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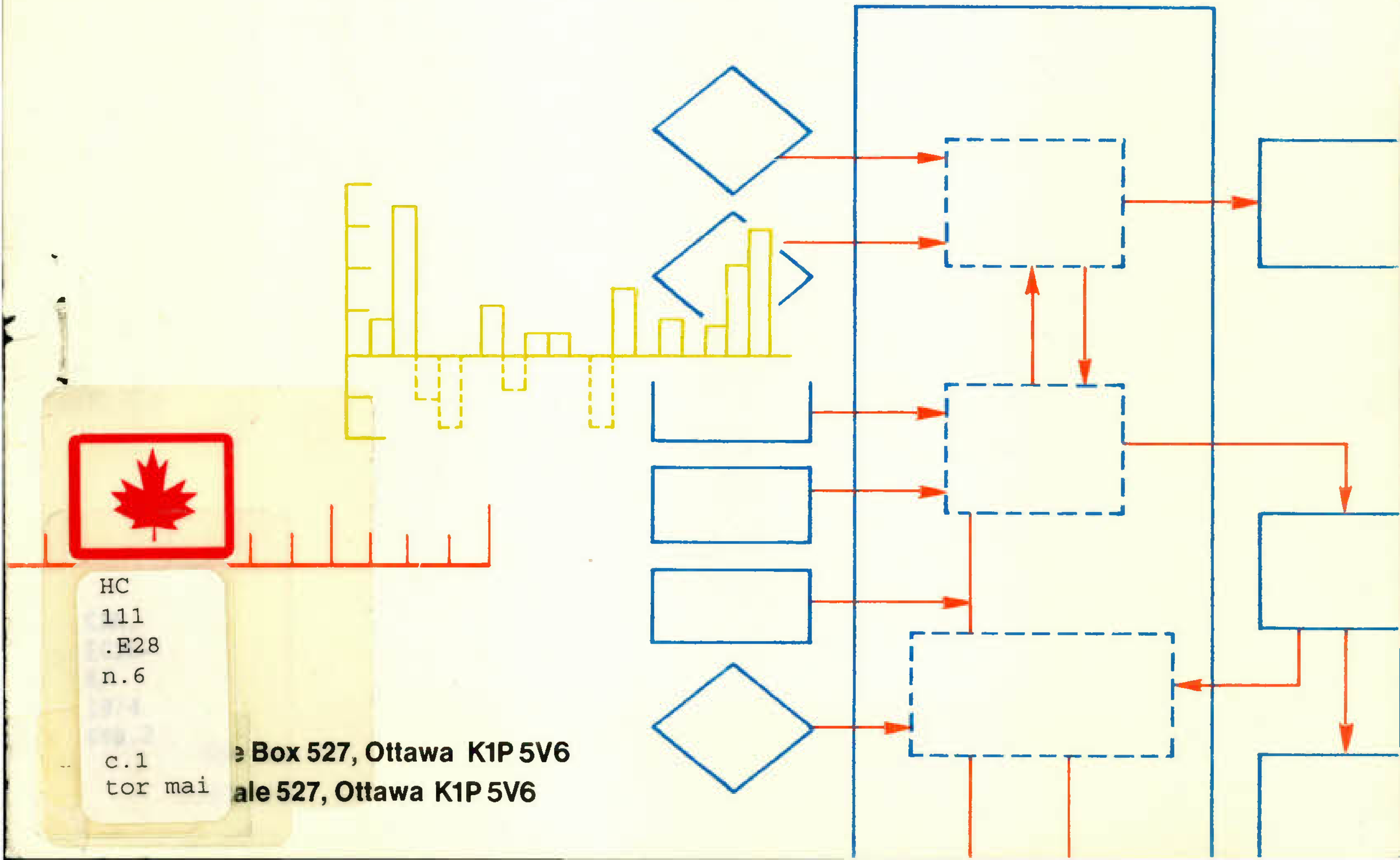

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DISCUSSION PAPER NO. 6

Social Indicators in Education: A Conceptual Framework

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January 1974

ABRÉGÉ

Les indicateurs sociaux dans l'enseignement: cadre théorique

par

Jeff Greenberg

Ce document constitue la première étape d'un projet de recherche pour l'élaboration et l'analyse d'indicateurs sociaux dans le domaine de l'enseignement. Ces indicateurs sont essentiellement fondés sur la constatation que les taux de persévérance ou d'achèvement des études ne sont pas suffisants pour mesurer les progrès de l'éducation. Comme le système d'enseignement attribue, à divers degrés, des qualités différentes à chaque individu, l'auteur rejette l'idée que les chiffres purs et simples sont la seule mesure utile des progrès de l'enseignement, pour se pencher plutôt sur divers aspects relatifs à la mesure de ces qualités. Compte tenu de cette prémisse, les indicateurs sociaux de l'enseignement peuvent être alors considérés comme les attributs ou les compétences acquises par une personne grâce au système d'enseignement.

Le document se divise en trois parties. La première se compose de quatre sections où l'on trouve une discussion du cadre général devant servir d'appui aux indicateurs d'enseignement. La seconde partie consiste en trois appendices, A, B, et C, qui donnent une liste des variables théoriques et des variables instrumentales utilisées pour les représenter dans les calculs empiriques. La dernière partie, composée des appendices D et E, présente une analyse critique du cadre théorique employé dans l'élaboration de ce projet.

ABSTRACT

Social Indicators in Education: A Conceptual Framework

by

Jeff Greenberg

This paper represents the first stage of a research project to develop and analyse social indicators in education. The basic premise upon which educational indicators are developed is that retention rates or level of completion rates are insufficient measures of educational progress. Because the education system imparts varying amounts of different qualities to each individual the emphasis is placed upon considerations of measurement of these qualities. Given this approach social indicators in education can then be considered as the attributes or skills that the individual internalizes as a result of the education system.

The paper consists of three parts. The first part is made up of the first four sections in which there is a discussion of the framework upon which the education indicators should be based. The second part which consists of Appendices A, B, and C, lists the theoretical variables and the proxies used to represent these for empirical purposes. The last part, Appendices D and E, is a discussion of the technical framework to be used in this empirical part of this project.

SOCIAL INDICATORS IN EDUCATION:
A CONCEPTUAL FRAMEWORK

Table of Contents

*I wish to thank David Henderson, Allan Maslove and Robin Rowley for their helpful comments.

Section 1

INTRODUCTION

In its *Eighth Annual Review*, the Economic Council of Canada stressed the importance of both economic and social indicators to measure the direction of change in the socio-economic climate in Canada. This paper is a first attempt to consider the primary education system as a major area of concern for which social indicators should be developed. What follows then is a preliminary discussion of what an education system is, what is to be measured as indicators of change in such a system and the first stages in introducing the empirical work.

The paper is divided into four sections and five appendices. The first section is an introduction into how we perceive social indicators and what they are meant to be.[1] In the second section we consider the problem of what the goals in education should be. It is in this section that we address ourselves to the problem of defining the education system and to the concept of equality of opportunity and equality of result within this framework. In section 3 there is a discussion of our definition of the education system and the framework we plan to use in the future. Also included in this section is a consideration of a taxonomy of educational inputs and outputs. In the last section we consider three possible avenues for the empirical research which is now under progress.

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[1] For a complete explanation of the definition of a social indicator, see D. W. Henderson [1973].

Economic growth has been considered for some time an essential step in the process of improving the lot of mankind. The preoccupation of economists with growth is certainly evident in the 1950's and 1960's if the professional literature[2] is any indication. Theories of growth such as that of the work of Rostow [1971] have monitored development as being tied either directly or indirectly to output per capita. But more than this, growth has been a principal recommendation by economists as a means of raising the standard of living[3] and of narrowing income differentials.[4] The constant thread throughout the literature is that changes in the standard of living or economic welfare of any system is synonymous with changes in output per capita; and output per capita is always based on some measure such as GNP or a derivative of this.

Measures such as GNP as originally conceptualized were not meant to be anything more than measures of market activity. They were not and are not, in broad terms, indicators of welfare, or even of economic welfare. They measure, on the one hand, payment for services rendered in the production of goods and services over a specified time period, and on the other hand, valuation of final goods and services at market prices. They are not measures of consumption, nor of wealth, nor of valuation of non-market-oriented production of goods

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[2] See Hahn and Mathews [1964], and Jorgenson and Griliches [1967], for good sources of references to this research.

[3] Hicks, J. R. [1969].

[4] Johnson, H. G. [1973].

and services.[5] For what they are, which is a measure of market activity, they are quite reasonable. However, if a society is concerned with a better yardstick for evaluating welfare or more specifically economic welfare, then certainly GNP-type data are not sufficient.

Two general frameworks have been advanced as solutions to the deficiencies which plague existing market activity measures as welfare indicators. The first, referred to as social accounting, is based upon expansion of the content and coverage of the existing accounts so that they incorporate activities that are now excluded, and expansion of the intermediate product nature of many expenditures which are now considered as final outputs. In general, the idea is to incorporate within a double entry bookkeeping system as many of the dimensions of welfare as can be conceptualized and measured. The second, referred to as social indicator research, is based upon devising a series of supplementary indicators which can be used to identify dimensions of welfare which are not or cannot be fitted within the framework of a market activity measure.[6] Common examples such as education retention rates, crime statistics, pollution counts, etc., certainly display a strong affinity toward the well-being of any society but represent data series which cannot, at present, be accommodated within the framework of the National Accounts.

With reference to social accounting, the work done by Tobin and Nordhaus [1972] is a good example. What they have attempted to do is expand the concept of GNP so

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[5] Of course an exception to this is imputed rent to homes.

[6] For a complete survey of social indicator research and definitions, see D. W. Henderson [1973].

that it reflects more closely the notion of economic welfare. To do this they have made adjustments in three general areas: reclassification of GNP expenditures as consumption, investment and intermediate products,[7] imputation for the services of consumer capital, for leisure and for the product of household work; correction for some of the externalities of urbanization.

Effectively, what they seem to be doing is trying to alleviate one of the principal shortcomings of GNP -- namely, that it is a measure of production, and not consumption. It is reasonably clear that the goal of life? economics is consumption and that production is simply an intermediate step in this process. The problem which Tobin and Nordhaus face then is creating a consumption measure from an index of production and including in that new measure those aspects which contribute to consumption not already considered in GNP.

Three issues stand out as being omitted from GNP statistics which should be considered as contributing to consumption. First, those goods and services that contribute to consumption which have no actual marketable value such as household production, leisure and all non-work activities, should be evaluated. Second, externalities which are a result of the developed state of our society such as urban crowding and pollution also contribute to consumption (both positively and negatively), and should be considered as contributing to welfare. Tobin and Nordhaus consider these two aspects and

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[7] For a definition of intermediate products, see Tobin and Nordhaus [1972], p. 5.

attempt to obtain measures of these to include with the augmented GNP data. And, thirdly, non-legal activities such as gambling, prostitution and drugs which are certainly considerable in their influence upon welfare, should be given some consideration.

Since the purpose of this discussion is simply to comment on the type of work being done in social accounting, a lengthy criticism of their work is not called for. However, a couple of brief comments should be made about the assumptions and the realism of the results. Specifically, to estimate the consumption value of work, non-work (household time), and leisure, a utility function is maximized subject to a time constraint (i.e., a 24-hour day). A number of problems arise because of this. First, work is considered a "bad" and something to be compensated for. That is, work satisfaction is not allowed for. One could take an extreme case in determining the value of satisfaction obtained from work, as Hawrylyshyn [1973] and Usher [1973] do by suggesting that this satisfaction could be equal to the market wage.[8] In this sense, there would be an addition to welfare equal to the value of the wage multiplied by the amount of work performed. Then, the satisfaction obtained from work is such that it is equal to compensation paid for performing the job. Certainly evaluation of work satisfaction at the market wage is absurd but it is a base from which comments can begin to flow. The traditional view put

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[8] Actually, this suggestion is made by Usher and Hawrylyshyn in the determination of the value of volunteer work.

forward that teachers and nurses are not underpaid because when we consider the satisfaction they are supposed to derive from their work, then the wage they receive is a fair one.[9] Whatever a proper evaluation for worker satisfaction is, it should be considered (which Tobin and Nordhaus have not done) because treating work as a "bad" is a throwback to the nineteenth and eighteenth centuries and is plainly wrong.

This notion of satisfaction derived from work also becomes important in the evaluation of non-work activities. For example, in Tobin and Nordhaus' work, the imputed wage for the evaluation of household time is considered to be the average hourly earnings for women. They choose this because "the majority of those keeping house are women".[10] Implied in this aggregation is the assumption of separability. That is, all men and women evaluate household work in the same way and therefore there is no loss in assuming them to be a homogeneous group. No allowance is made for differing imputed wages (or satisfaction) for different groups in spite of the fact that the estimates used for household work show that women spend only three times as much time at household work[11] than men. Certainly the assumption of separability is not valid on the grounds that the time spent by males in the household work is trivial.

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[9] Teachers' and nurses' associations (unions) have certainly destroyed this argument in recent years.

[10] Tobin and Nordhaus [1972], p. 45. A better choice for an imputed wage would be the market wage given to these performing comparable jobs.

[11] *Ibid.*, p. 43.

Secondly, the concept of marginalism is frequently employed. It is used to maximize the utility function from which welfare gains are derived[12] and to estimate the value of consumption[13] at some base-time period. More explicitly, the concept is used to indicate the principle that the individual can, on the margin, exchange leisure or non-market activity for market consumption. This concept conveys the notion of perfectly competitive markets with prices given to the individual who then chooses the number of hours he wishes to work. Tobin and Nordhaus seem to have ignored the inflexibility that exists in the choice of hours worked due to institutional constraints such as unions, collective agreements, and management goals. Certainly in the case of maximizing the utility function the inclusion of a constraint on hours worked should be considered.

These criticisms are not being made to indicate the relative merits of the work but are being made in order to keep in mind the realism of the results. Too often the conclusions of any work are evaluated without consideration of the assumptions. W. Leontief [1971], in his presidential address to the American Economics Association, addressed himself to this very point by stating:

> "By the time it comes to interpretation of the substantive *conclusions*, the assumptions on which the model has been based are easily forgotten. But it is precisely the empirical validity of these *assumptions* on which the usefulness of the entire exercise depends."[14]

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[12] *Ibid.*, p. 41.

[13] *Ibid.*, p. 39.

[14] Leontief [1971], p. 2.

Next, we look at social indicator work which we defined as the construction of socio-economic indicators used to identify dimensions of welfare. This approach takes its start as being different from social accounting because it recognizes the fact that many aspects which lend themselves to influencing welfare cannot at present be conceived as being comparable to measures derived from the market. To evaluate this type of work and identify how our own work on education fits into the scheme of social indicators, it seems best to devise a simple but general framework into which our work and that of others within the social indicator field can be incorporated.

Let us suppose that some overall index of welfare can be considered which we shall refer to as the Bliss Index (B). This index can be considered as a composite measure of performance of the entire socio-economic system. We shall further consider that it is made up of the relative weightings of the indices for the levels of output from each of n sub-systems of the entire social system such as the health, education, urban systems, hereafter referred to as S_d:

$$B = B(S_d) \qquad d = (1...D) \qquad (1.1)$$

The work done on quality of life estimates and on "happiness" indices based upon aggregative measures can usually be considered as concerned with equation 1.1. This type of research stresses the importance of the macro aspects of social indicators. Those pursuing this approach are concerned with the "grand scheme" and see the necessity of devising an overall macro theory. In a sense, they see problems of disaggregation of the outputs of social systems as being secondary in importance in terms of present research.

The indices of the outputs of the subsystems S_d can also be considered as composite measures of the system they represent. These outputs in turn can be considered to be composed of the various outputs, weighted appropriately, of the particular system they represent. For example, an overall indicator of a health system might be a weighted composition of various indices of mortality and morbidity. Such a relationship can be expressed as:

$$S_d = S_d(M_{df}) \qquad d = (1...D) \quad f = (1...F) \qquad (1.2)$$

where M_{df} represents any element of the set in which there are F elements.

Further, each M_{df} is determined by a technical relationship which relates the K inputs of that subsystem I_{kf} to output M_{df}:

$$M_{df} = M(I_{kf}) \qquad k = (1...K) \qquad (1.3)$$

As an example, suppose we hypothesize any subsystem d such as education. Then equation 1.3 represents an input-output relationship between the K inputs and some specific output, M_{df}, of the educational system such as achievement in language proficiency. Equation 1.2 would then be the aggregation of all the outputs of the education system into one aggregative indicator of education. Equation 1.1 would in turn be the further aggregation of the overall indicators of each subsystem such as health, education, urban indicators, etc., into a measure of welfare for the entire social system.

In effect, these three equations represent a multi-stage production function approach in which the outputs of one equation become the inputs of another equation.

To explain more clearly the operation of this model, we will compare this social indicator model with the economic concept of the determination of the GNE deflator. The GNE deflator, P, and the composition of P is made up by weighting the various R components of this index such as the consumer price index, P_c, and other component price indices P_r. This can be expressed as follows:

$$P = P(P_r) \qquad r = (1...c...R) \qquad (1.4)$$

Referring to the consumer price index as an example, P_c is made up by weighting the value of the various goods and services $(P_a \cdot G_a)$ according to the weights in the given basket of goods and services:

$$P_c = P_c(P_a \cdot G_a) \qquad a = (1...A) \qquad (1.5)$$

where P_a is the price of good G_a.[15]

Finally, the production of each good can then be represented by the production function for this good:

$$G_a = G_a(L, K) \qquad (1.6)$$

where L and K refer to human and non-human resources respectively.

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[15] Since the determination of P_a is not of interest to this model, we will simply assume it to be exogenously determined.

If we list the two models side by side, we can see the analogy more clearly:

$$B = B(S_d) \quad (1.1) \qquad P = P(P_r) \quad (1.4)$$

$$S_d = S_d(M_{df}) \quad (1.2) \qquad P_c = P_c(P_a \cdot G_a) \quad (1.5)$$

$$M_{df} = M_{df}(I_{kf}) \quad (1.3) \qquad G_a = (L, K) \quad (1.6)$$

Equation 1.3 which is the relationship between inputs and the various outputs of any subsystem such as education is analogous to the production of goods and services of equation 1.6. Equation 1.2 which is the relationship between the various outputs of a social subsystem and an overall indicator of this system is analogous to the determination of a price indicator for a certain basket of goods and services as seen in equation 1.5. Lastly, equation 1.1 depicts the determination of some overall quality of life indicator from the indicators of each subsystem and this is comparable to the determination of some overall price index from its component price index as seen in equation 1.4. It is from the model expressed by equation 1.1-1.3 that the various approaches to social indicator work can be examined.

In our own work we feel the necessity of beginning at the micro foundations. Using the above model, we feel that equation 1.3 is the important equation to begin with. It is at this level, the determination of how a subsystem such as education operates, that we feel we can make the most progress.

Section 2

OPPORTUNITY IN EDUCATION

Before proceeding into the analysis of how a system such as education operates, we feel that this research project should be put into a proper framework of its overriding social issue. The concern for the opportunity to receive an "equitable" income and a reasonable level of education has reached a level of prominence especially in the United States in both politics (the Johnson Administration's War on Poverty) and in research (Coleman [1966] and Jencks [1972]). The basic assumption involved in the War on Poverty seemed to be that equality of opportunity would achieve equality of results. Although Coleman does not make this same connection, he does point out that changing the school system alone would not equalize the opportunity. Jencks takes up the point that even if all children are provided with the cognitive skills, equality of income or even some movement towards it may not come about.

The first problem that must follow from these two major works is just what is meant by opportunity and result within the framework of an education system let alone equality of each. However, defining an education system represents a "Pandora's Box" since any explicit definition of an education system must open a multitude of avenues for further research and critical discussion. For the present, let us state that in our view the education system encompasses all those factors both in and out of school that influence the cognitive[16] and affective[17] development of the student during the years in which the student attends school. This is somewhat of a broad definition in that it includes factors outside the school as an integral part of the education system, namely home and outside environmental variables. By doing this we eliminate the possibility of educators controlling all those factors affecting the cognitive and affective development of the child. Opportunity within the context of the above framework can now be defined as the availability to the participants in the education system of relevant factors which come to bear in the production of education services. This definition of opportunity in the education system differs greatly from opportunity in the school. Clearly the difference occurs in the fact that the school represents only one vector of inputs into the education system.

The definition we have chosen and the reason for its choice will become clear later in this section. But for the present, we wish to point out that although there does not appear to be anything unusual about this broad definition, it does represent a marked departure from what is accepted in international circles. Both the OECD and UNESCO have recently adopted a definition of education which is stated as "organized and sustained instruction designed to communicate a combination

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[16] The cognitive domain is that area of education that deals with the recall and recognition of knowledge and the development of intellectual abilities and skills. (Bloom [1966], p. 7.)

[17] The affective domain is defined as the area that considers interests, attitudes and values, and the development of appreciations and adequate adjustment. (Bloom [1956], p. 7.)

of knowledge, skills and understanding, valuable for all the activities of life."[18] This definition relegates education to taking place only in the school and does not provide for the fact that learning in the broad sense as represented by experience outside the school is an important aspect in determining how the child performs in the school.

There is good reason for making this distinction about the various definitions of education and clinging to our broader one. If from the research we interpret that school variables are the most important single set of factors in the determination of cognitive and affective skills, then there is only minor loss in choosing the narrow rather than the broad definition. But if the converse appears to be true and the school inputs represent a relatively weaker influence than the home and environment, then opportunity in the school rather than in the school system represents a rather meaningless, if not faulty, concept for the decision-maker.

Within the context of the above definition of education and the opportunity of education, let us now proceed to discuss the results (or outputs) of such a system. Since, in general, one of the purposes of this research project is to take a disaggregated view of education, our discussion of results will centre around the distribution of them. The distribution of the results of education naturally ? reflect s the distribution of the inputs. Further, if the distribution of the results ~~are~~ is not in keeping with what society expects from its education system, then some interference with the operation of this system would seem to be necessary to alter the distribution of inputs.

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[18] OECD [1971], p. 5.

Equality of results is just such an imposed distribution which requires interference with the system to achieve this particular goal. On the other hand, an equitable distribution of the results of education (as we define it below) does not imply the imposition of a specific allocation of the outputs of education. It does imply a prior distribution of the inputs of the education system such that equality of opportunity is the only end goal. Within this context, equality of opportunity and an equitable distribution of results can be interpreted as allowing the individual to choose as much education as he or she wishes up to the limits of their capabilities and according to their desires.

Yet, by proposing equality of opportunity as a goal, we are not suggesting an equal distribution of each specific input. For example, if equality of opportunity is a goal as we have defined it, and if we do not feel it is justifiable that students be disadvantaged by their home background then our argument leads us to the conclusion that the schools and teachers variables should compensate for "disadvantaged" background. This would suggest an unequal distribution of these specific variables. On the other hand, individuals are a creation of their background and in no way do we wish to consider proposals to change this. Heterogeneity of individuals is an important contribution to our culture. It also contributes to the different motivations and capacities of individuals. Yet, there are obvious influencing aspects of the background of the individual which are deterents which should be compensated such as poverty, inadequate diets, and other facets such as these.

The situation of the working poor in the cities is also an unfortunate disadvantage which can hopefully be counter-influenced in the schools. These are simply examples which point out the difficulty of trying to provide counter-influences to some of the drawbacks of the home and environment while leaving intact the benefits derived from heterogeneity of backgrounds.

Effectively equality of opportunity implies that all barriers to entry to education are removed and the individual is free to choose, with some constraints, as much education as he wishes or is capable of mastering. Yet if we are not careful this definition of equality of opportunity can lead to somewhat undesirable conclusions. Suppose we hypothesize a system in which both home and school variables carry relatively equal ? weight and that unfavourable influences of the home and environment exist but can be offset by school variables. We could then imagine a world in which all students were removed from the home such that there would be no home or environment -- just school. Yet even if this "Brave New World" were possible to create, it is not clear that we would achieve homogeneity within the student body. Fortunately, in our society where the family remains an important institution such a solution, even if successful, would not be tolerable. Nevertheless, if equality of opportunity is an issue and if home, school and environment factors are important in the education process, then we can still derive interesting and feasible results. If we use the same assumptions about the influence of various factors and we wish to overcome the disadvantage of home and environment, then it could be achieved by exogenous increases in the school and teacher variables. In effect this implies that schools in locations which contain a large proportion of disadvantaged students should be changed so that they can offset these hindering influences. Whether this makes the schools better or worse is a subjective issue and not of importance. What is important is that the school system be flexible such that it can recognize this type of problem and deal with it. Only within the broad framework provided by the definitions of opportunity and education systems can an issue such as equality of opportunity make any sense.

Section 3

THE EDUCATION SYSTEM

The approach we have described earlier is to examine an education system as one of many social subsystems which influence the general welfare of the members of society. In an examination of this system we will assume that the production of educational outputs occurs within a closed system. The interaction of factors from other systems such as health and urban systems[19] will be treated as exogenous. This assumption is made for simplifying reasons only. In spite of the fact that it is unrealistic we will proceed with this assumption because to do otherwise would lead us to very large estimation problems later.

Our interest in the educational system is purely academic in that we are concerned with the technical operations of the system rather than evaluation. In this sense, we are concerned with the outputs of such a system and its relevant inputs.

An examination of the outputs of an education system is the study of the results of production by a service industry of a joint product. The jointness comes about from the fact that the output has two uses; one that is marketable and one that is not. This idea is often expressed as the investment and consumption aspects of education. That portion which is

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[19] See Maslove [1973] for a discussion of Urban Indicators.

considered investment is indirectly marketable in the sense that employers do not directly hire the attributes that an individual obtains from education but must hire the individual who possesses them. Embodied within this concept is the idea that employers implicitly assume that different quantities of the marketable services are embodied in the individual according to the level of schooling the individual has obtained. On the other hand, the consumption portion of education is not marketable and can only be used by the individual who owns it. This aspect of education can be considered as adding to the utility of the individual by his consumption of this service and also as a factor causing a shift in his tastes towards a different basket of goods and services.

Much of the literature dealing with the marketable part of the output of education has focused upon two areas. The first deals with the wages paid to labour with different quantities of the marketable services resulting from different levels of schooling.[20] Wages, income or earnings per worker when categorized by years of schooling is certainly positively related to levels of schooling completed. If we believe in the marginal productivity principle, then we argue that education increases the productive capacity of labour and the owner of this increased productivity receives payment for this service as a factor of production in the form of higher wages. This is the argument, albeit naively presented here, that human capital theorists have used to attribute the contribution made by education to growth in output.

Vs. Screening hypoth.

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[20] A good survey for this is Mincer [1970].

While this first research area emphasized how education led to increased productivity at a macro level, a second area of interest arose concerning qualitative differences among income earners with identical years of schooling, but with varying abilities and backgrounds (Taubman and Wales [1973], Griliches and Mason [1972]). Both of these two studies take into consideration the fact that differences in intelligence and background influence the income received. The importance of this research is that it considers as a variable the heterogeneity of backgrounds of individuals. But, while these studies take into consideration qualitative values such as I.Q., and family background in their influence upon wages, they still use years or levels of schooling as the prime condition for analysis.

Welch [1969] considers this point by conceptualizing the fact that it is not individuals with varying levels of schooling that are utilized in the production of goods and services, but the marketable attributes that the education system has imparted to them. Using a Lancaster [1966] approach, Welch treats the attributes of education as the end product where each level of schooling is made up of various quantities of these attributes. However, it is the individual who has control over the marketable attributes (services) and who receives compensation for their employment.

Because both the marketable and non-marketable outputs of an education system are not goods, but services, and because these services are not directly observable, the conceptualization of them represents a difficult problem. However, assuming we are able to conceptualize the outputs of education, we are still left with the task of finding appropriate output measures. The process of operationalizing the conceptual output typically involves collecting empirical data which are judged to be relevant to the conceptual output. The appropriateness of an empirical measure must be judged in terms of its apparent relevance to the conceptual output. In contrast, the conceptual output, which is basically derived from value judgments, is not subject to the same sort of analysis.

When discussing the measurement of educational outputs, we must give appropriate attention to the question of the use of a multiple versus single "overall" measure of educational output. Although a single measure possesses certain obvious advantages owing to its conceptual simplicity it is certainly ? ? unrealistic due to the computational difficulties and the misleading impression it leaves when considering the impact of any school system. With respect to the computation difficulties, the adding up of the measures of output requires the reduction of all the outputs to a common base through the use of a numeraire (such as pounds or dollars). Unfortunately, no such common dimension exists. — £s + $'s don't exist??? Secondly, if we were able to bridge that problem through the use of proxies which were all measured in the same terms, we would be confronted with the problem of assigning weights to the output measures. The assigning of weights depends upon the degree of "importance" which is derived from value judgments. This subjective weighting is more appropriately the responsibility of individual students, educators, planners and legislators who have some specific goal in mind.

Since we have stressed earlier that our goal is not evaluation of an education system, we will stay clear of the problem of assigning relative weights to the various outputs. Our concern is academic in the sense that we are concerned with the determination of how and what the education system produces. Consequently, our role is to make sure that the coverage of student outputs is broad enough to encompass most of the major concerns of the various groups.

Because of the difficulty arising from the fact that the education system is a service industry which produces outputs that are somewhat ambiguous, a great deal of research has been carried on using inputs as surrogates for outputs. The most common inputs used are expenditure per student data. However, to use this type of input variable, it is necessary to make two rather critical assumptions. Firstly, the relative efficiencies of the service producing units must be equal. That is, if schools represent the observation, then each school must be on the production possibility frontier in the use of its resources. A second condition for the use of inputs as proxies is the assumptions that there exist linear homogeneity in production. Clearly when scale factors exist there is no justification for expecting the reader to believe that changes in these inputs (such as expenditures per student) reflect changes in outputs. This problem of linear homogeneity in analysing education systems is something which cannot be assumed but must be validated empirically. Consequently, in order to use input surrogates for output measures, the stringency imposed by the efficiency assumption and the homogeneity assumption makes this approach somewhat suspect. to say the least!

A second, albeit highly aggregative surrogate for a measure of the output of an educational system, has been the proportion of an age cohort completing a specific level of schooling. But this measure gives us very little insight into the education of any students, since it is not really the output of the education production process but merely an incomplete and suspect representative of a composite of all the attributes produced by the education system as of some time period.

In a conceptual sense, what we consider as outputs of an education process are the ways in which individuals act, think or feel as the result of participating in some unit of instruction (Bloom [1956], p. 12). This concept implies that the characteristics that are internalized by individuals as a result of education are the outputs. In order to define the outputs of education, we will borrow heavily from Bloom's Taxonomy of Educational Objectives [1956, 1964] and from Inkeles [1966].

A principal objective of this paper is to construct a taxonomy of educational objectives and to classify within a unified framework all the major types of educational outcomes so that any specific outcome can fit into a slot within the taxonomy. The idea then is to build this framework general enough so that those associated with education can make use of it. A second reason for creating this taxonomy is so that the specific educational outputs of schools which are known and quantifiable can be placed within this classification system.

Before proceeding to discuss the taxonomy it should be made clear that this taxonomy is more of a classification system. A taxonomy, in the Aristotelian sense, has certain structural rules which *must* be validated by demonstrating its consistency with the theoretical views of research findings of the field it attempts to order.[21] On the other hand, a classification system may be validated by reference to some arbitrary set of values based upon such things as communicability, usefulness, etc.

The taxonomy of educational objectives is basically divided into three broad sets: (1) Adequacy of Cognitive Development, (2) Characterization Development (or affective skills), and (3) Psychomotor Development.[22] (We will ignore Psychomotor Development from now on because of our lack of knowledge in this area.) The first two general sets can be further subdivided into more specific sets as we have done in Appendix A(i). However, this classification is quite theoretical and operationalizing these terms is somewhat difficult. Therefore, all the conceptual outputs in Appendix A(i) are further divided such that they can be operationalized. Those subsets of Adequacy of Cognitive Development appear in Appendix A(ii) and those relating to Characterization Development appear in Appendix A(iii).

This taxonomy can also be looked at as a form of disaggregation. Looking at Appendices A(ii) and A(iii), we note that on the left-hand side, the title "Area of Social Concern" is used to label two most highly aggregative concepts: adequacy of "Cognitive Development" and "Characterization Development".

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[21] Bloom [1956], p. 17.

[22] See Section 2 for a definition of Cognitive and Affective Development.

In education we believe that there are three areas of general but prime social concern which must be considered. Cognitive and Characterization Development are two of the three areas with which we will be concerned.

However useful it is to specify areas of social concern, we must still be able to develop indicators of these areas. Dealing with Appendix A(ii) we have specified that the aggregative indicators of cognitive development are "Levels of Thought" and "Processes of Thought". Taking "Levels of Thought" by itself does not represent much of an indicator unless we can quantify it. The process we have decided upon is to determine the attributes or characteristics of this indicator (second column from right of Appendix A(ii)), namely ability and achievement and then try to obtain indicators of these characteristics or proxies which represent them (right hand column). In this way, moving from the left hand column to the right hand column of Appendices A(ii) and A(iii), it can be seen how we intend to quantify the areas of social concern with disaggregated indicators.[23]

Leaving the discussion of the outputs of an education system we now turn to observing how these outputs fit into an education system as we have defined it in Section 2. The education system we discussed was one that encompassed all those factors both in and out of school that influence the cognitive and affective development of the student during the years in which the student attends school. We also assume that

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[23] It is theoretically possible to begin at the right hand side of these two Appendices and combine these disaggregative indicators into a single indicator for the particular area of social concern. To do this, one would require a weighting system and a common numeraire by which these indicators could be added.

the education system is a closed system in that any factors from other systems such as health or urban structures are considered to be exogenous. Given these assumptions, we can perceive an education system as being composed of outputs and the broad groups of variables which affect the output, namely school and external environment. A general classification system is set up in Appendix B(i) showing how these two sets of variables can be subdivided into smaller sets. Note that as each subdivision becomes more detailed, a greater degree of relevancy in terms of data collection appears. The school vector is decomposed into teacher and non-teacher subsets which can be seen in Appendices B(ii) and B(iii) respectively. Similarly, we have divided the external environment into a family classification and a setting classification as is seen in Appendices B(iv) and B(v). These classifications of the school and the external environment which are completely laid out in Appendices B(i)-(v) are intended to be as broad as possible such that they encompass most factors that might influence the outputs of the education system.

It can be observed that in the input classification the purpose behind continuous subclassifying is to obtain variables to which data can be applied. In appendix C(i) and C(ii) there is a list of variables for which data has been collected for outputs and inputs respectively. At present, we are now at the stage of organizing this data in order to empirically estimate our concept of the education system.

Section 4

AREAS OF RESEARCH

In this section, we will discuss the various avenues the empirical research can take using the data listed in Appendix C. The basic conceptual model from which all the empirical analysis flows is as follows:

$$A_{it} = f(A_{it-1}, Q_{it^*}, P_{it}, S_{it}, T_{it}, F_i) \qquad (4.1)$$

A_{it} = a vector of test scores relating to the performance of individual i at time t.

Q_{it^*}= a vector of I.Q. scores relating to the performance of of individual i at time t^* where t^* is some period earlier than t.

P_{it} = a vector of peer influence variables for individual i at time t.

S_{it} = a vector of non-teacher school-related variables at time t.

T_{it} = a vector of teacher-related variables for individual i at time t.

F_i = a vector of family-related variables which are considered to be atemporal.

This model simply states that the performance of an individual in any test is affected by his past performance, his I.Q., the influence of his peers, the facilities provided by the schools, the ability of his teachers to teach and the socio-economic climate surrounding him in the home.

The vector of test scores, A_{it}, is the educational outputs which we shall refer to as social indicators. These outputs are achievement scores derived from a number of different tests. The test scores which we shall use as indicators are primarily taken from the Sequential Test of Educational Progress. The other vector of test scores, A_{it-1}, is the score on the same test taken one time period earlier. Wherever possible we will include as an input a variable relating to some past I.Q. score. This score will be taken from as far back as possible in the students' background. In most cases this will be four years previous, but in a few cases it will be closer to the present time period. Ideally we would like to have the I.Q. score of the individual as he enters the school system so that we can capture the influences of home, environment and heredity before the school has any impact on him. But since we cannot, we will use the I.Q. score taken as far back as possible.

The peer group variables, P_{it}, will be a composite variable made up from the class of the student in question. When possible, we will use a ratio of the student's I.Q. score to the mean I.Q. of the class in which he is a member. Because of the uncertainty of the direction of effect and the non-linearities in the relationship between the influences of the peers upon the learning process of the student, such a variable should be considered in a dichotomous fashion. In this way, if we hypothesize that above-average students are slowed down by peer influences and below-average students are sped up, we can account for both of these phenomena by treating this peer variable in a dichotomous fashion.

Other vectors of inputs which must be included are the non-teacher, teacher and socio-economic vectors. The particular variables to be considered from the teacher and non-teacher vectors are listed in Appendix C. For the socio-economic variables, we are limited at present to parental occupation for this data set.

Using the general model of equation 4.1, the research will be broken up into three parts where each of the parts might be considered a submodel. These three sections are:

1. a "value-added" model;
2. an educational change model; and
3. a series of estimations using standardized groups.

In the value-added model, the idea is to determine the direct and indirect effects of the variables in all six input vectors upon the vector of the outputs.[24] This approach relies upon the use of a recursive path model.[25] The following system of equations is a typical example which can be estimated.

$$\begin{aligned} A_{it-1} &= a_{it-2}A_{it-2} + \overset{*}{a}_{it-1}A_{it^*} + p_{it-1}P_{it-1} \\ &\quad + s_{it-1}S_{it-1} + t_{it-1}T_{it-1} + f_iF_i + q_{it^*}Q_{it^*} \end{aligned} \tag{4.2}$$

$$\begin{aligned} A_{it} &= a_{it}A_{it-1} + \overset{*}{a}_{it}A_{it^*} + p_{it}P_{it} + s_{it}S_{it} \\ &\quad + t_{it}T_{it} + f_iF_i + r_{it^*}Q_{it^*} \end{aligned} \tag{4.3}$$

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[24] See Appendix E for a discussion of these direct and indirect effects.

[25] See Appendix D entitled "Path Models: An Interpretation".

Equation 4.2 is a linear equation stating that the vector of achievement scores in period t-1 is related to achievement in the previous year, A_{it-2}, some past I.Q. score ($A_{it}*$), peer, school, and teacher influences and family influences.[26] Equation 4.3 is a similar equation except that the time period is now time t and all explanatory variables are now time t. By estimating this model recursively and by also estimating its reduced form, we can obtain direct effects (the actual values of the coefficients), total effects (the coefficients of the reduced form), and indirect effects (derived relationship between the structural coefficients and the reduced form coefficients).[27]

In the second area, namely the educational change model, we plan to take advantage of the dynamic nature of the Sequential Test of Educational Progress and estimate a rate of change model. The equation to be estimated would appear as follows:

$$\dot{A}_i = a^*A_{it}{}^* + p_{it}P_{it} + s_{it}S_{it} + t_{it}T_i + f_iF_i \qquad (4.4)$$

$$\dot{A}_i = \frac{A_{it}-A_{it-1}}{A_{it}} \qquad (4.5)$$

Our purpose in doing this is to determine what factors influence the rate of change of educational progress.

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[26] We are assuming family influences to be atemporal since the time period considered is only two years.

[27] For a complete explanation of this procedure, see Appendix E.

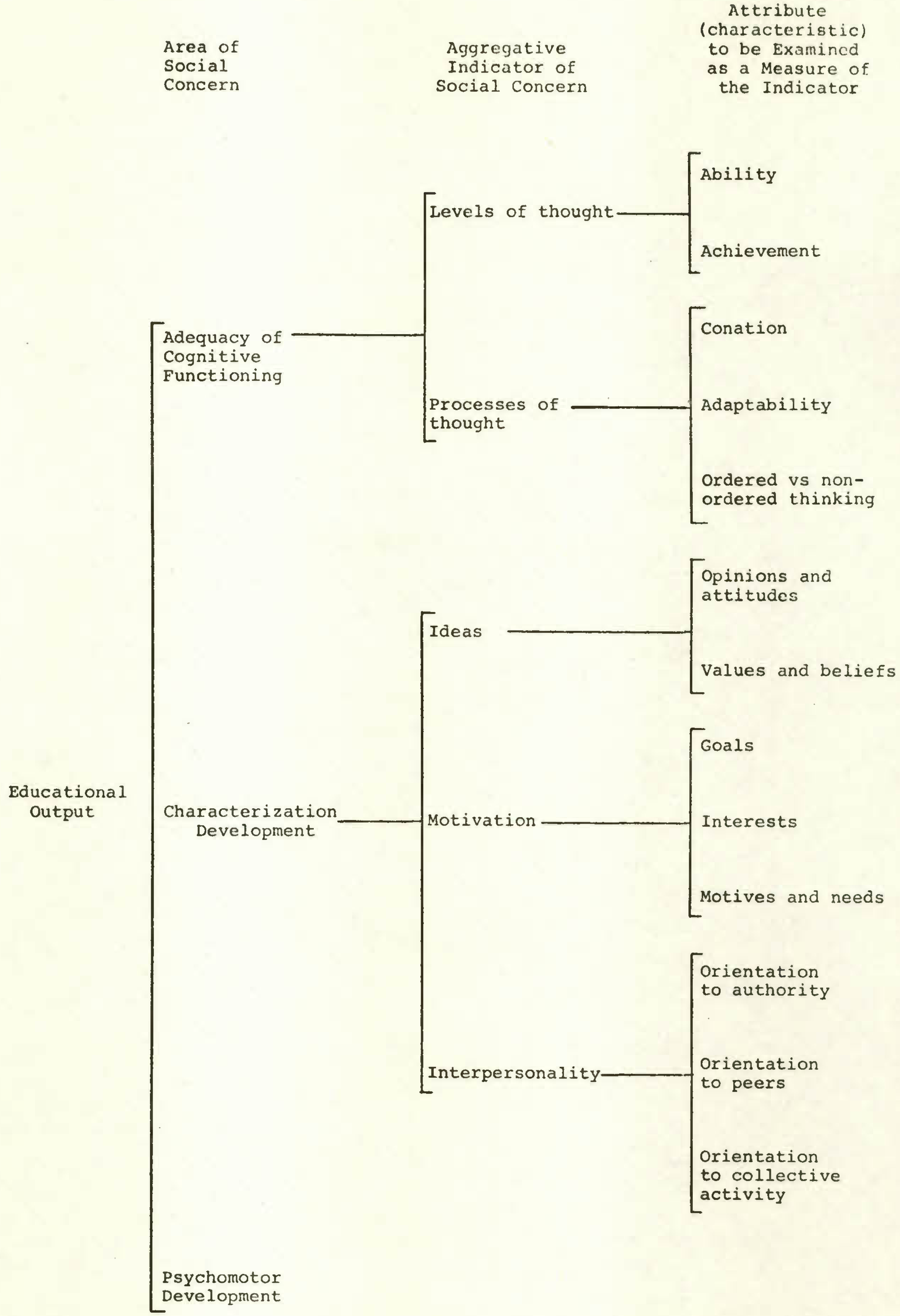
In this model we are looking for the variables which are most important in their relationship with educational progress. It would seem that if we are concerned with changes in education indicators, then we must consider models which deal with the dynamics of education rather than treating educational progress as a static concept. It is in working with the educational change model that we hope to make some significant progress in the area of education indicators.

In the third area of concern we will estimate a series of equations using selected samples of the population. Each of these samples will represent a homogeneous group. One such homogeneous subsample will be students stratified by the occupational groups of the parent. Our purpose in this case is to determine whether the "production" process of education is any different for students who come from different socio-economic backgrounds based upon parental occupations. Similar such procedures will be followed using I.Q. and student/teacher ratios as a basis for slicing. In general, we will be attempting to discover whether grouping students by any specified criteria has an effect upon the learning process.

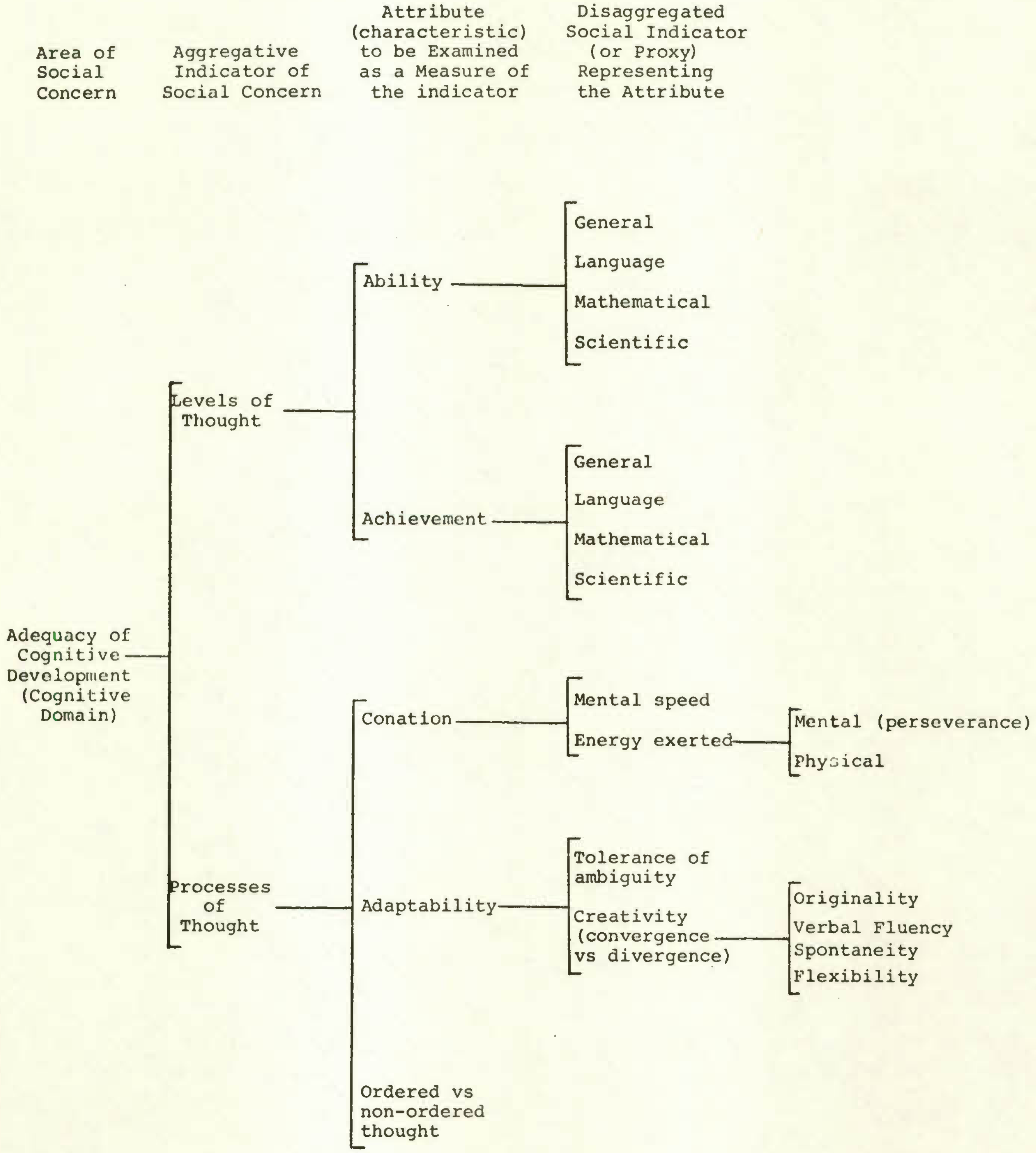
All the data to be used for this study, which have now been collected and are in the stage of being processed into useable shape, pertain to the individual. Ongoing research, which is now in progress, will contain the empirical findings based upon the theory set down in this paper.

Appendix A(i)

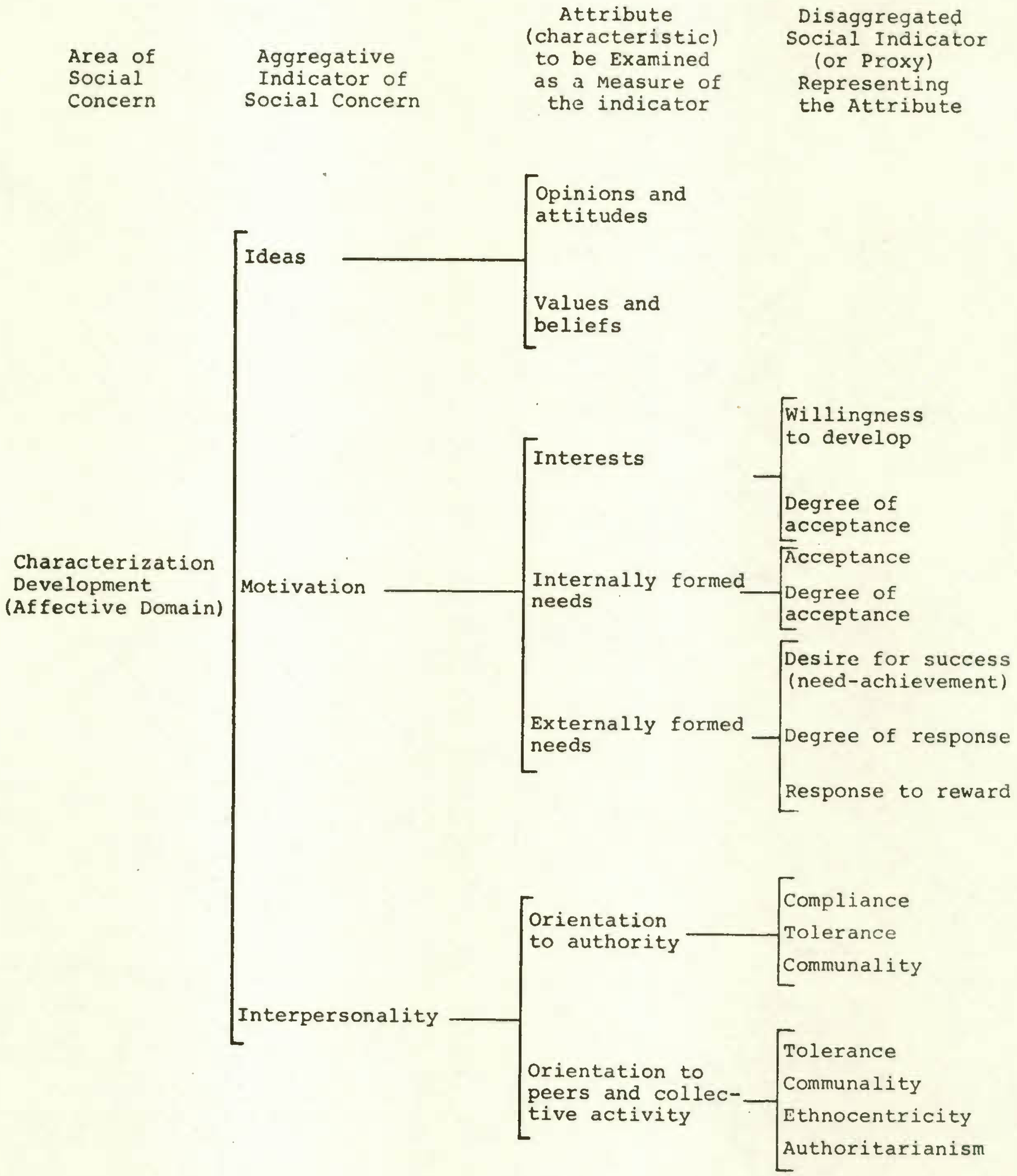
TAXONOMY OF OUTPUTS



Appendix A(ii)

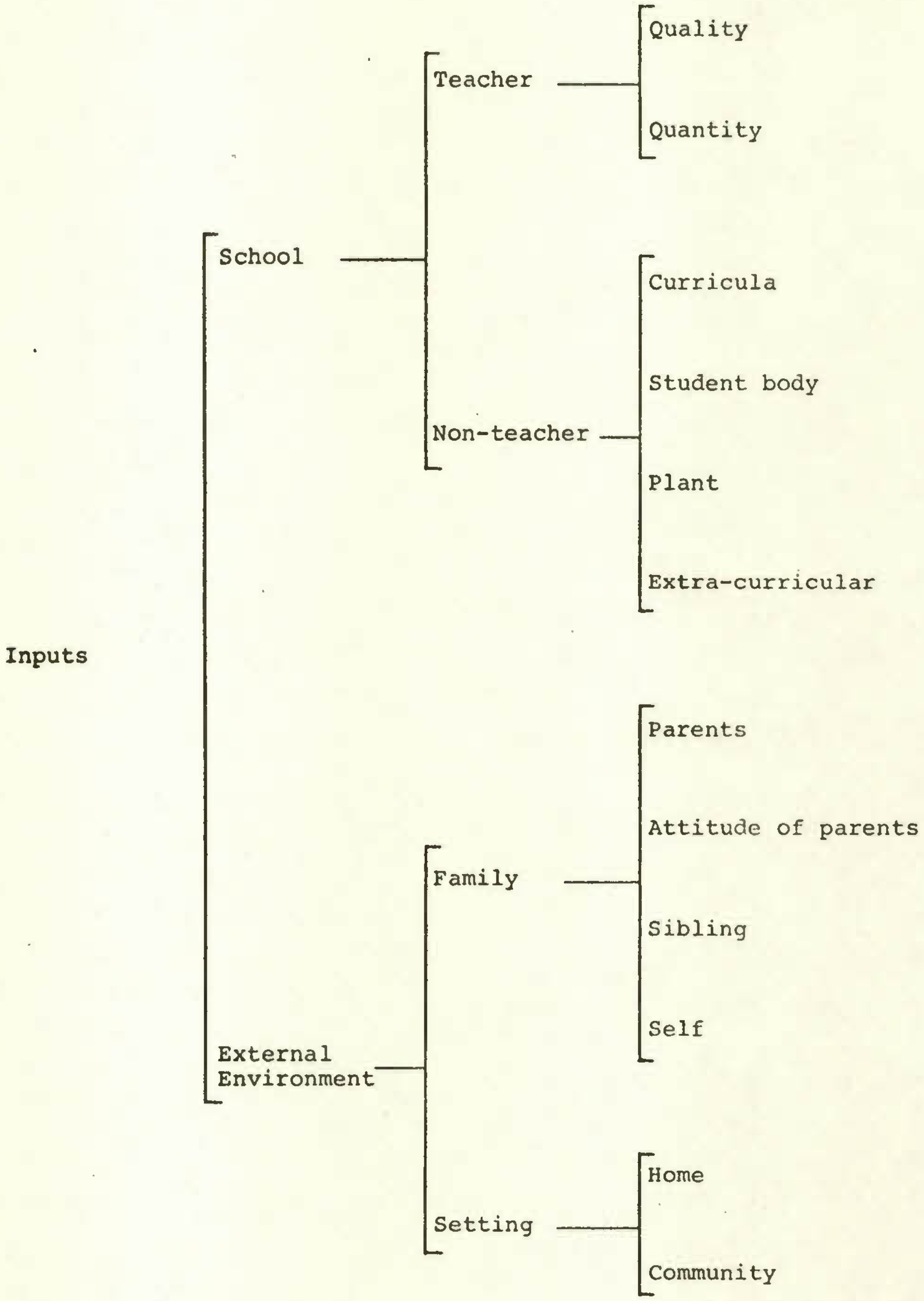


Appendix A(iii)



Appendix B(i)

TAXONOMY OF INPUTS



Appendix B(ii)

- Teacher
  - Quality
    - Age
    - Experience
    - Background
      - Home of teacher's parent with respect to school now teaching in
      - Socio-economic (parent income)
    - Sex
    - Vocabulary score of teacher
    - Academic qualification
      - Degree level
      - If no degree, number of post-secondary courses
      - Degree granting school quality
    - Upgrading
      - Courses taken beyond degree
        - academic
        - self interest
      - Number of courses taken in last two years (for teachers with more than two years' experience
  - Quantity
    - Student/teacher ratio
    - Number of class hours per teacher
    - Number of different courses per teacher
    - Extra curricular
      - Number of events per teacher
      - Hours spent per teacher
    - Students per guidance counsellor
    - Hours spent by guidance per student
    - Age of guidance staff

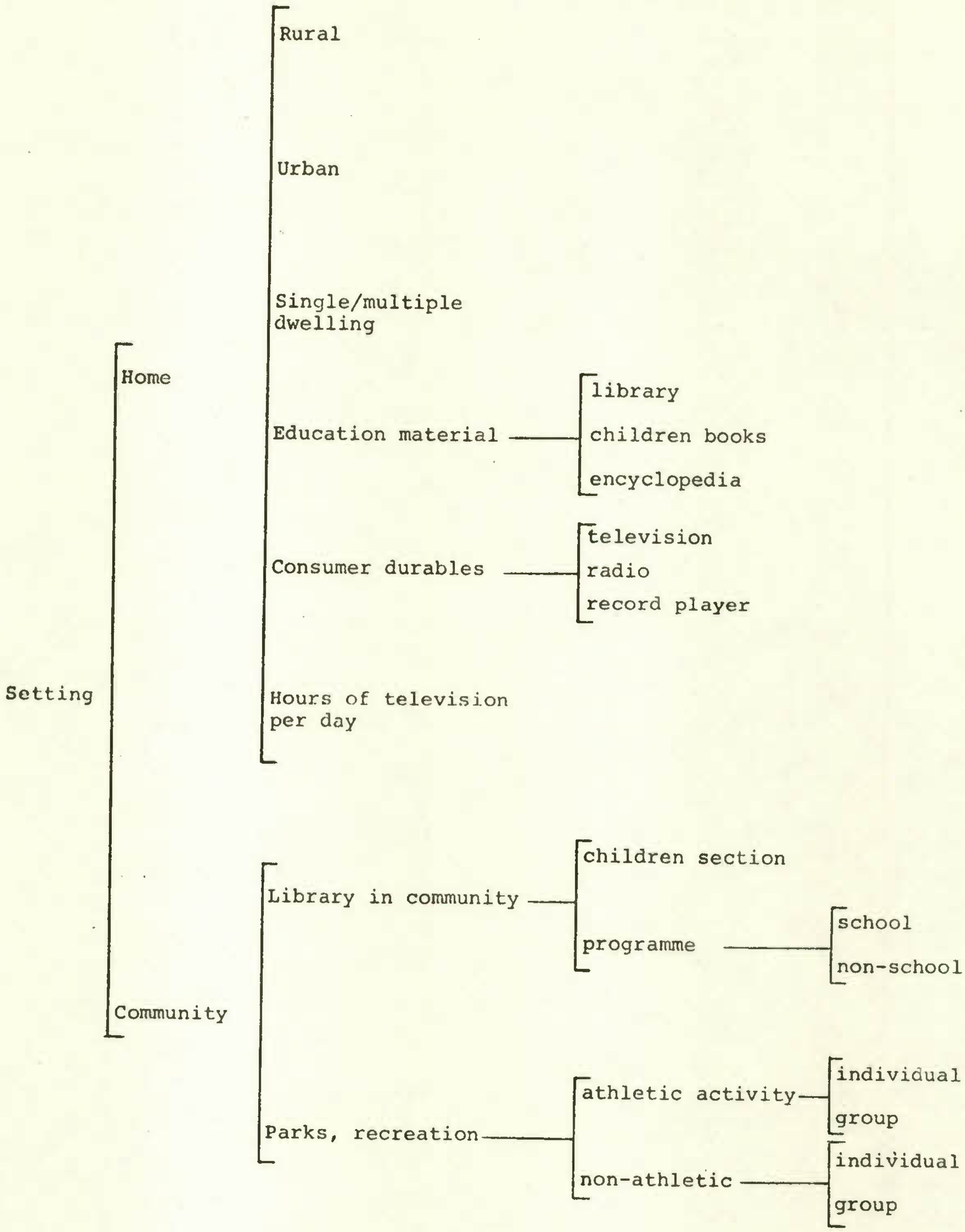
Appendix B(iii)

- Non-teacher
  - Curriculum
    - Comprehensiveness of curriculum
      - Technical
      - Commercial
      - Academic
      - Accelerated
      - Special
    - Number of classes per grade
    - Multigrade school
    - Number of days in session
    - Variety of courses with respect to curriculum
  - Student body
    - Peer groups
      - Class I.Q.
      - Class stratification
    - Size
      - Class
      - School
    - Proportion of matriculation to total student body
    - Degree of ethnic distribution
    - Volume of activities student participates in
      - Collective
      - Individual
  - Plant
    - Facilities
      - Library
        - Volumes per student
        - Access (scheduled, unscheduled)
      - Athletic facilities
      - Teaching aids
        - Audio-visual
      - Cafeteria
        - Existence
        - Hot food
      - Students per teaching area
      - Age of plant
      - Texts
        - Free
        - Over 5 years old
    - Location
      - Rural
      - Urban
        - Centre
        - Suburb
      - Income area
        - Upper
        - Middle
        - Low
        - Mix
  - Extra-curricular
    - School - volume of students per club or activity
    - Community - number of outside events student participates in the school

Appendix B(iv)

- Family
  - Parents
    - Broken/non-broken home
    - Both alive
    - Mother at home
    - Age differential
      - Between parents
      - Parents-child
    - Family income
    - Occupation
      - Mother
      - Father
    - Education
      - Mother
      - Father
    - Languages spoken
    - English in home
    - Ethnicity of parents
  - Parental attitude
    - Educational aspiration for child
    - Read to child at bedtime
  - Sibling
    - Number
    - Sex
      - Same
      - Opposite
    - Age differential
      - Closest
      - Opposite
    - School drop-out rate of brother
  - Individual
    - Native intelligence (preschool IQ)
    - Physical attributes
      - Health
      - Defects
        - Physical
        - Mental

Appendix B(v)


Setting
Home
Community
Rural
Urban
Single/multiple dwelling
Education material
library
children books
encyclopedia
Consumer durables
television
radio
record player
Hours of television per day
Library in community
children section
programme
school
non-school
Parks, recreation
athletic activity
individual
group
non-athletic
individual
group

Appendix C(i) & (ii)

DATA

The data for this study have been compiled through the co-operation and efforts of the Ministry of Education of Ontario. Much of the input data on the teacher and non-teacher variables are taken from the Ministry files, while all the output data are taken from a school board in Ontario.

Appendix C (i)

OUTPUT DATA

Sequential Test of Educational Progress (STEP)

-- a test package designed to evaluate the growth and the rate of growth of development of the basic cognitive skill inherent in most students.

1972-1973 -- grade 8, 7
1971-1972 -- grade 8, 7, 6, 4
1970-1971 -- grade 8, 7, 6, 5
1969-1970 -- grade 7, 6, 5, 4
1968-1969 -- grade 5, 4

administered in February- March

In this battery of tests we have for each student for the above years and grade:

1.	Raw reading score	STRR
1a	Adjusted reading score	STAR
2.	Raw writing score	STRW
2a	Adjusted writing score	STAW
3.	Raw science score	STRS
3a	Adjusted science score	STAS
4.	Raw mathematics score	STRM
4a	Adjusted mathematics score	STAM

Objective Test of Educational Development (OTED)

-- mastery of language and arithmetic skills specific to this school board.

Differs from STEP which measures broader outcome in these skills.

1971-1972 -- grade 6
1970-1971 -- grade 6 administered
1968-1969 -- grade 3 in May
1967-1968 -- grade 3

For the above grade and year by class, we have scores on the individual for the following tests:

5.	Reading score	OTRS
6.	Language score	OTLS
7.	Composition score	OTCS
8.	Language average	OTLA
9.	Arithmetic average	OTAA

California Test of Mental Maturity (CTMM) and Short Form Test of Academic Aptitude (SFTAA)

-- the SFTAA is an adaptation of the CTMM.

-- both provide information about the functional capacities that are basic to learning problem solving and responding to new situations.

-- these I.Q. scores are *not* to be interpreted as innate ability measures but as an index of academic aptitude.

-- it measures student's ability to:

1) engage in abstract reasoning;
2) to discern relationships among symbols;
3) to identify verbal concepts; and
4) to recall and interpret material read to him.

CTMM

1970-1971 -- grade 7, 4	
1969-1970 -- grade 4	administered in November
1968-1969 -- grade 4	

SFTAA

1972-1973 -- grade 7, 4	administered in November

For above grade and year for both CTMM and SFTAA, we have individual score by class for:

10.	Language I.Q.	CTLQ
11.	Non-language I.Q.	CTNQ
12.	Total I.Q.	CTTQ
13.	Chronological age in months	CTCA

Appendix C (ii)

INPUT DATA

School

The following data exist for each school in Ontario for 1972*. The source of this information is the Ministry of Education of Ontario.

14.	Total enrolment	ITEN
15.	Enrolment by grade and sex	IEGS
16.	Enrolment by age and sex	IEAS
17.	Total teaching staff	ITTS
18.	Full-time equivalent of teachers employed part-time	IPTS
19.	Full-time equivalent of full-time or part-time volunteer teacher aid	IVTA
20.	Full-time teachers by years of experience	ITYE
21.	Full-time teachers by salary range	ITSR
22.	Special equipment (radio, T.V., Video)	ISPE
	School facilities	
23.	- Gym	ISFG
24.	- Lunch room	ISFL
25.	- Size of site/acres	ISFA
	Mode of organization	
26.	- open space teaching	IMOS
27.	- continuous program	IMCP
28.	Existence of Parent-Teacher Association	IPTA

*Many other variables related to the school exist for this time period and for the years 1968-71. For information about these data, please see the author.

Teacher Data

The following data can be linked to the individual student.

29.	Teacher sex	ITSX
30.	Elementary years of experience	IEYX
31.	Total salary	ITSY
32.	Certificate status	ICTS
33.	Level of degree	ILDG

Parent Data

34.	Father occupation	IFOC
35.	Mother occupation	IMOC
36.	Father marital status	IFMS
37.	Mother marital status	IMMS
38.	Value of home	IVHM

Appendix D

PATH MODELS: AN INTERPRETATION

The purpose of this paper is to explain what a path model is and how it is to be interpreted. Path models were introduced by Sewall Wright [1918], a geneticist, as a method for dealing with "causal" systems. Here, cause is used in the sense that many events can be traced back continuously in time and space through successions of previous events.[28] For statistical purposes, variations in these events may be traced back, *in principle*, to variations in previous events, with varying degrees of relative importance. We stress "in principle" because in practice it may be more difficult to disentangle unidirectional sequences from the effects of reciprocal interaction. Wright [1920] formalized this type of causal model in which directional parameters were referred to as path coefficients. In a subsequent paper, [Wright, 1921] he gave a general account of this model stating that path analysis is a "method of measuring the direct influence along each separate path in a system and thus finding the degree to which variations of a given effect is determined by each particular cause.... In cases in which the causal relations are uncertain, the method can be used to find the logical consequences of any particular hypothesis in regards to them."[29] This paragraph is stressed because a

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[28] For a philosophical discussion of cause and effect as a concept in science, see Niles [1922] and Wright [1921, 1924, 1934].

[29] Wright [1921].

common misinterpretation of path analysis as described by Wright is that it can be used to determine direction or causation. This was never its intention. It can only be used to test certain hypotheses about causation. In fact, this is one of its virtues. It forces the researcher to establish a very clear framework of relationships based upon the ordering of the variables in terms of temporal direction prior to numerical estimation. Each equation of the path model must be rigorously specified prior to estimation. Only then may its causal validity be tested.

The main application of path analysis has been in the field of genetics where the method has been used as a powerful aid in determining estimated coefficients for inbreeding. Sociologists have also made use of path models especially to distinguish between the direct and indirect effects of environment and heredity upon values, income, job success, and quality of education.[30] Economists too have made use of one form of path models albeit under a different name. The work of Wold and Jureen [1953] on demand systems using recursive models is, in fact, identical to the use of simple path models. In supply and demand models where the separate identification of the supply and demand equation is the end concern recursive models can be used to help explain the shift in the supply or demand curve through the effects of something other than price.[31] The earlier work by Henry Schultz [1938] and E. J. Working [1927] should also be noted here.

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[30] For example, see Jencks [1972] and Coleman [1968].

[31] Wold and Jureen [1954], p. 12, and Goldberger [1972].

Recursive models which can be interpreted as path models are such that each equation of the system is a statement about unilateral causal dependence. There exists no feedback. In dealing with any dynamic model involving time dimensions there may be a logical order or sequence of events such that feedbacks are not possible. This logical ordering of events established before any examination of the data becomes the framework for the causal model which we are referring to a path or recursive models.[32] Estimation of the model then is to determine the variation in each of the endogenous variables brought about by the variation in each of the variables which is a priori assumed to affect them. These determining variables can be prior determined endogenous variables or exogenous variables.

The technique which Sewall Wright originally devised and which is rarely used in economics but is common to sociology is based to a large extent on the construction of a directional diagram.[33] In the illustrative Figure 1, endogenous variables denoted by X_3 and X_4 are represented as completely determined by certain other variables which may be previously determined within a simple model by exogenous variables. It is assumed here that collinearity may exist among all the determining variables in each of the linear equations formalizing this model. Unidirectional arrows in the diagram are used to indicate direction of causation from each determining variable

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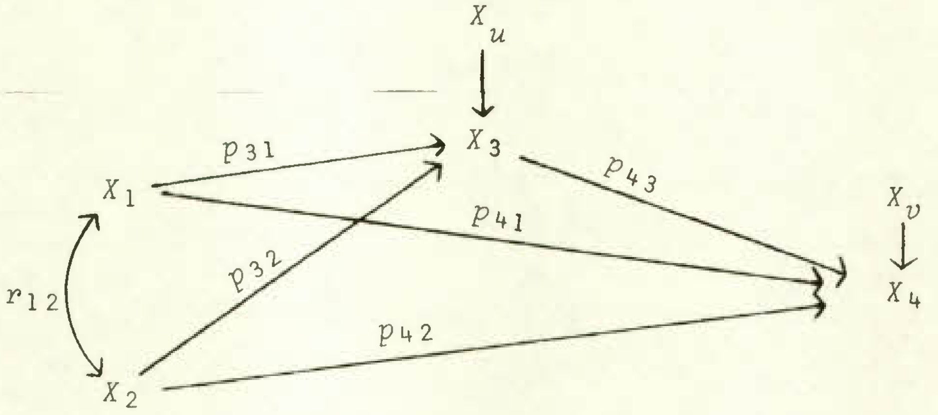
[32] Path models need not be recursive, but if the coefficients are to be uniquely identified, they must be.

[33] See Duncan [1966] for examples in Sociology and Wold and Jureen [1951], p. 67, for an interpretation of Tinbergen's arrow scheme of recursive models in Economics.

to the affected endogenous variables. Correlations between exogenous variables are shown by curved two-headed arrows. The quantities entered on the diagram are symbolic or numerical values for path coefficients, p_{ij}, or correlation coefficients r_{ij}. To complete the diagram error (or disturbance) terms are introduced representing additional sources of variation. It is assumed that the error terms are uncorrelated both with each of the determining variables and with each other. The link between the error terms X_u, X_v and the variables to which they are connected are shown by a one-headed arrow.

Figure 1



With a linear specification, this model can be written algebraically as:

$$X_3 = p_{31}X_1 + p_{32}X_2 + p_{30}X_u$$

$$X_4 = p_{41}X_1 + p_{42}X_2 + p_{43}X_3 + p_{4v}X_v \qquad (D.1)$$

In this form of the model,[34] all determined variables are on the left-hand sides of the equations, while determining variables are on the right-hand sides. The model can also be expressed in matrix form:

$$\begin{pmatrix} 1 & 0 \\ p_{43} & 1 \end{pmatrix} \begin{pmatrix} X_3 \\ X_4 \end{pmatrix} + \begin{pmatrix} p_{31} & p_{32} \\ p_{41} & p_{42} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} p_{30}X_u \\ p_{4v}X_v \end{pmatrix} \qquad \text{(D.1a)}$$

Here endogenous variables and exogenous variables are separately grouped. Only the terms involving the error are placed on the right-hand side. It should also be noted that the asymmetric coefficient matrix on the endogenous variables is triangular. This is a feature of this representation.

Since we will be concerned with estimation of a path model, we may wish to obtain particular estimates which are consistent and assymptotically unbiased. Consider the more general model with an obvious extension of notation.

$$\begin{pmatrix} B_{11} & \cdots\cdots\cdots & B_{1m} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ B_{m1} & \cdots\cdots\cdots & B_{mm} \end{pmatrix} \begin{pmatrix} Y_{1t} \\ \cdot \\ \cdot \\ \cdot \\ Y_{mt} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \cdots & \gamma_{1r} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \gamma_{m1} & \cdots & \gamma_{mr} \end{pmatrix} \begin{pmatrix} X_{1t} \\ \cdot \\ \cdot \\ \cdot \\ X_{rt} \end{pmatrix} = \begin{pmatrix} u_{1t} \\ \\ \\ \\ u_{mt} \end{pmatrix} \qquad \text{(D.2)}$$

Equation D.2 represents any linear structural model where all Y variables are endogenous and all X variables are exogenous.[35] The vector $\{U_i\}$ are the errors. Equation D.2 can be expressed as:

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[34] No constant terms appear because the X_i's are assumed to be deviations from their respective means.

[35] The similarity of this form of this model and that of the path model should be noted.

$$BY_t + \Gamma X_t = U_t \qquad (D.3)$$

where B is an (mxm) matrix, Γ is an (mxr) matrix, Y is an (mx1) vector, X is an (rx1) vector, and U is an (mx1) vector.

A reduced form equation of the model is derived by isolating on the left-hand side only the endogenous variables. The reduced form of equation D.3 is obtained as follows:

$$Y_t = \Pi X_t + V_t \qquad (D.4)$$

where

$$\begin{aligned} \Pi &= - B^{-1}\Gamma \\ V_t &= B^{-1}V_t \end{aligned} \qquad (D.5)$$

Least squares estimates of the new parameters, Π, will always exist. However, if the structural model is to be identified we must be able to determine values for B and Γ given Π. For a unique value of B and Γ to exist the necessary and sufficient conditions is that no other structural model exists which will yield the identical reduced form as that in equation D.4.

Suppose we consider an alternative structural model which is simply equation D.3 premultiplied by a non-singular matrix A which has the dimension (mxm):

$$(AB)Y_t + (A\Gamma)X_t = AU_t \qquad (D.6)$$

$$Y_t = (AB)^{-1}(A\Gamma)X_t + (AB)^{-1}AU_t$$

$$Y_t = B^{-1}(A^{-1}A)\Gamma X_t + B^{-1}(A^{-1}A)U_t$$

$$Y_t = B^{-1}\Gamma X_t + B^{-1}U_t \qquad (D.7)$$

$$Y_t = \Pi X_t + V_t.$$

Equation D.7 which is the reduced form of the structural model of equation D.6 is identical to the reduced form derived from the structural model of equation D.3. The implication of this is that if we estimate the coefficients for the structural equations D.2, these estimates will also be consistent with the estimates of equation D.6. Or, put another way, the estimates of equation D.6 and equation D.2 are consistent with either model. Therefore, if we can premultiply any structural model such as equation D.2 by some non-singular matrix such that both the original model and the transformed model will yield the identical reduced form, then it is not possible to obtain unique estimates for the coefficients of the structural model.[36] Therefore, if the estimation of the coefficients of equation D.2 is our object, all we can say about them is that B and Γ are related in the following way:

$$\Gamma = -B\Pi \qquad \text{D.5}$$

and this relationship holds both for the original structure and the transformed structure. We require some a priori restriction on B and Γ that is consistent with the structural model to be estimated. By imposing a restriction on B and Γ we are effectively providing another equation to combine with equation D.5 to obtain values for Γ and B.

One way of establishing the coefficients of this model uniquely, given knowledge of Π, is to impose conditions upon the association of the disturbance vector and the determining endogenous variables. As is well known, whenever the endogenous variables which are the "independent" variables in

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[36] Johnston [1972], p. 354.

the equation to be estimated are contemporaneously correlated with the disturbance term, least squares estimates of the coefficients will be biased and inconsistent. The issue then to be pursued is to discover a set of prior constraints to impose upon the relationship between these endogenous variables and the disturbance terms which will not only provide us with a unique solution for the B's and Γ's, but also allow us to apply least squares techniques to each equation in turn and retain optimal properties. Suppose we post-multiply equation D.3 by the transpose of the disturbance term, U_t', and take the mathematical expectations:

$$\begin{aligned} E(Y_t \cdot U_t') &= -E(B^{-1}\Gamma \cdot X_t \cdot U_t') + E(B^{-1}U_t \cdot U_t') \\ &= -(B^{-1}\Gamma)E(X_t \cdot U_t') + B^{-1}E(U_t \cdot U_t') \end{aligned} \qquad \text{(D.8)}$$

Assuming all exogenous variables X_t are uncorrelated with all disturbance terms such that $E(X_t \cdot U_t') = 0$, then

$$E(Y_t \cdot U_t') = B^{-1}\Sigma \qquad \text{(D.9)}$$

where $\Sigma = E(U_t \cdot U_t')$.

We will also assume that Σ is a diagonal matrix such that zeros appear in all cells except the principal diagonal. This assumption implies that the residuals are uncorrelated with each other $[E(U_{it} \cdot U_{jt}) = 0]$ for all $i \neq j$.

In order to understand what is implied by equation D.8 let us suppose that our original model $BY_t + \Gamma X_t = U_t$ from which equation D.9 is derived, is representative of the following system of equations:

$$Y_{1t} + B_{12}Y_{2t} + B_{13}Y_{3t} + \gamma_{11}X_t = U_{1t} \tag{D.10a}$$

$$B_{21}Y_{1t} \quad Y_{2t} + B_{23}Y_{3t} + \gamma_{21}X_t = U_{2t} \tag{D.10b}$$

$$B_{31}Y_{1t} \quad B_{32}Y_{2t} + Y_{3t} + \gamma_{31}X_t = U_{3t} \tag{D.10c}$$

Written in matrix notation, equation D.10 can be written as:

$$\begin{pmatrix} 1 & B_{12} & B_{13} \\ B_{21} & 1 & B_{23} \\ B_{31} & B_{32} & 1 \end{pmatrix} \begin{pmatrix} Y_{1t} \\ Y_{2t} \\ Y_{3t} \end{pmatrix} + \begin{pmatrix} \gamma_{11} \\ \gamma_{21} \\ \gamma_{31} \end{pmatrix} X_t = \begin{pmatrix} U_{1t} \\ U_{2t} \\ U_{3t} \end{pmatrix} . \tag{D.11}$$

Since X_t is exogenous in equation D.10, we will assume that $E(X \cdot U') = 0$.

If we post-multiply equations D.10 by $[U_{1t} \; U_{2t} \; U_{3t}]$ and take expected values, we obtain the following:

$$\begin{pmatrix} E(Y_{1t} \cdot U_{1t}) & E(Y_{1t} \cdot U_{2t}) & E(Y_{1t} \cdot U_{3t}) \\ E(Y_{2t} \cdot U_{1t}) & E(Y_{2t} \cdot U_{2t}) & E(Y_{2t} \cdot U_{3t}) \\ E(Y_{3t} \cdot U_{1t}) & E(Y_{3t} \cdot U_{2t}) & E(Y_{3t} \cdot U_{3t}) \end{pmatrix} = \begin{pmatrix} B^{11}\sigma_{11} & B^{12}\sigma_{22} & B^{13}\sigma_{33} \\ B^{21}\sigma_{11} & B^{21}\sigma_{22} & B^{23}\sigma_{33} \\ B^{31}\sigma_{11} & B^{32}\sigma_{22} & B^{33}\sigma_{33} \end{pmatrix} \tag{D.12}$$

where $\sigma_{ii} = E(U_{1t} \cdot U_{1t})$; $E(X \cdot U') = 0$; and B^{ij} is any element of B^{-1}. Let us, for the moment, leave the results of equation D.12, which we shall use soon, and turn our attention to estimating the model of equation D.10.

Suppose we wish to estimate equation D.10c. If we are using least squares principle in order to obtain consistent estimates, U_3 must be uncorrelated to Y_1 and Y_2 in all the data periods. That is, we must assume $E(Y_{1t} \cdot U_{3t}) = E(Y_{2t} \cdot U_{3t}) = 0$.

For this to be true B_{13} and B_{23} must also be equal to zero. To show this, let us assume B_{13} does not equal zero. Then equation D.10c could be substituted into equation D.10a such that we obtain the following:

$$
\begin{aligned}
Y_{1t} &= B_{12}Y_{2t} + B_{13}(B_{31}Y_{1t} + B_{32}Y_{2t} + \gamma_{31}X_t + U_{3t}) \\
&\quad + \gamma_{11}X_{1t} + U_{1t} \\
Y_{1t} &= \frac{B_{12} + B_{13}B_{32}}{\phi} Y_{2t} + \frac{B_{13}\gamma_{31}}{\phi} X_{3t} + \frac{B_{13}}{\phi} U_{3t} \\
&\quad + \frac{\gamma_{11}}{\phi} X_{1t} + \frac{U_{1t}}{\phi}
\end{aligned}
\tag{D.13}
$$

where $\phi = 1 - B_{13}B_{31}$.

In equation D.13 the term U_{3t} is a determining variable of Y_{1t}. Therefore, we cannot assume a priori that $E(Y_{1t} \cdot U_{3t}) \neq 0$. In all, in order to estimate this model recursively by using least squares principles for each equation independently, it can be shown that B_{12}, B_{13} and B_{23} must be zero. With the new assumption about the values of the B's, if we now post-multiply equation D.10 [by U_{1t}, U_{2t}, U_{3t}] the results we obtained in equation D.12 would appear as:

$$
\begin{pmatrix}
E(Y_{1t} \cdot U_{1t}) & E(Y_{1t} \cdot U_{2t}) & E(Y_{1t} \cdot U_{3t}) \\
E(Y_{2t} \cdot U_{1t}) & E(Y_{2t} \cdot U_{2t}) & E(Y_{2t} \cdot U_{3t}) \\
E(Y_{3t} \cdot U_{1t}) & E(Y_{3t} \cdot U_{2t}) & E(Y_{3t} \cdot U_{3t})
\end{pmatrix}
=
\begin{pmatrix}
\sigma_{11} & 0 & 0 \\
B^{21}\sigma_{11} & \sigma_{22} & 0 \\
B^{31}\sigma_{11} & B^{32}\sigma_{22} & \sigma_{23}
\end{pmatrix}
\tag{D.14}
$$

$$
= B^{-1}\Sigma
$$

where Σ =

$$= \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ B^{21} & 1 & 0 \\ B^{31} & B^{32} & 1 \end{pmatrix} \qquad \text{(D.15)}$$

Then, if the model of equation D.10 is to be consistently estimated using least squares principles, it must be of the following form"

$$\begin{pmatrix} 1 & 0 & 0 \\ B_{21} & 1 & 0 \\ B_{31} & B_{32} & 1 \end{pmatrix} \begin{pmatrix} Y_{1_t} \\ Y_{2_t} \\ Y_{3_t} \end{pmatrix} + \begin{pmatrix} \gamma_{11} \\ \gamma_{21} \\ \gamma_{31} \end{pmatrix} X_t = \begin{pmatrix} U_{1_t} \\ U_{2_t} \\ U_{3_t} \end{pmatrix} \qquad \text{(D.16)}$$

This system of equations is now a recursive model and is characterized in two ways: first, the matrix of coefficients on the endogenous variables must be triangular and, second, the dispersion matrix, Σ, must be a diagonal.

Effectively, what is implied by these two conditions is that the errors must be uncorrelated with each other in all time periods and there must also be no contemporaneous correlation between the error in each equation and the determining variables.

By using equation D.10, we are dealing with a special case. However, there is no loss in generalizing, and the same rules hold when applied to equation D.2. Any structural model such as equation D.2 which satisfies the condition of having a triangular matrix of coefficients for the endogenous variables and diagonal dispersion matrix, Σ, can be solved recursively by using least squares procedures.

In summary, we can state that in order for the explanatory variables in any equation to be contemporaneously independent of error in that equation we require $B^{-1}\Sigma$ to be triangular. That is, we require B to be triangular and if this is true then the system of equations we are dealing with must be recursive.

Appendix E

PREDICTION VERSUS DETERMINATION

When dealing with a recursive model or any model in which one or more variables are considered exogenous, a reduced form of the structure will always exist.

$$BY_t + \Gamma X_t = U_t \tag{E.1}$$

$$Y_t = \Pi X_t + V_t \tag{E.2}$$

$$\begin{aligned} \Pi &= -B^{-1}\Gamma \\ V_t &= B^{-1}U_t \end{aligned} \tag{E.3}$$

Equation E.2 is the reduced form of the model of equation E.1. We can often obtain conditionally unbiased estimates of the coefficients of the reduced form and consistent estimates of the structural model if it is recursive. Yet an occasion may arise where estimating one or the other of these forms is all that is needed.

Situations can arise where the reason for estimating a model is solely to determine the effects on a particular endogenous variable of a change in one or more of the exogenous variables. For example, there is no need to know that the increased expenditures may lead to increased capital formation and perhaps to an increase in the demand for labour. In some cases policymakers are not concerned with how exogenous shocks to the system make themselves wholly felt but only that the changes bring about the desired effects. If this is the purpose of estimation, then we need only estimate part of the reduced form of the model. Thus we would only have to estimate part of equation E.2 of the above model and not concern ourselves with equations E.1 or E.3. This is what we shall refer to here as prediction. There is no need for knowing anything about the model other than what the total effect of changing an exogenous variable has on the variable of interest.[37]

On the other hand, if the policy-maker or the researcher is concerned with how increased government expenditures cause a decline in unemployment, it then becomes necessary to determine, for example, how much variation in capital formation is caused by variation in government expenditures. We require this so that we may establish the indirect effect of government expenditures upon unemployment reduction.

To explain more fully, suppose we hypothesize a simple model of income distribution. Let us assume that income, X_4, is determined by changes in education levels, X_3, and the socio-economic situation in the home, X_2. In turn, education, X_3, is determined by the school, X_1, and by the home, X_2. This model can be written as follows:

$$X_3 = p_{31}X_1 + p_{32}X_2 + X_u \qquad \text{(E.4a)}$$

$$X_4 = p_{41}X_1 + p_{42}X_2 + p_{43}X_3 + X_v \qquad \text{(E.4b)}$$

The variables X_3 and X_4 are endogenous while X_1 and X_2 are exogenous. Since it can be shown that the matrix of coefficients on X_3 and X_4 is triangular, then if we assume $E(X_u \cdot X_v) = 0$

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[37] Note that we are ignoring the problem of predetermined but non-exogenous variables here. However, the argument can be readily extended to this case also.

where X_u and X_v are the errors, consistent estimates for the parameters of this model can be using least squares procedures. If we estimate each equation separately, any estimated coefficient p_{ij} provides information about the variation in X_i brought about by the variation in X_j assuming all other variables in that equation are held constant. This coefficient provides the information of the *direct* effect of a change in an independent variable upon the variable of interest.

However, suppose a decision-maker wishes to know by how much income differentials are altered if a change occurs in the socio-economic status of the home (i.e., variation in X_4 as X_2 changes). In effect he is asking what is the total effect upon X_4 as X_2 changes. The coefficient p_{42} of equation E4.b will not provide the answer. This can only be obtained by determining its reduced form and estimating its parameters:[38]

$$X_4 = P_{41}X_1 + P_{42}X_2 + X_w \qquad \text{(E.5)}$$

where

$$P_{41} = p_{41} + p_{43}p_{31} \qquad \text{(E.6a)}$$

$$P_{42} = p_{42} + p_{43}p_{32} \qquad \text{(E.6b)}$$

$$X_w = p_{43}X_u + X_v \qquad \text{(E.6c)}$$

The coefficient P_{42} of the reduced form (equation E.5) does provide the answer. This coefficient represents the combined effect of X_2 acting directly upon X_4 (p_{42}) plus the indirect effect of X_2 working through X_3 upon X_4 ($p_{43}p_{32}$).

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[38] Upper case letters are the coefficients of the reduced form while lower case letters are the coefficients of the structural model.

The indirect effect from estimating both the reduced form and the structural model is also of interest.[39] It provides the information which indicates how a policy action which acts directly upon the determination of one variable causes subsidiary effects upon another variable.

The estimation of the reduced form serves the purpose of providing answers with respect to the magnitude of change in exogenous variables necessary to induce changes in the endogenous variables. This is what we have referred to as *prediction*. It provides no insight into the structural model at all. On the other hand, estimation of a model either recursively or simultaneously, which we refer to as *determination*, provides insight into the operation of the system. The results can be used to determine the total or combined effects observed by estimation of the reduced form whereas the reduced form estimates by themselves cannot be used to derive the coefficients of the model. Within this context, the estimates from the full model are more general and serve a more versatile function. In a sense, the reduced form estimates are aggregates of the coefficients of the structural model. Because of this, estimating the reduced form must be done with caution since valuable information can be lost.

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[39] For examples of this in sociology, see Duncan [1966], p. 7.

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