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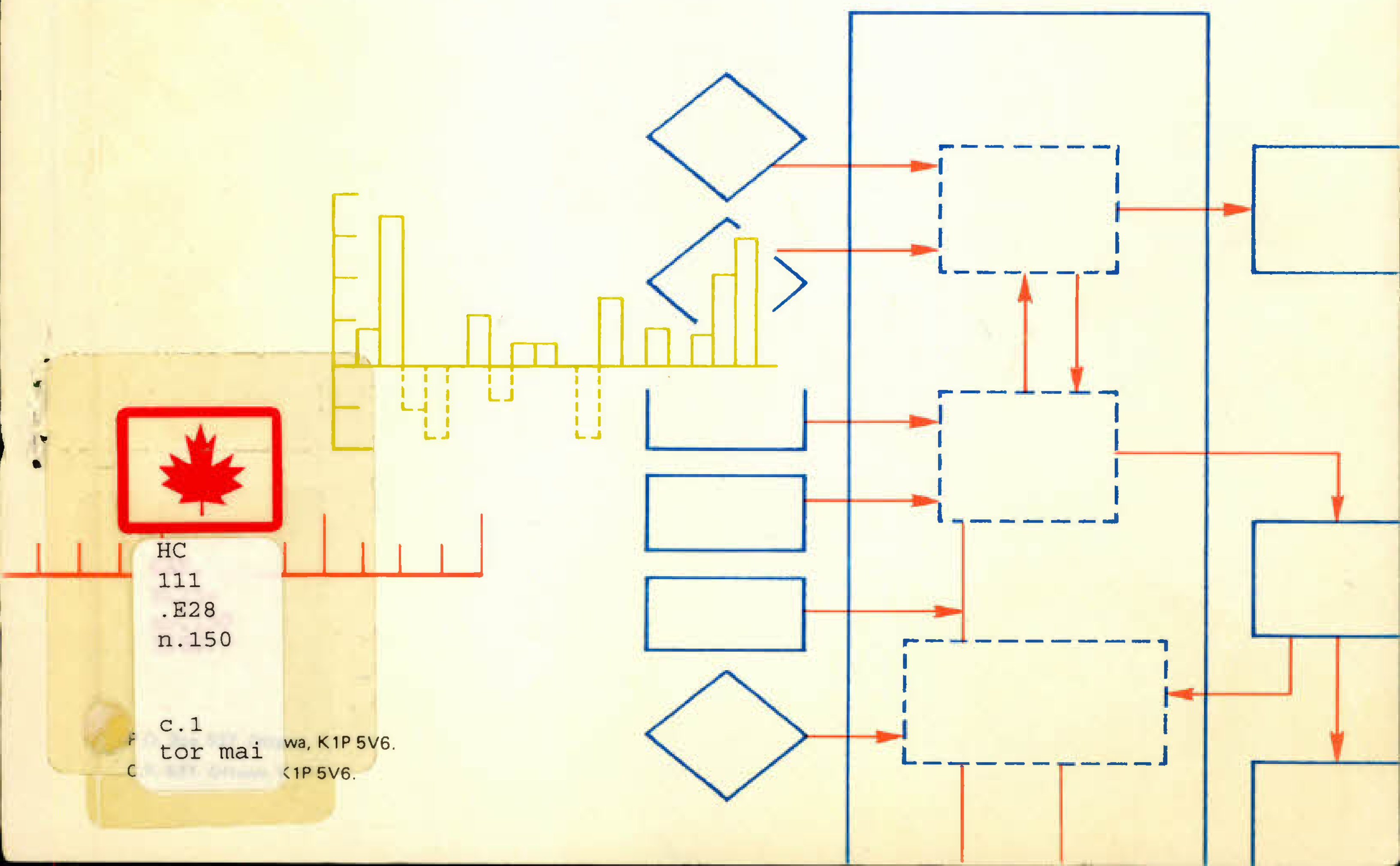

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DISCUSSION PAPER No. 150

Interest Rates, Inflation Expectations and Spurious Elements in Measured Real Income and Saving

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Background Paper to the Sixteenth Annual Review

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RÉSUMÉ

Les variables du calcul du revenu et de l'épargne enregistrent des mouvements trompeur à chaque modification du taux prévu de l'inflation des prix. Ces fluctuations découlent du fait que les mesures traditionnelles comprennent, comme composante du revenu, les intérêts nominaux perçus, moins les dépenses d'intérêt. Les taux de l'intérêt nominal contiennent une prime à l'inflation qui compense les détenteurs d'avoirs à revenu fixe pour les pertes de capital attribuables aux changements prévus dans les niveaux des prix. Cette prime s'accroît ou diminue en fonction du taux prévu d'inflation des prix, et a pour effet d'introduire un élément de gain de capital dans les valeurs calculées du revenu et de l'épargne. Il en résulte de sérieuses distorsions dans la perception du comportement de l'économie au cours d'intervalles caractérisés par des anticipations inflationnistes variables. Le présent document analyse la nature de ces distorsions et identifie les variables économiques globales les plus susceptibles d'en subir l'influence. Les variables les plus touchées sont les taux de l'épargne globale, les taux de bénéfices et les excédents ou déficits de l'État. Nous proposons une méthode générale permettant de purger les valeurs calculées de leur éléments trompeurs. A titre d'illustration, elle est appliquée à des données observées pour le taux de l'épargne personnelle aux États-Unis.

ABSTRACT

Measured income and saving variables exhibit spurious movements whenever the expected rate of price inflation changes. These movements arise because conventional measures include nominal interest receipts, net of interest expenses, as a component of income. Nominal interest rates contain an inflation premium which compensates the holders of fixed-income assets for capital losses due to expected changes in price levels. This inflation premium rises and falls with the expected rate of price inflation - thereby introducing a capital gains element into measured values of income and saving. The effect of this is to cause serious distortions in perceived economic behaviour over time-intervals when inflation expectations are variable. This paper analyzes the nature of these distortions and identifies the aggregate economic variables most susceptible to their influence. Variables most affected are aggregate saving rates, profit rates and government surpluses or deficits. A general method of purging measured values of their spurious elements is proposed. The method is illustrated in an application to observed data for the rate of personal saving in the United States.

I. Introduction

The high-inflation 1970s have seen some unusual movements in key macroeconomic variables; personal saving rates have risen to record high levels, not only in the U.S. but throughout the industrialized world. Profit rates have fallen below the averages of earlier decades, though there remains some debate as to whether this represents something more than merely a cyclical phenomenon.[1] Government deficits have increased sharply. The fact that these developments occurred during a period in which both actual and expected rates of price inflation were accelerating rapidly is not purely coincidental. The purpose of this paper is to point out a general set of dependencies which directly relate the foregoing economic events to increases in the expected rate of price inflation. The dependencies exist because of peculiarities in the conventional measure of income.

Conventional accounting practice is to include interest receipts, net of interest expenses, as part of measured real income and to exclude captial gains and losses. When the expected rate of price inflation increases, nominal rates of interest rise by an inflation premium which compensates the owner of a fixed income asset (at least partly) for the capital depreciation suffered on his asset holdings. This causes measured income to change because the actual capital loss (creditor) or gain (debtor) on the fixed income asset is not deducted from interest earnings or expenses in computing income. The measured income of creditors will rise with expected inflation while that of debtors will decline. The effects appear in both nominal and real measured incomes. Measured real saving will move in the same direction as measured real income, and saving rates will be similarly affected.

If price inflation was completely neutral in its impacts on the economy and could be perfectly anticipated, the described effects would still occur. Any movements in measured real income and saving as a result of changes in inflation under these circumstances would have to regarded as spurious-- they would not reflect any changes in economic behavior or well-being. We should expected "correct" measures of real income and saving to be invariant to price inflation in this setting.

In the remainder of this paper it will be argued that conventional measures of income and saving will contain spurious elements related to inflation expectations even in a world in which inflation is non-neutral in its impacts and can not be perfectly anticipated. It will also be argued that a number of recent observations which have puzzled economists, such as those mentioned in the introductory paragraph, are largely attributable to spurious movements associated with the rapid rise in inflation expectations during the early 1970s.

It is interesting that J.R. Hicks [1939] recognized some of the difficulties associated with conventional measures of income and saving during periods of inflation. Hicks [1939, p. 174] proposed a method of measuring income which circumvented the problem, but decided that the measure was impossible to use in a practical sense. He argued that macro-economics would be better served by avoiding the concepts of income and saving entirely and advocated an approach based on the concept of net worth. The rapid acceptance of Keynesian economics overwhelmed Hicks' preferred approach, and income and saving have become staple tools of the macro-economist. Stable inflation during the 1950s and 60s proved no obstacle to the use of these tools. It is only during the high-inflation 1970s that the problems foreseen by Hicks have arisen in serious magnitude.

Some connection between inflation and income measurement problems have been indentified in isolated instances in the recent literature. A number of writers (e.g., J. Pesando and S. Rea [1977]) have observed that in order to maintain the real value of their assets constant during an inflationary period, consumers will have to save an increased fraction of their incomes. This comes close to the recognition of an inflation-related spurious element in measured personal saving. In a different context, M. Feldstein and L. Summers [1977] observed that the rate of profit, when measured as the ratio of profits-before-interest payments to capital stock, does not have the downward shift during the early and mid-1970s that appears in the conventional profits-to-capital stock ratio. This too comes close to the identification of an inflation-related spurious element, though Feldstein and Summers did not recognize it as such.[2]

In the remainder of this paper we shall attempt to develop an analysis which reveals the nature and magnitude of these and other spurious elements in measured variables. We begin in the next section by considering the impact of a fully-anticipated price inflation on the measured income and saving of an individual in an inflation-neutral setting. The individual's net interest income is also computed using Hicks' suggested measure. The magnitude of the spurious element in the conventional measure of income is shown to be equal to the algebraic difference between the two income concepts. This provides a measure of the spurious element which is shown to be robust to conditions of uncertainty and non-neutrality.

In Section III the analysis is extended to an economy-wide basis and measures of inflation-related spurious elements in aggregate real incomes and saving are derived. A method for computing "inflation-adjusted" income and saving variables is proposed. The method is applied to one measure of the U.S. personal saving rate in Section IV.

Some extensions of the analysis are proposed in Section V.

II. Spurious Inflation Impacts on an Individual's Real Income and Saving

Consider the effects of perfectly-anticipated price inflation on an illustrative individual whose entire net worth at the start of some time period $t = 1$ is assumed to consist of a stock of nominal fixed-income assets A_0, carried forward from the end of the preceding time period. Assume that the nominal interest rate earned on these assets during any time period t is i_{t-1}, determined at the end of period $t-1$, and further assume that this rate is re-negotiated at the end of every period with perfect foresight regarding all future price inflation. Let us also assume that interest receipts are subject to a proportional tax at the rate τ. The symbol P_t will be used to denote the price level (known with certainty) during any period t .

Suppose that this individual expects to live until the end of time period $t = N$. Any assets, $A_N \geq 0$, which he holds at the time of his death will be passed on to his heirs in the form of a bequest. The real value of any such bequest will be (A_N/P_N) at the time of his death.

The individual may be presumed to have a utility function of the form $U = U(C_1, C_2, \cdots, C_N; \frac{A_N}{P_N})$, where C_t denotes his real consumption during period t .[3] At the start of period 1 he will choose the values of C_t $(t = 1, 2, \cdots, N)$ and (A_N/P_N) which maximize utility, subject to the restriction that $A_N \geq 0$ and the following budget constraint.

$$(1)\quad A_t = (1 + (1-\tau)i_{t-1})\, A_{t-1} - P_t C_t \ ; \quad t = 1,2,\cdots,N$$

The budget constraint defines the limitations imposed upon his choices by the market rate of interest, price levels, the tax rate and his initial stock of assets. The constraint is expressed in terms of nominal assets and the nominal rate of interest, but since the arguments in his utility function are all real variables, it would be more appropritate to express it in real terms. This is accomplished by simply dividing both sides of the equation by the price level P_t.

$$(2)\quad \frac{A_t}{P_t} = \left[\frac{(1 + (1-\tau)i_{t-1})}{(P_t/P_{t-1})}\right] \frac{A_{t-1}}{P_{t-1}} - C_t \ ; \quad t = 1,2,\cdots,N$$

The term in square brackets in this expression defines the after-tax real rate of interest faced by the individual at the end of period $t-1$; or, to be more precise, it measures the rate of transformation of real saving in period $t-1$ into real consumption in period t.[4] Letting ra_{t-1} denote this after-tax real rate, the budget constraint may be written as

$$(3)\quad \frac{A_t}{P_t} = (1 + ra_{t-1}) \frac{A_{t-1}}{P_{t-1}} - C_t \ ; \quad t = 1,2,\cdots,N .$$

Suppose this individual lives in a completely inflation neutral world, where changes in the perfectly-anticipated rate of price inflation have no impacts upon real decisions or constraints faced by him or any other economic agent. Suppose, initially, that the rate of price inflation expected to prevail into the indefinite future is $p = 0$, so that the future price level will remain at the constant value, P_0 , established during period $t=0$. For simplicity, assume that the nominal interest rate is also expected to remain constant at the value i_0 . With i_0 constant and $p=0$, the after-tax real rate of interest will also be constant at the value ra_0 $(=(1-\tau)i_0)$.

Let C_{0t} and A_{0N} denote the values of real consumption and nominal bequest, respectively, which maximize the individual's utility under these circumstances.[5]

With C_{ot} and A_{ON} determined, the budget constraint implies that there is only one time profile of real asset accumulation which is consistent with the optimal consumption-bequest plan. Let A_{ot} represent the nominal asset holding at the end of period t which is consistent with this plan.

Under the conditions specified here, the individual will have measured real income, after-tax, of Y_{ot}, equal to $(1-\tau)i_o(A_{ot-1}/P_o)$ during period t. He will have a personal saving rate of s_{ot}, equal to $1 - (C_{ot}/Y_{ot})$ in the same period. Consider how these measured values would differ if, instead of zero price inflation, some other rate of inflation, $p \neq 0$, were to be perfectly anticipated for time period $t=1$ and beyond.

Since the individual is assumed to live in an inflation-neutral world, we know that a change in the value of p can have no effects on the real constraints he faces or the real choices he makes. Indeed, this is the meaning of "neutrality." To translate that into the concepts developed above, a change in the value of p must leave his real budget constraint (equation (3)) unaltered. This can only happen if the after-tax real rate of interest is invariant to such a change. With ra_o unchanged by a change in p, the individual's optimal real consumption-bequest plan will not vary; C_{ot} will remain the optimal value of real consumption during period t. Similarly, the real value of the bequest in period $t=N$ will remain equal to (A_{ON}/P_O). Of course the nominal value of the optimal bequest will change with p because changes in the rate of price inflation will alter the price level P_N.

To keep the notation as unencumbering as possible, let us denote the nominal value of the optimal bequest when inflation occurs at the rate p simply as A_N. From the preceding it is clear that A_N will be equal to $P_N(A_{ON}/P_O) = (1+p)^N A_{ON}$. The change in the nominal value of optimal asset holdings will not be restricted to the terminal period. The nominal stock of assets at the end of every time period $t > 0$ will change in such a way as to keep the real value of assets at the levels which satisfy the optimal real consumption-bequest plan. Formally,

$$(4) \quad A_t = P_t\left(\frac{A_{ot}}{P_o}\right) = (1+p)^t A_{ot} \; ; \quad t = 1,2,\cdots,N ,$$

where A_t denotes the new optimal asset holding in nominal terms and P_t denotes the new price level.

With inflation neutrality,the nominal rate of interest will adjust to changes in p in such a way as to keep the after-tax real rate of interest constant at ra_o . The nominal rate must contain an "inflation premium" which compensates the holder of fixed income assets for both price changes and any changes in tax liabilities which might accompany them. The nominal rate associated with sustained inflation at rate p will be denoted simply as i . Its relationship to ra_o , p and τ is implicit in the definition of the after-tax real rate appearing in equation (2). Explicitly, the relationship is

$$(5) \quad i = \frac{ra_o(1+p)+p}{(1-\tau)} = i_o(1+p) + \frac{p}{1-\tau} \; .$$

A change in sustained inflation from zero to p will cause the nominal rate to change by $i_o p + \frac{p}{1-\tau}$. The first component $(i_o p)$ compensates for the effects of inflation on the purchasing power of interest receipts. The second component $\left(\frac{p}{1-\tau}\right)$ compensates for the capital depreciation incurred in the real value of the stock of fixed assets. This latter term is "marked up" by the tax factor $\frac{1}{1-\tau}$ because interest earnings are subject to tax, whereas capital depreciation is not.[6]

When $p \neq o$, the measured value of the individual's real income in period t is given by

$$(6) \quad Y_t = (1-\tau) i \frac{A_{t-1}}{P_t} = Y_{ot} + p\left(\frac{A_{t-1}}{P_t}\right) .$$

There has been a change in the measured value of his real income which must be regarded as spurious, because he is no better or worse off when $p \neq o$. The magnitude of the spurious change is

$$(7) \quad d_t = p\left(\frac{A_{t-1}}{P_t}\right) .$$

It is clear that $d_t \gtreqless o$ as $p \gtreqless o$; a positive rate of inflation will cause a spurious increase in measured real income, while a deflation will cause a spurious decrease.[7]

Note that since inflation is assumed to be perfectly anticipated, the right-hand-side of equation (7) may be written as $\left(\frac{p}{1-p}\right)\left(\frac{A_{t-1}}{P_{t-1}}\right)$. From this it is clear that the magnitude of the spurious element in any time period is independent of the date at which inflation at the rate p first begins. The only time-varying component of d_t is the quantity (A_{t-1}/P_{t-1}) , the real value of optimal assets at the end of period $t-1$. We have seen that this is determined solely by the optimal consumption-bequest plan and is independent of current and past rates of inflation. A return to anticipated inflation of $p = o$ at the end of period $t-1$ would eliminate the spurious element in period t and beyond.

Later in this paper we will propose some procedures designed to "purge" observed values of measured real income of their spurious elements. The result that d_t is independent of past rates of inflation is a critical finding which makes such adjustments feasible. It means that adjustments may be made to any observed value of real income in isolation; we do not have to consider the entire sequence of inflation rates which preceded the observation in time.

The individual's measured saving rate in period t will also be affected by the spurious element d_t .

$$(8) \quad s_t = 1 - \frac{C_{ot}}{Y_{ot}+d_t}$$

when $p > o$, s_t will excede s_{ot} and the opposite occurs when $p < o$.

The individual's measured saving rate will rise and fall with the perfectly anticipated rate of inflation, even though his real consumption is unaffected by such variations.

The spurious element d_t arises because the conventional income measure includes the flow of interest earnings associated with fixed income assets, but it excludes captial gains or losses in the real value of such assets. The nominal rate of interest in an inflation-neutral world includes an inflation premium which fully compensates for any changes in purchasing power of fixed income assets. When p rises, i rises to compensate for capital depreciation, and the flow of interest earnings rises by proportionately more than the price level. Since conventional accounting procedures do not deduct capital depreciation from interest flows, nominal measured income also rises disproportionately to the price level. The size of the capital depreciation in time period t is pA_{t-1} in nominal terms. In real terms the value is $p(A_{t-1}/P_t)$ -- precisely equal to the value of d_t identified in equation (7).

Conventional saving is measured as the difference between income and expenditure. Any spurious movement in measured income must also appear in measured saving. From a slightly different perspective, the conventional definition assures that measured saving will be equal to net asset accumulation. When inflation is $p > o$, our illustrative individual must accumulate $p(A_{t-1}/P_t)$ more real assets during period t than he would have in the zero inflation case. He must purchase this incremental quantity of real assets in order to offset capital depreciation and achieve the deserved real stock (A_t/P_t) by the end of the period. He therefore "saves" more during inflation in a conventional accounting sense.

The message conveyed by these results is that movements in measured real income may not provide reliable indications of changes in an individual's economic well-being during periods of inflation variability. Movements in measured saving rates may not reflect behavioral changes in the consumption-saving decision.

Inflation-Adjusted Income

Real world individuals have interest-bearing liabilities as well as assets. They also earn incomes from human capital and other real assets, but inflation appears to give rise to spurious movements only in interest flows. (This issue is examined more closely in the following section.) The preceding analysis suggests an adjustment which could be made to observed values of an individual's total after-tax income to eliminate these spurious affects. Let YD_t denote an individual's conventionally-measured real disposable income, let w_t denote his after-tax real income from non-interest sources, and let NA_{t-1} denote his net holdings of fixed income assets in nominal terms (A_{t-1} , less all debt liabilities for which interest expenses are deducted in computing conventional income).[8] YD_t is defined as the sum of w_t and $(1-\tau)i(NA_{t-1}/P_t)$. The "inflation-adjusted" measure of income suggested by the earlier results is

$$(9)\quad \overline{YD}_t = YD_t - d_t' ,$$

where d_t' is defined as $p(NA_{t-1}/P_t)$ in a straight-forward extension of equation (7).

$\overline{YD}_t$ would provide a measure of real income which is invariant to changes in perfectly anticipated inflation in an inflation-neutral world. Whether it would be appropriate to a non-neutral world with inflation uncertainty remains to be determined. As a prelude to that question, note that the definition of the after-tax real rate of interest allows $\overline{YD}_t$ to be written as

$$(10)\quad \overline{YD}_t = w_t + (1+p)ra(NA_{t-1}/P_t) .$$

The adjustment procedure amounts to replacement of the after-tax interest flow which appears in conventional income as $(1-\tau)i(NA_{t-1}/P_t)$ with the alternative measure $(1+p)ra(NA_{t-1}/P_t)$. In other words, the adjustment procedure implies that interest flows are better measured using real, as opposed to nominal, rates of interest. With inflation neutrality, the after-tax real rate is invariant to p , and equal to $(1-\tau)i_0$; hence measured income YD_t will be equal to $\overline{YD}_t$ only when the rate inflation is equal to zero.

It is interesting that J.R. Hicks considered some of these same issues in his classic Value and Capital [1939]. Hicks did not differentiate between real and nominal rates of interest but was concerned with defining a concept of income which would not vary with changes in price levels. He concluded that, "income ... must be defined as the maximum amount of money which an individual can spend this week, and still expect to spend the same amount in real terms in each ensuing week".[9] An alternative, but equivalent, way of stating this when the after-tax real rate of interest is constant over time, is to define real income as the maximum amount the individual can consume in any period, and still leave his end of period real net worth unchanged.[10]

Hicks did not confine his definition to income from fixed-income assets and was convinced that it could never be computed in the real world. Computation would require knowledge of all future receipts and price levels -- obviously an impossibility. However when applied only to interest flows, it is not impossible to compute, even in an uncertain world.[11] In an inflation-neutral world with perfect certainty, it is obviously simple to compute Hicks income from net fixed-income assets, NA_{t-1} . The value is $(1+p)ra(NA_{t-1}/P_t)$ -- precisely the concept we have argued is appropriate to inflation-adjusted income, $\overline{YD}_t$.

We are now in a position of being able to interpret the inflation-adjustment procedure as replacement of conventional net interest income with Hicks income derived from interest-bearing net assets. The spurious element appearing in the conventional measure is the difference between net interest receipts and Hicks income. Let us now extend these ideas to a world of uncertainty and non-neutrality. We shall continue to define as spurious any inflation-related movement in conventionally measured interest income which does not also appear in Hicks income. If a change in inflation expectations causes conventional interest income to change by some amount X_1 and Hicks income by some other amount X_2, then the difference, X_1-X_2, ought to be regarded as spurious.

Uncertainty and Non-Neutrality

Inflation expectations are seldom held with certainty in the real world. Differences between *ex ante* expectations and *ex post* realizations can, and often do, occur. Real world inflation is also not neutral in its impacts upon real opportunities and decisions. The effects of inflation on real money balances and upon the tax liabilities of corporations provide two examples of how a change in the rate of inflation will cause a re-distribution of real resources.[12] In addition, there is a growing body of empirical evidence which suggests that a change in inflation expectations alters nominal interest rates on approximatley a one-to-one basis; i.e., a rise in the expected rate of inflation by some amount, Δp^e, will cause nominal interest rates to rise by approximately this same amount.[13] Under these circumstances, the after-tax real rate of interest facing any investor with a positive tax rate will vary inversely with the expected rate of inflation.

Under conditions of uncertainty, Hicks income during period t is defined as the maximum amount an individual can consume during the period, given the realization P_t, and still expect to be able to consume the same amount in real terms in each ensuing time period.[14] Under the assumption that the individual expects to be able to earn the same *ex ante*, after-tax real of interest, ra_{t-1}, in period t and all future time periods, Hicks income for an individual with initital net assets of NA_{t-1} is $(1+p^e_t)ra_{t-1}(NA_{t-1}/P_t)$.[15] The variable p^e_t is the rate of price inflation expected to prevail during period t , with the expectation held as of the start of the period. The spurious element in conventional income in this case is $p^e_t(NA_{t-1}/P_t)$ -- analagous to the earlier expression, except that p^e_t is now the appropriate inflation variable. The interesting result is that the spurious element, and hence the appropriate adjustment factor to use in "correcting" measured income, depends only upon p^e_t . This is independent of the fact that inflation may be non-neutral in its effects upon interest rates.

Does this result make sense? A simple illustration should convince us that it does. Suppose that inflation was formerly zero but rises to some expected rate $p^e_t > 0$ at the start of period t . Consider the kind of inflation premium which appears to operate in the real world: the nominal rate of interest rises from i_0 to $i = i_0+p^e_t$, a one-to-one response. If inflation expectations are realized in period t , the real value of conventionally- measured interest income will change in proportion to $\left(\frac{1+p^e-i_0p^e}{i_0(1+p^e)}\right)$. For values of $i_0 < 1$ this will surely be positive in sign, and conventionally-measured income will rise. But if the individual faces a tax rate $\tau > 0$, ra_{t-1} will decline as a result of this and he will be worse off. Hicks income will decline. The conventional measure does show a spurious increase under these circumstances, and if we believe that Hicks income provides a better measure of interest income, then the magnitude of the spurious effect is $p^e(NA_{t-1}/P_t)$ as derived above.

How does unexpected inflation affect this result? Suppose $p^e_t = 0$, but inflation at the rate $p_t>0$ occurs unexpectedly during period t . The individual is surely worse off. Hicks income will decline by the amount

$p_t\ ra_{t-1}(NA_{t-1}/P_{t-1})$. Conventionally-measured income will also decline in this case by the amount $p_t(1-\tau)i_o(NA_{t-1}/P_{t-1})$. Since ra_{t-1} will be equal to $(1-\tau)i_o$ when p_t^e is equal to zero, both income measures will decline by the same amount. There is nothing spurious in this case because inflation was not anticipated. This is also a plausible result.

III Spurious Elements on an Economy Wide Basis

Since the spurious elements in the conventionally-measured real income of an individual are robust to conditions of uncertainty and non-neutrality, we can probe for their existence in an economy-wide setting by considering the impacts of a fully-anticipated price inflation in a hypothetical, inflation-neutral economy. Let us assume that this hypothetical economy consists of a government sector, a corporate sector and a personal sector. The economy will be assumed to be completely closed, so that no foreign transactions of any sort enter the analysis.[16] We shall further assume that all production in this economy is conducted by corporations, and that all corporate debt and equity claims are held by domestic residents.

In order to ensure inflation neutrality in the tax system, taxes will be assumed to be levied on nominal receipts at the same constant marginal = average tax rate, τ, for both persons and corporations. Corporations are permitted to deduct depreciation expenses at replacement costs. The corporate and personal tax systems are assumed to be fully integrated, so that double taxation does not arise. This integration is assumed to take the following form, though any other would serve equally as well:

1) Interest payments on corporate debt are an allowable tax deduction for corporations, but interest receipts of persons are taxed at full rates.

2) Corporate taxes are levied on nominal profits before dividend payments, but dividend receipts of persons are tax exempt.

3) Inflation-related capital gains accruing to equity-owners are not taxed.

To further insure that inflation is neutral in its impacts on real opportunities, we assume that the after-tax real rate of interest, ra, is constant and invariant to changes in the rate of inflation. The nominal rate of interest is, therefore, given by

$$(11) \qquad i = \frac{ra(1 + p)}{(1 - \tau)} + \frac{p}{(1 - \tau)} ,$$

where p is the fully anticipated rate of price inflation. To avoid the non-neutrality problems which can arise in the real world where inflation alters the real rate of return derived from holding money balances, we shall assume that money, if it exists at all, yields interest at the nominal rate, i. Money is, therefore, indistinguishable from government debt in this economy.

Finally, in order to keep the notation simple as possible, we make the following additional assumptions - none of which is critical to the analysis.

1) Corporations pay all receipts in excess of expenses out as dividends.

2) The population of this economy is static and real economic growth is zero.

3) In consideration of (2), net investment is zero. Gross investment represents replacement expenditures and is equal to tax-allowable depreciation expenses.

4) All government and corporate debt outstanding is held by persons.

This economy is assumed to have been in a long-run, no-inflation equilibrium prior to the advent of a sustained bout of fully-anticipated price inflation. For convenience let the price level be $P_0 = 1.0$ during this zero-inflation period. Inflation at the rate p begins at the start of time period $t=1$, and all wages and prices rise at the same rate. Debt contracts are renegotiated at the end of every period so that interest payments in period t are based on the nominal rate of interest determined at the end of $t-1$.[17] As a consequence of these assumptions, the nominal rate of interest is $i_0 = ra/(1-\tau)$ prior to period $t=0$, and rises to the new value i, given above, for $t = 0, 1, 2, \ldots$.

Let us compare the conventionally-measured values of aggregate variables before- and after-inflation with the aid of the following notation. For any nominal income or saving flow variable, X, X_t will denote the value in period $t \geq 1$, while $\overline{X}$ will denote its (constant) value during the pre-inflation era. Real values are given by X_t/P_t. Note that since $P_0 = 1$, $\overline{X}$ represents both the nominal and the real value of variable X prior to the onset of inflation. Since conventionally-measured real values coincide with Hicks' measure when $p = 0$, and all real assets are constant over time, $\overline{X}$ also measures real income prior to inflation using the Hicks' definition. This economy is inflation neutral; thus $\overline{X}$ also measures Hicks' income after the onset of inflation. Any differences $(X_t/P_t - \overline{X})$ which show up in the post-inflation period must, therefore, be spurious movements in the conventionally-measured value.

One final point: For any asset or liability variable, A, A_t will denote the nominal stock at the end of time period t, for $t \geq 0$. $\overline{A}$ will denote its value during the pre-inflation period. $A_0 = \overline{A}$ for all such variables. The price level, P_t, will be equal to $(1+p)^t$ for all $t \geq 0$.

Aggregate Income

The before- and after-inflation values for total nominal income, or GNP, are given in equations (12) and (12A) respectively.

$$(12) \qquad \overline{Y} = \overline{C} + \overline{I} + \overline{G}$$

$$(12A) \qquad Y_t = C_t + I_t + G_t = (1+p)^t \, \overline{Y}$$

Real consumption, investment and government expenditures are unaffected by a completely anticipated inflation in an inflation-neutral economy, so the nominal value of income rises proportionate with the price level. The conventionally-measured value of real GNP remains equal to $\overline{Y}$. There are no spurious effects here.

The Government Sector

We shall assume, consistent with the assumptions made earlier, that the government was in a zero net balance position prior to the advent of inflation. During this period outstanding government debt was constant at $\overline{DG}$. The nature of the tax system in this economy is such that total tax revenues are effectively generated by applying the tax rate, τ, to nominal GNP less investment outlays, plus interest on the government debt. Expressions for the government surplus, SG, before and after inflation are, therefore, given by

$$(13) \qquad \overline{SG} = \tau(\overline{Y} - \overline{I} + i_0\overline{DG}) - \overline{G} - i_0\overline{DG} = 0$$

$$(13A) \qquad SG_t = \tau(Y_t - I_t + iDG_{t-1}) - G_t - iDG_{t-1} < 0$$

The effect of inflation is to increase interest charges on the government debt disproportionately to increases in tax receipts and expenditures. Assuming $\overline{DG} > 0$, initially, the government sector is forced into a deficit position with the onset of inflation. The government must borrow in order to maintain constant real expenditures on goods and services. It is easy to verify that the government debt will assume the following time path:

$$DG_t = (1 + p)^t \, \overline{DG} \; ;$$

i.e., the nominal value of the debt will rise with the price level. The real value of the debt $DG_t/(1 + p)^t$ has, of course, been unaffected. There has been no transfer of real wealth between the government sector and the rest of the economy.

Substituting for DG_t in equation (13A), the post-inflation government surplus may be expressed as

$$(14) \quad SG_t = (1+p)^t \overline{SG} - pDG_{t-1} = (1+p)^t \left[\overline{SG} - \frac{p}{(1+p)} \overline{DG} \right]$$

The real value of this conventional measure is $SG_t/(1+p)^t = \overline{SG} - \frac{p}{(1-p)} \overline{DG}$, which is clearly less than Hicks value, $\overline{SG} = 0$. There has also been a spurious downward movement in the measured real value of the government surplus. Note that the magnitude of the spurious component is independent of the length of time inflation has been proceeding. Its size is the same in $t = 1$ as it is in any later period. In other words, spurious elements in real SG depend only on p and do not cumulate over time. It will appear to observers of this economy that the government is incurring a real deficit as a consequence of inflation, starting in period $t = 1$. This is only an illusion arising from the measurement error. In truth, the government sector is neither better off nor worse off than before the inflation occurred - a fact confirmed by the constancy of its real debt.

The spurious element here also appears in the government saving rate, which we define as the ratio of SG to nominal GNP.

$$(15) \quad sg_t = \overline{sg} - pDG_{t-1}/Y_t$$

In the real world, variations in the expected rate of price inflation cause spurious movements in the observed values of this saving rate. This makes inter-temporal comparisons of values of sg both difficult and confusing. The obvious solution is to compute inflation-adjusted values by adding to observed values of sg_t, the quantity $p_t^e DG_{t-1}/Y_t$, where p_t^e is the one-period expected rate of inflation and DG is measured at market value.[18] If the outstanding government debt was issued in the form of long-term liabilities some time prior to the end of period $t-1$, there is a chance that the expectations of the rate of inflation may have been revised in the period of time which has since elapsed. If so, the coupon rate of interest on outstanding debt will not equal the nominal rate of interest prevailing in the market at the end of period t-1. In that event, the book value of government debt will not equal its market value. It is important to realize that the appropriate correction factor is based upon the market value of the government debt at the end of time period t-1 . The use of book values would be incorrect.

A final consideration in regard to the government sector applies to the conventionally-measured value of real tax collections. These are given by

$$(16) \qquad \overline{TX} = \tau(\overline{Y} - \overline{I} + i_0\overline{DG})$$

$$(16A) \qquad TX_t/P_t = \overline{TX} + \frac{\tau}{(1-\tau)}\, pDG_{t-1}/P_t$$

There is a spurious element here which makes the real tax burden faced by the private sector of the economy appear to rise with increased price inflation. Holders of government debt in this economy have been fully compensated for inflation via the tax-adjusted inflation premium in the nominal interest rate. Their real tax burden cannot have truly increased. Hicks' measure of this, $\overline{TX}$, shows that to be the case, and we must regard the element, $(\frac{\tau}{1-\tau})pDG_{t-1}/P_t$, which appears in the conventional measure to be a spurious effect. Once again, the correction factor to apply to real world observations is suggested by the expression for the spurious element.

The Corporate Sector

The market value, V , of corporate enterprises represents the present discounted value of future after-tax interest and dividend payments. The appropriate present value computation involves discounting real flows by the after-tax real rate of interest. The resulting market values are expressed in real terms.

$$(17) \qquad \overline{V} = \sum_{j=1}^{\infty} \left[\frac{(1-\tau) i_0 \overline{DC} + (1-\tau)(\overline{Y} - \overline{W} - \overline{I} - i_0 \overline{DC})}{(1+ra)^j} \right] = \frac{(1-\tau)(\overline{Y}-\overline{W}-\overline{I})}{ra}$$

$$(17A) \qquad V_t = \sum_{j=1}^{\infty} \left[\frac{(1-\tau) i (DC_{t-1}/P_t) + (1-\tau)(Y_t - W_t - I_t - i\, DC_{t-1})/P_t}{(1+ra)^j} \right] = \overline{V}$$

W here denotes total wages and salaries and DC denotes the nominal value of corporate debt. The result that $V_t = \overline{V}$ obtains because Y_t, W_t and I_t all grow at the same rate as the price level. The reader should recall that tax depreciation is assumed to equal economic depreciation and both are equal to I.

These results show that inflation has no affect on the real value of corporation enterprises. V is, of course, distributed between debt and equity on some basis determined by the equity owners. Let us assume that prior to the onset of inflation, the debt/equity ratio was constant, so that $\overline{V} = \overline{DC} + \overline{E}$, where E denotes the nominal value of equity. After inflation $(1+p)^t V_t = DC_t + E_t$, and the relationships between DC_t and $\overline{DC}$ and E_t and $\overline{E}$ remain to be established.

We have made no assumptions which would lead to any particular value for the debt/equity ratio being preferred to any other value.[19] In the absence of compelling arguments to the contrary, we shall assume that those factors which led to the choice of the debt/equity split prior to inflation continue in force after inflation begins and that the debt/equity ratio remains constant. This implies

$$DC_t = (1 + p)^t \overline{DC} ; \qquad E_t = (1 + p)^t \overline{E}$$

i.e., the real values of debt and equity remain constant over time.

Profits before taxes, Π, can now be compared between the two regimes.

$$(18) \qquad \overline{\Pi} = \overline{Y} - \overline{W} - \overline{I} - i_0 \overline{DC}$$

$$(18A) \qquad \Pi_t = (1 + p)^t \overline{\Pi} - \frac{p}{(1 - \tau)} DC_{t-1}$$

There is a reduction in conventionally-measured profits as a result of the acceleration in price inflation. In real terms, the reduction is $[\frac{p}{(1-\tau)} DC_{t-1}/P_t]$ and it must be spurious, for we have just seen that the real values of debt and equity have not been affected by the inflation. The magnitude of this spurious element, like that of its government sector counterpart, is a function of p but is independent of the length of time the economy has been inflating.

Movements in aggregate U.S. profit rates since the advent of high inflation in the early 1970s have been a topic of some discussion in the recent literature. William Nordhaus [1974] initiated this discussion by observing a downward trend in corporate profit rates over the post-war period that has seemingly accelerated during the 1970s. We have little to say about movements in the profit rate during the earlier years, but the 1970s experience is surely in part a spurious response to accelerated rates of price inflation. If the profit rate, π, is defined as the ratio of before-tax profits to the replacement value of the capital stock, K, the following relationship emerges from our hypothetical economy.

$$(19) \qquad \pi_t = \Pi_t/K_t = \bar{\pi} - \frac{p}{(1-\tau)} \left[\frac{DC_{t-1}}{K_t}\right]$$

The higher the expected rate of price inflation, the further will π_t be depressed below its zero-inflation value. If the expected rate of inflation accelerates, as it did in the first half of this decade, π_t will show a spurious downward trend. Note that these effects do not disappear if we choose to measure the profit rate as the ratio of Π_t to some other nominal value, say P_tV_t or Y_t.

Inflation-induced spurious movements in corporate profits can not totally account for the profit rate performance of recent years. A great many other variables are involved here; many of them cyclical in nature. Nonetheless it is interesting that in a recent study, Martin Feldstein and Lawrence Summers [1977] have concluded that when the profit rate is measured as the ratio of before-tax profits plus interest payments to the value of the capital stock, the experience of non-financial corporations in the 1970s does not appear out of line with earlier experience. The inclusion of interest payments in the numerator of the profit rate measure makes sense to Feldstein and Summers because it offsets, at least partially, the effects of shifts in the debt/equity ratio on profit performance. From the point of view of our simple model it makes additional sense because it eliminates the spurious element which arises in conventionally-measured interest payments.

From the assumption that corporations distribute all residual receipts to equity owners, dividends, Z, in our hypothetical economy may be expressed as

$$(20) \qquad \bar{Z} = (1 - \tau)\,\bar{\Pi}$$

$$(20A) \qquad Z_t = (1 - \tau)\,\Pi_t + (DC_t - DC_{t-1}) = (1 + p)^t\,\bar{Z}$$

Increments to debt provide corporations with additional resources sufficient to compensate for higher after-tax interest payments. The nominal flow of dividends will therefore rise at the rate of price inflation, keeping the real value of dividends constant over time. This result is completely consistent with the earlier finding that, when the debt ratio is maintained at its pre-inflation value, the real value of E_t will be constant over time. Since real E_t is the discounted present value of future real values of Z_t, equation (20) implies $E_t = (1+p)^t\,\bar{E}$ and vice versa.

That dividends do not undergo a spurious movement when expected inflation rises is a significant finding. It means that spurious increases in the real incomes of holders of coroprate debt will not be offset by spurious decreases in the real incomes of corporations' equity-owners. The correction factors suggested in Section II are applicable to the aggregate incomes of persons. Note that this finding also eliminates any skepticism which might have remained about whether our so-called spurious movements are really spurious. If the rise in corporations' interest payments were a real effect due, say, to a rise in the real rate of interest, then either dividend payments would decline or the value of equity would decline, or both. The fact that the real value of neither dividends nor equity changes in our example proves that we must be dealing with spurious effects. There is no reason for spurious effects to be offsetting.

At this point in the analysis the remaining post-inflation results for our hypothetical economy are straightforward.

The net corporate saving rate, defined as the ratio of retained earnings plus depreciation to nominal GNP is given by

$$(21) \qquad sc_t = [(1-\tau)\Pi_t - Z_t]/Y_t = \overline{sc} - p(DC_{t-1}/Y_t) .$$

The Personal Sector

Gross assets, including the present value of human capital, of the personal sector are given in nominal terms by[20]

$$(22) \qquad \overline{GA} = \overline{DG} + \overline{DC} + \overline{E} + (1-\tau)\overline{W}/ra .$$

$$(22A) \qquad GA_t = DG_t + DC_t + E_t + (1+p)^t(1-\tau)\overline{W}/ra = (1+p)^t\, GA .$$

This verifies that inflation has no effect on the real value of total assets, which is equal to $\overline{GA}$, at all points in time. The same applies to each of the subcomponents as a result of our constant debt/equity ratio assumption. If corporations do alter debt/equity ratios after the onset of inflation, the real values of DC_t and E_t will differ from their pre-inflation values but their sum, V_t , will be invariant.

After-tax, real disposable income is an aggregate extension of our Section II finding.

$$(23) \qquad \overline{YD} = (1 - \tau)(\overline{W} + i_0\overline{DG} + i_0\overline{DC}) + \overline{Z}$$

$$(23A) \qquad YD_t/P_t = \overline{YD} + p(DG_{t-1} + DC_{t-1})/P_t$$

We can define two alternative measures of the rate of personal saving. The first is the ratio of personal saving to personal disposable income which is the familiar definition. Let us denote this sp* , and define sp as the ratio of personal saving to nominal GNP.

$$(24) \qquad sp_t^* = \frac{(YD_t - C_t)}{YD_t} = \frac{(\overline{YD} - \overline{C} + p(DG_{t-1} - DC_{t-1})/P_t)}{\overline{YD} + p(DG_{t-1} - DC_{t-1})/P_t} > \overline{sp^*}$$

$$(25) \qquad sp_t = \frac{(YD_t - C_t)}{Y_t} = \overline{sp} + p(DG_{t-1} + DC_{t-1})/Y_t > \overline{sp}$$

Both sp* and sp show spurious increases due to positive price inflation. They are distinguished from one another for two reasons. First, the correction necessary to purge each of its spurious element differs. The real world is more complicated than the simplified economy shown here, and the inflation-adjustments are therefore more complicated than those implied by equations (24) and (25). Real world individuals are debtors as well as creditors, which implies that corrections to measured YD and personal saving must be based on net assets. But interest payments made on consumer debt are regarded as a transfer from persons to businesses in the National Income and Product Accounts. This means that the net assets concept necessary to purge personal saving of its spurious element is lower than the net assets concept necessary to purge disposable income by the outstanding amount of consumer debt.[21] An inflation-adjusted value for sp* may be computed as the ratio of adjusted personal saving to adjusted disposable income. An inflation-adjusted value for sp may be computed as the ratio of adjusted personal saving to GNP.

The second reason for distinguishing between sp* and sp is that the latter, together with the earlier-defined variables sg and sc, provides a picture of the way in which a change in expected inflation alters the apparent allocation of net saving among the various sectors of the economy. From the way the saving rates have been defined, the sum sg + sc + sp must equal zero. A rise in the expected rate of inflation will reduce the measured saving of governments and corporations and increase that of persons by an offsetting amount. These are, of course, spurious effects and do not result from any changes in real behaviour.

IV. An Inflation-Adjusted Measure of the U.S. Personal Saving Rate

The preceding sections have shown that spurious elements in certain measured real income and saving variables arise when the *ex ante* real and nominal after-tax rates of interest diverge. This occurs whenever the expected rate of price inflation is not equal to zero and is independent of whether nominal rates fully incorporate an inflation premium. This finding has some important implications for both the inter-temporal comparison of economic variables and empirical research.

When the expected rate of inflation is variable over time, some portion of the period-to-period movements in observed values of conventionally-measured economic variables will be spurious in nature. For example, the onset of a period of accelerating inflation, such as occurred in the early and mid-1970s, will cause spurious declines in profits and spurious increases in government deficits, real disposable incomes and personal saving rates - provided that at least some fraction of the more rapid inflation is anticipated.

If they are unaware of the existence of these spurious elements, economists are apt to find observed movements in economic variables puzzling and may incorrectly infer that there have been some fundamental changes in economic behaviour. There may even be some temptation to conclude that traditional economic theory is inadequate to explain observed phenomena. The remedy for this is to identify the spurious components and compute "inflation-adjusted" values of the affected variables. We have made suggestions as to how this might be done in a number of instances. To add substance to those suggestions and shed some new light on an inflation-related observation that seems to have baffled economists in recent years, we shall use the remaining portion of this section to compute an inflation-adjusted value for one measure of the U.S. personal saving rate -- the ratio of personal saving to disposable income, sp^* .

After two decades of relative stability, observed values of sp^* began to rise in the late 1960s and reached historically high values in the early and mid-1970s before declining towards more "normal" values in 1976-78. The late 1960s and early-to-mid 1970s was, of course, a period of accelerating inflation, and a number of research efforts have been aimed at explaining the apparent correlation between personal saving and the rate of inflation. Most studies (e.g., T. Juster and P. Wachtel [1972a], [1972b]; P. Wachtel [1977]) have attributed the behaviour of the early 70s to increases in the risk associated with higher expected rates of inflation.[22] Theoretically, housholds ought to alter their consumption-saving decisions in response to changes in inflation risk, σ. However, theory does not predict the direction of the effect; $\partial sp/\partial\sigma$ could be either positive or negative depending upon individuals' attitudes towards risk and the distribution of lifetime earnings. Empirical studies which include σ among the traditional arguments of the saving function are bound to find a positive relationship because σ is the one determinant of sp* which has systematically risen and fallen with the rate of price inflation. Thus σ seems to "explain", the behaviour of sp* in the early 70s, but this may be nothing more than the spurious outcome of an inappropriately measured dependent variable.

Using the notation of the previous section, inflation-adjusted values for the ratio of personal saving-to-GNP were computed annually for 1958-78 from

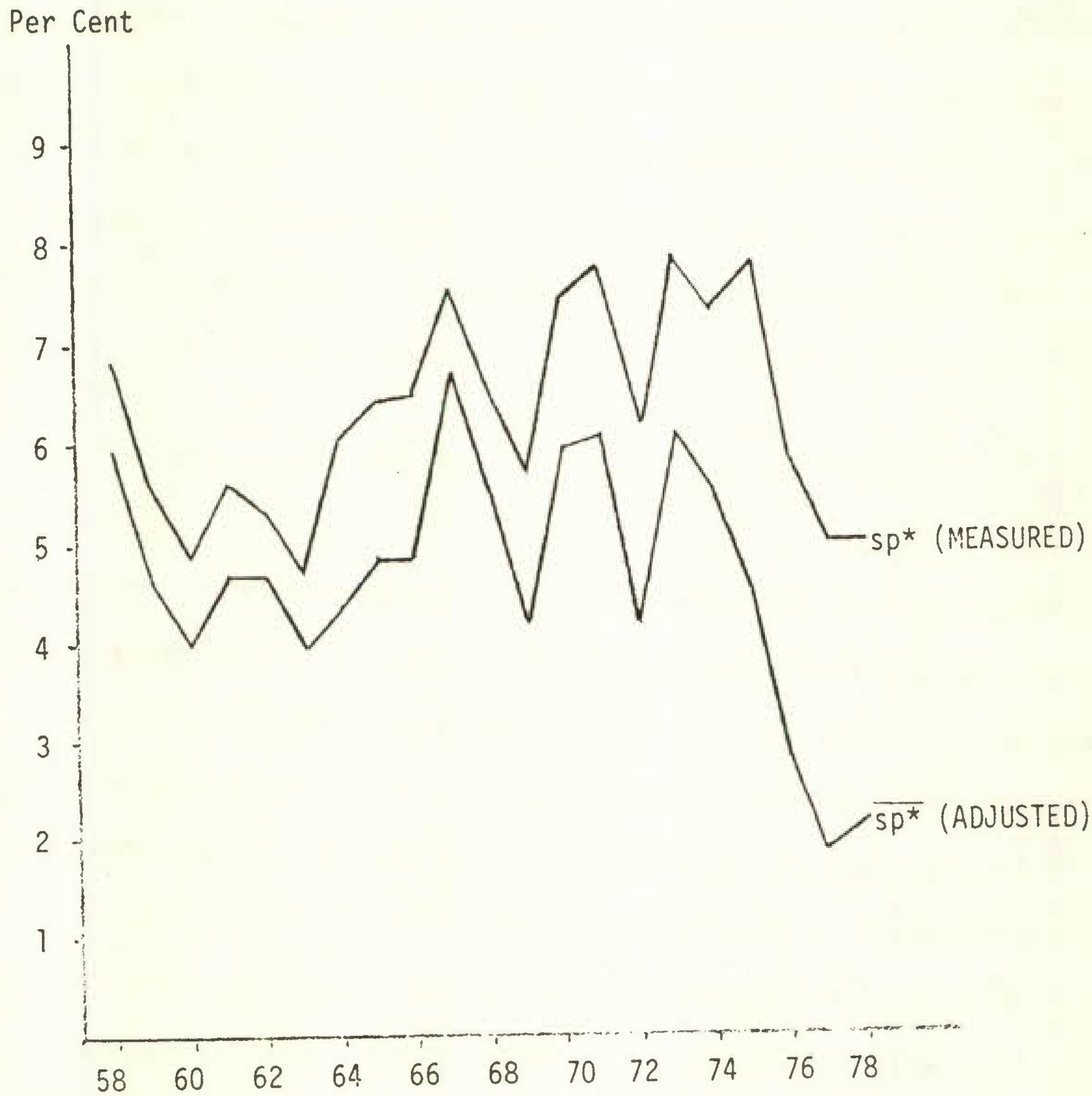
$$(26) \qquad \overline{sp^*_t} = \frac{\text{measured saving in } t - p^e_t(NA_{t-1} - CC_{t-1})}{YD_t - p^e_t NA_{t-1}} ,$$

where CC denotes the end-of-period stock of consumer debt. Measured saving and income data were taken from the NI&P Accounts. Estimates of the one-year expected rate of price inflation were estimated from an autoregressive process. Estimates for the market values of NA and CC were computed using book-value data for the household sector obtained from the Flow of Funds Accounts. Details of the computations are given in the Appendix.

The inflation-adjusted value of the personal saving rate is plotted along with its conventionally-measured counterpart in Figure I.[23] The conventional measure, sp* , shows an upward drift during period from about 1968 to 1975. This variable returned in 1976-78 to values considered to be more "normal" by historical standards. The inflation-adjusted series, $\overline{sp^*}$, did not show an

FIGURE 1

Measured versus Inflation-Adjusted Personal Saving
as a Per Cent of Disposable Income



upward trend during 1968-75, but did drop very sharply in the 1976-78 sub-interval. Whether this sharp decline represents some true behavioural result or is simply an error in the adjustment procedure is a question for future research. However it should be noted that measured saving rates for the U.S. have declined relative to those in other industrialized countries since 1975, and when a similar inflation-adjustment is applied to Canadian saving data, the resulting series looks very stable during the entire 1968-78 sub-period.[24]

It would be presumptious to draw more conclusions from these computations, but it does seem clear that spurious elements can account for many of the anomalies in observed saving patterns of the early 1970s. The magnitudes involved are quite substantial. There is every reason to suppose that inflation-adjusted values for other income and saving variables would also reveal spurious elements of significant size.

V. Some Extensions

This paper has identified spurious elements which arise in measured real income and saving variables because of the difference which exists between the *ex ante* values for the nominal and the real after-tax rates of interest. The focus has been on aggregate variables in a closed economy. Extension of the analysis to an open economy would be relatively straightforward, and it is interesting to speculate on the additional spurious elements which would arise in an international setting.

Measured imports include the value of interest paid to non-residents, while exports include interest received from abroad. An anticipated rise in world-wide inflation would cause spurious increases in both real imports and real exports in an open economy. If that economy is a net debtor to the rest of the world, the increase in imports would exceed that in exports, and its real GNP would undergo a spurious decline. This would not be a behavioural effect, so employment would not be altered directly. The economy would appear to have suffered a decline in productivity which might be mis-interpreted as an indication of poor economic performance. Of course the opposite would occur if the economy were a net-creditor abroad.

Spurious effects would occur in the balance of payments. The net-debtor economy would see a spurious worsening in its current account position, offset by an increased inflow of long-term capital; i.e., an increased outflow of new debt issues. This might also be mis-interpreted as a symptom of poor economic performance, though it, too, is not a behavioural effect. Whether there would be speculative repercussions in the foreign exchange market depends on whether participants in that market are able to "see through" the spurious elements.

The spurious consequences in an open economy of an increase in the expected rate of domestic inflation relative to inflation in the rest of the world are less obvious unless a purchasing-power parity determination of the foreign exchange rate holds in the short run. In that case, the spurious effects would be much the same as above.

These considerations, together with those described in the main body of the paper, have important implications for the way economists interpret measured movements in real variables. We have learned to look through the veil of inflation when dealing with nominal magnitudes. It appears we must learn to look through another veil at real magnitudes.

FOOTNOTES

1. For a discussion of this issue, see the papers by Nordhaus [1974] and Feldstein and Summers [1977].

2. They attribute the result to a change in debt/equity ratios, which may be partly correct.

3. The bequest enters the utility function in real, as opposed to nominal terms because the individual is presumed to be concerned with the well-being of his heirs. His heirs will be better off, the larger the stream of real consumption they can obtain over their lifetimes. This is affected by the real value of the bequest.

4. The concept of a real rate of interest is, of course, attributable to the classic work of Irving Fisher [1896], [1930]. Fisher dealt with a taxless world in which the real rate, r, is related to the nominal rate, i, and the rate of price inflation, p, through the famous Fisher equation:

$$r = \frac{i-p}{(1+p)} .$$

Fisher emphasized that $(1+r)$ measures the rate at which consumers can transform current real saving into future real consumption. Indeed that is the basis of Fisher's equation. In subsequent literature, the p in the denominator of the original equation has often been ignored, and the approximation, $r \doteq i - p$, is the more familiar equation. In a world where nominal interest receipts are taxed at the rate τ, the rate of transformation of current real saving into future real consumption is $(1+ra)$, where ra is the after-tax real rate of interest. The analogue to Fisher's equation in a world with taxes is

$$ra = \frac{(1-\tau)i-p}{(1+p)} .$$

This is the basis of the concept of the after-tax real rate used in this paper. The p term in the denominator of this expression is retained throughout the analysis.

5. The specific nature of the bequest motive has no bearing on the analysis to follow; in particular it does not matter whether the individual chooses to leave a non-zero bequest. What does matter is that he is able to define an optimal real consumption stream which dominates any other feasible stream.

6. The point that the inflation premium embodied in the nominal rate of interest should contain the tax-adjusted term $\left(\frac{p}{1-\tau}\right)$ in an inflation-neutral world appears to have first been emphasized by Darby [1975].

7. The restriction that the results apply only during $t \leq N$ has been discarded, beginning with equation (6). The spurious element, d_t , will persist for as long as inflation remains at p . The heirs of the illustrative individuals will also live in this inflation-neutral world. Their measured real incomes will also contain spurious elements analogous to d_t .

8. In general, any interest expenses incurred in the process of earning receipts are deducted in computing measured income.

9. Hicks [1939], p. 174.

10. The second statement is essentially the wording used by Milton Friedman [1957] in defining his concept of permanent income.

11. This issue is considered later in this section.

12. A number of recent studies have investigated the impacts of inflation on the taxation of corporations. The consensus finding is that corporations face higher real tax burdens when inflation rises because depreciation expenses are allowable only at historical costs. For a discussion of this and other

issues, see Shoven and Bulow [1976], Jenkins [1977], and Feldstein and Summers [1978].

13. See, for example, the studies by Yohe and Karnosky [1969], Eckstein and Feldstein [1970], and Feldstein and Summers [1978] using U.S. data, and the studies by Carr and Smith [1972] and Carr, Pesando and Smith [1976] using Canadian data. A paper by Sargent [1976] presents a summary of recent empirical literature.

14. Hicks was careful to distinguish between ex ante and ex post versions of his income concept under conditions of uncertainty. The definition in the text is the ex post version, based on the realization of the actual price level P_t . This seems the appropriate concept for the issues being examined in this paper, insofar as other components of income can be measured only ex post; i.e., after they are realized. The ex ante version defines income as the maximum amount the individual expects to be able to consume at the start of period t , and still be able to expect to consume the same amount in real terms in each ensuing time period. The ex ante version is based on the expectation that P_t will equal $(1+P_t^e)P_{t-1}$.

15. There is practically no empirical evidence to judge whether it is reasonable to assume individuals expect to earn the same after-tax real rate of interest, in all future periods. The issue is really one involving the term structure of after-tax real rates of interest. Most empirical studies of the term structure have looked only at nominal rates and have found support for the Expectations Hypothesis. However, there is nothing inconsistent with these findings and the assumption. If the marginal investor does expect to earn the same after-tax real rate over all holding periods, then nominal rates on long-term securities can still differ from those on short-term

securities, provided that long-term inflation expectations differ from short-term inflation expectations. In fact, the relationship between long-term rates and current and future expected short-term rates implied by the Expectations Hypothesis will still hold for nominal interest rates under these cirumstances. If the assumption is not valid, Hicks income becomes $(1+p_t^e)\overline{ra}\left(\frac{1+ra_{t-1}}{1+\overline{ra}}\right)(A_{t-1}/P_t)$, where $\overline{ra}$ is the after-tax real rate of interest the individual expects to earn in all subsequent periods starting with $t+1$. Measured income still contains a spurious element in this case, but its indentification is more complicated than the expression given in the text.

16. This assumption is made strictly for reasons of simplification. The inclusion of a foreign sector could be readily accommodated by an extension of the analysis which follows. Some comments regarding spurious elements arising in the foreign sector are offered in the final section of the paper.

17. This assumption is not critical to the analysis. It merely allows us to dispense with the distinction between market values and book values of assets and liabilities.

18. A logical question at this point is, why not compute adjusted government saving using the actual rate of inflation, p_t , instead of the expected rate, p_t^e ? This would be simple to compute since p_t can be observed directly. The answer to this question hinges on what is considered to be the most useful concept of income. A correction based upon p_t effectively measures interest income using the <u>ex post</u>, after-tax, real rate of interest. A correction based upon p_t^e effectively measures interest income using the <u>ex ante</u>, expected value of the after-tax real rate of interest. The latter concept has an economic interpretation; it is Hicks income, or in more familiar terms, it is the permanent income derived from fixed-income assets. The

former concept has no ready interpretation. Since permanent income has been found to be a useful concept in consumer theory, the adjustment based on p_t^e is conceptually more appealing.

19. Capital gains on equities are not taxed in our hypothetical economy, but full integration of the corporate and personal tax systems make it impossible for equity-owners to avoid being taxed on the corporate earnings which give rise to capital gains. The tax structure does not therefore give preference to any particular debt/equity ratio. In fact, debt and equity claims are perfect substitutes because there is also no uncertainty.

20. The personal sector as a whole may regard its net worth to be different from gross assets as shown here. In particular, government debt may not be regarded as net worth in the formulation of consumption-saving decisions. That issue has no bearing upon the existence of spurious elements in measured income and saving flows.

21. The NI&P Accounts also include some complicated imputations involving mortgage interest paid by the owner-occupants of residential dwellings. The net effect of these imputations is to make mortgage liabilities a "normal" fixed-income liability; i.e., they reduce the net assets of the personal sector in the same way as do other fixed-income liabilities.

22. One exception has been a study by Angus Deaton [1977]. Deaton offers an argument why unexpected inflation might lead to an increase in saving, above and beyond its impacts via lowered real wealth and real income. He does not present an explanation of why expected inflation might affect saving.

23. It is important to give the correct interpretation to $\overline{sp_t^*}$. This adjusted saving rate is an estimate of the measured saving rate which would have prevailed with $p_t^e = o$, but with ra_{t-1} and all other constraints faced by

consumers the same as those which actually prevailed during period t. It is not correct to interpret $\overline{sp^*_t}$ as simply the measured rate of saving associated with $p^e_t=o$. The latter intepretation would be correct only in an inflation-neutral world, where changes in p^e_t do not alter consumers' budget constraints. In a non-neutral world, computed values for $\overline{sp^*_t}$ will vary with p^e_t, because changes in inflation expectations will alter ra_{t-1}, and perhaps the real value of net worth -- thereby changing the optimal consumption stream. However any period-to-period variations in $\overline{sp^*_t}$ which are observed will reflect true behavioural responses. Thus $\overline{sp^*_t}$, rather than sp^*, is the appropriate variable to use in any empirical study of the determinants of saving behaviour.

24 The inflation-adjusted Canadian saving rate for 1968-78 also has about the same average value as the measured saving rate for the low-inflation period of the early 1960's. On an inflation-adjusted basis there is nothing "unusual" about Canadian saving in the past decade -- a result which makes the U.S. experience in 1976-78 all the more puzzling.

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APPENDIX

Computation of an Inflation-Adjusted Personal Saving Rate

An annual series for an inflation-adjusted rate of personal saving was computed for 1954-76 as

$$(1)\quad \overline{sp}^{*}_{t} = \frac{S_t - p^e_t(NA_{t-1} - CC_{t-1})}{YD_t - p^e_t NA_{t-1}} ,$$

where S is personal saving, p^e is the one-term expected rate of price inflation, NA is the market value of net interest-bearing assets of the household sector , CC is the market value of consumer debt and YD is personal disposable income. Values for S and YD were taken directly from the NI&P accounts. Values for p^e and NA were constructed.

The obvious source of data for household assets is the Flow of Funds Accounts. However, FFA data are recorded at book values rather than market values and a method of converting the former into the latter had to be devised. The market value, M_t, of an interest bearing asset is related to its book value, B_t, as follows

$$(2)\quad M_t = \frac{c}{i_t} B_t + B_t \left(1 - \frac{c}{i_t}\right)\left(\frac{1}{1 + (1-\tau)i_t}\right)^n ,$$

where c is the coupon rate of interest payable on the asset, n is its remaining term to maturity, i_t is the nominal rate of interest prevailing in the market at time t, and τ is the effective rate of tax applied to the coupon payments on this asset. For large values of n,

$$(3)\quad M_t \cong \left(\frac{c}{i_t}\right)B$$

Approximation (3) was used to convert FAA book values of household assets to market values.

The coupon rate of interest payable on net interest-bearing assets of households is not directly observable. But since coupon rates are normally set equal to market rates of interest at the time assets are issued, we can hypothesize that the effective coupon rate prevailing on total net assets at time t is some weighted average of current and past market rates; i.e.,

$$(4) \quad c_t = \sum_{j=0}^{N} w_j i_{t-j} \; ; \quad \Sigma w_j = 1.0.$$

The weights, w_j, represent the share of outstanding net assets acquired j periods prior to time t.

Under this hypothesis, the term $p_t^e NA_{t-1}$ in equation (1) was computed as

$$(p_t^e / i_{t-1}) (\sum_{j=1}^{N+1} w_j i_{t-j}) NAB_{t-1} ,$$

where NAB_{t-1} represents the book value of household's net assets. In performing the computations, i was measured by the yield on long term U.S. government bonds and the summation term was computed as a simple four-year average of i with $w_j = 1/4$ for $j = 1, 2, 3, 4$. The analogous computation was applied to the book value of consumer debt in order to estimate CC_{t-1}.

There is little to justify the choice of the long-term bond yield other than the fact that it is available over a longer time period than other nominal interest rates. With this yield, the computations could be performed as far into the past as 1958. A shorter-term interest rate might have been preferable, but data prior to the early 1960's are not available. In order to test the sensitivity of the computations to a different interest rate over the latter part of the simple period, an alternative series for $\overline{sp}^*$ was computed using the yield on 9 - 12 month U.S. Treasury bills for the interval 1964-78. The resulting series did not differ appreciably from the values obtained with the long-term bond yield.

The use of a four year simple average of past interest rates to approximate the coupon rate of interest was an arbitrary assumption. Coupon rates on some household assets and liabilities, such as short-term securities and bank loans, adjust very quickly to changes in market interest rates. The rates on others, such as long-term bonds and mortgages, adjust very slowly. Setting N in the range of 2 to 6 years seemed a priori reasonable and the value N = 3 was a mid-range compromise. Computations of $\overline{sp^*}$ based on alternative values for N within this range did not change the qualitative nature of the results. Future researchers may gain some additional refinement by disaggregating net assets and using a different coupon rate adjustment for the various sub-components.

The FFA definition of the household sector differs conceptually from the definition of the personal sector in the NI&PA. The main difference regards the treatment of government pension funds. Securities held by these funds are regarded as assets of the household sector in the FFA, but the NI&PA considers flows in and out of the funds as transfers between persons and governments. In order to use the net asset concept compatible with NI&PA saving and income, the book value of household net assets was computed as FFA total net interest-bearing assets of the household sector, less household holdings of public pension assets.

Inflation expectations were estimated with standard time-series analysis, from the proposition that in an efficient market all useful information pertaining to future inflation should be contained in observations on past rates of inflation. After some preliminary examination of the data to determine stationarity, the following specification was deemed best to describe the time series properties of observed inflation rates.

$$(5)\quad (p_t - p_{t-1}) = \sum_{j=1}^{N} \alpha_j (p_{t-j-1} - p_{t-j-2}) + \varepsilon_t$$

The variable ε_t is a white-noise disturbance and the summation-term represents the portion of any change in inflation between periods $t-1$ and t which is predictable on the basis of past observations.

Observations, on p_t were obtained from annual data on the rate of change in the implicit price deflator for consumer expenditures from the NI&P Accounts for the period 1947-78. The equation was first estimated using OLS for the period 1952-78 to determine the optimal value of N . A value $N=3$ gave maximum $\bar{R}^2$ over this interval and was used in all remaining regressions.

The equation was then estimated for 1952-57, and the estimated parameters used to predict the value of inflation for 1958. Actual data for 1958 were added to the sample and the equation was re-estimated for 1952-58. Results of this regression were used to make a prediction for 1959. The process was repeated in step-wise fashion until predicted values were obtained for each year in the 1958-78 interval. These predicted values were used as measures of p_t^e in the inflation adjustement procedure described above.

All relevant data are contained in the following table.

TABLE 1

Data Series Used to Compute
Inflation-Adjusted Personal Saving Rate

	sp* (%)	$\overline{sp*}$ (%)	p^e (%)	i (%)	S ($Bil)	YD ($Bil)	NAB $(Bil)	CC $(Bil)
1958	6.8	5.9	2.3	3.43	21.7	317.1	221.2	52.7
1959	5.6	4.6	2.3	4.07	18.8	336.1	231.8	60.7
1960	4.9	4.0	2.3	4.02	17.1	349.4	242.7	65.1
1961	5.6	4.7	2.1	3.90	20.2	362.9	261.1	67.6
1962	5.3	4.6	1.5	3.95	20.4	383.9	275.6	73.8
1963	4.7	3.9	1.5	4.00	18.8	402.8	291.1	82.8
1964	6.0	5.3	1.5	4.15	26.2	437.0	311.4	92.6
1965	6.4	5.8	1.4	4.21	30.3	472.1	331.8	103.2
1966	6.5	5.8	1.6	4.65	33.0	510.4	352.8	109.7
1967	7.5	6.7	2.3	4.85	40.9	544.5	393.5	115.4
1968	6.5	5.5	2.4	5.26	38.0	588.2	426.3	126.9
1969	5.6	4.2	3.5	6.12	35.1	630.4	451.6	137.7
1970	7.4	5.9	4.0	6.58	50.6	686.0	489.5	143.1
1971	7.7	6.0	4.5	5.74	57.3	742.8	538.6	157.8
1972	6.2	4.1	4.4	5.63	49.4	801.3	595.4	177.6
1973	7.8	6.0	3.9	6.30	70.3	901.6	613.3	203.7
1974	7.3	5.5	4.9	6.98	71.7	984.6	654.9	213.6
1975	7.7	4.4	9.7	6.99	83.6	1,086.6	773.3	223.3
1976	5.8	2.8	7.3	6.79	68.6	1,184.5	865.2	248.8
1977	5.0	1.8	6.9	7.06	65.0	1,305.1	903.1	289.4
1978	4.9	2.2	6.9	7.89	72.0	1,458.4	957.8	340.0

Sources: The series for i, S, YD, NAB, CC were provided by Data Resources Inc.