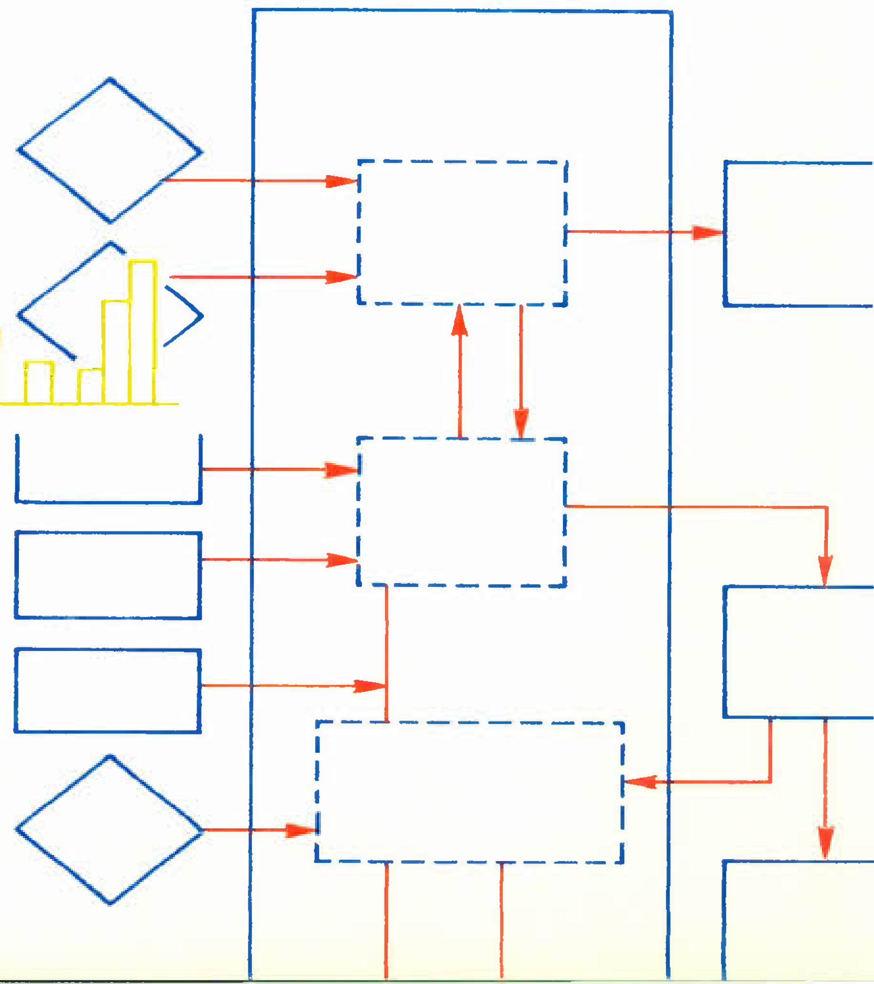
Economic Council of Canada
Conseil économique du Canada

[redacted]



HC
111
.E28
n.61

c.1
tor mai

Post Office Box 527, Ottawa K1P 5V6
Case Postale 527, Ottawa K1P 5V6

DISCUSSION PAPER NO. 61

CANDIDE and MDM: A Comparison of Two Large-Scale Econometric Models

by T. S. Barker

Ont. Ministry of Treasury, Economics & Intergovernmental Affairs
UTEAS
APR 3 1978
78/2332
LIBRARY

Discussion papers are distributed by the Council so that the author might have the advantage of professional comments. Any other use of these papers and the material contained therein is subject to prior agreement of the author.

July 1976

APR 3 1978
LIBRARY

CAN.
EC25-
61/
1976

TABLE OF CONTENTS

TABLES

I. INTRODUCTION *

This paper compares the characteristics of two large-scale econometric models, one of Canada, the other of the United Kingdom. The two models have so far developed quite independently, CANDIDE by the incorporation of an input-output table into a time-series macroeconomic model of the type associated with L. R. Klein and MDM (Multisectoral Dynamic Model) by the incorporation of time-series behavioural equations in an input-output model of the type constructed orginally by W. W. Leontief.

The Canadian model, CANDIDE, was built and is maintained by a project of the Economic Council of Canada. The model was dynamic from the beginning in 1970 and has been extensively described (McCracken,1973; Bodkin and Tanny,1975; Waslander, 1976). The British model, MDM, is a development of the static model built by the Cambridge Growth Project in the Department of Applied Economics, University of Cambridge, England. The static model is described most recently in a book co-authored by members of the project (Barker, 1976) but the dynamic version has only just been built and is not as yet fully documented. In fact, although a dynamic solution from 1973 to 1980 has been made, the model cannot yet be called fully operational. In this paper the version of the model producing this solution is compared with the current operational version of CANDIDE version 1.2M (Economic Council of Canada, 1976).

---

*The author is visiting the Economic Council of Canada through the summer of 1976. I am grateful to Tom Schweitzer for his help in explaining the details of CANDIDE and his patience in answering my many questions.

The two projects and models are compared in four different areas. First there are the differences in the overall approach which account for many of the detailed differences in programming and specification. Second there are operational differences between the models, differences in the organization of the solution and the size of the models. Third, there are differences in estimation. Fourth, there are the differences in the specification of the economic relationships. These four areas are covered in the sections which follow. The main relationships in MDM are listed in an Appendix.

II. DIFFERENCES IN THE OVERALL APPROACH

1. Scalar Versus Matrix Equations

The relationships in CANDIDE are for the most part estimated and solved as scalar equations; for example, business fixed investment in structures as estimated by 39 stochastic equations and 15 nonstochastic ones. Although each equation tends to have a similar set of determining variables there are frequent departures from the pattern, with output or price variables being dropped and other variables such as the unemployment rates or dummies being added.

The relationships in MDM are conceived, estimated and solved as relationships between matrix, vector or scalar variables. For example, a theoretical model of investment behaviour is estimated for a time series of investment matrices of 3 asset types by 40 industries, with independent variables being, inter alia, vectors of outputs by the 40 industries, and vectors of effective prices of the capital goods relative to the prices of the outputs for the 40 industries. The matrix of investment by industry is solved in the model given vectors of outputs and prices.

The MDM procedure is less flexible and will give a lower goodness of fit over the sample period unless the special determining variables are introduced into the general equation. However, it is neater and does force the economist to choose his explanatory variables carefully, rather than include or discard variables as to maximize the fit of a particular equation.

2. Parameter Restrictions

All model-builders face the problem of parameters being insignificant or having signs or magnitudes contrary to *a priori* expectations. CANDIDE tends to exclude parameters with t-values less than unity, although excepting the constant term from this rule, but MDM tends to exclude 'non-economic' parameters (time trends, dummy variables) with t-values less than the 5 per cent significance values, again excepting the constant. Thus MDM often includes income and price parameters although they may be very insignificant by the t-test. However, both models exclude parameters having the 'wrong' sign -- this applies particularily to coefficients of relative prices.

3. Functional Form

There are two major differences in approach. First CANDIDE tends to adopt linear relationship between variables whereas MDM adopts the log-linear form. (There are important exceptions -- the employment functions in CANDIDE are log-linear whilst the consumption function in MDM is linear.) The second difference is in the treatment of lags: CANDIDE includes Almon lags for the effects of the independent variables extensively throughout the model; MDM includes one-period lags in independent variables and introduces longer lags by including the lagged dependent variables. However, the investment equations in MDM include up to a 3-year lag in output.

4. <u>The Input-Output Matrix and Classification Converters</u>

There is a major difference here, which properly belongs in the later section on specification, but which sharply distinguishes the time-series macro-model CANDIDE from the dynamic input-output model MDM. In CANDIDE the input-output matrix, the mix matrix (showing which industries produce what commodities), the final demand classification converters and a set of ratios of net to gross output (all for the base year 1961) are transformed into two sets of coefficients, the first used in the calculation of real value-added by industry from the net final demands and the second used in the calculation of final demand deflators from value-added deflators and import prices. The calculated industrial value-added and final demand deflators are regressed on actual values over the sample period and the adjustment coefficients are used to forecast the value-added and the deflators from the calculated values.

In MDM these matrices and converters are explicitly included, in principle for each year (estimated over the sample period, projected forward for future years), although in practice as yet only the input-output table has been changed through time and the other matrices have been held at values of the base year 1970. There are no fixed ratios of net to gross output by industry: real net output is calculated by the double deflation method within the model, i.e., the value of inputs at their constant prices is subtracted from the value of output at constant prices. The model is solved by explicitly calculating commodity components of final demand, the industrial demand for commodities and the various deflators associated with the real flows.

III. OPERATIONAL DIFFERENCES BETWEEN THE MODELS

1. The Organization of the Solutions

The basic difference in the organization of the solutions is that CANDIDE is solved by a set of general FORTRAN programs described in Simulation System for Econometric Models, by M. C. McCracken and C. A. Sonner, 1973, whilst MDM is solved by one FORTRAN program specifically written with a set of mnemonic conventions so that the operations are easy to follow by economists and so that the character of the model can be exploited to save computer resources in solution and simulation. The programs available to model-builders of CANDIDE and MDM are listed in Table 1. The organization of the model solutions is shown in Table 2.

In CANDIDE the coefficients and exogenous data must be strictly ordered before the model can be solved. In MDM the procedure is more flexible: there are 5 input streams of coefficients and data, but the most important of these draws matrices and vectors directly from the databank. The databank information is randomly accessed. On the other hand, the ordering of the solution equations can be varied in CANDIDE whereas it is fixed in MDM.

Table 1

Programs Available for Model-Building

Name of Program	Operations
	CANDIDE* (Economic Council of Canada)
MASSAGER	1. Operations on vectors. 2. Series generation. 3. Regression analysis, including nonlinear regression, instrumental variable regression and various models of autocorrelation. 4. User-supplied routines. 5. Operations on time-series data held in databank. 6. Operations (10 options) on matrices.
DATABANK	Creates, deletes and adds time-series files. Each time-series is identified by a mnemonic.
DATAPREP) MODGEN) SOLUTION) CONVERT) BAR)	A set of related programs for solving an econometric model. See Table 2.
	MDM (Department of Applied Economics, Cambridge, U.K.)
GEM	1. Operations (up to 100 options) on scalars, vectors and matrices. 2. Reading from and writing to one or two databanks.
REXF	Regression package.
NLFIML	A set of FORTRAN subroutines available to perform Non-Linear Full Information Maximum Likelihood estimation.
ARMA etc.	A set of regression packages for estimating autoregressive error structures.

*The programs listed are maintained and supported by Informetrica Limited.

Table 2

The Organization of the Model Solutions

CANDIDE	MDM
Preparation of Regression Coefficients and Technical Coefficients	
File coefficients in strict order by equation block.	File coefficients as direct access matrices in the databank or in one of the special input files for MDM.
Preparation of the Data	
Create a MNEMONIC table of the variables on the model ordered in blocks of equations and enter data on the databank, which is limited to holding time series vectors. The software package MASSAGER provides operations (including regressions) on the time-series vectors held in the databank.	File data on exogenous and endogenous variables in random order in the databank or in one of the special input files for MDM. The databank files are matrices addressed by number or mnemonic. No operational distinction is made between data or coefficients, whether time-series or cross-section. Data is read into the databank directly or through a matrix manipulation package, GEM. Data is retrieved directly in MDM or indirectely through GEM.
Addressing and Sorting the Data	
DATAPREP is a program which checks the listing of the MNEMONIC table and the coefficient file, provides the addresses necessary to convert a MNEMONIC into a working vector, and re-orders the source data from the databank for solution.	No sorting required since data and coefficients can be directly accessed when required in MDM. The mnemonics are FORTRAN names in the solution subroutines and no addressing is needed.
Specifying the Models	
The equations determining each endogenous variable are specified using a model code in the order given by the MNEMONIC table. The model code is translated into FORTRAN instructions by a program MODGEN. These instructions together with the addresses and data supplied by DATAPREP are input with controls into the program solution.	The model is specified in FORTRAN subroutines, each subroutine working in an 'economically' distinct area (for example investment or import prices).
Solving the Models	
The blocks of equations are grouped into a prologue group which is solved first, a simultaneous group which is solved iteratively and an epilogue group which is solved last. Two methods of solution are provided, the Gauss-Seidel method (which is preferred) and the Ritz method. The ordering of the blocks of equations in the iterative solution is variable. Convergence criteria can be set for the percentage change or absolute value of the change in every variable or in any specified equation. The program can display solution results at each iteration.	The subroutines in the model are grouped into INPUT subroutines which read in controls, data and coefficients, and process the coefficients ready for the solution (for example calculate all the lagged effects, error terms, time trends and exogenous effects and add them into constants in each equation) the SOLVE subroutines which calculate the solution for a particular year and the OUTPUT subroutines which display summaries of the solution. The SOLVE subroutines provide a complete solution which can be dumped on a back-up disc file. The solution is done by the Gauss-Seidel iterative procedure with an upper limit on the number of iterations. The ordering of the subroutines in the solution is fixed. There is one convergence criterion for testing the sum of squares of differences between successive solutions of the 57 absorption prices (scaled by 1000), the 42 consumers' expenditure categories and the 57 commodity outputs. Several 'fix-ups' are built into the solution routines in case the solution goes out of range. These are printed along with the solution results at each iteration.
Display of the Solution	
The solution can be printed with observed and calculated values but it is organized not as a collection of time-series but as cross-section data, period by period. The program CONVERT transforms the output from the solution file into a DATABANK - structured file. The program BAR can analyse two simulation runs to give a multiplier analysis and can process and list results in the form of growth rates etc.	Three summaries of the solution are provided: (1) the macro-results for each year together with a 4-commodity and 4-industry breakdown, (2) the cross-section results for 15 real flows and unit-values for each year, (3) the time-series solution for about 12 macro variables. A further set of programs provides analyses of the complete solution held on the disc including full cross-section data and specified time-series data. Note that no comparisons with the observed data have been formally calculated as yet.

2. The Size of the Models

Table 3 shows the dimensions of both models. On the count of variables or equations both models are of a similar size. The significant differences come in the exogenous variables where CANDIDE has far fewer policy instruments than MDM and far more 'other' exogenous variables. There are two reasons for these differences. First, MDM has a large number of tax rates disaggregated by expenditure or income component: there are detailed breakdowns of tax rates by industry, consumers' expenditure category, imports and income groups. Second, CANDIDE has import and export prices as exogenous whereas they are endogenous in MDM; CANDIDE also has a much more detailed treatment of demographic variables.

The number of lagged variables is much the same in both models. However, it should be pointed out that, although the first period lags are available in MDM solution, not all of them are actually used.

Coefficients

There is a striking difference in the number of coefficients which is due to three factors. Firstly, the input-output matrix is much larger in CANDIDE, 105 x 75 compared with 57 x 40 in MDM. Secondly, CANDIDE holds two versions of the same information, one set of coefficients to tranform final demand into industrial value-added, the other set to transform industrial prices to final demand deflators. MDM holds one version of the information. Thirdly, the stochastic equations in CANDIDE typically have several more independent variables than those in MDM because of the Almon lags and the willingness of the original investigators to include extra variables in the equations.

Table 3

The Dimensions of the Models and Programs

	CANDIDE 1.2M		MDM	
Variables				
Exogenous - policy instruments	91		794	
- others	439	530	51	845
Lagged - exogenous - one period	) 757		845	
more than one period	)		0	
- endogenous - one period	) 3,764		2,551	
more than one period	)	4,511	480	3,876
Endogenous		2,209		2,551
		7,250		7,272
Equations				
Stochastic		645		603
Nonstochastic - technical		) 1,564	673	
- identities		)	1,275	1,948
		2,209		2,551
Coefficients				
Input-output coefficients and 'mix' matrix		16,275		2,959
Final demand converters		3,630		1,445
Other coefficients		11,438		6,706
		31,343		11,210
FORTRAN Mnemonics				
Economic variables		2,739		171
Parameters		2		58
Dimensions and controls		60		84
		2,801		313
Program				
Core size (K)		478		176
Number of iterations in solution		4-25*		4-25*
Time taken per solution (SECS) (Task time or CPU time)		2-5*		2-7*
Disaggregation				
Commodity output		105		57
Industrial output		62		40†
Consumers' expenditure categories		42		42
Government expenditure categories (current and capital)		12		10
Real assets		3		9
Financial sectors		--		8
Industrial investment-equipment		42		40†
Industrial employment		11		40†

*Approximate range depending on starting point.
†These 40 categories are the same for industrial output, employment, investment as well as wages, profits and prices.

Mnemonics

The disparity in the numbers of mnemonics in each model is partly due to the matrix approach adopted in MDM where, for example,industrial output is written as Y(I), the subscript being the I^{th} industry. Note also that MDM adopts mnemonics for parameters and sets of parameters, whereas CANDIDE uses either B for coefficients or C for constant adjustments.

Core storage

Both models are solved on IBM machines, CANDIDE on IBM370/168 and MDM on IBM 370/165. Both models solve in a similar number of iterations and take a similar time per solution. However, CANDIDE requires nearly three times the core storage of MDM, an extra 300K. Part of this difference is due to the extra 20,000 coefficients in CANDIDE; but this alone will only account for 78K of the difference. A further reason for the difference is that CANDIDE holds the observed values of the endogenous variables in store as well as the solved values; in projections outside the sample period this store is presumably unset. MDM takes observed values of all variables from the databank when required. CANDIDE also holds the first period lagged values of both endogenous and exogenous variables. In MDM the variable stores are initialized at these lagged values and the effects of the lags incorporated into the constant terms in the equations so that the same store can then be used to hold the current values of the variables. If the solution carries on for a further year then the variables are already correctly initialized with the exogenous variables being observed lagged values and the endogenous variables being calculated lagged values.

Another reason for the greater storage requirement of CANDIDE is the number of FORTRAN instructions generated by having scalar rather than matrix equations. A crude indication of the extra number of instructions is given by the number of lines in the solution program: CANDIDE has 12,872 lines whilst MDM has 2453 lines, both counts including comment statements and excluding those programs and routines which print the final results.

Finally it is possible that MDM has a more efficient overlay structure. (In fact due to the much greater number of instructions in CANDIDE, the use of OVERLAY* ought to save much more space than it does.) In MDM, 7 grouping of INPUT, SOLVE and OUTPUT routines are overlaid; in CANDIDE, the overlay structure does not seem to have been closely related to the ordering of the solution.

Disaggregations

The comparisons of the levels of disaggregation show that the extra numbers of coefficients in CANDIDE come from the greater detail in commodity and industrial outputs, not in final demand categories or employment.

3. Checks for Errors in the Solutions

CANDIDE has a special solution for checking the coding of the model and the reading of coefficients and data. This is the 'residual check' solution for the sample period. Here each equation in the model is solved using observed values for endogenous

*An IBM facility for saving core storage.

as well as exogenous variables. The calculated values of the endogenous variables are then compared with the values previously filed when the equations are estimated or the observed values when the equations are identities. There should be no discrepancies between the residual check solution and these data files. MDM has not been solved for the sample period as yet so no residual check solution is possible.

4. Adjustments

Both CANDIDE and MDM have procedures for introducing changes to the data and coefficients before or during the solution of the model. Such changes are made to make some endogenous variables exogenous in alternative simulations or to provide alternative or revised estimates of coefficients or exogenous data without re-creating the input files.

The procedures are such that the original input files remains unaltered by the adjustments. CANDIDE has two adjustment facilities:

(1) Overrides. Coefficients, observed values of exogenous and endogenous variables and constant adjustments can all be replaced by specified values in a particular run of the model.

(2) Constant adjustments. In the solution of the model each equation contains a constant adjustment, with a default value of zero. The constant adjustments are read as overrides.

There are three ways of making adjustment in MDM:

(1) ADJUST facility. At any point in the program a call can be made to the subroutine ADJUST in order to replace any elements of a matrix, vector or scalar, multiply them by a factor or add to constant to them. The elements can be any mnemonic variable or parameter in the model, although the mnemonics must be specified in advance. The adjustments are held on a special file.

(2) Temporary data sets. Any of the input data streams can be edited to a temporary file before a particular solution of the model. This allows the user to choose alternative sets of parameter estimates (by changing databank codes read as input), to change any or all of 10 key assumptions and 10 key policy instruments, or to replace any of the values of the exogenous variables or the adjustments.

(3) FIXUP subroutine. A special subroutine can be entered after the INPUT subroutines and before the SOLVE subroutines which allows the user to change any variable or parameter in the model.

IV. DATA AND ESTIMATION

Both projects have a fixed data base which is unchanged for 1-2 years at a time. At present the CANDIDE base is annual data 1948-1973 for exogenous variables and 1950-1973 for endogenous variables; the MDM base is 1954-1972 for all variables, although some equations have been estimated including data for 1973 and 1974. The CANDIDE price base is 1961, the date of the input-output tables included in the model; the MDM price base is 1970 with input-output tables for that year updated from the 1968 tables.

Single equation estimation methods are used in both projects. However, the estimation of autoregressive structures appears to be far more widespread in MDM than in CANDIDE.

V. DIFFERENCES IN SPECIFICATION

Table 4, on page 19, gives some of the main differences in specification between the models under 11 headings. In CANDIDE (and to a lesser extent MDM) the relationship described on the table are typical of those in the model rather than being the actual equations estimated.

Some of the differences have already been discussed in Section II above, for example, the use of linear functions in CANDIDE as against log-linear ones in MDM. But the table also brings out other general differences. There is more reliance in CANDIDE on time-series regressions in the explanation of industrial value-added, final demand prices, indirect and direct tax revenues and the value of investment allowances, whereas MDM uses cross-section data and models the accounting or institutional rules to calculate these variables. This is mainly due to the fact that the complete Cambridge model has not been simulated over the sample period: if this were to be done, then the values of variables calculated by these rules could be regressed on observed values and better fits could be obtained.

Another general difference is that the national accounting identites play a much smaller role in CANDIDE than in MDM. CANDIDE maintains the identities at the aggregate level of gross national product and expenditure; MDM maintains the identities for all the components of product and expenditure separately in the basis of a Social Accounting Matrix calculated for each year in constant and current prices. For example in MDM consumers' expenditures in the 42 categories in constant prices are each broken down by commodity, tax, wage and import content.

Apart from these general differences, there are several significant specific differences in the economics of the models.

(1) CANDIDE has a savings function which contains the change in the unemployment rate as an independent variable; MDM has a more orthodox consumption function with a wealth term. In consequence the dynamic multiplier for consumption with respect to an exogenous change in real income has different characteristics. In CANDIDE consumption rises, then falls, then rises again following a sustained increase in real income; in MDM consumption tends to rise smoothly.

(2) Investment in MDM is partly determined by investment allowances against taxable corporate income. There is no corresponding relationship in CANDIDE.

(3) In CANDIDE wage rates are determined by disaggregated Phillips' curves, whereas in MDM they are exogenous; i.e., independent of the level of unemployment and the cost of living. The MDM treatment is justified in the projections where an incomes and prices policy is assumed to be effective. When the past is being simulated, the intention is to introduce an aggregate wage rate equation.

(4) Export and import prices are exogenous in CANDIDE, endogenous in MDM. This must make a considerable difference in the responses of the models to changes in the exchange rate, since for a 10 per cent depreciation of the currency CANDIDE will give a 10 per cent increase in import prices (MDM 9 per cent) and a 10 per cent increase in export prices (MDM 4 per cent increase).

(5) The interest rate is linked with the budget deficit in CANDIDE, in MDM the interest rate is exogenous implying that the method of financing any deficit has little effect on the interest rate. However, in MDM the public debt is accumulated and interest payments on the debt are calculated.

(6) Finally CANDIDE makes government expenditures endogenous so that they tend to rise and fall with other components of natural expenditure. In MDM they are exogenous.

Table 4

Specification Differences Between the Models

CANDIDE	MDM
1. Accounting Identities	
National product is reconciled with national expenditure in aggregate both in constant and current prices.	Supplies equal demands for each commodity (57) both in constant and current prices. Value of each industry's outputs (40) equals value of its inputs plus labour costs, taxes and profits. Value of each financial sector's (8) incomings equals the value of its outgoings plus net savings.
2. Private Consumption	
Functions are estimated for the main components of personal savings and expenditures. Discretionary saving is related to personal disposable income and the change in the unemployment rate. The real expenditure components are linearly related to total real expenditure (usually lagged one year), to relative prices and to the lagged dependent variable, all in per capita terms. Several demographic variables and dummy variables also enter the functions.	Total consumers' expenditure in constant prices is a linear function of real personal disposable income in the current year and in the previous year, of real private wealth and of the lagged dependent variable. The function is estimated in per capita terms. Nondurable components of the total are calculated by the linear expenditure system or as log-linear functions of total expenditure (including a time trend in the coefficient) and the price of the component relative to the total. Durable components are related to real disposable income, current and lagged, to relative prices and to a hire purchase term, current and lagged. An adjustment is made to the components of consumers' expenditure so that they add to the total.
3. Public Consumption	
Functions are estimated which relate components of public consumption to demographic factors, to national expenditures and to personal expenditures on health.	This expenditure is exogenous.

Table 4 (cont'd)

CANDIDE	MDM
4. Industrial Investment	
Investment by business is divided into investment in structures and investment in equipment. Linear functions are estimated for each type of investment for 42 industries. These relate the investment to lagged capital stock, output, and the price of output relative to the rental cost of capital. Both the output and price effects are estimated with distributed lags. The rental cost of capital is a function of the current price of capital goods, a discount rate and the rate of replacement. When the theoretical specification derived from the neoclassical adjustment model gave unsatisfactory results alternative specification including dummies, time units, the corporate bond yield and the unemployment rate were estimated.	Investment is divided into 9 assets but only 3 of these (buildings, equipment and 'other vehicles') are important for most of the 40 industries, whilst 4 others (buses, ships, aircraft railway rolling stock) are important for one industry, transport. The investment in each asset for each industry is related log-linearly to industrial output, the effective price of the investment relative to the price of the industry's output and a time trend. Lags up to 3 years are included for both the output and relative price terms. The 'effective' relative price of investment is calculated to reflect the price of investment goods and the discounted unit-value of the tax allowances generated by the investment expenditure.
5. Exports and Imports of Goods and Services	
Exports and imports are divided into 30 and 14 groups respectively going to and coming from the U.S. and the rest of the world. The flows are linearly related to activity variables such as U.S. industrial production or Canadian industrial output and relative prices (after with a distributed lag). Where possible an indicator of capacity utilization is introduced into the equations. Exports of wheat and other grains and of crude peteroleum and natural gas are exogenous. Only one of the import groups, fuels and fuel products is exogenous. A special feature of the functions is the introduction of a dummy variable into those for automotive flows so as to reflect the influence of the Automotive Agreement with the U.S.	Exports and imports are both explained by log-linear equations relating trade to the level of activity abroad and at home, to relative prices and to measures of excess demand. Exports are divided into 16 commodity groups sold to each of 10 world areas. Each element of this matrix is related to industrial production of the destination, to the capacity to import of the area (where relevant), and to three sets of relative prices - the export price relative to the domestic absorption price, the export price relative to the foreign competitors' price, and the foreign competitors' price to the price in the foreign domestic market. The estimated parameters for the price terms are restricted to eliminate money illusion. Imports for 32 commodity groups are derived from

Table 4 (cont'd)

CANDIDE	MDM
	log-linear functions relating the ratio of imports to domestic output to domestic final expenditure per head, to the deviation of demand from trend and to current and lagged relative prices. Other imports are exogenous or determined as residuals.
6. Employment	
There are two sets of functions for 11 industry groups, one for employment, the other for total hours worked so that hours worked per man per week is implicit. Both set of functions are similar: required employment is derived by estimating an inverted Cobb-Douglas production function with expected output, the lagged capital stock and a time trend as the independent variables. Actual employment adjusts to required employment after a lag. Expected output is a distributed lag function of present and past actual output.	There are two sets of functions for the 40 industries, one for average hours worked per person per week the other for total hours worked, so that employment is implicit. The change in average hours worked is a log-linear function of the change in normal hours (with an adjustment lag) and the unemployment rate. The change in logs of total hours worked is a function of the change in logs of output and the investment-output ratio.
7. Wages	
Wage-rates by 12 industries are estimated as functions of excess demand in the labour markets (unemployment rate or change in employment), real productivity and the consumer price index. The last two variables usually enter the equations with a distributed lag. The wage bill is calculated from the wage rates and the hours worked on the employment.	The average wage for the economy is exogenous. Wage rates for each industry and for employment in government are calculated from a set of wage differentials. The wage bill for each industry is calculated from the average wage and the numbers employed.
8. Prices	
Export and import prices are exogenous with changes in the exchange rate fully passed on to the foreign or domestic purchasers.	Export prices are related to competitors' prices, the exchange rate and prices on the domestic market. Import prices are similarly endogenized.

Table 4 (cont'd)

CANDIDE	MDM
8. Prices (cont'd)	
Industrial value-added deflators are estimated as linear functions unit labour costs, export and import prices. 34 of the 91 deflators in the model are estimated, the remainder being set equal to estimated deflators or being an implicit identity. Final demand deflators are first calculated from the industrial value-added deflators and import unit-values. The projected unit-values are formed from equations estimated by regressing actual values of the deflators over the sample period on the calculated values.	Gross output deflators for 39 of the 40 industries are log-linear functions of 'key input' prices, usually unit labour costs and one or two material prices after imported material prices. The price of crude mineral oil is set at world oil prices plus a premium for the above-average quality of North Sea oil. Final domestic demand deflators are calculated from the gross output unit-values and import unit-values, imposing the restriction that for each commodity the value of supplies must equal that of demands.
9. Money and the Balance of Payments	
A short-term rate of interest is determined by a reduced-form equation relating it to tne real supply of high-powered money, the percentage change in gross national expenditure deflator and the real gross national product. Other interest rates in the model are functions of this short-term rate. An option exists in the model to make the supply of high-powered money responsive to the level of the federal budget deficit. Capital flows on the balance of payments are divided into short-term and long-term flows. Total net flows are estimated as a function of the current balance of payments, the Canada-U.S. interest rate differential and net direct investment, with dummy variables for exchange rate regimes and controls. Long-term flows are either exogenous or determined by interest rate differentials and economic activity. Short-term flows are a residual.	The rate of interest is exogenous. Capital flows on the balance of payments are exogenous.

Table 4 (cont'd)

CANDIDE	MDM
10. The Tax System	
Indirect taxes: 36 indirect tax rates are distinguished, mostly applicable to various categories of consumers' expenditure. Revenues are calculated from regression equations relating revenues to the tax rates and the real or nominal expenditures. Direct taxes: The personal tax system changed after 1972. Before 1972 federal and provincial personal tax revenues were separately regressed on taxable income and an appropriate overall tax rate. Corporation tax revenues are a function of the rate times the taxable profits, again distinguishing between federal and provincial revenues, tax rates and profits. Investment allowances: 10 functions for different industry groups relate capital consumption allowances to a measure of the undepreciated capital stock in the previous period and its rate of change.	Indirect taxes: 9 types of indirect tax are distinguished (local authority, fuel oil, other oil, other taxes, subsidies, purchase tax, selective employment tax, value-added tax and import duties). The rates applicable to each industry and each component of final demand are exogenous so that the revenues are endogenous. Direct taxes: The personal income tax system is modelled in great detail with 8 types of income divided between 25 income groups. The exemption limits and personal allowances are calculated for each type of income and income group, then the tax payable on the income received is calculated with an adjustment for tax numbers. Corporation tax revenue is calculated from corporate profits, the corporation tax rate and the standard income tax rates with allowable deductions for investment allowances and tax paid abroad. Petroleum revenue tax and North Sea royalties are calculated separately. Investment allowances: The present value of investment allowances attracted by each industry (40) and each asset (9) are calculated on the assumption that the industries will have sufficient profits to give the allowances their full value.
11. Income and Expenditure Accounts	
The income and expenditure accounts are implicit. However, net savings are calculated for the personal sector, the budget deficits are calculated for federal and provincial government and the balance of payments is calculated for the overseas sector.	7 institutional sectors are distinguished (households, nonprofit-making bodies, life assurance companies, companies, public corporations, government sector and the overseas sector). Current incomings and outgoings are calculated for each sector with net savings as a residual.

Appendix

A MULTISECTORAL DYNAMIC MODEL OF THE BRITISH ECONOMY: THE MAIN RELATIONSHIPS

In the equations below, capital Roman letters represent matrices, small Roman letters represent vectors, capital Greek letters represent operators and small Greek letters represent scalars. The parameters and variables are defined in order of appearance following the equations.

1. Social Accounting Matrix Identities

Commodity supplies and demand, constant and current prices

$$q + \hat{m}(i+t_{mo}) \equiv A_{qy}y + A_{qc}c + A_{qg}g + A_{qv}v + \Delta^{*}s + A_{qx}x \qquad (1)$$

$$\hat{P}_q q + \hat{P}_m \hat{m}(i+t_m) \equiv \hat{P}_h(A_{qy}y+A_{qc}c+A_{qg}g+A_{qv}v+\Delta^{*}s) + \hat{P}_x A_{qx}x \qquad (2)$$

Industry supplies and demands, constant and current prices

$$y \equiv A_{yq}q \qquad (3)$$

$$\hat{y}p_y \equiv \hat{y}A_{qy}p_h + \hat{y}\hat{t}_y p_y + \hat{y}m_y + \hat{y}w_y + p_r \qquad (4)$$

Consumers' expenditures, constant and current prices

$$c \equiv \hat{c}A'_{qc}i + \hat{c}t_{co} + \hat{c}m_c + \hat{c}\ell_c \qquad (5)$$

$$\hat{c}p_c \equiv \hat{c}A'_{qc}p_h + \hat{c}(\hat{t}_{c1}i+\hat{t}_{c2}p_c) + \hat{c}\hat{m}_c p_{mc} + \hat{c}\hat{\ell}_c p_{\ell c} \qquad (6)$$

Government expenditures, constant and current prices

$$g \equiv \hat{g}A'_{qg}i + \hat{g}t_{go} + \hat{g}m_g + \hat{g}\ell_g \qquad (7)$$

$$\hat{g}p_g \equiv \hat{g}A'_{qg}p_h + \hat{g}\hat{t}_g p_g + \hat{g}\hat{m}_g p_{mg} + \hat{g}\hat{\ell}_g p_{\ell g} \qquad (8)$$

Investment expenditures, constant and current prices

$$v \equiv \hat{v}A'_{qv}i + \hat{v}t_{vo} \qquad (9)$$

$$\hat{v}p_v \equiv \hat{v}A'_{qv}p_v + \hat{v}\hat{t}_v p_v \qquad (10)$$

Exports, constant and current prices

$$x \equiv \hat{x}A'_{qx}i \qquad (11)$$

$$\hat{x}p_x \equiv \hat{x}A'_{qx}p_x \qquad (12)$$

2. Consumers' Expenditure Functions

Consumption function

$$i'e = \varepsilon = \alpha_0 + \alpha_1\omega_1 + \alpha_2\mu_1 + \alpha_3\mu_2 \tag{13}$$

$$\omega_1 = \Lambda^{-1}(\omega_1+\delta) \tag{14}$$

$$\mu_1 = \alpha_4\mu + (1-\alpha_4)\ \Lambda^{-1}\mu_1 \tag{15}$$

$$\mu = \mu_1 + \mu_2 \tag{16}$$

Linear expenditure system

$$e_1 = \hat{p}_{e_1}^{-1}(b_1+b_2\tau)(\varepsilon_1\pi_{\varepsilon_1} - p_{e_1}{}'\ b_3) + b_3 \tag{17}$$

Log-linear expenditure functions

$$e_2 = b_1 + (b_2+b_3\tau)\ \log\ \varepsilon_1 + b_4\cdot\log\ \pi_{\varepsilon_1}^{-1}\ p_{e_1} \tag{18}$$

Durable expenditure functions

$$e_3 = b_1 + b_2\mu + \hat{b}_3\omega_2 p_{e_3}\pi_{\varepsilon}^{-1} + b_4\Lambda^{-1}\mu + \hat{b}_5\Lambda^{-1}e_3 \tag{19}$$

Identities

$$e = \hat{b}_1[(\hat{b}_2e_1+(I-\hat{b}_2)e_2) + \omega_3^{-1}e_3] \tag{20}$$

$$\varepsilon_1 = i'(e_1+e_2) \tag{21}$$

$$c = \omega_3 e \tag{22}$$

3. Investment Functions

For investment by industry in each fixed asset

$$\log\ y_{vj} = b_1 + b_2\tau + \hat{b}_7\log[\sum_{i=0}^{3}\hat{b}_{3+i}\Lambda^{-i}(y-\hat{b}_{13}\Lambda^{-1}y)] + \hat{b}_{12}\log[\sum_{i=0}^{3}\hat{b}_{8+i}\Lambda^{-1}(\hat{p}_y^{-1}p_{vj}^{*})] \tag{23}$$

For stock building by commodity

$$\Delta^* s = \hat{b}_1 y - \hat{b}_2 \Lambda^{-1} y \quad (24)$$

4. Export and Import Functions

Exports

$$\log x_j = b_1 + b_2 \log \omega_{6j} + \hat{b}_3 \log \omega_{7j} + \hat{b}_4 \log \hat{p}_x^{-1} p_{hs}$$

$$+ \sum_{i=0}^{1} \hat{b}_{5+i} \Lambda^{-i} \log(\hat{p}_{fc}^{-1} p_x \theta) + \sum_{i=0}^{1} \hat{b}_{7+i} \Lambda^{-1} \log(p_{fc} \pi_{fj})$$

$$+ \hat{b}_9 z + b_{10} \tau \quad (25)$$

Import-output ratios

$$\log \hat{q}^{-1}(m + \hat{m} t_{mo}) = b_1 + b_2 \mu_3 + b_3 + \hat{b}_3 \log d_k + b_4 \tau$$

$$+ \sum_{i=0}^{1} \hat{b}_{i+5} \Lambda^{-1} \log p_m^* + \sum_{i=0}^{1} \hat{b}_{i+7} \Lambda^{-1} \log p_q \quad (26)$$

5. Employment Functions

Normal average hours worked

$$\log t_e^* = b_1 + b_2 \tau \quad (27)$$

Change in actual average hours worked

$$\Delta^* \log t_e = b_6 + \hat{b}_7 \Delta^* \log t_e^* + \hat{b}_8 \Lambda^{-1} (\log t_e^* - \log t_e)$$

$$+ b_9 \log \tilde{u} \quad (28)$$

Change in actual total hours worked

$$\Delta^* \log y_e + \Delta^* \log t_e = b_1 + \hat{b}_3 \Delta^* \log y$$

$$+ (\hat{b}_4 + \hat{b}_5 \tau) \hat{y}^{-1} y_v i \quad (29)$$

Participation function

$$u = (b_1+b_2\tau)\omega_4 + b_3(\omega_5 - (1-b_1-b_2\tau)\omega_4) \qquad (30)$$

Unemployment rate

$$\tilde{u} = u/(u+\omega_5) \qquad (31)$$

Total employment

$$\omega_5 = i'y_e + i'c_e + i'g_e \qquad (32)$$

6. Wages, Prices and Profits

Industrial wages

$$\omega = (\hat{b}_1+\hat{b}_2\tau)\hat{y}^{-1}y_e\rho \qquad (33)$$

Industrial prices

$$p_y = b_1 + \hat{b}_2\omega + A_{py}p_y + A_{pm}p_m \qquad (34)$$

Export prices

$$\log p_x = b_9 + (\hat{b}_7-\hat{b}_3) \log (p_f\theta^{-1}) + \hat{b}_3 \log p_{hsc} + \hat{b}_4 \log (\hat{\bar{h}}^{-1}h)$$
$$+ \hat{b}_5 \log (\hat{\bar{q}}_f^{-1}q_f) + b_6\tau + (I-\hat{b}_7) \log \Lambda^{-1}p_x \qquad (35)$$

$$\log p_f = \hat{b}_1 \log p_{fh} + (I-\hat{b}_1) \log p_{fc} \qquad (36)$$

$$\log p_{hsc} = \hat{b}_2 \log p_{hs} + (I-\hat{b}_2) \log p_{hc} \qquad (37)$$

Import prices

$$\log p_m = b_1 + \hat{b}_2 \log p_f\theta^{-1} + (I-\hat{b}_2) \log p_q \qquad (38)$$

Other price vectors are derived from identities (2), (6), (8) and (10) above. Industrial profits are derived from identity (4).

DEFINITIONS

General

^	above a vector transforms it into a diagonal matrix
'	denotes transposition
log	is the logarithm to the base e
Λ^{-i}	is an operator lagging its operand i periods
Δ^*	is equivalent to $(I-\Lambda^{-1})$
i	is the unit vector
I	is the unit matrix
$b_1, b_2 \ldots$	parameter vectors specific to each equation

All flows are in £ million constant prices and all price indices are unit-values, 1970 = 1.000, unless otherwise indicated.

1. Social Accounting Matrix Identities		Equation Number
q	domestic commodity outputs	(1)
m	imports of goods and services	
t_{mo}	tax rates on imports in the base year	
A_{qy}	coefficient matrix, inputs of commodities per unit of output of industries (input-output matrix)	
y	domestic industry outputs	
A_{qc}	converter matrix, commodities to consumers' expenditures	
c	consumers' expenditures	
A_{qg}	converter matrix, commodities to government expenditures	
g	government expenditures	
A_{qv}	converter matrix, commodities to investment assets	
v	investments by asset category	
$\Delta^{*}s$	changes in stocks by commodity	
A_{qx}	converter matrix, commodities to export groups	
x	exports of goods and services in groups	
p_q	prices of commodity outputs	
p_m	prices of imports	
t_m	tax rates on imports	
p_h	prices of domestic absorption	
p_x	prices of export groups	
A_{yq}	coefficient matrix, industrial output per unit of commodity output (mix matrix)	(3)
p_y	prices of industrial outputs	(4)
t_y	tax rates on value of industrial output	
m_y	direct imports per unit of industrial output	
w_y	wage payments per unit of industrial output	
p_r	profits by industry	
t_{co}	tax rates on consumers' expenditures in base year	(5)
m_c	imports per unit of consumers' expenditure	
ℓ_c	labour costs (constant prices) per unit of consumers' expenditure	
p_c	price of consumers' expenditures	(6)
t_{c_1}	specific taxes per unit of consumers' expenditure	
t_{c_2}	_ad valorem_ taxes per unit of consumers' expenditure	
p_{mc}	prices of directly imported imports, consumers' expenditures	
$p_{\ell c}$	prices of labour inputs, consumers' expenditures	

		Equation Number
t_{go}	tax rates on government expenditures in base year	(7)
m_g	imports per unit of government expenditures	
ℓ_g	labour costs (constant prices) per unit of government expenditures	(8)
t_g	tax rates on government expenditures	
p_{mg}	prices of directly imported inputs, government expenditures	
$p_{\ell g}$	prices of labour inputs, government expenditures	
t_{vo}	tax rates on investment, base year	(9)
p_v	prices of investments	(10)
t_v	tax rates on investment	
p_x	prices of exports	(12)

2. Consumers' Expenditures Functions

e	consumers' expenditures per capita	(13)
ε	total consumers' expenditures per capita	
α_0 to α_4	parameters	
ω_1	permanent component of wealth per capita	
μ_1	permanent component of personal disposable income per capita	
μ_2	transitory component of personal disposable income per capita	
δ	net personal savings per capita	(14)
μ	personal disposable income per capita	(15)
e_1	consumers' expenditures on nondurables per capita	(17)
p_{e_1}	prices of consumers' expenditures (a re-ordering of the elements of p_c)	
τ	time trend	
ε_1	total consumers' expenditure on nondurables per capita	
π_{ε_1}, π_ε	price index appropriate to ε_1, ε	
e_2	consumers' expenditures on nondurables (log-linear elements) per capita	(18)
e_3	consumers' expenditures on durables	(19)
ω_2	hire purchase deposit term	
p_{e_3}	price indices of durables	
ω_3	population, de facto home, million	(20)
c	consumers' expenditures on goods and services	(22)

Equation Number

3. Investment Functions

y_{vj}	industrial investments by industry in asset j	(23)
τ	time trend	
y	industrial outputs	
p_y	prices of industrial outputs	
p^*_{vj}	user cost of investment by industry in asset j	
Δ^*s	changes in stocks by commodity	(24)

4. Export and Import Functions

x_j	exports by export group to destination j	(25)
ω_{6j}	industrial output for export destination j	
ω_{7j}	capacity to import in export destination j	
p_x	prices of exports	
p_{hs}	prices of home sales	
p_{fc}	prices of foreign competitors	
θ	exchange rate $ per £	
π_{fj}	domestic prices in export destination j	
z	dummy variables for economic integration (EEC and EFTA)	
τ	time trend	
q	commodity output	(26)
m	imports	
t_{mo}	tax rates on imports in the base year	
μ_3	domestic final expenditure per capita	
d_k	deviation from trend of commodity demands	
p^*_m	prices of imports adjusted for taxes on imports	
p_q	prices of domestic commodity outputs	

5. Employment Functions

t^*_e	normal average hours worked per worker per week	(27)
τ	time trend	
t_e	actual average hours worked per worker per week	(28)
$\tilde{u}$	unemployment rate	
y_e	employment by industry, thousands	(29)
y	industrial output	

		Equation Number
Y_v	investments, asset by industry	
u	unemployment in aggregate, thousands	(30)
ω_4	working population, thousands	
ω_5	employment in total, thousands	
c_e	employment in the personal sector, thousands	(32)
g_e	employment in the government sector, thousands	

6. Wages, Prices and Profits

w	wages per unit of industrial output, current prices	(33)
τ	time trend	
y	industrial outputs	
y_e	industrial employment, thousands	
ρ	average earnings per man-year	
p_y	prices of industrial output	(34)
p_m	prices of imports	
p_x	prices of exports	(35)
θ	exchange rate \$ per £	
p_f	foreign prices	
p_{hsc}	prices of home inputs or home sales	
$\bar{h}$	trend in home sales by export group	
h	home sales by export group	
$\bar{q}_f$	trend in foreign output	
q_f	foreign output	
p_{fh}	prices of foreign goods in export market	(36)
p_{fc}	prices of foreign competitors	
p_{hs}	prices of home sales by export group	(37)
p_{hc}	prices of home inputs by export group	
p_q	prices of domestic commodity outputs	(38)

REFERENCES

Barker, T.S., 1976, editor, Economic Strucutre and Policy, Chapman and Hall, London, forthcoming.

Bodkin, R. G. and S. M. Tanny, 1975, editors, CANDIDE Model 1.1, CANDIDE Project Paper No. 18, Ecnomic Council of Canada, Ottawa.

Economic Council of Canada, 1976, "CANDIDE Model 1.2M," mimeo, Economic Council of Canada, Ottawa.

McCracken, M.C., 1973, An Overview of CANDIDE Model 1.0, CANDIDE Project Paper No. 1, Economic Council of Canada, Ottawa.

McCracken, M. C. and C. A. Sonner, Users' Manual: Simulation System for Econometric Models, Informetrica Limited, Ottawa.

Waslander, H., 1975, "Summary: CANDIDE Model 1.2," memeo, Economic Council of Canada, Ottawa.

HC/111/.E28/n.61
Barker, T. S
CANDIDE and MDM : a
comparison of two dibj
c.1 tor mai