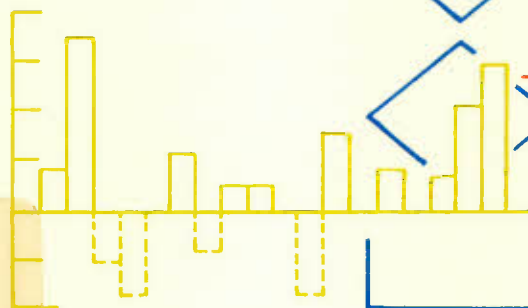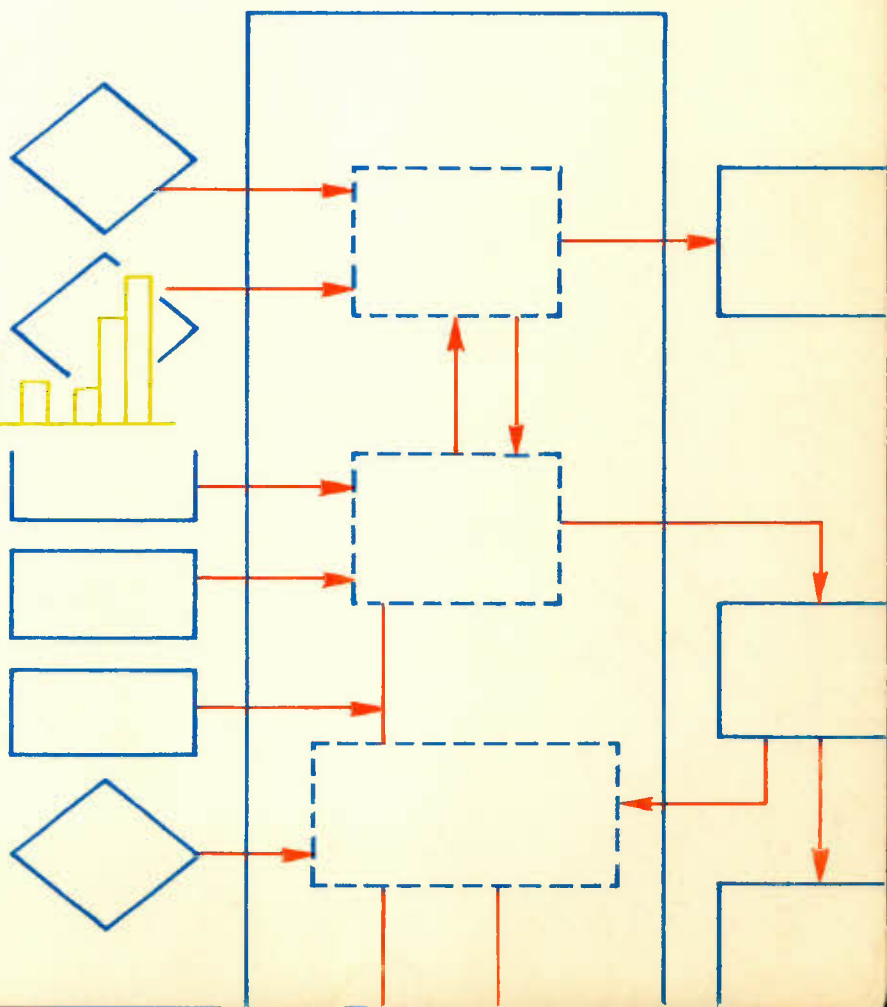# Economic Council of Canada
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DISCUSSION PAPER NO. 107

<u>Decomposition Of An Aggregate</u>
<u>Measure Of Income Distribution</u>

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ABSTRACT

Aggregate Gini coefficients, which are measures
of the inequality of the distribution of income, can be
decomposed in terms of types of income, provided the consti-
tuent coefficients are defined over family units ordered
according to their total income. This decomposition pro-
vides a valuable means of examining cyclical shifts affect-
ing income equality. However, other decompositions associat-
ed with specific socio-economic or demographic criteria are
shown to yield collective expressions which contain both the
distributional coefficients for the particular subgroups
into which the population is classified, and significant
interaction terms. These interaction terms prevent the
identification of a clear relationship between the overall
distribution of income and the distribution of income for
each of the specified subgroups. Further research in this
area should focus on distributions within structurally homo-
geneous groups, using informal procedures for linking these
distributions to form impressions of aggregate developments.*

RÉSUMÉ

Les coefficients agrégés de Gini, qui sont des
mesures de l'inégalité de la répartition du revenu, peuvent
être décomposés suivant les types de revenu, à la condition
que les coefficients désagrégés soient définis sur la base
des unités familiales classées elles-mêmes selon leur revenu
global. Cette désagrégation est utile dans l'examen des
variations conjoncturelles influant sur l'égalité du revenu.
Cependant, il existe d'autres désagrégations faites à partir
de critères spécifiques socio-économiques ou démographiques
dont les résultats contiennent à la fois les coefficients
de répartition des sous-groupes particuliers suivant les-
quels est classée la population et des termes appréciables
d'interaction. Ces termes d'interaction empêchent d'identi-
fier le rapport précis qui existe entre la répartition du
revenu pour l'ensemble de la population étudiée et la répar-
tition du revenu pour chacun des sous-groupes de cette popula-
tion. Dans ce domaine, les recherches à venir devraient être
orientées sur des répartitions de revenu au sein de groupes
structurellement homogènes, en utilisant des approches suffi-
samment flexibles qui permettent de se faire une idée assez
juste des changements observés dans la mesure agrégée*.

Introduction

The Lorenz curve and its attendant summary statistic, the Gini coefficient, have enjoyed many years of popular use in the description of income distributions by size of income. Indeed Morgan [1962] asserted that at the time he wrote it "has generally been agreed, after much discussion, that the best single measure of inequality is the proportion of the triangular area on a Lorenz diagram which falls between the Lorenz curve and the diagonal, often called the Gini Index of concentration". However this use has almost always been accompanied by criticism, usually on the ground that significant dynamic, socio-economic and demographic factors have been ignored in calculations so that the interpretation of the estimated Lorenz curve and its Gini coefficient is difficult whenever comparisons are made and meaningless otherwise. The most notable criticisms have been summarized by Garvy [1952] for the particular context that we shall stress below and by Paglin [1975, 1977] and Pyatt [1975] from alternative approaches. One primary shift in the focus of criticism during recent years stems from the greater availability of compatible data for the factors affecting incomes in terms of both size and sufficiency, factors such as age, family size, education, number of earners, sex of family heads and experience of unemployment. Researchers are now able to add empirical substance to claims for the superiority of different techniques for measuring inequality.

Three approaches to amending the measures of inequality can be discerned. First, the reference line of

perfect equality in incomes can be replaced by an alternative standard that reflects some important characteristics of the population over which the coefficient is calculated. Paglin [1975] illustrates how dynamic characteristics might be affected by the age of population units and life-cycle factors.[1] A second approach restricts the use of the Gini coefficient to more homogeneous groups that can be delineated according to socio-economic or demographic criteria. There are two variants to this approach. One assigns data to subpopulations using the criteria directly and then treats each group separately. The second retains all data but amends them, for example, by use of family equivalent scales to reflect differences in family composition, or by extensions of earning periods to reduce the impact of factors associated with the variability of individual family incomes through time. Early examples of the first variant are provided by Fisher [1952a, b] and Morgan. Benus and Morgan [1975] describe some aspects of the second variant and cite earlier research that adopts such amendments to data.

Chart 1 illustrates the Gini coefficients that stem from the use of the first variant of this approach when Canadian families,[2] in each of the given years, are assigned to

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[1] Adverse comments on Paglin's procedure are contained in the *American Economic Review* for June, 1977. See the contribution of Danziger *et al.*, Johnson, Kurien, Minarik and Nelson and, also, the response by Paglin. A Canadian perspective on Paglin's procedure is proposed by Armstrong *et al.* [1977].

[2] The term families in this paper refers to economic family units, i.e., both economic families and unattached individuals. For the definition of an economic family, see Statistics Canada, *Income Distribution by Size in Canada* (1965).

Chart 1

GINI COEFFICIENTS FOR THE TOTAL INCOME OF FAMILY UNITS
BY AGE OF HEAD, 1965-1975
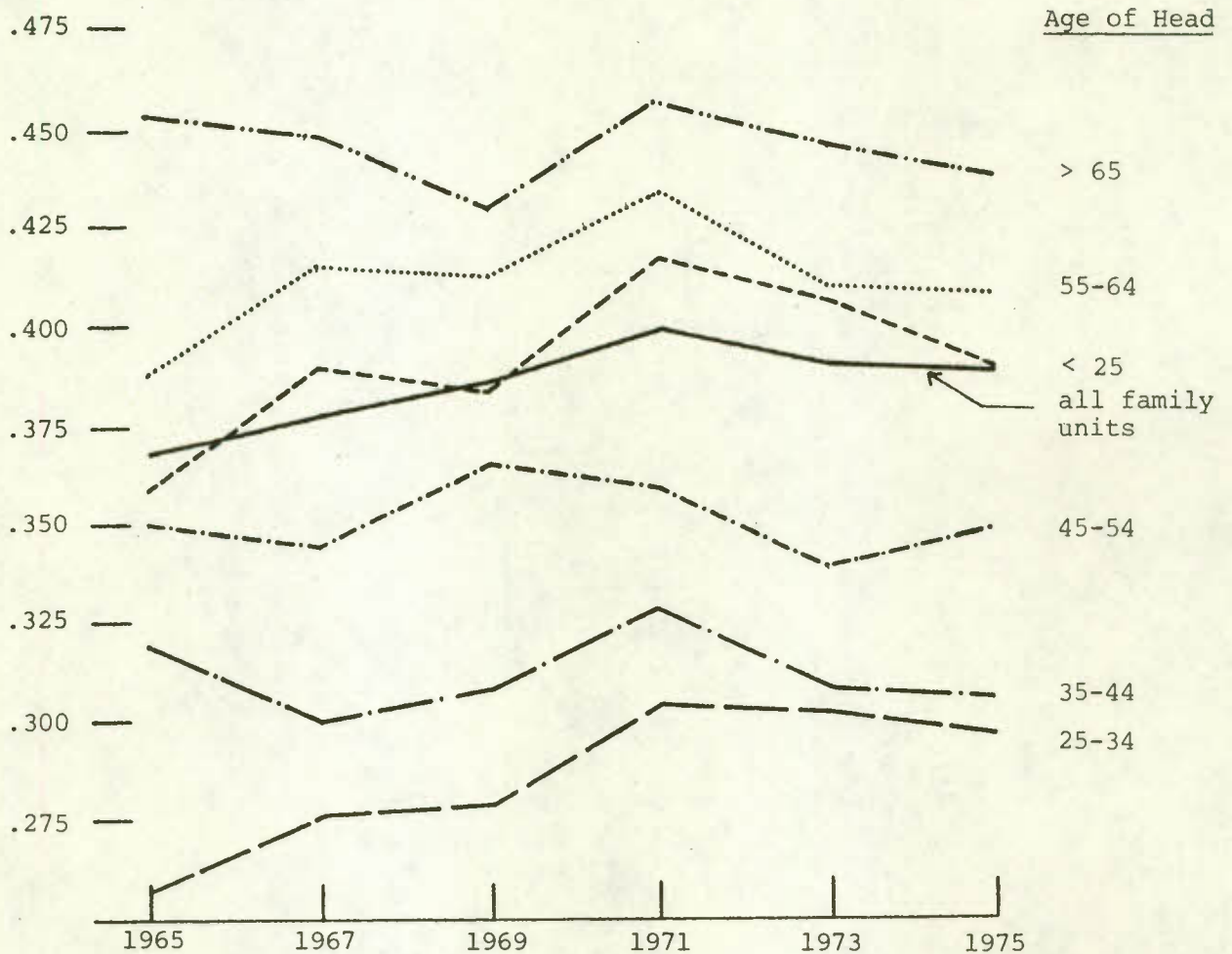
Source: Surveys of Consumer Finances, Statistics Canada, and
         estimates by the authors.

six sub-populations according to the age of their heads. The
graphs contained in this chart show distributional coefficients
representing inequality of incomes within the six groups during
the decade 1965-1975. These six graphs are "layered" in the
sense that, apart from families with young heads (aged less
than 25 years when the Surveys of Consumer Finances were under-
taken), their levels increase with the age of their heads.
Thus income inequalities are apparently monotonically asso-
ciated with age throughout the period described (apart from
the young head exception). Many explanations of this layering
effect can be suggested. In his early exploration of incomes
for family heads in the United States, Morgan provides a list
of potential influences before concluding that "extra earners
tend to reduce inequality both within age groups and for the
whole population, but ... the major factors causing age dif-
ferences in inequality ... are the differential impact of
less than full year work and some spreading of the earning
rates as differential advancement occurs".[3] Since, as we
show below, employment income is a principal contributor to
inequality among income types in Canada, it seems that Morgan's
conclusions will also have some validity in the Canadian con-
text. However, the more rapid growth of the labour force in
Canada relative to the United States probably strengthened the
impact of supplement earners.

The six graphs for the different age groups in Chart
1 are interesting quite apart from the layering effect. Plots

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[3] Morgan [1962], p. 275.

of their changes in level through time reveal that there occur significant differences in experience among the six age groups. For example, the graph for the Gini coefficients of the families with heads aged between 45 and 54 years frequently moves counter to those graphs for other groups and its behaviour through time is not adequately represented by changes in the aggregate Gini coefficient. The separate graphs may also have special significance for particular families falling within groups. The primary concern of some families may be with inequality within their own age group rather than with the experience of other groups or with aggregate experience.[4]

The potential confounding influence of changes in the relative size of age groups should also be noted. The age-composition of the population of Canadian families has changed markedly since 1965.[5] Families with young heads (under 25 years of age) were 7.8 per cent of families in 1965 but had risen to 10.9 per cent by 1975. Those with heads having ages in the range of 35 to 44 years declined from 22.4 per cent of the population in 1965 to 18.0 per cent in 1975. Since levels of inequality within these groups were markedly different, as were their levels of average incomes, these demographic shifts could have markedly affected changes in the aggregate Gini coefficients.[6]

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[4] This is elaborated by Hoffman and Podder [1976], p. 342.

[5] See Appendix D, "Distribution Of Family Units By Various Characteristics Canada, Selected Years, 1965-73" in Henderson and Rowley [1977].

[6] Similar treatments based on many different socio-economic and demographic criteria are given in Henderson and Rowley [1977].

The estimation of separate group coefficients and a knowledge of attendant population proportions, as characterizing this second approach, may thus be more valuable than the traditional reliance on single aggregate measures.

The third approach to amendments shifts away from the Gini coefficient to alternative measures that are perhaps more amenable either to decompositions associated with particular socio-economic or demographic factors or to the explicit recognition of other factors affecting incomes. Use of the Theil-Bernoulli index has, for example, often been advocated on such grounds. Hoffman and Podder [1976, Section III] illustrate this approach with data for families in the United States.[7]

Our purpose in this paper is to explore the decomposition of the aggregate Gini coefficient into sets of constituent coefficients that are defined in terms either of subpopulations fixed by specific socio-economic criteria or of different types of income. We also explore the reconstitution of the aggregate coefficient from its components with sets of appropriate weights. In this context, we show that it is possible to derive a simple decomposition of the aggregate index into the sum of separate indexes with known weights although the interpretation of this decomposition for the subpopulations defined by specific socio-economic criteria is severely restricted. In particular, we show that, provided

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[7] See also Theil [1967] and Pyatt [1976, pp. 249-251].

families are always ordered according to their total incomes,
it is possible to represent an aggregate Gini coefficient
for distribution of total incomes as a linear combination of,
for example, three distributional coefficients (similar to
Gini coefficients) that are identified with the allocation
of three distinct types of income. Weights for this linear
combination are the relative magnitudes of these income types
(employment income, government transfers, other income) in
total income. The decomposition can readily be extended to
a finer classification of income types but its utility is
then adversely affected by data inaccuracies as the split
becomes finer. We also illustrate the use of this income-
type decomposition to clarify sources of intertemporal chan-
ges in the aggregate Gini coefficient. Finally, we consider
decompositions associated with general partitions of families
according to socio-economic or demographic criteria. Another
linear formula for decomposition of the aggregate Gini coef-
ficient is established but this indicates how unlikely it is
for such a decomposition to add meaningfully to the inter-
pretation of the aggregate Gini coefficient. A classifica-
tion based on the ages of family heads is used here to illus-
trate that approximations to aggregate coefficients based
only on group's Gini coefficients and attendant population
weights are potentially misleading due to the neglect of
particular interaction terms.

Three classifications of data are necessary for the
derivations to be presented below. Each is identified with a
partition of the population of family units or their incomes

into distinct sets of mutually exclusive and exhaustive categories. Two of these classifications concern the separation of the population of families into groups, one according to income classes determined by levels of family incomes and the other according to a non-income criterion such as age of head, education of head, number of earners, family size or sex of head. The third classification concerns the division of income into three types according to its source. For generality, we can assume the existence of n income "classes", m socio-economic "groups" (on sub-populations), and r income "types".

These classifications may be identified with five sets of simple proportions and with three sets of more complex ones. Let $\{a_i$ for $i = 1, 2, \ldots, n\}$ represent the proportion of the population of families that are in the ith income class, $\{b_j$ for $j = 1, 2, \ldots, m\}$ represent the proportion or income received by the jth socio-economic group, $\{c_j$ for $j = 1, 2, \ldots, m\}$ represent the proportion of the population of families that falls within the jth group, $\{y_i$ for $i = 1, 2, \ldots, n\}$ represent the proportion of income received by the ith income class, and $\{c_k^*$ for $k = 1, 2, \ldots, r\}$ represent the proportion of income of the kth type in total income. Each of these sets of proportions sum to unity. In addition, let $\{a_{ji}\}$ represent the proportion of the jth socio-economic group in the ith income class, $\{y_{ji}\}$ represent the proportion of the jth group's income that is received by the families in the ith income class, and $\{y_{ki}^*\}$ represent the proportion of income of the kth type received by families in the ith income class. Here the ranges of subscripts i, j and k are as those

indicated for the simple proportions. Summation over $\{i\}$
(that is, over income classes) for each set of complex pro-
portions yields unity again.

Among these sets of proportions, three relations
can be established. These are:

$$(1) \quad y_i = \sum_{k=1}^{r} c_k^* y_{ki}^*$$

$$(2) \quad a_i = \sum_{j=1}^{m} c_j a_{ji}$$

$$(3) \quad y_i = \sum_{j=1}^{m} b_j y_{ji}$$

The first of these relations merely indicates that the in-
come of the families in the ith income class is derived from
r different sources, the second assigns members of this given
income class into different socio-economic groups, whereas
the third spreads the income received by families in the in-
come class over the different groups that make up the class.

The familiar aggregate Gini coefficient for total
income and defined over all families is based on $\{a_i\}$ and
$\{y_i\}$. It is usually calculated on the basis of a trapezoidal-
approximation formula that was described and made popular by
Morgan [1962, p. 281]. This formula and its generalizations
yield

$$(4) \quad G = 1 - 2 \sum_{s=1}^{n} a_s \left(\sum_{i \leq s} y_i \right) + \sum_{i=1}^{n} a_i y_i$$

$$(5) \quad G_j = 1 - 2 \sum_{s=1}^{n} a_{js} \left(\sum_{i \leq s} y_{ji} \right) + \sum_{i=1}^{n} a_{ji} y_{ji}$$
$$\text{for } j = 1, 2, \ldots, m$$

and (6) $G_k^* = 1 - 2 \sum\limits_{s=1}^{n} a_s \left(\sum\limits_{i \leq s} y_{ki}^* \right) + \sum\limits_{i=1}^{n} a_i y_{ki}^*$

$$\text{for } k = 1, 2, \ldots, r$$

for the aggregate Gini coefficient G, coefficient for separate socio-economic groups G_j, and coefficients for separate income types G_k^*. Here it is assumed that the ordering of families is always based on their total incomes. This simplifies the formula for G_k^* and permits their combination to yield G. Thus:

$$(7) \quad \sum\limits_{j=1}^{r} c_k^* G_k^* = \left(\sum\limits_{k=1}^{r} c_k^* \right) - 2 \sum\limits_{s=1}^{n} a_s \left[\sum\limits_{i \leq s} \left(\sum\limits_{k=1}^{r} c_k^* y_{ki}^* \right) \right]$$

$$+ \sum\limits_{i=1}^{n} a_i \left(\sum\limits_{k=1}^{r} c_k^* y_{ki}^* \right)$$

$$= 1 - 2 \sum\limits_{s=1}^{n} a_s \left(\sum\limits_{i \leq s} y_i \right) + \sum\limits_{i=1}^{n} a_i y_i$$

$$= G \text{ using (1).}$$

The distributional curves (similar to Lorenz curves) that are summarized by the coefficients G_k^* are illustrated for 1973 by a three-fold partition of income types in Chart 2. The vertical axis records the percentages of total income and of its three components received by percentages of family units that have been ordered according to the levels of their total incomes. Only the curve for total income is a true Lorenz curve. The other curves are novel yet interesting since they reveal the primary role of employment income in determining income disparities among families, the counteracting influence of government transfers and the markedly

asymmetric impact of income as it affects families with high
total incomes and other families. Further evidence is pre-
sented in Tables 1 and 2. Table 1 contains values for the
constituent coefficients G_k^* and their weights c_k^* for each of
the years 1967, 1969, 1971, 1973 and 1975. The constituent
coefficients for government transfer income are negative
reflecting the general redistributive impact of the trans-
fers. However, the overall contribution of transfers to the
reduction of income inequalities in any year (Table 1) is
apparently shown to be persistently small, always markedly
less in absolute magnitude than the overall contribution of
employment income or other income to income inequalities.
The entries in Table 2 deal with the dynamic evolution of
the aggregate Gini coefficients as reflected by changes in
the three components. Four successive two-year periods are
considered, namely 1967-69, 1969-71, 1971-73 and 1973-75.
It is clear that although the individual contribution of
other income to income inequality in any year may be small,
changes over time in its contribution markedly affect the
evolution of the aggregate index. Prior to 1973 changes in
the distribution of other income and changes in the distri-
bution of employment income have, as might reasonably have
been expected, reinforced each other. Their negative associa-
tion with transfers is also consistent with the provision of
supplemental benefits during moderate cyclical downturns. How-
ever, the changes in 1973-75 signal a change in the relation
of employment income and other income. During this two-year
period, changes in the labour market markedly increased

Chart 2

THE DISTRIBUTION OF TOTAL INCOME AND ITS COMPONENTS
AMONG CANADIAN FAMILY UNITS IN 1973[1]
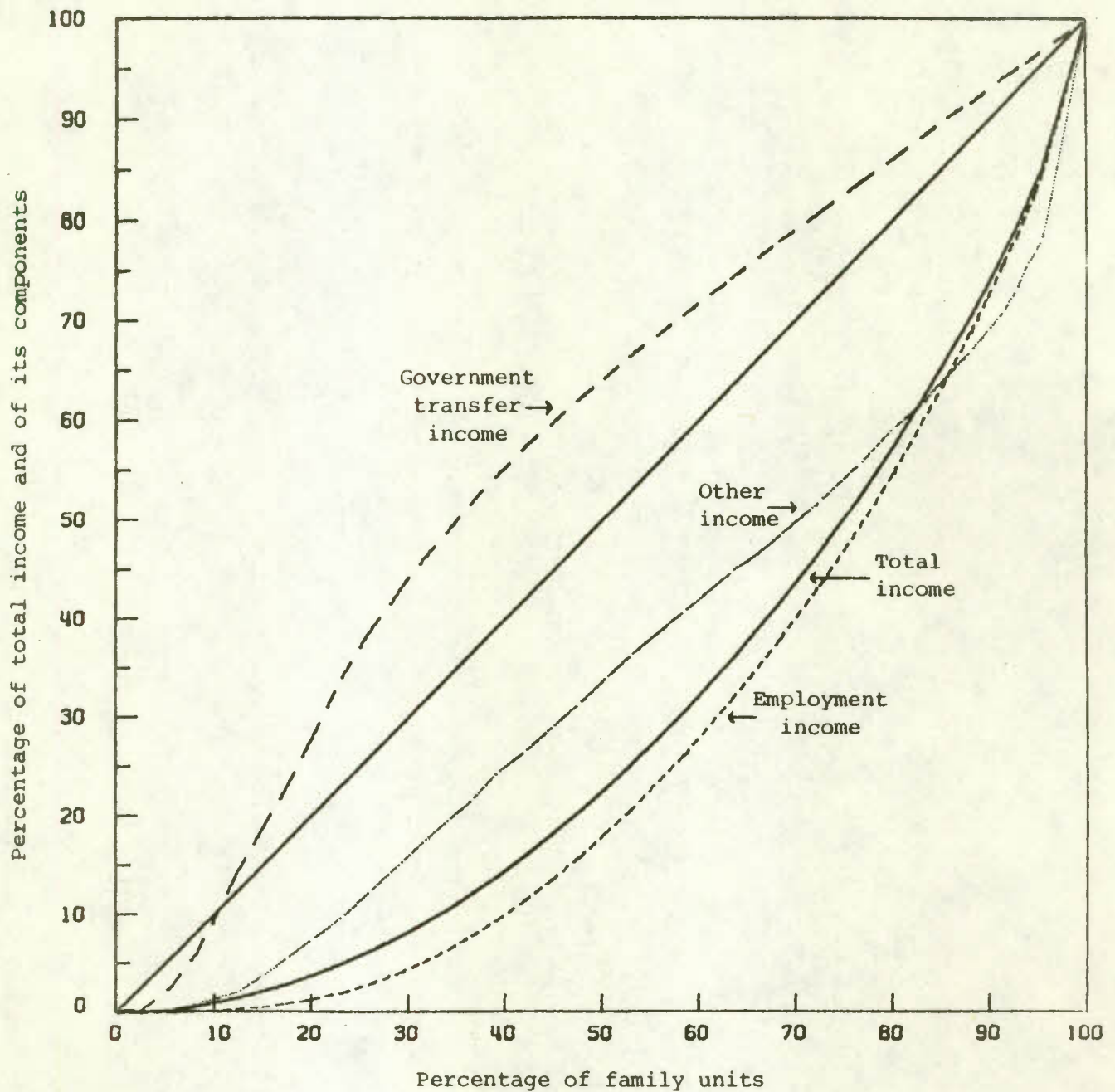
1 The distributions associated with each component of total income are
defined among family units that have been ordered according to their
total income.

Source: Surveys of Consumer Finances, Statistics Canada, and estimates
by the authors.

Table 1

DECOMPOSITION OF GINI COEFFICIENT[1] BY TYPES OF INCOME
CANADA, SELECTED YEARS, 1967-75

	1967	1969	1971	1973	1975
Employment Income					
(a) Coefficient[1]	.4274	.4376	.4474	.4474	.4580
(b) Proportion of total income	.8796	.8707	.8630	.8559	.8421
(c) Contribution to aggregate					
Gini coefficient	.3759	.3810	.3861	.3829	.3856
Government Transfer Income					
(a) Coefficient[2]	-.1813	-.2195	-.1970	-.1585	-.1268
(b) Proportion of total income	.0662	.0661	.0659	.0776	.0918
(c) Contribution to aggregate					
Gini coefficient	-.0120	-.0145	-.0130	-.0123	-.0116
Other Income					
(a) Coefficient[2]	.2558	.2896	.3644	.3026	.2566
(b) Proportion of total income	.0542	.0632	.0711	.0665	.0661
(c) Contribution to aggregate					
Gini coefficient	.0139	.0184	.0259	.0201	.0170
Aggregate Coefficient (sum of (c)'s)	.3778	.3849	.3990	.3907	.3910
Gini Coefficient Calculated Directly From The Distribution Of Total Income To All Family Units	.3784	.3853	.3994	.3911	.3909

[1] The Gini coefficient for total income and all family units.

[2] The coefficient in this table are calculated from the distribution among family units of each of the components of total income, respectively, where the family units are ordered according to the size of their total income.

Source: Surveys of Consumer Finances, Statistics Canada, and estimates by the authors.

Table 2

PROPORTIONAL CONTRIBUTION OF THE COMPONENTS OF TOTAL INCOME
TO CHANGE IN AGGREGATE GINI COEFFICIENTS

	1967-69	1969-71	1971-73	1973-75
Direction of change in value of Gini coefficients[1]	increasing (+)	increasing (+)	decreasing (−)	increasing (+)
Proportional contribution of the components of total income to the change in the Gini coefficient:				
Employment income	+0.718	+0.357	-0.384	+11.000
Government transfers	-0.353	+0.109	+0.082	+2.640
Other income	+0.635	+0.534	-0.698	-12.640
All income components	+1.000	+1.000	-1.000	+1.000

[1] An increase in the Gini coefficient suggests an increase in the inequality of distribution.

Source: Surveys of Consumer Finances, Statistics Canada, and estimates by the authors.

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inequality in the distribution of employment income while the inequality of other incomes fell substantially to a level compatible with that experienced in 1967.

Success with this simple decomposition leads one to hope that similar relationships can be established for coefficients identified with sub-populations of families. This would permit systematic introduction of supplemental information that describes demographic and other non-economic shifts and so permit the isolation of various economic influences on income distribution. Unfortunately, while it seems that the Gini coefficient can be *disaggregated*, it cannot be exactly disaggregated to account simply for the distribution of families according to socio-economic or

demographic criteria and then reconstituted using an uncomplicated formulation.[8] Suppose a generalization of the group distributional coefficient is defined by:

$$(8) \quad G_{jt} = 1 - 2 \sum_{s=1}^{n} a_{js} \left(\sum_{i \leq s} y_{ti} \right) + \sum_{i=1}^{n} a_{ji} y_{ti}$$

$$\text{for } j, t = 1, 2, \ldots, m$$

where G_{jj} is clearly G_j, the jth group's distributional coefficient. G_{jt} is the coefficient for the jth socio-economic group or sub-population after its income populations $\{y_{ji}\}$ have been replaced by those for the tth group. This coefficient has no apparent meaning when its subscripts are unequal. It can be shown, as in our Appendix, that (2) and (3) imply:

$$(9) \quad G = \sum_{j=1}^{m} \sum_{t=1}^{m} c_j b_t G_{jt}$$

$$= \sum_{j=1}^{m} c_j b_j G_j + \sum_{\substack{j=1 \\ }}^{m} \sum_{\substack{t=1 \\ t \neq j}}^{m} c_j b_t G_{jt}.$$

Hence the aggregate Gini coefficient can be represented as the sum of m^2 components where m is the number of classes of families. Only m of these components are the familiar groups' distributional coefficients. The remaining components are interaction terms. Table 3 illustrates this decomposition for 1973 when six categories are associated with the age of the heads of families. Entries in the column for

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[8] Pyatt [1976], pp. 247-249, also explores conditions for the exact decomposition of the Gini coefficient and its approximation.

G_{jt} can easily be identified with points on the graphs contained in Chart 1 whenever the subscripts j and t are equal. Thus, for example, the distributional coefficients for families with young heads (less than 25 years) and old heads (age in excess of 64 years) were 0.40726 and 0.44606 in 1973; their respective contributions to the aggregate Gini coefficient, which is the sum of 36 elements in the final column of the table (0.391075), were 0.002659 and 0.007245, far less than many of the interactive components that seem, as we have noted, to have little meaning. The contribution to the aggregate Gini coefficient of all six terms for which j and t are equal (0.061291) represents only 15.7 per cent of this coefficient, the remainder being represented by the interactive components.

In conclusion, a disaggregation of Gini coefficients in terms of types of incomes provides a valuable means of examining the cyclical shifts affecting income inequality, provided the constituent coefficients are defined over families ordered according to their total income. However, other decompositions associated with socio-economic or demographic criteria, where the Gini coefficients are disaggregated as linear combinations of constituent distributional coefficients, yield collective expressions containing significant interaction terms which do not appear to permit any further clarification of inequality. In this context, it seems that research should place less stress on a search for analytical expression for decomposition and should focus on distributions within structuarally homogeneous groups along the lines exemplified by Fisher [1952a, b] using informal procedures for linking these distributions to form impressions of aggregate developments.

Table 3

DECOMPOSITION OF GINI COEFFICIENT BY AGE OF HEAD CATEGORIES, 1973

jt	G_{jt}	c_j	b_t	$c_j b_t$	$c_j b_t G_{jt}$
11	.40726	.105424	.061930	.006529	.002659
12	.72339	.105424	.231671	.024424	.017688
13	.80195	.105424	.233925	.024661	.019777
14	.82470	.105424	.227300	.023963	.019762
15	.72997	.105424	.149110	.015720	.011475
16	.35195	.105424	.096065	.010128	.003565
21	-.14423	.220104	.061930	.013631	-.001966
22	.30340	.220104	.231671	.050992	.015471
23	.46699	.220104	.233925	.051488	.024044
24	.52918	.220104	.227300	.050030	.026475
25	.38044	.220104	.149110	.032820	.012486
26	-.09373	.220104	.096065	.021144	-.001982
31	-.30026	.186222	.061930	.011533	-.003463
32	.13010	.186222	.231671	.043142	.005613
33	.30892	.186222	.233925	.043562	.013457
34	.38448	.186222	.227300	.042328	.016274
35	.23473	.186222	.149110	.027768	.006518
36	-.21303	.186222	.096065	.017889	-.003811
41	-.29600	.174465	.061930	.010805	-.003198
42	.09802	.174465	.231671	.040418	.003962
43	.26765	.174465	.233925	.040812	.010923
44	.34243	.174465	.227300	.039656	.013579
45	.20381	.174465	.149110	.026014	.005302
46	-.20799	.174465	.096065	.016760	-.003486
51	-.01640	.144705	.061930	.008962	-.000147
52	.34546	.144705	.231671	.033524	.011581
53	.47864	.144705	.233925	.033850	.016202
54	.53237	.144705	.227300	.032891	.017510
55	.41154	.144705	.149110	.021577	.008880
56	.01573	.144705	.096065	.013901	.000219
61	.51833	.169080	.061930	.010471	.005427
62	.75846	.169080	.231671	.039171	.029710
63	.81459	.169080	.233925	.039552	.032219
64	.83182	.169080	.227300	.038432	.031969
65	.75900	.169080	.149110	.025212	.019136
66	.44606	.169080	.096065	.016243	.007245
Sum		6.000000	6.000006		.391075

jt   The six age-of-head categories are as follows: less than 25 years, 25-34 years, 35-44 years, 45-54 years, 55-64 years, and 65 or more years. These are represented by j or t = 1, 2, 3, 4, 5 and 6, respectively.

c_j Proportion of family units in each age-of-head category.

b_t Proportion of total income received by these categories.

G_{jt} Cross-distributional coefficients.

Source: Surveys of Consumer Finances, Statistics Canada, and estimates by the authors.

APPENDIX

THE DERIVATION OF A LINEAR RELATIONSHIP ASSOCIATING GROUPS' DISTRIBUTIONAL COEFFICIENTS

From the definition of G_{jt}, summation yields

$$(1) \quad \sum_{j=1}^{m} \sum_{t=1}^{m} c_j b_t G_{jt} = \sum_{j=1}^{m} \sum_{t=1}^{m} c_j b_t$$

$$- 2 \sum_{j=1}^{m} \sum_{t=1}^{m} \sum_{s=1}^{n} c_j b_t a_{js} \left(\sum_{i \le s} y_{ti} \right)$$

$$+ \sum_{j=1}^{m} \sum_{t=1}^{m} \sum_{i=1}^{n} c_j b_t a_{ji} y_{ti}$$

$$= S_1 - 2S_2 + S_3 \text{ with an obvious}$$

choice of notation.

$$(2) \quad S_1 = \sum_{j=1}^{m} c_j \sum_{t=1}^{m} b_t = 1$$

$$(3) \quad S_2 = \sum_{j=1}^{m} \sum_{s=1}^{m} c_j a_{js} \sum_{i \le s} \left(\sum_{j=1}^{m} b_t y_{ti} \right)$$

$$= \sum_{j=1}^{m} \sum_{s=1}^{m} c_j a_{js} \sum_{i \le s} y_i \quad \text{as} \quad y_i = \sum_{t=1}^{m} b_t y_{ti}$$

$$= \sum_{s=1}^{m} \left(\sum_{j=1}^{m} c_j a_{js} \right) \sum_{i \le s} y_i$$

$$= \sum_{s=1}^{m} a_s \left(\sum_{i \le s} y_i \right) \quad \text{as} \quad a_s = \sum_{j=1}^{m} c_j a_{js}$$

$$(4) \quad S_3 = \sum_{j=1}^{m} \sum_{i=1}^{n} c_j a_{ji} \left(\sum_{t=1}^{m} b_t y_{ti} \right)$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{n} c_j a_{ji} y_i \quad \text{as} \quad y_i = \sum_{t=1}^{m} b_t y_{ti},$$

$$= \sum_{i=1}^{n} \left(\sum_{j=1}^{m} c_j \, a_{ji} \right) Y_i$$

$$= \sum_{i=1}^{n} a_i \, Y_i \quad \text{as} \quad a_i = \sum_{j=1}^{m} c_j \, a_{ji}.$$

Thus,

$$(5) \quad G \equiv 1 - 2 \sum_{s=1}^{m} a_s \left(\sum_{i \leq s} Y_i \right) + \sum_{i=1}^{n} a_i \, Y_i$$

$$= \sum_{j=1}^{m} \sum_{t=1}^{m} c_j \, b_t \, G_{jt}.$$

BIBLIOGRAPHY

ARMSTRONG, D.E., P.H. FRIESEN and D. MILLER (1977), "The
         Measurement of Income Distribution in Canada:
         Some Problems and Some Tentative Data", *Canadian
         Public Policy*, Vol. 3, No. 4, pp. 479-488.

BENUS, J. and J.N. MORGAN (1975), "Time Period, Unit of
         Analysis, and Income Concept in the Analysis of
         Income Distribution", in (ed.) J.D. Smith, *The
         Personal Distribution of Income and Wealth* (New
         York: National Bureau of Economic Research), pp.
         209-224.

DANZIGER, S., R. HAVEMAN and E. SMOLENSKY (1977) "The
         Measurement and Trend of Inequality: Comment",
         The American Economic Review, Vol. 67, No. 3,
         pp. 505-512.

FISHER, J.A. (1952a), "Income, Spending, and Saving Patterns
         of Consumer Units in Different Age Groups", in
         Studies in Income and Wealth, Vol. 15 (Princeton,
         New Jersey: Princeton University Press for the
         National Bureau of Economic Research), Part IV.

FISHER, J.A. (1952b), "Post-War Changes in Income and
         Savings Among Consumers in Different Age Groups",
         Econometrica, pp. 47-70.

GARVY, G. (1952), "Inequality of Income: Causes and Measure-
         ment", in *Studies in Income and Wealth*, Vol. 15
         (Princeton, New Jersey: Princeton University Press
         for the National Bureau of Economic Research), Part
         II.

HENDERSON, D.W. and J.C.R. ROWLEY (1977), *The Distribution
         and Evolution of Canadian Family Incomes, 1965-1973*,
         Discussion Paper No. 91, Economic Council of Canada
         (Ottawa: Supply and Services Canada).

HOFFMAN, S. and N. PODDER (1976), "Income Inequality", in (eds)
         G.J. Duncan and J.N. Morgan, *Five Thousand American
         Families -- Patterns of Economic Progress*, Vol. IV
         (Ann Arbor: Institute for Social Research, The
         University of Michigan), Chap. 12.

JOHNSON, W.R. (1977), "The Measurement and Trend of Inequa-
         lity: Comment", *The American Economic Review*, Vol.
         67, No. 3, pp. 502-504.

KURIEN, C.J. (1977), "The Measurement and Trend of Inequality:
         Comment", *The American Economic Review*, Vol. 67, No.
         3, pp. 517-519.

MINARIK, J.J. (1977), "The Measurement and Trend of Inequality: Comment", *The American Economic Review*, Vol. 67, No. 3, pp. 513-516.

MORGAN, J. (1962), "The Anatomy of Income Distribution", *The Review of Economics and Statistics*, Vol. 44, No. 3, pp. 270-283.

NELSON, E.R. (1977), "The Measurement and Trend of Inequality: Comment", *The American Economic Review*, Vol. 67, No. 3, pp. 497-501.

PAGLIN, M. (1975), "The Measurement and Trend of Inequality: A Basic Revision", *The American Economic Review*, Vol. 65, No. 4, pp. 598-609.

PAGLIN, M. (1977), "The Measurement and Trend of Inequality: Reply", *The American Economic Review*, Vol. 67, No. 3, pp. 520-531.

PYATT, G. (1976), "On The Interpretation and Disaggregation of Gini Coefficients", *Economic Journal*, Vol. 86, pp. 243-255.

THEIL, H. (1967), *Economics and Information Theory*, (Amsterdam: North-Holland), Chap. 4.