2010. This leads all households to fare better relative to those under Policy (a) or (b). There is some amount of cross-cohort subsidization: current bondholders give to future cohorts, making the young reap smaller losses than the old (and in the case of the youngest rich, slight gains). Under Policy (d), the more than fair compensation to current bondholders at the expense of foreign bondholders makes the currently old (and most of the rich) in a position to have gains as a result of higher inflation. The young and the poor suffer losses. Finally, Policy (e) is a little bit worse than Policy (d) because the partial confiscation of the debt held by foreigners does not take place.

To summarize, we see that the poor and the old (not the oldest) are the biggest losers, because their dependence on money is larger. The actual details vary with the use of the government windfall from the inflation tax on money and bonds as well as if and how it is distributed among the various household groups.

7.3.2 Aggregate measures of welfare: W_h, W_j and W

While the main theme of this paper is to build measures of welfare costs of inflation jointly by income class and cohort, it is instructive to generate more aggregate measures comparing cohorts or classes directly and even providing a society-wide measure of the cost of inflation according to the criteria described above.

Figure 9 summarizes the welfare costs of the various cohorts, W_h, as defined in Equation (11).

Table 7: Welfare Cost in Percent of Own Consumption ($\lambda^{\ell}_{hj} \cdot 100$) at 5% Inflation

	Age in 2010					
Class	80 ($h=1$)	70 ($h=2$)	60 ($h=3$)	50 ($h=4$)	40 ($h=5$)	30 ($h=6$)
	(a) No changes in the value of bond holdings, seigniorage wasted by government					
Poor	0.00	1.34	1.42	1.29	1.11	0.94
Poor-Middle	0.00	0.92	0.95	0.87	0.77	0.66
Middle	0.00	0.72	0.75	0.69	0.62	0.54
Middle-Rich	0.00	0.55	0.59	0.55	0.51	0.45
Rich	0.00	0.35	0.39	0.38	0.36	0.32
	(b) All the windfall is uselessly spent					
Poor	0.99	1.60	1.73	1.50	1.18	0.96
Poor-Middle	0.63	1.21	1.21	1.20	0.83	0.67
Middle	0.34	1.05	0.94	0.86	0.66	0.56
Middle-Rich	0.44	0.85	0.84	0.72	0.57	0.47
Rich	0.39	0.54	0.64	0.53	0.41	0.35
	(c) All the windfall is given back proportionally to income via lower income taxes					
Poor	0.67	1.23	1.30	1.02	0.69	0.51
Poor-Middle	0.33	0.86	0.80	0.73	0.36	0.23
Middle	0.06	0.69	0.52	0.39	0.19	0.12
Middle-Rich	0.14	0.49	0.43	0.28	0.11	0.04
Rich	0.11	0.19	0.24	0.10	-0.03	-0.08
	(d) Seigniorage reduces taxation, bondholders transferred all debt reduction					
Poor	-1.46	0.81	0.82	0.78	0.79	0.71
Poor-Middle	-0.99	0.35	0.40	0.20	0.46	0.43
Middle	-0.58	0.10	0.29	0.25	0.35	0.30
Middle-Rich	-0.73	-0.01	0.05	0.12	0.21	0.20
Rich	-0.66	-0.06	-0.13	-0.03	0.09	0.08
	(e) Seigniorage reduces taxation, no change in bond holdings					
Poor	-0.14	1.17	1.23	1.07	0.88	0.73
Poor-Middle	-0.13	0.75	0.76	0.65	0.55	0.45
Middle	-0.12	0.55	0.55	0.48	0.40	0.33
Middle-Rich	-0.13	0.39	0.40	0.35	0.30	0.24
Rich	-0.13	0.19	0.21	0.18	0.15	0.12

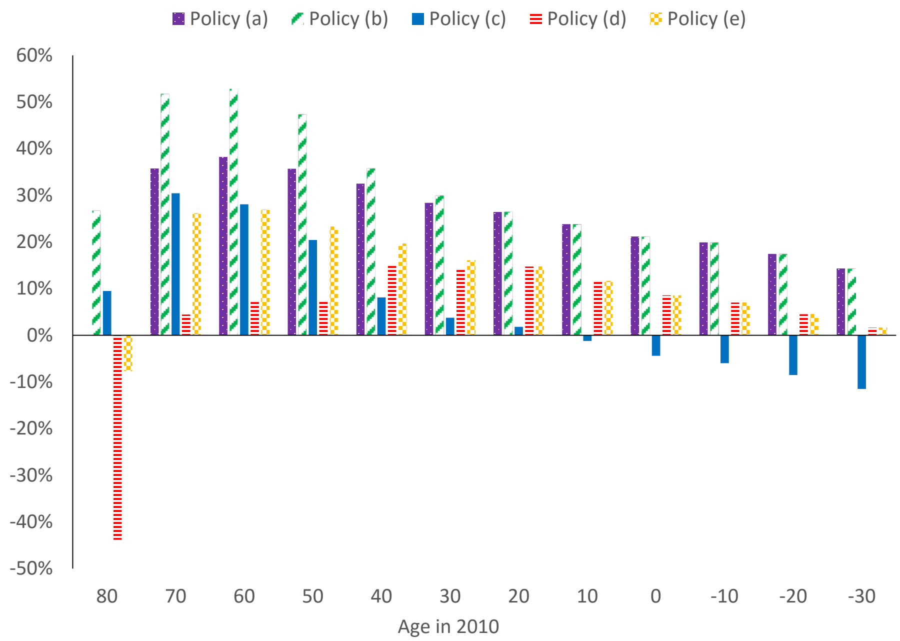


Figure 9: Welfare Cost of Inflation by Cohort (W_h), in Percent of Own Annual Consumption in 2010 Value

It includes results for some future cohorts that are not active in 2010, labeled younger than 30 as their age in 2010. We can see how future cohorts are better protected by higher inflation to the point that under Policy (c), which redistributes the government windfall in the form of lower taxation, they actually are better off with permanently higher inflation. Policies that compensate or more than compensate bondholders induce lower losses to the older households in 2010.

Table 8 displays the welfare costs by class aggregating over various cohorts using the expression in Equation (12). In all cases the poor lose more—a lot more—than the rich. The bulk of the difference arises from the higher dependency on money for transactions of the poor. The differences are more severe the more compensated households are for their policies with the inflation windfall. All the policies that include compensation do so in a way that favours the rich, some of it in the form of lower taxation (which is proportional to income) and the rest in the form of compensation to bondholders (with richer households holding more). Still, all policies are ranked similarly by all classes. Taxation alleviation is the least bad (Policy (c)), and windfall waste is the worst (Policy (b)). The table also reports two aggregate measures of welfare (W) , one defined in Equation (13) and another that simply averages all W_j's. Both confirm the rankings among the policies. The first measure (in bold) aggregates the costs of inflation by adding the

Table 8: Welfare Cost in Aggregate (W) and by Class (W_j), in Percent of 2010 Annual Consumption of the Respective Group

Policy	W_j Poor	Poor-Middle	Middle	Middle-Rich	Rich	W (Weighted Average)	Unweighted Average
(a)	44.86	31.21	25.72	20.97	14.70	**21.78**	27.49
(b)	49.32	35.72	28.85	24.65	17.99	**25.35**	31.31
(c)	27.65	14.23	7.42	3.43	-2.97	**4.15**	9.95
(d)	28.46	14.81	11.23	5.81	0.24	**6.81**	12.11
(e)	34.60	21.02	15.56	10.89	4.77	**11.73**	17.37

amount of the good that has to be given to the different groups to be indifferent. This is the right measure of welfare. The last column aggregates the costs of inflation weighting W_j's by the number of households instead of their consumption. This measure is larger because poor people suffer more but they are cheaper to buy out. Last but not least, we think that these costs are still large despite the effects of financial technology suppressing the welfare cost through cohort effects. Even in the best case scenario (all windfall is used to lower taxation) the welfare cost of inflation amounts to more than half a month of TOTAL consumption.

7.3.3 Comparison with the literature on the welfare effects of inflation

How do our results compare to those of the literature? We compare explicitly our calculations to those in two classic studies based on the definition of money close to ours: Lucas (2000) and Erosa and Ventura (2002). These studies differ from ours in terms of the period used, the country where the data come from, the extent to which heterogeneity of agents is taken into account, and the considerations about the future importance of money holdings for transactions, features about which we can do little, but they also implement the inflation changes under different ways of allocating the government windfall. Lucas (2000) ignores all considerations of the windfall, so we should compare his findings to those under our policy (a), while Erosa and Ventura (2002) ignore the windfall arising from the lower value of the debt but accounts for increased seigniorage, which means that we should compare their calculations to our policy (e). Still, in order to make their calculations comparable to ours, we need to make further adjustments such as imposing the same elasticity of money demand that we obtained to Lucas's method (.6 versus .5), normalizing the size of the inflation increase to 3% and reporting the costs in terms of 2010 consumption.

Details are in Appendix H.

Table 9: Welfare Cost of Inflation in Percentage of 2010 Consumption by Various Studies, Permanent Increase from 2.02% to 5% per Year

Others	This Paper
Lucas (2000)	Policy (a)
25.4%	21.8%
Erosa-Ventura (2002)	Policy (e)
17.1%	11.7%

Table 9 reports the comparison. Our calculations imply a lower cost of inflation than those in Lucas (2000) and Erosa and Ventura (2002). The main residual difference between ours and their calculations is that they ignore cohort effects, which reduce the future costs. In the framework of Lucas (2000), the presence of cohort effects implies a continuous inward shift in the demand curve for money over time. This is not accounted for when estimating the entire demand curve from the time series of aggregate money. Similar intuition applies to the study of Erosa and Ventura (2002), which does not account for financial innovation and its reflection in the cohort effects in the future. Needless to say we think our calculations are more reliable, but the precise role of cohort effects deserves more attention.

7.4 Welfare implications without cohort effects

To gain further insight into the importance of accounting for cohort effects in the welfare analysis, we remake the welfare calculations under the lens of a model that ignores cohort effects and imputes all differences in the money holdings of different age groups to age effects (we set h to be the same for all cohorts). We then calibrate γ_i's to match the levels of money-consumption ratios across age groups as in the data while keeping other parameters as in the baseline economy. Table 10 displays the implied values of parameters and moments. Notice the ratio between the values of the largest and smallest γ's is about 40, while it is much smaller, about 10, in the baseline economy with cohort effects (Table 2).

We use Policy (a) for the comparison, and we report the implied measures of welfare (the λ_{hj}^{ℓ})

Table 10: Calibration without Cohort Effects

Parameter	Value	Target	Data	Model
γ_1	0.00047	$\frac{1}{5}\sum_j\left(\frac{m}{c}\right)_{1,j}$	0.1796	0.1805
γ_2	0.00149	$\frac{1}{5}\sum_j\left(\frac{m}{c}\right)_{2,j}$	0.1889	0.1882
γ_3	0.00217	$\frac{1}{5}\sum_j\left(\frac{m}{c}\right)_{3,j}$	0.2586	0.2575
γ_4	0.00385	$\frac{1}{5}\sum_j\left(\frac{m}{c}\right)_{4,j}$	0.3160	0.3139
γ_5	0.00820	$\frac{1}{5}\sum_j\left(\frac{m}{c}\right)_{5,j}$	0.4076	0.4050
γ_6	0.01972	$\frac{1}{5}\sum_j\left(\frac{m}{c}\right)_{6,j}$	0.5849	0.5765

for the calibration that abstracts from cohort effects and for the baseline economy in Table 11. As we would have expected, abstracting from cohort effects implies a much larger cost of inflation, especially for the later cohorts, for whom the calculated costs are 25% larger.

The large difference between the two cases becomes more apparent by looking at aggregate measures of welfare costs, either by cohort as reported in Figure 10 or by class in Table 12. The welfare cost by cohort stays around 40% from the Age 20 cohort in 2010 onward while the cost with cohort effects continue to decline over future cohorts. The aggregate welfare cost is 47% larger when abstracting from cohort effects, and this relative magnitude is observed across all classes.[27] Clearly, abstracting from cohort effects induces a severe bias upward of the costs of inflation.

8 Conclusion

To summarize, in this paper we have documented how money holdings per unit of consumption increase with age and decrease with consumption levels. Using a model where transactions are made with costly credit or with cash, and taking advantage of the observed variations in money holdings and nominal interest rates over the last decades, we decompose the increased money holdings by age into age and cohort effects (the latter due to improvements in transaction technology). This allows us to calculate both the interest rate elasticity of money demand and the welfare cost of inflation using new methods. We found that the elasticity is about .6, on the upper end of the estimates in the literature, and that the welfare cost of inflation, is about 25% of one year of consumption for a permanent increase of inflation from 2% to 5%—a large

[27]This table provides larger differences than those in Table 12 because it looks further into the future.

Table 11: Welfare Cost in Percent of Own Consumption at 5% Inflation for Policy (a) with and without Cohort Effects

Class	Age in 2010: 80 ($h=1$)	70 ($h=2$)	60 ($h=3$)	50 ($h=4$)	40 ($h=5$)	30 ($h=6$)
	Without Cohort Effects					
Poor	0.00	1.54	1.78	1.72	1.53	1.37
Poor-Middle	0.00	1.05	1.19	1.15	1.08	0.97
Middle	0.00	0.82	0.94	0.91	0.87	0.80
Middle-Rich	0.00	0.63	0.74	0.73	0.71	0.66
Rich	0.00	0.40	0.49	0.49	0.50	0.47
	With Cohort Effects (Baseline)					
Poor	0.00	1.34	1.42	1.29	1.11	0.94
Poor-Middle	0.00	0.92	0.95	0.87	0.77	0.66
Middle	0.00	0.72	0.75	0.69	0.62	0.54
Middle-Rich	0.00	0.55	0.59	0.55	0.51	0.45
Rich	0.00	0.35	0.39	0.38	0.36	0.32
	Average difference between the above two panels					
	0.00	0.11	0.21	0.24	0.26	0.27

value but somewhat lower than existing estimates that use other methods because our explicit calculation of the improving transaction technology that shows up as cohort effects dampens the cost of inflation over future cohorts.

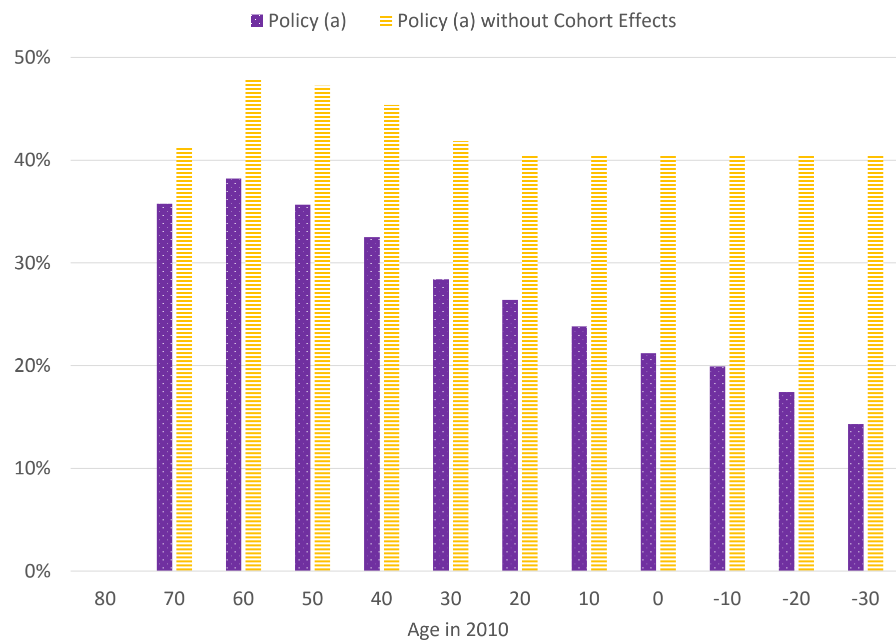


Figure 10: Welfare Cost of Inflation by Cohort (W_h), in Percent of Own Annual Consumption in 2010 Values

Table 12: Welfare Cost in Aggregate (W) and by Class (W_j) without Cohort Effects, in Percent of 2010 Annual Consumption of the Respective Group

Policy	Aggregate	Poor	Poor-Middle	Middle	Middle-Rich	Rich
(a) No Cohort Effect	31.92	66.88	45.69	37.36	30.44	21.67
(a) Baseline	21.78	44.86	31.21	25.72	20.97	14.70

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Appendix

A Description of Data

A.1 Canadian Financial Monitor

The Canadian Financial Monitor (CFM) by Ipsos Reid Canada collects detailed information on household balance sheets (including those of bank accounts), income and consumption. The survey has a sample size of approximately 12,000 households per year responding to a mail-in questionnaire. The survey started in 1999. This section describes the data set, defines relevant variables, provides summary statistics and discusses its comparison to macroeconomic data.

Statistics on individual households' money and consumption in our paper are obtained from the CFM. We define "money" to consist of the sum of cash (amount in wallets and for emergencies), current balances in chequing accounts and chequing/saving accounts as well as the balance in business accounts in all financial institutions.[28] All accounts are in Canadian dollars.

In addition, since 2008, the CFM has collected information on monthly consumption expenditures.[29] We define consumption to consist of the household's sum of gross spending on durable, semi-durable, non-durable goods and services, but exclude the property tax and housing services from owned and rented residential properties. Specifically, we define the four categories of consumption to consist of:

- Durable goods: A new or used automobile/RV/motorcycle/boat/truck; Home appliances and electronics (small or large) (e.g., stove, stereo, TV, DVD player, etc.); Home furnishings (e.g., bed, couch, bedding, textiles, tables, chairs, etc.);

- Semi-durable goods: Clothing/footwear (e.g., pants, socks, shoes, boots, outerwear, hats, gloves, pantyhose, belts, ties, suits, dresses/skirts, etc.);

- Non-durable goods: Groceries, including beverages (e.g., packaged goods, produce, milk, bread, detergent, diapers, etc.); Snacks and beverages from convenience stores (e.g., chocolate, soft drinks, salty snacks, gum, etc.); and

- Services: Hydro bills (e.g., heat, water, electricity); Other utilities (e.g., cable, satellite, phone, internet, cell phone, PDA); Insurance premiums (e.g., life, home, cottage, auto, medical/dental/illness); Domestic and child care services/school (e.g., cleaning, day care, school supplies/trips, tuition, etc.); Food and beverages at/from restaurants/clubs/bars (e.g., visits to fast food restaurants, takeout service, casual/fine dining, coffee/donut shops, etc.); Recreation (e.g., movies, theatres, concerts, sporting events, fitness clubs, etc.); Health services (e.g., drugs, hospital care, vision care, eye glasses, contact lenses, chiropractor, etc.); Automobile maintenance/gas (e.g., service, parts, oil, tires, gas/diesel, etc.); Public and other transportation (e.g., bus, subway, train, plane, parking, etc.); Gifts or donations (e.g., birthday or other celebration/event, charity, etc.); Health and beauty aids/personal grooming (e.g., hair cuts, vitamins/other supplements, salons, lotions/creams, soaps, perfumes, etc.); and Vacation/trip (e.g., auto/air/rail travel, camping, hotels, all-inclusive packages, honeymoon, reunion, etc.).

[28] These are the categories in the CFM.

[29] We annualize monthly consumption into its annual value for our statistics.

Table A1 provides summary statistics of the variables used to construct money and consumption in the paper.[30]

Table A1: Summary Statistics from the CFM over the Period 2009Q3-2012Q2

Definition	Mean	S.D.	25th	50th	75th
Money, all combined	6,112	11,867	552	2,330	6,455
Cash	417	705	58	174	380
Chequing Account	5,325	8,543	742	2,446	6,210
Chequing/Savings Account	7,507	16,067	746	2,486	7,419
Business Account	8,684	13,367	753	3,404	10,608
Consumption, all combined	29,522	22,083	14,964	24,036	37,716
Durable Goods	6,314	10,428	372	1,388	6,322
Semi-Durable Goods	1,940	2,310	730	1,323	2,267
Non-Durable Goods	5,524	4,152	2,790	4,674	6,885
Services	18,309	14,349	9,222	14,935	23,256
Age of Household Head	50	16	38	49	63

A.2 National Household Survey

The National Household Survey (NHS) by Statistics Canada is an accompanying survey to the census. It is a voluntary survey and provides data to support federal, provincial, territorial and local government planning and program delivery. Specifically, it contains information on Aboriginal peoples; education, training and learning; ethnic diversity and immigration; families, households and housing; income, pensions, spending and wealth; labour; languages; population and demography and society and community. Table A2 displays the number of households in different age groups used in Section 2.[31]

[30] Some notes regarding the calculations of the table are warranted. First, all variables are deflated to their 2010 value by Core CPI from Statistics Canada. The Core CPI series (V41690926) can be found at http://www.statcan.gc.ca/pub/62-001-x/2015008/t075-eng.htm. Second, the numbers are based on the three-year period from 2009Q3 to 2012Q2. Given the long-term nature of our question in the paper, a three-year period is chosen to smooth out business-cycle-related fluctuations. In addition, the CFM began collecting "cash" information starting only in 2009Q3. Finally, statistics are conditional on households reporting a positive number in the survey.

[31] There is also an age category below age 25 with 458,915. We do not consider this group in this study. For more information about the census and NHS, follow http://www23.statcan.gc.ca/imdb/p2SV.pl?Function=getSurvey&SDDS=5178.

Table A2: Number of Households by Age Group, 2011 National Household Survey

25-34	35-44	45-54	55-64	65-74	75+	Total
1,912,825	2,388,765	3,023,355	2,560,680	1,620,080	1,354,635	12,860,340

B Characterization of the Solution to the Household Problem

The Lagrangian for the household problem (2) is given by:

$$\mathcal{L} = \sum_{i=1}^{I} \beta_{ij} u(c_{hij}) + \psi_{hij}\left[m_{hij} - c_{hij}(1-s_{hij})\right]$$
$$+\mu_{hij}\left[(1+r_t(1-\tau_t))a_{hij} + m_{hij} + (1-\tau_t)w_t z_{ij} - c_{hij} + q_t\xi(s_{hij}) + a_{h,i+1,j} + (1+\pi_{t+1})m_{h,i+1,j}\right].$$

The first-order necessary conditions (FOCs), respectively, for c_{hij}, s_{hij}, $m_{h,i+1,j}$ and $a_{h,i+1,j}$, are

$$c_{hij}: \quad u'(c_{hij}) - \psi_{hij}(1-s_{hij}) - \mu_{hij} = 0, \tag{B1}$$
$$s_{hij}: \quad \psi_{hij}c_{hij} - \mu_{hij}q_t\gamma_i\eta^h\left(\frac{s_{hij}}{1-s_{hij}}\right)^{\theta_i} = 0, \tag{B2}$$
$$m_{h,i+1,j}: \quad -\beta_{ij}\mu_{hij}(1+\pi_{t+1}) + \beta_{i+1,j}(\psi_{h,i+1,j}+\mu_{h,i+1,j}) = 0 \quad \textit{and} \tag{B3}$$
$$a_{h,i+1,j}: \quad -\beta_{ij}\mu_{hij} + \beta_{i+1,j}\mu_{h,i+1,j}(1+r_{t+1}(1-\tau_{t+1})) = 0. \tag{B4}$$

Focusing on the interior solutions where $\psi_{hij} > 0$ and $\mu_{hij} > 0$, FOCs (B1) and (B2) give us

$$\psi_{hij} = \frac{u'(c_{hij})q_t\gamma_i\eta^h(s_{hij}/(1-s_{hij}))_i^\theta}{c_{hij}+(1-s_{hij})q_t\gamma_i\eta^h(s_{hij}/(1-s_{hij}))^{\theta_i}} \quad \textit{and} \tag{B5}$$
$$\mu_{hij} = u'(c_{hij}) - (1-s_{hij})\frac{u'(c_{hij})q_t\gamma_i\eta^h(s_{hij}/(1-s_{hij}))_i^\theta}{c_{hij}+(1-s_{hij})q_t\gamma_i\eta^h(s_{hij}/(1-s_{hij}))^{\theta_i}}. \tag{B6}$$

In addition, FOCs (B3) and (B4) combine to be

$$\psi_{hij} + \widetilde{R}_t\mu_{hij} = 0.$$

Substituting out ψ_{hij} and μ_{hij} from this equation using (B5) and (B6), we have

$$\frac{c_{hij}+(1-s_{hij})q_t\gamma_i\eta^h(s_{hij}/(1-s_{hij}))^{\theta_i}}{q_t\gamma_i\eta^h(s_{hij}/(1-s_{hij}))_i^\theta} = 1 - s_{hij} - \frac{1}{1-\widetilde{R}_t}.$$

This expression, combined with (B4) and (B6), gives us the characterization of optimal consumption dynamics as

$$\frac{u'(c_{hij})}{1+(1-s_{hij})\widetilde{R}_t} = \frac{\beta_{i+1,j}}{\beta_{ij}}[1+r_{t+1}(1-\tau_{t+1})]\frac{u'(c_{h,i+1,j})}{1+(1-s_{h,i+1,j})\widetilde{R}_{t+1}}.$$

Furthermore, in order to derive the optimal money-consumption ratio, we combine (B2) and the cash-in-advance constraint, Condition (3), to obtain

$$\frac{m_{hij}}{c_{hij}} = \frac{1}{1 + \left(\frac{\psi_{hij} c_{hij}}{\mu_{hij} q_t \gamma_i \eta^h}\right)^{1/\theta_i}}.$$

Substituting out ψ_{hij}/μ_{hij} from this expression using (B3) and (B4), we derive Equation (8) as

$$\frac{m_{hij}}{c_{hij}} = \frac{1}{1 + \left[\widetilde{R}_t c_{hij}/(q_t \gamma_i \eta^h)\right]^{1/\theta_i}}.$$

C Proof of Proposition 1

This section provides the derivations of the partial derivatives and their signs stated in Proposition 1.

Proof. We have the equality, $\frac{m_{hij}}{c_{hij}} = \frac{1}{1+\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i}}$, from Equation (8).

For (i), taking a derivative of this equation with respect to γ_i, we have

$$\begin{aligned}
\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial\gamma_i &= \left(1 + \left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i}\right)^{-2} \frac{1}{\theta_i}\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i - 1} \widetilde{R}_t c_{hij}/(q_t\eta^h)^{-1}\cdot\gamma_i^{-2} \\
&> 0,
\end{aligned}$$

where $c_{hij} > 0$ by household optimization and $\widetilde{R}_t > 0$ is the exogenous range in consideration. In addition, $q_t = w_t$ and $w_t > 0$ from firm optimization. Hence, all terms on the right-hand side are positive.

Regarding (ii), we have

$$\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial\theta_i = \frac{\theta_i^{-2}\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i}}{\left(1+\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i}\right)^2}\cdot \ln\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h} \quad \begin{aligned} &> 0 \;\; \textit{if} \;\; \widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h) > 1 \;\; \textit{or} \\ &\le 0 \;\; \textit{if} \;\; \widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h) \le 1. \end{aligned}$$

$\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h) > 1$ implies $\frac{m_{hij}}{c_{hij}} < \frac{1}{2}$ and $\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h) \le 1$ implies $\frac{m_{hij}}{c_{hij}} \ge \frac{1}{2}$ from Equation (8).

For (iii), we have similarly

$$\begin{aligned}
\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial\eta &= \left(1 + \left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i}\right)^{-2} \frac{1}{\theta_i}\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i - 1} \widetilde{R}_t c_{hij}(q_t\gamma_i)^{-1}\cdot h\cdot\eta^{-h-1} \\
&> 0.
\end{aligned}$$

For (iv), without loss of generality, we assume h to be a continuous variable for this proof.

Then, we have

$$\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial h = \left(1+\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i}\right)^{-2}\frac{1}{\theta_i}\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i-1}\widetilde{R}_t c_{hij}(q_t\gamma_i)^{-1}\cdot\eta^{-h}\cdot\ln\eta$$
$$< 0 \quad as \quad \ln\eta<0 \quad for \quad \eta\in(0,1).$$

Finally for (v), we have

$$\partial\left(\frac{m_{hij}}{c_{hij}}\right)/\partial c_{hij} = -\left(1+\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i}\right)^{-2}\frac{1}{\theta_i}\left[\widetilde{R}_t c_{hij}/(q_t\gamma_i\eta^h)\right]^{1/\theta_i-1}\widetilde{R}_t(q_t\gamma_i\eta^h)^{-1}$$
$$< 0.$$

□

D Derivation of the Nominal Interest Rate Elasticity of the Money Demand Equation

There are a few steps in deriving the analytical expression for the elasticity, $\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}$. Given that

$$\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}} = \frac{\partial(m_{hij}/c_{hij})}{\partial\widetilde{R}_t}\frac{\partial\widetilde{R}_t}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}, \tag{D7}$$

we first derive the left term of the right-hand side, $\frac{\partial(m_{hij}/c_{hij})}{\partial\widetilde{R}_t}$, the partial derivative of the individual money-consumption ratio with respect to the after-tax nominal interest rate. Let us define $\Phi_{hijt}\equiv\frac{c_{hij}}{q_t\gamma_i\eta^h}$. From Equation (8), we have $\frac{m_{hij}}{c_{hij}}=\frac{1}{1+\left(\widetilde{R}_t\Phi_{hijt}\right)^{1/\theta_i}}$. Taking a partial derivative of this expression with respect to $\widetilde{R}_t$ gives us

$$\frac{\partial(m_{hij}/c_{hij})}{\partial\widetilde{R}_t} = -\frac{1}{\theta_i}\Phi_{hijt}^{1/\theta}\cdot\left[1+\left(\widetilde{R}_t\Phi_{hijt}\right)^{1/\theta}\right]^{-2}\widetilde{R}_t^{1/\theta-1}.$$

Next, from the definition of the after-tax nominal interest rate, $\widetilde{R}_t\equiv(1+\pi_t)[1+r_t(1-\tau_t)]-1$, and by substituting the inflation term, $1+\pi_t$, out using the Fisher equation, $1+R_t=(1+\pi_t)(1+r_t)$, we can calculate the middle term of the right-hand side of Equation (D7) to be

$$\frac{\partial\widetilde{R}_t}{\partial R_t}=\frac{1}{1+r_t}[1+r_t(1-\tau_t)].$$

Using Equation (8) for the last term of Equation (D7) and combining all three terms with $\Phi_{hijt}\equiv\frac{c_{hij}}{q_t\gamma_i\eta^h}$, we have

$$\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}} = -\frac{1}{\theta_i}\cdot\left(\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}\right)^{1/\theta_i}\cdot\left[1+\left(\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}\right)^{1/\theta_i}\right]^{-1}\cdot\frac{1+\widetilde{R}_t}{\widetilde{R}_t}\cdot\frac{R_t}{1+R_t}.$$

E Proof of Proposition 2

Proof. Let us first define the following term, which is the only term, containing γ_i, θ_i, η, h and c_{hij} from Equation (9):

$$\Xi_{hijt} \equiv \left(\frac{\widetilde{R}_t c_{hij}}{q_t \gamma_i \eta^h}\right)^{1/\theta_i}.$$

Note that $\Xi_{hijt} > 0$ given the assumptions on the parameters. Then, Equation (9) becomes

$$\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}} = -\frac{1}{\theta_i}\cdot \Xi_{hijt}\cdot(1+\Xi_{hijt})^{-1}\cdot\frac{1+\widetilde{R}_t}{\widetilde{R}_t}\cdot\frac{R_t}{1+R_t}.$$

For statements (i), (iii), (iv) and (v) of the proposition, we do this in two steps. First, we derive the expression for $\partial\left[\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right]/\partial\Xi_{hijt}$ as follows:

$$\begin{aligned}
\partial\left[\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right]/\partial\Xi_{hijt} &= -\frac{1}{\theta_i}\cdot\frac{1+\widetilde{R}_t}{\widetilde{R}_t}\cdot\frac{R_t}{1+R_t}\cdot\left[(1+\Xi_{hijt})^{-1} - \Xi_{hijt}\,(1+\Xi_{hijt})^{-2}\right] \\
&= -\frac{1}{\theta_i}\cdot\frac{1+\widetilde{R}_t}{\widetilde{R}_t}\cdot\frac{R_t}{1+R_t}\cdot(1+\Xi_{hijt})^{-2} \\
&< 0.
\end{aligned}$$

Next, we derive the expressions for $\partial\Xi_{hijt}/\partial\gamma_i$, $\partial\Xi_{hijt}/\partial\eta$, $\partial\Xi_{hijt}/\partial\theta_i$, $\partial\Xi_{hijt}/\partial h$ and $\partial\Xi_{hijt}/\partial c_{hij}$, respectively, as

$$\begin{aligned}
\partial\Xi_{hijt}/\partial\gamma_i &= -\frac{1}{\theta_i}\cdot\left(\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}\right)^{1/\theta_i-1}\cdot\frac{\widetilde{R}_t c_{hij}}{q_t\eta^h}\cdot\gamma_i^{-2} = -\frac{1}{\gamma_i\theta_i}\cdot\Xi_{hijt} < 0; \\
\partial\Xi_{hijt}/\partial\eta &= -\frac{1}{\theta_i}\cdot\left(\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}\right)^{1/\theta_i-1}\cdot\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i}\cdot h\cdot\eta^{-h-1} = -\frac{h}{\eta\theta_i}\cdot\Xi_{hijt} < 0; \\
\partial\Xi_{hijt}/\partial h &= -\frac{1}{\theta_i}\cdot\left(\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}\right)^{1/\theta_i-1}\cdot\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i}\cdot\eta^{-h}\cdot\ln\eta = -\frac{1}{\theta_i}\cdot\ln\eta\cdot\Xi_{hijt} > 0;\ \ \textit{and} \\
\partial\Xi_{hijt}/\partial c_{hij} &= \frac{1}{\theta_i}\cdot\left(\frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}\right)^{1/\theta_i-1}\cdot\frac{\widetilde{R}_t}{q_t\gamma_i\eta^h} = \frac{1}{\theta_i c_{hij}}\cdot\Xi_{hijt} > 0.
\end{aligned}$$

Note that $\ln\eta < 0$ for $\eta\in(0,1)$.

Combining the results from the two steps above, we have, respectively,

$$\partial\left[\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right]/\partial\gamma_i > 0;$$
$$\partial\left[\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right]/\partial\eta > 0;$$
$$\partial\left[\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right]/\partial h < 0; \quad and$$
$$\partial\left[\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right]/\partial c_{hij} < 0.$$

For statement (ii), let us define $\chi_{hij} \equiv \frac{\widetilde{R}_t c_{hij}}{q_t\gamma_i\eta^h}$. Then, we have

$$\begin{aligned}\partial\left[\frac{\partial(m_{hij}/c_{hij})}{\partial R_t}\frac{R_t}{m_{hij}/c_{hij}}\right]/\partial\theta_i &= \frac{1+\widetilde{R}_t}{\widetilde{R}_t}\frac{R_t}{1+R_t}\theta_i^{-3}\left(1+\chi_{hij}^{-1/\theta_i}\right)^{-2}\left[\theta_i\left(1+\chi_{hij}^{-1/\theta_i}\right)+\ln\chi_{hij}\right] \\ &> 0 \quad if \quad \theta_i\left(1+\chi_{hij}^{-1/\theta_i}\right)+\ln\chi_{hij} > 0.\end{aligned}$$

Then, a sufficient condition of this inequality is given by $\chi_{hij} > 1$, implying that $\frac{m_{hij}}{c_{hij}} \in (0, \frac{1}{2})$ from Equation 8. □

F Derivation of Table 4

Table 4 contains the share of aggregate bond holdings by household type. We derive them using bond holding data from the CFM and the number of households by type from the National Household Survey as follows. We use the latter data set to aggregate bond holdings to the household sector level.[32] We first obtain the average bond holding by household type from the CFM over the 2009Q3-2012Q2 period as shown in Table F3. The table depicts the increasing patterns of bond holding with class and age.

Table F3: Average Bond Holding by Household, CFM 2009Q3-2012Q2

Class	30	40	50	60	70	80
Poor	133	385	802	1,107	1,061	2,389
Poor-Middle	231	688	2,413	1,771	2,122	2,766
Middle	489	579	1,735	1,865	3,438	2,112
Middle-Rich	858	1,420	2,503	3,587	4,389	4,064
Rich	1,699	1,990	4,251	6,472	5,232	6,800

These average bond holdings are multiplied by the number of households in each group from Table A2 to obtain the aggregated bond holding for each household group. Table F4 summarizes

[32] We do this because the CFM provides a relative weight for each individual sample but does not give the weights for aggregation.

the resulting numbers in the unit of millions of dollars. The sum of all the numbers from the table represents the aggregate bond holding of the household sector based on the CFM bond data and the number of households from the NHS. They aggregate to $28.5 billion.[33] The final numbers in Table 4 are calculated as a ratio of the number in this table to the aggregate bond holdings by all households, $28.5 billion.

Table F4: Aggregate Bond Holding by Household Group ($Mil), 2009Q3-2012Q2 CFM and 2011 NHS

Class	30	40	50	60	70	80
Poor	51	184	485	567	344	647
Poor-Middle	88	329	1,459	907	688	749
Middle	187	276	1,049	955	1,114	572
Middle-Rich	328	678	1,513	1,837	1,422	1,101
Rich	650	951	2,570	3,315	1,695	1,842

G Nominal Interest Rate Elasticity of Money Demand by Age and Class

Table G5: Nominal Interest Rate Elasticity of Money Demand by Age and Class

	1940	1950	1960	1970	1980	1990	2000	2010	Average
Age									
30	0.504	0.543	0.538	0.516	0.526	0.552	0.549	0.549	0.535
40	0.566	0.610	0.604	0.580	0.591	0.620	0.617	0.617	0.601
50	0.553	0.597	0.592	0.568	0.579	0.608	0.604	0.605	0.588
60	0.575	0.622	0.617	0.593	0.604	0.634	0.630	0.630	0.613
70	0.634	0.686	0.682	0.656	0.669	0.701	0.696	0.697	0.678
80	0.704	0.764	0.761	0.733	0.749	0.785	0.778	0.779	0.757
Class									
Poor	0.569	0.619	0.619	0.597	0.611	0.638	0.630	0.631	0.614
Poor-Middle	0.577	0.625	0.623	0.599	0.612	0.640	0.633	0.633	0.618
Middle	0.581	0.629	0.625	0.602	0.614	0.642	0.635	0.635	0.620
Middle-Rich	0.585	0.633	0.629	0.604	0.616	0.645	0.638	0.638	0.623
Rich	0.591	0.638	0.632	0.607	0.618	0.648	0.642	0.641	0.627

[33] The aggregate bond value of the household sector from this calculation matches closely to $28 billion obtained from the National Balance Sheet Account, found in CANSIM 378-0121 at the Statistics Canada.

H Details of the Calculations of Welfare Costs of Inflation in Lucas (2000) and Erosa and Ventura (2002)

First, Lucas (2000) obtains the welfare cost of inflation as the area under the inverse demand function from the change in nominal interest rate. He parameterizes the demand function to match the historical observations of the M1-to-income ratio and the interest rate. For our purpose of comparing welfare results, we use a constant interest rate elasticity of money demand of -0.623, the average from our study, instead of the -0.5 that he used. Using the rest of the parameter values as in his paper, we first obtain the welfare cost of moving from 2.02% to 5% inflation to be about 0.35% of income per period.[34] Next, based on this number, we need to derive the long-term welfare impact in terms of its present value as a fraction of annual aggregate consumption. For the purpose of comparison to our result, we assume that this effect persists annually for 120 years and derive their present value of welfare impacts. We do this because the last cohort considered in our welfare calculation dies in 120 years. We discount future values using our implied 2010 annual real interest rate of 2.41% and re-normalize the result to be a fraction of annual aggregate consumption instead of income by dividing it by 0.56, the Canadian household consumption-to-GDP ratio in 2010.[35] As a result, we obtain the welfare cost of 25%.

Moreover, we obtain the per-period aggregate welfare cost of changing inflation by 2.98 percentage points to be 0.26% of income from Erosa and Ventura (2002). We derive this number from Table 7 in their paper.[36] There are two exogenous types of households: high productivity and low productivity. They provide the welfare cost by type. Similarly to our results, the aggregate welfare cost depends on how it weighs these two types. Specifically, we derive their aggregate welfare cost of moving from 0% to 10% to be 0.80% with type-specific consumption as the weight. By taking a fraction, $\frac{2.98\%}{10\%}$, of this number, we obtain 0.24% of income per period when inflation changes by 2.98%. Following the same normalization of this number to aggregate Canadian consumption, we obtain 17% from the Erosa and Ventura (2002) study.

[34] This is visually observable from Figure 6 in his paper, where the interest rate of 3% is assumed to be with zero inflation. The number, 0.35%, is obtained by using the welfare cost function for the log-log demand function on page 251 between the interest rates of 5.02% (the real interest of 3% plus 2.02% inflation from our baseline calibration) and 8% (an increase in inflation by 2.98 percentage points).

[35] In the U.S. National Income and Product Accounts, the ratio of "Personal Consumption Expenditure" to GDP in 2010 is 0.68, whereas it is 0.56 in Canada. The main discrepancy is the fact that Americans pay much more out-of-pocket for health care services and goods than Canadians. Since our calibration is based on the Canadian consumption data, we use the Canadian consumption-to-GDP ratio for the purpose of this section. Canadian data are based on CANSIM Table 380-0064 from Statistics Canada.

[36] The table contains welfare results without idiosyncratic income risk and with transition dynamics, the closest comparison to our simulation environment. Hence, although their baseline model is with idiosyncratic income risk and without transition dynamics, we compare our result to that found in Table 7 of their paper.