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Canadian Participation in the Second International Mathematics Study

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Canadian Participation in the Second International Mathematics Study

Contents

Foreword

To meet increasing global competition, Canada's quest for productivity improvement must focus on technological advance and the skill needs of the knowledge economy. Education and training will thus be a cornerstone of any long-run strategy for economic development. Accordingly, the Economic Council of Canada has decided to undertake a study of this complex and controversial field which will be completed in late 1991. The broad framework of analysis will consider a variety of topics concerned with quality and the efficiency of resource allocation in the education and training system.

One specific question that has attracted considerable public attention has to do with the performance of Canadian students relative to their counterparts in other countries. The present working paper by David Robitaille of the University of British Columbia, examines attainment in mathematics, and a companion working paper by Robert Crocker of Memorial University, entitled "Science achievement in Canadian schools: National and international comparisons," does the same for achievement in science subjects. The authors interpret the international comparisons from a Canadian standpoint, extract the main conclusions, and indicate the relevant policy implications.

Judith Maxwell
Chairman

Introduction

For the past several years, a great deal of interest has been shown in the results of comparative analyses of the outputs of educational systems. For example, the September 18, 1989 issue of *Time* included an article discussing the results of an international, comparative study of 13-year olds' achievement in science and mathematics. As is frequently the case with such articles, the authors described the current situation in the United States vis-à-vis the teaching of science and mathematics in highly charged language, using terms such as "crisis," "antiquated curricula," and threats to "American supremacy."

International comparisons of the outcomes of schooling are a fairly recent development in the field of educational research, and such studies have been criticized by some. Critics have charged that such research is fundamentally flawed because it seeks, in the words of Torsten Husén, the noted Swedish educational researcher, "to compare the incomparable." The outcomes of schooling are many faceted and the result of the interaction of a host of factors, most of which are beyond the control of the school, are not readily amenable to codification and quantification as required in current research paradigms.

Proponents of international comparisons of educational outcomes would assert that the benefits to be derived from such analyses are significant, and that the information to be obtained from them is important enough to justify their being conducted. International comparisons of educational programs, practices, and outcomes assist each participating educational system to view itself in a wider context, and to see what alternative arrangements are possible. While carefully controlled experiments to compare the efficacy of different curricula, teaching practices, or administrative arrangements for education may be prohibitively expensive, time consuming, or have unpredictable effects on the educational careers of the students involved, comparative research offers a means of obtaining the needed information in a straightforward manner. Such studies use the world as an educational laboratory with built-in variation between and among systems on virtually every variable of educational significance.

The first international study of the outcomes of schooling was carried out in the mid-1960s, by a then newly established consortium of educational research centres known as the International Association for the Evaluation of Educational Achievement (now simply, IEA). In the ensuing almost 35 years, IEA has conducted a number of major surveys of educational practices and outcomes. In addition to the first mathematics survey [Husén, 1967], these have included the Six-Subject Survey [Peaker, 1975], the Second International Mathematics Study [Robitaille and Garden, 1989], the Written Composition

Study [Gorman, et al., 1988], the Classroom Environment Study [Ryan and Anderson, 1984], and the Second International Science Study [IEA, 1988].

At its 1989 meeting in August, the General Assembly of IEA approved a proposal to conduct a third study of mathematics during the 1993-94 school year and about 30 member institutions have indicated interest in participating in that project. The International Coordinating Centre for the third mathematics study is to be located in the Faculty of Education at the University of British Columbia, under the direction of the author.

Rationale for an International Study of Mathematics

Educational systems throughout the world place a great deal of importance on the teaching and learning of mathematics, and they annually allocate significant levels of resources to maintaining and improving their efficiency and effectiveness. Mathematics is seen as contributing to the overall intellectual development of students, as preparing them to function as well-informed citizens in contemporary society, and as providing students with the background and skills needed for success in the fields of commerce, industry, technology, and science.

Throughout the world, the study of mathematics occupies a central place in the school curriculum. It is estimated that, in most countries, between 12 and 15 per cent of class time is devoted to the study of mathematics. The only other subjects allocated as much time are those associated with the mother tongue and reading. The report of the Cockroft Committee [1982], *Mathematics Counts*, in the United Kingdom says the following about the special role of mathematics in the curriculum:

> An important reason for teaching mathematics must be its importance and usefulness in so many other fields. It is fundamental to the study of the physical sciences and of engineering of all kinds. It is increasingly used in medicine and in the biological sciences, in geography and economics, in business and management studies. It is essential to the operations of industry and commerce both in the office and workshop.

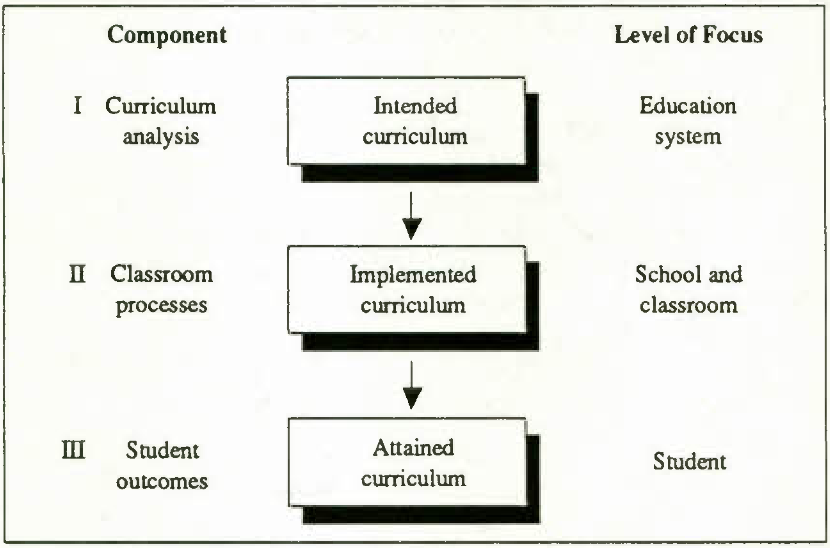
Beyond these practical considerations, it is generally believed that mathematics provides an exemplar of precise, abstract, and elegant thought. And, whereas the generalized effects of mathematical studies on a student's overall intellectual development are difficult to analyze, let alone measure, there is a universally held belief that the study of mathematics helps to broaden and hone intellectual capabilities.

Conceptualization of the Study

The Second International Mathematics Study had three major foci: the mathematics curriculum, the ways in which mathematics was taught, and the eventual outcomes of the instructional process as demonstrated by the achievement and attitudes of students. The study was therefore conceptualized as an examination of the mathematics curriculum at three levels: the *intended curriculum* as laid down by national or system level authorities; the *implemented curriculum* as interpreted and operationalized by teachers for their own students; and the *attained curriculum* which represents that part of the intended curriculum actually learned by students as manifested in their attitudes and achievement. This conceptualization is summarized in Figure 1.

Figure 1

Framework for the Second International Mathematics Study



Participation in the Study

IEA is a consortium of research centres which have formed an association for the purpose of carrying out comparative studies of education, usually surveys of educational outcomes and teaching practices. IEA currently has about 50 members, including institutions from a wide variety of industrialized countries, two socialist countries, and several developing countries. Each institution is free to participate in whichever IEA projects it chooses, and this has the effect of limiting the representativeness of the participants. Although institutions from over 20 countries participated in the Second International

Mathematics Study, they cannot be considered as either a representative or a random sample of educational systems throughout the world.

Canadian participation in IEA is complicated by the fact that there is no national institution with a mandate to carry out research in education at the national level. Other countries in which governance of education is situated at the state or provincial level – for example, Australia and the United States – have established institutions or ad hoc arrangements for facilitating participation in such projects on a national basis. The Council of Ministers of Education of Canada could conceivably play such a role if it desired to do so, but this has not happened thus far.

At the present time, institutions in the provinces of British Columbia, Ontario, and Quebec hold memberships in IEA. In the case of British Columbia, the membership is held by the B.C. Ministry of Education. In the other two provinces, the memberships reside in one of the provincial universities: the Ontario Institute for Studies in Education is the member institution for Ontario, and the Faculté des sciences de l'éducation at the Université du Québec à Montréal, for Quebec.

British Columbia and Ontario participated in the mathematics study. Quebec did not, since it did not belong to the organization at the time. It is hoped it will be possible to organize Canada-wide participation in the third IEA mathematics study in such a way that each province will be able to produce a report of the performance of its own students, but Canada-wide results will be available for the international reports of the project.

The following is a list of the jurisdictions which were involved in some or all aspects of the Second International Mathematics Study:

Australia	Israel
Belgium (Flemish)	Japan
Belgium (French)	Luxembourg
Canada (British Columbia)	Netherlands
Canada (Ontario)	New Zealand
Chile	Nigeria
England and Wales	Scotland
Finland	Swaziland
France	Sweden
Hong Kong	Thailand
Hungary	United States
Ireland	

The list of participants includes a number of countries of particular interest to Canadians because of the similarity of their educational systems or cultural traditions to ours, their importance to Canada politically or economically, or their reputation for educational excellence. In order to make the analyses and interpretation of the results more comprehensible and, it is hoped, more useful, the discussion in this paper has been limited to a subset of the participants who have been deemed to fit one or more of those criteria. That subset includes, in addition to the two "Canadas" (British Columbia and Ontario), the following: England and Wales, France, Japan, New Zealand, Sweden, and the United States.

Methodology

This section includes brief descriptions of the sampling techniques which were employed in constructing the national samples for the study, as well as of the instruments which were developed for use in the study. More detail is available in the international reports.

Sampling

Two populations of students were identified as subjects for the study, and they were identified as Population A and B. The international definitions of the two populations were as follows:

Population A: All students in the grade (year level) where the majority of students have attained the age of 13.00 to 13.11 years by the middle of the school year.

Population B: All students who are in the normally accepted terminal grade of the secondary education system, and who are studying mathematics as a substantial part of their academic program.

These populations were selected to correspond to important break points in most of the participating educational systems. At age 13, in almost all of the participating countries, virtually the entire age cohort of students is still in school and studying mathematics. Some streaming or tracking of students into different programs on the basis of their academic record, future plans, or some estimate of their academic ability may have occurred, but that kind of differentiation is not practiced universally at this level. For example, all students at the Population A level in France and Japan follow essentially the same program. Quite the reverse is true in the United States where Population A students are sorted into four quite distinct mathematics programs: algebra, enriched, regular, and remedial.

Sampling procedures were designed with two main purposes in mind. First, samples had to be selected so as to allow population parameters to be estimated to a reasonable degree of precision. Second, samples had to be chosen in such a way as to enable cross-national comparison of those population parameters to be made, given the appropriate background information. Consistent with IEA practice in all of its surveys, an International Sampling Referee was appointed to oversee the sampling process and to ensure that the desired degree of comparability was achieved. Details regarding the sampling strategy employed in each participating jurisdiction are given in Garden [1987].

Survey Methodology and Instrumentation

The Second International Mathematics Study, like other IEA projects, employed standard survey techniques for data collection. All of the questionnaires, tests, opinionaires, and attitude scales were developed by specialists from the participating countries especially for use in the study. The data were collected during the 1981-82 school year, except in British Columbia and Japan where data collection occurred during the previous year.

A number of participating jurisdictions wished to use the study as an opportunity to examine the nature and importance of linkages between the teaching practices employed by teachers and the attitudes and achievement of their students. This interest was not shared equally in all jurisdictions and led to the partitioning of the project into two sub-studies: the cross-sectional study and the longitudinal study. In the longitudinal study, students were administered tests and questionnaires at both the beginning and the end of the school year. In addition, the teachers of those students were asked to complete a number of highly detailed questionnaires during the school year concerning the teaching practices they employed in their teaching of a number of specific topics. Students and teachers in the cross-sectional study completed tests and questionnaires only once, at the end of the school year.

Data sources for the study included the following:

1. Case-study questionnaires completed in each National Centre to provide system-level background information. These included ratings of the suitability or appropriateness of each test item utilized in the study for students in that system.

2. Questionnaires completed by school officials concerning characteristics of their schools, teachers, the mathematics curriculum offered in their schools, as well as school and departmental policies which might affect the teaching of mathematics.

3. Questionnaires completed by teachers to provide background information about their experience, training, qualifications, beliefs, and attitudes. In

the longitudinal study, there were additional questionnaires to provide information about teaching practices, beliefs about effective teaching, and the specific techniques which teachers used in the teaching of a number of specific topics.

4. Judgments by teachers whether or not the mathematical content needed to respond to each item on the achievement tests had been taught to their students during that year, in a previous year, or not at all. This is referred to, in IEA parlance, as *Opportunity to Learn* data.

5. Questionnaires to be completed by students to obtain information about variables such as parents' educational backgrounds and occupations, time spent on homework, and their attitudes and beliefs about mathematics.

6. Achievement tests taken by students at the end of the school year. In the longitudinal study, these tests were administered as pretests at the beginning of the school year as well.

Findings at the School and System Level

The Second International Mathematics Study produced an enormous amount of data and a great many results. In this section of the paper, some of the more important of those results are discussed. When considering the implications of the findings, it is important to bear in mind that the data were collected several years ago and that, in each of the participating jurisdictions, changes which might well affect some of the findings may already have been introduced. For example, since the study was carried out, British Columbia has acted upon the findings of the recent Royal Commission of Inquiry into Education [Sullivan, 1988] and completely revamped the structure and content of the K-12 curriculum along the lines described in the research report on the curriculum prepared for that Royal Commission [Robitaille, et al., 1988].

Population A

Some descriptive statistics about Population A are summarized in Table 1. While these results show more similarities among these Population A systems than differences, some of those differences are rather interesting. Japanese students attend school six days per week for a total of over 240 days, and this is significantly greater than the total in any other participating jurisdiction. Student/teacher ratios in Japan and Ontario are higher than in other places, frequently by more than 25 per cent. In the case of Ontario, this is related to

Table 1

Structure of Population A

	School days per year	Class time for math	Student/ teacher ratio (s.d.)	Class size (s.d.)
		(Per cent)		
Canada (British Columbia)	195	15	16 (3)	27 (5)
Canada (Ontario)	186	15	21 (3)	29 (5)
England and Wales	192	13	17 (3)	25 (8)
France	185	12	16 (4)	24 (3)
Japan	243	13	22 (5)	39 (6)
New Zealand	190	14	19 (2)	28 (4)
Sweden	180	12	12 (2)	20 (6)
United States	180	14	15 (8)	26 (7)

the fact that Population A classes are located in elementary schools. The large standard deviation for the United States on this variable likely stems from the fact that, in that country, there are several different kinds of administrative arrangements.

In some jurisdictions, Grade 8 classes are in junior high schools. In others, they are in middle schools and, in some cases, in elementary schools. In general, student/teacher ratios are lower in secondary schools than in elementary schools because the former typically have specialist teachers such as librarians and counsellors on their staffs, and they are included in the calculation of the student/teacher ratio.

Median class size varies between 25 and 29 students in most places at this grade level. Swedish classes are much smaller than those elsewhere; Japanese classes, much larger. In fact, the median size of a Japanese Population A class is almost twice that of a Swedish one.

In British Columbia and Ontario, as well as in the United States and a number of other places, Population A corresponds to the Grade 8 level. In British Columbia and many other places, Grade 8 is part of the secondary school system. In Ontario, however, Grade 8 is normally the last year of elementary school. One implication of this is that fewer teachers at the Population A level in Ontario are specialists than is the case in systems where Population A is part of the secondary school system. While secondary schools tend to have a departmentalized structure, elementary schools do not, and almost all teachers at that level are expected to teach mathematics.

Some descriptive information about Population A teachers in the selected jurisdictions is summarized in Table 2. The results show that the average age

Table 2

Descriptive Data for Population A Teachers

	Mean age of teachers (s.d.)	Fully qualified	Years of experience (s.d.)	Weekly teaching
		(Per cent)		(Hours)
Canada (British Columbia)	39 (9)	59	14 (9)	21 (3)
Canada (Ontario)	37 (7)	4	15 (8)	21 (7)
England and Wales	38 (10)	62	12 (9)	19 (4)
France	38 (8)	29	14 (8)	18 (3)
Japan	37 (10)	91	17 (11)	17 (3)
New Zealand	36 (10)	56	10 (9)	19 (4)
Sweden	38 (9)	74	12 (9)	16 (3)
United States	39 (10)	63	14 (8)	21 (6)

and number of years of teaching experience of the participating teachers are highly comparable across countries. It is also worth noting that teachers in Canada and the United States teach more hours per week than their colleagues elsewhere. In some cases, the differences amount to an hour per day.

The most dramatic between-system differences are evident in the per cent of the Population A teaching force categorized as "fully qualified" to teach mathematics at that level. Principals or department heads supplied this information in response to a questionnaire item which asked them to indicate what per cent of their teachers of Population A were "fully qualified" to teach mathematics at that level. The expression "fully qualified" was not defined.

At one extreme is Japan where principals report that virtually all of their Population A mathematics teachers are fully qualified. In most of the remaining places, over 40 per cent of Population A mathematics teachers are judged to be less than fully qualified. British Columbia and Ontario report among the highest percentages of less than fully qualified teachers at the Population A level.

In Ontario, virtually none of the teachers are seen as fully qualified; this is not surprising given the fact that Grade 8 is normally part of the elementary school system in that province. Elementary school teachers are generalists who are expected to teach all, or almost all, subjects to their class. Few elementary schools would be likely to have a mathematics specialist as a member of the teaching staff.

In British Columbia, these results indicate that slightly more than 40 per cent of Population A mathematics teachers are less than fully qualified to teach mathematics at that level. Similar findings have been reported in other studies of mathematics at the provincial level over the past 12 years, and concern has been expressed in each case. No remedial action appears to have been undertaken.

Many of the less than fully qualified teachers would be teachers of other subjects – such as science, social studies, or physical education – who have been assigned a class or two of Grade 8 mathematics in order to complete a teaching load. There is a widespread, if unsubstantiated, belief that almost anyone can teach mathematics at this level.

Population B

The structure and composition of Population B are much more complex than those of Population A. In almost every country, students who are still in school at the senior level have been partitioned into a variety of more or less specialized courses and programs. In addition, there are much greater differences in curricula at this level. In most European countries, for example, Population B students take a full year of calculus. This is much less likely to be the case in either Canada or the United States.

The existence of such fundamental differences in the makeup of Population B and in the content of the curriculum, makes any comparison of the outcomes attained by students highly problematic. The Population B study might best be viewed as a series of parallel replications of the study, replications designed to produce a set of descriptions of what can be done at that level. Whether or not any particular alternative is superior to all of the others, depends on a set of educational, political, and economic choices which would have to be made first.

The age and grade level composition of Population B varies considerably from jurisdiction to jurisdiction, as does the proportion of students retained in school and studying "academic" mathematics. In British Columbia, where there is essentially only one academic mathematics course at the Grade 12 level, a large proportion of the grade cohort has been retained in Population B. In England, on the other hand, Population B was defined to include only those students studying mathematics in the second year of the sixth form at either the General Certificate of Education (G.C.E.), the Advanced (A-level), or the Scholarship level.

At the time of data collection for the Second International Mathematics Study, Ontario had two grade levels which might have been used for Population B: Grade 12 or Grade 13. For international purposes, it was decided to use Grade 13 students who were taking two mathematics courses, including calculus. However, a parallel sample of Grade 12 classes was also selected, and this provides data on a set of students who are largely comparable to the Population B samples in the United States and British Columbia.

Some information on the makeup of Population B in the participating systems is given in Table 3. France did not participate in the Population B study, and has therefore not been included in this analysis.

Table 3

Retention Rates: Population B

	Age cohort in school	Grade group	Age group
		(Per cent)	
Canada (British Columbia)	82	38	30
Canada (Ontario 12)	74	41	30
Canada (Ontario 13)	33	55	19
England and Wales	17	35	6
Japan	92	13	12
New Zealand	17	67	11
Sweden	24	50	12
United States	82	15	12

An example will help clarify the meaning of the data in the table. The entry for British Columbia indicates that 82 per cent of the age cohort of eligible persons is still in school at the Grade 12 level. Of that number, 38 per cent are taking sufficient mathematics to be included in the Population B definition for British Columbia. These two pieces of data are combined in the third column to indicate that 30 per cent of the Population B age group in British Columbia is studying Population B mathematics, the same percentage as in Ontario.

Extremely large between-system differences are depicted in this table. By combining high retention rates in school with high participation rates in senior mathematics, Ontario and British Columbia produce comparatively high yield. England and New Zealand have very low retention rates which result, especially in the case of England, in correspondingly low yield.

The complete picture is even more complex than these results may suggest. In particular, the per cent of students studying any mathematics at the Population B grade level in most countries is probably higher than that indicated in Table 3. For example, the participation rates for Grade 13 in Ontario are based only on those students who were taking at least two Grade 13 mathematics courses. Many others were likely taking one Grade 13 course, but that information is not available. In other countries, there might be a variety of mathematics courses available to students, depending upon their career choices. Only those students who met the definition of Population B adopted in their country have been accounted for in these results.

Several interesting results are portrayed in Table 3. In Japan, 92 per cent of the age group is still in school in the last year of the secondary program: that is to say, only 8 per cent of students have left school. In British Columbia and the United States, slightly more than 80 per cent of the age group is still in school, but this means a drop-out rate of almost 20 per cent. The drop-out rate in Ontario is somewhat higher. England and New Zealand represent the opposite extreme, retaining less than 20 per cent of the age group in school until the completion of secondary school.

Table 4 contains additional background data on Population B, similar to that reported earlier for Population A.

Table 4

Structure of Population B

	School days per year	Class time for math	Student/ teacher ratio (s.d.)	Class size (s.d.)
		(Per cent)		
Canada (British Columbia)	195	19	18 (2)	23 (8)
Canada (Ontario)	186	14	18 (2)	25 (7)
England and Wales	192	21	17 (3)	10 (5)
Japan	243	20	19 (6)	40 (9)
New Zealand	190	16	19 (6)	16 (5)
Sweden	180	17	13 (2)	22 (5)
United States	180	14	19 (5)	22 (10)

Once again, large between-system differences are evident. Japan has the largest class sizes, although they also have a fairly large range of different class sizes, as indicated by the large standard deviation. Classes at this level in England and Wales tend to be very small, with a median of only 10 and a standard deviation of 5. Class sizes in the three North American jurisdictions are all very much the same, in the low to mid-twenties.

Table 5 provides information on several variables concerning Population B teachers in the selected jurisdictions. Mathematics teachers at this level are experienced, and somewhat older than their Population A counterparts. The percentages of teachers who were considered to be fully qualified are higher at this level, but those from the two Canadian provinces are among the lowest.

Table 5

Descriptive Data for Population B Teachers

	Mean age of teachers (s.d.)	Fully qualified	Years of experience (s.d.)	Weekly teaching	Male
		(Per cent)		(Hours)	(Per cent)
Canada (British Columbia)	42 (9)	68	17 (8)	21 (4)	96
Canada (Ontario)	41 (9)	81	16 (8)	19 (5)	79
England and Wales	38 (9)	66	14 (9)	19 (4)	65
Japan	41 (10)	97	17 (9)	14 (2)	92
New Zealand	37 (9)	–[a]	13 (8)	20 (3)	76
Sweden	45 (7)	94	18 (8)	14 (3)	80
United States	41 (9)	92	17 (8)	20 (6)	51

[a]Data not available.

Canadian teachers have among the heaviest teaching loads in terms of hours of teaching per week. When this finding is combined with the previously reported one to the effect that Canadian class sizes are also among the largest within this set of jurisdictions, it appears that Canadian mathematics teachers have rather heavy workloads.

Findings at the Classroom or Teacher Level

Several of the instruments developed for use in the study were designed to collect information from teachers about a number of different aspects of what goes on in classrooms where mathematics is being taught. Some of these questionnaires dealt with general topics, such as academic and professional qualifications of teachers, years of teaching experience, and their objectives for the teaching of mathematics. Others were designed to obtain highly detailed information from teachers about how they approached the teaching of specific topics.

All of these instruments were completed by teachers at the Population A level in the eight jurisdictions which participated in the longitudinal phase of the study, including those in the two Canadian provinces and the United States. The corresponding questionnaires at the Population B level were utilized only in the United States and Canada, and few analyses of the data from those instruments has been carried out. Accordingly, in the remainder of this section, attention is focussed on findings at the classroom level for Population A only.

Orientation towards Teaching

More than one third of all participating Population A teachers reported that the majority of class time was taken up with the whole class working together as a group, either listening to the teacher or participating in discussions. Teachers of mathematics also said that they spent little time in small-group instruction.

Sixty per cent of all teachers reported that they teach all students the same content, and at the same pace. In other words, little attention is paid to individual differences between students at this level. Given a choice, teachers said they were more likely to vary the pace at which material was presented rather than to adapt the content itself.

A number of previous studies of the teaching practices employed by teachers of mathematics have concluded that the teaching of mathematics is largely a teacher-directed, "chalk and talk" affair. The findings from the Second International Mathematics Study add further confirmation. Teachers usually agreed with questionnaire items which stressed the importance of the active involvement of students in the learning process and the efficacy of using manipulative devices in the development of concepts and skills; however, they indicated elsewhere that they rarely put these opinions into practice in their day-to-day teaching.

The reasons for this lack of congruence between teachers' opinions about teaching and the actual teaching practices they employ are unclear. It may be that they find themselves so pressed for time to complete the prescribed curriculum and carrying such heavy teaching loads that they decide there is not sufficient time available to do what they believe is best. Or perhaps they are simply unwilling to do so.

Causes of Students' Failure

Teachers were given a list of nine possible causes for lack of satisfactory progress by students, and they were asked to rate the importance of each of these. The nine were as follows:

- misbehaviour by the student;
- lack of ability;
- indifference or lack of motivation;
- a debilitating fear of mathematics;

- absenteeism;
- insufficient time allocated in the timetable of the school for mathematics;
- a lack of proficiency on the part of the teacher;
- limited availability of resources for teaching; and
- too many students to handle.

Teachers rated students' lack of ability and their indifference as the most important causes of unsatisfactory progress. In other words, teachers see the most important causes of failure to succeed in mathematics as being inherent in the students, rather than in the system or in the behavior of the teacher. Japanese teachers were the only ones who indicated that their own professional limitations were likely to be a significant contributing factor in this process. Class size and availability of resources for teaching were not seen as being significant factors.

Streaming

In every school system, some sort of selection process is at work. Almost every subject becomes optional at some level in the secondary school system, and only those students with a particular interest or ability in that subject take those courses. In some cases, students and their parents are permitted to exercise some choice over which courses are taken; in others, students are directed by teachers and counsellors into certain subjects or sections of courses, and effectively barred from entering others. In most Canadian secondary schools, some combination of both of these approaches is common.

In the case of mathematics, a student may have several choices, ranging from some form of remedial mathematics course, to a course designed for students not planning to continue in the study of mathematics or science at the post-secondary level, to pre-university academic mathematics courses. The decision about which "stream" to follow can have a far-reaching effect on the career paths which are left open to students.

The question of whether or not students should be separated into different sections of the same course or into different courses on the basis of some measure of ability is highly contentious. [See, for example, Kulik and Kulik, 1982; Goodlad, 1984; Radwanski, 1987; Robitaille, et al., 1988.] Moreover, the decision about which course a particular student will be directed toward can have important implications for that student's future. In the United States, for example, only a small proportion of students is permitted to study algebra during the Population A year [McKnight, et al., 1987]. The rest, the vast majority of students, do so the following year. If the procedures by which students are directed into different courses are subject to error, and they are, then many students may be unjustly excluded from accelerated or pre-university courses, to the detriment of their future career choices.

In spite of the seriousness of such concerns and the lack of confirmatory research evidence, teachers in Canada and the United States continue to insist on the necessity for this kind of streaming. British Columbia provided an extreme example at the time of the study because over 70 per cent of schools reported that they streamed Population A students into different mathematics courses even though the official curriculum guide expressly stated that all students were expected to follow the same program in mathematics until the end of Grade 10.

Analysis of the achievement results from the Second International Mathematics Study demonstrates that it is not necessary to sort 13-year olds into different mathematics courses in order for them to achieve at high levels. Students in Japan and France did very well on the achievement tests in the Second International Mathematics Study, and yet there is apparently no streaming of students into different classes or programs at this level in those countries. In Japan and France, as well as in Ontario, only a small proportion of the total variance in test scores is attributable to difference between schools or between classes. Almost all the variance is attributable to differences between students. In the United States, on the other hand, more than 50 per cent of the variance is attributable to either school-level or classroom-level differences, rather than to student-level differences. That is to say, these results show that sorting of students into different courses or programs is another example of a self-fulfilling prophecy.

Findings at the Student Level

When results from studies of this kind are discussed in the news media, the focus tends to be on the "horse-race" or "international Olympics" aspects. There is a great deal of popular interest in finding out "Who won?" While such interest is understandable, there is a danger that too much attention will be focussed on that aspect of international studies, and that what is most valuable will be overlooked.

There is a sense in which the achievement scores are the least important of all the results emanating from these studies. We know that students from some countries achieve at higher levels than Canadian students do. However, unless we can take into account the influence of a number of important variables, that information is not very useful. We need to understand that cultural and sociological variables have a very significant effect on the way students learn and respond. We also need to understand that, while students' achievement levels might well be increased by systematically increasing the amount of pressure exerted upon them to excel, there are limits beyond which our society would not be prepared to go. Perhaps most importantly, we need to understand the importance of curriculum as a variable. In other words, what students study in school is an important determinant of what they achieve.

There is also a sense in which the achievement results are of crucial importance. They show us what students can achieve when they are challenged to do so, and they show us that different kinds of curricular arrangements are possible. We can learn what students are capable of learning and obtain information which will be useful in planning changes to our curriculum.

Achievement of Population A Students

The students who participated in the Second International Mathematics Study did not write a mathematics test as that term is usually understood. That is, it is not true that all students were asked to respond to exactly the same set of items. Instead, a large set of items was developed, and a process of matrix sampling was used to assign items to test forms and test forms to students. Five test forms were assembled for Population A, and administration procedures were designed to ensure that there were sufficiently many replications of each item in each participating classroom to permit accurate estimation of item-level and subtest results at the classroom level.

The process of item development was carried out in several stages early in the study. Based on the results of a preliminary survey of the mathematics curriculum in several systems, a content-by-cognitive-behaviour grid was developed. The content dimension was partitioned into five topic strands: arithmetic, algebra, geometry, descriptive statistics, and measurement. The cognitive behaviour dimension was partitioned into four hierarchical categories: computation, comprehension, application, and analysis.

The final item pool consisted of 157 items which were used in all participating countries. These were divided among the five strands as follows: arithmetic, 46 items; algebra, 30 items; geometry, 39 items; descriptive statistics, 18 items; and measurement, 24 items. Caution should be exercised in making judgments about the implications of students' performance in any of these areas without examining the content of the items. That is to say, misleading conclusions might be reached about students' performance in, say, algebra unless it were clear what topics were actually included under that label.

Every effort was made to include as many items as possible to make comparisons of the achievement test results fair and valid. However, given the substantial differences that exist between systems with regard to the content of the mathematics curriculum, it was not possible to construct an absolutely "level playing field." Even in those places where the vast majority of the items were judged to be appropriate, it may well be that important topics in their curriculum were not included.

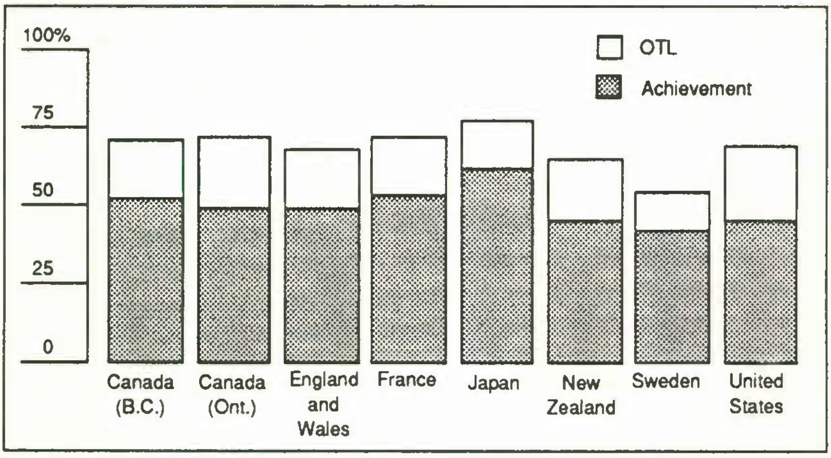
In the discussion that follows, repeated reference is made to *Opportunity to Learn* (OTL). This variable, which is used in all IEA studies, is intended to assist in the interpretation of students' performance by indicating what proportion of participating teachers said that they had either taught or reviewed the material students needed to respond correctly to a given item in the current school year. In the charts that follow, item-level OTL results have been aggregated to the subtest level.

Overall Performance

Chart 1 shows the overall performance of students from the eight selected systems on all 157 items. The mean per cent correct across all participating systems and all items is 47 per cent, which suggests that students found these items rather difficult. Students from some systems performed exceptionally well on certain items, but these were relatively few and far between. Average performance exceeds 70 per cent correct on only 16 items – about 10 per cent of the total – while 13 items resulted in average scores of less than 25 per cent.

Chart 1

Overall Performance: Population A



Item scores range from a high of 84 per cent correct on an item requiring students to estimate the length of a segment in centimetres to a low of 13 per cent correct on an item which required students to identify a vector representing the difference between two given vectors. OTL scores on the latter items are very low everywhere, so the poor performance is not surprising.

Of the 16 easiest items (i.e., the items with the highest mean per cent correct scores), five involved primarily the application of a standard computational algorithm, such as multiplication or division of whole numbers, fractions, and decimals. Three others concerned the use of metric units of measurement; and two, the ability to read information from graphs. In general, the items which students found easiest were those that required them to recognize a definition or rule, or to apply a standard computational technique.

Of the 13 most difficult items (i.e., those with the lowest average per cent correct scores), six were from the geometry subtest, and four of these were concerned with transformational geometry in a fairly formal way. Other difficult items concerned the surface area and volume of a rectangular prism, finding a missing term in a proportion, and the identification and application of the distributive and associative properties.

In general, and with the exception of the transformational geometry items which have very low OTL scores, indicating that the content was not taught, the more difficult items required students to read more text or to solve a problem which involved more than a routine application of a standard technique or algorithm. For example, two apparently similar items concerning the topic of circle graphs resulted in significantly different performance levels. On one, students were asked to read information from a circle graph, and 79 per cent selected the correct answer. On the second item, students were asked to find the size of the central angle required to represent a certain amount on a circle graph. The average score on that item is only 22 per cent correct.

The average performance of Grade 8 students in British Columbia was superior to that of students in Ontario, but not markedly so. Both groups of Canadian students performed better than students from the United States, but whether this would have been the case if all of Canada had been represented in the study is not possible to say.

Performance on Specific Topics

The Population A item pool was partitioned among five subtests or strands, representing major foci of the mathematics curriculum at this level. The five were arithmetic, algebra, geometry, descriptive statistics, and measurement. A summary of students' performance on each of those subtests and the corresponding Opportunity-to-Learn data are presented in Chart 2 on the following page.

Arithmetic — The OTL results show that, on the whole, the items on the arithmetic subtest dealt with content which the teachers had either taught as new material or reviewed. Almost all of the OTL scores are greater than 75 per cent. In both British Columbia and Ontario, the OTL rating for arithmetic is 87 per cent.

The items on this subtest dealt with a variety of arithmetic topics including whole numbers, common and decimal fractions, ratio and proportion, per cent, number theory, exponential notation, and square roots. Students from British Columbia and Ontario performed relatively well on this set of items, obtaining mean scores of 58 and 54 per cent respectively.

Algebra — The algebra subtest consisted of 30 items dealing with topics such as integers, order of operations, simplification of algebraic expressions, evaluation of expressions and formulae, and solution of linear equations and inequalities. Students in most places apparently found these items more difficult, because most of the averages shown in the graph are lower in algebra than they are in arithmetic. Algebra is not typically an important part of the Grade 8 mathematics curriculum in North America, and the lower scores are undoubtedly a reflection of this fact.

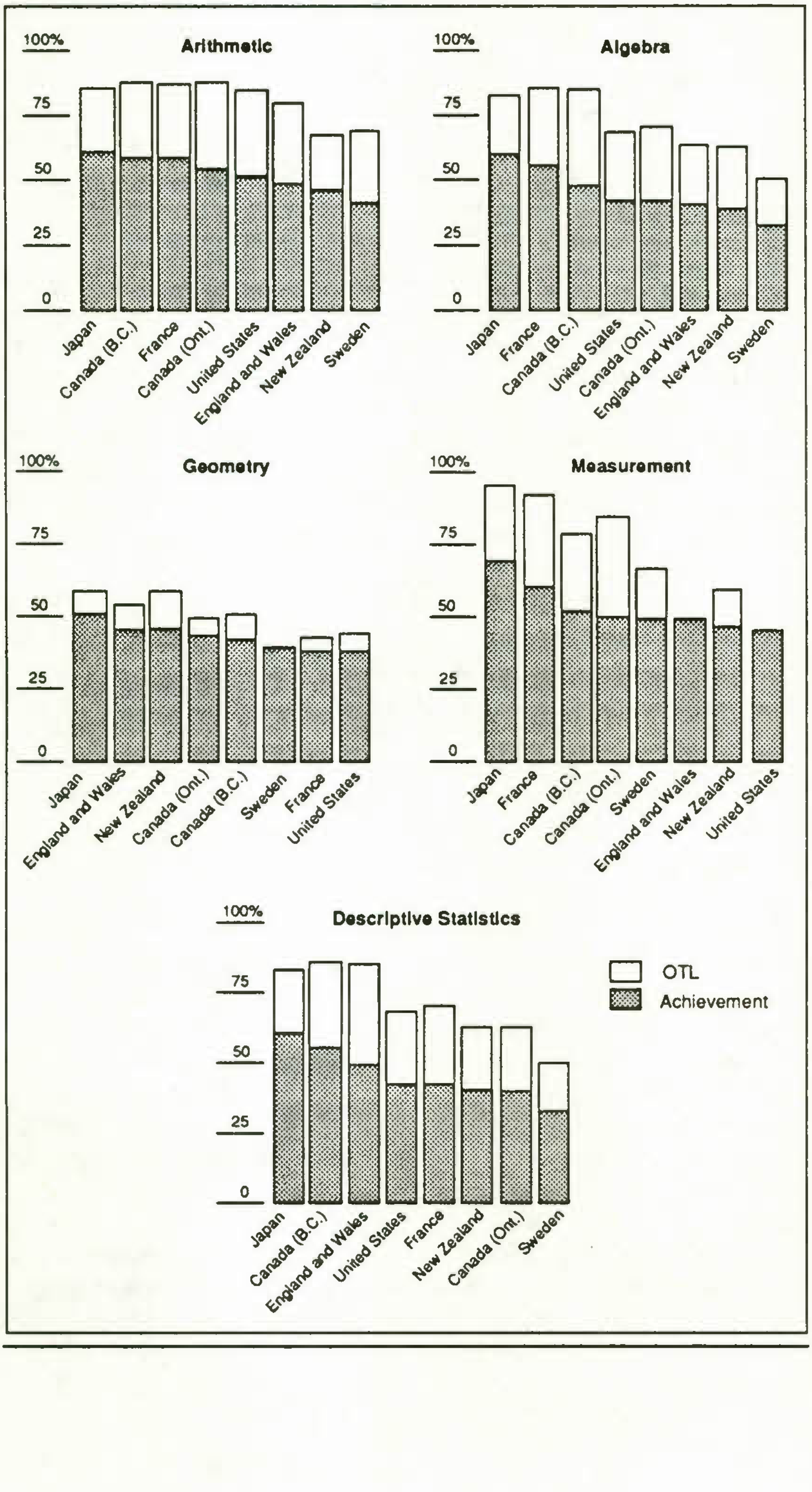
An examination of individual item results reveals rather poor levels of performance in all systems on what appears to be straightforward items. Students everywhere had difficulty with many of these items, and it may be the case that some of this material is either too difficult or too abstract for students of this age.

Geometry — There were 39 geometry items in the total pool of 157: 25 per cent of the total. They dealt with a wide variety of topics including properties of plane figures, similarity and congruence, the Pythagorean theorem, elements of proof, formal and informal transformations, coordinate geometry, spatial visualization, and elementary properties of 3-D figures in space. This variety of topics demonstrates the lack of consensus that exists between countries concerning the appropriate geometric content to be included in the mathematics curriculum, as well as the proper status of geometry in that curriculum.

It was extremely difficult to assemble a group of geometry items which all participating jurisdictions agreed were appropriate to their curricula. The geometric content which is taught ranges from an intuitive investigation of elementary properties of plane figures in some countries, to a very extensive range of topics in others. The teaching is based on a traditional approach to

Chart 2

Achievement on Subtests: Population A



Euclidean geometry in most places, to an emphasis on geometric transformations in others. Teaching methods range from highly intuitive to formally axiomatic. All of these differences make comparisons of achievement levels highly problematic.

Chart 2 shows that OTL scores are low everywhere, below 60 per cent. Complicating inter-jurisdictional comparisons is the fact that, whatever the set of items considered appropriate to a given jurisdiction, the overlap between those items and the ones considered appropriate in another, comparable jurisdiction might well be almost empty.

The lesson to be derived from this is that the geometry curriculum is in disarray. For the past 25 years there has been a great deal of discussion about what geometric content should or should not be included in the school mathematics curriculum, but little has been agreed upon. Mathematicians urge the school system to teach geometry, but they do not practice what they preach. Few prospective teachers of secondary mathematics take a geometry course as part of their academic training; in many cases, this is a simple reflection of the scarcity of such courses at the undergraduate level in North American universities where future teachers receive their academic preparation.

Descriptive Statistics — International and national groups in a number of countries interested in the advancement of mathematics education have urged teachers to give increased prominence to fundamental ideas from the areas of probability and statistics in the mathematics curriculum, beginning with the earliest school years. As the OTL figures for this subtest show in Chart 2, there is a rather large spread in the degree to which those ideas have become part of the mathematics curriculum. In Japan, British Columbia, and England and Wales, it appears that many of these topics are included in the mathematics curriculum. This is much less apparent in the other jurisdictions.

In a few jurisdictions other than those shown here, students' achievement scores were actually higher than the corresponding OTL ratings from teachers. That is to say, students correctly answered more items than their teachers said they should have been able to on the basis of the content which they had been taught. This is likely attributable to the fact that students gain experience with graphs, charts, and tables in contexts other than their mathematics classes. Mathematics teachers are apparently unaware, at least to a degree, of the influence of the impact of out-of-mathematics-class experiences on students' knowledge, experience, and achievement.

Measurement — The 24 items on the measurement subtest dealt with students' knowledge of the basic units of measurement in the SI metric system, finding areas and volumes, and estimation. The graph shows fairly high OTL scores for this material almost everywhere, and comparatively high achievement scores as well. Students from mainly English-speaking countries performed much more poorly on items dealing with the metric system than might have been expected, and the data show rather clearly that most Canadian students still do not "think metric." Indeed, since they live in a milieu in which they encounter frequent usage of the imperial system of measurement on a daily basis, one wonders whether they are well served by an educational system which insists on teaching the metric system only.

Growth in Students' Achievement

The eight systems included here, with the exception of England and Wales, participated in the longitudinal phase of the Second International Mathematics Study. This means that students were given a pre-test at the beginning of the school year, and a post-test at the end. Both administrations involved the same set of items, so it was possible to track students' growth in achievement over the course of an entire, 10-month school year. An effort was also made in the construction of the item pool to include as many items as possible which, on the basis of the curricular analyses which had been carried out earlier in the study, appeared likely to result in significant amounts of growth.

The findings from the analysis of the growth data are not very encouraging. Across all participating systems, the median amount of growth on an item is about 7 percentage points. That is, on the average item from this set of 157 items, the difference between students' performance at the beginning and at the end of the school year is about 7 percentage points. There are a small number of "high growth" items, but these invariably deal with lower level cognitive tasks such as computational skills or recognition of terminology.

Intriguing exceptions to this general trend are apparent in Japan and France. In both of these countries, students' performance increased dramatically on a number of items dealing with topics that were being taught there for the first time during the Population A year. In France, this had to do, for example, with items dealing with fractions which were dealt with for the first time at that level. In Japan, dramatic levels of growth occurred on some of the algebra items.

In the three North American systems, on the other hand, high-growth items are quite rare. In most cases, the differences between pre-test and post-test scores are very small.

Achievement of Population B Students

The composition of Population B varied dramatically among the countries participating in the study, and it is necessary to exercise caution in making comparisons and drawing conclusions on the basis of these results. At one extreme, we have the case of British Columbia, where Population B includes 30 per cent of the appropriate age cohort and almost 40 per cent of the grade cohort. At the other extreme, Population B represents only 6 per cent of the age cohort in New Zealand as well as in England and Wales. Not only must differences in the abilities of the students making up these groups be taken into account, but also the fact that the kinds of mathematics courses which can be offered to a highly select group of students are very different from the typical pre-calculus course offered to Grade 12 students in Canada and the United States.

The structure of the item pool for Population B reflected the emphasis accorded to the study of calculus at this level in many countries. Of the 136 items in the pool, 46 dealt more or less specifically with calculus. Needless to say, these items dealt with topics beyond the scope of almost all students at the Grade 12 level in Canada and the United States. In Ontario, however, the international Population B sample was drawn from Grade 13 classes which were taking a full calculus course.

In order to provide comparisons of interest within Canada, the analyses in this paper include a sample of Grade 12 classes from Ontario, in addition to the Grade 12 sample from British Columbia and the Grade 13 sample from Ontario. Also included is the performance of the American Population B sample which consisted of students in Grade 12 as well.

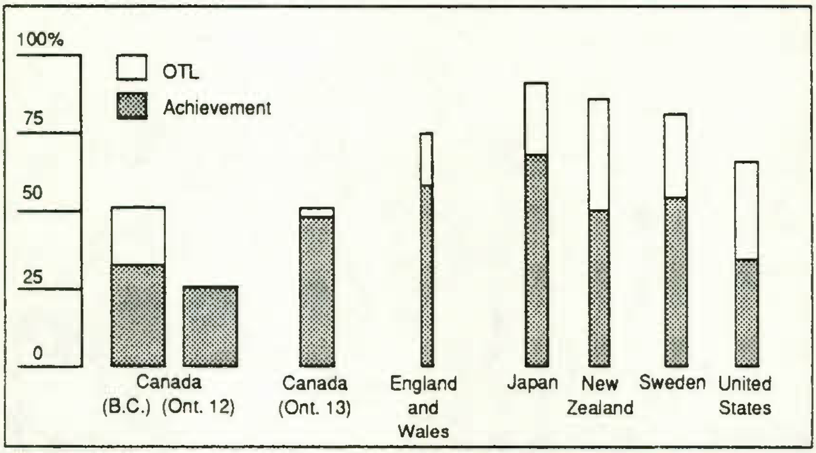
Comparing the performance of Canadian Grade 12 students to that of the American Population B students is somewhat unfair, since about 5 per cent of those American students were enrolled in an Advanced Placement calculus course. Comparisons are also complicated by the fact that British Columbia and Ontario both retain a much greater proportion of the age cohort in the study of academic mathematics at the Grade 12 level than is the case in the United States: 30 per cent in each of the two provinces, versus 12 per cent in the United States.

Overall Performance

Chart 3 summarizes the achievement results for Population B in the jurisdictions being discussed in this paper. Also shown in the chart is the OTL information from teachers as an indicator of the extent to which students in the systems involved had actually had an opportunity to learn the material involved as part of their formal mathematics course or courses. No OTL ratings were available for the Ontario Grade 12 sample, and those for the Grade 13 sample are likely underestimated because of the way that sample was drawn and the OTL information collected.

Chart 3

Overall Achievement: Population B



In the chart, differences in the retention rates of school systems and, hence, in the makeup of the Population B cohort, are indicated through the differing widths of the bars used to represent each system. Since British Columbia retains 30 per cent of the age cohort in Population B, the bar for British Columbia is five times as wide as that for England and Wales, where only 6 per cent of the age cohort is retained.

Chart 3 illustrates, in rather dramatic fashion, the different patterns of achievement which can result from decisions to prefer high retention rates over strict selectivity. There is no well-defined measure of mathematical "yield" for an educational system, but this chart raises a number of questions about what kind of final product a system can expect on the basis of decisions that are made concerning participation in the academic program at the secondary school level.

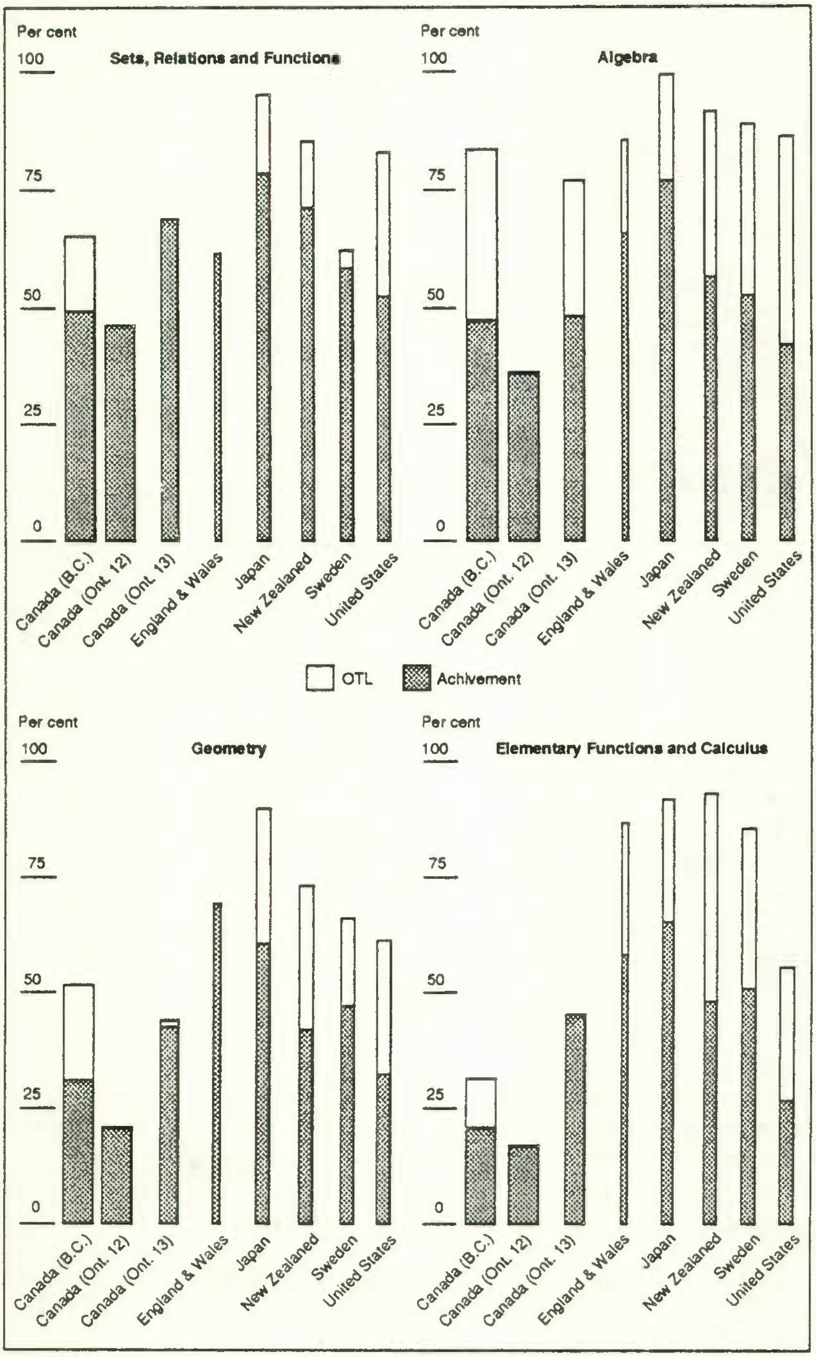
Performance on Specific Topics

There were 136 items in the Population B item pool, and these were divided among eight test booklets: 17 items to each form. All of the items were presented in multiple-choice format with five alternatives, just as they were for Population A. Many of the items were difficult and non-routine, requiring students to use higher order thinking strategies.

Chart 4 summarizes students' performance on four of the subtests into which the Population B items were partitioned. As with the previous chart, the widths of the bars are proportional to the mathematics retention rates of the systems under consideration.

Chart 4

Achievement on Subtests: Population B



Sets, Relations, and Functions — This subtest dealt with a number of concepts which are of fundamental importance to establishing the proper conceptual foundations of mathematics at this level. They included Venn diagrams, sets, functions, domain, and range. Overall, students performed better on this subtest than on any of the others. Grade 12 students from British Columbia and Ontario performed at about the same level, and considerably lower than students elsewhere.

Algebra — There were 26 items on the algebra subtest dealing with many of the topics traditionally associated with an algebra course at this level. These included solving equations of the first and second degree, simplifying rational and radical expressions, constructing equations as models of physical-world situations, and so on. The international mean on this subtest is 57 per cent, and Ontario Grade 13 students performed at approximately that level. The average performance of British Columbia Grade 12 students was again superior to that of Ontario Grade 12 students, but both performed at levels considerably lower than did students elsewhere.

Geometry — If, as was suggested earlier, it is difficult to identify a common international core of geometric content for the Population A mathematics curriculum, it is even more difficult at the Population B level. On the one hand, the field is much broader at this more advanced level and, for this study, included both analytic geometry and trigonometry. On the other hand, the approach taken to the study of geometry in countries such as Belgium and France is so unfamiliar to North American teachers and their students, that test items based on these approaches might not be recognized as geometric items at all.

The performance of North American students generally on this subtest was disappointing, and considerably lower than on the other subtests discussed thus far.

Elementary Functions and Calculus — Virtually no Grade 12 students in British Columbia or Ontario were studying calculus when this study was conducted, and this fact helps explain why the performance level on the elementary functions and calculus subtest in the two provinces was either at or below the chance level. In many other countries, including all of the non-North American ones shown in Chart 4, Population B students take a full calculus course equivalent to that offered in many North American universities.

Retention Rates and Achievement

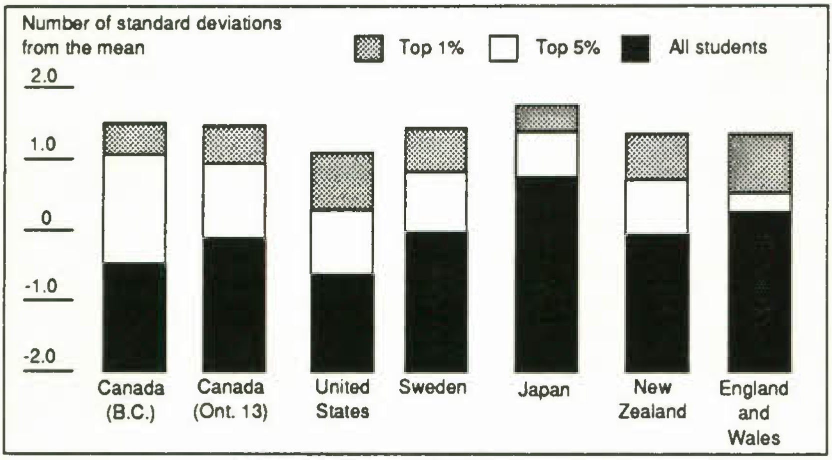
The relationship between retention rate and the achievement of students in Population B was investigated in some detail by Miller and Linn [1985].

Scores on all test forms were equated using a linear equation procedure and three major subtests were used as criterion measures. Only the analysis of the algebra subtest is reported here. Additional information about these analyses may be found in Robitaille and Garden [1989].

Chart 5 indicates the mean achievement level for all Population B students in the systems under consideration, as well as for the top 5 per cent and the top 1 per cent of students in those systems. The systems have been ordered along the horizontal axis in descending order of their retention rates, and the Ontario Grade 12 sample is not included because it was not part of Miller and Linn's analysis.

Chart 5

Retention Rates and Achievement



Husén [1967], in his analysis of comparable data from the first IEA mathematics study, concluded that higher retention rates did not have an adverse effect on the mathematics achievement of the most capable students. In addition, his results indicated that higher retention rates were associated with higher system performance. "The higher the proportion of students retained in the school system, the higher the proportion of the group making high scores is likely to be" [Husén, 1967]. Miller and Linn concluded that a lower retention rate results in higher mean achievement for the students remaining in school, but that the yield and quality of talented students are not negatively affected by higher retention rates. They did not find "conclusive evidence to show that talented students would do better or worse as a function of the national retention rates."

Some subsequent analyses of the link between retention rates and achievement of talented students have found that the more selective systems (i.e., those in which Population B represents a lower proportion of the age cohort) tend to have higher percentages of the Population B students in the high achievement range on tests such as these. However, the relationship between selectivity and the per cent of the age cohort in the high achievement range is very weak, about –0.10. Thus, it appears that high selectivity is not a necessary condition for excellence.

Japan is an illustrative example of a system which retains a comparatively large proportion of students at the Population B level and many of those students perform at extremely high levels of achievement. In fact, there were almost 800 perfect scores obtained by Japanese students in this study.

Gender

A good deal of concern has been expressed about the rate of participation of girls in mathematics and science courses at the senior secondary and tertiary levels. The concern has to do with the fact that many girls may be cutting themselves off from a wide variety of professional choices by deciding too early in their educational careers to discontinue the study of science or mathematics.

Results from the Second International Mathematics Study confirm that such a pattern still exists. In every one of the participating systems, the proportions of boys and girls at the Population A level are virtually identical. All boys and girls are still studying mathematics at age 13. By the time students reach the last year of secondary school, however, the situation has changed. In British Columbia and Ontario, girls constitute 40 per cent of the population of senior mathematics students. This is lower than the proportion reported in the United States (44 per cent), but higher than those reported elsewhere: e.g., England, 34 per cent; Sweden, 26 per cent; and Japan, 22 per cent.

Research is needed to learn more about this phenomenon. We need to learn why it is that many girls decide to discontinue the study of mathematics so early, and we need to know when they make that decision. We also need to learn which girls make that decision. For example, it is not at all clear that it is the less able female students who drop out of mathematics. Some work on these questions is currently under way in British Columbia in a project funded by the B.C. Ministry of Education.

Related to the question of the participation of girls in the study of mathematics and science is gender of teachers. It may be that one of the factors which influences the decisions made by female students about pursuing the study of mathematics or science is the gender of their mathematics and science teachers. If this is the case, then the findings from this study are cause for some concern.

Over 80 per cent of the mathematics teachers from British Columbia who participated in the study at the Population A level were male. In Ontario, two out of three were male. In the case of Population B, the results were even more dramatic: 95 per cent male teachers in British Columbia, and 80 per cent in Ontario. These percentages are significantly higher than those reported in most other jurisdictions.

Further Education Expected

Both populations of students were asked to indicate how many more years of formal education they expected to acquire, and Canadian students were among those expecting the most. Over 60 per cent of the Grade 8 students from British Columbia and Ontario indicated that they expected to pursue some sort of formal educational program at the post-secondary level. This rate was substantially lower than the corresponding figure for the United States (75 per cent), but considerably higher than that reported elsewhere. Only about 20 per cent of students from England and Wales said they expected to continue to the post-secondary level, and slightly less than 50 per cent of students in Japan did.

Some interesting changes in expectations were apparent among Population B students. Over 80 per cent of the Canadian Population B students said that they expected to continue their formal education at the post-secondary level. This high rate of expected participation is not at all surprising given the fact that these students are all enrolled in pre-university programs, or the equivalent. The corresponding figures from other jurisdictions are all high at this level, with the United States reporting the highest rate, 97 per cent.

Findings from Another International Survey

In 1988, the Educational Testing Service (ETS) in the United States carried out an international survey of the achievement of 13-year olds in mathematics and science. International participants in the project included the United States, the Republic of Korea, the United Kingdom, the Republic of Ireland, and Spain. Four Canadian provinces – British Columbia, Ontario, Quebec, and New Brunswick – also participated; in every province except British Columbia, there were two samples of students drawn: one from pre-university schools, and one from francophone schools.

The comparative performance of students from the 12 participating systems is shown in Chart 6. The vertical axis is a "mathematics proficiency" scale developed through the use of item-response theory, and represents an effort to portray performance in mathematics as a uni-dimensional trait.

Chart 6

Students' Performance in the IAEP Study

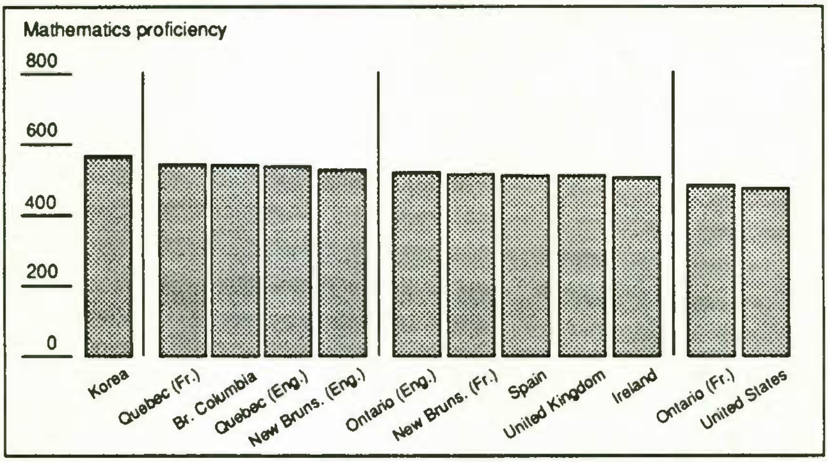


Chart 6 is divided by vertical lines into four parts. The report of the study [Lapointe, et al., 1989] states that the differences between groups separated by the vertical lines are statistically significant, but that the differences within each of the four groups are not. Thus the performance of Korean students was significantly better than that of students in all of the other systems. However, there were no significant differences in the performance levels attained by students in Quebec (French), British Columbia, Quebec (English), and New Brunswick (English).

These results tend to confirm the rank ordering of systems reported earlier. That is to say, the relative ordering of British Columbia, Ontario, and the United States were the same in this survey as they were in the IEA study. It is unfortunate that there was not a greater overlap of participants in the surveys, and that it was not possible to obtain participation from all of the Canadian provinces.

Implications of the Findings

The Second International Mathematics Study was a major undertaking, involving thousands of students and hundreds of teachers in each of the participating countries, including the two Canadian provinces. The findings have been reported in scholarly journals as well as in the popular media; and major organizations, including the National Council of Teachers of Mathematics which represents both Canadian and American teachers of mathematics, have made recommendations for sweeping changes in the way in which mathematics is taught and to whom it is taught on the basis of those findings.

Many of those recommendations deal specifically with aspects of the curriculum or teaching practices, and likely have little relevance in a paper of this kind. There are, however, a number of areas of more general interest which are discussed below.

Need for Canada-Wide Participation

It seems clear that Canada has much to gain from participation in comparative research studies of the kind sponsored by IEA. Unfortunately, because of the way in which the governance of education is organized in Canada, this is very difficult to do. In fact, it has yet to be done. Individual provinces have decided, independently of one another, to participate or not to participate in such projects, but there has been no national coordination. The recently completed IEA survey of science did include Canada-wide participation, but separate studies were conducted in French and in English schools two or three years apart.

Provincial educational systems across the country, under the coordination of the Council of Ministers of Education of Canada (CMEC), are currently engaged in a process of identification and development of a set of "educational indicators": that is, a set of measurable outcomes which could help policy-makers evaluate the success of the educational enterprise. For that reason at least, CMEC seems to be supportive of projects such as the IEA studies. However, the Council has no authority and no funds, so it can do little more than recommend to the individual provinces that they participate.

An alternative source of funding is the Social Sciences and Humanities Research Council of Canada (SSHRCC), whose mandate is to support research in a wide variety of areas including education. Over the past decade or more, SSHRCC has funded several IEA-related projects, including several aspects of the Second International Mathematics Study, as well as both the francophone and anglophone versions of the IEA science study. SSHRCC will probably continue to provide support for IEA projects on a competitive basis, but it is unlikely that they would be able to provide sufficient funds to support the entire cost of any such project, given their limited budget.

Gender Differences

Educational researchers have, for some time, been investigating differences in patterns of achievement and participation in mathematics and science courses by males and females. About 20 years ago, researchers found significant differences between the achievement levels attained by boys and by girls as age level increased. They found few differences at the elementary school level, but increasingly large differences in the secondary grades.

Critics of those early studies charged that the investigators had failed to take into account differences in "course-taking" behaviour between boys and girls, and there is substantial support for that criticism in these results. Many more girls decide to withdraw from the study of mathematics than boys, and little is known about their reasons for doing so, or even about which girls decide to withdraw. That is to say, we do not even know whether or not it is the most qualified girls who decide that mathematics is not for them.

In any event, the results from the Second International Mathematics Study reveal that there are now relatively few statistically significant differences between the performance levels attained by boys and girls in most countries, including Canada. The problems associated with under-representation of females in mathematics courses at the senior secondary school level persist, but the large differences in achievement levels reported in earlier studies appear to be narrowing.

Impact of Technology

The vast majority of teachers who participated in the Second International Mathematics Study, in Canada as well as in other countries (in this case, notably, Japan), indicated that the use of calculators or computers by students in mathematics classes should be either severely restricted or forbidden. Although the data for the Second International Mathematics Study were collected between 1980 and 1982, more recent studies in British Columbia and elsewhere indicate that teachers' and parents' opinions in this regard have not changed significantly.

The precise reasons underlying this resistance on the part of teachers to the introduction of technological innovations which might well facilitate their work are difficult to discern. Many people, parents as well as teachers, believe that the mastery of routine, yet "hard," topics such as the computational algorithms of arithmetic contributes to the moral and intellectual development of children. They express concern about how this generation will react when faced with a difficult computational situation without a calculator or, even worse, a calculator with exhausted batteries.

From an educational point of view, questions about what the mathematics curriculum would look like, and what kinds of mathematical knowledge and expertise would be expected of teachers of mathematics in a technologically enriched mathematics environment, require urgent attention. On the one hand, schools can ill afford to simply ignore this issue and pretend that the technology does not exist. On the other hand, most teachers of mathematics are comfortable with the existing, largely algorithm-based, mathematics curriculum. Many of them would be reluctant to adopt new curricula and new teaching techniques with which they are unfamiliar or with which they feel less secure.

Goals of the Mathematics Curriculum

Howson and Wilson [1989] have said that one of the major problems affecting efforts to make changes in the mathematics curriculum is that everyone seems to agree, and to have always agreed, that the subject is an essential ingredient in any program of general education. Moreover, the mathematics curriculum is very much the same all around the world, and this complicates attempts to make changes in any particular country.

The two reasons most frequently put forward in support of the importance of mathematics in the school curriculum have to do with its purported contribution to the development of "logical thinking," and its usefulness in the modern world. As to the first of these, there is no evidence that people trained in mathematics are more logical or better thinkers than anyone else when they are operating outside the realm of their particular expertise. The second reason put forward in defence of the place of mathematics in the school curriculum is its usefulness in so many different fields and activities. [See, for example, Cockroft, 1982.] Christine Keitel says that the usefulness of mathematics is a pretence. She says: "No modern society can exist without mathematics, but the overwhelming majority of people in a modern society can and do live quite well *while doing hardly any mathematics*" [Keitel, 1989].

Canadians need to examine the place of mathematics in the school curriculum in a fundamental way. We need to ask why almost all students are required to study mathematics in every grade. We need to determine what is really meant by the usefulness of mathematics, and to judge the extent to which that goal is being met by current curricula and teaching practices.

Teacher Supply and Qualifications

Large numbers of teachers who have been assigned responsibility for the teaching of mathematics, particularly in the elementary and lower secondary grades, have little or no academic background in mathematics at the post-secondary level. Results from the Learning Assessment Program in British Columbia show that many of these teachers have not had any professional preparation in methods of teaching mathematics either. That is to say, a great deal of mathematics is being taught in Canadian schools by teachers who are ill-equipped for that task. This situation is likely to become much worse as the current teacher shortage worsens.

Students' Performance

At the Population A level, Canadian students performed reasonably well in comparison with students from other countries. The overall results show that Grade 8 students from British Columbia were among the top third of participants, and that their performance surpassed that of students in any of the other English-speaking jurisdictions involved. These included Scotland, England and Wales, the United States, and New Zealand. The performance of students from Ontario was somewhat lower, placing them at about the mid-point of the range of total scores.

At the Population B level, Canadian students in Grade 12 had the lowest overall scores of all. Grade 13 students in Ontario did much better, but their performance lagged significantly behind that of students from other countries. The high rates of retention of students in the study of pre-university mathematics in the two Canadian provinces, and the concomitant impact on the curriculum which can be offered to those students, are major factors to consider in attempting to decide whether or not there are grounds for concern about our students' performance. Unfortunately, there is no way of moderating the test scores to take into account the influence of those variables.

On the whole, students performed satisfactorily on tasks which involved straightforward applications of more or less routine tasks. Performance levels fell off sharply, however, on items requiring higher level cognitive behaviours, and this raises concerns. Professional associations of teachers of mathematics have, for the past several years, been exhorting teachers to give increased emphasis to the applications of mathematics and to problem solving. The need for teachers in all countries to do so is clear from these results. Modern Canadian society has little need for people who are skilled calculators. On the other hand, we do have an urgent need for skilled problem solvers.

Policy Conclusions

The results from the Second International Mathematics Study are of considerable significance for teachers, teacher educators, and educational researchers. But education affects everyone in our society, and the cost of providing educational services to all citizens is one of the major costs of government at all levels. Hence these results are of significance to all Canadians, regardless of the degree of their personal involvement with the formal educational system. The findings of the Second International Mathematics Study suggest a number of policy initiatives and directions toward which, in the opinion of the author, Canadian educational policy should be directed. Seven such policy directions are discussed in this section of the paper.

National Support for Research in Education

In most of the IEA countries, there exists a national organization with a mandate to conduct educational research on a national level. This is true, not only in countries such as France and Japan which have highly centralized educational systems, but also in countries such as Australia, the Federal Republic of Germany, and the United States whose educational systems are as decentralized as is Canada's.

Given the importance of education in general and of the K-12 school system in particular to the state of the nation, Canada needs more national research in education in order to provide better information about what is going on in Canadian schools. At the present time, the Social Sciences and Humanities Research Council of Canada (SSHRCC) is virtually the only national source of funding for research in education on a regular basis, and the amount of funding available from SSHRCC for educational research each year is small.

Consideration should be given to the establishment of a national centre for educational research in Canada. Such an agency could be modelled on successful institutions of this kind in other countries, including the Australian Council for Educational Research and the National Foundation for Educational Research in the United Kingdom. Some link between such a centre and the Council of Ministers of Education might be established to provide governance and overall direction for the work of the centre.

Canada has much to gain from participation in both national and international research studies in the field of education. Our experience with the IEA mathematics and science studies has shown how difficult it is, in the present circumstances, to enlist the participation of schools and students from across the country in such projects. Moreover, the fact that so few sources of funding are available for such projects adds to that difficulty. The establishment of a national centre, at an appropriate level of funding, would greatly improve this situation, and facilitate the gathering of badly needed data about what is going on in the schools of the nation.

Participation Rates

In both British Columbia and Ontario, the proportion of students in the last year of secondary school who are enrolled in pre-university mathematics courses is higher than in most other countries. Whether or not this is true in other parts of the country is not known, but it seems likely.

Mathematics is an important component in the academic preparation required for an increasingly wide variety of careers and professions. Students who decide to discontinue taking mathematics courses prematurely will, therefore, effectively eliminate a great number of possible career choices. Schools should encourage students to continue studying mathematics, and provide a variety of courses at the senior secondary level to enable all qualified students to do so.

Gender Differences

Current research indicates that the gaps between achievement levels obtained by girls and boys in mathematics have narrowed over the past 20 years, and this trend was confirmed in the Second International Mathematics Study. At the Population A level, virtually no significant differences between the achievement of boys and girls was found in either Canadian province or in the United States. More differences were found at the Population B level but, even there, most of the statistically significant differences were not large enough to be considered important from an educational perspective.

Of greater concern is the fact that females continue to be under-represented in mathematics courses at the senior secondary level. In British Columbia and Ontario, girls constitute only 40 per cent of Population B. This proportion is higher than the comparable proportion in many other countries, but there is no obvious educational reason why males and females should not continue in the study of mathematics in equal numbers.

Little in the way of good information is available about the course-taking behaviour of students at the senior secondary level. Little is known about the reasons underlying students' decisions to continue or to discontinue the study of mathematics as soon as it is permissible for them to do so. It may be, for example, that many of the girls who "drop out" of mathematics are among the most academically able. This would certainly help explain the wider gap in achievement patterns among older students.

Given the importance of a mathematically literate population to the continued economic and scientific development of Canada, we can ill afford to have such a situation continue. We need to investigate this problem in detail, to identify causes, and to seek solutions. Professor Jim Gaskell of the Faculty of Education at UBC heads a team which is currently interviewing girls in senior secondary schools to learn more about the reasons underlying their course-taking decisions in mathematics and science. Results of that study will be available in the last quarter of 1990. Similar studies should be conducted in other parts of the country.

The Role of Technology

There is no doubt that widespread introduction of calculators and of computer technology into classrooms will and should change what goes on in mathematics classrooms in a fundamental way. So-called symbolic manipulation software packages, which are widely available today at moderate cost, effortlessly perform virtually every algorithmic procedure students have traditionally been required to master in mathematics courses in secondary school and in the junior university years.

Does it make sense to have elementary school students spend time in every grade from Grade 3 to Grade 8 learning and reviewing how to do long divisions? Certainly calculators are easily available to relieve us and them of that chore. Moreover, we already know that, at the end of Grade 8, over 30 per cent of students are likely to produce an incorrect answer when asked to solve a long-division exercise. Could not their time and their teachers' time be spent more fruitfully and efficiently?

Does it make sense to have secondary school students and teachers spend enormous amounts of time solving equations, working with trigonometric identities, learning graphing techniques, or memorizing formulas for differentiation and integration, when symbolic manipulation software and microcomputers can take over those routine chores?

An in-depth study of ways in which available technology can best be put to use in educational settings, particularly in the areas of mathematics and science, is needed. Moreover, means will have to be found to overcome regional disparities in ability to provide the financial resources needed to make the technology available to all students.

Students' Achievement

The performance of Canadian students on the achievement testing aspects of the Second International Mathematics Study was generally satisfactory. Results at the Population A level were better than those at Population B, but international comparisons at the latter level are of questionable validity because of the substantial between-system differences in retention rates which exist.

What is of concern is the fact that the kinds of test items on which all students, including Canadians, performed best tended to be those which involved routine, low-level tasks. Performance levels on problem-solving items tended to be much lower.

The teaching of mathematics in Canada needs to be re-oriented to place more emphasis on the development of students' problem-solving skills. Schools need to make better use of the technology available to deal with routine techniques and procedures, and to take advantage of the time saved in that way to devote more attention to the real-world applications of mathematics. This may make mathematics more difficult for some teachers to teach, particularly those with minimal academic preparation in mathematics, but it will make the subject more interesting and more relevant for students.

Teacher Supply

In British Columbia, principals reported that 40 per cent of their Grade 8 teachers of mathematics, and 30 per cent of their Grade 12 teachers of mathematics, were less than fully qualified to teach mathematics at that level. Almost 20 per cent of the teachers of Grade 13 mathematics in Ontario were also judged to be less than fully qualified. In other words, a substantial proportion of mathematics classes at the secondary school level in Canada are being taught by teachers who have had insufficient or no academic or professional preparation in mathematics.

Canada ranked lower on this variable than most other industrialized countries in the study, and this is a serious matter. If the teaching of mathematics in this country is to keep pace with technological and scientific advances, we need teachers of mathematics who have an in-depth knowledge of the subject matter and whose professional preparation has familiarized them with the appropriate teaching methodologies.

Grouping of Students by Ability

The practice of grouping students into different classes for one or more subjects on the basis of some measure of academic ability is very prevalent in Canadian schools at the junior secondary level. Although such "streaming" of students occurs in several subject areas, it seems to be most prevalent in mathematics.

The effects of various arrangements for streaming have been studied extensively, and the findings of this research are quite unequivocal. The only type of streaming which is beneficial for students is grouping high-ability students together. Placing all "low-ability" or "slow" students together has been shown to be particularly ineffective, since it results in a widening of the gap between the ability groups.

An argument frequently employed by teachers to justify the practice of streaming is that their teaching will be rendered more effective if the range of ability of the students in their class is narrower. This might be the case if it were possible to assess a student's ability level accurately, but it is extremely unlikely that this could be done within a single school and without the use of highly sophisticated assessment procedures. Moreover, we have evidence from countries such as France and Japan that heterogeneous grouping of junior secondary students in mathematics works very well indeed.

Many teachers believe that the content of the mathematics curriculum in the academic program is inappropriate for students who are not planning to attend college or university. The very real danger is that many students may be mis-categorized and directed into non-academic, terminal programs which severely limit their academic and career options.

Conclusion

Canadian participation in the Second International Mathematics Study, although limited to two provinces, has provided a wealth of comparative information on our educational system. It is encouraging to see that a group such as the Economic Council of Canada is interested in knowing more about the results of the study, because it underscores the importance of the educational system to all Canadians. If we are to provide the very best educational system we can for our children, then we must also support educational research as a national priority. For, it is only through research that the improvements we all seek will be realized.

IEA has decided to conduct replications of some of its surveys on a regular, periodic basis, and both mathematics and science, as well as written expression, are included in that group. The data-collection phase of the Third International Mathematics Study is scheduled for the 1993-94 school year, and it is hoped that it will be possible to arrange for Canada-wide participation in that study.

Preparatory work on the third IEA mathematics study is under way, and variables of interest are being identified. Groups such as the Economic Council which might wish to have questions related to variables of interest to them included on questionnaires, or other instruments used in the paper, should contact the author.

List of Tables, Charts, and Figure

Tables

Charts

Figure

References

COCKROFT, W. H., *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools* (London: HMS, 1982).

GARDEN, R. A., *Second International Mathematics Study: Sampling Report* (Washington, D.C.: Center for Education Statistics, 1987).

GOODLAD, J. I., *A Place Called School* (New York: McGraw Hill, 1984).

GORMAN, T. P., et al., *The IEA Study of Written Composition*, Vol. 1 (Oxford, U. K.: Pergamon Press, 1988).

HUSÉN, T. (ed.), *International Study of Achievement in Mathematics*, Vols. 1 and 2 (New York: John Wiley & Sons, 1967).

INTERNATIONAL ASSOCIATION FOR THE EVALUATION OF EDUCATIONAL ACHIEVEMENT, *Science Achievement in Seventeen Countries* (Oxford, U. K.: Pergamon Press, 1988).

KEITEL, C., "Mathematics education and technology," *For the Learning of Mathematics* 9, no. 1 (1989), pp. 7-13.

KULIK, C. L. and J. A. KULIK, "Effects of ability grouping on secondary school students: A meta-analysis of evaluation findings," *American Educational Research Journal* 19 (1982), pp. 415-27.

LAPOINTE, A. E., et al., *A World of Differences: An International Assessment of Mathematics and Science* (Princeton, N.J.: Educational Testing Service, 1989).

MCKNIGHT, C. C., et al., *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective* (Champaign, Ill.: Stipes Publishing Company, 1987).

MILLER, D. and R. LINN, "Cross-national achievement with differential retention rates," unpublished paper prepared for the National Center for Education Statistics, United States Department of Education, 1985.

PEAKER, G. F., *An Empirical Study of Education in Twenty-One Countries: A Technical Report* (Stockholm: Almqvist & Wiksell / New York: John Wiley, 1975).

RADWANSKI, G., *Ontario Study of the Relevance of Education and the Issue of Dropouts* (Toronto: Ministry of Education, 1987).

ROBITAILLE, D. F. and R. A. GARDEN, *The IEA Study of Mathematics II: Contexts and Outcomes of School Mathematics* (Oxford, U.K.: Pergamon Press, 1989).

ROBITAILLE, D. F., et al., *Curriculum in the Schools of British Columbia. Commissioned Papers:* Vol. 3 (Victoria, B.C.: The British Columbia Royal Commission on Education, 1988).

RYAN, D. W. and L. W. ANDERSON (eds.), "Rethinking research on teaching: Lessons learned from an international study," *Evaluation in Education* 8 (1984).

SULLIVAN, B. S., *A Legacy for Learners: The Report of the Royal Commission on Education* (Victoria, B.C.: The Queen's Printer, 1988).