STAFF STUDY No. 31

AN ANALYSIS OF CANADIAN MANUFACTURING PRODUCTIVITY

SOME PRELIMINARY RESULTS

by

Harry H. Postner

prepared for the
Economic Council of Canada

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PREFACE

I should like to express my appreciation to the Economic Council for giving me the opportunity to prepare a Study of this nature. Several Council staff members made useful comments on an early draft of the Study; I am particularly grateful to R. Agarwala, L. Auer, J. A. Dawson, M. Tenenhaus and D. A. White. Special thanks go to Professor R. G. Bodkin of the University of Western Ontario for his constructive suggestions on many aspects of the Study. It should be noted that the research contained herein is based on statistical data that were generally available as of January 1971.

Any remaining errors or inadequacies in this Study are the responsibility of the author.

CHAPTER 1

INTRODUCTION

This Staff Study is primarily written for professional economists in government, industry, or university, who are concerned with problems of productivity analysis. The Study provides part of the background material and technical analysis for Chapter 3 of the *Seventh Annual Review.*[1] In particular, the statistical data basis of this Study is generally the same as that used in the Annual Review. The reader who is interested in a nontechnical exposition of Canadian manufacturing productivity is referred directly to the Annual Review for a concise coverage.

1.1 Purpose and Scope of Study

There are two main purposes of this Study. The first is to develop a general model and methodology capable of identifying and measuring the sources of productivity growth for an industry. The second purpose is to apply the methodology in order to explain why productivity growth rates differ from industry to industry within the Canadian manufacturing sector. Productivity growth is analysed over the medium term by studying trend growth rates for periods of nine or ten years' duration. The manufacturing industries are examined at the two-digit Standard Industrial Classification (hereafter referred to as S.I.C.) level of aggregation.[2] The two main objectives of this Study now call for further comment.

Because of the medium-term growth nature of the analysis, it is clear that the economic complications of explaining annual variations in observed productivity lie outside the scope of this Study.[3] Similarly, the industrial aggregation level of the analysis precludes any emphasis on the engineering or work-study approach to examining the physical or psychological factors that may affect productivity at the level of the individual plant.[4] It is also evident that a good deal of the analysis will critically depend upon our "choice" of definitions. For example, if the labour

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[1] Economic Council of Canada, *Seventh Annual Review: Patterns of Growth* (Ottawa: Queen's Printer, 1970), Chapter 3, especially pp. 24-27.

[2] See Section 2.2 (Tables 2-1 and 2-2) for a list of the industries that are analysed.

[3] Some of the short-term complications are: lags in the input-output relations; delayed adjustment to changing equilibrium conditions; the problem of forward-expectations; and the unreliability of observed data in correctly reflecting annual changes.

[4] Some important aspects of industrial engineering are: methods and time analysis; work simplification; materials handling; and flow process analysis. The field of industrial psychology includes: supervisory methods; incentive systems; team size and organization; and labour-management relations.

productivity growth rate is defined so as to account already for changes in the skill composition (or quality) of the various types of labour employed, then such a labour productivity analysis will differ as compared to an analysis based upon an "output per unweighted man-hour" definition. The approach adopted in this Study is to choose definitions that conform, as much as possible, to common usage. This approach is easily understood and yields results amenable to economic policy formulation.

There is considerable emphasis in this Study on the problem of developing a general model and methodology. On the one hand, the model is eclectic in the sense that it combines and extends the work of various writers in the productivity literature.[5] On the other hand, the methodology is consistent and unrestricted in the sense that the major sources of productivity growth are simultaneously derived from a general production function. It turns out that it is *not* necessary to assume specific production functions nor to suppose that certain neoclassical competitive equilibrium conditions are satisfied in order to identify and measure sources of inter-industry productivity growth differentials. The contribution of such sources as "labour quality change", "changes in the capital-intensity of production", and "economies of scale", to labour productivity growth differentials can be simultaneously *estimated* from observable data. The general model provides the economic interpretation of the resulting estimates in a consistent framework.

The methodology of this Study is applied to two distinct time periods, 1947-56 and 1957-67. For the first time period, reasonably "complete" estimates of the coefficients (or sources) of inter-industry productivity growth differentials are obtained on the statistical basis of the two-digit 1948 S.I.C. For the second time period, the empirical results are on the statistical basis of the two-digit 1960 S.I.C., and the estimates are "incomplete" for reasons explained in the text.[6] Both applications and the resulting estimates should be regarded as preliminary and illustrative rather than definitive.[7] However, the emphasis is on the *economic* interpretation of the *analytical* results rather than the *statistical* interpretation of *descriptive results.*[8] The analytical results obtained in this Study are also compared with those obtained by other writers, even though the various results are often not strictly comparable.

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[5] The most suggestive references have proved to be: Zvi Griliches, "Production Functions in Manufacturing: Some Preliminary Results", in M. Brown, ed., *The Theory and Empirical Analysis of Production,* National Bureau of Economic Research, Studies in Income and Wealth, vol. 31 (New York: Columbia University Press, 1967), pp. 275-320, but especially pp. 308-310; L. R. Christensen and D. W. Jorgenson, "U.S. Real Product and Real Factor Input, 1929-1967", *Review of Income and Wealth,* March 1970, pp. 19-50, but especially pp. 29, 34-37; and H. F. Lydall, "Technical Progress in Australian Manufacturing", *Economic Journal,* December 1968, pp. 807-826. See also W. G. Salter, *Productivity and Technical Change* (Cambridge: Cambridge University Press, Second Edition, 1966).

[6] See Sections 2.1 and 3.3.

[7] See Chapter 7 on "Suggestions for Future Research".

[8] The descriptive (nonestimation) approach is well emphasized in N. H. Lithwick, G. Post and T. K. Rymes, "Postwar Production Relationships in Canada", in M. Brown, ed., *op. cit.,* especially pp. 158-190.

1.2 A Brief Outline of the Study

This Staff Study is roughly divided into two parts – the text and the Appendixes. The text is written primarily for the non-mathematical reader and presupposes some basic knowledge of economics and statistical methods. For example, it is assumed that the reader is acquainted with such elementary concepts as "production function", "elasticity of output with respect to an input", and the economist's use of "returns to scale". Similarly, it is supposed that the reader knows the meaning of, e.g., "random variable", "standard error", and the elementary properties of "ordinary least-squares regression estimates". Generally speaking,[9] the text attempts to provide an intuitive and verbal explanation of the productivity growth models that are formulated in Appendixes A and B. For a complete understanding of this Study, the reader is encouraged to work through both Appendixes *before* turning to Chapter 3. The mathematical requirements[10] for understanding Appendixes A and B are quite modest, and most of the mathematical results are translated into words.

Chapter 2 presents some of the statistical data which enter the productivity growth analyses. There is a discussion of the advantages and disadvantages of the two time periods that are studied. Chapter 3 is the key analytical chapter of this Study. It is shown that any analysis of *labour productivity* growth for an industry is very much dependent upon our "definitions" and "measurements". There are 10 possible sources of labour productivity growth – with "technological change" being only one of them. Many, but not all, of the sources of such growth are analytically tractable in the sense that their individual contributions (positive or negative) are measurable from *observable* data. Statistical data are already available to measure individually the impact of some of the sources (or parts of the sources) of labour productivity growth. It is equally enlightening to know what is not individually measured – i.e., the exact composition of the "residual". The particular estimation procedure and its justification, including limitations, are then spelled out in considerable detail. The empirical estimates of the coefficients of inter-industry labour productivity growth differentials require a careful economic interpretation. Some of these analytical results are summarized in the next section.

The *capital productivity* and *factor productivity* growth analyses of Chapters 4 and 5 are quite analogous to that of Chapter 3. Both chapters serve as a "check" on the empirical results of the labour productivity analysis, having as well some independent interest of their own. It is shown that the estimation results of all three chapters[11] are consistent with each other in the sense that certain required theoretical equalities are statistically acceptable.

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[9] The text contains a number of footnotes with mathematical and similarly technical material. These footnotes could be overlooked without loss of continuity.

[10] For example, R. G. D. Allen, *Mathematical Analysis for Economists* (London: Macmillan, 1938).

[11] That is, the estimates of the coefficients (or sources) of labour productivity growth, of capital productivity growth, and of factor productivity growth.

One of the sources of productivity growth for an industry is the so-called "resource (or factor) shift effect" within the industry. Chapter 6 illustrates this phenomenon with reference to the "total man-hours employed" and "total fixed capital stock" resource shifts between the two-digit industries of the Canadian manufacturing sector. The analysis is performed for both the 1947-56 and 1957-67 time periods, and the results are compared with those of a similar U.S. manufacturing study. The dominant theme of Chapter 7 is a familiar one to practising economists – if definitive analytical results are to be attained (say, for economic policy purposes), there is need for more and better statistical data. The chapter contains concrete research proposals for enriching and extending the results of this Study if suitable statistical data are made available.

There are four technical Appendixes. Appendix A is essentially an introduction to the foundations of productivity growth theory. The development is self-contained, and the models are relatively simple. It turns out that many of the assumptions made in Appendix A can be relaxed, and the special cases can be generalized. This is done in Appendix B, where the mathematical level is somewhat more advanced. Appendix C contains the derivation of the formulas used for calculating the compound growth rates of the ratio and product of two variables from the compound growth rates of the individual variables, when time is treated in a discrete manner. Finally, Appendix D gives the origins of all the statistical data employed in this Study. There are references to some alternative sets of data and their limitations.

1.3 Summary of Main Conclusions

The productivity growth models and methodology of this Study are adaptable to various types of empirical application. As already stated, the particular application shown here is an inter-industry productivity growth differential analysis. It is evident that such an application is best exemplified[12] where the industry productivity growth rates and the corresponding growth rates of certain explanatory variables are subject to considerable inter-industry variation. The two-digit industry aggregation level of our analysis already represents a considerable "averaging-out" of extreme growth rate observations that would normally be exhibited at, say, the three-digit level.[13] However, even within the limitations of our particular observations, some preliminary conclusions of the application are apparent.

For the period 1947-56, the inter-industry labour productivity growth rate differentials are largely explained, *on the average,*[14] by inter-industry differences in the rate of growth of "the quality of labour",[15] "the capital-intensity of production", "the size of the representative establishment", and "the total level of

[12] That is, "best exemplified" in terms of the possibility of obtaining highly statistically significant estimation results.
[13] See Sections 2.2 and 7.1 for the relevant discussion.
[14] The exact meaning of the term "on the average" is given in Section 3.3.
[15] At least to the extent that changes in "the quality of labour" are measured in this Study.

industry output". All of these elements are statistically significant positive sources, on the average, of labour productivity growth. Increasing "quality of labour" is largely associated with the growing proportion of nonproduction-labour to production-labour employment. The "capital-intensity of production" seems to be adequately indicated by the "gross" measure of fixed capital stock in the sense that an additional variable allowing for the "net" measure is statistically insignificant. The "representative establishments" of the manufacturing industries were operating (1947-56) at average output levels where increasing economies of scale were experienced. Finally, the growth of "total industry output" was found to be a significantly positive "proxy source" of labour productivity growth *over and above*[16] the impact of the other explanatory growth variables. An additional result, obtained from the factor productivity growth analysis, is that the ratio of factor shares as observed in the 1949 input-output table is a statistically acceptable approximation, on the average, to the ratio of the respective aggregate output elasticities[17] over the 1947-56 time period. All these results are invariant to whether labour input is measured in "man-hours" or "number employed".

Some partial and preliminary coefficient estimates were also attained for the second period, 1957-67. It was again found that the growth rate of "the capital-intensity of production" and "the total level of industry output" are highly significant positive sources of inter-industry labour productivity growth differentials. Other elements such as economies of scale and changes in the quality of labour and fixed capital have not yet been investigated in this context. The evidence also suggests that the ratio of factor shares as observed in the 1961 input-output table is an acceptable approximation, on the average, to the ratio of the respective output elasticities over the 1957-67 time period.

The resource shift exercise shows that differential growth rates of "total man-hours employed" and "total fixed capital stock" between the various two-digit industries make a negligible net contribution to the factor productivity growth of "Total Manufacturing". This result holds for both the 1947-56 and 1957-67 periods.

For a more complete statement of the main analytical results of this Study, the reader is referred to the relevant sections in the text[18], where some possible economic interpretations of the results are developed.

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[16] The term "over and above" has a special technical meaning in this Study (see Section 3.3).

[17] The elasticities refer to the "aggregate output elasticity with respect to aggregate labour input" and the "aggregate output elasticity with respect to aggregate fixed capital input".

[18] See particularly Sections 3.4, 4.2, 5.2, and 6.3.

CHAPTER 2

SOME DESCRIPTIVE RESULTS

The main purpose of this chapter is to present some of the data which enter the analyses of the following chapters. There is also a discussion of the reasons for choosing the particular time periods studied and the particular method of exhibiting growth rates.

2.1 The Two Basic Time Periods

As stated in the Introduction, this Study is primarily concerned with analysing medium-term productivity trends within the Canadian manufacturing sector during the postwar era. In identifying such productivity trends, it is desirable to choose individual time periods that are long enough so as not to be dominated by temporary disturbances. On the other hand, the time periods should not be so long as to be subject to changes of trend. It was therefore decided to study productivity trends for two distinct time periods of about 10 years' duration.

Trends during a particular time period are conveniently measured by calculating average annual growth rates for the period from observations on the relevant variables. There are two basic methods for estimating such growth rates:[1] (1) the "terminal-year method", whereby the initial and terminal observations of the period are manipulated to yield an average annual (compound) growth rate (the intermediate observations are ignored); and (2) the "least-squares fit method", whereby the relevant growth rate is set equal to the antilog (minus one) of the estimated coefficient of the time variable in a least-squares linear regression of the logarithm of all the annual observations (for the period) upon time. Thus the second method is less sensitive to the particular initial and terminal observations of a time period, since it yields growth rates that reflect all the observations.

Nevertheless, there are a number of reasons for working with growth rates calculated by the terminal-year method. This method produces growth rates of individual variables that can be directly combined so as to yield the growth rate of the ratio of two variables, or the growth rate of the product of two (or more) variables. (The least-squares fit method may also have this property.)[2] Such a property is particularly convenient because of the wide use of various combinations of ratio variables (and some product variables) in this Study. Of course, the least-squares fit method requires more observations and more computational time for any calculated growth rate. Also, the terminal-year method is by far the most

[1] See Appendix C for a precise formulation.
[2] Technical details are discussed in Appendix C.

commonly used method of determining medium- or long-term growth rates, so that "terminal-year results" are comparable to other studies.[3] However, if the terminal-year method is employed, it is desirable that the initial and terminal years of a chosen time period have approximately equal unemployment rates so as to minimize the effect of cyclical influences on the productivity (and other) growth rates.

In view of all this, it was decided to use the terminal-year method[4] in the analyses of postwar Canadian manufacturing productivity trends for the *two basic time periods*, 1947-56 and 1957-67. Note that 1947 and 1956 are years of virtually full employment; 1957 and 1967 are years of moderate unemployment[5].

The first postwar period is particularly homogeneous in terms of both economic structure and availability of continuous statistical time series. For example, growth rates calculated by the two methods mentioned above are quite similar, and the $\bar{R}^2$ values of the least-squares fits are close to unity. The data for this period fall naturally under the 1948 S.I.C., so that 17 two-digit manufacturing industries are analysed. On the other hand, the 1957-67 period has a number of disadvantages. It is composed of two subperiods, 1957-61 and 1961-67. The former subperiod is one of a prolonged recession; the latter, of a prolonged "boom". Therefore, growth rates calculated by either of the two methods measure a weighted average of what are essentially two different trends. This "average trend" is quite sensitive to the particular method of calculating the growth rate. Moreover, there are discontinuities in some of the statistical time series relating to this period, and much of the data are subject to revision.[6] Therefore, the results shown for the second postwar period, 1957-67, are preliminary and should be interpreted with caution. The data for this period fall under the 1960 S.I.C., and 20 two-digit manufacturing industries are discussed.[7]

2.2 A Comparison of the Important Growth Rates

The following two tables present some of the growth rates which are used in the productivity analyses of Chapters 3, 4, and 5. All the growth rates are average annual (compound) rates of change for the time period designated and are expressed as percentages. The *growth rate variables* of the industry concerned are defined as follows:

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[3] For example, see Lydall, *op cit.;* Salter, *op. cit.;* and Lithwick, Post and Rymes, *op. cit.*

[4] For comparison purposes, a number of growth rates were also calculated by the least-squares fit method.

[5] The unemployment rates in "Total Manufacturing" were 3.2, 4.5 and 4.1 per cent in 1956, 1957, and 1967 respectively. (The corresponding 1947 figure is unavailable.) There are no unemployment rate data available for the individual industries. One would expect the durable manufacturing industries to show greater unemployment rate fluctuations than the nondurable industries.

[6] For further comments, see Appendix D.

[7] Some suggestions for future research relating to the 1960 S.I.C. data are made in Chapter 7.

q is total net output in constant dollars;

l is total labour man-hours employed;

k is total gross fixed capital stock in constant dollars;

w is average hourly money earnings of labour;

e is total number of establishments;

$c = [\alpha l + (1-\alpha)\,k]$ is weighted average of total man-hours and gross capital stock, with α equal to the labour share of value of net output as observed in the 1949 input-output table (1948 S.I.C.) or in the 1961 input-output table (1960 S.I.C.);

$k/l \equiv \left[100\left(\frac{100+k}{100+l}\right) - 100\right]$[8] $\simeq (k-l)$ is gross capital stock per man-hour;

$q/l \equiv \left[100\left(\frac{100+q}{100+l}\right) - 100\right] \simeq (q-l)$ is net output per man-hour (or labour productivity);

$q/k \equiv \left[100\left(\frac{100+q}{100+k}\right) - 100\right] \simeq (q-k)$ is net output per unit of gross capital stock (or capital productivity);

$q/c \equiv \left[100\left(\frac{100+q}{100+c}\right) - 100\right] \simeq (q-c)$ is net output per unit of weighted average of total man-hours and gross capital stock (or factor productivity).

Thus, for example, the "net output" of "Food and beverages" grew at an average annual rate of 3.56 per cent for the time period 1947-56 (see Table 2-1), while "total man-hours employed" of "Petroleum and coal products" declined at an average rate of 1.36 per cent per annum for the time period 1957-67 (see Table 2-2).

For the convenience of the reader, Tables 2-3 and 2-4 are also included. These tables provide the key summary statistics of the distributions of the various growth rate variables given in the previous tables. Some descriptive results of the four tables, taken together, are as follows:

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[8] See Appendix C for the derivation of this formula for calculating the growth rate of the ratio of two variables from the growth rates of the individual variables.

TABLE 2-1

SOME SELECTED DATA USED FOR PRODUCTIVITY ANALYSIS
(Average annual growth rates from 1947 to 1956)

Industry (1948 S.I.C.)	q	l	k	w	e	α(1)	c	k/l	q/l	q/k	q/c
Total Manufacturing	5.52	1.59	4.64	7.40	1.62	0.67	2.60	3.00	3.87	0.84	2.85
Food and beverages	3.56	0.60	4.58	6.82	−1.11	0.63	2.07	3.96	2.94	−0.98	1.46
Tobacco products	5.60	−1.40	5.76	8.70	−6.23	0.62	1.32	7.26	7.10	−0.15	4.22
Rubber products	2.44	−0.56	3.99	7.10	4.74	0.79	0.40	4.58	3.02	−1.49	2.03
Leather products	0.70	−1.62	1.14	5.72	−2.24	0.79	−1.04	2.81	2.36	−0.43	1.76
Textile products	3.16	−0.70	2.83	7.06	3.40	0.67	0.46	3.55	3.89	0.32	2.69
Clothing	3.33	−0.06	2.78	5.01	−1.16	0.77	0.60	2.84	3.39	0.54	2.71
Wood products	4.31	1.02	2.76	6.93	1.46	0.74	1.47	1.72	3.26	1.51	2.80
Paper products	4.88	1.48	4.70	8.18	1.38	0.53	2.99	3.17	3.35	0.17	1.84
Printing and publishing	7.28	3.32	3.11	7.17	7.31	0.76	3.27	−0.20	3.83	4.04	3.88
Iron and steel products	5.66	1.65	4.89	7.89	3.36	0.70	2.62	3.19	3.94	0.73	2.96
Transportation equipment	5.87	3.01	2.03	7.05	0.81	0.69	2.71	−0.95	2.78	3.76	3.08
Non-ferrous metal products	4.70	2.32	4.23	7.82	1.62	0.50	3.28	1.87	2.33	0.45	1.37
Electrical apparatus and supplies	9.18	5.13	6.43	7.41	5.35	0.68	5.55	1.24	3.85	2.58	3.44
Non-metallic mineral products	9.18	4.32	8.70	7.64	3.57	0.57	6.20	4.20	4.66	0.44	2.81
Petroleum and coal products	12.08	3.20	8.17	8.85	3.28	0.44	5.98	4.82	8.60	3.61	5.76
Chemicals and products	8.70	2.83	7.25	8.10	0.87	0.55	4.82	4.30	5.71	1.35	3.70
Miscellaneous manufacturing	10.47	3.46	3.70	7.50	7.49	0.72	3.53	0.23	6.78	6.53	6.70

(1) The variable α is not a growth rate.
Note: For definition of variables, see the above text. Also see Appendix D for some alternative growth rate data for L.
Source: Based on data from Dominion Bureau of Statistics, as described in Appendix D.

TABLE 2-2

SOME SELECTED DATA USED FOR PRODUCTIVITY ANALYSIS
(Average annual growth rates from 1957 to 1967)

Industry (1960 S.I.C.)	q	l	k	w	e	α(1)	c	k/l	q/l	q/k	q/c
Total Manufacturing	5.39	1.37	4.72	4.53	−0.36	0.69	2.41	3.30	3.97	0.64	2.91
Food and beverages	4.64	0.56	4.88	4.97	−1.97	0.67	1.99	4.30	4.06	−0.23	2.60
Tobacco products	4.40	−0.92	6.75	5.80	−3.11	0.60	2.15	7.74	5.37	−2.20	2.20
Rubber products	3.57	0.68	3.62	4.23	2.21	0.67	1.65	2.92	2.87	−0.05	1.89
Leather products	1.59	−0.77	0.57	4.38	−1.49	0.95	−0.70	1.35	2.38	1.01	2.31
Textile products	6.87	0.91	2.35	4.67	0.94	0.73	1.30	1.43	5.91	4.42	5.50
Knitting mills	5.60	−0.28	1.48	4.36	1.05	0.86	−0.03	1.76	5.90	4.06	5.63
Clothing	2.74	0.25	0.49	4.07	−0.56	0.89	0.28	0.24	2.48	2.24	2.45
Wood products	4.32	−0.02	3.72	5.28	−5.66	0.80	0.72	3.74	4.34	0.58	3.57
Furniture and fixtures	5.76	2.53	4.09	4.28	1.47	0.80	2.84	1.52	3.15	1.60	2.84
Paper products	3.85	1.76	5.59	4.46	1.08	0.54	3.52	3.76	2.05	−1.65	0.32
Printing and publishing	4.04	0.92	3.72	4.89	0.60	0.78	1.54	2.77	3.09	0.31	2.46
Primary metals	4.68	1.71	5.17	4.03	0.01	0.64	2.96	3.40	2.92	−0.47	1.67
Metal fabricating	4.89	2.54	3.37	4.11	4.64	0.76	2.74	0.81	2.29	1.47	2.09
Machinery industries	8.00	3.78	4.86	4.63	4.57	0.74	4.06	1.04	4.07	2.99	3.79
Transportation equipment	6.09	1.70	5.23	4.26	3.59	0.76	2.55	3.47	4.32	0.82	3.45
Electrical products	7.64	2.32	4.22	3.55	2.43	0.78	2.74	1.86	5.20	3.28	4.77
Non-metallic mineral products	4.39	1.91	4.86	4.52	1.14	0.66	2.91	2.89	2.43	−0.45	1.44
Petroleum and coal products	5.18	−1.36	4.36	5.05	1.06	0.33	2.47	5.80	6.63	0.79	2.64
Chemicals and products	6.71	1.37	5.82	4.56	0.90	0.55	3.37	4.39	5.27	0.84	3.23
Miscellaneous manufacturing	7.16	4.03	6.58	4.67	1.83	0.73	4.72	2.45	3.01	0.54	2.33

(1)The variable α is not a growth rate.
Note: For definition of variables, see the above text. Also see Appendix D for some alternative growth rate data for l.
Source: Based on data from Dominion Bureau of Statistics, as described in Appendix D.

TABLE 2-3

SUMMARY STATISTICS FOR 1947-56 GROWTH RATES

Variable	Mean	Median	Standard Deviation	Coefficient of Variation	Skewness Coefficient
q	5.95	5.60	3.00	0.50	0.35
l	1.65	1.65	1.97	1.19	0.00
k	4.53	4.23	2.08	0.46	0.43
w	7.35	7.41	0.93	0.13	−0.19
e	1.99	1.46	3.38	1.70	0.47
α	0.66	0.68	0.03	0.05	−2.00
c	2.72	2.71	2.02	0.74	0.01
k/l	2.86	3.17	2.82	0.99	−0.33
q/l	4.22	3.83	1.75	0.41	0.66
q/k	1.35	0.54	2.04	1.51	1.19
q/c	3.13	2.81	1.39	0.44	0.69

Note: The coefficient of variation equals the standard deviation divided by the mean. The skewness coefficient is approximated as: 3 (mean-median) / standard deviation.

Source: See Table 2-1 above.

TABLE 2-4

SUMMARY STATISTICS FOR 1957-67 GROWTH RATES

Variable	Mean	Median	Standard Deviation	Coefficient of Variation	Skewness Coefficient
q	5.11	4.79	1.59	0.31	0.60
l	1.18	1.66	1.44	1.22	−1.00
k	4.09	4.29	1.71	0.42	−0.35
w	4.54	4.49	0.47	0.10	0.11
e	0.74	1.10	2.40	3.24	−0.15
α	0.71	0.74	0.17	0.24	−0.53
c	2.19	2.51	1.34	0.61	−0.72
k/l	2.77	2.57	1.68	0.61	0.36
q/l	3.89	3.62	1.37	0.35	0.59
q/k	1.00	0.81	1.96	1.96	0.29
q/c	2.86	2.53	1.28	0.45	0.77

Source: See Table 2-2 above.

First, there is a considerable diversity of growth rates over the individual industries for each of the variables (for both periods), which is concealed by the data for "Total Manufacturing". Thus, for example, the capital-output ratio[9] for "Total Manufacturing" has been virtually constant during both time periods, but there have been significant and widely dispersed movements in this ratio for some of the individual industries.

Second, the *pattern* of dispersion of the growth rate variables, as measured by the coefficient of variation, is very similar in the two time periods. For example, of the 11 variables shown in Tables 2-1 and 2-2, the growth variables e, q/k and l are the most dispersed (in that order) for both periods, while the variables α and w are the least dispersed in both periods. In fact, such a pattern of dispersion has been reported by other investigators in comparable studies.[10]

Third, the two-digit level of industrial aggregation probably conceals an even greater *degree* of growth rate dispersion that would be revealed at, say, the three-digit level. This could be indicated by comparing the coefficients of variation obtained by Lydall in his 54-industry analysis with those obtained in the 17- and 20-industry analyses of this Study.

Fourth, the unweighted arithmetic means of the distributions for each of the growth variables are usually quite similar[11] to the corresponding variables for "Total Manufacturing". It should be noted that the "Total Manufacturing" growth rates for q, l, k and e, are weighted arithmetic averages of the corresponding growth rates for the individual industries.[12] But such a relationship generally does not hold for the "composite" growth rate variables such as c, q/l, q/k and q/c. Indeed, this point is the very essence of the so-called "resource shift effect" that is analysed in Chapter 6.

Fifth, the skewness coefficients[13] are usually small, with most of the growth rate distributions having a slightly positive skew. A more sensitive measure of skewness, together with more observations at a finer level of disaggregation, is needed in order to test whether such distributions are lognormal[14] or asymmetric.

Finally, some of the more obvious implications of the growth rate data are mentioned in the next chapter in order to motivate the productivity analysis.

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[9] The growth rate of the capital-output ratio is, of course, approximately equal to the negative of the capital productivity growth rate.

[10] See, for example, Lydall, *op. cit.*

[11] Most of the exceptions occur for the 1960 S.I.C. which, as we have already noted, is subject to revision.

[12] This statement neglects a relatively small approximation error in the relationship.

[13] See G. Udny Yule and M. G. Kendall, *An Introduction to the Theory of Statistics* (New York: Hafner Publishing Company, 1950), pp. 160-163.

[14] The basic reference is J. Aitchison and J. A. C. Brown, *The Lognormal Distribution* (Cambridge: Cambridge University Press, 1957).

CHAPTER 3

AN ANALYSIS OF LABOUR PRODUCTIVITY GROWTH

In this chapter, the labour productivity growth rate model formulated in Appendixes A and B is applied to the observed statistical data. Some of these data were presented in the previous chapter. We first consider in some detail the labour productivity growth model or, more generally, the sources of labour productivity growth for an industry. It is seen that many sources of such growth can be identified with observed growth rate differences between particular variables. But there are other sources of labour productivity growth for an industry that are difficult to quantify and must be either ignored[1] or crudely approximated by surrogate growth rate variables. We then note that the labour productivity growth model is adaptable to various types of empirical application. The reasons for the particular application shown in this Study are given. Some technical problems relating to this application are discussed. It is important to understand the correct economic interpretation of the applied productivity model and the corresponding estimates. A complete set of estimates and related statistical tests are given for the time period 1947-56. The estimates for the period 1957-67 are incomplete, for reasons explained in the text.

In Appendix A it is shown that the labour productivity growth model is quite analogous to that of the capital productivity and factor productivity growth models. Therefore, much of the discussion in this chapter also serves as an introduction to the analyses of the following two chapters. Finally, the mathematical footnotes used in this chapter are incidental and may be overlooked without loss of continuity.

3.1 The 10 Sources of Labour Productivity Growth

The concept of "output" used throughout this Study is that of real net output, or real value added. The methods used to measure this concept of output and its theoretical basis have been discussed by other writers and need not detain us here.[2] The level of labour productivity for a manufacturing industry at a particular time is *defined* simply as the ratio of the total output of the industry to total labour man-hours employed in that industry. This definition seems to conform to common usage of the term "labour productivity". One can now imagine many

[1] That is, classified with the "residual".

[2] See Dominion Bureau of Statistics, *Indexes of Real Domestic Product by Industry of Origin*, Cat. No. 61-505 (Ottawa: Queen's Printer, 1963); and C. A. Sims, "Theoretical Basis for a Double Deflated Index of Real Value Added", *Review of Economics and Statistics*, November 1969, pp. 470-471.

reasons why the level of labour productivity for an industry may change over time. In particular, one may suspect that relatively high labour productivity growth rates are associated with relatively high capital-intensity[3] growth rates, and conversely. Thus, in the context of the data presented in Chapter 2, one may wish to test whether those industries which experienced relatively high labour productivity growth rates were generally the same industries that experienced relatively high capital-intensity growth rates for the two time periods considered. (The answer is yes!) Thus the growth of capital-intensity could be a possible "source" of labour productivity growth.

However, a deeper and more systematic analysis of the sources of labour productivity growth for an industry may proceed along the following lines. Underlying the concept of industry labour productivity are the production relationships (or production functions) for each of the component manufacturing establishments of the industry. The aggregate output (index) of an industry reflects the distribution of the quantities of the various categories of output produced by the individual establishments, so that aggregate output itself is a function[4] of the distribution of the various types of labour man-hours employed; the amounts of the various types of fixed capital stock utilized; and the distribution of "other difficult-to-specify inputs" among the establishments. Seen in this light, the following is a list of possible sources of industry labour productivity growth: (1) changes in the quality of labour employed in the industry; (2) changes in the quality of output produced by the industry; (3) the growth of capital-intensity of production for the industry; (4) changes in the quality of fixed capital stock utilized in the industry; (5) changes in the average size of the manufacturing

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[3] The "capital-intensity" of production for an industry is *defined* simply as the ratio of total fixed capital stock to total labour man-hours employed in that industry.

[4] This function is the so-called *industry production function.* More precisely, suppose that the industry is composed of two establishments. Let $f_1(\ )$ be the production function for establishment number one, so that

$$Q_1 = f_1(L_{11}, L_{21}, K_{11}, K_{21})$$

where Q_1 is output; L_{11} (L_{21}) is man-hours employed of first (second) type of labour; K_{11} (K_{21}) is amount of capital stock utilized of first (second) type of fixed capital—all of establishment number one. Similarly, let $f_2(\ )$ be the production function for establishment number two, so that

$$Q_2 = f_2(L_{12}, L_{22}, K_{12}, K_{22})$$

where the symbols are defined analogously to those of establishment number one. Then the basic assumption of any industry production function analysis is that there exist functions ψ, g, h and f, such that

$$\psi\,[f_1(L_{11}, L_{21}, K_{11}, K_{21}),\ f_2(L_{12}, L_{22}, K_{12}, K_{22})\,]$$
$$= f[g(L_{11}, L_{21}, L_{12}, L_{22}),\ h(K_{11}, K_{21}, K_{12}, K_{22})\,]$$

for all non-negative values of L_{ij}, K_{ij} ($i,j = 1,2$). Thus

$$Q^* = \psi(Q_1, Q_2)$$

is the aggregate output of the industry;

$$L^* = g(L_{11}, L_{21}, L_{12}, L_{22})$$

is the aggregate labour input of the industry;

$$K^* = h(K_{11}, K_{21}, K_{12}, K_{22})$$

is the aggregate fixed capital input of the industry; and the function $f(\ )$ in

$$Q^* = f(L^*, K^*)$$

is the industry production function. For an example of the conditions under which such functions exist, see Harry H. Postner, *Estimation of the Elasticity of Capital-Labour Substitution in Postwar Canadian Manufacturing Industries,* unpublished Ph.D. thesis, University of Minnesota, 1970, Chapter 4.

establishments that comprise the industry; (6) changes in the distribution of the various types of labour employed among the establishments; (7) changes in the distribution of the various types of fixed capital utilized among the establishments; (8) changes in the distribution of the various categories of output produced among the establishments; (9) the growth of "other inputs" of the industry and its establishments; and finally (10) shifts in the production relations of the manufacturing establishments. We will now discuss, in turn, each of the above *10 possible sources* of industry labour productivity growth. In doing so, frequent reference will be made to the components of the labour productivity growth rate model that is formulated in Appendixes A and B.

ONE – Recall that the definition of "labour productivity" given above does not distinguish between different types of labour employed. In effect, all labour is treated as if it were homogeneous. However, it is well known that some types of labour are "more productive" than other types and thus should be given a greater weight in measuring the correct "aggregate labour input" of an industry. Indeed, the growth rate of the correct aggregate labour input could be written as the summation of two expressions: (1) the growth rate of the unweighted total of the various types of labour employed; and (2) an expression that measures the growth rate of the "quality of labour employed". The latter expression has a particularly simple meaning–we say that the quality of labour employed in an industry has "increased" if and only if the employment of the "more productive" types of labour has a higher growth rate than the employment of the "less productive" types.[5] Similarly, we could measure a "decrease" in the quality of labour employed. Thus, e.g., if the quality of labour employed in an industry has increased, the use of the unweighted total of labour employed (in the measurement of labour productivity) will underestimate the actual labour input. In this sense, an increase (decrease) in the quality of labour employed in an industry is a positive (negative) source of labour productivity growth.

It should be noted that the above "quality change" source of labour productivity growth does *not* distinguish between the different establishments that comprise the industry. Thus, e.g., any particular type of labour (cross-classified by education, age, sex, occupation, etc.) would be treated as if it were homogeneous[6], even though employment occurs in different establishments.

TWO – The total output of an industry is measured in constant prices. This means that the quantity (in physical units) of each category of output is multiplied by its observed unit price at some chosen base year, and the unweighted total of the value (in constant prices) of the various types of output produced is the relevant measure of industry total output in the definition of labour productivity. In effect, it is implicitly assumed that relative base-year prices correctly reflect the

[5] See Sections A.2, B.1, and B.2 for more precise definitions. Briefly, e.g., the term "more productive" should be interpreted as meaning "*marginally* more productive" (at the particular combination of inputs concerned).

[6] That is, aggregated by taking the unweighted total of labour employed for that particular type of labour.

corresponding marginal rates of transformation[7] of the different outputs measured in physical units. If this condition does not hold, then the growth rate of the correct "aggregate output" of an industry may not equal the growth rate of the observed industry total output. The difference between the two growth rates represents the rate of change of the "quality of output produced". Thus, e.g., the quality of industry output has "increased" if and only if the amount produced of the "relatively underpriced" types of physical output has a larger growth rate than the amount produced of the "relatively overpriced" types.[8] In the latter case, the growth rate of total industry output will underestimate the growth of the correct aggregate output measure. Therefore, in contrast to the labour quality change discussion, an increase (decrease) in the quality of output produced by an industry is a negative (positive) source of labour productivity growth *as defined.*

THREE – We have already defined the capital-intensity of production for an industry as the ratio of total *fixed capital* stock (valued in constant prices) to total man-hours employed in that industry. Implied in this definition are two assumptions: (1) the aggregate *working capital*[9] input of an industry, as a factor of production, is complementary to both the aggregate labour input and the aggregate fixed capital input in the industry production function; and (2) the aggregate labour input and fixed capital input are always "scarce" relative to working capital input.[10] These assumptions allow us to disregard the role of working capital in discussing labour productivity growth. Considering the nature of working capital stock, the assumptions are not unreasonable and are highly convenient for the purposes of this Study.

It is well known that an increase (decrease) in the capital-intensity of production is a positive (negative) source of labour productivity growth for an industry. What is not known is how to measure changes in capital-intensity or, more particularly, the growth rate of total fixed capital stock. There are two relevant measures of fixed capital stock[11]–*net* fixed capital stock and *gross* fixed capital

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[7] See James M. Henderson and Richard E. Quandt, *Microeconomic Theory* (New York: McGraw-Hill, 1958), pp. 67-75.

[8] See Section B.5 for a more precise definition.

[9] Fixed (or durable) capital consists of physical plant, machinery and equipment. Working capital includes stocks of goods which are themselves a product of the production process. Thus working capital consists of "stocks of goods in process" and "warehouse stocks of finished goods". On all this, see Trygve Haavelmo, *A Study in the Theory of Investment* (Chicago: University of Chicago Press, 1960), for one of the rare treatments of working capital as a factor of production.

[10] More precisely, the assumptions amount to first supposing that the industry production function at time t could be written as

$$Q^* = \min[f(L^*, K^*; t),\ \phi(C^*; t)]$$

where Q^*, L^*, K^* have the same meaning as before; C^* is the aggregate working capital input of the industry; and the explicit variable t allows for shifts over time in the functions f and ϕ. Thus the formulation is general enough to permit shifts in storage operations and changes in the period of production. It is further supposed that

$$\phi(C^*; t) > f(L^*, K^*; t)$$

for all L^*, K^*, C^* and t, so that the relevant industry production relation is simply

$$Q^* = f(L^*, K^*; t).$$

[11] This is not the place to discuss "survival curves" and "depreciation formulas"; see Appendix D.

stock. The correct growth rate of any particular type of fixed capital is always some unknown weighted average of the growth rates of its net and gross measures. It turns out[12] that the capital-intensity source of labour productivity growth can be expressed as the summation of two terms–one term representing the gross stock of fixed capital-intensity source, and the other term accounting for a possible measurement error in neglecting the net stock of capital measure. Thus, e.g., if the growth rate of total net stock of fixed capital is greater than that of total gross stock of fixed capital, then the gross capital-intensity source of labour productivity growth will underestimate the correctly measured capital-intensity of production source of this growth.

FOUR – The definition of capital-intensity of production given above does not distinguish between the different types of fixed capital utilized in the industry. This is equivalent to assuming that relative base year *asset* prices[13] are equal to the corresponding ratios of the marginal productivities of the different capital items measured in physical units.[14] However, the observed relative prices reflect not only the relative marginal productivities, but the relative *durabilities* of the various types of fixed capital as well. Indeed, for any two capital items (measured in physical units), the ratio of their marginal productivities will *tend* to be greater than the corresponding ratio of their constant asset prices if the first type of fixed capital is "less durable" than the second type.[15] In this case, the marginal product of the first type of capital will be greater than that of the second type when they are both measured in constant prices, and we say that the first type of capital is "more productive" than the second type. More generally, e.g., we say that the "quality of fixed capital stock utilized" in an industry has "increased" if and only if the growth rate of the "more productive" types of fixed capital is larger than the growth rate of the "less productive" types.[16] Again, in this case, the use of the unweighted total of fixed capital stock in constant prices, in the measurement of capital-intensity of production, will underestimate the contribution of the correct aggregate fixed capital input. Therefore, an increase (decrease) in the quality of fixed capital stock utilized in an industry is a positive (negative) source of labour productivity growth.

Before continuing, two comments are in order. First, the above discussion overlooked the fact that the correct measure of the individual fixed capital stock items is generally unknown.[17] Fortunately, it turns out that the fixed capital quality change expression can be written as the summation of *two* further expressions–one representing the growth rate of the "quality of *net* fixed capital stock utilized" and the other representing the growth rate of the "quality of *gross* fixed capital stock utilized".[18] Indeed, e.g., the quality of net (gross) fixed capital

[12] See Sections A.3 and B.2 for further details.

[13] Recall that total fixed capital stock is valued in constant asset prices.

[14] The reasoning is analogous to that spelled out in Section B.5.

[15] See Haavelmo, *op. cit.*, pp. 97-102, for a clear account of the problem of aggregating fixed capital of different durabilities.

[16] See Sections A.3 and B.2 for the technical details.

[17] That is, the correct *weights* to be given to the net and gross measured growth rates are unknown.

[18] See again Sections A.3 and B.2 for a more complete discussion.

stock utilized in an industry has "increased" if and only if the growth rate of the net (gross) stock of the "more productive" types of net (gross) fixed capital is larger than the growth rate of the net (gross) stock of the "less productive" types. Also, if two types of fixed capital are "net measured" by the same depreciation formula *and* if the correct weights to be given to their net and gross measured growth rates are the same, then the ratio of the marginal productivities of the two types of net fixed capital would equal the corresponding ratio for the gross fixed capital measure. Thus we may often expect the two quality change expressions to move in similar directions.

The second comment is that the above two "quality change" sources of labour productivity growth do not distinguish between the various manufacturing establishments that comprise an industry. This means that any particular type of fixed capital stock (cross-classified by machinery or structure, durability, vintage, etc.) would all be treated as if it were homogeneous, even though utilization occurs in different establishments.

FIVE–Suppose that the typical or "representative" establishment of an industry is operating at an output level where it exhibits increasing (decreasing) economies of scale with respect to its *own* labour and fixed capital input. Then it is possible to show that an increase (decrease) in the "size of the establishment", as measured by the total of the various types of labour employed in the establishment, will raise the labour productivity level of the establishment.[19] Now the total labour employed in a typical establishment is defined as the ratio of the total labour employed in the industry over the total number of establishments that comprise the industry. Similarly, the labour productivity level of a typical establishment is the ratio of the total output of the industry per establishment over the total labour employed in the industry per establishment. Clearly, the labour productivity level of the typical establishment is then identical to that of the industry. Thus, e.g., an increase (decrease) in the average size of the establishment, as measured above, is a positive (negative) source of industry labour productivity growth, when the typical establishment of the industry exhibits *increasing* internal economies of scale.[20]

SIX–The sixth source of labour productivity growth distinguishes the individual establishments of the industry that employ any particular type of labour.[21] It is well known that a particular type of labour may be "more productive" when employed in one establishment than in another. Thus, e.g., a redistribution of employment among the establishments for *any* particular type of labour is a positive (negative) source of labour productivity growth for an industry if and only if the growth rate of its employment in the establishments where the particular labour type is "relatively more productive" is higher (lower) than the

[19] This relationship holds, *ceteris paribus.* See Sections A.4, A.5, and B.4 for the technical discussion.

[20] If there are *decreasing* economies of scale, substitute "positive" for "negative" and "negative" for "positive" in this statement.

[21] See again the discussion of the "first" source of productivity growth.

employment growth rate where the labour type is "less productive". Such an employment redistribution is sometimes referred to as a "resource shift effect"[22] but could be regarded as just a special case of "labour quality change". The employment redistribution and its effect on productivity growth occurs for each particular type of labour.

SEVEN–Similarly, we could distinguish the individual establishments that utilize any particular type of fixed capital stock.[23] So, e.g., a redistribution among the establishments of *any* particular type of fixed capital is a positive (negative) source of industry labour productivity growth if and only if the growth rate of its stock utilized in the establishments where the particular fixed capital type is "relatively more productive" is higher (lower) than the corresponding growth rate where the fixed capital type is "less productive". Such a fixed capital "resource shift effect" is further discussed in Chapter 6. It should be noted that the redistributive aspect of so-called resource shift effects is purely a result of differential growth rates and does not necessarily imply any physical transfer of fixed capital stock or labour from one establishment to another.

EIGHT–At any point in time, the individual establishment produces a number of different categories of output. If, at some later date, the establishment produces a smaller number of output categories, we say that output specialization within the establishment has "increased". Now it is possible for output specialization within an *industry* to remain constant or even decrease, while output specialization within each of its component establishments increases. This occurs when there has been an appropriate redistribution among the establishments of the various categories of output produced by an industry. It is often stated[24] that such a redistribution is a positive source of labour productivity growth for an industry. In contrast to the previous seven sources of labour productivity growth, the writer has been unable to explicitly formulate such an output redistribution effect in a consistent framework–e.g., one provided by the supposition of an industry production function. The main problem seems to be the mathematical difficulty of handling "zero values". The writer welcomes suggestions for the solution of this problem.[25] Nevertheless, it may be possible to simulate the effect of an increase in output specialization by the use of certain proxy variables. This is explained in the next two sections, where a stepwise regression procedure is worked out.

NINE–There are "other inputs" of an industry (and its establishments) that are normally not accounted for by the *current own* aggregate labour input and the *current own* aggregate fixed capital input of the industry. These include:[26] public goods and service externalities; technological external economies or diseconomies;

[22] This phenomenon is discussed in further detail in Chapter 6.
[23] See again the discussion of the "fourth" source of productivity growth.
[24] See the references in the next section.
[25] Dr. L. Auer of the Economic Council staff has recommended a parametric programming approach to this problem.
[26] Such matters are discussed in F. M. Bator, "The Anatomy of Market Failure", *Quarterly Journal of Economics,* August 1958, pp. 351-379; and K. J. Arrow, "The Economic Implications of Learning by Doing", *Review of Economic Studies,* April 1962, pp. 155-173.

cumulative gross investment; and cumulative production output. Each of these elements is an "input variable" in a more completely specified industry production function. However, their impact on labour productivity growth is difficult to formulate and quantify, thus requiring special investigation. In this Study, the "other inputs" are part of the "unknown" sources of industry labour productivity change.

TEN–The tenth and final source of labour productivity growth is merely all other such sources not accounted for by the previous nine sources. It is by definition a "residual". Changes in this residual over time are often referred to as "technological change". More specifically, technological change includes changes in the organization of production[27] with existing types of labour and fixed capital input, and the introduction of new methods of production[28] requiring new types of labour and fixed capital. Such phenomena are also difficult to formulate and quantify but could be crudely approximated by allowing for a time shift parameter in the production relationships of the establishments. Then the industry production function would also contain an explicit time variable as one of its "input variables". This variable is supposed to account for "technical change" in the industry production function. An "improvement" in technology is by definition a positive source of labour productivity growth.

Two concluding remarks on this final source of labour productivity change are in order. First, the double-deflation method of measuring real net output has the disadvantage of assigning *any* increase or decrease in *gross* output due to "technical change" to *net* output.[29] It is important to remember that at least some "technical change" should be associated with raw materials and intermediate inputs. Second, it is clear that our formulation of "technical change" is general enough to encompass "non-neutral disembodied technical change".[30] But it is also sufficiently general to include "capital-embodied technical change" *if* gross investment is assumed to grow at a steady exponential rate.[31]

3.2. Application of the Labour Productivity Growth Model

In the previous section, we outlined the possible sources of labour productivity growth for an industry. Frequent reference was made to the related labour productivity growth rate model[32] that is formulated in Appendixes A and B. The problem now is to apply the model to the available statistical data. More precisely, we wish to test (say, by statistical methods) the significance, and estimate

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27 This is the field of industrial engineering.

28 This is the field of applied research and development.

29 See the discussion in Sims, *op. cit.*

30 These well-known concepts are fully discussed in Murray Brown, *On the Theory and Measurement of Technological Change* (Cambridge: Cambridge University Press, 1966).

31 This is shown in R. J. Gordon, *Measurement Bias in Price Indexes for Capital Goods*, Paper presented at Eleventh Congress of International Association for Research in Income and Wealth, Israel, 1969.

32 The exact relationship of the model to the 10 sources of growth will be apparent in the course of this section.

the relative importance, of the various (known and "unknown") sources of labour productivity growth. It will be seen that the formal growth model is adaptable to various possibilities of empirical application. There are *four such possibilities.* The first three are discussed briefly; the fourth, which is the one actually applied, is explained at greater length.

The *first possibility* is essentially a nonstatistical method application. For each industry, it involves the assumption that the various types of labour employed and fixed capital stock utilized are always paid a service price equal to the respective value of their marginal products. If the particular service prices are observed, if constant returns to scale are assumed, and if the gross stock measure of fixed capital is used, then sources number one, three, and four of labour productivity growth are directly obtained.[33] These methods of "adjusting for labour quality change" and "adjusting for capital quality change" are associated with the names of Denison[34] and Griliches-Jorgenson,[35] respectively. In this Study, it is preferred to *test* the above assumptions by noting the implications of statistical estimates. Our estimates of sources of productivity growth are therefore "unbiased" in the sense that they do not depend on possibly false assumptions. On the other hand, the estimates can be "inefficient" in the sense that they do not make use of possibly relevant information.

The *second possibility* is a time series analysis for *each* industry. This would involve using the annual observations for the variables to obtain a time series of annual growth rates. With suitable and sufficient data, we could then estimate the sources of labour productivity growth over a particular time period for each industry. Unfortunately, the "suitable and sufficient" data are not available. It is

[33] That is, obtained in so far as the statistical data are available. For example, one term in the "labour quality change" expression derived in Section B.2 is

$$(1) \qquad \left(\frac{NP}{Q^*L}\right)\left(\sum_i \frac{N_i}{N}\frac{\partial Q^*}{\partial N_i} - \sum_j \frac{P_j}{P}\frac{\partial Q^*}{\partial P_j}\right)\left(\frac{\dot{N}}{N} - \frac{\dot{P}}{P}\right)$$

where the symbols are defined in the Appendix. Suppose that each type of labour is paid the value of its marginal product, so that

$$(2) \qquad q\frac{\partial Q^*}{\partial N_i} = s_i \text{ and } q\frac{\partial Q^*}{\partial P_j} = w_j$$

where q is the price index of aggregate (real) output, s_i is the (money) salary rate of the i-th type of nonproduction labour, w_j is the wage rate of the j-th type of production labour. Then substituting (2) in (1), it is found that the term becomes

$$(3) \qquad \left[\frac{NP}{(qQ^*)L}\right]\left(\frac{1}{N}\sum_i N_i s_i - \frac{1}{P}\sum_j P_j w_j\right)\left(\frac{\dot{N}}{N} - \frac{\dot{P}}{P}\right),$$

where $N^{-1}\sum_i N_i s_i$ is the observed average salary rate and $P^{-1}\sum_j P_j w_j$ is the observed average wage rate. Thus the contribution of this term to labour quality change can be obtained directly under the stated assumption.

[34] Edward F. Denison, *The Sources of Economic Growth in the United States and the Alternatives Before Us* (New York: Committee for Economic Development, 1962).

[35] D. W. Jorgenson and Zvi Griliches, "The Explanation of Productivity Change", *Review of Economic Studies,* July 1967, pp. 249-282.

well known that such time series estimates would be affected by multicollinearity and would be sensitive to cyclical fluctuations, so the underlying sources of growth may be concealed. Even if the data (or the estimation procedure) could be adjusted to allow, say, for cyclical and other short-term influences, we would still be faced with one basic problem–namely, the unreliability of the available capital stock data to reveal *annual* changes.[36] Because of the crucial role played by the various fixed capital stock growth rates in this Study, it was decided not to attempt such an annual time series analysis.[37]

The *third possibility* is a regional cross-section trend analysis for *each* industry. This would involve obtaining regional data for each of the industries. The trend growth rates[38] of the regional variables (say, over a 10-year period) would form the required observations for such an analysis. Unfortunately, the necessary real output and capital stock data are not available. Even if they were available, there are too few official regions or provinces in Canada to provide sufficient data for estimation.[39] Thus this application is not feasible. However, some future research possibilities with regional data are mentioned later (Chapter 7).

The *fourth possibility* (and the one actually applied) is an inter-industry cross-section trend analysis for the manufacturing sector. This means that the trend growth rates (over a nine- or ten-year time period) for all the industry variables are the required observations. The growth rates of each of the industry variables constitute one *set* of observations in this analysis. Now, in Chapter 2 it was seen that labour productivity trend growth rates differ from industry to industry. The main problem, then, is to test the significance, and estimate the relative importance, of the sources of *inter-industry* labour productivity growth rate *differentials.* For example, in the previous section, it was stated that an increase in the capital-intensity of production is a positive source of labour productivity growth for an industry. Thus we should expect to find[40] that the industries that experienced high (low) labour productivity growth rates were generally the same industries that experienced high (low) capital-intensity rates of growth. In this sense, inter-industry differentials in the growth rates of the capital-intensity of production "should" be a significantly positive source of inter-industry labour productivity growth rate differences. Similarly, e.g., inter-industry "quality of labour" growth rate differentials may be a positive source of labour productivity growth rate differences from industry to industry. Furthermore, the labour productivity growth model of Appendixes A and B shows that it is possible to formulate and quantify many of

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[36] See Appendix D for the methods used to develop capital stock data. These methods are only appropriate for measuring medium-term *trend* growth rates.

[37] It should also be noted that annual observations on factor shares for two-digit manufacturing industries are not available. These observations would be required for a factor productivity time series analysis.

[38] The methods for calculating trend growth rates were explained in Chapter 2.

[39] This is in contrast to the situation in the United States where an interstate cross-section analysis is certainly feasible. See, e.g., G. H. Hildebrand and T. C. Liu, *Manufacturing Production Functions in the United States, 1957* (Ithaca, New York: Cornell Studies in Industrial and Labour Relations, No. 15, 1965).

[40] That is, "expect to find" *ceteris paribus.*

the sources of labour productivity growth for an industry. Hence we should be able to discover the relative importance of those sources which are responsible for the inter-industry labour productivity growth rate differentials.

Generally speaking, suitable statistical data are available to calculate the trend growth rates for the required industry variables.[41] An inter-industry (cross-section) trend analysis for the manufacturing sector is certainly feasible, and is the particular method applied in this Study. There are a number of technical problems relating to this method. These problems will become apparent in the remaining discussion of this chapter. Right now, it is convenient to state the basic applied equation of the labour productivity growth analysis and to explain briefly the included variables. It should be noted that the basic equation uses all the labour data and fixed capital stock data that are currently available at the two-digit manufacturing level.

The *basic applied equation* is[42]

$$\begin{aligned}(q/l) \;=\;& a_1(n/p) + a_2(n^m/n^f) + a_3(p^m/p^f) \\ &+ a_4(k^g/l) + a_5(k^n/k^g) + a_6(m^g/s^g) \\ &+ a_7(m^n/s^n) + a_8(l/e) + a_9(q) + a_0\end{aligned}$$

where all the variables are trend growth rates over a particular time period. The growth rate variables for the industry concerned are expressed as average annual percentage rates of change and are defined as follows:

- (q/l) is the labour productivity growth rate;
- (n/p) is the growth rate of the ratio of total nonproduction labour employment to total production labour employment;
- (n^m/n^f) is the growth rate of the ratio of total male nonproduction labour employment to total female nonproduction labour employment;
- (p^m/p^f) is the growth rate of the ratio of total male production labour employment to total female production labour employment;
- (k^g/l) is the gross fixed capital-intensity of production growth rate;
- (k^n/k^g) is the growth rate of the ratio of total *net* stock of fixed capital to total *gross* stock of fixed capital;
- (m^g/s^g) is the growth rate of the ratio of total gross stock of machinery and equipment to total gross stock of buildings and structures;

[41] Some limitations of the available data are discussed later in this chapter and in Appendix D. Also see Chapter 7 on suggestions for further research.

[42] This equation is the applied discrete counterpart of the theoretical continuous labour productivity growth model that was formulated in Section A.5 (equation 46). Also, a word on notation. Let $X(t)$ or just X denote a variable at time t. Then its rate of growth (at t) is denoted by $(dX/dt)/X$, which is often represented simply by x. In our context, x would be the average annual (compound) growth rate of X over the time period designated. Then

$$(x/y) \equiv \left[100\left(\frac{100+x}{100+y}\right) - 100\right] \simeq (x-y) \; ,$$

for any similarly defined y. See again Chapter 2 and Appendix C.

(m^n/s^n) is the growth rate of the ratio of total net stock of machinery and equipment to total net stock of buildings and structures;

(l/e) is the growth rate of the ratio of total labour employment to total number of establishments;

(q) is the total net output growth rate.

And,

a_i are unknown coefficients. $(i = 0,1,\ldots,9)$

We will now explain the relationship of the above basic applied equation to the "10 sources" of labour productivity growth.

The first three terms on the right-hand side of the equation account for *part of*[43] "labour quality change"—source number one of labour productivity growth. Thus, e.g., if nonproduction labour employment has been "typically more productive"[44] than production labour employment over the time period designated[45] then the coefficient a_1 will be positive. Furthermore, if the growth rate of nonproduction employment is greater than that of the production type, then (n/p) is also positive. In this case, the first term, $a_1(n/p)$, is a positive source of labour productivity growth. It should be noted that other writers have also introduced nonproduction labour and production labour as separate variables in productivity-type studies as a means of catching important changes in the *skill composition* of the labour force.[46] An analogous interpretation should be given to the terms a_2 (n^m/n^f) and a_3 (p^m/p^f) as well. The question of how much "labour quality change" is actually accounted for by the three terms is an empirical one. This is discussed in the next two sections, together with the presentation of the empirical estimates.

The fourth and fifth terms in the basic equation reflect the growth of capital-intensity of production (source number three). The coefficient a_4 is always positive, so that an increase in the gross capital-intensity is a positive source of labour productivity growth. The coefficient a_5 is also positive,[47] so that the

[43] Exactly which part of "labour quality change" is theoretically accounted for by these three terms is explained in Section B.2. Briefly, e.g., the terms do not account for intra-male nonproduction labour quality change. Also, note that the term with (n/p) already takes into account the cross-classified "possible terms" with variables (n^m/p^m), (n^f/p^f), (n^m/p^f) and (n^f/p^m). See Section B.2.

[44] The precise meaning of "typically more productive" in this context is given in Section B.2.

[45] The applied equation is a discrete approximation to the continuous productivity model of the Appendix. In the spirit of such an approximation, each unknown coefficient of the equation should be thought of as an average of its values, in the related continuous model, over the time period concerned. Consult Christensen and Jorgenson, *op. cit.*, pp. 26-27, for further details.

[46] This is the main argument in G. E. Delehanty, *Non-Production Workers in U.S. Manufacturing* (Amsterdam: North-Holland Publishing Company, 1968). He shows that (n/p) is a key indicator of the growth of highly trained technical and professional employment. See also Hildebrand and Liu, *op. cit.*, p. 50.

[47] Unless the net stock measure should receive a zero weight for all types of fixed capital; see Section A.3. An alternative interpretation of the term $a_5(k^n/k^g)$ is given in Griliches, *op. cit.*, p. 281.

particular contribution of the fifth term depends on the relative growth rates of the net stock and gross stock measures of total fixed capital.

The sixth term, $a_6(m^g/s^g)$ is *part of*[48] the "gross fixed capital quality change" source (number four) of labour productivity growth. Thus, e.g., if the gross stock of machinery and equipment (measured in constant prices) has been "typically more productive"[49] than the gross stock of buildings and structures (also measured in constant prices) over the time period concerned, then the coefficient a_6 will be positive. In this case, the term makes a positive contribution to labour productivity growth if $(m^g/s^g) > 0$. Again, it should be noted that other writers have introduced machinery capital stock, and structures capital stock, as distinct variables in productivity studies as a means of catching major changes in the *durability composition* of total fixed capital stock.[50] A completely analogous interpretation should be given to the "net fixed capital quality change" term $a_7(m^n/s^n)$.[51] These terms are further discussed, together with the empirical estimates.

The eighth term, $a_8(l/e)$, could be identified with the economies-of-scale source (number five) of labour productivity growth. The magnitude and sign of the coefficient a_8 generally depend on the output of the typical establishment of the industry. If, e.g., the average output over the time period concerned was such that the typical establishment exhibited increasing economies of scale, then a_8 will be positive. In this case, an increase in the size of the typical establishment (i.e., $(l/e) > 0$) would be a positive source of labour productivity growth.[52] This method of measuring the contribution of economies of scale has been applied by other writers as well.[53]

The ninth term, $a_9(q)$, has a special status in this Study, which must be explained. The idea is that those industries which have relatively high total output growth rates should generally be the same industries which have relatively high labour productivity growth rates *over and above* that part explained by the "conventional" sources of labour productivity growth. The major reason for this is that the growth of industry output is often accompanied by increases in the degree of output specialization at the establishment level.[54] The latter phenomenon is

48 See Section B.2.

49 See Section B.2.

50 This is in essence the nonestimation approach of Griliches, *op. cit.*, pp. 314-315 and 337-338. For the Canadian manufacturing sector, machinery and equipment has an assumed expected life span of about one-half that of buildings and structures. See also J. E. La Tourette, "Aggregate Factors in the Trends of Capital-Output Ratios", *Canadian Journal of Economics*, May 1970, pp. 265-266.

51 Note that "possible terms" with (m^n/m^g) and (s^n/s^g) are already accounted for in the term with (k^n/k^g). See Section A.3 for the precise formulation.

52 This is worked out in Sections A.5 and B.4.

53 See, e.g., Griliches, *op. cit.*, pp. 304-308, who considers various size classifications of establishments in order to simulate the effect of variable homogeneity. We do not have the required capital stock data to carry out such an analysis.

54 The classic explanation of this matter seems to be George J. Stigler, "The Division of Labor Is Limited by the Extent of the Market", *Journal of Political Economy*, June 1951, pp. 185-193. A closely related statement is in P. J. Verdoorn, "Complementarity and Long-Range Projections", *Econometrica*, October 1956, p. 434.

source number eight of labour productivity growth. In effect, the variable q in $a_9(q)$ is a proxy variable for "increased specialization",[55] and we should expect a_9 to be positive. However, as a proxy variable, it should not receive the same "priority" as the other well-defined variables in the estimation procedure. Hence, a stepwise regression procedure is applied in the next section.

The tenth term, a_0, is, of course, the residual–namely, the difference between (q/l) and the summation of the first nine terms on the right-hand side of the basic applied equation. As such, it contains the following sources (or parts of sources) of labour productivity growth: (1) that part of source number one not already accounted for; (2) all of source number two; (3) that part of source number four not accounted for; (4) all of source number six; (5) all of source number seven; (6) that part of sources number eight, nine, and ten which is not accounted for by the use of the proxy variable q; and finally (7) the interaction effects of the various sources of productivity growth due to the application of a discrete approximation to a continuous model.[56] Two comments are now in order.

First, it was noted that all of the "output quality change" source of labour productivity growth is contained in the residual. The reason for this is that a possible output quality change term in the basic applied equation would be only theoretically relevant in the inter-industry context of this Study.[57] The contribution of this term could be minimized by choosing constant output prices close to the middle of each of the time periods analysed. Second, it may appear that the residual includes "so much" as to "dominate" the terms that were specified in the basic applied equation. However, this is strictly an empirical matter. If this were true, it will show up in the empirical estimates and related statistical tests. To this matter, we now turn.

3.3 Estimation Procedure and Statement of Empirical Results

In this section, we first present the stochastic counterpart of the exact basic applied equation that was developed in the previous section. A standard estimation procedure is then applied. Some technical problems relating to the applied procedure in an inter-industry trend analysis context are mentioned. It is important to have a well-defined criterion for selecting the "best" of the estimated equations.

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[55] The variable q could also approximate part of source number nine of productivity growth. For example, if output grows at a constant exponential rate, then the rate of growth of current output is equal to the growth rate of *cumulative* output. The latter could be a "learning input variable" in a more complete productivity analysis.

[56] It should be noted that the growth rates of interaction variables are of a smaller order of magnitude than the included noninteraction growth rate variables. Also, our method of calculating the trend growth rates takes into account the compounding effects of ratio and product growth rates. For further details, see R. W. Conley, "Some Remarks on Methods of Measuring the Importance of Sources of Economic Growth", *Southern Journal of Economics,* January 1969, pp. 224-230.

[57] Briefly, the application of an output quality change term requires a distinction between different categories of output (see Section B.5). Such a distinction is nonmeasurable across industries, in contrast to distinguishing, say, machinery capital stock from structures capital stock. However, an output quality change term certainly is feasible in a time-series analysis or regional cross-section analysis for any particular industry.

Finally, the empirical results are stated in a number of tables. The economic interpretation of the empirical estimates is offered in Section 3.4.

For each industry in the analysis, we have the basic labour productivity growth equation of the previous section–namely,

$$(1)\quad (q/l)_j = a_{0j} + a_{1j}(n/p)_j + a_{2j}(n^m/n^f)_j + a_{3j}(p^m/p^f)_j + a_{4j}(k^g/l)_j + a_{5j}(k^n/k^g)_j + a_{6j}(m^g/s^g)_j + a_{7j}(m^n/s^n)_j + a_{8j}(l/e)_j + a_{9j}(q)_j$$

where j distinguishes the unknown coefficients and the trend growth rate variables of the j-th industry in the inter-industry analysis ($j = 1, \ldots, N$). This is an exact equation which holds because of the residual nature of the coefficient a_{0j}. It was already noted that a_{0j} represents the summation of "many omitted terms" that reflect various unobserved sources (both known and "unknown") of labour productivity change. This is so for each of the N industries. In this situation, it is natural to characterize each a_{0j} ($j = 1, \ldots, N$) as a random (or stochastic) drawing from the *same* probability distribution with mean (or expected value) $\bar{a}_0$ and finite variance. Then a_{0j} could be rewritten as

$$(2)\quad a_{0j} = \bar{a}_0 + \epsilon_{0j} \qquad (j = 1, \ldots, N)$$

where ϵ_{0j} ($j=1,\ldots,N$) is a random disturbance, all from the same distribution with mean zero and finite variance. Substitution of (2) into the exact equation (1) yields the basic stochastic labour productivity growth equation for each industry.

Now if the true values of the slope coefficients corresponding to each growth rate variable were the same for all industries; i.e., if

$$(3)\quad a_{ij} = a_{ij*} = a_i \qquad (j, j^* = 1, \ldots, N), \qquad (i = 1, \ldots, 9),$$

then we could proceed with estimating the 10 coefficients a_0 and a_i (i=1,...,9) without further discussion. This is equivalent to assuming that the labour productivity growth rate for each of the industries would be *equally* affected by equal changes in the growth rate of, e.g., the gross capital-intensity of production, or the average size of establishment, and so on. There is no reason for this assumption to hold true.[58] *Three approaches* to this problem can be proposed.

[58]The structures of each of the first eight slope coefficients are known (see Sections A.5, B.2, and B.4). Indeed, the values of these coefficients generally depend upon the functional form of the industry production function and the values of all the arguments of the function. Thus one should expect corresponding slope coefficients to differ from industry to industry. However, it should also be noted that the values of the eight slope coefficients do *not* depend upon units of measurement. For example, the structure of the first coefficient is known to be

$$\left(\frac{NP}{Q^*L}\right)\left[\sum_i\left(\frac{N_i}{N}\right)\frac{\partial Q^*}{\partial N_i} - \sum_j\left(\frac{P_j}{P}\right)\frac{\partial Q^*}{\partial P_j}\right] = \left[\sum_i\frac{N_i}{Q^*}\frac{\partial Q^*}{\partial N_i}\right]\frac{P}{L} - \left[\sum_j\frac{P_j}{Q^*}\frac{\partial Q^*}{\partial P_j}\right]\frac{N}{L}$$

where $N + P = L$ (see Section B.2). This expression is invariant with respect to the units of measurement of all the variables.

First, we may wish to characterize each of the nine *sets* of coefficients a_{ij} $(j = 1, \ldots, N)$, $(i = 1, \ldots, 9)$, as a random drawing from the same probability distribution with mean $\bar{a}_i$ and finite variance. Then a_{ij} could be rewritten as

$$(4) \quad a_{ij} = \bar{a}_i + \epsilon_{ij} \qquad (j = 1, \ldots, N) \;,\; (i = 1, \ldots, 9)$$

analogous to (2). Substitution of (4) into (1) yields a random coefficient[59] labour productivity growth equation for each industry, and we may estimate the 10 *mean* coefficients $\bar{a}_i$ $(i = 0, 1, \ldots, 9)$. In the opinion of the present writer, this approach is not applicable, because the generation of the coefficients a_{ij} $(i = 1, \ldots, 9)$ $(j = 1, \ldots, N)$ seems essentially to be a nonstochastic process.[60]

Second, it is straightforward to see that if the *ratios* of the true values of the slope coefficients corresponding to each growth variable were known – i.e., if

$$(5) \quad a_{ij}/a_{i1} = \lambda_{ij} \qquad (j = 1, \ldots, N) \;,\; (i = 1, \ldots, 9)$$

were known, then the observed growth rate variables could be suitably transformed[61] by the known factors λ_{ij}. In this case, we estimate the mean coefficient $\bar{a}_0$ and the nine unknown coefficients a_{i1} $(i = 1, \ldots, 9)$. This approach is difficult to apply because the required ratios are not known. However, some intelligent assumptions could often be made and used to advantage. This approach is not pursued in the present Study.

Third, we can *initially* suppose that the values of the slope coefficients corresponding to each growth variable are the same, or approximately the same, for the individual industries. Then the basic stochastic labour productivity growth equation for the j-th industry $(j = 1, \ldots, N)$ is misspecified as

$$\begin{aligned}(6) \quad (q/l)_j = \bar{a}_0 &+ a_1(n/p)_j + a_2(n^m/n^f)_j + a_3(p^m/p^f)_j \\ &+ a_4(k^g/l)_j + a_5(k^n/k^g)_j + a_6(m^g/s^g)_j \\ &+ a_7(m^n/s^n)_j + a_8(l/e)_j + a_9(q)_j + \epsilon_{0j}\end{aligned}$$

Further, suppose that the ordinary least-squares (O.L.S.) estimation procedure is applied to estimate the "10 coefficients" $\bar{a}_0, a_i$ $(i = 1, \ldots, 9)$.[62] The relevant

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[59] A simple ramdom coefficient model is discussed in Lawrence R. Klein, *A Textbook of Econometrics* (Evanston, Illinois: Row, Peterson and Company, 1953), pp. 216-218. A more recent and extensive account is C. Hildreth and J. Houck, "Some Estimators for a Linear Model with Random Coefficients", *Journal of the American Statistical Association,* June 1968, pp. 584-595.

[60] Even if the process were stochastic, it is difficult to see in what economic sense the stochastic process could have the convenient statistical properties required for estimation.

[61] That is, the basic stochastic labour productivity growth equation for the j-th industry $(j = 1, \ldots, N)$ becomes

$$\begin{aligned}(q/l)_j = \bar{a}_0 &+ a_{11}\lambda_{1j}(n/p)_j + a_{21}\lambda_{2j}(n^m/n^f)_j + a_{31}\lambda_{3j}(p^m/p^f)_j \\ &+ a_{41}\lambda_{4j}(k^g/l)_j + a_{51}\lambda_{5j}(k^n/k^g)_j + a_{61}\lambda_{6j}(m^g/s^g)_j \\ &+ a_{71}\lambda_{7j}(m^n/s^n)_j + a_{81}\lambda_{8j}(l/e)_j + a_{91}\lambda_{9j}(q)_j + \epsilon_{0j} \;.\end{aligned}$$

[62] That is, the coefficients are estimated from observations on the dependent growth variable and independent growth variables for the N industries.

problem is to relate the O.L.S. *estimates* of $\overline{a}_0$, a_i $(i = 1, \ldots, 9)$, to the unknown true values of the coefficients $\overline{a}_0$, a_{ij} $(j = 1, \ldots, N)$, $(i = 1, \ldots, 9)$. In particular, it would be essential to know under what conditions, and in what sense, the estimate of a_i is an indicator of the corresponding true values of the industry coefficients a_{ij} $(j = 1, \ldots, N)$ for each of the coefficients $(i = 1, \ldots, 9)$. This is, in effect, the approach adopted in this Study.

The statistical theory of the adopted approach has been worked out by other writers to whom the reader is referred for complete details.[63] Briefly, it turns out that the O.L.S. estimate of a_i $(i = 1, \ldots, 9)$ can be regarded as an unbiased and consistent estimate of a general weighted average (with weights that sum to *unity*) of the corresponding individual industry coefficients a_{ij} $(j = 1, \ldots, N)$.[64] If the independent growth variables tend to be uncorrelated, *and* if there is no particular correlation between the distribution of a_{ij} values $(j = 1, \ldots, N)$ and the squared average deviations of the i-th observed independent variable for the various industries, then the O.L.S. estimate of a_i becomes an unbiased and consistent estimate of the simple arithmetic average of the N industry coefficients a_{ij} (see Section 3.4 for further discussion).

We are now prepared to state the estimation results. It was already mentioned in the previous section that the term a_9 $(q)_j$ in equation (6) has a special status in this Study. The growth variable $(q)_j$ in a "loosely defined" proxy variable, which also happens to be highly correlated with the dependent variable $(q/l)_j$ and mildly correlated with some of the independent growth variables. If such a variable is introduced simultaneously with the other regressor variables in the O.L.S. procedure, it would "dominate" the regression. However, because of the proxy nature of the variable $(q)_j$ and its economic interpretation as being "over and above" the influence of the other "well-defined" independent variables, it is natural to employ the following two-step O.L.S. procedure.[65] In the *first step*, we estimate the coefficients $\overline{a}_0, a_i$ $(i = 1, \ldots, 8)$ of the rewritten basic stochastic equation

$$(7) \quad (q/l)_j = \overline{a}_0 + a_1(n/p)_j + a_2(n^m/n^f)_j + a_3(p^m/p^f)_j + a_4(k^g/l)_j + a_5(k^n/k^g)_j + a_6(m^g/s^g)_j + a_7(m^n/s^n)_j + a_8(l/e)_j + \epsilon_{0j}^* \; (j = 1, \ldots, N)$$

where $\epsilon_{0j}^* = \epsilon_{0j} + a_9(q)_j$. The particular second step in the estimation procedure will be explained shortly.

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[63] A special case is shown in Klein, *op. cit.*, pp. 218-220. The more general results are presented in A. Zellner, "Estimation of Cross-Section Relations: Analysis of a Common Specification Error", *Metroeconomica*, April-December 1962, pp. 111-117. See also Griliches, *op. cit.*, pp. 277-278.

[64] Strictly speaking, we should add to this statement – "plus a general weighted average (with weights that sum to *zero*) of each set of noncorresponding industry coefficients a_{i^*j} $(i \neq i^*)$". Thus the statement in the text holds to the extent that the distributions of noncorresponding coefficient values are uncorrelated with their zero-sum weights of the appropriate weighted averages.

[65] Stepwise regression theory is explained in Arthur S. Goldberger, *Econometric Theory* (New York: John Wiley and Sons, 1964), pp. 194-196.

TABLE 3-1

ESTIMATES OF COEFFICIENTS OF MANUFACTURING LABOUR PRODUCTIVITY GROWTH DIFFERENTIALS, 1947-56
(Man-hour data, N=16)

Coefficients of:	Regression Numbers 1	2	3	4	5	6	7
Pure constant	0.701	0.771	1.302	0.785	1.332	0.906	0.828
	(1.311)	(1.226)	(0.813)	(0.901)	(0.795)	(0.829)	(0.780)
(n/p)	0.425	0.399	0.293	0.422	0.191	0.387	0.449
	(0.290)	(0.267)	(0.122)	(0.223)	(0.145)	(0.201)	(0.139)
(n^m/n^f)	– 0.437	– 0.340		– 0.529		– 0.431	– 0.501
	(0.529)	(0.437)		(0.392)		(0.318)	(0.266)
(p^m/p^f)	0.098			0.108			
	(0.262)			(0.231)			
(k^g/l)	0.572	0.587	0.553	0.549	0.557	0.566	0.566
	(0.210)	(0.195)	(0.170)	(0.171)	(0.167)	(0.161)	(0.155)
(k^n/k^g)	0.079	0.170		0.033	0.343	0.135	
	(0.491)	(0.403)		(0.387)	(0.276)	(0.307)	
(m^g/s^g)	– 0.121	– 0.137					
	(0.360)	(0.338)					
(m^n/s^n)	0.075	0.089					
	(0.255)	(0.238)					
(l/e)	0.314	0.266	0.226	0.316	0.167	0.262	0.294
	(0.228)	(0.180)	(0.142)	(0.201)	(0.147)	(0.158)	(0.134)
R^2	0.696	0.690	0.573	0.691	0.625	0.683	0.677
$\bar{R}^2$	0.348	0.418	0.466	0.485	0.489	0.525	0.560

Note: The dependent variable is (q/l). N = 16 is the number of observations. The figures in parentheses are the estimated standard errors of the estimated coefficients shown above. The standard errors have a non-negative bias (see Zellner, *op. cit.*, p. 114). The $\bar{R}^2$ values are the estimated population coefficients of determination for the multiple regressions and have a nonpositive bias (Zellner, *op. cit.*, p. 115). See Section 3.2 for the meaning of the variables.

Table 3-1 presents the first-step O.L.S. estimates of the coefficients together with the estimated standard errors and $\bar{R}^2$ values. The results in this table are calculated from the trend growth rate observations for the 16 manufacturing industries[66] (i.e., N = 16) covering the time period 1947-56. The various labour employment growth variables are measured in "man-hours". Table 3-2 is identical to Table 3-1, except that the various labour growth rates are measured in terms of "number employed". The two tables show the estimates when all eight independent variables are included and when various combinations of nonsignificant independent variables are excluded.

[66] That is, all the two-digit manufacturing industries (1948 S.I.C.) except "Miscellaneous manufacturing" which is excluded because of its residual nature.

TABLE 3-2

ESTIMATES OF COEFFICIENTS OF MANUFACTURING LABOUR PRODUCTIVITY GROWTH DIFFERENTIALS, 1947-56
(Number-employed data, N=16)

Coefficients of:	Regression Numbers 1	2	3	4	5	6	7
Pure constant	0.277	0.264	0.513	0.834	0.827	0.532	0.516
	(1.349)	(1.239)	(0.900)	(0.776)	(0.771)	(0.787)	(0.741)
(n/p)	0.449	0.452	0.431	0.242	0.317	0.427	0.444
	(0.292)	(0.266)	(0.224)	(0.148)	(0.123)	(0.201)	(0.137)
(n^m/n^f)	– 0.299	– 0.310	– 0.425			– 0.416	– 0.436
	(0.511)	(0.426)	(0.375)			(0.317)	(0.258)
(p^m/p^f)	0.012		0.011				
	(0.248)		(0.218)				
(k^g/l)	0.611	0.609	0.581	0.597	0.578	0.583	0.580
	(0.214)	(0.198)	(0.171)	(0.166)	(0.164)	(0.161)	(0.152)
(k^n/k^g)	0.092	0.080	0.026	0.256		0.038	
	(0.522)	(0.420)	(0.406)	(0.281)		(0.319)	
(m^g/s^g)	– 0.176	– 0.172					
	(0.365)	(0.336)					
(m^n/s^n)	0.123	0.120					
	(0.264)	(0.242)					
(l/e)	0.272	0.277	0.279	0.174	0.226	0.274	0.285
	(0.227)	(0.187)	(0.200)	(0.152)	(0.140)	(0.166)	(0.135)
R^2	0.700	0.700	0.690	0.636	0.609	0.690	0.689
$\bar{R}^2$	0.357	0.437	0.483	0.504	0.511	0.534	0.576

Note: See Table 3-1.

Before carrying out the second step in the two-step estimation procedure, it is necessary to decide which of the regressions shown in Tables 3-1 and 3-2 will serve as the first-step O.L.S. estimates. Clearly, it is desirable to eliminate independent variables that are not significant[67] in the first-step results. But it is equally desirable to have a well-defined criterion for selecting the "best" of the regression equations. Such a criterion should *not* necessarily eliminate an independent variable if its estimated coefficient has the "wrong" sign. The criterion used in this Study is that of Theil.[68] Briefly, we choose as "best" that regression equation which yields the maximum corrected (estimated) multiple correlation coefficient (i.e., the maximum $\bar{R}^2$ value). The "best" regression equation in the labour productivity growth analysis (1947-56) is regression number 7[69] in Tables 3-1 and 3-2.

[67] For example, at the 5 per cent significance level.
[68] See H. Theil, *Economic Forecasts and Policy* (Amsterdam: North-Holland Publishing Company, 1961), pp. 212-214.
[69] Other combinations of independent variables were also deleted, with the result that regression number 7 is the "best", using the Theil criterion. A formal sequential deletion procedure is stated in Y. Haitovsky, "A Note on the Maximization of $\bar{R}^2$", *The American Statistician*, February 1969, pp. 20-21.

Then the *second step* in the estimation procedure is to calculate the O.L.S. estimate of coefficient a_9 in the following residual stochastic equation

$$(8)\quad (q/l)_j - \hat{\bar{a}}_0 - \hat{a}_1(n/p)_j - \hat{a}_2(n^m/n^f)_j - \hat{a}_4(k^g/l)_j - \hat{a}_8(l/e)_j = a_9(q)_j + \epsilon_{0j} \qquad (j = 1, \ldots, 16)$$

where, e.g., $\hat{a}_1$ is the O.L.S. estimate of the coefficient a_1 from the first step, and so on.[70] The estimation results of the second step in the two-step procedure are as follows. With man-hour data, it is found that

$$\hat{\hat{a}}_9 = 0.205 \quad (0.069) \;,$$

and with number-employed data, it is found that

$$\hat{\hat{a}}_9 = 0.186 \quad (0.074) \;.$$

Now combining the results of the stepwise regression procedure, we have as our *complete estimates* of the coefficients of inter-industry labour productivity growth differentials, *1947-56*:

$$(9)\quad \begin{array}{l} (q/l) = \underset{(0.780)}{0.828} + \underset{(0.139)}{0.449(n/p)} - \underset{(0.266)}{0.501(n^m/n^f)} + \underset{(0.155)}{0.566(k^g/l)} \\ \qquad + \underset{(0.134)}{0.294(l/e)} + \underset{(0.069)}{0.205(q)} \qquad \bar{R}^2 = 0.704 \end{array}$$

when man-hour data are used, and

$$(10)\quad \begin{array}{l} (q/l) = \underset{(0.741)}{0.516} + \underset{(0.137)}{0.444(n/p)} - \underset{(0.258)}{0.436(n^m/n^f)} + \underset{(0.152)}{0.580(k^g/l)} \\ \qquad + \underset{(0.135)}{0.285(l/e)} + \underset{(0.074)}{0.186(q)} \qquad \bar{R}^2 = 0.682 \end{array}$$

when number-employed data are used. (The figures in parentheses again represent the estimated standard errors[71] of the corresponding estimated coefficients.) With 10 degrees of freedom (i.e., 16 minus 6), the 5 per cent significance level for the student t value in a one-tailed test is 1.812. The relevant 5 per cent significance level for adjusted $\bar{R}^2$ equals 0.332.

Some estimation results were also obtained for the time period 1957-67. It was already noted in Section 2.1 that the trend growth rate observations for the

[70] Note the use of the "best" first-step regression equation. Also, the left-hand side of (8) is an O.L.S. residual term; the summation of such residual terms for all observations (N = 16) equals zero. Since the pure constant is suppressed in (8), it is necessary to perform the second-step O.L.S. calculations with the independent variable $(q)_j$ measured in terms of its average deviations.

[71] Strictly speaking, the estimated standard errors of the pure constant and the first four slope coefficients should be adjusted upwards by a factor equal to (16 - 5)/(16 - 6) to allow for the loss of an additional degree of freedom in the second step of the procedure. Similarly, the estimated standard error of the last coefficient may be multiplied by (16 - 2)/(16 - 6) to reflect the loss of four degrees of freedom in the first step. However, the standard errors are likely to be biased upwards for other reasons (see Zellner, *op. cit.*, p. 114), so that the suggested adjustments may not be necessary.

two-digit (1960 S.I.C.) manufacturing industries covering this period are subject to certain economic and statistical difficulties. Briefly, the calculated growth rates are some weighted average of essentially two different economic trends. There is a 1961 statistical discontinuity in the data, and some of the key growth rates (such as "net real output") are liable to significant revision. It was therefore decided not to present a complete analysis of labour productivity growth for the period 1957-67 with the currently available data.[72] However, it is still of interest to test whether such economically important independent variables as (k^g/l) and (q) are statistically significant for this period, even with the available data.

The required trend growth rate observations were calculated for the 19 manufacturing industries[73] (i.e., $N = 19$) covering the 1957-67 time period. Then the *incomplete and preliminary estimates* of the coefficients of inter-industry labour productivity growth differentials, *1957-67*, are:[74]

$$(11)\quad (q/l) = \underset{(0.904)}{0.244} + \underset{(0.133)}{0.399}(k^g/l) + \underset{(0.153)}{0.541}(q) \qquad \bar{R}^2 = 0.474$$

when man-hour data are used, and

$$(12)\quad (q/l) = \underset{(0.899)}{0.175} + \underset{(0.139)}{0.312}(k^g/l) + \underset{(0.153)}{0.544}(q) \qquad \bar{R}^2 = 0.467$$

when number-employed data are used. With 16 degrees of freedom (i.e., 19 minus 3), the 5 per cent significance level for the student t value in a one-tailed test is 1.746. The required 5 per cent significance level for the adjusted $\bar{R}^2$ value is equal to 0.219.

Finally, it should be noted that the complete estimates shown in equations (9) and (10) for the period 1947-56 are *not* comparable to the incomplete estimates shown in equations (11) and (12) for the period 1957-67. Indeed, the latter estimates should be regarded with appropriate caution.

3.4 Interpretation of the Empirical Estimates

The estimation results of the previous section require careful economic interpretation. It is also desirable to compare our results with those obtained by other writers who employ either estimation or nonestimation methods. The discussion in this section is mainly with relevance to the complete estimates for the time period 1947-56, as shown in Tables 3-1 and 3-2 and equations (9) and (10). We will first begin with a number of general comments.

Of the nine independent growth rate variables originally introduced to explain labour productivity growth differentials, we have attained statistical

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[72] See Chapter 7 on suggestions for future research with 1960 S.I.C. statistical data.
[73] That is, all the two-digit industries (1960 S.I.C.) except "Miscellaneous manufacturing".
[74] The estimates were obtained by a one-step O.L.S. procedure. Since (k^g/l) and (q) are virtually uncorrelated, the procedure is approximately equivalent to the "more correct" two-step O.L.S. procedure (see Goldberger, *op. cit.*, p. 195).

significance[75] for five of them–namely, (n/p), (n^m/n^f), (k^g/l), (l/e) and (q). None of the remaining variables–i.e., (p^m/p^f), (k^n/k^g), (m^g/s^g), (m^n/s^n)–exhibit statistical significance in *any* of the regressions. If more industry observations were available (e.g., for a corresponding analysis at the three-digit manufacturing level), one might expect the estimated $|t|$ values[76] to be greater, if only because the number of degrees of freedom is larger. It is also true that the estimated standard errors in our inter-industry analysis have a non-negative bias, when the actual values of corresponding individual industry coefficients differ.[77] However, it is doubtful whether either or both of these statistical factors are sufficient to account for the nonsignificance of some of the independent variables.[78] Therefore, it seems preferable to attempt an *economic* interpretation of the estimates rather than to concentrate on possible *statistical* ambiguities. We will now discuss each of the estimated coefficients in turn. The following remarks are with particular reference to the "man-hour data" results (Table 3-1 and equation (9)), but are generally applicable to the "number-employed data" results as well.

The estimated coefficient of the growth variable (n/p) is significantly positive. This indicates that nonproduction labour employment has been "typically more productive" than production labour employment, on the average,[79] over the time period 1947-56. Indeed, an increase of 10 percentage points in the (n/p) growth rate, *ceteris paribus*, results in an increase of about 4.5 percentage points[80] in the labour productivity growth rate, on the average. One should expect the industries with relatively high (n/p) growth to also show relatively high (q/l) growth, other things being equal. The significantly positive sign of the (n/p) coefficient conforms with what would be derived from neoclassical competitive equilibrium assumptions, since the average salary rate of nonproduction labour is greater than the average wage rate of production labour.[81] But the magnitude ot the *point* estimate is considerably greater than that derived from such equilibrium assumptions.[82] We can interpret this to mean that there is some evidence (subject to sampling error)

[75] That is, the estimated coefficients of the various growth variables are statistically different from zero at the 5 per cent significance level.

[76] The estimated $|t|$ is the absolute value of the ratio of the estimated coefficient over its estimated standard error.

[77] See again Section 3.3 and Zellner, *op. cit.,* p. 114. This non-negative bias persists even under "favourable conditions" for estimating an average or weighted average of individual industry coefficients.

[78] There is also the possibility of so-called single equation bias of O.L.S., and related specification error bias (see Goldberger, *op. cit.*, pp. 196-197, 288-290). Indeed, it would be interesting to re-estimate the coefficients using an instrumental variable estimation procedure.

[79] That is, as an average or weighted average of the probably different individual industry coefficients.

[80] Of course, Figure 4.5 is subject to standard (sampling) error.

[81] See mathematical footnote at the beginning of Section 3.2. Average salary and wage rate data for manufacturing industries are available from Dominion Bureau of Statistics, *General Review of the Manufacturing Industries of Canada,* Cat. No. 31-201 (Ottawa: Queen's Printer, various annual issues).

[82] See again mathematical footnote in Section 3.2. As a rough check, one can form the required "share of salary bill" and "share of wage bill" ratios for "Total Manufacturing" from data in DBS, *General Review . . . , ibid.*, and DBS, *Comptes Nationaux, Revenus et Dépenses, 1926-1956,* Cat. No. 13-502 F (Ottawa: Queen's Printer, 1962).

that nonproduction labour is "typically underpaid" *relative to* production labour, on the average, for the industries of the manufacturing sector.[83]

The estimated coefficient of the independent growth variable (n^m/n^f) is significantly negative, just "making" the 5 per cent significance level. Taken literally, this means that female nonproduction labour has been "typically more productive" than male nonproduction labour, on the average, for the period concerned. This is quite a surprising result in view of the fact that the average male salary rate is considerably larger than the average female salary rate in all the manufacturing industries. The present writer believes that an explanation of this paradox probably lies in the high degree of *complementarity* between male and female nonproduction labour. For example, if professional and technical employees (primarily male) are to make any contribution to production, they must be "supported" by clerical and office employees (primarily female).[84] As evidence that this is so, we should expect the (n^m) growth rates to be "nearly equal" to the (n^f) growth rates. Such evidence is clearly apparent from the statistical data.[85] In this case, where complementarity phenomena may be *dominant,*[86] the neoclassical (substitution) equilibrium-type correspondence between relative salary rates and relative marginal productivities is no longer valid or even approximately valid. The negative coefficient of the growth variable (n^m/n^f) could then indicate that female nonproduction labour has been "relatively scarce" (or in relatively short supply) compared with male nonproduction labour, on the average, for the period concerned. However, since the (n^m) and (n^f) growth rates are nearly equal, the term with growth variable (n^m/n^f) has a negligible impact as a source of labour productivity change (see the discussion later in this section).[87]

The estimated coefficient of the growth variable (p^m/p^f) is positive but not significantly different from zero. This means that there is *not* sufficient evidence to claim that male production labour is "typically more productive" than female production labour, at least on the basis of the particular sample. However, the sign and magnitude of the *point* estimates of the (p^m/p^f) coefficient in regression numbers 1 and 4 of Table 3-1 certainly conform to what would be approximately derived from neoclassical equilibrium assumptions. Other writers have shown significantly that the relatively higher male wage rate is a reflection of relatively higher male marginal productivity in a production labour analysis.[88]

The estimated coefficient of the independent variable (kg/l) is positive and highly significant. This agrees with *a priori* considerations, since the true value of

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[83] A similar conclusion seems to have been reached by Hildebrand and Liu, *op. cit.*, pp. 122-129, whose framework is not strictly comparable with ours.

[84] See Section B.3 for a more precise formulation of *complementarity* among factor inputs, including the possibility of a "mixture" of complementarity and substitution elements.

[85] The average growth rate observation of (n^m/n^f) for the 17 two-digit industries equals 0.33, with a standard deviation equal to 1.32. The corresponding figures for (n/p) are 3.74 and 2.81; and for (p^m/p^f), 1.60 and 1.90.

[86] See again Section B.3 for the technical discussion.

[87] To the present writer's knowledge, none of the productivity estimation studies in the literature attempt to distinguish between male and female nonproduction labour.

[88] See, e.g., Griliches, *op. cit.*, pp. 280, 299-300.

this coefficient for each industry represents the elasticity of aggregate output with respect to aggregate fixed capital input (see Section A.5). The magnitude of the coefficient point estimate is greater than what could reasonably be expected from competitive equilibrium assumptions.[89] However, such assumptions include the supposition of constant returns to scale. If the estimated (k^g/l) coefficient is "normalized" in order to account for the estimated degree of returns to scale (as will be seen shortly), then the normalized point estimate is of the same order of magnitude as expected from equilibrium conditions. This topic is discussed again in Chapter 5.

The estimated coefficient of the growth variable (k^n/k^g) is always positive but never significantly different from zero. This would indicate that there is not sufficient evidence to give the "net stock" measure of fixed capital any positive weight in the "correct" measure of fixed capital input. Naturally this result is very much a reflection of the particular survival curve and depreciation formula used to measure gross and net fixed capital stock in this Study.[90] It would be interesting to see whether the result is preserved under alternative and more realistic measurements. Right now, it is possible to obtain a crude point estimate of the positive weight that net stock might receive *if* the coefficient were significant.[91] Using regression number 6 from Table 3-1, the estimate equals 0.24. Then the "correct" measure of the growth rate of any particular type of fixed capital could be:

0.24 (net stock growth rate) + 0.76 (gross stock growth rate).

Other writers who introduce a variable such as (k^n/k^g) in productivity studies have sometimes shown its positive significance.[92]

The estimated coefficients of the growth variables (m^g/s^g) and (m^n/s^n) are not significantly different from zero. The point estimates for the (m^g/s^g) coefficient are negative and those for the (m^n/s^n) coefficient are positive, but this peculiarity is probably due to statistical multicollinearity[93] and has little economic significance.[94] The failure to obtain a statistically significant estimate for either of the coefficients is a major disappointment of this Study. It is tempting to "blame" the results on the possibility of factor complementarity between machinery capital

[89] Competitive assumptions imply that the (k^g/l) coefficient should reflect the "capital share" of net output in the various industries. The average of such capital shares is about 0.35 over the time period concerned. See Appendix D and DBS, *Comptes Nationaux . . . , op. cit.*

[90] This is mentioned in Appendix D.

[91] If the weight is the same for each type of fixed capital, then the coefficient of (k^n/k^g) is equal to the coefficient of (k^g/l) multiplied by the common weight (see Sections A.3 and B.2). An estimate of the weight could be derived by dividing the point estimate of the (k^n/k^g) coefficient by the point estimate of the (k^g/l) coefficient.

[92] See e.g., La Tourette, *op. cit.*, pp. 266-273; and B. F. Massell, "Determinants of Productivity Change in U.S. Manufacturing", *Yale Economic Essays,* No. 2, 1962, pp. 343-346.

[93] For example, the correlation coefficient between the observed (m^g/s^g) and (m^n/s^n) growth rates equals 0.88; the correlation between (m^g/s^g) and (q/l) is 0.18; and the correlation between (m^n/s^n) and (q/l) is 0.10.

[94] If the unknown correct *weight* for the net stock growth rate of machinery input is typically greater than that of the structures input, then it is certainly possible to have a negative (m^g/s^g) coefficient and a positive (m^n/s^n) coefficient (see Section A.3).

input and structures capital input. However, there is no empirical evidence to support this possibility.[95]

A tentative interpretation of the results could be the following. It was already insinuated in Sections 3.1 and 3.2 that significantly positive coefficients should be expected, since machinery and equipment is typically less durable than buildings and structures.[96] But this inference is empirically valid on condition that relative constant asset prices are correctly measured from observations. If it should happen that the machinery price deflator is biased upwards relative to the structures price deflator, then the observed relative asset prices may approximate the relative marginal productivities of machinery and structures (measured in physical units). In this case, there would be little, if any, statistical evidence showing machinery capital input to be "typically more productive" than structures capital input, when both are measured in *observed* constant prices. It is well known that currently used fixed capital price deflators are often crude approximations to the desired (correct) price indexes.[97] When improved price deflators are available, it would be interesting to repeat the analysis of this Study. Other writers have introduced a variable such as (m^g/s^g) in productivity estimation studies, but their results are not comparable to ours.[98]

To continue, the estimated coefficient of the independent growth variable (l/e) is significantly positive. This indicates that, on the average, the representative establishments of the manufacturing industries operate at output levels where increasing economies of scale are evident, at least for the time period 1947-56. The magnitude of the estimated point coefficient is larger than that estimated in other studies, but the estimate is not significantly different compared with other studies.[99] Also, since the two component growth rates of (l/e) are, on the average, nearly equal, the term with growth variable (l/e) makes a negligible contribution as a source of labour productivity growth (this is seen shortly).

Turning now to the pure constant of the productivity regression analysis, the estimated constant is always positive, but never quite significant. It could very well be that a significantly positive estimate could be attained with more observations (i.e., a larger number of degrees of freedom). One would normally expect the pure constant to be positive, since it accounts for the net effect of all the "omitted terms", including interaction effects, on labour productivity change. More precisely, the point estimate of 0.83 in regression number 7, Table 3-1, indicates

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[95] The average growth rate of (m^g/s^g) equals 4.15; the standard deviation equals 3.03. The figures for (m^n/s^n) are 6.01 and 4.24, respectively.

[96] See again Haavelmo, *op. cit.*, pp. 97-101.

[97] A complete discussion is in Dominion Bureau of Statistics, *Fixed Capital Flows and Stocks: Manufacturing 1926-1960*, Cat. No. 13-522 (Ottawa: Queen's Printer, 1967), pp. 71-87.

[98] Consult, e.g., La Tourette, *op. cit.*, pp. 265-266, 269-270; and Massell, *op. cit.*, pp. 342-346. No other writers appear to introduce *both* (m^g/s^g) and (m^n/s^n) in a productivity analysis, even though (k^g/l) and (k^n/k^g) are simultaneously used.

[99] Two comparable studies are Griliches, *op. cit.*, pp. 276, 279, 298-299, 310; and Hildebrand and Liu, *op. cit.*, pp. 23-24, 106-109, 129. A remarkably similar estimate of economies of scale is reported by Lydall, *op. cit.*, pp. 820-821, who also carried out an inter-industry analysis.

that, on the average, all the "other terms" that are *not* specified in the estimation analysis contributed about 0.83 percentage points to the annual growth rate of labour productivity for the period concerned.

Finally, the estimated coefficient of the "proxy" growth variable (q) in the complete equation (9) is significantly positive as expected.[100] In fact, the magnitude of the point estimate is quite similar to that obtained by other writers in comparable studies.[101] However, since this term involves a rather loosely defined proxy variable (introduced mainly for illustrative purposes), we will not include the impact of this variable in the following table of sources of labour productivity growth. In effect, its contribution, properly measured, is contained in the pure constant term.[102]

To conclude this chapter, we present Table 3-3. The table shows the percentage of inter-industry labour productivity growth differentials that could be attributed, on the average, to the various sources, both measured and unmeasured, of such growth. For example, the percentage attributed to the growth variable (n/p) is estimated[103] as

$$(\overline{n/p})\ (0.449)\ /\ (\overline{q/l})$$

where $(\overline{n/p})$ is the average of the $(n/p)_j$ growth rate observations for the 16 industries, $(\overline{q/l})$ is the average of the $(q/l)_j$ observations, and (0.449) is the estimated regression coefficient of (n/p) from regression number 7 of Table 3-1. All the other percentage sources are estimated analogously. It is easy to see that this is a very natural method of attributing growth sources, in view of a well-known property of O.L.S. estimates.[104] In fact, the estimated coefficients themselves could be regarded as unbiased estimates of the averages of the corresponding individual industry coefficients (see the discussion in Section 3.3).

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[100] It may be objected that the positive significance of the estimated coefficient is "spurious" due to a possible measurement error in the output growth variable. This problem has been investigated by Salter, *op. cit.*, pp. 109-113, 191-194, in a related context, with the conclusion that the expected degree of upward bias is negligible. In *our* context, the extent of such bias is mitigated by the two-step estimation procedure.

[101] This is clearly seen in Lydall, *op. cit.*, p. 821; and Salter *op. cit.*, pp. 122-123, 210-211.

[102] It will be recalled that the variable $(q)_j$ is measured in terms of its average deviations in the second step of the two-step O.L.S. procedure. The arithmetic average of such measurements (over the 16 industries) is, of course, zero.

[103] After multiplication by 100. This method of attributing sources of growth is similar to that used by Denison, *op. cit.* A recent discussion, with further references, is available in Conley, *op. cit.*, pp. 224-225.

[104] The pure constant estimate can be obtained directly from the averages of observations and the slope estimates; see Goldberger, *op. cit.*, pp. 183-184.

TABLE 3-3

ESTIMATES OF THE PERCENTAGE OF MANUFACTURING LABOUR PRODUCTIVITY GROWTH DIFFERENTIALS ATTRIBUTED TO THE VARIOUS SOURCES, 1947-56
(Man-hour data, N = 16)

Source of Growth	As Estimated by Regression Number 1		Number 6		Number 7	
(n/p)	39.3%		35.8%		41.5%	
(n^m/n^f)	− 3.5		− 3.1		− 3.6	
(p^m/p^f)	3.9					
(subtotal)		(39.7)		(32.7)		(37.9)
(k^g/l)	42.6		42.2		42.2	
(k^n/k^g)	1.9		3.3			
(subtotal)		(44.5)		(45.5)		(42.2)
(m^g/s^g)	−12.6					
(m^n/s^n)	11.3					
(subtotal)		(−1.3)				
(l/e)	− 0.4		− 0.3		− 0.4	
Pure constant	17.2		22.3		20.4	
Grand Total		99.7%		100.2%		100.1%

Note: The regression numbers refer to the estimation results of Table 3-1. See Section 3-2 for the meaning of the growth source variables. The grand totals do not sum to 100.0% because of rounding.

The meaning of the results shown in Table 3-3 is largely self-evident. The relative importance of the various sources of inter-industry labour productivity growth differentials is quite invariant with respect to the particular regressions that are shown. The "capital-intensity of production" source of growth is the most important, with "changes in the quality of labour"[105] as a close second. Both of these positive growth sources are considerably more important than the net effect of all the "omitted terms" which we have been unable to individually measure. It is also interesting to observe that the percentage contribution of the "economies-of-scale" source is relatively negligible, even though our coefficient estimates show the existence of substantial increasing returns to scale. This is because the relevant measure of size of establishment growth (namely, (l/e)) is virtually nil, on the average (i.e., $(\overline{l/e})$ equals − 0.05) for the period concerned. Thus, while "scale effects" could be a *potentially* important source of labour productivity growth, their actual (or *realized*) impact is relatively nil. Finally, it should be recognized that the estimation results are subject to sampling error[106] and depend upon the particular measurements of the variables. It is appropriate, then, to regard the empirical results with some caution.

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[105] In so far as changes in the "quality of labour" are explicitly measured in this Study.
[106] But the sampling errors are *known* (see Tables 3-1 and 3-2).

CHAPTER 4

CAPITAL PRODUCTIVITY GROWTH ANALYSIS

In this chapter, the capital productivity growth rate model that is formulated in Section A.6 is applied to the observed trend growth rate data of the manufacturing industries. It is clear that an inter-industry analysis of *capital productivity* growth differentials would be quite analogous to that of *labour productivity* growth differences. Therefore, almost all the discussion in the previous chapter concerning sources of productivity growth, the applied methodology, and the estimation procedure is directly relevant to this chapter as well. Thus the emphasis in this brief chapter is on the statement of empirical results. Indeed, the empirical estimates presented in this chapter serve as a partially independent check on those of Chapter 3. The capital productivity growth analysis also serves to bridge the gap between the more traditional labour productivity and factor productivity growth analyses, having as well some interest of its own.

4.1. Sources of Capital Productivity Growth and Application of the Model

The capital productivity level for any manufacturing industry at a particular time is simply *defined* as the ratio of the total net output of the industry to the total gross fixed capital stock utilized in that industry. This definition seems to agree with the most common usage of the term "capital productivity".

Perhaps the easiest way to understand why the level of capital productivity for an industry may vary over time is to merely interchange the roles of aggregate labour input and aggregate fixed capital input in the discussion of Chapter 3. In fact, the *list* of sources of capital productivity growth, as defined, is identical to that of labour productivity growth, as defined, with one exception. The *third* source of productivity growth in Section 3.1 should now be written as: "the growth of labour-intensity of production for the industry". Analogously, the labour-intensity of production is *defined* as the ratio of total labour man-hours employed to total fixed capital stock utilized in an industry. An increase (decrease) in the labour-intensity of production is a positive (negative) source of capital productivity growth for an industry. Also, if the growth rate of total net stock of fixed capital is different than that of total gross stock, then the labour-intensity source of capital productivity growth should be expressed as the summation of two terms–one representing a labour-intensity source with a gross capital stock measure, and the other accounting for the measurement error in neglecting the net stock of capital data.

Indeed, the impact and interpretation of the various sources of capital productivity growth are quite analogous to that of labour productivity. Thus, e.g., it is possible to show[1] that an increase (decrease) in the average size of establishment, *as measured by* the ratio of total gross capital stock to total number of establishments for an industry, is a positive source of industry capital productivity growth if the representative establishment is operating at an output level where increasing (decreasing) economies of scale are evident. Again, in Chapter 2, it was seen that capital productivity trend growth rates differ from industry to industry. So the main application in the present chapter is to test the significance, and estimate the relative importance, of the various sources of inter-industry capital productivity growth rate differentials.

The basic applied nonstochastic equation of the capital productivity growth analysis is then[2]

$$\begin{aligned}(q/k^g) &= b_1(n/p) + b_2(n^m/n^f) + b_3(p^m/p^f)\\ &\quad + b_4(l/k^g) + b_5(k^n/k^g) + b_6(m^g/s^g)\\ &\quad + b_7(m^n/s^n) + b_8(k^g/e) + b_9(q) + b_0\end{aligned}$$

where all the variables are trend growth rates for the industry concerned over a particular time period and are defined as follows:

(q/k^g) is the capital productivity growth rate;

(l/k^g) is the growth rate of labour-intensity of production with a gross capital stock measure;

(k^g/e) is the growth rate of the ratio of total gross capital stock to total number of establishments.

And,

b_i are unknown coefficients. $(i = 0,1,\ldots,9)$

The other growth rate variables are described in Section 3.2.

The relationship of the above basic applied equation to the 10 sources of capital productivity growth is, of course, completely analogous to that of the applied labour productivity growth analysis. Furthermore, in Appendix A, it is seen that both the labour and capital productivity growth models are theoretically derived from the same fundamental industry output growth equation. In this case, the following *theoretical* equalities should hold between the unknown coefficients of the basic applied (nonstochastic) labour productivity growth equation and those of the above capital productivity growth equation:[3]

$$a_i = b_i \qquad (i = 0,1,\ldots,8;\ i \neq 4)$$

$$a_4 + b_4 - 1 = a_8 = b_8.$$

[1] See Sections A.6 and B.5.
[2] This equation is the applied discrete counterpart of the theoretical continuous capital productivity growth model formulated in Section A.6 (equation 48).
[3] See again Sections A.5 and A.6.

It is in this sense that the *empirical regression* estimates of this chapter serve as a partially independent check[4] on those of the previous chapter. Also, because of the proxy, and purely illustrative, nature of the "additional" growth variable (q) in the basic applied equations, there is *no* theoretical foundation for the equality: $a_9 = b_9$. But the signs of the two coefficients should be the same, and their order of magnitude should be similar.

4.2 Statement and Interpretation of the Empirical Estimates

The development of an estimation procedure (and its rationale) for the inter-industry capital productivity growth differential analysis is completely analogous to that of the labour productivity case.[5] Tables 4-1 and 4-2 present the *first-step* ordinary least squares (O.L.S.) estimates of the "average" coefficients b_i ($i = 0,1,...,8$) together with their estimated standard errors. The results in these tables are again calculated from the trend growth rate observations for the 16 manufacturing industries (1948 S.I.C.) covering the time period 1947-56. The two tables show the estimates when all eight regressor variables are included and when various combinations of nonsignificant variables are eliminated. The "best" regression equation (according to Theil's criterion) in this first-step analysis is again the same regression number 7 in both Tables 4-1 and 4-2. Then the estimation results of the *second step* in the two-step procedure are:

$$\hat{b}_9 = 0.148 \quad (0.072),$$

$$\hat{\hat{b}}_9 = 0.175 \quad (0.071)$$

with man-hour data and number-employed data, respectively.

Thus the *complete estimates* of the coefficients of inter-industry capital productivity growth differentials, *1947-56*, are as follows:

$$\begin{aligned}(q/k^g) = \underset{(0.747)}{0.929} &+ \underset{(0.132)}{0.435}(n/p) - \underset{(0.252)}{0.481}(n^m/n^f) + \underset{(0.214)}{0.766}(l/k^g) \\ &+ \underset{(0.121)}{0.277}(k^g/e) + \underset{(0.072)}{0.148}(q) \qquad \bar{R}^2 = 0.657\end{aligned}$$

and:

$$\begin{aligned}(q/k^g) = \underset{(0.710)}{0.604} &+ \underset{(0.131)}{0.433}(n/p) - \underset{(0.246)}{0.423}(n^m/n^f) + \underset{(0.214)}{0.744}(l/k^g) \\ &+ \underset{(0.123)}{0.271}(k^g/e) + \underset{(0.071)}{0.175}(q) \qquad \bar{R}^2 = 0.667\end{aligned}$$

with man-hour data and number-employed data, respectively.[6]

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[4] Note that the estimated (stochastic) regression counterparts of the two basic (exact) applied equations are *not* theoretically equivalent. For a clear discussion of such matters, consult Herman Wold, *Demand Analysis* (New York: John Wiley and Sons, 1953), pp. 34-35.

[5] See Section 3.3.

[6] With 10 degrees of freedom, the 5 per cent significance level for the student t value in a one-tailed test is 1.812. The relevant 5 per cent significance level for adjusted $\bar{R}^2$ equals 0.332.

TABLE 4-1

ESTIMATES OF COEFFICIENTS OF MANUFACTURING CAPITAL PRODUCTIVITY GROWTH DIFFERENTIALS, 1947-56

(Man-hour data, N=16)

Coefficients of:	Regression Numbers 1	2	3	4	5	6	7
Pure constant	0.774	0.845	1.387	1.413	0.879	0.998	0.929
	(1.249)	(1.170)	(0.781)	(0.766)	(0.858)	(0.794)	(0.747)
(n/p)	0.422	0.396	0.286	0.188	0.414	0.378	0.435
	(0.277)	(0.255)	(0.117)	(0.140)	(0.211)	(0.192)	(0.132)
(n^m/n^f)	− 0.436	− 0.338			− 0.517	− 0.418	− 0.481
	(0.498)	(0.414)			(0.369)	(0.302)	(0.252)
(p^m/p^f)	0.101				0.111		
	(0.248)				(0.218)		
(l/k^g)	0.790	0.724	0.717	0.655	0.809	0.736	0.766
	(0.336)	(0.279)	(0.235)	(0.236)	(0.283)	(0.234)	(0.214)
(k^n/k^g)	0.055	0.149		0.325	0.019	0.123	
	(0.467)	(0.385)		(0.265)	(0.367)	(0.293)	
(m^g/s^g)	− 0.112	− 0.128					
	(0.342)	(0.321)					
(m^n/s^n)	0.073	0.087					
	(0.242)	(0.227)					
(k^g/e)	0.301	0.255	0.218	0.163	0.301	0.249	0.277
	(0.206)	(0.163)	(0.130)	(0.134)	(0.181)	(0.143)	(0.121)
R^2	0.709	0.702	0.588	0.638	0.704	0.696	0.691
$\bar{R}^2$	0.376	0.441	0.485	0.506	0.507	0.544	0.578

Note: The dependent variable is (q/k^g). $N = 16$ is the number of observations. The figures in parentheses are the estimated standard errors of the estimated coefficients shown above. The standard errors have a non-negative bias (see Zellner, *op. cit.*, p. 114). The $\bar{R}^2$ values are the estimated population coefficients of determination for the multiple regressions and have a nonpositive bias (Zellner, *op. cit.*, p. 115). See Sections 3.2 and 4.1 for the meaning of the variables.

Again, it is of interest to test whether the theoretically important growth variables (l/k^g) and (q) are statistically significant for the second time period 1957-67, using some of the available data for the 19 manufacturing industries (1960 S.I.C.). Then the *incomplete (and preliminary) estimates* of the coefficients of inter-industry capital productivity growth differentials, *1957-67*, are:

$$(q/k^g) = \underset{(0.883)}{0.316} + \underset{(0.139)}{0.694}(l/k^g) + \underset{(0.148)}{0.529}(q) \qquad \bar{R}^2 = 0.676$$

$$(q/k^g) = \underset{(0.879)}{0.241} + \underset{(0.145)}{0.721}(l/k^g) + \underset{(0.149)}{0.543}(q) \qquad \bar{R}^2 = 0.674.$$

TABLE 4-2

ESTIMATES OF COEFFICIENTS OF MANUFACTURING
CAPITAL PRODUCTIVITY GROWTH DIFFERENTIALS, 1947-56
(Number-employed data, N=16)

	Regression Numbers						
Coefficients of:	1	2	3	4	5	6	7
Pure constant	0.327	0.322	0.586	0.909	0.905	0.613	0.605
	(1.290)	(1.185)	(0.858)	(0.749)	(0.742)	(0.754)	(0.710)
(n/p)	0.454	0.455	0.429	0.241	0.310	0.423	0.433
	(0.280)	(0.255)	(0.214)	(0.143)	(0.118)	(0.192)	(0.131)
(n^m/n^f)	– 0.315	– 0.319	– 0.425			– 0.411	– 0.423
	(0.486)	(0.406)	(0.356)			(0.302)	(0.246)
(p^m/p^f)	0.005		0.017				
	(0.236)		(0.208)				
(l/k^g)	0.718	0.721	0.746	0.623	0.692	0.737	0.744
	(0.348)	(0.299)	(0.287)	(0.244)	(0.229)	(0.249)	(0.214)
(k^n/k^g)	0.054	0.049	0.004	0.237		0.022	
	(0.501)	(0.403)	(0.388)	(0.271)		(0.305)	
(m^g/s^g)	– 0.161	– 0.160					
	(0.349)	(0.320)					
(m^n/s^n)	0.118	0.117					
	(0.252)	(0.231)					
(k^g/e)	0.269	0.271	0.273	0.172	0.218	0.266	0.271
	(0.207)	(0.171)	(0.182)	(0.140)	(0.128)	(0.151)	(0.123)
R^2	0.693	0.693	0.683	0.624	0.598	0.683	0.683
$\bar{R}^2$	0.343	0.425	0.472	0.487	0.497	0.525	0.568

Note: See Table 4-1.

The empirical results of the inter-industry capital productivity growth analysis are remarkably consistent with those of the labour productivity analysis. A simple comparison of Tables 3-1 and 4-1 implies that the required theoretical equalities stated in Section 4.1 are statistically acceptable.[7] Indeed, the "best" regression equations in the two tables contain the same significant independent growth variables as *analogous* sources of productivity growth. For example, the estimate of the (l/k^g) coefficient (i.e., the estimate of b_4), is positive and highly significant, as expected.[8] Also, the magnitude of the point estimate of this coefficient is greater than that of the corresponding estimate of a_4–a result that

[7] Professor Ronald Bodkin has correctly pointed out that a formal statistical hypothesis test of the required theoretical equalities should involve an application of multivariate analysis. The relevant statistical theory is contained in Goldberger, *op. cit.,* pp. 201-212 and T. W. Anderson, *Introduction to Multivariate Statistical Analysis* (New York: John Wiley and Sons, 1958), Chapter 8.

[8] Recall that the true value of this coefficient for each industry represents the elasticity of aggregate output with respect to aggregate labour input.

conforms with certain competitive equilibrium assumptions.[9] Furthermore, a crude point estimate of the weight that the *net* stock growth measure of capital might receive could be obtained from the regression number 6 estimates of Table 4-1. The estimate equals[10] 0.24, which is identical to that obtained in the labour productivity analysis (Section 3.4).

It is again possible to prepare a table similar to Table 3-3, showing the percentage of inter-industry capital productivity growth differentials that could be attributed, on the average, to the various sources of this growth, 1947-56. However, such a table might appear to be ambiguous because of the large negative sources of growth in this case. Instead we will merely note the following. The average (q/k^g) growth rate observation for the 16 industries equals 1.03. The positive sources of this growth, ranked in order of relative importance, are: (1) changes in the quality of labour, (2) the net effect of all the "omitted terms" (or the residual), (3) increasing economies of scale. On the other hand, the prime negative source of capital productivity growth is the changes in the labour-intensity of production. Indeed, the average (l/k^g) growth rate observation is negative and equal to –2.90. It should be noted that the positive percentage contribution of economies of scale to capital productivity growth is *not* negligible because the relevant measure of size of establishment growth (namely, (k^g/e) is quite large, on the average (equal to 2.98). This contrasts with the situation in the case of labour productivity growth, where the relative impact of economies of scale is negligible (see Section 3.4). Thus the relative realized importance of "scale effects" in explaining inter-industry productivity growth differentials depends upon which *particular* productivity growth we are analysing.

Finally, it was already emphasized that the "complete" estimates for the period 1947-56 are not comparable with the "incomplete" 1957-67 empirical results. But the capital productivity growth estimates of the *latter period* should be compared with the corresponding results for labour productivity growth. Such a comparison shows that the *preliminary* point estimate of the elasticity of aggregate output with respect to aggregate labour input is considerably greater than the preliminary estimate of the elasticity of output with respect to aggregate capital input. In fact, the ratio *and* the magnitude of the two point estimates appear to approximately agree with what would be derived if competitive equilibrium conditions are assumed to hold during the 1957-67 time period. This is further discussed in the next chapter.

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[9] This is further discussed in Chapter 5. See again Lydall, *op. cit.*, pp. 820-821, whose estimates, in this respect, are very similar to ours.

[10] The formula used is: $\hat{b}_5 / (\hat{b}_8 + 1 - \hat{b}_4)$.

CHAPTER 5

FACTOR PRODUCTIVITY GROWTH ANALYSIS

It is now quite straightforward to apply an inter-industry empirical analysis of factor productivity growth differentials. The basic growth rate model is formulated in Section A.6 and is merely a weighted average of the labour productivity and capital productivity growth models. Therefore, much of the fundamental discussion in Chapter 3 is again directly relevant. In this chapter, the emphasis is on the empirical results and the additional light that a factor productivity analysis throws on the sources of productivity growth. There is a comparison between the factor productivity results of this Study and those of other writers. Also, the role of the output growth variable (q) in explaining inter-industry productivity growth differences is further developed in this chapter.

5.1 Sources of Factor Productivity Growth and Application of the Model

The factor productivity growth rate[1] for a manufacturing industry is simply *defined* as the *difference* between its total net output rate of growth, on the one hand, and a particular weighted average of its growth rates of total man-hours employed and total gross fixed capital stock utilized, on the other hand. The two "particular weights" are, respectively: (1) the *observed* factor share of labour in the value of net output at some base time point; and (2) the remainder, which could be called the "observed factor share of capital".[2]

From the above definition, it is easy to see that the factor productivity growth rate is just the same weighted average of the labour productivity and capital productivity growth rates.[3] Then the factor productivity growth model (which explains the sources of factor productivity growth) is merely the weighted average of the labour productivity growth model and the capital productivity growth model. Indeed, the *list* of sources of factor productivity growth, as defined, is identical to that of labour productivity growth, as defined. However, the third and fifth sources of productivity growth now require a different interpretation.

The third productivity growth source in Section 3.1 is "the growth of capital-intensity of production". It turns out[4] that the contribution of gross

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[1] It is not meaningful to define the "factor productivity level", since its value for inter-industry comparative purposes would generally depend on the choice of measurement units for labour and fixed capital.

[2] Note that the remainder contains the "factor share" of both fixed and working capital. The labour factor share is also arbitrary to some extent (see Appendix D).

[3] This is explicitly shown in Section A.6.

[4] See again Section A.6 for the technical details.

capital-intensity growth to factor productivity growth depends upon the ratio of the "particular weights" (i.e., the ratio of the observed factor shares). If the ratio of the observed factor shares for the chosen base time point *correctly reflects* the ratio of the aggregate output elasticities with respect to the corresponding aggregate factor inputs at a certain time, then gross capital-intensity growth will make *no* contribution to factor productivity change. Otherwise, the gross capital-intensity growth variable is a positive or negative source of factor productivity change, depending upon the relationship between the two ratios.[5]

The fifth productivity growth source in Section 3.1 is "changes in the average size of the manufacturing establishments that comprise the industry". It now turns out that the relevant indicator of such "changes in average size" is the *difference* between the weighted average of the growth rates of total man-hours and total gross fixed capital, on the one hand, and the rate of growth of the total number of establishments, on the other. Again, e.g., an increase (decrease) in the average size, so indicated, is a positive source of factor productivity growth if the typical establishment exhibits increasing (decreasing) economies of scale.

In Chapter 2 it was seen that factor productivity trend growth rates differ from industry to industry.[6] Thus the principal application in this chapter is to test the significance, and estimate the relative importance, of the various sources of inter-industry factor productivity growth rate differentials. The basic applied nonstochastic equation of this factor productivity growth analysis is then[7]

$$\begin{aligned}(q/c) &= d_1(n/p) + d_2(n^m/n^f) + d_3(p^m/p^f) \\ &\quad + d_4(k^g/l) + d_5(k^n/k^g) + d_6(m^g/s^g) \\ &\quad + d_7(m^n/s^n) + d_8(c/e) + d_9(q) + d_0\end{aligned}$$

where, again, all the variables are trend growth rates for the industry concerned over a certain time period and are defined as follows:

(q/c) is the factor productivity growth rate;

(c/e) is the growth rate of the relevant indicator of "average size of establishment".

And,

d_i are unknown coefficients. $(i = 0, 1, \ldots, 9)$

The other growth rate variables have already been described in Section 3.2.[8]

---

[5] It is assumed that the gross capital-intensity growth rate is always positive.

[6] See the last column in Tables 2-1 and 2-2.

[7] The following equation is the applied discrete counterpart of the theoretical continuous factor productivity growth model formulated in Section A.6 (equation number 51).

[8] The individual growth rate variable (c) in the above equation is actually equal to $[\alpha l+(1-\alpha)k^g]$. where α is the labour share of net output, as observed in the 1949 input-output table (1947-56 analysis) or as observed in the 1961 input-output table (1957-67 analysis). See also Appendix D for details.

Now, the above basic equation can be considered as the particular weighted average of the basic applied labour productivity and capital productivity equations of the previous two chapters. In this case, the following *theoretical* equalities should hold between the unknown coefficients of the three nonstochastic productivity growth equations:

$$a_i = b_i = d_i \qquad (i = 0,1,\ldots,8; i \neq 4)$$

$$\alpha a_4 - (1-\alpha)b_4 = d_4 \ .$$

Indeed, the second equality implies:[9]

$$d_4 \gtreqless 0 \text{ if and only if } \frac{\alpha}{(1-\alpha)} \gtreqless b_4/a_4 \ .$$

Thus the *regression* estimates of this chapter serve as a partially independent check on those of the previous two chapters. But they also provide a *direct test* of the hypothesis that the observed factor shares are, on the average, proportional to the respective aggregate output elasticities. Finally, one should expect the constant coefficient d_9 to have the same sign and be of a similar order of magnitude as the corresponding coefficients a_9 and b_9.

5.2 Statement and Interpretation of the Empirical Estimates

The estimation method for the inter-industry factor productivity growth analysis is the now familiar, two-step ordinary least squares (O.L.S.) procedure. The "best" regression equation in the first step is again chosen according to Theil's criterion.[10] Then, using the trend growth observations for the 16 manufacturing industries (1948 S.I.C.), the *complete estimates* of the "average" coefficients of factor productivity growth differentials, *1947-56*, are as follows:[11]

$$\begin{aligned}(q/c) = \underset{(0.515)}{1.674} &+ \underset{(0.115)}{0.301(n/p)} - \underset{(0.228)}{0.357(n^m/n^f)} \\ &+ \underset{(0.094)}{0.192(c/e)} + \underset{(0.069)}{0.147(q)} \qquad \bar{R}^2 = 0.370\end{aligned}$$

and,

$$\begin{aligned}(q/c) = \underset{(0.548)}{1.349} &+ \underset{(0.118)}{0.307(n/p)} - \underset{(0.229)}{0.316(n^m/n^f)} \\ &+ \underset{(0.098)}{0.200(c/e)} + \underset{(0.075)}{0.134(q)} \qquad \bar{R}^2 = 0.323\end{aligned}$$

---

[9] Also, using the theoretical equality stated in Chapter 4—i.e., $a_4 + b_4 - 1 = a_8 = b_8$—it is possible to show that the following implication holds: $[\alpha/(1-\alpha) = b_4/a_4]$ implies that $a_4 = (1-\alpha)(a_8+1)$ *and* $b_4 = \alpha(b_8+1)$.

[10] Refer to Section 3.3 and Theil, *op. cit.*, pp. 212-214.

[11] The figures in parentheses represent the estimated standard errors of the corresponding estimated coefficients and have a non-negative bias (see Zellner, *op. cit.*, p. 114).

with man-hour data and number-employed data, respectively. With 11 degrees of freedom (i.e., 16 minus 5), the 5 per cent significance level for the student t value in a one-tailed test is 1.796. The required 5 per cent significance level for adjusted $\bar{R}^2$ equals 0.306.

Some preliminary estimation results were also obtained for the second time period, 1957-67. Then using some of the available data for the 19 manufacturing industries (1960 S.I.C.), the *incomplete estimates* of the coefficients of factor productivity growth differentials, *1957-67*, are found to be:

$$
\begin{array}{lllllll}
(q/c) & = & 0.865 & - & 0.183(k^g/l) & + & 0.511(q) & \\
 & & (0.915) & & (0.135) & & (0.155) & \bar{R}^2 = 0.384 \\
(q/c) & = & 0.284 & & & + & 0.521(q) & \\
 & & (0.828) & & & & (0.158) & \bar{R}^2 = 0.354
\end{array}
$$

when man-hour data are used, and

$$
\begin{array}{lllllll}
(q/c) & = & 0.713 & - & 0.203(k^g/l) & + & 0.534(q) & \\
 & & (0.919) & & (0.142) & & (0.156) & \bar{R}^2 = 0.401 \\
(q/c) & = & 0.115 & & & + & 0.540(q) & \\
 & & (0.841) & & & & (0.161) & \bar{R}^2 = 0.364
\end{array}
$$

when number employed is used.[12]

The empirical estimates of the inter-industry factor productivity growth analysis are consistent with those of the labour productivity and capital productivity analyses. A comparison of the "complete estimates" (1947-56) for this chapter with the "complete estimates" for the previous two chapters implies that the required theoretical equalities stated in Section 5.1 are statistically acceptable. Thus, e.g., the economic interpretation given to the significant estimated coefficients of the growth variables (n/p), (n^m/n^f), (l/e) and (q) in Section 3.4 is further confirmed by the empirical estimates of this section. Similarly, the economic interpretation of the nonsignificant coefficients of (p^m/p^f), (k^n/k^g), (m^g/s^g) and (m^n/s^n) is maintained. Two related comments are now in order.

First, the various regression estimates show conclusively that the growth variable (k^g/l) is not statistically significant in the factor productivity analysis.[13] Recalling the development in Section 5.1, this estimated result could be given the following economic interpretation. For the period 1947-56, the ratio of factor shares, as observed in the 1949 input-output table, is a statistically acceptable approximation, on the average,[14] to the ratio of the respective aggregate output

[12] The relevant significance levels for the student t test and $\bar{R}^2$ value are 1.746 and 0.219, respectively.

[13] A table analogous to Tables 3-1 and 4-1 could be presented showing the various first-step regression estimates that lead up to the "best" regression. With man-hour data, the various estimated coefficients of (k^g/l) range between 0.064 and 0.176 and are never close to 5 per cent significance. With number-employed data, the estimated range is between 0.096 and 0.213, also never statistically significant.

[14] That is, as an average, or weighted average, of the different individual industry coefficients of the growth variable $(k^g/l)_j$ $(j = 1, \ldots, 16)$, in the factor productivity model.

elasticities over that period. This is so, even though the observed factor shares are a rather rough measure of the actual labour share and actual fixed capital share in the value of net output.[15] Thus, while observed total factor shares can approximate the ratio of the respective aggregate output elasticities, this does *not* imply, e.g., that observed relative shares of two types of labour are a statistically acceptable measure of their respective disaggregated output elasticity ratio. In fact, the economic analysis of the empirical estimates in Section 3.4 shows the occurrence of this possibility (see, particularly, the discussion of the estimated coefficient of (n^m/n^f)).

The second comment concerns the proportion of the inter-industry factor productivity growth differentials that is accounted for by the empirical estimates. The $\bar{R}^2$ value of the "best" regression (man-hour data) shows that 37 per cent[16] of these differentials are explained by significant explicit inter-industry productivity growth source differentials. This proportion is considerably smaller than that attained in the corresponding labour productivity and capital productivity analyses. There are two reasons for this: (1) the magnitude of inter-industry factor productivity variation "to be explained" is only about one-half that of labour productivity and capital productivity variation;[17] and (2) the highly significant explanatory growth variable (k^g/l) of Chapters 3 and 4 is no longer significant in the factor productivity case, so that the O.L.S. residuals are not correspondingly reduced.[18]

To continue, we present Table 5-1 showing the percentage of inter-industry factor productivity growth differentials that could be attributed to the individual sources (both explicitly measured and unmeasured) of such growth. The methodology of the first two columns is completely analogous to that used to prepare Table 3-3 of Section 3.4. The last two columns explicitly introduce the impact of the proxy variable (q) as a source of factor productivity growth. In effect, its contribution, loosely measured, is subtracted from the pure constant term.[19]

The economic interpretation of the results shown in Table 5-1 is quite clear. The relative importance of the various sources of factor productivity growth is largely invariant with respect to the man-hour or the number-employed estimates.

---

[15] The result that factor shares are proportional to aggregate output elasticities seems to have also been obtained, on the average, by Hildebrand and Liu, *op. cit.*, pp. 111-122, and Griliches, *op. cit.*, p. 298, but their estimates are not strictly comparable to ours.

[16] It should be recalled that the $\bar{R}^2$ value, as an estimate of the population coefficient of determination, has a non-positive bias in our context (see Zellner, *op. cit.*, p. 115).

[17] The sums of squared deviations about the mean for the 16 labour-productivity, capital-productivity, and factor-productivity growth rate observations are equal to 44.95, 41.69, and 19.41, respectively (using man-hour data).

[18] See again Goldberger, *op. cit.*, pp. 159-160 and 217, for a more precise discussion.

[19] This is done by no longer observing the variable $(q)_j$ in terms of its average deviations in the second step of the two-step procedure. Then this second step yields a pure constant as well as the slope coefficient of (q). Since the dependent variable is the residual of the first step, and recalling a well-known property of O.L.S. estimates, it is seen that the estimated coefficient of (q) is the same as before. A "net" pure constant is then obtained as the summation of the pure constants from the first and second steps (see Goldberger, *op. cit.*, pp. 182-184 and 194-197).

TABLE 5-1

ESTIMATES OF THE PERCENTAGE OF MANUFACTURING FACTOR PRODUCTIVITY GROWTH DIFFERENTIALS ATTRIBUTED TO THE VARIOUS SOURCES, 1947-56

	As Estimated by			
Source of Growth	Man-hours	Number Employed	Man-hours	Number Employed
(n/p)	38.8%	45.1%	38.8%	45.1%
(n^m/n^f)	− 3.6	− 5.7	− 3.6	− 5.7
(c/e)	7.1	9.9	7.1	9.9
(q)			28.7	28.6
Original pure constant	57.4	50.9		
Net pure constant			28.8	22.3
Grand Total	99.7%	100.2%	99.8%	100.2%

Note: The estimates are from the "best" complete estimates shown in this section. See Sections 3.2 and 5.1 for the meaning of the growth source variables. The grand totals do not sum to 100.0% because of rounding.

The original pure constant, containing the net effect of all the "omitted terms" that are not individually measured, is the most important positive source of factor productivity growth, while "changes in the quality of labour"[20] is a close second. About one-half of the former source seems to be associated with the proxy growth variable (q). Thus increased specialization, learning phenomena, and technical change facilitated by the growth of total industry output, are all important positive sources of factor productivity growth to the extent that they are approximated by the variable (q). It is also interesting to note that the percentage contribution of increasing economies of scale to factor productivity growth is *not* negligible, on the average, as was the case in the labour productivity analysis (see Table 3-3). Finally, it should be realized that our result – that a substantial proportion of factor productivity growth cannot be identified with measurable and well-defined sources – is very much in conformity with the current productivity research literature.[21]

So far the discussion has focused on the "complete" 1947-56 estimation results. One comment could be made on the "incomplete" factor productivity results shown earlier in this section for the second time period 1957-67. The four regression estimates taken together lead one to suspect that the incomplete nature of the specified regression equations has biased downwards the estimated coefficient of the growth variable (kg/l).[22] It would be interesting to see whether the negativity and "near-significance" of this coefficient is preserved under a more complete specification with appropriately revised statistical data.

[20] In so far as "changes in the quality of labour" are explicitly measured.

[21] The best and most recent summary of the literature is in Christensen and Jorgenson, *op. cit.*, pp. 43-49.

[22] The ratio of the estimated aggregate output elasticities from the incomplete labour productivity and capital productivity analyses (1957-67) is almost identical to the ratio of the average respective factor shares, as observed in the 1961 input-output table.

CHAPTER 6

RESOURCE SHIFT ANALYSIS AND PRODUCTIVITY GROWTH

In Chapter 3, a special case of industry "labour quality change" was referred to – namely, the "labour resource shift effect".[1] This can occur when there are differential growth rates for any particular type of labour between the establishments that comprise the industry. Similarly, we referred to the "fixed capital resource shift effect".[2] Theoretically, it is possible to measure the contribution of such resource shift effects to the productivity growth of an industry by applying the framework of Appendixes A and B. The technique assumes the existence of a well-defined industry production function.

However, an alternative approach is to express the productivity growth of an industry as a suitably weighted average of the productivity growth rates of its component establishments, *plus* a remainder term. The latter term is the total "resource shift effect" and could be measured if suitable data were available.[3] Note that this approach does *not* require the existence of an industry production function. The main purpose of this chapter is to illustrate this method by measuring the *contribution* of "resource shift effects" between the various two-digit industries *to* the productivity growth of the *Total Manufacturing* sector. The resource shift analysis is carried out in terms of labour productivity growth and factor productivity growth for both the 1947-56 and 1957-67 time periods.

6.1 A Methodology for Resource Shift Analysis

It is well known that the *labour productivity* of two particular industries could remain constant, or even fall, over time, while the labour productivity level for the *total* of the two industries increases, *if* there has been an appropriate redistribution of the total labour employed *between* the two industries.[4] In this

[1] See source number 6 of productivity growth in Section 3.1.
[2] See source number 7 of productivity growth in Section 3.1.
[3] For example, one would need input and output data at the level of the individual establishments in order to calculate the component productivity growth rates.

[4] Let Q_i^0 (Q_i^1) represent the i-th industry output at time zero (time one), (i = 1,2). Let $Q^0 = Q_1^0 + Q_2^0$ denote the total industry output at time zero. Similarly, $Q^1 = Q_1^1 + Q_2^1$. Also let $L^0 = \sum_i L_i^0$, $L^1 = \sum_i L_i^1$, where L_i^0 (L_i^1) is labour employed in i-th industry at time zero (time one). Then by choosing $Q_1^0 = Q_2^0 = 50$, $L_1^0 = 75$, $L_2^0 = 25$, $Q_1^1 = 40$, $Q_2^1 = 160$, $L_1^1 = 60$, $L_2^1 = 80$, it is easily seen that

$$Q_1^0/L_1^0 = Q_1^1/L_1^1 \text{ and } Q_2^0/L_2^0 = Q_2^1/L_2^1 ,$$

but

$$100/100 = Q^0/L^0 < Q^1/L^1 = 200/140 .$$

case, productivity change within the two industries would make a zero, or even negative, contribution to total industry productivity growth. Indeed, the growth of total industry productivity would be due entirely to a relative shift in the proportion of total labour employed towards the particular industry with the higher average labour productivity level. This "labour resource shift effect" makes a positive contribution to total industry labour productivity growth.

The major task of a *general* labour productivity resource shift analysis is to show the total industry labour productivity growth rate as the summation of *two* expressions: (1) the weighted average of the labour productivity growth rates *within* each of the particular industries; and (2) the contribution of the "labour resource shift effect". The latter expression has a very simple interpretation. We say that the "resource shift effect" is positive (negative) if and only if the growth rates of labour employed in the relatively high labour productivity industries is greater (smaller) than the growth rates of labour employed in the relatively low productivity industries. It is possible to obtain an expression with this *net* interpretation for the case of any number of industries.[5] Note, e.g., that the labour productivity growth rate of the total of the industries is greater than the suitably weighted average of the individual industry productivity growth rates *if* the "labour resource shift effect" is positive.

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[5] Suppose at first there are two industries. Let $Q_i(t)$, or simply Q_i , represent the output of the i-th industry at time t $(i = 1,2)$. Let $Q = Q_1 + Q_2$ denote the total output of the two industries at time t. Similarly, let $L = \sum_i L_i$ where L_i is labour employed in i-th industry at time t $(i = 1,2)$. Then it is straightforward to see that the total industry labour productivity growth rate can be written as the summation of *two* expressions; i.e.,

$$(1)\quad \left(\frac{\dot{Q}}{Q} - \frac{\dot{L}}{L}\right) = \sum_{i=1}^{2} \frac{Q_i}{Q}\left(\frac{\dot{Q}_i}{Q_i} - \frac{\dot{L}_i}{L_i}\right) + \sum_{i=1}^{2} \frac{Q_i}{Q}\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}}{L}\right)$$

where the dots signify derivatives with respect to time. The first expression on the R.H.S. of the equation is simply a weighted average of the labour productivity growth rates of the two industries, and its interpretation is obvious. If one substitutes $\dot{L}/L = (L_1/L)\,(\dot{L}_1/L_1) + (L_2/L)\,(\dot{L}_2/L_2)$ in the second expression on the R.H.S. of (1), the expression becomes

$$(2)\quad \sum_{i=1}^{2} \frac{Q_i}{Q}\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}}{L}\right) = \left(\frac{1}{QL}\right)(Q_1L_2 - Q_2L_1)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)$$

$$= \left(\frac{L_1L_2}{QL}\right)\left(\frac{Q_1}{L_1} - \frac{Q_2}{L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right).$$

Thus, e.g., the second expression is positive if and only if $\dot{L}_1/L_1 \gtrless \dot{L}_2/L_2$ *and* $Q_1/L_1 \gtrless Q_2/L_2$, which is a required property of an expression that represents the "labour resource shift effect" contribution.

In the more general case of n industries, it is found that

$$(3)\quad \left(\frac{\dot{Q}}{Q} - \frac{\dot{L}}{L}\right) = \sum_{i=1}^{n} \frac{Q_i}{Q}\left(\frac{\dot{Q}_i}{Q_i} - \frac{\dot{L}_i}{L_i}\right) + \sum_{\lambda<j}\left(\frac{L_\lambda L_j}{QL}\right)\left(\frac{Q_\lambda}{L_\lambda} - \frac{Q_j}{L_j}\right)\left(\frac{\dot{L}_\lambda}{L_\lambda} - \frac{\dot{L}_j}{L_j}\right)$$

where $(\lambda = 1, \ldots, n-1)$, $(j = 2, \ldots, n)$. The interpretation of equation (3) is analogous to that of equations (1) and (2). Note that the individual terms in the second expression on the R.H.S. of (3) may not all have the same sign; it is the *net* contribution that matters.

A similar analysis is applicable in the *factor productivity* growth context. Now, e.g., the factor productivity growth rate for the total of two industries will depend not only on the factor productivity growth rates of the individual industries and the redistribution of total labour employed between the two industries, but on the redistribution of total fixed capital stock between the industries as well. Thus the major task of a general factor productivity resource shift analysis is to show the total industry factor productivity growth rate as the summation of *three* expressions: (1) the weighted average of the factor productivity growth rates *within* each of the particular industries; (2) the contribution of the "labour resource shift effect"; and (3) the contribution of the "fixed capital resource shift effect". If one assumes that the observed factor shares of the value of the individual industries' output are *proportional* to their respective production function elasticities,[6] it turns out that the latter two expressions have a familiar interpretation. For example, the "labour resource shift effect" is positive (negative) if and only if the growth rates of labour employed in the industries where the *marginal* productivity of labour is relatively high are greater (smaller) than the growth rates of labour employed in the industries where the *marginal* productivity of labour is relatively low. An analogous interpretation holds for the "fixed capital resource shift effect", under the additional assumption that fixed capital input is correctly measured by the gross capital stock data.[7]

[6]This assumption is supported by the empirical results of Chapter 5; see Section 5.2.
[7]This assumption is supported by the empirical results of Chapter 3; see Section 3.4.

Again, it is possible to obtain "resource shift effect" expressions with the required properties for the general case of any number of industries.[8] Indeed, e.g., the factor productivity growth rate of the total of the industries is greater than the suitably weighted average of the individual industry productivity growth rates *if* both the net "labour resource shift effect" and the net "fixed capital resource shift effect" are positive. Also, it is interesting to note that in the labour productivity resource shift analysis, the *criterion* for inter-industry labour productivity differentials is the "average labour productivity level". On the other hand, in the factor productivity "labour resource shift effect" analysis, the corresponding criterion is a "weighted average of the marginal products of the various types of labour employed". Of course, the factor productivity resource shift analysis reduces to the labour productivity analysis in the special case where the labour factor shares of the individual industries are all equal to unity.

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[8] Using the same notation as in the previous mathematical footnote of this section, suppose at first that there are two industries. In addition, let $K = \sum_i K_i$ where K_i is the gross fixed capital stock in the i-th industry at time t (i = 1,2). Further, let α_i represent the labour factor share of the i-th industry, so that $\sum_i \alpha_i Q_i/Q = \alpha$ is the labour factor share of the total of the two industries at time t (i = 1,2). Then it is easy to see that the total industry factor productivity growth rate can be written as the summation of *three* expressions, i.e.,

$$(4)\quad \left[\frac{\dot{Q}}{Q} - \alpha\frac{\dot{L}}{L} - (1-\alpha)\frac{\dot{K}}{K}\right] = \sum_{i=1}^{2} \frac{Q_i}{Q}\left[\frac{\dot{Q}_i}{Q_i} - \alpha_i\frac{\dot{L}_i}{L_i} - (1-\alpha_i)\frac{\dot{K}_i}{K_i}\right] + \sum_{i=1}^{2} \frac{Q_i}{Q}\alpha_i\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}}{L}\right) + \sum_{i=1}^{2} \frac{Q_i}{Q}(1-\alpha_i)\left(\frac{\dot{K}_i}{K_i} - \frac{\dot{K}}{K}\right).$$

The first expression on the R.H.S. of (4) is simply a weighted average of the factor productivity growth rates of the two industries and has the obvious economic interpretation. The second expression requires some further analysis.

Suppose that factor shares are proportional to the respective production function elasticities and that each industry exhibits the same degree of returns to scale. This amounts to

$$(5)\quad \alpha_i = \left(\frac{1}{r}\right)\sum_{j=1}^{m} \frac{L_{ij}}{Q_i}\frac{\partial Q_i}{\partial L_{ij}} \qquad (i = 1,2)$$

where $\partial Q_i/\partial L_{ij}$ is the marginal product of the j-th type of labour employed in the i-th industry; r is the returns to scale coefficient; $\sum_{j=1}^{m} L_{ij} = L_i$; and (without loss of generality) m is the number of different types of labour employed in the i-th industry (i = 1,2). Now substituting (5) and the relation $\dot{L}/L = (L_1/L)(\dot{L}_1/L_1) + (L_2/L)(\dot{L}_2/L_2)$ in the second expression on the R.H.S. of equation (4), it is found that

$$(6)\quad \sum_{i=1}^{2} \frac{Q_i}{Q}\alpha_i\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}}{L}\right) = \left(\frac{L_1 L_2}{QL}\right)\left(\frac{\alpha_1 Q_1}{L_1} - \frac{\alpha_2 Q_2}{L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)$$

with the implication that

$$\left(\frac{\alpha_1 Q_1}{L_1} - \frac{\alpha_2 Q_2}{L_2}\right) \gtreqless 0 \text{ if and only if } \sum_{j=1}^{m} \frac{L_{1j}}{L_1}\frac{\partial Q_1}{\partial L_{1j}} \gtreqless \sum_{j=1}^{m} \frac{L_{2j}}{L_2}\frac{\partial Q_2}{\partial L_{2j}}.$$

Thus, e.g., the second expression is positive if $\dot{L}_1/L_1 > \dot{L}_2/L_2$ *and*, loosely speaking, the "average" marginal product of labour in the first industry is greater than the "average" marginal product of labour in the second industry. This satisfies the required property of an expression that represents the "labour resource shift" contribution in a factor productivity growth context. The third expression in (4) can be subject to a completely similar analysis. The generalization to any number of industries is straightforward.

6.2 Application of the Analysis

In this and the next section, the methodology of resource shift analysis is applied in order to measure the contribution of "resource shift effects" between the various two-digit manufacturing industries, as they contribute to the productivity growth of the Total Manufacturing sector. In particular, e.g., the "output" of the Total Manufacturing sector is *defined* as simply the total of the outputs of the component two-digit industries. (Similar definitions hold for the "labour employed" and "fixed capital stock" of the Total Manufacturing sector.) No explicit functional dependence is assumed between the "output" so defined, and the "labour employed" and "fixed capital stock" so defined.[9] It is in this sense that the analysis does *not* require the assumed existence of a well-defined Total Manufacturing "production function". We will first discuss the application of the *labour productivity resource shift analysis.*

The basic applied relationship is

$$(1)\quad (q/l)_T \simeq \sum_{i=1}^{N} \frac{Q_i}{Q}(q/l)_i + \sum_{i=1}^{N} \frac{Q_i}{Q}(l_i/l)$$

where:

$(q/l)_T$ is the labour productivity trend growth rate (over a particular time period) for the Total Manufacturing sector;

$(q/l)_i$ is the labour productivity trend growth rate for the i-th (two-digit) industry of the manufacturing sector;

(l_i/l) is the trend growth rate of the ratio of labour employed in the i-th industry to total labour employed in all manufacturing;

Q_i/Q is the simple average of the ratios of the i-th industry output to "Total Manufacturing" output, the ratios being calculated for the initial and terminal years of the particular time period;

N is the number of two-digit industries for the S.I.C. relevant to the particular time period.

Two comments are now in order. The above relationship is the applied discrete counterpart[10] of the theoretical continuous labour productivity resource shift equation given in the first mathematical footnote of the previous section. As such, it is subject to an approximation error, so that the L.H.S. of the relation is

[9] For example, in the simplest case, suppose the manufacturing sector is composed of two industries, each of which employs one type of labour and uses one type of capital stock. Let $Q_1 = f_1(L_1,K_1)$ represent the production function of the first industry. Similarly $Q_2 = f_2(L_2,K_2)$ represents the second industry. Then $Q = Q_1 + Q_2$ is the defined output of the manufacturing sector. Similarly $L = L_1 + L_2$ and $K = K_1 + K_2$. It is *not* assumed that there exists a function g such that

$$Q = f_1(L_1,K_1) + f_2(L_2,K_2) = g(L,K) = g(L_1 + L_2, K_1 + K_2)$$

for all non-negative L_i,K_i $(i = 1,2)$.

[10] See Henri Theil, *Economics and Information Theory* (Amsterdam: North-Holland Publishing Company, 1967), Chapter 5.

nearly equal to the summation of the two expressions on the R.H.S. The second comment is that the "labour (man-hours) employed" data used in this analysis differ somewhat from the corresponding data used in the productivity analyses of Chapters 3, 4 and 5.[11]

The basic applied relationship for the *factor productivity resource shift analysis* is

$$(2) \quad (q/c)_T \simeq \sum_{i=1}^{N} \frac{Q_i}{Q}(q/c)_i + \sum_{i=1}^{N} \frac{Q_i}{Q}\alpha_i(l_i/l) + \sum_{i=1}^{N} \frac{Q_i}{Q}(1-\alpha_i)(k_i/k)$$

where:

- $(q/c)_T$ is the factor productivity trend growth rate for the Total Manufacturing sector;
- $(q/c)_i$ is the factor productivity trend growth rate for the i-th industry of the manufacturing sector;
- (k_i/k) is the trend growth rate of the ratio of fixed capital stock utilized in the i-th industry to total fixed capital stock utilized in all manufacturing;
- α_i is the labour factor share of the i-th industry as observed in the 1949 input-output table (1948 S.I.C.), or as observed in the 1961 input-output table (1960 S.I.C.).

The remaining notation has the same meaning as before.

Thus the relationship (2) is the applied discrete approximation of the theoretical continuous factor productivity resource shift equation shown in the second mathematical footnote of Section 6.1. Here, both the "labour (man-hours) employed" data and the "fixed capital stock" data differ somewhat from the corresponding statistical data used for the inter-industry productivity growth differential analysis.[12]

6.3 Empirical Results and Their Interpretation

The results of applying the *labour productivity resource shift analysis* to the 17 two-digit manufacturing industries (1948 S.I.C.) for the period *1947-56* are as follows. We have

$$(q/l)_T \simeq \sum_{i=1}^{17} \frac{Q_i}{Q}(q/l)_i + \sum_{i=1}^{17} \frac{Q_i}{Q}(l_i/l) ,$$

which becomes:

$$3.89 \quad \simeq \quad 3.82 \quad + \quad 0.09$$

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[11] The question of alternative data sources is mentioned in Appendix D.
[12] See again Appendix D.

Thus almost all of labour productivity growth for the Total Manufacturing sector is due to the growth of labour productivity *within* the individual industries. The "labour resource shift" makes a negligible *net* contribution to total labour productivity change. It should be noted that this "negligible *net* contribution" reflects a considerable cancelling out of *positive* and *negative* elements.[13] For example, "Petroleum and coal products" is an industry with a relatively high labour productivity level[14] and with an above-average labour employment trend growth rate – this will tend to *augment* the "labour resource shift effect". Similarly, the resource shift contribution is increased by the presence of the "Textile products" and "Clothing" industries which are relatively low labour productivity industries with below-average labour employment growth rates. However, "Food and beverages" is a high labour productivity industry with a relatively low employment rate of growth, and both "Transportation equipment" and "Electrical apparatus and supplies" are low productivity industries with relatively high labour employment growth rates. The existence of industries with the latter two characteristics will tend to *diminish* the resource shift contribution to productivity growth.

The preliminary results of applying the *labour productivity resource shift analysis* to the 20 two-digit industries (1960 S.I.C.) for the time period *1957-67* are as follows.[15] Again, we have

$$(q/l)_T \simeq \sum_{i=1}^{20} \frac{Q_i}{Q}(q/l)_i + \sum_{i=1}^{20} \frac{Q_i}{Q}(l_i/l) \quad ,$$

which becomes:

$$3.77 \simeq 3.68 + (-0.01) \quad .$$

The labour productivity growth for "Total Manufacturing" is entirely due to the increase in productivity *within* the 20 individual industries. If one were to examine the average labour productivity levels for the two-digit industries over the period 1957-67, together with their corresponding labour employment trend growth rates, it would be found that the "picture" is similar in *some* respects to that described for the 1947-56 time period. For example, "Textile products", "Knitting mills", and "Clothing" are, again, industries with relatively low labour productivity and below-average growth rates for employment. Also, "Electrical products" is once more a low labour productivity industry with a relatively high labour rate of growth.[16] However, "Petroleum and coal products", which again has the highest labour productivity level, now exhibits a negative growth rate for employment in

[13] See the generalized "labour resource shift" expression in equation (3) of the first mathematical footnote in Section 6.1.

[14] That is, the "labour productivity level" of the "Petroleum and coal products" industry, as an average over the period 1947-56, is relatively high compared with the other two-digit industries. The particular labour productivity levels can be calculated, if desired, from the data sources given in Appendix D.

[15] The 1960 S.I.C. results are preliminary because they are based on unrevised statistical data.

[16] It should be remembered that 1948 S.I.C. and 1960 S.I.C. industries are not strictly comparable.

the period 1957-67. And "Transportation equipment", still having an above-average labour growth rate, now tends to augment the "labour resource shift effect" because of its relatively high labour productivity level during this period. However, the *net* resource shift contribution is virtually nil because of the apparent cancellation of the positive and negative elements in the "labour resource shift" expression.[17]

The results of applying the *factor productivity resource shift analysis* to the 17 two-digit industries for the period *1947-56* are as follows:

$$(q/c)_T \simeq \sum_{i=1}^{17} \frac{Q_i}{Q}(q/c)_i + \sum_{i=1}^{17} \frac{Q_i}{Q}\alpha_i\,(l_i/l) + \sum_{i=1}^{17} \frac{Q_i}{Q}(1-\alpha_i)(k_i/k) \quad ,$$

which becomes:

$$2.64 \simeq 2.63 + 0.02 + (-0.03)$$

The analogous preliminary results as applied to the 20 two-digit industries for the period *1957-67* are:

$$2.81 \simeq 2.68 + 0.01 + 0.02$$

Thus the *net* effect of relative shifts in labour employed and fixed capital stock *between* the two-digit industries accounted for a negligible proportion of factor productivity growth for "Total Manufacturing" in both the 1947-56 and 1957-67 time periods. In fact, each of the potential "resource shift effects" are virtually zero. This again reflects a cancelling-out of positive and negative elements in the relevant resource shift expressions. One could examine the *product* of the labour factor shares[18] and the average labour productivity levels over the particular time periods, together with the corresponding trend growth rates of labour employed, in order to formulate examples of industries that have tended to augment or diminish the "labour resource shift" expression.[19] Similarly, an examination of the *product* of the capital factor shares and average capital productivity levels,[20] together with corresponding growth rates of fixed capital stock, would reveal the industries that have tended to augment or diminish the "fixed capital resource shift" expression. In any event, it is seen that the net changes giving rise to the growth of "Total Manufacturing" factor productivity have occurred *within* the individual industries over the two postwar decades.

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[17] The results of the labour productivity resource shift analysis were "checked" using a method described in Salter, *op. cit.*, pp. 184-185. In fact, Salter gives two methods which, when applied to our data, yield identical results. These results are virtually the same as those reported in the text of this Study for both the 1947-56 and 1957-67 time periods. It should be noted that Salter's method cannot be extended to a factor productivity analysis. Our method is a special case of the more general factor productivity resource shift analysis.

[18] Labour factor shares are only observed in the years 1949 and 1961 from the relevant input-output table.

[19] Recall that the product of the labour share and average labour productivity level yields a "weighted average of the marginal products of the various types of labour employed". See again the second mathematical footnote in Section 6.1 for the assumptions involved.

[20] The particular capital productivity levels can be calculated, if desired, from the data sources given in Appendix D.

It is interesting to compare the results of our factor productivity resource shift analysis with those of a similar U.S. manufacturing analysis.[21] Briefly, it was found that "resource shift effects" accounted for about one-third of U.S. factor productivity growth in total manufacturing for the period 1946-57. The "resource shift effects" were largely dominated by the magnitude of the "fixed capital resource shift" contribution, but both the labour and capital net effects were positive and not negligible. These results, of course, differ significantly from those obtained in this Study of Canadian manufacturing. It may be valuable for future research to consider the reasons for this discrepancy.

In conclusion, let it be noted that all these results are dependent upon the particular industrial disaggregation level of the analysis. For example, the labour productivity growth rate of *each* of the two-digit industries can be expressed as the summation of two further expressions: (1) the weighted average of the productivity growth rates *within* its component three-digit industries; and (2) the net contribution of the "labour resource shifts" *between* these industries. Thus, while the net effect of relative shifts in labour employed between Canadian two-digit industries may be, and in fact is, negligible, there might very well be a substantially positive contribution from relative shifts in labour employed *between* the component industries *within* some of the two-digit manufacturing industries.[22] At an even finer level of disaggregation, we could consider the net effect of relative shifts in labour employed and fixed capital stock between the particular production activities (or processes) of the individual manufacturing plant. Indeed, this is what an important part of "technological change" is all about.

[21] See Massell, *op. cit.*, pp. 319-330. The methodology used in the Massell article is similar to the one used in this Study, and the applications are approximately comparable. However, the rationale and development of the resource shift methodology are somewhat different in this Study.

[22] More precisely, using the same notation as in the first mathematical footnote of Section 6.1, we had

$$(1)\qquad \left(\frac{\dot{Q}}{Q} - \frac{\dot{L}}{L}\right) = \sum_{j=1}^{n} \frac{Q_i}{Q}\left(\frac{\dot{Q}_i}{Q_i} - \frac{\dot{L}_i}{L_i}\right) + \sum_{i=1}^{n} \frac{Q_i}{Q}\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}}{L}\right).$$

Without loss of generality, suppose that the i-th industry is composed of m sub-industries, so that $Q_i = \sum_{j=1}^{m} Q_{ij}$, where Q_{ij} is the output of the j-th sub-industry within the i-th industry. Similarly, let $L_i = \sum_{j} L_{ij}$ $(i = 1,\ldots,n)$. Then

$$(7)\qquad \left(\frac{\dot{Q}_i}{Q_i} - \frac{\dot{L}_i}{L_i}\right) = \sum_{j=1}^{m} \frac{Q_{ij}}{Q_i}\left(\frac{\dot{Q}_{ij}}{Q_{ij}} - \frac{\dot{L}_{ij}}{L_{ij}}\right) + \sum_{j=1}^{m} \frac{Q_{ij}}{Q_i}\left(\frac{\dot{L}_{ij}}{L_{ij}} - \frac{\dot{L}_i}{L_i}\right)$$

for $i = 1,\ldots,n$. Substituting (7) in (1) it is found that

$$(8)\qquad \left(\frac{\dot{Q}}{Q} - \frac{\dot{L}}{L}\right) = \sum_{i,j} \frac{Q_{ij}}{Q}\left(\frac{\dot{Q}_{ij}}{Q_{ij}} - \frac{\dot{L}_{ij}}{L_{ij}}\right) + \sum_{i,j} \frac{Q_{ij}}{Q}\left(\frac{\dot{L}_{ij}}{L_{ij}} - \frac{\dot{L}_i}{L_i}\right) + \sum_{i} \frac{Q_i}{Q}\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}}{L}\right).$$

The first expression on the R.H.S. of (8) is the contribution of productivity growth within, say, the three-digit industries. The second expression is the contribution of "labour resource shifts" between three-digit industries within the *same* two-digit industry. The third expression is the familiar "labour resource shift" between two-digit industries (within the Total Manufacturing sector). Only the contribution of the *last* expression has been shown to be negligible.

CHAPTER 7

SUGGESTIONS FOR FUTURE RESEARCH

Throughout this Study, a number of general comments have been made concerning the incomplete and preliminary nature of the current investigation. The purpose of this Chapter is to discuss these comments in greater detail. The related suggestions for future research fall naturally into two categories. First, there is the need to obtain additional results within the framework of the current Study. This is particularly important for the 1960 S.I.C. two-digit level of disaggregation. Second, it is seen that the methodological apparatus is sufficiently flexible to provide a considerable enrichment and extension of the current analytical results. Some concrete research proposals are made along these lines.

7.1 The Need for Additional Results

There are various types of additional results that would be desirable to obtain within the analytical framework of this Study. These potential investigations are now discussed in order of priority and feasibility.

The most important requirement is to attain "complete estimates" of the coefficients of inter-industry productivity growth differentials for a second postwar time period based on the 1960 S.I.C. two-digit manufacturing industries. Such estimates would then correspond with the "complete estimates" for the period 1947-56 (shown in Chapters 3, 4 and 5), based on the 1948 S.I.C. This writer believes that a second time period such as 1957-67 might be difficult to analyse.[1] However, a time period such as 1961-69 or 1961-70 has a number of advantages. This period is particularly homogeneous, in terms of both economic structure and the availability of continuous statistical time series. For example, the revised net output data that should be forthcoming in 1971 will provide more accurate[2] output growth rates for the period beginning in 1961. Also, the expected availability of complete 1960 S.I.C. two-digit fixed capital *formation* data, from 1917 on, will provide the basis for superior fixed capital *stock* growth rates for the period 1961 on.[3]

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[1] See the discussion in Sections 2.1 and 3.3. Briefly, there is both a structural (trend) break and a statistical discontinuity in the year 1961 for most of the manufacturing industries.

[2] The revisions furnish a more accurate measure of manufacturing output as a "total activity" concept in line with the labour and capital stock data. They also yield a sharper indication of *net* output and employ a new price deflator. Such revisions could critically change some of the output growth rates.

[3] Recall that capital stock data are calculated by the perpetual inventory method and that the structures component has an average assumed life of about 45 years.

If "complete estimates" for a second time period are obtained, it will then be possible to test for significant changes in the sources (or coefficients) of differential productivity growth in the first time period compared with the second. A convenient statistical test is that of G. C. Chow.[4] Further research could also proceed by recalculating the relevant trend growth rates by the alternative, and probably superior, least-squares fit method.[5] For example, the latter method would be highly desirable for time periods such as 1947-57 and 1961-69 because of the critical difference between the unemployment rates (at least for "Total Manufacturing")[6] for the initial and terminal years of the two mentioned periods.

It has already been emphasized that the various fixed capital stock growth rates of this Study are dependent upon the particular *assumed* survival curves, depreciation formulas, and asset price deflators (see also Appendix D). Therefore, it is of some importance to know the sensitivity of the productivity coefficient estimates to alternative assumptions. However, it is conjectured that the impact of such alternative assumptions may not be large because of the medium-term growth rate nature of this Study.[7] Also, a sensitivity analysis would not reveal the "correct" assumptions to be made in measuring capital stock growth rates.

In this Study, as in most others, there is need for more observations in order to provide greater degrees of freedom for estimation.[8] The two-digit 1948 S.I.C. contains 17 industries; the corresponding 1960 S.I.C. yields 20 possible observations. A much larger number of observations for each of the two S.I.C.'s could be obtained by analysing the manufacturing productivity trends at the three-digit level of disaggregation.[9] The chief "bottleneck" for the feasibility of such an analysis is the nonavailability of fixed capital formation time series data at a disaggregation level finer than that of the two-digit classification. It is hoped that at least some such data will be made available, together with the relevant life assumptions, in the near future.

Finally, it is well known that regional economic analysis is of considerable interest in Canada. Therefore, a natural application of the methodological framework of this Study is to repeat the manufacturing productivity trend analysis for *each* of the five major regions of the Canadian economy.[10] We could then

[4] See G. C. Chow, "Tests of Equality between Sets of Coefficients in Two Linear Regressions", *Econometrica*, July 1960, pp. 591-605.

[5] This was discussed in Section 2.1.

[6] Derek White, of the Economic Council staff, has pointed out that the reference cycles of *individual* two-digit industries often differ by intervals of as much as one year. This fact would tend to support the use of the least-squares fit method of calculating trend growth rates even when the initial and terminal years of the period have equal unemployment rates for "*Total* Manufacturing".

[7] For another view, see Lithwick, Post and Rymes, *op. cit.*, pp. 172-182.

[8] See again the comments in Section 3.4 and Section 7.2. Note that in the inter-industry study of Lydall, *op. cit.*, a total of 54 industries are analysed.

[9] At least on the basis of a mixture of selected three-digit and two-digit manufacturing industries. See the net output indexes published in Dominion Bureau of Statistics, *Indexes of Real Domestic Product. . . , op. cit.*, pp. 36-40 and 21-28.

[10] Jim Gander, of the Economic Council staff, suggested the idea of an intra-regional productivity growth analysis. The five major regions are: Atlantic Region, Quebec, Ontario, Prairie Region, British Columbia.

statistically test for significant differences in the sources of inter-industry differential productivity growth in any one region compared with the other regions. Unfortunately, the required statistical data for such an analysis is not currently available, but the increasing importance of regional studies is evident.[11]

7.2 Some Possible Extensions of the Present Study

There are a number of direct extensions of this Study that could be briefly discussed. It is seen that most of these investigations call for additional explanatory variables in the productivity regression equations, so that a larger number of industries should normally be analysed in order to provide adequate degrees of freedom.

The first extension involves a sharper measure of "labour quality change". In the present Study, labour quality change was largely measured to the extent that it was revealed in changes in the ratio of nonproduction-labour employment to production-labour employment.[12] Such a measure is admittedly incomplete in the sense that other aspects of labour quality change are neglected.[13] Moreover, the nonproduction-labour/production-labour distinction is not amenable to economic policy formulation and the distinction itself is subject to a conceptual statistical time series break in the year 1961.[14] Thus it would seem desirable in future research to exploit any available data that could yield a classification distribution of the "education stock" of the labour force employed by individual manufacturing industries.[15] In a productivity growth context, such a distribution (possibly extrapolated) is required for both the initial and terminal years of any time period analysed. The general methodology of Appendix B, especially Section B.2, could then be applied to yield a new and consistent measure of labour quality change. For example, if four categories of "school years completed" are statistically available for *both* males and females, then the *measured* labour quality change expression will contain *seven* distinct terms. This contrasts with the *three* distinct labour quality change terms – (n/p), (n^m/n^f), and (p^m/p^f) – of the present Study.

The second suggested extension is also related to the problem of measuring labour quality change. So far in this Study it has been implicitly assumed that the input of each type of labour employed should be measured in terms of "man-hours".[16] This is equivalent to supposing that the elasticity of aggregate output with respect to the "number employed" of any particular labour type is

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[11] Witness, e.g., the recent work on regional income accounts and regional input-output tables.

[12] The use of this ratio is defended (at least in the context of U.S. manufacturing) in Delehanty, *op. cit.*, especially pp. 131-147, 183-206. See also Hildebrand and Liu, *op. cit.*, pp. 49-50.

[13] For technical details, see Section B.2.

[14] See Dominion Bureau of Statistics, *General Review of the Manufacturing Industries of Canada, 1961*, Cat. No. 31-201, and DBS, *Manufacturing Industries of Canada, Section A, Canada, 1964*, Cat. No. 31-203. Therefore, labour quality change terms for the period 1947-56 and 1957-67 may not be comparable if estimated on the above basis.

[15] See, e.g., the various data sources exploited by Griliches, *op. cit.*, pp. 312-313. It may be possible to apply similar methods to Canadian manufacturing industries.

[16] Empirical results were also obtained using "number employed", but the latter was regarded as inferior to the "man-hour" results.

always equal to the corresponding elasticity with respect to "average hours worked per week". Such an assumption, although widely made, is quite restrictive and should be relaxed.[17] It could be easily shown that the methodology of this Study is sufficiently flexible to allow for the relaxation of the assumption.[18] Indeed, it is possible to empirically estimate the "correct" weight that the "average hours" component of labour input should receive. The solution to the problem is somewhat analogous to that worked out in the case of "correctly" measuring fixed capital input from gross and net capital stock data. The productivity regression equations will contain additional explicit "average hours worked" growth variables in such further investigation.

A third extension of the present Study concerns the use of an alternative proxy variable to simulate "increased output specialization" – source number eight of productivity growth (see Section 3.1). In the U.S. census of manufactures, every manufacturing product is listed as "primary" to a specific four-digit industry. Every establishment is then classified within a particular four-digit industry on the basis of the largest volume of product shipments that are primary to that industry. The ratio of primary product shipments to total shipments yields a rather crude measure of "specialization" for an establishment.[19] The ratio of the summation of primary product shipments to the summation of total shipments – summation being over all establishments classified within *any* industry – yields a measure of "average specialization" for the various establishments of the industry. Thus the rate of growth of such a ratio could be a more sensitive proxy variable than the total industry output growth variable (q) used in this Study. In fact, *both* proxy variables could be introduced simultaneously in the second step of the productivity regression estimation procedure (see Section 3.3). Comparable specialization ratios have not yet been published for the Canadian census of manufacturing.

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[17] See the discussion in M. S. Feldstein, "Specification of the Labour Input in the Aggregate Production Function", *Review of Economic Studies*, October 1967, pp. 375-386.

[18] Briefly, in the simplest case, let

$$Q^* = f(L^*, K^*; t)$$

represent the production function (in Appendix notation). Suppose there is just one type of labour employed, and that

$$L^* = NH^{\alpha} \quad , \quad \alpha \geqq 0$$

where N is the number employed and H is average hours worked. Then, defining labour productivity as output per *man-hour* or Q^*/NH, it is straightforward to see that the labour productivity growth rate equation becomes

$$\left(\frac{\dot{Q}^*}{Q^*} - \frac{\dot{L}}{L}\right) = (\alpha - 1)\left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{H}}{H} + \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\left(\frac{\dot{K}^*}{K^*} - \frac{\dot{L}}{L}\right) + \frac{1}{Q^*}\frac{\partial Q^*}{\partial t}$$

where $L = NH$ and constant returns to scale prevail. Thus the coefficient of the growth variable $\dot{H}/H$ would indicate the relative size of the two mentioned elasticities. It is possible to extend these results to any number of labour types and a production function that allows for variable homogeneity. See Sections B.1, B.2 and B.4.

[19] For further details, consult Delehanty, *op. cit.*, pp. 129-130 and 154-155.

The fourth and final proposed extension of this Study seeks to estimate the relationship between productivity growth, changes in average wage rates, and price increases. For example, a simple industry *net* output price formation equation could be[20]

$$(p) = a + b(w) + c(q/l)$$

where (p) is the growth rate of net output price; (w) is the growth rate of average hourly money earnings of labour; and (q/l) is the labour productivity growth rate. Then empirical estimates of the coefficients (a,b,c) from an inter-industry trend analysis could indicate (1) whether money wage rate increases are more effective in raising prices than productivity increases are in lowering prices, and (2) to what extent price changes have been occurring, on the average, independently of money wage and productivity changes. One might wish to substitute the factor productivity growth rate variable (q/c) for (q/l) in the above equation. It should be noted that the success of this extension depends crucially on the development of a reliable price index of net output for each of the industries analysed. Such an index is not currently available but could be approximated with sufficient effort.[21]

[20] See Lydall, *op. cit.*, p. 823.
[21] See, e.g., Lithwick, Post and Rymes, *op. cit.*, pp. 222.

APPENDIX A

THE PRODUCTIVITY GROWTH MODELS AND THEIR INTERPRETATION

In this Appendix, we develop the basic productivity growth models that underlie the empirical analyses of Chapters 3, 4 and 5. The development is formal and self-contained so that the reader can turn directly to this Appendix if he so desires. Also, the basic models are unfolded in a series of steps designed to motivate the productivity analysis. It is shown that the models provide the economic interpretation of the estimated regression coefficients in a consistent framework. Finally, the models described in this Appendix are relatively simple. However, many of the restrictive assumptions can be relaxed, yielding a more general economic interpretation of the empirical estimates. The mathematical development of such general growth models is more advanced and is discussed in Appendix B.

A.1 A Neoclassical Production Function

Beginning with the simplest assumptions first, let the production relationship of an industry at continuous time t be defined by the differentiable two-factor production function

(1) $Q^*(t) \;=\; f[L^*(t)\,, K^*(t)\;; t],$

where $Q^*(t)$ is a scalar index of aggregate net output (or real value added) of the industry concerned at time t; $L^*(t)$ is an index of aggregate labour input; $K^*(t)$ is an index of aggregate fixed capital input; and the explicit variable t allows for shifts in the production function over time. In order to simplify the notation, we omit the implicit time variable from all the input and output variables, so that equation (1) is written

(1a) $Q^* \;=\; f(L^*\,, K^*\;; t)$.

To conform with neoclassical production function properties, it is assumed that $\dfrac{\partial f}{\partial L^*} > 0$ and $\dfrac{\partial f}{\partial K^*} > 0$ for all $L^* > 0, K^* > 0$, and t.

A.2 An Expression for Labour Quality Change

Suppose for now that we distinguish two homogeneous types of labour,[1] L_1 and L_2. Then the aggregate labour input index L^* is a function of the two types of labour; namely,

[1] It will shortly become apparent what is meant by "two homogeneous types of labour".

(2) $L^* = g(L_1, L_2)$

where the function g is also assumed to be differentiable with respect to its arguments. It is natural to require g to be homogeneous of degree unity,[2] and that $\frac{\partial g}{\partial L_1} > 0, \frac{\partial g}{\partial L_2} > 0$ for all $L_1 > 0,\ L_2 > 0$. Let L_1 and L_2 be expressed in identical units of measurement (such as "number employed" or "man-hours"), and call

(3) $L = L_1 + L_2$

the simple unweighted sum of the two types of labour. We could now develop an expression for "labour quality change".

By totally differentiating both sides of equation (1a) with respect to time t, we get

$$(4)\quad \frac{dQ^*}{dt} = \frac{\partial f}{\partial L^*}\frac{dL^*}{dt} + \frac{\partial f}{\partial K^*}\frac{dK^*}{dt} + \frac{\partial f}{\partial t}$$

or, in the more compact notation,

$$(4a)\quad \dot{Q}^* = \frac{\partial f}{\partial L^*}\dot{L}^* + \frac{\partial f}{\partial K^*}\dot{K}^* + \frac{\partial f}{\partial t}.$$

Divide both sides of (4a) by Q^* (assumed positive), so that

$$(5)\quad \frac{\dot{Q}^*}{Q^*} = \left(\frac{L^*}{Q^*}\frac{\partial f}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} + \left(\frac{K^*}{Q^*}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} + \frac{1}{Q^*}\frac{\partial f}{\partial t}.$$

This equation has the well-known interpretation that the rate of growth of aggregate output equals the summation of "the contribution of aggregate labour input", "the contribution of aggregate capital input" and "the contribution of the residual", respectively.

By similarly totally differentiating equation (2) and dividing by L^*, we have

$$(6)\quad \frac{\dot{L}^*}{L^*} = \left(\frac{L_1}{L^*}\frac{\partial g}{\partial L_1}\right)\frac{\dot{L}_1}{L_1} + \left(\frac{L_2}{L^*}\frac{\partial g}{\partial L_2}\right)\frac{\dot{L}_2}{L_2}.$$

Substitute the latter expression for $\dot{L}^*/L^*$ in (5), and noting that

$\left(\frac{\partial f}{\partial L^*}\frac{\partial g}{\partial L_1}\right) = \frac{\partial f}{\partial L_1}$ and $\left(\frac{\partial f}{\partial L^*}\frac{\partial g}{\partial L_2}\right) = \frac{\partial f}{\partial L_2}$, we end up with

$$(7)\quad \frac{\dot{Q}^*}{Q^*} = \left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right)\frac{\dot{L}_1}{L_1} + \left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\frac{\dot{L}_2}{L_2} + \left(\frac{K^*}{Q^*}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} + \frac{1}{Q^*}\frac{\partial f}{\partial t}.$$

[2] For example, if the quantity of each type of labour doubled, one would expect the aggregate labour input index to also double.

Rewrite equation (7) as

$$(8)\quad \frac{\dot{Q}^*}{Q^*} = \left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}}{L}\right) + \left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}}{L}\right)$$
$$+ \left[\left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right) + \left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\right]\frac{\dot{L}}{L}$$
$$+ \left(\frac{K^*}{Q^*}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} + \frac{1}{Q^*}\frac{\partial f}{\partial t}\,.$$

But from (3) it is known that

$$(9)\quad \frac{\dot{L}}{L} = \left(\frac{L_1}{L}\right)\frac{\dot{L}_1}{L_1} + \left(\frac{L_2}{L}\right)\frac{\dot{L}_2}{L_2}$$

so that, on substitution, the first two terms on the right-hand side (R.H.S.) of (8) become

$$\left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right)\left[\frac{\dot{L}_1}{L_1} - \left(\frac{L_1}{L}\right)\frac{\dot{L}_1}{L_1} - \left(\frac{L_2}{L}\right)\frac{\dot{L}_2}{L_2}\right]$$
$$+ \left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\left[\frac{\dot{L}_2}{L_2} - \left(\frac{L_1}{L}\right)\frac{\dot{L}_1}{L_1} - \left(\frac{L_2}{L}\right)\frac{\dot{L}_2}{L_2}\right]$$
$$= \frac{\dot{L}_1}{L_1}\left[\left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right)\left(1 - \frac{L_1}{L}\right) - \left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\frac{L_1}{L}\right]$$
$$+ \frac{\dot{L}_2}{L_2}\left[\left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\left(1 - \frac{L_2}{L}\right) - \left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right)\frac{L_2}{L}\right]$$
$$= \frac{\dot{L}_1}{L_1}\left[\left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right)\frac{L_2}{L} - \left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\frac{L_1}{L}\right]$$
$$+ \frac{\dot{L}_2}{L_2}\left[\left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\frac{L_1}{L} - \left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right)\frac{L_2}{L}\right]$$
$$= \left[\left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right)\frac{L_2}{L} - \left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right)\frac{L_1}{L}\right]\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)$$
$$(10)\quad = \left(\frac{L_1 L_2}{Q^* L}\right)\left(\frac{\partial f}{\partial L_1} - \frac{\partial f}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)\,.$$

If we use expression (10) in equation (8), noting that unitary homogeneity of the function g implies that

$$(11)\quad \left(\frac{L_1}{Q^*}\frac{\partial f}{\partial L_1}\right) + \left(\frac{L_2}{Q^*}\frac{\partial f}{\partial L_2}\right) = \left(\frac{L^*}{Q^*}\frac{\partial f}{\partial L^*}\right)\left[\frac{L_1}{L^*}\frac{\partial g}{\partial L_1} + \frac{L_2}{L^*}\frac{\partial g}{\partial L_2}\right]$$
$$= \frac{L^*}{Q^*}\frac{\partial f}{\partial L^*}\;,$$

then we could finally rewrite equation (8) as

$$(12)\quad \frac{\dot{Q}^*}{Q^*} = \left(\frac{L_1 L_2}{Q^* L}\right)\left(\frac{\partial f}{\partial L_1} - \frac{\partial f}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right) + \left(\frac{L^*}{Q^*}\frac{\partial f}{\partial L^*}\right)\frac{\dot{L}}{L}$$

$$+ \left(\frac{K^*}{Q^*}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} + \frac{1}{Q^*}\frac{\partial f}{\partial t}.$$

Comparing equations (5) and (12), it is evident that the "contribution of aggregate labour input" expression is replaced by the summation of two terms; i.e.,

$$(13)\quad \left(\frac{L^*}{Q^*}\frac{\partial f}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \left(\frac{L_1 L_2}{Q^* L}\right)\left(\frac{\partial f}{\partial L_1} - \frac{\partial f}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)$$

$$+ \left(\frac{L^*}{Q^*}\frac{\partial f}{\partial L^*}\right)\frac{\dot{L}}{L}.$$

Recalling that $L = L_1 + L_2$, the second term on the R.H.S. of (13) represents the "contribution of the growth of, say, total number employed of labour" to the rate of growth of aggregate output. Let us now analyse the first term on the R.H.S.

This term is the product of three components.[3] The third component, $(\dot{L}_1/L_1 - \dot{L}_2/L_2)$, is the rate of growth of the ratio L_1/L_2. It is positive, for example, if $\dot{L}_1/L_1 > \dot{L}_2/L_2$. The second component, $(\partial f/\partial L_1 - \partial f/\partial L_2)$, is the simple difference between the marginal product of the first type of labour and the marginal product of the second type of labour. It is positive, for example, if $\partial f/\partial L_1 > \partial f/\partial L_2$ at a particular value of L_1, L_2, K^* and t.[4] Loosely speaking, the second component is positive, for example, if the first type of labour is "more productive" than the second type (at a particular value of L_1, L_2, K^* and t). Therefore, it is clear that the whole term is positive if and only if a "more productive" type of labour is growing at a faster rate, algebraicly, than a "less productive" type. Similarly, the term is negative if and only if a "more productive" type of labour is growing at a slower rate than a "less productive" type. Of course, the term is zero when the two types of labour are "equally productive" or when the two types of labour have equal growth rates.[5] All of these are the properties we should expect to find in a term that is supposed to represent the "contribution of labour quality change" to output growth. But we should also expect one further property – namely, that the absolute value of the "quality change expression" should be greatest, *ceteris paribus,* when $L_1 = L_2$. Stated another way, for any non-zero values of $(\partial f/\partial L_1 - \partial f/\partial L_2)$, $(\dot{L}_1/L_1 - \dot{L}_2/L_2)$, L and Q^*, we should not expect "labour quality" to change by very much if one type of labour is

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[3] It is assumed that each type of labour is positively employed.

[4] Equivalently, it is positive if the marginal rate of substitution of the second type of labour for the first type is greater than unity at the particular combination of inputs considered at a particular time.

[5] In either of these cases, there is no need to "distinguish" two types of labour, and we could simply write $L^* = L = L_1 + L_2$ (for the particular L_1, L_2, K^* and t).

insignificant[6] compared with the other. This property is accounted for by the first component of the relevant term – namely, the component $(L_1 L_2/Q^*L)$. It is easy to show that this component is indeed maximized (for *given* Q^* and L) when $L_1 = L_2$.[7]

As a result of these considerations, the term

$$\left(\frac{L_1 L_2}{Q^* L}\right)\left(\frac{\partial f}{\partial L_1} - \frac{\partial f}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)$$

would represent the "contribution of the growth of labour quality" to the growth of aggregate net output. It should be noted that while L^* or $\dot{L}^*/L^*$ in equation (13) is not generally observable, both growth rates $(\dot{L}_1/L_1 - \dot{L}_2/L_2)$ and $\dot{L}/L$ *are* observable. This consideration will play a key role in the empirical analysis of inter-industry productivity growth rate differentials.

A.3 An Expression for Capital Quality Change

A "fixed capital quality change" expression could be developed in the same manner as the expression for labour quality change. However, there is one additional complication. Suppose that the aggregate fixed capital input index is a differentiable[8] function of two types of fixed capital, so that

(14) $K^* = h(K_1, K_2)$.

Further, suppose that

(15) $K_1 = a_1 K_{1n}^{\beta_1} K_{1g}^{(1-\beta_1)}$, and $K_2 = a_2 K_{2n}^{\beta_2} K_{2g}^{(1-\beta_2)}$

where K_{in} is *net* fixed capital stock of the i-th type of capital; K_{ig} is *gross* fixed capital stock of the i-th type of capital; $a_i > 0$ is an arbitrary constant; $1 \geqq \beta_i \geqq 0$ is a particular unknown constant ($i = 1,2$).[9] Let K_{1n} and K_{2n} be expressed in the same measurement units (such as "net capital stock in constant dollars"), and define[10]

(16) $K_n = K_{1n} + K_{2n}$

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[6] That is, insignificant in terms of the common unit of measurement of the two types of labour.

[7] The proof is trivial once it is recalled that $L = L_1 + L_2$.

[8] It is again assumed that the function h is homogeneous of degree unity with respect to its arguments and that $\partial h/\partial K_1 > 0$, $\partial h/\partial K_2 > 0$ for all $K_1 > 0, K_2 > 0$.

[9] All the manipulations and interpretations of this Appendix (and the next) would still follow if we considered the more general function

(15a) $K_i = h_i(K_{in}, K_{ig})$ $(i = 1, 2)$,

provided that h_i has the required differentiability and homogeneity properties. Also, this is not the place to mention "survival curves" and "depreciation formulas"; see Appendix D.

[10] A more rigorously constructed definition could be developed along the lines indicated in Section B.5.

as the simple summation of the two types of net capital stock. Similarly, define

$$(17)\quad K_g = K_{1g} + K_{2g} .$$

We could now develop an expression for "fixed capital quality change" that would be useful for empirical productivity analysis.

Consider the term in equation (5) or (12) representing the "contribution of aggregate capital input" to the growth of aggregate output, i.e.,

$$(18)\quad \left(\frac{K^*}{Q^*}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} .$$

But from (14) we derive

$$(19)\quad \frac{\dot{K}^*}{K^*} = \left(\frac{K_1}{K^*}\frac{\partial h}{\partial K_1}\right)\frac{\dot{K}_1}{K_1} + \left(\frac{K_2}{K^*}\frac{\partial h}{\partial K_2}\right)\frac{\dot{K}_2}{K_2}$$

and from (15) we have

$$(20)\quad \frac{\dot{K}_i}{K_i} = \beta_i\frac{\dot{K}_{in}}{K_{in}} + (1-\beta_i)\frac{\dot{K}_{ig}}{K_{ig}} . \qquad (i = 1, 2)$$

Clearly

$$(21)\quad \beta_i = \frac{K_{in}}{K_i}\frac{\partial K_i}{\partial K_{in}} , \text{ and } (1-\beta_i) = \frac{K_{ig}}{K_i}\frac{\partial K_i}{\partial K_{ig}} , \qquad (i = 1, 2)$$

so that

$$(22)\quad \left(\frac{\partial f}{\partial K^*}\frac{\partial h}{\partial K_i}\right)\left(\frac{\beta_i K_i}{K_{in}}\right) = \frac{\partial f}{\partial K_{in}} \qquad (i = 1, 2)$$

and

$$(23)\quad \left(\frac{\partial f}{\partial K^*}\frac{\partial h}{\partial K_i}\right)(1-\beta_i)\frac{K_i}{K_{ig}} = \frac{\partial f}{\partial K_{ig}} . \qquad (i = 1, 2)$$

Combining the results in (19), (20), (22) and (23), it is evident that the term (18) could be written as

$$(24)\quad \left(\frac{K^*}{Q^*}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} = \sum_{i=1}^{2}\left(\frac{K_{in}}{Q^*}\frac{\partial f}{\partial K_{in}}\right)\frac{\dot{K}_{in}}{K_{in}} + \sum_{i=1}^{2}\left(\frac{K_{ig}}{Q^*}\frac{\partial f}{\partial K_{ig}}\right)\frac{\dot{K}_{ig}}{K_{ig}}$$

$$= \sum_{i=1}^{2}\left(\frac{K_{in}}{Q^*}\frac{\partial f}{\partial K_{in}}\right)\left(\frac{\dot{K}_{in}}{K_{in}} - \frac{\dot{K}_n}{K_n}\right) + \sum_{i=1}^{2}\left(\frac{K_{ig}}{Q^*}\frac{\partial f}{\partial K_{ig}}\right)\left(\frac{\dot{K}_{ig}}{K_{ig}} - \frac{\dot{K}_g}{K_g}\right)$$

$$+ \sum_{i=1}^{2}\left(\frac{K_{in}}{Q^*}\frac{\partial f}{\partial K_{in}}\right)\frac{\dot{K}_n}{K_n} + \sum_{i=1}^{2}\left(\frac{K_{ig}}{Q^*}\frac{\partial f}{\partial K_{ig}}\right)\frac{\dot{K}_g}{K_g} .$$

But noting that equation (16) implies that

$$(25)\quad \frac{\dot{K}_n}{K_n} = \sum_{i=1}^{2}\left(\frac{K_{in}}{K_n}\right)\frac{\dot{K}_{in}}{K_{in}} ,$$

it is now evident that the term

$$(26)\quad \sum_i \left(\frac{K_{in}}{Q^*}\frac{\partial f}{\partial K_{in}}\right)\left(\frac{\dot{K}_{in}}{K_{in}} - \frac{\dot{K}_n}{K_n}\right)$$

$$= \left(\frac{K_{1n}K_{2n}}{Q^*K_n}\right)\left(\frac{\partial f}{\partial K_{1n}} - \frac{\partial f}{\partial K_{2n}}\right)\left(\frac{\dot{K}_{1n}}{K_{1n}} - \frac{\dot{K}_{2n}}{K_{2n}}\right)$$

by using a procedure completely analogous to that which yielded the "labour quality change" expression (10) above. Similarly, the term

$$(27)\quad \sum_i \left(\frac{K_{ig}}{Q^*}\frac{\partial f}{\partial K_{ig}}\right)\left(\frac{\dot{K}_{ig}}{K_{ig}} - \frac{\dot{K}_g}{K_g}\right)$$

$$= \left(\frac{K_{1g}K_{2g}}{Q^*K_g}\right)\left(\frac{\partial f}{\partial K_{1g}} - \frac{\partial f}{\partial K_{2g}}\right)\left(\frac{\dot{K}_{1g}}{K_{1g}} - \frac{\dot{K}_{2g}}{K_{2g}}\right)$$

Finally, it is straightforward to show, from equations (22) and (23) above, that

$$(28)\quad \sum_i \left(\frac{K_{in}}{Q^*}\frac{\partial f}{\partial K_{in}}\right) = \sum_i \beta_i\left(\frac{K_i}{Q^*}\frac{\partial f}{\partial K_i}\right)$$

and that

$$(29)\quad \sum_i \left(\frac{K_{ig}}{Q^*}\frac{\partial f}{\partial K_{ig}}\right) = \sum_i (1-\beta_i)\left(\frac{K_i}{Q^*}\frac{\partial f}{\partial K_i}\right),$$

so that the last two terms on the R.H.S. of (24) become

$$(30)\quad \sum_i \left(\frac{K_{in}}{Q^*}\frac{\partial f}{\partial K_{in}}\right)\frac{\dot{K}_n}{K_n} + \sum_i \left(\frac{K_{ig}}{Q^*}\frac{\partial f}{\partial K_{ig}}\right)\frac{\dot{K}_g}{K_g}$$

$$= \sum_i \beta_i\left(\frac{K_i}{Q^*}\frac{\partial f}{\partial K_i}\right)\frac{\dot{K}_n}{K_n} + \sum_i (1-\beta_i)\left(\frac{K_i}{Q^*}\frac{\partial f}{\partial K_i}\right)\frac{\dot{K}_g}{K_g}$$

$$= \sum_i \beta_i\left(\frac{K_i}{Q^*}\frac{\partial f}{\partial K_i}\right)\left(\frac{\dot{K}_n}{K_n} - \frac{\dot{K}_g}{K_g}\right) + \left(\frac{K^*}{Q^*}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}_g}{K_g},$$

where in the last step we used the assumption that the function h in (14) is homogeneous of degree unity.

Collecting the results in (26), (27), and (30), it is now seen that the "contribution of aggregate capital input" term (18) can be rewritten as the summation of four expressions – namely,

$$(31)\quad \left(\frac{K_{1n}K_{2n}}{Q^*K_n}\right)\left(\frac{\partial f}{\partial K_{1n}} - \frac{\partial f}{\partial K_{2n}}\right)\left(\frac{\dot{K}_{1n}}{K_{1n}} - \frac{\dot{K}_{2n}}{K_{2n}}\right)$$

$$+ \left(\frac{K_{1g}K_{2g}}{Q^*K_g}\right)\left(\frac{\partial f}{\partial K_{1g}} - \frac{\partial f}{\partial K_{2g}}\right)\left(\frac{\dot{K}_{1g}}{K_{1g}} - \frac{\dot{K}_{2g}}{K_{2g}}\right)$$

$$+ \sum_i \beta_i\left(\frac{K_i}{Q^*}\frac{\partial f}{\partial K_i}\right)\left(\frac{\dot{K}_n}{K_n} - \frac{\dot{K}_g}{K_g}\right) + \left(\frac{K^*}{Q^*}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}_g}{K_g}$$

The first expression represents the "contribution of *net* fixed capital quality change" to aggregate output growth. This economic interpretation is, of course, completely analogous to that of the labour quality change factor discussed previously. The second expression in (31) describes the "contribution of *gross* fixed capital quality change". The third and fourth expressions *together* represent the "contribution of a suitably weighted average of the growth rates of the simple totals of net and gross capital" to the rate of growth of output.[11] However, it is convenient for empirical productivity analysis to consider the two expressions separately. Note that in the special case where $\beta_i = 0$ $(i = 1, 2,)$, the first and third expressions vanish.[12] Similarly, $\beta_i = 1$ $(i = 1, 2)$ implies that the second expression vanishes, and the sum of the third and fourth expressions will be simply $(K^*/Q^*)\,(\partial f/\partial K^*)\,(\dot{K}_n/K_n)$.[13]

Finally, it is important to notice that while $\dot{K}^*/K^*$, $\dot{K}_1/K_1$, and $\dot{K}_2/K_2$ are generally unknown, all the growth rate variables in (31) – i.e., $\dot{K}_{in}/K_{in}$, $\dot{K}_{ig}/K_{ig}$ $(i = 1, 2)$, $\dot{K}_n/K_n$ and $\dot{K}_g/K_g$ – *are* observable.

A.4 A Production Function of the Representative Establishment

So far, the development of this Appendix has been based solely on an industry production function. However, it is revealing to take some account of the individual establishments that comprise the industry, and this is done in the following simple manner.

Let the production relationship of the representative (or typical) establishment of an industry, at time t, be defined by the differentiable two-factor production function[14]

$$(32)\quad Q^*/E = f(L^*/E,\ K^*/E;\ t),$$

where E is the number of establishments in the industry at time t and Q^*, L^*, and K^* have the *same* meaning as before. Thus, for example, L^*/E is the aggregate labour input of the representative establishment of the industry concerned. Then $L^*/E = g(L_1/E,\ L_2/E)$, since the function g is homogeneous of degree one in its arguments.

[11] The sum of the third and fourth expressions equals

$$\sum_i \left(\frac{K_i}{Q^*}\frac{\partial f}{\partial K_i}\right)\left[\beta_i\frac{\dot{K}_n}{K_n} + (1-\beta_i)\frac{\dot{K}_g}{K_g}\right].$$

[12] That is, aggregate capital input would be correctly measured by *gross* capital stock data alone.

[13] That is, aggregate capital input would be correctly measured by *net* capital stock data alone.

[14] Strictly speaking, we should not be using the same functional symbol f that was used to denote the industry production function in equation (1a), but it is inconvenient to introduce additional notation. This simplification should not cause any difficulty in understanding the subsequent analysis.

Suppose that the new function f is homogeneous of degree r in L^*/E and K^*/E, where r is a positive constant. Then it follows that

$$(33)\quad Q^* = Ef(L^*/E, K^*/E;t) = E^{(1-r)}f(L^*,K^*;t)$$

for all L^*, K^*, E, and t; and

$$(34)\quad \frac{\partial Q^*}{\partial E} = (1-r)E^{-r}f(L^*,K^*;t) \quad .$$

Thus it is clear that

$$(35)\quad \frac{\partial Q^*}{\partial E} \gtreqless 0 \text{ if and only if } r \lesseqgtr 1.$$

In words, if there are internal increasing (decreasing) economies of scale, an increase in the number of establishments, *ceteris paribus*, will decrease (increase) industry output because the scale of the typical establishment is now smaller.

Now, taking the logarithm of both sides of equation (33) and then totally differentiating with respect to t, we derive

$$(36)\quad \frac{\dot{Q}^*}{Q^*} = (1-r)\frac{\dot{E}}{E} + \left(\frac{L^*}{f}\frac{\partial f}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} + \left(\frac{K^*}{f}\frac{\partial f}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} + \frac{1}{f}\frac{\partial f}{\partial t} \, ,$$

where $f \equiv f(L^*, K^*; t)$. But

$$(37)\quad \left(\frac{L^*}{f}\frac{\partial f}{\partial L^*}\right) = \left(\frac{L^*E^{(1-r)}}{Q^*}\right)\left(\frac{\partial Q^*}{\partial L^*}\right)(E^{(r-1)}) = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right) \, .$$

Similarly

$$(38)\quad \left(\frac{K^*}{f}\frac{\partial f}{\partial K^*}\right) = \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right) \text{ and } \frac{1}{f}\frac{\partial f}{\partial t} = \frac{1}{Q^*}\frac{\partial Q^*}{\partial t} \, ,$$

so equation (36) can be more conveniently rewritten as

$$(39)\quad \frac{\dot{Q}^*}{Q^*} = (1-r)\frac{\dot{E}}{E} + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} + \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} + \frac{1}{Q^*}\frac{\partial Q^*}{\partial t} \, .$$

The latter equation is equivalent to the earlier basic "source of industry output growth" equation (5), when there are constant returns to scale ($r=1$).[15]

A.5 The Labour Productivity Growth Model

We are now in a position to review and collect the results of Sections A.2, A.3, and A.4. The fundamental output model is the production function of the representative establishment of an industry– namely,

$$(32)\quad Q^*/E = f(L^*/E, K^*/E;t),$$

[15] After simply replacing the production function symbol in equation (5) by the industry aggregate output symbol.

where[16] Q^* is aggregate net output of the industry at time t; L^* is aggregate labour input of the industry; K^* is aggregate fixed capital input of the industry; E is the number of establishments in the industry; and the explicit variable t allows for shifts in the production function over time. It is assumed, as before, that function f is homogeneous of degree r (a constant) in L^*/E and K^*/E, so that, using Euler's Theorem,[17] it is known that

$$(40)\quad \left(\frac{L^*/E}{Q^*/E}\right)\left(\frac{\partial Q^*/E}{\partial L^*/E}\right) + \left(\frac{K^*/E}{Q^*/E}\right)\left(\frac{\partial Q^*/E}{\partial K^*/E}\right) = r$$

for all L^*, K^*, E, and t. However,

$$(41)\quad \frac{\partial Q^*/E}{\partial L^*/E} = \left(\frac{\partial Q^*/E}{\partial Q^*}\right)\left(\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\partial L^*}{\partial L^*/E}\right) = E^{-1}\left(\frac{\partial Q^*}{\partial L^*}\right)E = \frac{\partial Q^*}{\partial L^*}\ .$$

$$(42)\quad \text{Similarly}\quad \frac{\partial Q^*/E}{\partial K^*/E} = \frac{\partial Q^*}{\partial K^*}\ ,$$

so that, using (41) and (42) in (40), it is known that

$$(43)\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right) + \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right) = r\ .$$

The latter relationship is particularly useful, since it was shown in Section A.4 that the basic "source of industry output growth equation" derived from the production function (32) is simply

$$(39)\quad \frac{\dot{Q}^*}{Q^*} = (1-r)\frac{\dot{E}}{E} + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} + \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} + \frac{1}{Q^*}\frac{\partial Q^*}{\partial t}\ .$$

To continue our review of previous results, it was found in Section A.2 that relations (2) and (3) – namely, $L^* = g(L_1,L_2)$ and $L = L_1 + L_2$, respectively – imply

$$(13a)\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right) + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L}\ ,$$

where in (13a) we have replaced the production function symbol of (13) by the industry aggregate output symbol.

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[16] To repeat the meaning of the symbols introduced previously.

[17] See R. G. D. Allen, *Mathematical Analysis for Economists* (London: Macmillan and Co., 1938), p. 319.

Similarly, the key results of Section A.3 could be rewritten as

$$\text{(31a)}\quad \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} = \left(\frac{K_{1n}K_{2n}}{Q^*K_n}\right)\left(\frac{\partial Q^*}{\partial K_{1n}} - \frac{\partial Q^*}{\partial K_{2n}}\right)\left(\frac{\dot{K}_{1n}}{K_{1n}} - \frac{\dot{K}_{2n}}{K_{2n}}\right)$$
$$+ \left(\frac{K_{1g}K_{2g}}{Q^*K_g}\right)\left(\frac{\partial Q^*}{\partial K_{1g}} - \frac{\partial Q^*}{\partial K_{2g}}\right)\left(\frac{\dot{K}_{1g}}{K_{1g}} - \frac{\dot{K}_{2g}}{K_{2g}}\right)$$
$$+ \sum_{i=1}^{2} \beta_i \left(\frac{K_i}{Q^*}\frac{\partial Q^*}{\partial K_i}\right)\left(\frac{\dot{K}_n}{K_n} - \frac{\dot{K}_g}{K_g}\right) + \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\frac{\dot{K}_g}{K_g} .$$

It is now straightforward to collect these results and present the fundamental labour productivity growth model. First, the continuous rate of growth of labour productivity for an industry is *defined* as the difference between the growth rate of the industry aggregate net output and the growth rate of the simple total of the two types of labour employed by the industry – i.e.,

$$\text{(44)}\quad \text{labour productivity growth rate} \equiv \dot{Q}^*/Q^* - \dot{L}/L \quad .$$

This definition seems to conform to the common usage of the term "labour productivity". Second, it is convenient to rewrite relation (43) as

$$\text{(43a)}\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right) = -\left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right) + 1 + (r-1) \quad ,$$

so that the last term on the R.H.S. of (13a) is

$$\text{(45)}\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L} = -\left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\frac{\dot{L}}{L} + \frac{\dot{L}}{L} + (r-1)\frac{\dot{L}}{L} \quad .$$

Then the labour productivity growth equation is constructed as follows. Substitute (13a) and (31a) in the basic output growth equation (39). Now use relation (45) in the expanded output growth equation and transfer the term $\dot{L}/L$ over to the left-hand side (L.H.S.). All this yields

$$\text{(46)}\quad \left(\frac{\dot{Q}^*}{Q^*} - \frac{\dot{L}}{L}\right) = \left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)$$
$$+ \left(\frac{K_{1n}K_{2n}}{Q^*K_n}\right)\left(\frac{\partial Q^*}{\partial K_{1n}} - \frac{\partial Q^*}{\partial K_{2n}}\right)\left(\frac{\dot{K}_{1n}}{K_{1n}} - \frac{\dot{K}_{2n}}{K_{2n}}\right)$$
$$+ \left(\frac{K_{1g}K_{2g}}{Q^*K_g}\right)\left(\frac{\partial Q^*}{\partial K_{1g}} - \frac{\partial Q^*}{\partial K_{2g}}\right)\left(\frac{\dot{K}_{1g}}{K_{1g}} - \frac{\dot{K}_{2g}}{K_{2g}}\right)$$
$$+ \sum_{i=1}^{2} \beta_i \left(\frac{K_i}{Q^*}\frac{\partial Q^*}{\partial K_i}\right)\left(\frac{\dot{K}_n}{K_n} - \frac{\dot{K}_g}{K_g}\right)$$
$$+ \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\left(\frac{\dot{K}_g}{K_g} - \frac{\dot{L}}{L}\right)$$
$$+ (r-1)\left(\frac{\dot{L}}{L} - \frac{\dot{E}}{E}\right) + \frac{1}{Q^*}\frac{\partial Q^*}{\partial t} .$$

Equation (46) is the fundamental industry labour productivity growth rate model of this Study. It is instructive to consider each term on the R.H.S. of (46).

The first term represents the "contribution of the growth of labour quality"[18] to the growth of labour productivity. Its contribution is positive, for example, if the "more productive" type of labour is growing at a faster rate than the "less productive" type. Indeed, the sign of the coefficient of the growth rate variable $(\dot{L}_1/L_1 - \dot{L}_2/L_2)$ – i.e., the sign of $(L_1L_2/Q^*L)(\partial Q^*/\partial L_1 - \partial Q^*/\partial L_2)$ – indicates which, if either, of the two types of labour is "more productive".[19] The second and third terms represent the "contribution of net capital quality change" and the "contribution of gross capital quality change", respectively.[20] The interpretation of these terms is completely analogous to that of the first term. Skipping for the moment to the fifth term, let the ratio K_g/L be called the "capital-intensity of production".[21] Then the fifth term describes the "contribution of the growth of capital-intensity" to the growth rate of labour productivity. The coefficient of the growth rate variable $(\dot{K}_g/K_g - \dot{L}/L)$ – namely, (K^*/Q^*) $(\partial Q^*/\partial K^*)$ – is always positive[22] and is simply the elasticity of aggregate output with respect to aggregate capital input. Now the fourth term accounts for a possible measurement error in the above definition of capital-intensity of production.[23] Thus the "contribution of the fifth term" to labour productivity growth is underestimated, for example, if $\dot{K}_n/K_n > \dot{K}_g/K_g$ *and* at least one of $\beta_i \neq 0$ $(i = 1,2)$. Indeed, the coefficient of the variable $(\dot{K}_n/K_n - \dot{K}_g/K_g)$ is always positive unless $\beta_i = 0$ $(i = 1,2)$, in which case the coefficient of the second term will also vanish. To continue, the sixth term on the R.H.S. of (46) represents the "contribution of economies of scale" to the growth rate of labour productivity. Its contribution is positive, for example, if there are increasing (decreasing) economies of scale[24] *and* if the size of the typical establishment, as measured by the ratio L/E, is increasing (decreasing). For this reason, it is natural to call L/E the *relevant* measure of "average size of establishment of an industry" when analysing the sources of labour productivity growth. Finally the last term accounts for the "contribution of the unknown elements" (or the "residual")[25] to the growth of labour productivity.

Thus the labour productivity growth rate equation (46) shows that if the empirical investigator ignores *all* quality change (e.g., assumes $L^* = L_1 + L_2$), and neglects net capital input (i.e., assumes $K^* = K_{1g} + K_{2g}$), and supposes constant returns to scale (i.e., assumes $r = 1$); then the contribution of the first four terms,

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[18] See the discussion in Section A.2.

[19] That is, "more productive" at the particular combination of values of all the variables.

[20] See the discussion in Section A.3.

[21] This definition seems to conform to common usage and is convenient for later definitions in Section A.6.

[22] It was assumed that $\partial Q^*/\partial K^* > 0$ for all $L^* > 0$, $K^* > 0$, and t.

[23] See the discussion in Section A.3. Another interpretation of the fourth term is to be found in Chapter 3.

[24] See the discussion in Section A.4.

[25] Some of the important components of the "residual" are explained in Chapter 3.

together with the contribution of the sixth term, all "get lumped" with the residual. Again, it is important to note, for empirical analysis, that all the growth rate variables in equation (46) are observable, with the possible exception of the variable $\dot{Q}^*/Q^*$.[26]

A.6 The Capital Productivity and Factor Productivity Growth Models

The growth rate of capital productivity for an industry is *defined* as the simple difference between the growth rate of aggregate output and the growth rate of total gross capital stock for that industry – i.e.,

(47) capital productivity growth rate $\equiv \dot{Q}^*/Q^* - \dot{K}_g/K_g$

where $K_g = K_{1g} + K_{2g}$. This definition conforms to the ordinary usage of the term "capital productivity".

The capital productivity growth equation is constructed in a manner completely analogous to that of the labour productivity growth model. First, rewrite relation (43) as

$$\text{(43b)} \quad \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right) = -\left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right) + 1 + (r-1) \ .$$

Then following the procedure given in Section A.5, using (43b) instead of (43a) and transferring the term $\dot{K}_g/K_g$ over to the L.H.S., it is found that

$$\begin{aligned}
\text{(48)} \quad \left(\frac{\dot{Q}^*}{Q^*} - \frac{\dot{K}_g}{K_g}\right) = & \left(\frac{L_1 L_2}{Q^* L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right) \\
& + \left(\frac{K_{1n}K_{2n}}{Q^* K_n}\right)\left(\frac{\partial Q^*}{\partial K_{1n}} - \frac{\partial Q^*}{\partial K_{2n}}\right)\left(\frac{\dot{K}_{1n}}{K_{1n}} - \frac{\dot{K}_{2n}}{K_{2n}}\right) \\
& + \left(\frac{K_{1g}K_{2g}}{Q^* K_g}\right)\left(\frac{\partial Q^*}{\partial K_{1g}} - \frac{\partial Q^*}{\partial K_{2g}}\right)\left(\frac{\dot{K}_{1g}}{K_{1g}} - \frac{\dot{K}_{2g}}{K_{2g}}\right) \\
& + \sum_{i=1}^{2} \beta_i \left(\frac{K_i}{Q^*}\frac{\partial Q^*}{\partial K_i}\right)\left(\frac{\dot{K}_n}{K_n} - \frac{\dot{K}_g}{K_g}\right) \\
& + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\dot{L}}{L} - \frac{\dot{K}_g}{K_g}\right) \\
& + (r-1)\left(\frac{\dot{K}_g}{K_g} - \frac{\dot{E}}{E}\right) + \frac{1}{Q^*}\frac{\partial Q^*}{\partial t} \ .
\end{aligned}$$

Equation (48) is the fundamental industry capital productivity growth rate model of this Study. It is again instructive to briefly consider each term on the R.H.S. of (48).

[26] This latter consideration is discussed in Section B.5.

The first three terms are identical to that of the labour productivity growth equation (46) and have completely analogous interpretations. Let the ratio L/K_g be called the "labour-intensity of production". Then the fifth term measures the "contribution of the growth of labour-intensity" to the growth of capital productivity. Indeed, as the fourth term indicates, the contribution of labour-intensity is overestimated if $\dot{K}_n/K_n > \dot{K}_g/K_g$ – i.e., when the measurement error accounted for by this term is positive.[27] Finally, the sixth term, or the "contribution of economies of scale" term, clearly shows that the ratio K_g/E is the economically *relevant* measure of "average size of establishment of an industry" when the investigator is analysing the sources of capital productivity growth for an industry.

Before continuing, let it be noted that the exact equation (46) is theoretically equivalent to the exact equation (48). But such a relationship does not generally hold for the stochastic counterparts of equations (46) and (48), which are estimated in the empirical chapters of this Study.

We now turn to the formulation of the factor productivity growth model. The growth rate of factor productivity for an industry is *defined* as the difference between the growth rate of aggregate output and a particular weighted average of the growth rates of total labour employed and total gross capital stock for the industry concerned. More precisely,

(49) factor productivity growth rate $\equiv \dot{Q}^*/Q^* - \alpha(\dot{L}/L) - (1-\alpha)\ (\dot{K}_g/K_g)$

where $L = L_1 + L_2$, $K_g = K_{1g} + K_{2g}$, and α is the *observed* factor share of labour in the value of net output at time t.[28] This definition certainly conforms to the most frequent usage of the term "factor productivity" among statistical economists.

It is useful to note that the factor productivity growth rate is simply

$$(50)\quad \frac{\dot{Q}^*}{Q^*} - \alpha\frac{\dot{L}}{L} - (1-\alpha)\frac{\dot{K}_g}{K_g} = \alpha\left(\frac{\dot{Q}^*}{Q^*} - \frac{\dot{L}}{L}\right) + (1-\alpha)\left(\frac{\dot{Q}^*}{Q^*} - \frac{\dot{K}_g}{K_g}\right),$$

or the same weighted average of the labour productivity and the capital productivity growth rates. Indeed, the factor productivity growth equation is constructed by simply substituting equations (46) and (48) in the R.H.S. of (50). This yields

[27] Assuming that at least one of $\beta_i \neq 0$ $(i = 1,2)$.
[28] Then $(1-\alpha)$ is called the observed factor share of capital. It is assumed that $0 < \alpha < 1$.

$$
\begin{aligned}
(51)\quad \left[\frac{\dot{Q}^*}{Q^*} - \alpha\frac{\dot{L}}{L} - (1-\alpha)\frac{\dot{K}_g}{K_g}\right] &= \left(\frac{L_1 L_2}{Q^* L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right) \\
&+ \left(\frac{K_{1n}K_{2n}}{Q^* K_n}\right)\left(\frac{\partial Q^*}{\partial K_{1n}} - \frac{\partial Q^*}{\partial K_{2n}}\right)\left(\frac{\dot{K}_{1n}}{K_{1n}} - \frac{\dot{K}_{2n}}{K_{2n}}\right) \\
&+ \left(\frac{K_{1g}K_{2g}}{Q^* K_g}\right)\left(\frac{\partial Q^*}{\partial K_{1g}} - \frac{\partial Q^*}{\partial K_{2g}}\right)\left(\frac{\dot{K}_{1g}}{K_{1g}} - \frac{\dot{K}_{2g}}{K_{2g}}\right) \\
&+ \sum_{i=1}^{2} \beta_i \left(\frac{K_i}{Q^*}\frac{\partial Q^*}{\partial K_i}\right)\left(\frac{\dot{K}_n}{K_n} - \frac{\dot{K}_g}{K_g}\right) \\
&+ \left[\alpha\left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K}\right) - (1-\alpha)\left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\right]\left(\frac{\dot{K}_g}{K_g} - \frac{\dot{L}}{L}\right) \\
&+ (r-1)\left[\alpha\frac{\dot{L}}{L} + (1-\alpha)\frac{\dot{K}_g}{K_g} - \frac{\dot{E}}{E}\right] + \frac{1}{Q^*}\frac{\partial Q^*}{\partial t}.
\end{aligned}
$$

Equation (51) is the fundamental factor productivity growth rate model for each of the industries of this Study. It is clear that the first four terms and the last term on the R.H.S. of (51) are identical to those of equations (46) and (48) and have analogous economic interpretations.[29] The fifth and sixth terms are different and must be analysed.

Intuitively, one would not expect the growth of capital-intensity, as defined, to make any contribution to the growth of factor productivity, as defined, since the latter expression is "supposed" to take account of the former. This is true so long as the weights in the factor productivity expression are chosen "correctly". In fact, it is easily seen that the coefficient of the capital-intensity growth rate variable in the fifth term is such that

$$
(52)\quad \left[\alpha\left(\frac{K^*}{Q^*}\;\frac{\partial Q^*}{\partial K^*}\right) - (1-\alpha)\left(\frac{L^*}{Q^*}\;\frac{\partial Q^*}{\partial L^*}\right)\right] \gtreqless 0 \text{ if and only if}
$$

$$
\frac{\alpha}{(1-\alpha)} \gtreqless \left(\frac{L^*}{Q^*}\;\frac{\partial Q^*}{\partial L^*}\right)\Big/\left(\frac{K^*}{Q^*}\;\frac{\partial Q^*}{\partial K^*}\right).
$$

In words, the weights are "correct" if and only if the ratio of the observed factor shares equals the ratio of the corresponding output elasticities at that time. If, for example, the elasticity of aggregate output with respect to aggregate labour input is *relatively* overestimated by the observed factor share of labour, then the fifth term would make a positive contribution to factor productivity growth, as defined, so long as $\dot{K}_g/K_g > \dot{L}/L$.

---

[29] Of course, the fourth term is interpreted *mutatis mutandis.*

Clearly, the economies-of-scale term implies that the economically *relevant* measure of the "average size of establishment", in a factor productivity context, must account for both labour and capital and is actually equal to

$$(53)\quad bL^{\alpha}K_g{}^{(1-\alpha)}E^{-1}$$

where b is an *arbitrary* positive constant. Thus, just as comparative factor productivities at a point of time are undefined,[30] so are the comparative "average sizes of establishments" undefined. But both concepts are well defined (and, in fact, simultaneously defined) in the *growth rate* context of this Study.

Finally, some simple relationships between the coefficient of the fifth term and the coefficient of the sixth term are shown in Chapter 5.

[30] More precisely, factor productivity ratios are only comparable *between* industries at a point of time if the observed factor shares of the industries are equal. Otherwise, the *relative* ratios will depend upon the particular units of measurement for L and K_g. On the other hand, factor productivity growth rates over a certain time period are comparable between industries because such growth rates are independent of the choice of measurement units for L and K_g.

APPENDIX B

SOME GENERALIZATIONS OF THE PRODUCTIVITY GROWTH MODELS

It is traditional in economics to state that theoretical results are not "interesting" unless they can be generalized. Indeed, some of the results in Appendix A were obtained under quite restrictive assumptions and for very special cases.[1] In this Appendix, some of the assumptions are relaxed, and the special cases are generalized. It turns out that the consideration of more general productivity growth models enriches the economic interpretation of the empirical results of Chapters 3, 4 and 5. In fact, the empirical results, in turn, often suggest the direction in which theoretical generalizations should proceed. The mathematical development in this Appendix is somewhat more advanced than that in Appendix A, but the mathematical level is still quite elementary. In order to save space, some of the mathematical proofs are merely sketched.

B.1 A Generalization of the Labour Quality Change Expression

In Section A.2, we obtained an expression for labour quality change in the special case where there were *two* homogeneous types of labour – i.e.,

(1) $L^* = g(L_1, L_2)$

where L^* is aggregate labour input, and L_i is the number of the i-th type of labour employed ($i = 1, 2$). Suppose now that we distinguish m types of labour, so that

(1a) $L^* = g(L_1, L_2, \ldots, L_m)$

where the function g is differentiable and homogeneous of degree one with respect to its m arguments. Let

(2) $L = \sum_{i=1}^{m} L_i$

represent the simple total of the number of the m types of labour employed. We could now develop a general expression for labour quality change.

[1] It should be noted that no specific neoclassical production function (such as the Cobb-Douglas or the C.E.S.) was assumed.

By reviewing the manipulations in Section A.2, it is evident that the "contribution of aggregate labour input" term[2] could be written as

$$(3)\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \sum_{i=1}^{m}\left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}}{L}\right) + \sum_{i=1}^{m}\left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\frac{\dot{L}}{L}$$

Also, equation (2) above implies that

$$(4)\quad \frac{\dot{L}}{L} = \sum_{j=1}^{m}\left(\frac{L_j}{L}\right)\frac{\dot{L}_j}{L_j} \quad ,$$

so that substituting (4) in the first term on the R.H.S. of (3), we obtain:

$$\sum_i\left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\left[\frac{\dot{L}_i}{L_i}-\sum_j\left(\frac{L_j}{L}\right)\frac{\dot{L}_j}{L_j}\right]$$

$$= \sum_i\left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\left[\frac{\dot{L}_i}{L_i}-\left(\frac{L_i}{L}\right)\frac{\dot{L}_i}{L_i}-\sum_{\substack{j\\ j\neq i}}\left(\frac{L_j}{L}\right)\frac{\dot{L}_j}{L_j}\right]$$

$$= \sum_i\left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\left[\left(1-\frac{L_i}{L}\right)\frac{\dot{L}_i}{L_i}-\sum_{\substack{j\\ j\neq i}}\left(\frac{L_j}{L}\right)\frac{\dot{L}_j}{L_j}\right]$$

$$= \sum_i\left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\left[\sum_{\substack{j\\ j\neq i}}\left(\frac{L_j}{L}\right)\frac{\dot{L}_i}{L_i}-\sum_{\substack{j\\ j\neq i}}\left(\frac{L_j}{L}\right)\frac{\dot{L}_j}{L_j}\right]$$

$$= \sum_i\left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\left[\sum_{\substack{j\\ j\neq i}}\left(\frac{L_j}{L}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right)\right]$$

$$= \sum_i\left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\left[\sum_{\substack{j\\ j>i}}\left(\frac{L_j}{L}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right)+\sum_{\substack{j\\ j<i}}\left(\frac{L_j}{L}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right)\right]$$

$$= \sum_{i<j}\left(\frac{L_iL_j}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_i}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right)+\sum_{j<i}\left(\frac{L_jL_i}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_i}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right).$$

The two last terms are now rewritten as

$$\sum_{i<j}\left(\frac{L_iL_j}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_i}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right)+\sum_{i<j}\left(\frac{L_iL_j}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_j}{L_j}-\frac{\dot{L}_i}{L_i}\right) \quad ,$$

so the summation of the two terms finally equals

$$\sum_{i<j}\left(\frac{L_iL_j}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_i}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right)+\sum_{i<j}\left(\frac{L_iL_j}{Q^*L}\right)\left(-\frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right)$$

---

[2] That is, the contribution *to* the aggregate output growth rate. Also, we replace the production function symbol with the aggregate output symbol, so that the results of this Appendix conform to the productivity growth rate models of Sections A.5 and A.6.

$$(5) \quad = \sum_{i<j} \left(\frac{L_i L_j}{Q^* L}\right)\left(\frac{\partial Q^*}{\partial L_i} - \frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}_j}{L_j}\right) .$$

Also, using the assumed homogeneity of function g, it is evident that the second term on the R.H.S. of equation (3) is

$$(6) \quad \sum_i \left(\frac{L_i}{Q^*}\frac{\partial Q^*}{\partial L_i}\right)\frac{\dot{L}}{L} = \sum_i \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{L_i}{L^*}\frac{\partial g}{\partial L_i}\right)\frac{\dot{L}}{L} = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L} .$$

Thus, combining the results in (5) and (6), it has been shown that the "contribution of aggregate labour input" term is equal to the summation of two expressions–i.e.,

$$(7) \quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \sum_{i<j} \left(\frac{L_i L_j}{Q^* L}\right)\left(\frac{\partial Q^*}{\partial L_i} - \frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}_j}{L_j}\right) + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L} ,$$

where $i = 1, 2, \ldots, m-1$ and $j = 2, 3, \ldots, m$. Again, recalling the earlier analysis in Section A.2, it is apparent that the first expression on the R.H.S. of (7) represents the "contribution of the growth of labour quality" to the growth rate of aggregate output. Indeed, the theoretical analysis of each of the $\binom{m}{2}$[3] terms of this expression is quite analogous to that of the single term yielded in the earlier special case where $m = 2$. The summation of the $\binom{m}{2}$ terms is the required generalization of the "labour quality change" expression.

Two comments are now in order. First, it is instructive to observe the implications when two or more labour types are "equally productive". Without loss of generality, suppose that $\partial Q^*/\partial L_1 = \partial Q^*/\partial L_2$ for any particular values of L_i $(i = 1, \ldots, m), K^*$, and t. Then the generalized labour quality change expression becomes

$$(8) \quad \sum_{i<j} \left(\frac{L_i L_j}{Q^* L}\right)\left(\frac{\partial Q^*}{\partial L_i} - \frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}_j}{L_j}\right)$$

$$= \sum_{2<j} \left(\frac{L_1 L_j}{Q^* L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_j}{L_j}\right)$$

$$+ \sum_{2<j} \left(\frac{L_2 L_j}{Q^* L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}_j}{L_j}\right)$$

$$+ \sum_{2<i<j} \left(\frac{L_i L_j}{Q^* L}\right)\left(\frac{\partial Q^*}{\partial L_i} - \frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}_j}{L_j}\right)$$

[3] For any positive integer $m \geq 2$, the expression $\binom{m}{2} = m!/(m-2)!\,2!$, it being understood that $0! = 1$.

But the first two expressions on the R.H.S. of (8) sum to

$$(9)\quad \sum_{2<j}\left(\frac{\partial Q^*}{\partial L_1}-\frac{\partial Q^*}{\partial L_j}\right)\left[\left(\frac{L_1L_j}{Q^*L}\right)\left(\frac{\dot{L}_1}{L_1}-\frac{\dot{L}_j}{L_j}\right)+\left(\frac{L_2L_j}{Q^*L}\right)\left(\frac{\dot{L}_2}{L_2}-\frac{\dot{L}_j}{L_j}\right)\right]$$

$$=\sum_{2<j}\left(\frac{\partial Q^*}{\partial L_1}-\frac{\partial Q^*}{\partial L_j}\right)\left\{\frac{(L_1+L_2)L_j}{Q^*L}\left[\left(\frac{L_1}{L_1+L_2}\right)\frac{\dot{L}_1}{L_1}+\left(\frac{L_2}{L_1+L_2}\right)\frac{\dot{L}_2}{L_2}-\frac{\dot{L}_j}{L_j}\right]\right\}.$$

However, it is easily recognized that

$$(10)\quad \left(\frac{L_1}{L_1+L_2}\right)\frac{\dot{L}_1}{L_1}+\left(\frac{L_2}{L_1+L_2}\right)\frac{\dot{L}_2}{L_2}=\frac{(L_1\dot{+}L_2)}{(L_1+L_2)}\quad ,$$

so that substituting (10) in (9) and using (8), we find that the original labour quality change expression is reduced to

$$(11)\quad \sum_{2<j}\left[\frac{(L_1+L_2)L_j}{Q^*L}\right]\left(\frac{\partial Q^*}{\partial L_1}-\frac{\partial Q^*}{\partial L_j}\right)\left[\frac{(L_1\dot{+}L_2)}{(L_1+L_2)}-\frac{\dot{L}_j}{L_j}\right]$$
$$+\sum_{2<i<j}\left(\frac{L_iL_j}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_i}-\frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_i}{L_i}-\frac{\dot{L}_j}{L_j}\right)\quad .$$

This means that the $\binom{m}{2}$ terms of the original quality change expression have been reduced to the $(m-2)$ terms of the first expression in (11) *plus* the $\binom{m-2}{2}$ terms of the second expression in (11). It is straightforward to show that

$$(m-2)+\binom{m-2}{2}=\binom{m-1}{2}\quad ,$$

as expected. Indeed, the revised quality change expression (11) is precisely what one would derive if we had assumed that

$$L^* = g(L_1, L_2, \ldots, L_m) = g^*[(L_1+L_2), L_3, \ldots, L_m]$$

for all L_i $(i = 1, \ldots, m)$–that is, if we had distinguished $(m-1)$ homogeneous types of labour. From now on it will be assumed that there exist, say, m distinct types of labour, if it is possible that $\partial Q^*/\partial L_i \neq \partial Q^*/\partial L_j$ $(i{=}j)$ $(i,j = 1, \ldots, m)$ for at least some combination of values for L_i K^* and t $(i = 1, \ldots, m)$.

The second comment concerns the utility of the generalized "labour quality change" expression. The question naturally arises as to how useful the general expression (5) is for empirical investigation. It is quite clear that the $\binom{m}{2}$ terms of (5) are not compatible with a regression analysis because the $\binom{m}{2}$ observable growth rate variables $(\dot{L}_i/L_i - \dot{L}_j/L_j)$ are linearly dependent. In fact, there exist $(m-1)$ basic growth rate variables such that each of the $\binom{m}{2}$ growth variables can be expressed as a linear combination of the $(m-1)$ basic growth rates.[4]

Suppose that the basic growth rate variables are chosen to be

$$(12) \quad (\dot{L}_1/L_1 - \dot{L}_j/L_j). \qquad (j = 2, \ldots, m)$$

Then

$$(13) \quad \left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}_\lambda}{L_\lambda}\right) = \left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_\lambda}{L_\lambda}\right) - \left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_i}{L_i}\right) \qquad (i = 2, \ldots, m-1;\ \lambda = 3, \ldots, m)$$

and the generalized labour quality change expression becomes

$$(14) \quad \sum_{1<j} \left(\frac{L_1 L_j}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_j}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_j}{L_j}\right) + \sum_{1<i<\lambda} \left(\frac{L_i L_\lambda}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_i} - \frac{\partial Q^*}{\partial L_\lambda}\right)\left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}_\lambda}{L_\lambda}\right),$$

which, after substituting (13), could be shown to equal

$$(15) \quad \sum_{1<j} \left[\left(\frac{L_1 L_j}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_j}\right) - \sum_{\substack{\lambda \\ \lambda>j}} \left(\frac{L_j L_\lambda}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_j} - \frac{\partial Q^*}{\partial L_\lambda}\right) + \sum_{\substack{\mu \\ 1<\mu<j}} \left(\frac{L_\mu L_j}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_\mu} - \frac{\partial Q^*}{\partial L_j}\right)\right]\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_j}{L_j}\right).$$

Thus expression (15) is an equivalently alternative way of writing the generalized labour quality change expression, with $(m-1)$ linearly independent observable growth rate variables. Unfortunately, even expression (15) is not particularly useful for empirical analysis, for the following reasons. To fix ideas, suppose that $m = 3$ (i.e., there are three distinct types of labour). Then (15) is

$$(15a) \quad \left[\left(\frac{L_1 L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_2}\right) - \left(\frac{L_2 L_3}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_2} - \frac{\partial Q^*}{\partial L_3}\right)\right]\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right) + \left[\left(\frac{L_1 L_3}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_3}\right) + \left(\frac{L_2 L_3}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_2} - \frac{\partial Q^*}{\partial L_3}\right)\right]\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_3}{L_3}\right).$$

It is now apparent that the coefficients of the basic growth rate variables are difficult to interpret.[5] The coefficients reflect the relative marginal productivities

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[4] In the language of matrix algebra, the $(m-1)$ basic growth rate variables constitute a "basis". This "basis" is, of course, not unique.

[5] See again the discussion in Sections A.2 and A.5.

of the nonbasic growth rate variables as well as the basic ones. For example, if $\partial Q^*/\partial L_1 > \partial Q^*/\partial L_2$ and $\dot{L}_1/L_1 > \dot{L}_2/L_2$, it is *not* necessarily true that the first term in (15a) makes a positive contribution to output growth or to productivity growth. Moreover, the choice of the basic growth rate variables is not unique and, in this sense, the choice is arbitrary.

All these difficulties vanish, of course, in the "uninteresting" special case where $m=2$.[6] Thus we are faced with the following dilemma. On the one hand, we need $m>2$ on the grounds of realism and generality. On the other hand, when $m>2$, the general labour quality change expression is either not compatible with empirical estimation or is difficult to interpret and is arbitrary. A solution to this dilemma is offered in the next section.

B.2 Application of the Labour and Capital Quality Change Expressions

In order to fix ideas, suppose that there are four homogeneous types of labour[7] – i.e., $m = 4$. Then the complete labour quality change expression is

$$(16)\quad \left(\frac{L_1L_3}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1}-\frac{\partial Q^*}{\partial L_3}\right)\left(\frac{\dot{L}_1}{L_1}-\frac{\dot{L}_3}{L_3}\right)+\left(\frac{L_1L_4}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1}-\frac{\partial Q^*}{\partial L_4}\right)\left(\frac{\dot{L}_1}{L_1}-\frac{\dot{L}_4}{L_4}\right)$$
$$+\left(\frac{L_2L_3}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_2}-\frac{\partial Q^*}{\partial L_3}\right)\left(\frac{\dot{L}_2}{L_2}-\frac{\dot{L}_3}{L_3}\right)+\left(\frac{L_2L_4}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_2}-\frac{\partial Q^*}{\partial L_4}\right)\left(\frac{\dot{L}_2}{L_2}-\frac{\dot{L}_4}{L_4}\right)$$
$$+\left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1}-\frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1}-\frac{\dot{L}_2}{L_2}\right)+\left(\frac{L_3L_4}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_3}-\frac{\partial Q^*}{\partial L_4}\right)\left(\frac{\dot{L}_3}{L_3}-\frac{\dot{L}_4}{L_4}\right),$$

where $L = L_1 + L_2 + L_3 + L_4$. The key result of this section follows. Expression (16) could be equivalently rewritten as the summation of three terms

$$(17)\quad \left[\frac{(L_1+L_2)(L_3+L_4)}{Q^*L}\right]\left[\left(\frac{L_1}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_1}+\left(\frac{L_2}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_2}\right.$$
$$\left.-\left(\frac{L_3}{L_3+L_4}\right)\frac{\partial Q^*}{\partial L_3}-\left(\frac{L_4}{L_3+L_4}\right)\frac{\partial Q^*}{\partial L_4}\right]\left[\frac{(L_1\dot{+}L_2)}{(L_1+L_2)}-\frac{(L_3\dot{+}L_4)}{(L_3+L_4)}\right]$$
$$+\left[\frac{L_1L_2}{Q^*(L_1+L_2)}\right]\left(\frac{\partial Q^*}{\partial L_1}-\frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1}-\frac{\dot{L}_2}{L_2}\right)$$
$$+\left[\frac{L_3L_4}{Q^*(L_3+L_4)}\right]\left(\frac{\partial Q^*}{\partial L_3}-\frac{\partial Q^*}{\partial L_4}\right)\left(\frac{\dot{L}_3}{L_3}-\frac{\dot{L}_4}{L_4}\right),$$

where $\frac{(L_1\dot{+}L_2)}{(L_1+L_2)} = \left(\frac{L_1}{L_1+L_2}\right)\frac{\dot{L}_1}{L_1}+\left(\frac{L_2}{L_1+L_2}\right)\frac{\dot{L}_2}{L_2}$

[6] Note that $\binom{m}{2} = (m-1) = 1$, when $m = 2$.

[7] To suppose that $m = 4$ is more useful than to just "fix ideas". The empirical productivity analysis of Chapters 3, 4, and 5 happens to account for exactly four types of labour because of statistical data limitations.

and $$\frac{(L_3 \dot{+} L_4)}{(L_3+L_4)} = \left(\frac{L_3}{L_3+L_4}\right)\frac{\dot{L}_3}{L_3} + \left(\frac{L_4}{L_3+L_4}\right)\frac{\dot{L}_4}{L_4} \,.$$

The proof of this key result will now be sketched.[8] Expand the first term in (17) by multiplying the first two components together and regroup like "quality-change" terms. Then multiply by the third component, collect "quality-change" terms and the remainders. These "quality-change" terms are precisely the first four terms of (16). Now add all the remainders with a $(\dot{L}_1/L_1 - \dot{L}_2/L_2)$ component and note the cancellations. Do the same for the $(\dot{L}_3/L_3 - \dot{L}_4/L_4)$-type remainders. The latter two terms differ from the last two terms of (16) by precisely the last two terms in (17), respectively.

The reader should now note that the summation of the three terms in (17) constitutes a highly convenient form of the labour quality change expression. The three terms are compatible with empirical regression analysis since the growth rate variable components of the terms are generally linearly independent and observable. Moreover, each term has a straightforward economic interpretation and the sign[9] of each of the coefficients has a significant economic meaning. Indeed, each term in (17) is *completely* analogous to that of the single-term labour quality change expression yielded when $m = 2$.[10] All this is clear for the second and third terms in (17). The first term is also analogous to an "$m = 2$-type quality change expression" if the weighted average of the first two types of labour is considered as a "single type", and the weighted average of the last two types is also considered as a "single type" – the weights in each case being proportional to the quantity employed of the particular types of labour. Thus, for example, if this weighted average of the growth rates of the first two types of labour is greater than the weighted average of the last two types, *and* if the *same* weighted average of the marginal productivities of the first two types of labour is greater than the *same corresponding* weighted average of the marginal productivities of the last two types of labour, then the relevant term would make a positive contribution to productivity growth. Loosely speaking, the sign of the coefficient of the growth rate variable in the first term of (17) would indicate which, if either, of the two weighted averages of types of labour is "more productive".

Form (17) of the labour quality change expression is most revealing once it is realized that statistical observations on types of labour employed are usually available in terms of partition (or classification) totals and subtotals. For example, we may know total employment of nonproduction workers; total employment of

[8] This writer worked out the proof in the case $m = 4$. Professor M. Tenenhaus showed that the result could be extended in the required manner for the case of any number of labour types. The validity of the more general result is implicitly assumed in the subsequent analysis of this section.

[9] That is; positive, negative, or zero.

[10] See again the discussion in Sections A.2 and A.5.

production workers; total employment of male (or female) nonproduction workers; and total employment of male (or female) production workers. More formally, and using vector notation,[11] the general aggregate labour input index is as before:

$$(1a)\quad L^* = g(L_1, L_2, \ldots, L_m) \equiv g(\vec{L}) \equiv g(\vec{N},\vec{P}),$$

where $\vec{N} \equiv (\vec{N}^m, \vec{N}^f)$, $\vec{P} \equiv (\vec{P}^m, \vec{P}^f)$, $\vec{N}^m \equiv (N_1, \ldots, N_{n^*})$,

$$\vec{N}^f \equiv (N_{n^*+1}, \ldots, N_n), \quad \vec{P}^m \equiv (P_1, \ldots, P_{p^*}),$$

$$\vec{P}^f \equiv (P_{p^*+1}, \ldots, P_p), \quad (n+p = m).$$

Suppose we have statistical observations for

$$N = \sum_{i=1}^{n} N_i, \; N^m = \sum_{i=1}^{n^*} N_i, \; P = \sum_{j=1}^{p} P_j, \; P^m = \sum_{j=1}^{p^*} P_j,$$

when there are n distinct types of nonproduction labour, $n^* < n$ types of male nonproduction labour, p types of production labour, and $p^* < p$ types of male production labour. By analogy with the procedure that yielded the equivalence of expressions (16) and (17), it is now possible to see that the generalized labour quality change expression

$$(5)\quad \sum_{i<j} \left(\frac{L_i L_j}{Q^* L}\right) \left(\frac{\partial Q^*}{\partial L_i} - \frac{\partial Q^*}{\partial L_j}\right) \left(\frac{\dot{L}_i}{L_i} - \frac{\dot{L}_j}{L_j}\right) \qquad \begin{pmatrix} i = 1, \ldots, m-1; \\ j = 2, \ldots, m; \\ m = n + p \end{pmatrix}$$

is equivalent to the summation of three terms plus "many other terms" – i.e.,[12]

$$\begin{aligned}(18)\quad & \left(\frac{NP}{Q^* L}\right) \left[\sum_{i=1}^{n} \left(\frac{N_i}{N}\right) \frac{\partial Q^*}{\partial N_i} - \sum_{j=1}^{p} \left(\frac{P_j}{P}\right) \frac{\partial Q^*}{\partial P_j}\right] \left(\frac{\dot{N}}{N} - \frac{\dot{P}}{P}\right) \\ & + \left[\frac{(N^m)(N^f)}{Q^* N}\right] \left[\sum_{i=1}^{n^*} \left(\frac{N_i}{N^m}\right) \frac{\partial Q^*}{\partial N_i} - \sum_{i=n^*+1}^{n} \left(\frac{N_i}{N^f}\right) \frac{\partial Q^*}{\partial N_i}\right] \left(\frac{\dot{N}^m}{N^m} - \frac{\dot{N}^f}{N^f}\right) \\ & + \left[\frac{(P^m)(P^f)}{Q^* P}\right] \left[\sum_{j=1}^{p^*} \left(\frac{P_j}{P^m}\right) \frac{\partial Q^*}{\partial P_j} - \sum_{j=p^*+1}^{p} \left(\frac{P_j}{P^f}\right) \frac{\partial Q^*}{\partial P_j}\right] \left(\frac{\dot{P}^m}{P^m} - \frac{\dot{P}^f}{P^f}\right) \\ & + \text{"many other terms"},\end{aligned}$$

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[11] For example, the vector $\vec{x}$ simply denotes $(x_1, \ldots, x_q)$ when there are q components of $\vec{x}$.

[12] To be clear, $L = N + P$, $N^f = N - N^m$, and $P^f = P - P^m$.

Note that $\dot{N}/N = \sum_{i=1}^{n} (N_i/N)(\dot{N}_i/N_i)$, $\dot{P}/P = \sum_{j=1}^{p} (P_j/P)(\dot{P}_j/P_j)$,

$(\dot{N}^m)/(N^m) = \sum_{i=1}^{n^*} \left[(N_i)/(N^m)\right] (\dot{N}_i/N_i)$, and similarly for the other growth rate variables.

of which, in the latter, $\binom{n^*}{2}$ terms account for "intra-male nonproduction labour quality change"; $\binom{n-n^*}{2}$ terms account for "intra-female nonproduction labour quality change"; $\binom{p^*}{2}$ terms account for "intra-male production labour quality change"; and finally, $\binom{p-p^*}{2}$ terms account for "intra-female production labour quality change".

However, the relevant consideration is that the first three terms in (18) are in a highly convenient form for empirical analysis. For example, the first term makes a positive contribution to productivity growth if the growth rate of nonproduction labour employment is greater than that of production labour employment, *and*, very loosely speaking, if nonproduction labour is "typically more productive" than production labour. Indeed, the structure of the second component in this first term indicates the exact economic meaning of the words "typically more productive" in this particular context. Completely analogous interpretations could be given to the second and third terms in (18) as well. Thus the three terms with observable growth rate variable components $(\dot{N}/N - \dot{P}/P)$, $(\dot{N}^m/N^m - \dot{N}^f/N^f)$ and $(\dot{P}^m/P^m - \dot{P}^f/P^f)$ all have well-defined economic meanings in the most general case of any number of labour types. But note that the "contributions of the 'many other terms' to productivity growth" all get lumped with the "residual" referred to in Sections A.2 and A.5.[13]

Finally, it is possible to obtain a generalization of the fixed capital quality change expression by reasoning along the lines indicated in Section B.1. Such a generalization is again difficult to apply to empirical work. But there exists an equivalent form of this generalization that contains terms compatible with empirical estimation and capable of direct economic interpretation. More precisely, let the general aggregate fixed capital input index be

$$(19)\quad K^* = h(K_1, K_2, \ldots, K_q) \equiv h(\vec{K}) \equiv h(\vec{M}, \vec{S}) \ ,$$

where $\vec{M} \equiv (M_1, \ldots, M_r)$, $\vec{S} \equiv (S_1, \ldots, S_s)$, $(r + s = q)$,

and suppose that

$$(20a)\quad M_i = a_i M_{in}^{\beta_i} M_{ig}^{(1-\beta_i)} \qquad (a_i > 0;\ 1 \geq \beta_i \geq 0;\ i = 1, \ldots, r)$$

$$(20b)\quad S_j = b_j S_{jn}^{\gamma_j} S_{jg}^{(1-\gamma_j)} \qquad (b_j > 0;\ 1 \geq \gamma_j \geq 0;\ j = 1, \ldots, s) \ .$$

[13] This raises questions concerning alternative statistical classification procedures. For example, the male-female breakdown may not be "very revealing" in terms of identifying sources of productivity growth. Also, the above theoretical analysis is applicable to any other classification procedure—e.g., education levels, occupational groups, and their cross-classifications.

Further, let

$$\text{(21a)}\quad K_n = M_n + S_n = \sum_{i=1}^{r} M_{in} + \sum_{j=1}^{s} S_{jn}$$

$$\text{(21b)}\quad K_g = M_g + S_g = \sum_{i=1}^{r} M_{ig} + \sum_{j=1}^{s} S_{jg}$$

Then the "contribution of aggregate fixed capital input" term – namely,

$$\text{(22)}\quad \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\frac{\dot{K}^*}{K^*},$$

could be equivalently rewritten, after using (19), (20) and (21), as

$$\text{(23)}\quad \left(\frac{M_n S_n}{Q^* K_n}\right)\left[\sum_i \left(\frac{M_{in}}{M_n}\right)\frac{\partial Q^*}{\partial M_{in}} - \sum_j \left(\frac{S_{jn}}{S_n}\right)\frac{\partial Q^*}{\partial S_{jn}}\right]\left(\frac{\dot{M}_n}{M_n} - \frac{\dot{S}_n}{S_n}\right)$$

$$+ (\text{"many other terms"})$$

$$+ \left(\frac{M_g S_g}{Q^* K_g}\right)\left[\sum_i \left(\frac{M_{ig}}{M_g}\right)\frac{\partial Q^*}{\partial M_{ig}} - \sum_j \left(\frac{S_{jg}}{S_g}\right)\frac{\partial Q^*}{\partial S_{jg}}\right]\left(\frac{\dot{M}_g}{M_g} - \frac{\dot{S}_g}{S_g}\right)$$

$$+ (\text{"many other terms"})$$

$$+ \left[\sum_i \beta_i \left(\frac{M_i}{Q^*}\frac{\partial Q^*}{\partial M_i}\right) + \sum_j \gamma_j \left(\frac{S_j}{Q^*}\frac{\partial Q^*}{\partial S_j}\right)\right]\left(\frac{\dot{K}_n}{K_n} - \frac{\dot{K}_g}{K_g}\right) + \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\frac{\dot{K}_g}{K_g}.$$

Thus, if M_{in} (M_{ig}) denotes the i-th type of net (gross) machinery and equipment capital stock, and if S_{jn} (S_{jg}) denotes the j-th type of net (gross) buildings and structures capital stock, then the first and third specified terms in (23) are meaningful "fixed capital quality change" terms.[14] For example, if the coefficient of $(\dot{M}_n/M_n - \dot{S}_n/S_n)$ is positive, this would mean that

$$\sum_i \left(\frac{M_{in}}{M_n}\right)\frac{\partial Q^*}{\partial M_{in}} > \sum_j \left(\frac{S_{jn}}{S_n}\right)\frac{\partial Q^*}{\partial S_{jn}},$$

or, in words, this would indicate that machinery and equipment net capital stock is "typically more productive" than buildings and structures net capital stock. But again, all the "many" intra-machinery and intra-structures quality change terms get lumped with the residual.

B.3 Complementarity among Factor Inputs

So far, the discussion in both Appendixes A and B has presupposed a neoclassical factor substitution type of production model. This is evident in the assumptions that (a) production function f is differentiable[15] with respect to the

[14] However, the summation of these two specified terms constitute only part of the "fixed capital quality change" expression. Nevertheless, it can be shown that they account for $2rs$ of the $2\binom{r+s}{2}$ terms of the generalized quality change expression.

[15] That is, differentiable for *all* positive values of its arguments.

aggregate factor input indexes L^* and K^*; (b) the aggregate labour input function g is differentiable with respect to the m types of labour L_i; (c) the aggregate fixed capital input function h is differentiable with respect to the q types of fixed capital K_j; and (d) all the partial derivatives are finite and positive for positive values of their arguments. In this section, we retain assumption (a) and its "corresponding" assumption in (d), but we relax assumptions (b) and (c) and their "corresponding" assumptions in (d). In particular, it is shown that such a generalization enriches the economic interpretation of the various quality change expressions. We will consider the simplest example first.

Suppose, as before, that[16]

$$(24)\quad Q^* = f(L^*, K^*; t)$$

where Q^* represents aggregate (net) output; L^* is aggregate labour input; K^* is aggregate fixed capital input; and t allows for shifts in the production relation over time. Further, let there be two types of labour and suppose that the aggregate labour input function g is

$$(25)\quad L^* = g(L_1, L_2) = \min(L_1, \lambda L_2) \qquad (\text{for all } L_1 > 0, L_2 > 0)$$

where λ is a positive constant. This means that

$$(25a)\quad L^* = g(L_1, L_2) = \begin{cases} L_1 & \text{for } L_1 \leqq \lambda L_2 \\ \lambda L_2 & \text{for } L_1 \geqq \lambda L_2 \end{cases} .$$

Thus this function g is still homogeneous of degree unity with respect to L_1 and L_2, but the particular function is not differentiable for values $L_1 = \lambda L_2$. In fact,

$$(26a)\quad \frac{\partial g}{\partial L_1} = \begin{cases} 1 & \text{for } L_1 < \lambda L_2 \quad (\text{Case I}) \\ 0 & \text{for } L_1 > \lambda L_2 \quad (\text{Case II}) \\ \text{undefined} & \text{for } L_1 = \lambda L_2 \quad (\text{Case III}). \end{cases}$$

Similarly,

$$(26b)\quad \frac{\partial g}{\partial L_2} = \begin{cases} 0 & \text{for } L_1 < \lambda L_2 \quad (\text{Case I}) \\ \lambda & \text{for } L_1 > \lambda L_2 \quad (\text{Case II}) \\ \text{undefined} & \text{for } L_1 = \lambda L_2 \quad (\text{Case III}) \end{cases}$$

The specification of the function g as min $(L_1, \lambda L_2)$ is usually referred to as the condition of *complementarity* among factor inputs (i.e., the two types of labour are complements in production). It is instructive to analyse the labour quality change expressions in the above three basic cases. The relations (25a), (26a), and (26b) are implicitly used throughout the following analyses.[17]

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[16] It is easily seen that the consideration of the number of establishments (E) does not alter the following analysis. See again Section A.4.

[17] It is again assumed that L_1 and L_2 are measured in identical units (e.g., number employed) and that $L = L_1 + L_2$.

Case I–($L_1 < \lambda L_2$): The "contribution of aggregate labour input" expression

$$(27)\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \left(\frac{L_1}{Q^*}\frac{\partial Q^*}{\partial L_1}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}}{L}\right) + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L}$$

and the first term on the R.H.S. of (27) could be rewritten as

$$(28)\quad \left(\frac{L_1}{Q^*}\frac{\partial Q^*}{\partial L_1}\right)\left[\frac{\dot{L}_1}{L_1} - \left(\frac{L_1}{L}\right)\frac{\dot{L}_1}{L_1} - \left(\frac{L_2}{L}\right)\frac{\dot{L}_2}{L_2}\right]$$

$$= \left(\frac{L_1}{Q^*}\frac{\partial Q^*}{\partial L_1}\right)\left[\left(\frac{L_2}{L}\right)\frac{\dot{L}_1}{L_1} - \left(\frac{L_2}{L}\right)\frac{\dot{L}_2}{L_2}\right] = \left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)$$

$$= \left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right),$$

where, in the last equality, we used the fact that $\partial Q^*/\partial L_2 = (\partial Q^*/\partial L^*)(\partial g/\partial L_2) = 0$ when $L_1 < \lambda L_2$. The last term in (28) is identical with the labour quality change expression derived in Section A.2 for the "substitution case" of two types of labour employed. Note, e.g., that when $\dot{L}_1/L_1 > \dot{L}_2/L_2$, the term makes a positive contribution to output growth and productivity growth, because in this case the faster-growing type of labour is also the "relatively scarce" type of labour and is therefore "more productive". Indeed, it is again natural to identify the last term in (28) as the relevant labour quality change expression.

Case II–($L_1 > \lambda L_2$): Now the "contribution of aggregate labour input" term

$$(29)\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \lambda\left(\frac{L_2}{Q^*}\frac{\partial Q^*}{\partial L_2}\right)\lambda^{-1}\left(\frac{\dot{L}_2}{L_2}\right)$$

$$= \left(\frac{L_2}{Q^*}\frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}}{L}\right) + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L},$$

and the first term on the extreme R.H.S. of (29) becomes

$$(30)\quad \left(\frac{L_2}{Q^*}\frac{\partial Q^*}{\partial L_2}\right)\left[\frac{\dot{L}_2}{L_2} - \left(\frac{L_1}{L}\right)\frac{\dot{L}_1}{L_1} - \left(\frac{L_2}{L}\right)\frac{\dot{L}_2}{L_2}\right] = \left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1}\right)$$

$$= \left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_2} - \frac{\partial Q^*}{\partial L_1}\right)\left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1}\right) = \left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L_1} - \frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right)$$

using the fact that $\partial Q^*/\partial L_1 = 0$ when $L_1 > \lambda L_2$. Here, e.g., the relevant term makes a negative contribution to output growth if $\dot{L}_1/L_1 > \dot{L}_2/L_2$, because in this case the slower-growing type of labour is "relatively scarce" and is therefore "more productive". But the symbolic expression for labour quality change is the same as in Case I.

Case III–($L_1 = \lambda L_2$): The first thing to note in this case is

$$(31)\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\left[\min\left(\frac{\dot{L}_1}{L_1}, \frac{\dot{L}_2}{L_2}\right)\right] \leqq \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L}$$

and the equality holds in the latter relationship if and only if $\dot{L}_1/L_1 = \dot{L}_2/L_2$. There are in fact three subcases to consider.

Case IIIa–($L_1 = \lambda L_2$ and $\dot{L}_1/L_1 < \dot{L}_2/L_2$): Now

$$\text{(31a)}\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}_1}{L_1} = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}}{L}\right) + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L},$$

and the first term on the extreme R.H.S. of (31a) becomes

$$\text{(32a)}\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}}{L}\right) = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\left[\frac{\dot{L}_1}{L_1} - \left(\frac{L_1}{L}\right)\frac{\dot{L}_1}{L_1} - \left(\frac{L_2}{L}\right)\frac{\dot{L}_2}{L_2}\right]$$

$$= \left(\frac{L^*L_2}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right) = \left(\frac{L_1L_2}{Q^*L}\right)\left(\frac{L^*}{L_1}\frac{\partial Q^*}{\partial L^*} - 0\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right).$$

The final term in (32a) has a straightforward economic interpretation. One may regard (L^*/L_1) $(\partial Q^*/\partial L^*)$ as the marginal product of the first type of labour, at $L_1 = \lambda L_2$; the marginal product of the second type of labour equals zero. In this case, the slower-growing type of labour (namely, L_1) is always "relatively scarce" and is therefore "more productive". Thus the relevant term is always negative as expected from relationship (31). It is natural, then, to *define*

$$\text{(33a)}\quad \frac{\partial Q^*}{\partial L_1} = \frac{L^*}{L_1}\frac{\partial Q^*}{\partial L^*} \quad \text{and} \quad \frac{\partial Q^*}{\partial L_2} = 0$$

when $L_1 = \lambda L_2$ and $\dot{L}_1/L_1 < \dot{L}_2/L_2$. Indeed, this case is quite analogous to Case I, because when $L_1 < \lambda L_2$, it is easily seen that the relations (33a) hold. Substituting (33a) in (32a) yields the familiar labour quality change expression.

Case IIIb–($L_1 = \lambda L_2$ and $\dot{L}_1/L_1 > \dot{L}_2/L_2$): Then

$$\text{(31b)}\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}_2}{L_2} = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}}{L}\right)$$

$$+ \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L},$$

and now

$$\text{(32b)}\quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}}{L}\right) = \left(\frac{L^*L_1}{Q^*L}\right)\left(\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1}\right)$$

$$= \left(\frac{L_1L_2}{Q^*L}\right)\left(0 - \frac{L^*}{L_2}\frac{\partial Q^*}{\partial L^*}\right)\left(\frac{\dot{L}_1}{L_1} - \frac{\dot{L}_2}{L_2}\right).$$

Since the second type of labour is "relatively scarce", it is natural to interpret (L^*/L_2) $(\partial Q^*/\partial L^*)$ as the marginal product of the second labour type, at $L_1 = \lambda L_2$, and give the first type of labour a zero marginal product. In fact, we *define*

$$\text{(33b)} \quad \frac{\partial Q^*}{\partial L_1} = 0 \text{ and } \frac{\partial Q^*}{\partial L_2} = \frac{L^*}{L_2}\frac{\partial Q^*}{\partial L^*}$$

when $L_1 = \lambda L_2$ and $\dot{L}_1/L_1 > \dot{L}_2/L_2$. These are precisely the relations that also hold in Case II, because $(L^*/L_2)\ (\partial Q^*/\partial L^*) = \lambda^{-1}\ (L^*/L_2)\ (\partial Q^*/\partial L^*)\lambda = (\lambda L_2/\lambda L_2)\ (\partial Q^*/\partial L^*)\ (\partial L^*/\partial L_2) = \partial Q^*/\partial L_2$, *when* $L_1 > \lambda L_2$. The usual labour quality change expression is preserved by using (33b) in (32b).

Case IIIc–($L_1 = \lambda L_2$ and $\dot{L}_1/L_1 = \dot{L}_2/L_2$): Here

$$\text{(31c)} \quad \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*} = \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}}{L}$$

and any labour quality change expression will vanish because $\dot{L}_1/L_1 = \dot{L}_2/L_2$. However, it is still desirable to define the marginal products of the two types of labour in this case. Such marginal products are required in order to consider more general aggregate labour input functions that exhibit a "mixture" of complementarity and substitution elements. (This is seen shortly.) It is appropriate to *define*

$$\text{(33c)} \quad \frac{\partial Q^*}{\partial L_1} = \frac{1}{2}\left(\frac{L^*}{L_1}\frac{\partial Q^*}{\partial L^*}\right) \text{ and } \frac{\partial Q^*}{\partial L_2} = \frac{1}{2}\left(\frac{L^*}{L_2}\frac{\partial Q^*}{\partial L^*}\right)$$

when $L_1 = \lambda L_2$ and $\dot{L}_1/L_1 = \dot{L}_2/L_2$–i.e., when the particular complementarity condition involves *two* types of labour.[18] From (33c), it is instructive to observe that

$$\frac{\partial Q^*}{\partial L_1} \gtreqless \frac{\partial Q^*}{\partial L_2} \text{ if and only if } \lambda \lesseqgtr 1 \quad .$$

Intuitively, this is what one should expect. For example, the first type of labour is "more productive" than the second type of labour (in this Case IIIc) if and only if the number of units required of the first type of labour is less than that required of the second type of labour. But labour quality does not change, since the two types of labour have equal growth rates.

Thus, in all possible five cases,[19] the basic theoretical analysis of Appendix A still holds, and the familiar labour quality change expression is relevant after introducing appropriate definitions. It is evident that the condition of factor complementarity can be regarded as merely a limiting case of factor substitution. To show this more generally, consider the following "mixture" of complementarity and substitution elements

[18] A more general complementarity condition involving more than two labour types is mentioned later. For the time being, it is assumed that complementarity is always of the "two-way" sort.

[19] It is understood that Case I involves ($L_1 < \lambda L_2$ and $\dot{L}_1/L_1 \gtreqless \dot{L}_2/L_2$); Case II involves ($L_1 > \lambda L_2$ and $\dot{L}_1/L_1 \gtreqless \dot{L}_2/L_2$).

(34) $L^* = g[\min(L_1,\lambda L_2),L_3] = g(L_1^*,L_3)$[20]

where the aggregate labour input function g is differentiable and has positive partial derivatives with respect to L_1^* and L_3, for all $L_i > 0$ $(i = 1,2,3)$, $(L_1 \gtreqless \lambda L_2)$.

Then, using the analysis of Section B.2, it could be shown that the relevant labour quality change expression is the summation of two terms—namely,[21]

$$(35)\quad \left[\frac{(L_1+L_2)L_3}{Q^*L}\right]\left[\left(\frac{L_1}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_1}+\left(\frac{L_2}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_2}-\frac{\partial Q^*}{\partial L_3}\right]\left[\frac{(\dot{L}_1+\dot{L}_2)}{(L_1+L_2)}-\frac{\dot{L}_3}{L_3}\right]$$
$$+\left[\frac{L_1L_2}{Q^*(L_1+L_2)}\right]\left(\frac{\partial Q^*}{\partial L_1}-\frac{\partial Q^*}{\partial L_2}\right)\left(\frac{\dot{L}_1}{L_1}-\frac{\dot{L}_2}{L_2}\right),$$

where we define, if necessary,

$$(35a)\quad \frac{\partial Q^*}{\partial L_1} = \begin{cases} \dfrac{\partial Q^*}{\partial L_1^*}\dfrac{\partial L_1^*}{\partial L_1} = \dfrac{L_1^*}{L_1}\dfrac{\partial Q^*}{\partial L_1^*} & \text{for } L_1 < \lambda L_2 \text{ and } \dot{L}_1/L_1 \gtreqless \dot{L}_2/L_2 \\ \dfrac{L_1^*}{L_1}\dfrac{\partial Q^*}{\partial L_1^*} & \text{for } L_1 = \lambda L_2 \text{ and } \dot{L}_1/L_1 < \dot{L}_2/L_2 \\ \dfrac{1}{2}\left(\dfrac{L_1^*}{L_1}\dfrac{\partial Q^*}{\partial L_1^*}\right) & \text{for } L_1 = \lambda L_2 \text{ and } L_1/L_1 = \dot{L}_2/L_2 \\ 0 & \text{otherwise,} \end{cases}$$

and where

$$(35b)\quad \frac{\partial Q^*}{\partial L_2} = \begin{cases} \dfrac{\partial Q^*}{\partial L_1^*}\dfrac{\partial L_1^*}{\partial L_2} = \dfrac{L_1^*}{L_2}\dfrac{\partial Q^*}{\partial L_1^*} & \text{for } L_1 > \lambda L_2 \text{ and } \dot{L}_1/L_1 \gtreqless \dot{L}_2/L_2 \\ \dfrac{L_1^*}{L_2}\dfrac{\partial Q^*}{\partial L_1^*} & \text{for } L_1 = \lambda L_2 \text{ and } \dot{L}_1/L_1 > \dot{L}_2/L_2 \\ \dfrac{1}{2}\left(\dfrac{L_1^*}{L_2}\dfrac{\partial Q^*}{\partial L_1^*}\right) & \text{for } L_1 = \lambda L_2 \text{ and } \dot{L}_1/L_1 = \dot{L}_2/L_2 \\ 0 & \text{otherwise.} \end{cases}$$

The reader should verify for himself that the weighted average of the marginal products of the two types of labour that exhibit the complementarity condition is such that

$$(36)\quad \left(\frac{L_1}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_1}+\left(\frac{L_2}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_2}=\left(\frac{L_1^*}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_1^*}\quad \text{for } L_1 \gtreqless \lambda L_2 \text{ and } \dot{L}_1/L_1 \gtreqless \dot{L}_2/L_2,$$

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[20] To be clear, $L_1^* = \min(L_1,\lambda L_2)$ in this example.

[21] In this example, $L = L_1 + L_2 + L_3$.

so the first term in (35) could always be rewritten as

$$\left[\frac{(L_1+L_2)L_3}{Q^*L}\right]\left[\left(\frac{L_1^*}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_1^*}-\frac{\partial Q^*}{\partial L_3}\right]\left[\frac{(L_1\dot{+}L_2)}{(L_1+L_2)}-\frac{\dot{L}_3}{L_3}\right].$$

Note that $L_1^* \overset{<}{=} L_1+L_2$ for $L_1 \gtreqless \lambda L_2$, which, together with (36), implies that

$$(37)\quad \left(\frac{L_1}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_1}+\left(\frac{L_2}{L_1+L_2}\right)\frac{\partial Q^*}{\partial L_2} \leq \frac{\partial Q^*}{\partial L_1^*} \quad \text{for } L_1 \gtreqless \lambda L_2 \text{ and } \dot{L}_1/L_1 \gtreqless \dot{L}_2/L_2 .$$

Finally, the second term in (35) is essentially the same as the one analysed earlier in this section.

A more important "mixture" of complementarity and substitution elements could be revealed in another example. Suppose that the aggregate labour input index is a function of four types of labour such that

$$(38)\quad L^* = g[\min(L_1,\lambda L_2),L_3,L_4] = g(L_1^*,L_3,L_4)^{22}$$

where g is differentiable and has positive partial derivatives with respect to L_1^*, L_3 and L_4, for all $L_i > 0$ ($i = 1, \ldots, 4$), ($L_1 \gtreqless \lambda L_2$). Further, suppose that statistical observations are only available for the simple summations (L_1+L_3) and (L_2+L_4). Thus the available statistical data *also* contain a "mixture" of complementarity and substitution elements.[23] Let $L = \sum_{i=1}^{4} L_i$. Then the one measurable term of the relevant labour quality change expression is[24]

$$(38)\quad \left[\frac{(L_1+L_3)(L_2+L_4)}{Q^*L}\right]\left[\left(\frac{L_1}{L_1+L_3}\right)\frac{\partial Q^*}{\partial L_1}+\left(\frac{L_3}{L_1+L_3}\right)\frac{\partial Q^*}{\partial L_3} - \left(\frac{L_2}{L_2+L_4}\right)\frac{\partial Q^*}{\partial L_2}-\left(\frac{L_4}{L_2+L_4}\right)\frac{\partial Q^*}{\partial L_4}\right]\left[\frac{(L_1\dot{+}L_3)}{(L_1+L_3)}-\frac{(L_2\dot{+}L_4)}{(L_2+L_4)}\right]$$

where $\partial Q^*/\partial L_1$ and $\partial Q^*/\partial L_2$ are defined as in (35a) and (35b), respectively.

It can now be recognized that the magnitude and the sign of the unknown *coefficient* of the observed growth rate difference in term (38) could be particularly sensitive to the relative values and relative growth rates of the two labour types that exhibit the complementarity condition—namely, L_1 and L_2. For example, if $L_1 < \lambda L_2$, then the coefficient is more likely to be positive, *ceteris paribus*, than if $L_1 > \lambda L_2$. Similarly, e.g., the coefficient is more likely to be positive if $L_1 = \lambda L_2$ and $\dot{L}_1/L_1 < \dot{L}_2/L_2$ than if $L_1 = \lambda L_2$ and $\dot{L}_1/L_1 \geq \dot{L}_2/L_2$. Thus the answer to whether (L_1+L_3) is "typically more, less, or equally productive", compared with

[22] Again, $L_1^* = \min(L_1,\lambda L_2)$.
[23] This would be essentially the situation in most empirical investigations.
[24] Recall the discussion in Section B.2.

(L_2+L_4), may depend critically on whether $L_1 \gtreqless \lambda L_2$ and on whether $\dot{L}_1/L_1 \gtreqless \dot{L}_2/L_2$. Indeed, this dependence is particularly apparent to the extent that both $(\partial Q^*/\partial L_1^* > \partial Q^*/\partial L_3, \partial Q^*/\partial L_1^* > \partial Q^*/\partial L_4)$ *and* $(L_1^* > L_3, L_1^* > L_4)$, for in this special case the elements of factor complementarity are more likely to "dominate" the elements of factor substitution in the relevant coefficient.[25]

The key result of this section can now be stated. It is that the familiar derived labour quality change expressions are sufficiently flexible to reflect any mixture of complementarity and substitution elements after introducing appropriate definitions. Most important for empirical analysis, it is *not* necessary to know in advance which, if any, particular labour types exhibit the complementarity condition. The existence of any case of complementarity will merely be indicated by the magnitude and the sign of the coefficients of the relevant growth rate variables.

To conclude this section, two brief comments are in order. First, all the factor complementarity phenomena introduced so far have been of a "two-way" sort. However, it is easy to extend the results to any sort of complementarity condition, once it is realized, e.g., that

$$(39) \quad \min(L_1, \lambda L_2, \mu L_3) = \min[\min(L_1, \lambda L_2), \mu L_3] \qquad (\lambda > 0, \mu > 0)$$

for all $L_i > 0$ $(i = 1,2,3)$. Second, the method of representing labour factor complementarity as a limiting case of labour factor substitution applies equally well for conditions of fixed capital complementarity.

B.4 A Production Function with Variable Homogeneity

The reader will recall that the fundamental productivity growth rate equations of Sections A.5 and A.6 were developed from the production function of the representative establishment – namely, equation (32) in Section A.4, which is

$$(40) \quad Q^*/E = f(L^*/E, K^*/E; t).$$

The function f was assumed to be homogeneous of degree r (a positive *constant*) with respect to L^*/E and K^*/E. This constancy assumption was crucial to the shown derivation of the basic equations (36) and (39) of Sections A.4 and A.5, respectively. However, this is a highly restrictive assumption[26] that should be relaxed, if possible. Fortunately, there is an alternative derivation[27] of the productivity growth models, which permits a larger class of representative

[25] The estimated coefficient of the observed growth rate difference between male and female nonproduction labour employment is given such an interpretation in Section 3.4. It would be correct to think of (L_1+L_3) as total male nonproduction labour employment and and (L_2+L_4) as total female nonproduction labour employment.

[26] See P. A. Meyer, "An Aggregate Homothetic Production Function", *Southern Economic Journal*, January 1970, pp. 229-238.

[27] The previous derivation in Appendix A was motivated by expository considerations.

establishment production functions and allows for a richer interpretation of the "economies-of-scale contribution" to productivity growth. The alternative derivation will now be sketched.

Consider the following production function of the representative establishment

$$(41)\quad Q^*/E = F[f(L^*/E, K^*/E;t)] \equiv F[Z^*/E] \quad ,$$

where the function F is differentiable *and* monotonically increasing in its single argument, and f is again homogeneous of degree r (a constant) in L^*/E and K^*/E.[28] Clearly, the production function described by (41) is more general than that of (40) and coincides with (40) only in the special case where the function F is also homogeneous of some constant degree with respect to Z^*/E. To show this explicitly, put

$$(42)\quad Q^{**} \equiv Q^*/E,\ L^{**} \equiv L^*/E,\ K^{**} \equiv K^*/E,\ Z^{**} \equiv Z^*/E \quad .$$

Then the degree of homogeneity of the production function (41) equals[29]

$$(43)\quad \left(\frac{L^{**}}{Q^{**}}\frac{\partial Q^{**}}{\partial L^{**}}\right) + \left(\frac{K^{**}}{Q^{**}}\frac{\partial Q^{**}}{\partial K^{**}}\right)$$

$$= \left(\frac{Z^{**}}{Q^{**}}\frac{dF}{dZ^{**}}\right)\left(\frac{L^{**}}{Z^{**}}\frac{\partial f}{\partial L^{**}}\right) + \left(\frac{Z^{**}}{Q^{**}}\frac{dF}{dZ^{**}}\right)\left(\frac{K^{**}}{Z^{**}}\frac{\partial f}{\partial K^{**}}\right)$$

$$= \left[\frac{Z^{**}}{F(Z^{**})}F'\ (Z^{**})\right] r = [G(Z^{**})]\, r = G[F^{-1}(Q^*/E)]\, r = H(Q^*/E),$$

which is a function of a variable–namely, the industry aggregate output per establishment (Q^*/E). Thus returns to scale of production function (41) are, *in part*, technologically determined through the functional form H and, *in part*, economically determined through the variable Q^*/E.[30]

To develop the productivity growth rate equations, totally differentiate both sides of (41) with respect to time t and then divide both sides by Q^{**}, yielding

$$(44)\quad \frac{\dot{Q}^{**}}{Q^{**}} = \left(\frac{L^{**}}{Q^{**}}\frac{\partial Q^{**}}{\partial L^{**}}\right)\frac{\dot{L}^{**}}{L^{**}} + \left(\frac{K^{**}}{Q^{**}}\frac{\partial Q^{**}}{\partial K^{**}}\right)\frac{\dot{K}^{**}}{K^{**}} + \frac{1}{Q^{**}}\frac{\partial Q^{**}}{\partial t} \quad .$$

But

$$(45)\quad \frac{\dot{Q}^{**}}{Q^{**}} = \left(\frac{\dot{Q}^*}{Q^*} - \frac{\dot{E}}{E}\right), \frac{\dot{L}^{**}}{L^{**}} = \left(\frac{\dot{L}^*}{L^*} - \frac{\dot{E}}{E}\right), \frac{\dot{K}^{**}}{K^{**}} = \left(\frac{\dot{K}^*}{K^*} - \frac{\dot{E}}{E}\right) \quad ,$$

and

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[28] It is understood that the function f is differentiable with respect to $L^*/E, K^*/E$, and t.

[29] See R. G. D. Allen, *op. cit.*, p. 319.

[30] If F is homogeneous of constant degree, then $G(Z^{**})$ is also a constant, and returns to scale are, *in full*, technologically determined.

$$(46)\quad \left(\frac{L^{**}}{Q^{**}}\frac{\partial Q^{**}}{\partial L^{**}}\right) = \left(\frac{L^{*}}{Q^{*}}\frac{\partial Q^{*}}{\partial L^{*}}\right), \left(\frac{K^{**}}{Q^{**}}\frac{\partial Q^{**}}{\partial K^{**}}\right) = \left(\frac{K^{*}}{Q^{*}}\frac{\partial Q^{*}}{\partial K^{*}}\right),$$

$$\frac{1}{Q^{**}}\frac{\partial Q^{**}}{\partial t} = \frac{1}{Q^{*}}\frac{\partial Q^{*}}{\partial t},$$

so that using relations (43), (45) and (46) in (44), we finally derive

$$(47)\quad \frac{\dot{Q}^{*}}{Q^{*}} = [1-H(Q^{*}/E)]\frac{\dot{E}}{E} + \left(\frac{L^{*}}{Q^{*}}\frac{\partial Q^{*}}{\partial L^{*}}\right)\frac{\dot{L}^{*}}{L^{*}} + \left(\frac{K^{*}}{Q^{*}}\frac{\partial Q^{*}}{\partial K^{*}}\right)\frac{\dot{K}^{*}}{K^{*}} + \frac{1}{Q^{*}}\frac{\partial Q^{*}}{\partial t}.$$

Equation (47) is identical to the basic "source of industry output growth equation" (39) of *Appendix A,* Sections A.4 and A.5, *except* for the replacement of the constant r by the variable $H(Q^*/E)$.

So the fundamental productivity growth models of Appendix A still hold for a production function with variable homogeneity (i.e., relation (41)), except that the constant r in each of the three fundamental models[31] should be replaced by the function H with variable Q^*/E. A broader interpretation of the term representing the "economies-of-scale contribution" to productivity growth would then be as follows. For example, if the coefficient $[H(Q^*/E)-1]$ of the labour productivity growth rate model term

$$[H(Q^{*}/E)-1](\dot{L}/L - \dot{E}/E)$$

is positive, this would indicate that the representative establishment is operating at an output level where returns to scale are increasing. In this case, an increase (decrease) in the "size of the establishment", as measured by L/E, would make a positive (negative) contribution to labour productivity growth. Thus the measure Q^*/E determines the direction of returns to scale through the function H at a particular time, while L/E is the relevant measure for *change* in the "average size of the establishment" as it may contribute to labour productivity growth at that time. Analogous remarks apply to the capital productivity and factor productivity growth rate models as well.

B.5 A Quality Change Expression for Output

Up to this point, we have emphasized at some length that the aggregate labour input L^* and the aggregate fixed capital input K^* are generally not observable. This has necessitated the separation, e.g., of the term for "contribution of aggregate labour input to the growth of output" into two expressions – one being a "labour quality change" expression, and the other representing the "growth contribution of the simple total of the various types of labour".[32] Both these

[31] That is, equations (46), (48) and (51) of Appendix A. To be sure, all the shown manipulations still follow because

$$\left(\frac{L^{*}}{Q^{*}}\frac{\partial Q^{*}}{\partial L^{*}}\right) + \left(\frac{K^{*}}{Q^{*}}\frac{\partial Q^{*}}{\partial K^{*}}\right) = H(Q^{*}/E)\ .$$

[32] See again Sections A.2 and B.1.

expressions are compatible with empirical estimation. However, nothing has been said[33] about the problem of observing the aggregate net output index Q^*. In fact, the various productivity concepts of the growth models in Sections A.5 and A.6 are defined in terms of Q^*. Therefore, we now turn to a brief analysis of this potentially "embarrassing" question.

Suppose that there are two homogeneous types of output–namely, Q_1^* and Q_2^*–each of which is measured in *physical* units. Further, let

$$(48)\quad Q^* = \psi^*(Q_1^*, Q_2^*)$$

where the function ψ^* is assumed to be differentiable and homogeneous of degree unity with respect to its arguments. Now, in empirical investigation, we observe[34]

$$(49)\quad Q_1 = \lambda_{10} Q_1^* \text{ and } Q_2 = \lambda_{20} Q_2^*$$

where λ_{i0} is the base period price per unit of the i-th type of output ($i = 1,2$). Thus λ_{10} and λ_{20} are constants, and Q_1 and Q_2 are now measured in *common* constant-dollar units. Then equation (48) could be rewritten as

$$(50)\quad Q^* = \psi^*(Q_1/\lambda_{10}, Q_2/\lambda_{20}) = \psi(Q_1, Q_2) \quad ,$$

and the observed simple total of the two types of output, measured in constant dollars, would be

$$(51)\quad Q = Q_1 + Q_2 \quad .$$

It also follows that the function ψ in (50) is differentiable and homogeneous of degree one in Q_1 and Q_2.

If we now totally differentiate, with respect to time t, the two extreme sides of equation (50) and divide by Q^*, we have

$$(52)\quad \frac{\dot{Q}^*}{Q^*} = \left(\frac{Q_1}{Q^*}\frac{\partial\psi}{\partial Q_1}\right)\frac{\dot{Q}_1}{Q_1} + \left(\frac{Q_2}{Q^*}\frac{\partial\psi}{\partial Q_2}\right)\frac{\dot{Q}_2}{Q_2}$$

$$= \left(\frac{Q_1}{Q^*}\frac{\partial\psi}{\partial Q_1}\right)\left(\frac{\dot{Q}_1}{Q_1} - \frac{\dot{Q}}{Q}\right) + \left(\frac{Q_2}{Q^*}\frac{\partial\psi}{\partial Q_2}\right)\left(\frac{\dot{Q}_2}{Q_2} - \frac{\dot{Q}}{Q}\right) + \frac{\dot{Q}}{Q}$$

$$= \left(\frac{Q_1 Q_2}{Q^* Q}\right)\left(\frac{\partial\psi}{\partial Q_1} - \frac{\partial\psi}{\partial Q_2}\right)\left(\frac{\dot{Q}_1}{Q_1} - \frac{\dot{Q}_2}{Q_2}\right) + \frac{\dot{Q}}{Q} \quad ,$$

by manipulations completely analogous to those which yielded the labour quality change expression in Section A.2. Substituting (52) in the basic output growth equation of Section A.2,[35] we arrive at

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[33] Except for a footnote at the end of Section A.5.
[34] It is assumed that $\lambda_{i0} > 0$ ($i = 1,2$).
[35] The number of establishments (E) is not relevant for this analysis because of the unitary homogeneity of the function ψ.

$$(53)\quad \frac{\dot{Q}}{Q} = \left(\frac{Q_1 Q_2}{Q^* Q}\right)\left(\frac{\partial\psi}{\partial Q_2} - \frac{\partial\psi}{\partial Q_1}\right)\left(\frac{\dot{Q}_1}{Q_1} - \frac{\dot{Q}_2}{Q_2}\right) + \left(\frac{L^*}{Q^*}\frac{\partial Q^*}{\partial L^*}\right)\frac{\dot{L}^*}{L^*}$$
$$+ \left(\frac{K^*}{Q^*}\frac{\partial Q^*}{\partial K^*}\right)\frac{\dot{K}^*}{K^*} + \frac{1}{Q^*}\frac{\partial Q^*}{\partial t} \quad .$$

It is straightforward to see the conditions under which observed $\dot{Q}/Q \neq$ unobserved $\dot{Q}^*/Q^*$. From (49) and (50) we have

$$(54)\quad \frac{\partial\psi}{\partial Q_1} = \frac{1}{\lambda_{10}}\frac{\partial\psi^*}{\partial Q_1^*} \text{ and } \frac{\partial\psi}{\partial Q_2} = \frac{1}{\lambda_{20}}\frac{\partial\psi^*}{\partial Q_2^*} \quad ,$$

so that

$$(55)\quad \left(\frac{\partial\psi}{\partial Q_1} - \frac{\partial\psi}{\partial Q_2}\right) \gtreqless 0 \text{ if and only if } \frac{\partial\psi^*}{\partial Q_1^*} \Big/ \frac{\partial\psi^*}{\partial Q_2^*} \gtreqless \lambda_{10}/\lambda_{20} \quad .$$

In words, if *relative* base year prices do not correctly reflect the corresponding marginal rate of transformation[36] of the two types of *physical* output at the particular values of Q_1^* and Q_2^*, then $\dot{Q}/Q$ may not equal $\dot{Q}^*/Q^*$. Thus, for example, if $\partial\psi^*/\partial Q_1^* \div \partial\psi^*/\partial Q_2^* > \lambda_{10}/\lambda_{20}$ *and* $\dot{Q}_1^*/Q_1^* > \dot{Q}_2^*/Q_2^*$, then the "relatively undervalued (or underpriced)" output type has a larger growth rate than the "relatively overvalued" type of output. In this case, $\dot{Q}^*/Q^* > \dot{Q}/Q$, and it is natural to state that the quality of aggregate output has "improved" over and above that indicated by observed relative base year output prices. *In this sense,*

$$(56)\quad \left(\frac{Q_1 Q_2}{Q^* Q}\right)\left(\frac{\partial\psi}{\partial Q_1} - \frac{\partial\psi}{\partial Q_2}\right)\left(\frac{\dot{Q}_1}{Q_1} - \frac{\dot{Q}_2}{Q_2}\right)$$

is a "quality change expression for output". Indeed, the sign of the coefficient of the observed growth rate difference $(\dot{Q}_1/Q_1 - \dot{Q}_2/Q_2)$ in equation (53) would reveal which, if either, output is "relatively undervalued". Therefore, if it is incorrectly assumed[37] that $\dot{Q}^*/Q^* = \dot{Q}/Q$ in any of the fundamental productivity growth rate models, then the *negative* of the "quality change expression for output" (56) gets lumped with the residual.

Finally, the results of this section were obtained for the special case of two types of physical output. However, it is certainly possible to generalize the results by a procedure completely analogous to that used in Section B.1. Again, statistical observations on output types are usually available in terms of classification totals and subtotals, all measured in constant dollars. Then the manipulations of Section B.2 are applicable. These manipulations yield "output quality change terms" in a form convenient for empirical analysis and economic interpretation.

[36] The reader may prefer to think of the underlying production function of the representative establishment in implicit form; namely,

$$G(Q^*/E, L^*/E, K^*/E; t) \equiv [Q^*/E - f(L^*/E, K^*/E; t)] = 0 \ ,$$

where

$$Q^* = \psi^*(Q_1^*, Q_2^*) \ .$$

[37] See Section 3.2 for an explanation of the empirical problems of applying the "quality change expression" in the framework of this Study.

APPENDIX C

SOME TREND GROWTH RATE FORMULAS

The main purpose of this Appendix is to provide a convenient reference for the derivation of certain trend growth rate formulas used in this Study. There is also an analysis of the comparative properties of the two basic methods for calculating the trend growth rates.

C.1 The Terminal-Year Method

In this Study we are dealing with annual observations, and time t is treated as a discrete annual variable. Let X_0 represent the initial observation on a variable for a particular time period. Let X_T represent the terminal observation, where T is a positive integer and the time period is composed of T years. Then the average annual compound trend growth rate of the variable X over the time period is equal to $100x$ per cent, where

$$(1) \quad X_T = X_0 (1+x)^T \quad .$$

The growth rate, $100x$ per cent, can be obtained directly from compound growth rate tables, given X_T/X_0 and T. This is the so-called "terminal-year method" of calculating trend growth rates. Similarly, suppose that

$$(2) \quad Y_T = Y_0 (1+y)^T .$$

The first problem of this section is to derive the formula for calculating the corresponding trend growth rate of the variable Z, where

$$(3) \quad Z_0 = Y_0/X_0 \quad \text{and} \quad Z_T = Y_T/X_T \quad ,$$

when the individual growth rates $100x$ and $100y$ are *known*.[1] Using relations (1), (2), and (3), it is found that

$$Z_T = \frac{Y_T}{X_T} = \frac{Y_0(1+y)^T}{X_0(1+x)^T} = Z_0 \left(\frac{1+y}{1+x}\right)^T = Z_0 \left(\frac{100+100y}{100+100x}\right)^T$$

$$= Z_0 \left[\frac{100 + 100\left(\dfrac{100+100y}{100+100x}\right) - 100}{100}\right]^T$$

$$= Z_0 \left(\frac{100+100z}{100}\right)^T = Z_0(1+z)^T \quad ,$$

[1] For example, we may know the growth rate of "total output" and the growth rate of "total man-hours employed", from which we may wish to calculate the "labour productivity" growth rate.

so the required trend growth rate is $100z$ per cent, where

$$(4) \quad 100z = 100\left(\frac{100+100y}{100+100x}\right) - 100 \quad .$$

Note that $100z$ could also be expressed as

$$(5) \quad 100z = \frac{100(100y-100x)}{100+100x} = \frac{100y-100x}{1+x} \quad ,$$

so that

$$(6) \quad 100z \simeq 100y - 100x$$

when x is close to zero. The accuracy of the approximation (6) is independent of the magnitude of the percentage growth rate of the numerator in the original ratio.

The second problem is to derive the formula for calculating the trend growth rate of the variable W, where

$$(7) \quad W_0 = X_0 Y_0 \quad \text{and} \quad W_T = X_T Y_T$$

when $100x$ and $100y$ are *known*.[2] Now using relations (1), (2) and (7), we find that

$$\begin{aligned} W_T &= X_T Y_T = X_0 Y_0 (1+x)^T (1+y)^T = W_0 [(1+x)(1+y)]^T \\ &= W_0 \left[\frac{(100+100x)(100+100y)}{100^2}\right]^T \\ &= W_0 \left[\frac{100 + \dfrac{(100+100x)(100+100y)}{100} - 100}{100}\right]^T \\ &= W_0 \left(\frac{100+100w}{100}\right)^T = W_0 (1+w)^T \quad , \end{aligned}$$

and the required trend growth rate is $100w$ per cent, where

$$(8) \quad 100w = \frac{(100+100x)(100+100y)}{100} - 100 \quad .$$

Note that $100w$ could also be expressed as

$$(9) \quad 100w = \frac{100(100x+100y+100xy)}{100} = 100x + 100y + 100xy$$

so that

$$(10) \quad 100w \simeq 100x + 100y$$

[2] For example, we may know the growth rate of "total number employed" and the growth rate of "average hours worked", from which we may wish to calculate the growth rate of "total man-hours employed".

when either x or y or, *a fortiori*, both x and y are close to zero. Moreover, it is evident that formula (8) is easily generalized, so that, e.g.,

$$(11)\quad 100w^* = \frac{(100+100x)(100+100y)(100+100z)}{100^2} - 100$$

when, say, $W^* = XYZ$ and $100x$, $100y$, $100z$ are the individual percentage growth rates.

C.2 The Least-Squares Fit Method

Let X_t represent the t-th annual observation on the variable X for $t = 0,1,...,T$. Suppose that X is growing at *approximately* a constant annual rate of $100x$ per cent over the time period designated. Then

$$(12)\quad X_t = X_0(1+x)^t u_t \qquad (t = 0,1,\ldots,T)$$

where u_t is the random disturbance term at time t. Taking the logarithm of both sides of (12) yields

$$(13)\quad \log X_t = \log X_0 + t\log(1+x) + \log u_t \quad .$$

The problem is to estimate $100x$ or just x from the observations. The least-squares estimate of $\log(1+x)$ is[3]

$$(14)\quad \widehat{\log(1+x)} = (\sum_t t\log X_t)/(\sum_t t^2)$$

and x is estimated as

$$(15)\quad \hat{x} = \text{antilog}[\widehat{\log(1+x)}] - 1 \quad .$$

This is the so-called "least-squares fit method" of calculating (or estimating) trend growth rates. Similarly, suppose that

$$(16)\quad Y_t = Y_0(1+y)^t v_t \quad \text{for all } t,$$

$$(17)\quad \widehat{\log(1+y)} = (\sum_t t\log Y_t)/(\sum_t t^2)$$

and the percentage annual growth rate $100y$ or just y is estimated as

$$(18)\quad \hat{y} = \text{antilog}[\widehat{\log(1+y)}] - 1 \quad .$$

Now, analogous to the first problem of Section C.1, consider the growth variable Z, where

$$(19)\quad Z_t = Y_t/X_t \quad \text{for all } t,$$

and suppose that

$$(20)\quad Z_t = Z_0(1+z)^t \epsilon_t$$

[3]Without loss of generality, we have redefined the origin of time at the middle of the time period, so that t is rewritten as $t = (-\frac{1}{2}T, \ldots, -1, 0, 1, \ldots, \frac{1}{2}T)$ and $\sum_t t$, so defined, equals zero.

where ϵ_t is the random disturbance term at time t. Then the annual growth rate of the variable Z, as estimated by the "least-squares fit method", is simply[4]

$$\begin{aligned}
\hat{z} &= \text{antilog}[\widehat{\log(1+z)}] - 1 \\
&= \text{antilog}[(\sum_t t\log Z_t)/(\sum_t t^2)] - 1 \\
&= \text{antilog}[(\sum_t t\log Y_t)/(\sum_t t^2) - (\sum_t t\log X_t)/(\sum_t t^2)] - 1 \\
&= \text{antilog}[\widehat{\log(1+y)} - \widehat{\log(1+x)}] - 1 \\
&= \text{antilog}[\log(\hat{y}+1) - \log(\hat{x}+1)] - 1 \\
&= \text{antilog}[\log(\hat{y}+1)/(\hat{x}+1)] - 1 \\
&= \frac{(\hat{y}+1)}{(\hat{x}+1)} - 1.
\end{aligned}$$

Evidently, the percentage annual growth rate of the variable $Z = Y/X$, as estimated by the "least-squares fit method", can also be obtained *directly* from the correspondingly estimated individual percentage growth rates $100\hat{x}$ and $100\hat{y}$ by the familiar formula

$$(21) \quad 100\hat{z} = 100\left(\frac{100+100\hat{y}}{100+100\hat{x}}\right) - 100 \quad .$$

Similarly, the percentage annual growth rate of the variable $W = XY$, as estimated by the "least-squares fit method", becomes

$$(22) \quad 100\hat{w} = \frac{(100+100\hat{x})(100+100\hat{y})}{100} - 100 \quad .$$

[4]Using relations (19), (17), (14), (18), (15) and the well-known properties of logarithms.

APPENDIX D

NOTES ON SOURCES OF STATISTICAL DATA

The main purpose of this Appendix is to state the sources and adaptations of the statistical data used in the various analyses of this Study. It should be clear by now that the emphasis in this Study is *not* on the development of new statistical time series. However, some such development was reluctantly undertaken when needed, in order to complete the two-digit disaggregation level of the analyses for the two time periods. It is believed that the developed and adapted time series are fair approximations for the purposes of *trend* growth analyses. The growth rates calculated from these series should be regarded as preliminary, pending the publication of revised and more complete statistical data.

The Appendix also provides the *individual* growth rates used to calculate the various growth rates of the ratios of variables used in this Study. Thus the reader could calculate the growth rates of any other ratios of the individual variables that might be desired.

D.1. The Statistical Data Sources

Each of the sources and adaptations of the statistical data used in both the productivity analyses and the resource shift analyses[1] is now briefly stated in turn. The reader is referred directly to the original sources (usually DBS publications) for a more complete discussion.[2]

OUTPUT DATA – The 1948 S.I.C. data for the period 1947-56 are from DBS 61-506, *Indexes of Real Domestic Product by Industry*, Table 2, pp. 36-40. The 1960 S.I.C. data for the period 1957-67 are from *ibid.*, Table 1, pp. 21-28. The latter does not provide 1957 index data for "Wood products", "Primary metals", "Metal fabricating" and "Machinery industries". These data were developed by a special weighted calculation from the major corresponding 1948 S.I.C. components of "Wood products", "Iron and steel products" and "Non-ferrous metal products", shifting the Table 2 indexes to a 1961=100 base. Thus the output growth rates, 1957-67, of the four mentioned 1960 S.I.C. industries should be regarded as approximations. In fact, all the 1960 S.I.C. output indexes for the period 1961-67, as shown in Table 1, are preliminary and subject to revision.

[1] The statistical data used in the two types of analyses are the same, unless indicated otherwise.

[2] A thorough discussion of many of the 1948 S.I.C. data sources can also be found in Lithwick, Post and Rymes, *op. cit.*, pp. 200-248.

FIXED CAPITAL STOCK DATA – The 1948 S.I.C. data for the period 1947-56 are basically from DBS 13-522, *Fixed Capital Flows and Stocks, Manufacturing,* various tables, pp. A–1 to A–112. This publication gives data, e.g., for "*Combined* tobacco and tobacco products, rubber products, and leather products industries". The required data for each of the component two-digit industries were developed[3] on the basis of relative values of their "buildings and equipment" stock in 1947 (see Department of National Revenue, *Taxation Statistics, 1949*) used to allocate the pre-1947 historical capital formation series for the combined industries, together with the post-1946 series for the three individual industries. Similarly, the required capital stock data were developed for "Non-ferrous metal products", "Electrical apparatus and supplies", "Non-metallic mineral products" and "Petroleum and coal products". All 1948 S.I.C. capital stock growth rates are based on Set I *fixed* life assumptions, constant 1949 dollars, and mid-year stocks. The net stock measure is one of *straight-line depreciation*.[4] The stock of "machinery and equipment" used in this Study includes that of "capital items charged to operating expenses". The stock of "buildings and structures" coincides with that listed as "construction". It should be noted that the price index for non-residential construction has recently been revised, but the impact of the revision is only apparent from 1957 on. (See DBS 11-003, *Canadian Statistical Review,* January 1970, Table 8, p. 110.) The 1960 S.I.C. capital stock growth rates do use the revised price index for construction.

The 1960 S.I.C. data for 1957-67 on "Combined tobacco, rubber, and leather industries", "Textile products", "Paper products", "Printing and publishing", "Transportation equipment" and "Chemicals and products" were assumed to coincide with their 1948 S.I.C. counterparts. (See DBS 11-003, *op. cit.,* February 1970, p. 4). The required unpublished data are available as an extension of the DBS 13-522, *op. cit.,* series. (See DBS 11-003, *op. cit.,* February 1970, p. 5.) The data for certain other industries[5] were developed by allocating the pre-1948 historical capital formation series (1948 S.I.C.) on the basis of relative values of the major corresponding industries' "buildings and equipment" stock in 1947[6] and then using the perpetual inventory method with the unpublished 1960 S.I.C. capital formation series which are available for 1948 on. (See, again, DBS 11-033, *op. cit.,* February 1970, p. 6.) The required capital stock data for the remaining industries were developed by the perpetual inventory method, using the pre-1948 (1948 S.I.C.) historical capital formation series[7] and the 1960 S.I.C. series for 1948 on. All the

3 That is, using the perpetual inventory method.

4 Capital stock data based on more realistic survival curves and depreciation formulas are not yet available. See the discussion in Zvi Griliches, "Capital Stock in Investment Functions: Some Problems of Concept and Measurement", in C. Christ, ed., *Measurement in Economics* (Stanford: Stanford University Press, 1963), pp. 115-137.

5 Namely, "Wood products", "Furniture and fixtures", "Knitting mills", "Clothing", "Primary metals", "Metal fabricating", "Machinery industries", "Non-metallic mineral products", and "Petroleum and coal products".

6 As observed in Department of National Revenue, *Taxation Statistics, 1949.*

7 Except for "Electrical products", where the data were obtained by splitting off "Non-ferrous metal products" according to relative 1947 values as indicated in *Taxation Statistics, op. cit.*

1960 S.I.C. capital stock growth rates are again based on Set I fixed lives, mid-year stocks, straight-line depreciation, but are in constant 1961 dollars.

Finally, the fixed capital stock growth rates used in the resource shift analyses of Chapter 6 are the simple average of the net stock and gross stock measured rates of growth[8] as derived from the above sources for both time periods.

NUMBER EMPLOYED DATA – The 1948 S.I.C. data for 1947-56 are all from DBS 31-201, *General Review of the Manufacturing Industries of Canada*, various annual issues. This includes the data for male and female production workers, and male and female nonproduction employees (or "administrative and office employees"). The 1960 S.I.C. growth rates for the period 1957-67 were formed as a weighted average of the 1957-61 growth rates and the 1961-67 growth rates[9] because of a statistical discontinuity in the year 1961. The 1957-61 data are from DBS 31-201, *op. cit.*, 1961, and the 1961-67 data are from DBS 31-203, *Manufacturing Industries of Canada, Section A, Summary for Canada, 1967.* Only "total employees" are used in the 1960 S.I.C. analyses.

The total labour employed data used in the resource shift analyses for the period 1947-56 are from DBS 72-201, *Review of Employment and Payrolls*, various annual issues. Under the 1960 S.I.C., for the period 1957-67, the required data are from DBS 72-201, *Review of Employment and Average Weekly Wages and Salaries, 1957-67.* This series lacks the coverage property of the census of manufactures data but has the advantage of statistical continuity.

HOURS WORKED DATA – The 1948 S.I.C. data for 1947-56 are all from DBS 31-201, *op. cit.*, various annual issues. This again includes the data for male and female production workers, and male and female nonproduction employees. The 1960 S.I.C. data for 1957-67 are from DBS 72-202, *Review of Man-Hours and Hourly Earnings, 1957-67.* The present use of the latter publication for "total employees' average hours worked" implicitly assumes that the growth rate of "nonproduction employees' average hours worked" is equal to that of "production workers' average hours worked".

The hours worked data used in the resource shift analyses are all from DBS 72-202, *op. cit.*, various relevant annual issues.

NUMBER OF ESTABLISHMENTS DATA – The 1948 S.I.C. data for 1947-56 are from DBS 31-201, *op. cit.*, various annual issues. The 1960 S.I.C. growth rates were again formed as a weighted average of the 1957-61 growth rates and the 1961-67 growth rates, with weights equal to 0.4 and 0.6, respectively. The 1957-61 data are from DBS 31-201, *op. cit.*, and the 1961-67 data are from DBS 31-203, *op. cit.*, 1967.

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[8] In effect it is assumed that the two alternative growth rates should receive equal weight (see Section 3.1).

[9] The 1957-61 growth rate has a weight of 0.4; the 1961-67 growth rate has a weight of 0.6.

LABOUR FACTOR SHARE DATA – The 1948 S.I.C. data for the year 1949 are from DBS 13-513, *Supplement to the Inter-Industry Flow of Goods and Services, Canada, 1949,* Table 1. First, the manufacturing industries shown in the table were combined into the 17 major two-digit industry groups. Then the labour factor share for each industry[10] was set equal to the ratio of the summation of "wages, salaries, and supplementary labour income" *plus* one-half of "net income of unincorporated business", to "gross domestic product at factor cost".

The 1960 S.I.C. data for the year 1961 are from DBS 15-501, *The Input-Output Structure of the Canadian Economy, 1961, Volume 1,* Table 8. First, the manufacturing industries listed on pages 178-180 were combined into the 20 major two-digit industry groups. Then the labour factor share for each two-digit industry was set equal to the ratio of the summation of "wages and salaries" *plus* one-half of "net-income of unincorporated business" *to* the summation of "wages and salaries" *plus* one-half of "net income of unincorporated business" *plus* "surplus".[11]

INDUSTRY SHARE OF TOTAL OUTPUT DATA – The resource shift analyses of Chapter 6 require the ratios of industries' output to "Total Manufacturing" output. The 1948 S.I.C. data for the period 1947-56 were calculated from time series of "gross domestic product by industry of origin" in constant 1949 dollars, kindly supplied by the Industrial Output Section of the Dominion Bureau of Statistics. For the period 1957-67 (1960 S.I.C.), the required time series in constant 1961 dollars were developed from the industries' share of manufacturing output as observed in DBS 15-501, *op. cit.,* for the year 1961, combined with the real output indexes (1961=100) for the period 1957-67 from DBS 61-506, *op. cit.*

D.2. Some Statistical Tables

The following four tables are largely self-explanatory. Taken in conjunction with Tables 2-1 and 2-2, they provide all the basic growth rates and related data used to obtain the principal analytical results of this Study.

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[10] Except for "Non-ferrous metal products", where a special calculation was required on the basis of the census of manufactures data for "total earnings" and the "gross domestic product by industry of origin" in 1949 dollars supplied by the Industrial Output Section of the Dominion Bureau of Statistics.

[11] See DBS 15-501, *op. cit.,* pp. 29-33, for the defence of this procedure for calculating the required labour factor share data.

APPENDIX TABLE D-1

CAPITAL STOCK DATA USED FOR PRODUCTIVITY ANALYSIS
(Average annual growth rates from 1947 to 1956)

Industry (1948 S.I.C.)	k^g	k^n	m^g	s^g	m^n	s^n
Total Manufacturing	4.64	6.06	6.70	2.96	8.89	3.61
Food and beverages	4.58	5.98	5.54	3.54	7.92	3.98
Tobacco products	5.76	5.84	9.70	3.10	12.12	2.33
Rubber products	3.99	3.64	7.06	1.88	8.01	0.81
Leather products	1.14	0.29	1.43	0.99	1.87	−0.42
Textile products	2.83	3.71	4.61	0.47	5.10	1.43
Clothing	2.78	1.64	7.07	0.07	8.01	−3.57
Wood products	2.76	3.72	5.61	0.63	9.78	−1.30
Paper products	4.70	6.98	8.11	2.46	13.66	2.23
Printing and publishing	3.11	5.28	3.35	2.77	6.42	3.62
Iron and steel products	4.89	6.11	7.12	2.48	7.54	4.27
Transportation equipment	2.03	5.03	4.66	−0.14	6.91	3.17
Non-ferrous metal products	4.23	3.58	4.22	4.25	3.73	3.47
Electrical apparatus and supplies .	6.43	5.88	7.10	5.68	6.73	5.08
Non-metallic mineral products ...	8.70	11.61	15.90	4.87	19.92	6.61
Petroleum and coal products	8.17	11.40	8.91	7.93	12.64	11.00
Chemicals and products	7.25	9.40	13.87	3.92	16.94	4.65
Miscellaneous manufacturing	3.70	2.91	5.60	2.59	5.70	1.24

Note: For definition of variables, see Section 3.2.

Source: Based on data from Dominion Bureau of Statistics, as described in Section D.1.

APPENDIX TABLE D-2

LABOUR MAN-HOUR DATA USED FOR PRODUCTIVITY ANALYSIS
(Average annual growth rates from 1947 to 1956)

Industry (1948 S.I.C.)	n	p	n^m	n^f	p^m	p^f
Total Manufacturing	4.75	0.88	4.78	4.48	0.97	−0.08
Food and beverages	1.29	0.42	0.96	1.95	0.01	0.69
Tobacco products	−2.29	−1.26	−2.38	−2.14	−2.01	−1.95
Rubber products	3.77	−1.53	4.06	3.13	−0.76	−3.85
Leather products	−0.74	−1.72	−1.15	0.20	−2.57	−0.52
Textile products	5.98	−1.68	6.27	5.30	−1.16	−2.73
Clothing	−1.00	0.23	−1.41	−0.30	−0.81	0.81
Wood products	2.57	0.71	2.32	3.24	0.81	−0.91
Paper products	3.63	1.11	3.80	3.62	1.60	−0.74
Printing and publishing	5.76	1.99	4.98	7.19	2.60	−0.39
Iron and steel products	4.96	0.97	5.46	3.62	0.85	−1.79
Transportation equipment	9.72	1.62	10.09	8.27	1.48	1.26
Non-ferrous metal products	5.01	1.73	5.30	4.21	1.88	−2.04
Electrical apparatus and supplies .	9.90	3.48	10.90	7.48	3.61	2.40
Non-metallic mineral products ...	8.50	3.64	8.79	7.48	3.77	−1.54
Petroleum and coal products	9.52	1.04	9.46	9.92	0.86	−0.95
Chemicals and products	6.16	1.37	6.65	5.80	1.99	−0.64
Miscellaneous manufacturing	6.32	2.73	7.08	6.00	3.87	2.13

Note: For definition of variables, see Section 3.2.

Source: Based on data from Dominion Bureau of Statistics, as described in Section D.1.

APPENDIX TABLE D-3
DATA USED FOR RESOURCE SHIFT ANALYSIS, 1947-56

Industry (1948 S.I.C.)	q	k	l	α	Q_i/Q
Total Manufacturing	5.52	5.34	1.57	0.67	1.0000
Food and beverages	3.56	5.24	0.99	0.63	0.1342
Tobacco products	5.60	5.80	−1.19	0.62	0.0091
Rubber products	2.44	3.82	−0.68	0.79	0.0190
Leather products	0.70	0.72	−1.90	0.79	0.0180
Textile products	3.16	3.20	−1.23	0.67	0.0543
Clothing	3.33	1.90	0.24	0.77	0.0596
Wood products	4.31	3.02	0.76	0.74	0.0772
Paper products	4.88	5.85	1.06	0.53	0.0901
Printing and publishing	7.28	4.15	2.31	0.76	0.0451
Iron and steel products	5.66	5.51	1.19	0.70	0.1487
Transportation equipment	5.87	3.54	3.24	0.69	0.1006
Non-ferrous metal products	4.70	3.92	3.15	0.50	0.0544
Electrical apparatus and supplies	9.18	6.21	5.19	0.68	0.0588
Non-metallic mineral products	9.18	10.21	3.50	0.57	0.0317
Petroleum and coal products	12.08	9.82	3.91	0.44	0.0219
Chemicals and products	8.70	8.30	2.81	0.55	0.0558
Miscellaneous manufacturing	10.47	3.30	2.42	0.72	0.0217

Note: For definition of variables, see Sections 2.2 and 6.2.
Source: Based on data from Dominion Bureau of Statistics, as described in Section D.1.

APPENDIX TABLE D-4
DATA USED FOR RESOURCE SHIFT ANALYSIS, 1957-67

Industry (1960 S.I.C.)	q	k	l	α	Q_i/Q
Total Manufacturing	5.39	4.64	1.56	0.69	1.0000
Food and beverages	4.64	4.71	0.94	0.67	0.1330
Tobacco products	4.40	6.40	−1.00	0.60	0.0073
Rubber products	3.57	3.38	1.23	0.67	0.0175
Leather products	1.59	0.65	−0.43	0.95	0.0104
Textile products	6.87	2.27	0.87	0.73	0.0358
Knitting mills	5.60	0.93	−0.39	0.86	0.0075
Clothing	2.74	−0.02	0.64	0.89	0.0321
Wood products	4.32	3.64	0.60	0.80	0.0419
Furniture and fixtures	5.76	4.36	1.95	0.80	0.0182
Paper products	3.85	5.41	1.63	0.54	0.0997
Printing and publishing	4.04	4.09	1.34	0.78	0.0509
Primary metals	4.68	5.06	1.40	0.64	0.0825
Metal fabricating	4.89	3.95	1.91	0.76	0.0800
Machinery industries	8.00	4.00	3.14	0.74	0.0474
Transportation equipment	6.09	5.54	1.85	0.76	0.1097
Electrical products	7.64	3.71	2.49	0.78	0.0680
Non-metallic mineral products	4.39	4.55	1.89	0.66	0.0331
Petroleum and coal products	5.18	3.57	−0.11	0.33	0.0257
Chemicals and products	6.71	6.11	1.59	0.55	0.0677
Miscellaneous manufacturing	7.16	7.04	3.77	0.73	0.0318

Note: For definition of variables, see Sections 2.2 and 6.2.
Source: Based on data from Dominion Bureau of Statistics, as described in Section D.1.

LIST OF TABLES

PUBLICATIONS OF ECONOMIC COUNCIL OF CANADA

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PUBLICATIONS DU CONSEIL ÉCONOMIQUE DU CANADA

Annual Reviews	Author—Auteur	Exposés annuels
First Annual Review: Economic Goals for Canada to 1970 (EC21-1/1964, $3.50)	Council – Conseil	Premier exposé annuel: Objectifs économiques du Canada pour 1970 (EC21-1/1964F, $3.50)
Second Annual Review: Towards Sustained and Balanced Economic Growth (EC21-1/1965, $2.75)	Council – Conseil	Deuxième exposé annuel: Vers une croissance économique équilibrée et soutenue (EC21-1/1965F, $2.75)
Third Annual Review: Prices, Productivity and Employment (EC21-1/1966, $2.75)	Council – Conseil	Troisième exposé annuel: Les prix, la productivité et l'emploi (EC21-1/1966F, $2.75)
Fourth Annual Review: The Canadian Economy from the 1960's to the 1970's (EC21-1/1967, $2.75)	Council – Conseil	Quatrième exposé annuel: L'économie canadienne des années 1960 aux années 1970 (EC21-1/1967F, $2.75)
Fifth Annual Review: The Challenge of Growth and Change (EC21-1/1968, $2.75)	Council – Conseil	Cinquième exposé annuel: Défi posé par la croissance et le changement (EC21-1/1968F, $2.75)
Sixth Annual Review: Perspective 1975 (EC21-1/1969, $2.75)	Council – Conseil	Sixième exposé annuel: Perspectives 1975 (EC21-1/1969F, $2.75)
Seventh Annual Review: Patterns of Growth (EC21-1/1970, $2.50)	Council – Conseil	Septième exposé annuel: Les diverses formes de la croissance (EC21-1/1970F, $2.50)
Performance and Potential: Mid-1950's to Mid-1970's (EC21-1/1970-1, $1.50)	Council – Conseil	La tenue et le potentiel de l'économie: du milieu des années 1950 au milieu des années 1970 (EC21-1/1970-1F, $1.50)
Eighth Annual Review: Design for Decision-Making – An Application to Human Resources Policies (EC21-1/1971, $3.00)	Council – Conseil	Huitième exposé annuel: L'État et la prise des décisions—Application aux politiques de développement des ressources humaines (EC21-1/1971F, $3.00)
Performance and Potential: 1971 (EC21-1/1971-1, $1.50)	Council-Conseil	La tenue et le potentiel de l'économie: 1971 (EC21-1/1971-1F, $1.50)

Staff Studies	Author—Auteur	Études preparées par le personnel
1. Population and Labour Force Projections to 1970 (EC22-1/1, $.75)	Frank T. Denton Yoshiko Kasahara Sylvia Ostry	1. Projections de la population et de la main-d'œuvre jusqu'à 1970 (EC22-1/1F, $.75)
2. Potential Output, 1946 to 1970 (EC22-2/2, $1.00)	B. J. Drabble	2. Potentiel de production, 1946 à 1970 (EC22-1/2F, $1.00)
3. An Analysis of Post-War Unemployment (EC22-1/3, $.75)	Frank T. Denton Sylvia Ostry	3. Une analyse du chômage depuis la fin de la guerre (EC22-1/3F, $.75)
4. Housing Demand to 1970 (EC22-1/4, $.75)	Wolfgang M. Illing	4. Demande d'habitations pour 1970 (EC22-1/4F, $.75)
5. Business Investment to 1970 (EC22-1/5, $1.00)	Derek A. White	5. Investissements privés pour 1970 (EC22-1/5F, $1.00)
6. Special Survey of Longer Range Investment Outlook and Planning in Business (EC22-1/6, $.25)	B. A. Keys	6. Enquête spéciale sur les intentions et la programmation à moyen terme des entreprises privées (EC22-1/6F, $.25)
7. Canada and World Trade (EC22-1/7, $.50)	M. G. Clark	7. Le Canada et le commerce mondial (EC22-1/7F, $.50)
8. Export Projections to 1970 (EC22-1/8, $.75)	J. R. Downs	8. Projections des exportations jusqu'à 1970 (EC22-1/8F, $.75)
9. Federal Tax Revenues at Potential Output, 1960 and 1970 (EC22-1/9, $.50)	D. J. Daly	9. Rendement des impôts fédéraux dans l'hypothèse de la réalisation du potentiel de production, 1960 et 1970 (EC22-1/9F, $.50)
10. National Saving at Potential Output to 1970 (EC22-1/10, $.50)	Frank W. Wildgen	10. L'épargne nationale dans l'hypothèse de la réalisation du potentiel de production pour 1970 (EC22-1/10F, $.50)
11. Changes in Agriculture to 1970 (EC22-1/11, $.50)	John Dawson	11. Changements dans le secteur agricole jusqu'à 1970 (EC22-1/11F, $.50)
12. The Contribution of Education to Economic Growth (EC22-1/12, $1.50)	Gordon W. Bertram	12. Apport de l'éducation à la croissance économique (EC22-1/12F, $1.50)
13. Internal Migration in Canada, 1921–1961 (EC22-1/13, $1.60)	Isabel B. Anderson	13. Migration à l'intérieur du Canada, 1921-1961 (EC22-1/13F, $1.60)
14. Interregional Disparities in Income (EC22-1/14, $1.75)	S. E. Chernick	14. Disparités interrégionales du revenu (EC22-1/14F, $1.75)

Staff Studies (*continued*)	Author—Auteur	Études préparées par le personnel (*suite*)
15. An Analysis of Interregional Differences in Manpower Utilization and Earnings (EC22-1/15, $1.00)	Frank T. Denton	15. Analyse des différences interrégionales dans l'utilisation de la main-d'œuvre et le revenu gagné (EC22-1/15F, $1.00)
17. Business Cycles in Canada (EC22-1/17, $2.25)	Derek A. White	17. Les cycles économiques au Canada (EC22-1/17F, $2.25)
18. Manpower Planning in Industry: A Case Study (EC22-1/18, $1.00)	B. A. Keys H. H. Wright	18. La planification de la main-d'œuvre dans l'industrie (Étude de cas) (EC22-1/18F, $1.00)
19. Population, Family, Household and Labour Force Growth to 1980 (EC22-1/19, $1.00)	Wolfgang M. Illing Yoshiko Kasahara Frank T. Denton M. V. George	19. La croissance de la population, du nombre de familles et de la main-d'œuvre, jusqu'en 1980 (EC22-1/19F, $1.00)
20. Enrolment in Schools and Universities, 1951-52 to 1975-76 (EC22-1/20, $1.25)	Wolfgang M. Illing Zoltan E. Zsigmond	20. Inscriptions aux écoles et aux universités, 1951-1952 à 1975-1976 (EC22-1/20F, $1.25)
21. Scale and Specialization in Canadian Manufacturing (EC22-1/21, $1.75)	D. J. Daly B. A. Keys E. J. Spence	21. Échelle et spécialisation dans l'industrie manufacturière canadienne (EC22-1/21F, $1.75)
22. Medium-Term Business Investment Outlook: 1967 Survey (EC22-1/22, $.35)	B. A. Keys	22. Perspectives à moyen terme des investissements des entreprises: Relevé de 1967 (EC22-1/22F, $.35)
23. Canadian Income Levels and Growth: An International Perspective (EC22-1/23, $2.25)	Dorothy Walters	
24. Canadian Agricultural Productivity (EC22-1/24, $1.50)	L. Auer	
25. Enrolment in Educational Institutions, by Province, 1951-52 to 1980-81 (EC22-1/25, $3.00)	Z. E. Zsigmond C. J. Wenaas	25. Inscriptions dans les institutions d'enseignement, par province, de 1951-1952 à 1980-1981 (EC22-1/25F, $3.00)
26. Personal Consumer Expenditures in Canada, 1926-75: Part 1 (EC22-1/26-1, $1.50)	Thomas T. Schweitzer	26. Dépenses personnelles de consommation au Canada, 1926-1975: Partie 1 (EC22-1/26-1F, $1.50)
Personal Consumer Expenditures in Canada, 1926-75: Part 2 (EC22-1/26-2, $1.50)	Thomas T. Schweitzer	Dépenses personnelles de consommation du Canada, 1926-1975: Partie 2 (EC22-1/26-2F, $1.50)

Staff Studies *(concluded)*	Author—Auteur	Études préparées par le personnel *(fin)*
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An Analysis of Canadian Manufacturing Productivity

Some Preliminary Results

by Harry H. Postner

Economic Council of Canada