A paper prepared for the

Economic Council of Canada

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Un document préparé pour le

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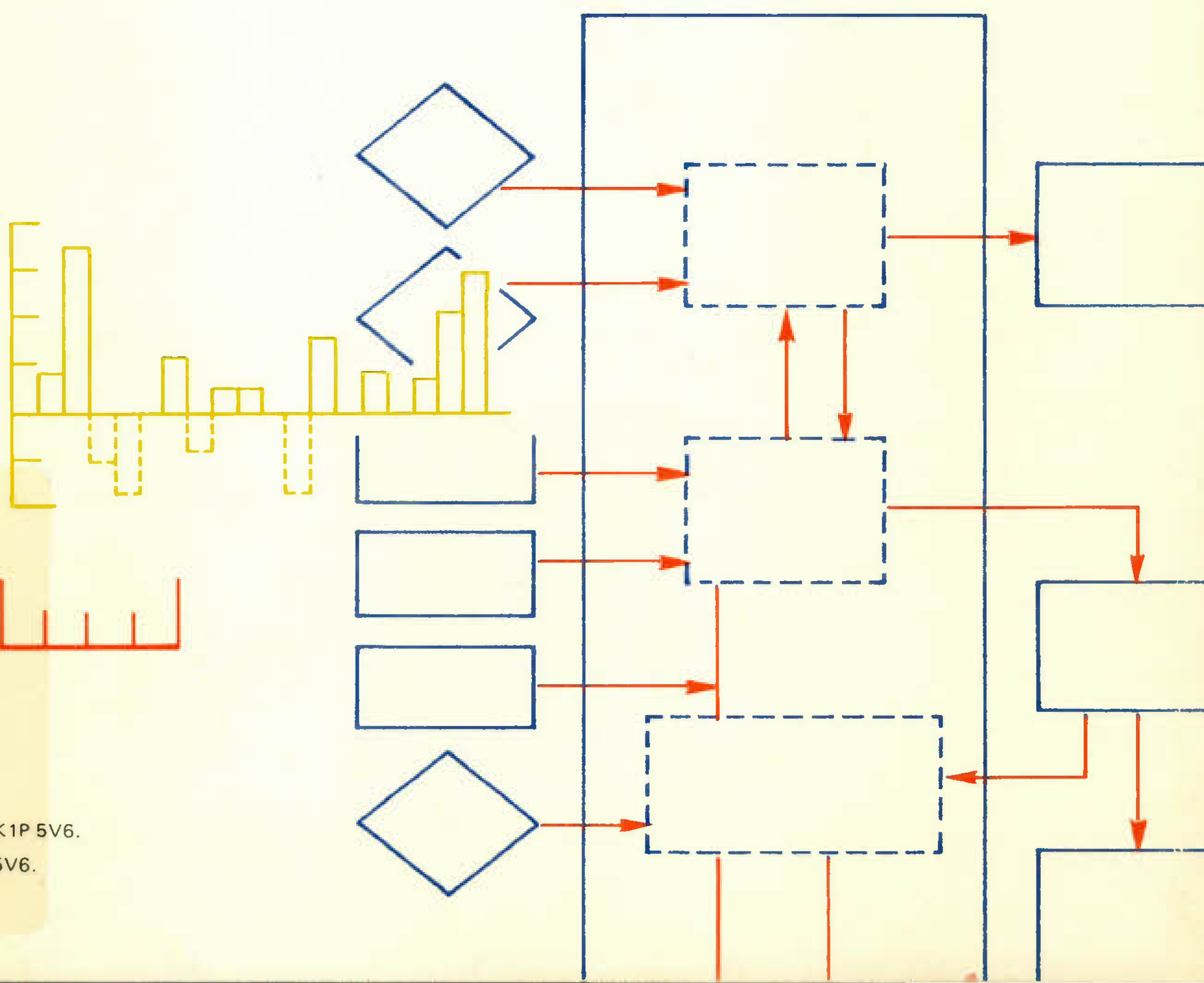

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DISCUSSION PAPER NO. 168

Pensions in a General Equilibrium Model of Canada

by Joel Fried and
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April 1980

Résumé

Dans le présent document, nous avons construit un modèle néo-classique sur le chevauchement des générations, destiné à mieux faire comprendre les conséquences macro-économiques et distributives des diverses modifications apportées au système de pensions du Canada. La caractérisation de l'économie a exigé l'analyse de la demande ou de l'offre de trois types d'agents privés : a) les épargnants à faible revenu dont les cotisations au Régime de pensions du Canada (RPC) génèrent des effets de substitution et de richesse; b) les épargnants à revenu élevé pour qui les modifications apportées aux cotisations au RPC ne produisent que des effets de richesse; et c) les consommateurs à revenu élevé dont les liquidités sont restreintes durant leur jeunesse, de sorte qu'ils n'épargnent pas avant l'âge mûr. La caisse de retraite est caractérisée par : (1) une politique d'investissement (choix à opérer entre des titres des secteurs publics et privés); (2) une retenue sur le salaire; (3) une cotisation maximale par période; et (4) un taux de rendement sur les cotisations. Les points (2) et (4) déterminent le point (5), le niveau des prestations destinées à un agent, mais, alors que chacun des points (2), (4) et (5) peut être considéré comme un instrument de politique, seulement deux d'entre eux peuvent être établis de façon autonome. En outre, il est supposé que le gouvernement applique également un second

programme de sécurité de la vieillesse (SV) financé par un impôt général sur le revenu ou par des emprunts publics. Enfin, on tient pour acquis que le capital est parfaitement mobile sur le plan international.

Après l'élaboration de la structure du modèle dans les sections II à IV, nous considérons, dans la section V, les effets qualitatifs de divers changements de paramètres sur l'offre de travail, le revenu, l'épargne intérieure, la balance des paiements, l'investissement étranger et la taille du secteur financier. Les changements de paramètres considérés sont : (1) une "poussée" démographique; (2) une augmentation du capital-actions détenu par le RPC; (3) une diminution de la taille du RPC au moyen d'une réduction de la cotisation maximale et du niveau des prestations (tout en maintenant un taux de rendement constant des cotisations); une diminution du taux de rendement sur les cotisations au moyen (4) d'une augmentation de la retenue à la source et (5) d'une réduction du niveau des prestations; et (6) une hausse des prestations de sécurité de la vieillesse financée par une augmentation du taux de l'impôt sur le revenu des particuliers.

ABSTRACT

In this paper a neoclassical overlapping generations model is constructed to serve as a guide to the macroeconomic and distributive consequences of various changes in Canada's pension program. Characterization of the economy entailed developing the demands and/or supplies of three types of private agents: a) low income savers for whom contributions to the pension fund (C.P.P.) generate substitution and wealth effects; b) high income savers for whom changes in C.P.P. contributions generate only wealth effects; and c) high income consumers who are liquidity constrained in their youth so that they do not save until middle age. The pension fund is characterized by: (1) an investment policy (choosing between government and private securities); (2) a payroll tax rate; (3) a maximum contribution per period; and (4) a rate of return on contributions. (2) and (4) imply (5) the level of benefits for an agent but, while (2), (4) and (5) can each be considered a policy instrument, only two of them can be independently set. In addition, it is supposed that the government also runs a second old age security program (O.A.S.) financed by a general income tax or by government borrowing. Finally it is supposed that capital is perfectly mobile internationally.

After developing the structure of the model in Sections II to IV, in Section V we consider the qualitative effects of various parameter changes on labour supply, income, domestic saving, the balance of payments, foreign investment and the size of the financial sector. The parameter changes considered are: (1) a demographic "bulge"; (2) an increase in equity holding by the C.P.P.; (3) a decrease in the scale of the C.P.P. by decreasing maximum contribution and benefit levels (while maintaining a constant rate of return on contributions); a decrease in the rate of return on contributions by (4) increasing the payroll tax rate and by (5) decreasing benefit levels; and (6) an increase in O.A.S. payments financed by an increase in the income tax rate.

PENSIONS IN A GENERAL EQUILIBRIUM MODEL OF CANADA

I. Introduction

This paper is concerned with the consequences on the Canadian economy of changes in pensions and demography. The issues involved are of current interest because of the major demographic shifts that have occurred since World War II and the recognition that such shifts will have a significant impact on total government spending on the elderly over the coming decades. The issues we regard as important in structuring a pension plan in the face of the demographic shifts that are occurring are the effects of the plan on: (1) the level of domestic savings and foreign investment; (2) the supply of labour; (3) the efficacy of short-run stabilization policy; and (4) the intergenerational redistribution of wealth and welfare.

Some have argued that existing pension programs have significantly reduced personal saving and the capital stock. For instance Feldstein (6) has indicated that the U.S. social security system has reduced personal saving by some 60 percent and that the funding of that system has implied a reduced capital stock in that country of over 25 percent relative to what it would have been in the absence of social security. While these figures have been sharply criticized (c.f. (1), (2), (9)), changes anywhere near this magnitude suggest serious adjustments for any economy. For the Canadian economy, a decrease in saving of this amount would affect the capital stock somewhat; but it would also imply a major increase in capital inflows, a situation that many would consider to be undesirable. Our analysis, therefore, will examine the effects of pension programs on both saving and the capital account of the balance of payments.

The supply of labour has also been thought to be affected by the wealth and relative price effects generated by government pension programs in the U.S. While the evidence is mixed (c.f. (4) (16)) most studies seem to suggest that there has been a reduction in the labour force aged 65 and older. Information on the supply of labour under 65 is quite ambiguous so it will be our purpose to sort out the theoretical effects of pension plans on total labour supply in Canada.

While our interest is in the longer-run consequences of pension programs, the policies themselves may have implications about the efficacy of fiscal and monetary policies in responding to short-run disturbances. To get a handle on this issue a financial sector is included in our model. While we are unable to say how pensions will affect the conduct of short-run stabilization policy we can indicate some of the consequences of pension and demographic changes on longer-run trends in financial markets. In this way, policymakers may be able to distinguish between transitory and permanent shifts in financial variables and, we hope, be able to respond in a better fashion.

Almost any change in pension programs will have an effect on the intergenerational distribution of welfare. The information of who gains and who loses from any such change is certainly of importance to policymakers if they are to make informed decisions. To analyze the redistributive effects of the pension plan we shall study a model of overlapping generations, and examine what happens to each generation as a result of any policy changes. The results can be used to supplement other studies of the redistributive effects of pension programs. We shall also be concerned with the timing of these effects, because the demographic changes will themselves have effects that vary over time. Furthermore, since pension changes will affect households differently depending upon their tastes and incomes, we shall include in the model three different types of household.

Our model of the Canadian economy has been drastically simplified in the interest of manageability. A principal assumption used is that Canada can be characterized as a small open economy with reasonably free movement of goods and capital across its borders. This fixes both relative output prices and the after tax rate of return on capital. This means that the capital stock and its rate of change is independent of the level of domestic savings, an implication we find useful in simplifying our analysis. If the reader feels uncomfortable with this implication we would point out that in the model a fall in domestic saving corresponds to an increase in foreign saving--a capital account inflow. If one wished, then, one could relate an increase in the capital account to an increase in the interest rate, a decrease in the wage rate and a fall in domestic output. However, we will not consider a formal model with these properties in this paper.

A second simplifying assumption is that government bonds and the liabilities of financial intermediaries are treated as perfect substitutes in agents' portfolios. This is meant to limit the number of interest rates we have to contend with and does not qualitatively affect our results. Without it, the primary change would be changes in the spread between government bonds and intermediaries' liabilities in response to changes in government and pension fund investment policies.

A third set of assumptions deal with government non-pension policy rules. Since government behavior unrelated to pension programs is not our central concern, we have tried to specify simple rules for these policies. On the expenditure side, per capita government purchases of goods and transfers to households are assumed to grow at the underlying rate of productivity growth in the economy as a whole. On the revenue side there is a constant marginal tax rate applied to all taxable sources of income. Government portfolio policy consists of increasing the outstanding per capita real supply of government bonds at the rate of productivity growth. One could instead adopt a rule of maintaining a balance of payments equilibrium, or if the assumption of perfect substitutability between government bonds and intermediary deposits were relaxed, a real money supply rule. These alternatives are not explored here.

The fourth set of assumptions deals with some empirical data to generate predictions. The major assumption here is that the yield on pension contributions is greater than both the net yield on the best alternative yield that private agents can obtain with their savings and the rate of return on the pension funds portfolio. The former assumption, which is the same as assuming that the present value of the pension plan is positive for all agents, will generate significantly different implications under an expansion of the pension program than if the present value of the plan is negative. The latter assumption deals with the sustainability of the fund in response to deviations of the population relative to trend.

In modeling the economy we consider six sectors: producers, households, financial intermediaries, foreigners, the pension fund and the government. Producers produce commodities, issue equities to purchase capital, and use labour and capital services. Households demand consumption, leisure, liquidity, and equities over a three period lifetime, while providing labour to firms, tax revenue to the government, and contributions to the pension fund. They receive transfers from the government over their lifetime and benefits from the pension fund in the last period of their life. Financial intermediaries hold money, bonds, and equities, and supply bonds that pay a competitive rate of interest. The pension fund is concerned with intergenerational transfers other than old age security payments, and operates with three principal instruments: the contribution rate, the rate of return on contributions, and the maximum contribution level per period. It also makes a portfolio decision on how to invest contributions. Government fiscal operations involve expenditures on goods, transfers, (including old age security payments) to households, and raising revenue by a general income tax. On the financial side, it issues money and bonds, and holds (positive or negative amounts of) international reserves. The basic framework and objects of choice are discussed in the following section and the behavior of the six sectors is spelled out in detail in Section III.

In Section IV we consider: (1) changing the rate of return on contributions; (2) a decrease of the plan by decreasing (a) the maximum pensionable earnings ceiling and (b) the payroll tax rate; (3) increasing contributions with no increase in benefits; (4) increasing the overall tax rate; and (5) changing the cost of liquidity, the rate of return on private saving and the real wage. The effects of these changes on all three types of agents are examined and the information obtained is then used in Section V to determine the aggregate effect of items (1), (2a), (3) and (4) above on labor supply, output, saving, the net worth of the pension fund, the balance of payments, capital inflows and the gross debt of financial intermediaries. We also look at the effect of the demographic bulge itself and the portfolio policy of the fund on these aggregates in Section V. While ambiguities in the aggregate responses still remain, and can only be removed through simulations using specific empirical data, the amount that can be said is more than we initially anticipated. Section VI provides some extensions and additional qualifications to our results.

II. The basic elements

The agents in the model consist of:

-- households of different types (i) and ages (a)

-- producers

-- financial intermediaries (FI's)

-- the government

-- the pension fund (CPP)[1]

-- foreigners.

The tradeable objects consist of:

-- three physical commodities

  -- consumption goods (C)

  -- capital goods (K)

  -- government goods (G)

-- labour services

-- three financial assets

  -- money (M)

  -- bonds (B)

  -- equity (E).

This section briefly describes the tradeable objects. First, the three physical commodities are all produced by the same technology. They are also produced abroad by the same technology, and can be traded internationally. We assume no tariffs or transportation costs; thus all relative commodity prices are constant and equal to unity, both at home and abroad.

Although the assumption of constant relative prices (and thus no terms-of-trade effects) may be unrealistic for the short run, the alternative of modifying the supply side to permit relative prices to change would obscure the pension and demographic issues we wish to consider. For example, we might suppose that there are adjustment costs associated with gross investment that drive a wedge between the (constant) supply price of capital goods and their demand price.[2] This would not qualitatively affect the steady-state properties of the model but, in the short run, it would imply that an unanticipated increase in the rate of growth of the effective labour supply would reduce the wage-rental ratio, whereas in our model the wage-rental ratio would be unaffected. Another possible modification would be to suppose that one of the three commodities is not traded internationally, and has a different supply technology. A possible candidate might be government goods. In this case, changes in government purchases would alter relative output prices and the real wage rate in terms of consumption goods (the real rental rate on capital would, however, remain fixed). Depending upon the labour intensity in producing government goods relative to other goods the wage rate will increase or decrease with an increase in government spending. However, since these issues of relative factor prices are not our primary concern, neither of these modifications is considered in the current study.

Factor inputs consist of capital goods and the labour services of different households. The households differ in their productivity as workers, but their labour-services are perfect substitutes for each other in production. Thus we may measure each household's labour services in homogeneous efficiency-units. In particular, each hour of labour services of an (i,a) household at date 0 generates q_a^i efficiency-units. Furthermore, labour-embodied technical change is assumed to occur at the rate g. Thus an hour of (i,a) labour-services at date t generates $q_a^i(1-g)^{-t}$ efficiency-units. Each household is endowed with one unit of time per period, part of which can be supplied to the market, and part of which can be withheld in the form of leisure.

Financial assets are all measured in real terms. Money, M_t, is supplied only by the government, and is demanded by households and financial intermediaries (FIs). Bonds, B_t, are not traded internationally: they are issued by the government and by FIs, and held by households, FIs, and the pension fund. A unit of equity is a claim on the return to one unit of capital. Equity, E_t, is traded internationally. It is issued by producers, foreigners, and the government. (The government's holdings constitute its net international reserve position.) They may also be issued by households, but only to the extent that they hold money and bonds as collateral. In other words, capital markets are imperfect in the sense that no household may borrow against its future wage income, pension benefits, or other government transfers. Equity is held by households, FIs, foreigners, and (perhaps) the pension fund.

The demand for money and bonds by households is a derived demand. Households are interested in money or bonds only because these assets "produce" liquidity, according to a constant elasticity of substitution (CES) production function. Equity, while it is marketable, is assumed not to generate liquidity services.

We shall follow the convention of expressing the total amount of asset Y held by sector z at date t as Y_{zt}, and the amount issued by sector z as ${}_zY_t$, where the sectors are households (z=h), producers (z=p), the government (z=g), financial intermediaries (z=f), the pension fund (z=c) and foreigners (z=e).

III. The Actors

a) Households

Each household lives for three periods. Thus a equals 0,1 or 2. Also, as we explain more fully below, there are three different types of household. Thus i equals 1,2, or 3. Let PN^i_{ab} denote the population of type i, age a, and generation b (that is, they were born in the start of period b). Then the total population at time t is:

$$PN_t = \sum_{i=1}^{3} \sum_{a=o}^{2} PN^i_{a,t-a} \tag{3.1}$$

It is assumed that the population of each type i stands in a constant proportion to the total population over time. That is, $PN^i_{a,t-a} = \gamma^i \sum_i PN^i_{a,t-a}$. Also, population grows at the constant trend rate of growth n, except for the effects of the "bulge" generation (b=1). More specifically, assume that:

$$PN^i_{a,t-a} = \begin{cases} PN^i_{a,-a}(1-n)^{-t+\overline{PN}\gamma^i} & (t-a=1) \\ PN^i_{a,-a}(1-n)^{-t} & (t-a\neq 1) \end{cases} \quad a=0,1,2;\ i=1,2,3 \tag{3.2}$$

which implies that

$$PN_t = \begin{cases} PN_o(1-n)^{-t} + \overline{PN} & t=1,2,3 \\ PN_o(1-n)^{-t} & t=4,5,\ldots \end{cases} \tag{3.3}$$

where $\overline{PN}$ is the "surplus" population of the bulge generation. According to (3.2) and (3.3), the bulge affects the population only of its own generation. We are assuming, in effect, that the "surplus" members of the bulge generation do not reproduce. When they die at the end of period 3 the population returns to its trend level.

Each household acts as if it were maximizing a Stone-Geary, intertemporally additive preference function, where the arguments of the "instantaneous" utility function are current consumption, leisure, and liquidity. Thus the preference function of a household age 0, type i, generation b, is:

$$\Phi^i_{ob} = \sum_{a=o}^{2} (\Omega^i)^a [\alpha^i_c \ln(c^i_{a,b} - \underline{c}^i_{a,b}) + \alpha^i_\ell \ln(\ell^i_{a,b} - \underline{\ell}^i_a) + \alpha^i_L \ln(L^i_{a,b} - \underline{L}^i_{a,b})] \quad (3.4)$$

where

Ω^i is a discount factor equal to one minus the (constant) rate of time preference

c^i_{ab} is the demand for consumption goods planned for age a

ℓ^i_{ab} is the demand for leisure (measured in hours, not efficiency-units) at age a

L^i_{ab} is the demand for liquidity services at age a

$\underline{x}^i_{ab}$ is a non-negative constant which can be interpreted as the subsistence demand or "requirement" for age a, $x=c,\ell,L$

and α^i_x is a constant, $x=c,\ell,L$.

This functional form constitutes the entire class of preferences that generate linear demand functions (Samuelson, 1947). Thus it yields a model that is tractable, that gives at least some unambiguous predictions that are capable of being tested, and that is amenable to simulation and/or estimation even where it gives only ambiguous predictions.

In the interest of simplicity we shall assume that $\underline{L}^i_{ab} = 0$. We shall also assume that the consumption-"requirement" grows at the rate g; that is, $\underline{c}^i_{a,t-a} = \underline{c}^i_{a,o}(1-g)^{-t} > 0$. This assumption is made mainly for simplicity, but it may be rationalized by the commonplace observation that as productivity grows, so does our psychological notion of subsistence. Note, however, that the leisure-requirement, ℓ^i_a, is the same for all generations. Since ℓ^i_a is measured in units of time this means that the leisure requirement is also growing at the rate g if measured in efficiency-units.

We are also permitting households of different types to differ in their rates of time preference. We believe this distinction to be worth including in our model because, with imperfect capital markets, the CPP may have quite different effects on households, depending on their rates of time preference. In particular, the plan may "force" those with high rates to save through mandatory contributions, whereas it may reduce the saving of those with low rates who regard the pension plan as a substitute for personal saving.

At age 0 the household of type i, generation b, faces the following sequence of budget constraints:

$$A^i_{-1,b} = 0 \tag{3.5}$$

$$A^i_{ab} = R^{-1}_e\{[1-\ell^i_{ab})\, w^i_{ab} - CN^i_{ab} + TR^i_{ab} + BN^i_{ab} + OAS^i_{ab}](1-tx) - c^i_{ab} + A^i_{a-1,b} - M^i_{ab}(\bar{r}_e + \pi) - B^i_{ab}(\bar{r}_e - \bar{r}_b)\} \quad a=0,1,2 \tag{3.6}$$

$$A^i_{ab} = M^i_{ab} + B^i_{ab} + E^i_{ab} \geqq 0 \qquad a=0,1,2 \tag{3.7}$$

$$L^i_{ab} = [v(M^i_{ab})^{-\rho} + (1-v)(B^i_{ab})^{-\rho}]^{-1/\rho} \tag{3.8}$$

$$\ell^i_{ab},\ (1-\ell^i_{ab}),\ c^i_{ab},\ M^i_{ab},\ B^i_{ab} \geqq 0 \qquad a=0,1,2 \tag{3.9}$$

where

w^i_{ab} is the (expected) real wage rate at age a

tx is the (constant) marginal tax rate on all income

CN^i_{ab} is the household's CPP contribution at age a $(CN^i_{2b} \equiv 0)$

TR^i_{ab} is the net value of the household's general lump sum transfer from the government (all transfers except OAS benefits, CPP contributions, and CPP benefits)

π is the (expected) rate of inflation

$\bar{r}_b$ is the after-tax real yield on bonds

$\bar{r}_e$ is the after-tax real yield on equity

$\bar{R}_e$ is the after-tax equity yield factor, $\bar{R}_e = 1-\bar{r}_e$.

OAS^i_{ab} is the household's Old Age Security payment ($OAS^i_{ob} \equiv OAS^i_{1b} \equiv 0$)

BN^i_{ab} is the household's CPP benefit ($BN^i_{ob} \equiv BN^i_{1b} \equiv 0$)

A^i_{ab} is the total real value of financial assets held at age a (at the end of the period)

M^i_{ab}, B^i_{ab}, E^i_{ab} are the (end-of-period) real holdings of money bonds, and equity at age a

v, ρ are parameters, satisfying: $0 < v < 1$, $0 \neq \rho > -1$

Equation (3.5) states that the household starts life with no financial assets. Equations (3.6) state that the total assets at the end of a period equal total assets at the beginning, plus disposable income during the period, minus consumption expenditures, minus the interest-opportunity cost of liquid asset holdings, all multiplied by the return factor $\bar{R}_e^{-1}$. We are assuming that all transfers from the government and the pension fund are taxable at the rate tx, and that pension contributions are tax-deductible. Equations (3.7) embody the above-mentioned assumption of imperfect capital markets. They state that equity can only be issued by a household if backed by collateral in the form of other financial assets. Equations (3.8) describe the "production" of liquidity services.[3] Equations (3.9) are feasibility conditions, stating that the household cannot go short in leisure, labour, consumption goods, money, or bonds.

Since the preference functions (3.4) include no motive for bequests, (3.7) will hold with equality for a=2. In other words, the household will die holding liquid assets just equal in value to its equity debt. These liquid assets will revert to the household's creditors at the end of the period and be "recycled" at the beginning of the next period. If the "liquidity" constraint is not binding in any other period (that is, (3.7) holds with strict inequality for a=0 and 1), then the household will be acting as if the constraints (3.5), (3.6) and (3.7) were collapsed into the single present-value constraint:

$$\sum_{a=0}^{2} \bar{R}_e^a \{ [(1-\ell_{ab}^i) w_{ab}^i - CN_{ab}^i + TR_{ab}^i + BN_{ab}^i + OAS_{ab}^i](1-tx) - c_{ab}^i - M_{ab}^i(\bar{r}_e + \pi) - B_{ab}^i(\bar{r}_e - \bar{r}_b) \} = 0 \qquad (3.10)$$

As mentioned above, we believe it is important to distinguish between households with high and low rates of time preference. We also believe it is important to distinguish between households with high and low productivity, because the CPP affects households differently depending upon the size of their wage income. For households with high enough incomes to be paying the maximum CPP contributions, these mandatory contributions in effect constitute a lump-sum tax that does not affect their behaviour on the margin (although it may affect their wealth). But for households with lower incomes, the mandatory contributions constitute a marginal tax on labour income, which <u>will</u> affect their behaviour on the margin. We also suppose that the productivity profile, and hence the expected real wage profile, of each household is "humped". More precisely, we assume that[4]

$$q_{ob}^i < q_{1b}^i, \; q_{2b}^i < q_{1b}^i \qquad i=1,2,3; \text{ all } b. \qquad (3.11)$$

Because of the humped wage profile, households with a high rate of time preference may save at age 0 by more than they would with perfect capital markets. That is, they may find the liquidity constraint (3.6) binding at age 0.

We are now in a position to describe the different types (i) of household. There are three types: Type 1 has low productivity and low time preference, type 2 has high productivity and low time preference, and type 3 has high productivity and high time preference. (In the interest of simplicity we decided not to include a fourth type with low productivity and high time preference.) More precisely, we assume that:

$$q^1_{ab} < q^2_{ab} = q^3_{ab}; \quad \text{all } a,b \quad , \tag{3.12}$$

and that the liquidity constraint is binding for all households of type 3, age 0, and all households of all types, age 2, but not for any others.

b) Producers

Producers are assumed to be profit-maximizing firms that transform inputs of labour services and capital goods into final outputs of consumer, government and capital goods. The aggregate production transformation is a Cobb-Douglas function with constant returns to scale:

$$Y_t = N^*_t\, f(k^*_t) = N^*_t (K_t/N^*_t)^\xi \tag{3.13}$$

where Y_t is aggregate output in period t

K_t is the capital stock

N^*_t is the supply of labour (measured in <u>efficiency-units</u>)

$k^*_t \equiv K_t/N^*_t$

and ξ is a constant satisfying: $0 < \xi < 1$.

Competition implies that the real wage of an efficiency-unit of labour will equal its marginal product:

$$w_t^* = (1 - \xi)k_t^{*\xi} \qquad (3.14)$$

The demand for labour (in efficiency-units) is thus:

$$N_t^{*d} = [(1 - \xi)w_t^*]^{1/\xi} K_t \qquad (3.15)$$

Because labour-embodied technical change occurs at the rate g, the wage of any household of type i, age a, generation b, is:

$$w_{ab}^i = w_{a+b}^* \, q_a^i (1-g)^{-(a+b)} \qquad (3.16)$$

The gross real rental rate on a unit of capital is:

$$r_{k_t} \equiv f'(k_t^*) = \xi(k_t^*)^{\xi-1} \qquad (3.17)$$

Assuming no depreciation, no adjustment costs to install capital goods, and a financial policy whereby producers purchase net capital additions by issuing equity, the profit-maximizing producer that takes the after tax rate of return on equity ($\bar{r}_e$) as given will employ capital up to the point where this rate of return equals the net real rental rate on capital:[5]

$$\bar{r}_e = (1-tx)f'(k_t^*) \qquad (3.18)$$

From (3.17) and (3.18) the capital-labour ratio at time t can be expressed as:

$$K_t/N_t^* = \left\{\frac{\xi}{\frac{\bar{r}_e}{1-tx}}\right\}^{\frac{1}{1-\xi}} \equiv X_1(\bar{r}_e/(1-tx)) \qquad (3.19)$$

Thus, given the labour supply, N_t^*, producers will employ the quantity of capital:

$$K_t^d = \left[\frac{\xi}{\frac{\bar{r}_e}{1-tx}}\right]^{\frac{1}{1-\xi}} N_t^* = X_1(\bar{r}_e/(1-tx))N_t^* \tag{3.20}$$

Without costs of adjusting the capital stock, investment demand in period t will be

$$I_t = (K_t^d - K_{t-1}) = X_1(\bar{r}_e/(1-tx))N_t^* - K_{t-1} \tag{3.21}$$

c) Financial Intermediaries:

We suppose that the FIs are perfectly competitive and operate under constant returns with free entry. Thus they cannot have positive profits. Furthermore, a FI's only cost consists of the interest payments on its liabilities, and its only revenue comes from the returns on its assets. FIs hold equity, money and bonds in fixed proportions through the statutory imposition of a required primary reserve ratio (η_m) and a secondary reserve ratio (η_b). The nominal return (r_b) that FIs will offer on their deposits is determined by the returns they obtain on their assets. These nominal returns are 0 for money, r_b for bonds and $r_e + \pi$ for equity. Therefore the nominal bond rate of interest is determined by the equation:

$$\begin{aligned} r_b &= \eta_b \cdot r_b + (r_e + \pi)(1-\eta_m - \eta_b) \\ &= (r_e + \pi)(1-\eta_m - \eta_b)/(1-\eta_b) \end{aligned} \tag{3.22}$$

The <u>real</u> <u>pretax</u> rate of return on bonds is

$$(r_b - \pi) = r_e(1-\eta_m - \eta_b)/(1-\eta_b) - \pi\,\eta_m/(1-\eta_b), \tag{3.23}$$

and the <u>real</u> <u>after-tax</u> rate of return is

$$\bar{r}_b = r_b(1-tx) - \pi = \bar{r}_e(1-\eta_m-\eta_b)/(1-\eta_b) - \pi(tx + (1-tx)\eta_m/(1-\eta_b)). \tag{3.24}$$

For a given $\bar{r}_e$, an increase in π will reduce the real return on bonds, both pre- and after-tax; an increase in tx will increase the pre-tax and lower the after-tax real return on bonds.

Our assumption that FI deposits are regarded as perfect substitutes by asset holders is what makes the nominal return on government bonds (r_b) independent of their supply. If we were interested in the consequences of changes in r_b, this could be studied by varying the required reserve ratios, η_m and η_b. A more general alternative would be to suppose that FI deposits were regarded as imperfect substitutes for government bonds. Then an increase in the supply of government bonds would cause their return to rise, ultimately causing the yield on deposits to rise as well, provided assets are gross substitutes (c.f. (15)). Competition would still imply zero net profits to FIs but the spread between the return on deposits and government bonds would be affected by reserve requirements.

d) The Pension Fund

To examine the pension fund policy and population shifts we treat the fund's accounts in two separate parts, one to handle "normal" growth and another to handle the effects of the population bulge. Call the former the "normal account", and the latter the "reserve account" of the pension fund. Agents whose contributions and benefit payments are made through the reserve account are treated the same as their cohorts but the two accounts permit us to distinguish the effects of the demographic change from that of pension plan changes.

Of the two basic intergenerational programs for the Canadian economy; OAS and the CPP, the pension fund is concerned only with the latter. The OAS is under the control of the government. The CPP is funded with contributions in the form of a payroll tax at rate τ up to a fixed amount of earnings--the "yearly maximum pensionable earnings", YMPE. Define

$$\overline{CN}_t = \tau \; YMPE_t \tag{3.25}$$

as the maximum payroll tax. For the CPP this $YMPE_t$ is the average yearly industrial wage and thus is geared to normal productivity growth, g. Consequently we shall suppose that unless policy is altered,

$$\overline{CN}_t = (1-g)^{-t} \; \overline{CN}_o \; . \tag{3.26}$$

Both the <u>contribution rate</u>, τ, and the <u>maximum contribution level</u>, $\overline{CN}_t$, are policy instruments of the fund.

Benefits from CPP contributions also depend upon YMPE and are currently set at a maximum of one quarter of YMPE. Let $\overline{BN}_t$ denote this maximum at date t. Then

$$\overline{BN}_t = (1-g)^{-t} \; \overline{BN}_o \; . \tag{3.27}$$

The level of benefits received by any household is also related to the household's contributions relative to the maximum contribution. We shall interpret this to mean that the plan is intragenerationally neutral in the sense that the internal rate of return on contributions to the fund, r_c, is the same for all agents, and is defined by:

$$\overline{CN}_t + \overline{CN}_{t+1} \; (1-r_c) + \overline{BN}_{t+2} \; (1-r_c)^2 = 0 \tag{3.28}$$

Both $\overline{BN}_t$ and r_c are policy instruments of the fund (although variations in the fund's instruments are constrained by (3.28)).

We now wish to consider the portfolio policy of the fund. There are two financial assets that the fund might hold: equity and bonds. These have real rates of return r_e and $r_b - \pi$ respectively.[6] One question to be asked is: Under what circumstances is the policy of the "normal account" of the fund sustainable? A sustainable policy is defined as one where the real net worth of the account is just equal to zero, given the constant portfolio policy, the current level of contributions and benefits, the constant rates of return on bonds and equity, and the constant growth rates of population and productivity. Let $\hat{F}_t = \hat{E}_t + \hat{B}_t$ be the total value of financial assets required for the policy to be sustainable at the end of date t. Then:[7]

$$\hat{F}_t = \frac{CN_o(1-r_g)^{-t}}{(2-n)} \{2-n - \frac{(1-r_g)(1-n)}{1-r_c} - \frac{(1-r_g)^2}{(1-r_c)^2}\}/(r_g - r_f) \tag{3.29}$$

where r_g is the "real rate of growth of the economy", defined by:

$$(1-r_g) \equiv (1-n)(1-g), \tag{3.30}$$

and r_f is the real rate of return on the fund's portfolio:

$$r_f = \sigma_f r_e + (1-\sigma_f)(r_b - \pi), \tag{3.31}$$

where σ_f denotes the fraction of the fund's assets held in the form of equity.[8]

We may also define the required "reserve" fund as:

$\hat{F}^r_t = \hat{B}^r_t + \hat{E}^r_t$, where:

$$\hat{F}^r_o = \frac{\overline{PN}(1-g)^{-1} CN_o(1+(1-n) + (1-n)^2)}{PN_o(2-n)} [1+(1-g)^{-1} R_f - \frac{R_f^2}{R_c^2} - (1-g)^{-1} \frac{R_f^2}{R_c}] \tag{3.32}$$

$$\hat{F}^r_1 = R_f^{-1}(\hat{F}^r_o + CN_o(1-g)^{-1}\,\overline{PN}(1+(1-n) + (1-n)^2)/PN_o(2-n)) \quad (3.33)$$

$$\hat{F}^r_2 = R_f^{-1}(\hat{F}^r_1 + CN_o(1-g)^{-2}\,\overline{PN}(1+(1-n) + (1-n)^2)/PN_o(2-n)) \quad (3.34)$$

$$\hat{F}^r_t = 0 \qquad t \geqq 3 \quad (3.35)$$

$R_c = (1-r_c)$ is the return factor on contributions and $R_f = (1-r_f)$. The net worth of the Fund may be defined as:

$$V_{ct} = F_t - \hat{F}_t - \hat{F}^r_t = B_t + E_t - (\hat{B}_t + \hat{E}_t) - (\hat{B}^r_t + \hat{E}^r_t) \quad (3.36)$$

Thus, V_{ct} is the "excess" of the fund (or "deficiency" if negative), given the current array of interest and growth rates, its current portfolio policy, the level of contributions, and the current size of the fund. The pension plan as a whole is sustainable if $V_{ct} = 0$.

Note that as long as the fund is earning a rate of return on its portfolio at least as great as the real rate of growth of the economy ($r_f \geqq r_g$) then any rate of return on contributions is sustainable, no matter how large, provided that enough initial funding exists. Indeed, if the rate on contributions is less than r_g, then, according to (3.29), the initial funding can be negative, because in this case the fund is not paying even as high a return as a pay-go scheme. On the other hand, if $r_f < r_g$, then the fund cannot pay a higher rate than a pay-go scheme. In this case a sustainable policy requires $\hat{F}_t \equiv 0$.[9]

In a sense, any sustainable policy is "fully funded", because, given the current projections of rates of growth and rates of return, the plan is actuarially sound. There is a risk that future generations will be unwilling to pay the older generations. But this is conceptually the same as the risks involved in a change in the yield on equity or bonds. If, on the other hand, a policy is not sustainable, that is, if r_c is "too high" given actual asset holdings, then something must ultimately be altered. For convenience we shall suppose that the net worth of the fund simply decreases and that generations 6 and after must decide on how to remove that liability. Thus no generation that is alive in years 0 through 3 will be affected by the necessity to remedy the deficiency created by any parameter change that we shall be considering below. However, the size of the deficiency will provide an indicator of how much of an inter-generational transfer is provided to the current generations from the future.

Notice also that the reserve account requires no initial funding if $r_c = r_f$. If $r_c > r_f$ it requires initial funding and if $r_c < r_f$ it can operate with an initial liability. In other words, the effect of the bulge on the overall fund will be to leave it unchanged, with an excess, or with a deficiency, depending upon whether the return on contributions is equal to, less than, or greater than the return on the fund's portfolio.

e) The Government

Our major concern with the government is its effect on the inter-generational distribution of wealth and welfare. Consequently its role for purchasing goods and providing general transfers, and its methods of taxing, will be simplified as much as possible by the use of rules. In particular we assume that government purchases of goods per capita and the net value of general lump-sum transfers per capita are both growing at the underlying rate of productivity growth so that:

$$G_t = G_o (PN_t / PN_o)(1-g)^{-t} \tag{3.37}$$

and

$$TR_t = TR_o (PN_t / PN_o)(1-g)^{-t} \tag{3.38}$$

where G_t is the aggregate value of government purchases of goods in period t and TR_t ($= \sum_i \sum_a PN^i_{a,t-a} \; TR^i_{a,t-a}$) is the aggregate net value of general lump-sum transfers (other than Old Age Security payments). There are two points that need to be made regarding transfer payments. First, unless the average level of transfers is the same across age groups, demographic changes will violate the rule (3.38). Consequently we shall suppose:

$$\sum_i \gamma^i_a \; TR^i_{a,t-a} = TR_t / PN_t \, , \qquad a=0,1,2 \tag{3.39}$$

This still permits variations in transfer levels across (i) types. The second point is related to inflation policy. Under a floating exchange rate,[10] the method of increasing the rate of inflation is by increasing the rate of growth of nominal government debt. In this model, the method of accomplishing this is through increases in the level of nominal transfer payments which will imply a change in (3.38) whenever inflation policy is altered.

The total value of Old Age Security (OAS) benefit payments in t equals:

$$OAS_t = \sum_i PN^i_{2,t-2} \; OAS^i_{2,t-2} = \psi_t \sum_i PN^i_{2,t-2} \tag{3.40}$$

where ψ_t is the OAS benefit per recipient in period t. (Each type of household receives the same benefit.) This too grows at the same rate as productivity:

$$\Psi_t = \Psi_o(1-g)^{-t} \tag{3.41}$$

Other transfers that are included in the government accounts are interest payments on the national debt, $r_b\, {}_gB_t$, and interest income from international reserves, $\bar{r}_e\, E_{gt}$. The latter is a negative transfer and uses the real after tax rate of return under the assumption that equity is taxed by the country of issue.

Government expenditures are financed by tax revenue or by the sale of financial assets. Tax revenue is generated by a constant marginal tax rate, tx, applied to the tax base. The tax base consists of household income from wages, government transfers, pension fund benefits less contributions, nominal interest income from bonds and nominal dividend income from all equity issued by Canadian sources. Because the income from the pension plan's investments is tax exempt, and because of the assumption that FIs earn zero profits, the tax base is:

$$\begin{aligned} TB_t = {} & Y_t + TR_t + OAS_t + r_b\,({}_gB_t) \\ & - (r_b\, B_{ct} + r_e\, E_{ct} + CN_t - BN_t) \end{aligned} \tag{3.42}$$

where $CN_t \equiv \sum_i \sum_{a=o}^{1} PN^i_{a,t-a}\ CN^i_{a,t-a}$; $BN_t = \sum_i PN^i_{2,t-2}\ BN^i_{2,t-2}$.

A wide variety of rules could be invoked to constrain the financing of the government deficit by monetary issue, bond issue and/or foreign borrowing. While a money supply rule would be inconsistent with a fixed exchange rate (except by chance), we might adopt either a bond supply rule:

$$ {}_gB_t = {}_gB_o PN_t / PN_o (1-g)^t \tag{3.43}$$

or a foreign borrowing rule such as:

$$E_{gt} = E_{go}(1-r_g)^{-t}\ . \tag{3.44}$$

We shall adopt (3.43), although (3.44) would lead to qualitatively similar results. Given the level of the government debt at a point of time, and private demands for real money balances, (3.43) fixes both ${}_gB_t$ and E_{gt}.

In real terms the government deficit, DF_t, is equal to government spending plus transfers less taxes less the capital gain on the government debt due to inflation and is also equal to the real change in its outstanding financial debts less financial assets:

$$DF_t = G_t + TR_t + OAS_t + r_b\, {}_gB_t - \bar{r}_e\, E_{gt} - \pi({}_gB_t + M_t) - tx\, TB_t \qquad (3.45)$$

$$= ({}_gB_t - {}_gB_{t-1}) + (M_t - M_{t-1}) - (E_{gt} - E_{gt-1})$$

In conducting the experiments in Section V it will be assumed that, given the initial policy settings, and in the absence of policy and demographic change,

$$DF_t = r_g({}_gB_t + M_t) \qquad (3.46)$$

and that $E_{gt} = 0$ for all t.

f) The Foreign Sector

The foreign sector is the residual sector in the model. Any output that is not consumed domestically as described by the consumption function, or invested according to the investment function, or spent on government goods as described by the government spending rule is exported. Any equity sales by firms to finance any new investment that is not purchased from household savings, or by financial intermediaries or the pension fund, are purchased by the foreign sector, and are equal to the capital account surplus (deficit) if positive (negative). The service account deficit is described by after-tax interest payments on foreign holdings of equity issued by Canadian firms less government interest income from international reserve holdings. The balance of payments is equal to the trade account (net exports) less the service account deficit plus the capital account surplus. Since each sector faces a binding budget restriction the balance of payments surplus (deficit) will correspond to the net purchase (sale) of equity by the government.

The foreign sector also fixes the real after-tax rate of return on equity ($\bar{r}_e$) by the assumption of perfect mobility of (equity) capital internationally. Thus the real pre-tax rate domestically will be:

$$r_e = \bar{r}_e/(1-tx) \qquad (3.47)$$

This is the rate that the pension fund earns on equity. It is also the rate that determines the effective capital-labour ratio, as in (3.19) above.

IV. Household Demand Functions

In this section we state the explicit forms of the household demand functions, and derive some comparative-statics results. First, we assume that the liquidity constraint (3.7) is not binding for any household of age 0 or 1, except for type 3, age 0, for whom it is binding. Next we assume that in ages 0 and 1, type 1 households earn less than YMPE, whereas types 2 and 3 earn more. Then for a household of types 1 or 2, the demand functions for consumption, leisure, and liquidity, resulting from the maximization of (3.4) subject to (3.5)-(3.9), can be expressed as functions of initial lifetime wealth:

$$x^i_{ab} = \underline{x}^i_{ab} + \alpha^i_{ax} W^i_{ob}/p^i_{axb} \quad i=1,2;\ a=0,1,2;\ x=c,\ell,L \qquad (4.1)$$

or of current wealth:

$$x^i_{ab} = \underline{x}^i_{ab} + \beta^i_{ax} W^i_{ab}/p^i_{axb} \quad i=1,2;\ a=0,1,2;\ x=c,\ell,L \qquad (4.2)$$

The demands of type 3 age 0 can be expressed as functions of disposable income:

$$x^3_{ob} = \underline{x}^3_{ob} + \alpha^3_{ox} Y^3_{ob}/p^3_{oxb} \quad x=c,\ell,L \qquad (4.3)$$

The type 3 demands, age 1 and 2, can be expressed as functions of age 1 wealth:

$$x^3_{ab} = \underline{x}^3_{ab} + \beta^3_{1x} W^3_{1b}/p^3_{axb} \quad a=1,2;\ x=c,\ell,L \qquad (4.4)$$

or the type 3 demands of age 2 can be expressed as functions of age 2 wealth:

$$x^3_{2b} = \underline{x}^3_{2b} + \beta^3_{2x} W^3_{2b}/p^3_{2xb} \quad x=c,\ell,L \qquad (4.5)$$

where:

$$\alpha^i_{ax} = (\Omega^i)^a \alpha^i_x / \sum_{j=0}^{2} (\Omega^i)^j (\alpha^i_c + \alpha^i_\ell + \alpha^i_L) \quad i=1,2;\ a=0,1,2;\ x=c,\ell,L$$

$$\beta^i_{ox} = \alpha^i_{ox} \quad i=1,2;\ x=c,\ell,L$$

$$\beta^i_{1x} = \alpha^i_x / \sum_{j=1}^{2} (\Omega^i)^{j-1} (\alpha^i_c + \alpha^i_\ell + \alpha^i_L) \quad i=1,2,3;\ x=c,\ell,L$$

$$\beta^i_{2x} = \alpha^i_x / (\alpha^i_c + \alpha^i_\ell + \alpha^i_L) \quad i=1,2,3;\ x=c,\ell,L$$

$$\alpha^3_{ox} = \beta^3_{2x} \quad x=c,\ell,L$$

$$p^i_{acb} = \bar{R}^a_c \quad i=1,2,3;\ a=0,1,2$$

$$p^i_{aLb} = \bar{R}^a_e \lambda \quad i=1,2,3;\ a=0,1,2$$

$$\lambda = [u_1(\bar{r}_e + \pi)^{-\upsilon} + u_2(\bar{r}_e - \bar{r}_b)^{-\upsilon}]^{-\frac{1}{\upsilon}}$$

$$\delta = -\rho/(1+\rho)$$

$$u_1 = v^{-\upsilon/\rho}$$

$$u_2 = (1-v)^{-\upsilon/\rho}$$

$$p^1_{a\ell b} = \bar{R}^a_c w^1_{ab}(1-tx)\left(1-\tau\left(1-\left(\frac{\bar{R}_e}{\bar{R}_c}\right)^{2-a}\right)\right) \quad a=0,1$$

$$p^i_{a\ell b} = \bar{R}^a_c w^i_{ab}(1-tx) \quad i=2,3;\ a=0,1,2, \text{ and } (i,a)=(1,2)$$

$$W^1_{ab} = \bar{R}^a_e \quad A^1_{a-1,b} + \sum_{j=a}^{2} [(1-\underline{\ell}^1_{jb})p^1_{j\ell b} - \underline{c}^1_{jb}p^1_{jcb} + TR^1_{jb}\bar{R}^j_e(1-tx)]$$

$$+ \bar{R}^2_c\, t_{b+2}(1-tx) \quad a=0,1,2$$

$$W^i_{ab} = \bar{R}^a_c \; A^i_{a-1,b} + \sum_{j=a}^{2} [(1-\ell^i_{jb})p^i_{j\ell b} - \underline{c}^i_{jb}p^i_{jcb} + TR^i_{jb}\mu^j(1-tx)] - \sum_{j=a}^{1} \bar{R}^j_e \; \overline{CN}_{j+b}(1-tx)$$

$$+ \bar{R}^2_e(Y_{b+2} + \overline{BN}_{b+2})(1-tx) \quad i=2,3;\; a=0,1,2$$

$$Y^3_{ob} = (1-\ell^3_{ob})p^3_{o\ell b} - \underline{c}^3_{ob}p^3_{ocb} - \overline{CN}_b + TR^3_{ob}$$

The price of liquidity, λ, is just a CES function[11] of the individual asset costs $\bar{r}_e + \pi$ and $\bar{r}_e - \bar{r}_b$. Notice that the "effective" wage to type 1 households, age 0 or 1, depends upon the payroll tax rate, τ, and the return factor R_c on the pension fund. If $\frac{\bar{R}_e}{R_c} = 1$, then the return on pension contributions just equals the after tax yield on equity, and these effective wages are then independent of τ. Finally, note that each wealth variable, W^i_{ab}, is the present value of lifetime disposable income minus the present value of lifetime subsistence requirements, evaluated at age 0,[12] where disposable income includes the market value of <u>all</u> labour time, whether supplied on the market or consumed as leisure. Equations (4.1)-(4.5) state simply that a household's demand for a commodity over and above the subsistence requirement, is proportional to wealth and inversely proportional to the "effective" price of the commodity.

We are interested in the comparative statics results on the quantities demanded by each household following a change in one of the following: (a) a decrease in CPP benefits, holding constant the contribution rates; (b) an increase in the defined level of YMPE; (c) an increase in τ, holding constant the YMPE and the rate of return on contributions; (d) an increase in τ, holding constant the YMPE but allowing the rate of return on the <u>additional</u> contribution to be zero; (e) an increase in the general tax rate on all income; (f) an increase in the cost of liquidity (λ); (g) an increase in the after-tax real rate of return on equity ($\bar{r}_e$); and (h) a once-over increase in the marginal product of an efficiency-unit of labour.

The method for conducting these conceptual experiments is quite simple. From (4.1)-(4.5), it follows that the effect of any change on x^i_{ab} can be divided into a wealth-effect and a substitution-effect. For example:

$$\frac{\partial \ell^1_{ob}}{\partial R_c} = \alpha^1_{o\ell} \left[\frac{1}{p^1_{o\ell}} \cdot \underset{(-)}{\frac{\partial W^1_{ob}}{\partial R_c}} - \frac{W^1_{ob}}{(p^1_{o\ell})^2} \cdot \underset{(+)}{\frac{\partial p^1_{o\ell}}{\partial R_c}}\right] \qquad (4.6)$$

In other words, a decrease in the rate of return on contributions, as in (a) above, will have a negative wealth-effect and a positive substitution-effect on the demand for leisure by a household of type 1, age 0. One advantage of the Stone-Geary utility function is that substitution-effects will exist on x^i_{ab} only when the "own price" (p^i_{ax}) is affected by the change. Thus, for example, only a wealth-effect exists in the result:

$$\frac{\partial c^1_{ob}}{\partial R_c} = \alpha^1_{oc} \left[\frac{1}{p^1_{oc}} \cdot \frac{\partial W^1_{ob}}{\partial R_c}\right] < 0 \qquad (4.7)$$

Generally speaking, when a substitution-effect and wealth-effect both exist but conflict in sign, the sign of the <u>total</u> effect cannot be determined <u>a priori</u>.

The following remarks are intended to help the reader understand precisely how the conceptual experiments are performed. In each case we differentiate the open-form demand functions (that is, the functions expressed in terms of <u>initial</u> wealth (types 1,2) or disposable income (type 3, age 0) or middle-aged wealth (type 3, ages 1,2)). Thus each comparative-static result shows the effect of the change on a household born after (or simultaneously with) the parameter change:

(a) Differentiate with respect to R_c, holding τ, $\overline{CN}_o$ constant.

(b) Differentiate with respect to $\overline{CN}_o$, with $(d\overline{CN}_t/d\overline{CN}_o)$ $= (1-g)^{-t}$, and $(d\overline{BN}_t/d\overline{CN}_o) = \frac{(1-g)^{-(t-2)}}{R_c^2} + \frac{(1-g)^{-(t-1)}}{R_c}$ holding τ and R_c constant.

(c) Differentiate with respect to τ, with $(\partial\overline{CN}_o/\partial\tau)(\tau/\overline{CN}_o)$ $= 1$, and $(d\overline{CN}_t/d\overline{CN}_o)$, $(d\overline{BN}_t/d\overline{CN}_o)$ as in (b), holding R_c constant.

(d) In this case, suppose that the effective wage to type 1, age 0 or 1, instead of being $p^1_{a\ell}$ as above, is:

$$\hat{p}^1_{a\ell} = p^1_{a\ell} - \hat{\tau}(1-tx),$$

and differentiate with respect to $\hat{\tau}$, at a point where $\hat{\tau} = 0$, with $(d\overline{CN}_o/d\hat{\tau})(\tau/\overline{CN}_o) = 1$, $(d\overline{CN}_t/d\overline{CN}_o)$ as in (c), but $(d\overline{BN}_t/d\hat{\tau}) = 0$.

(e) Differentiate with respect to tx, holding $\bar{R}_e$ λ, and all w^i_{ab}'s constant.

(f) Differentiate with respect to λ.

(g) Differentiate with respect to $\bar{R}_e$ (and change sign), holding λ and w^i_{ab}'s constant.

(h) Differentiate with respect to w^i_{ab} all (i,a,b), with $(dw^i_{ab}/dw^j_{a'b'})$ $= (w^i_{ab}/w^j_{a'b'})$.

The results of these conceptual experiments are tabulated in Table 1 below. The last three rows also give the changes in lifetime household utility (as of age 0). Notice that this Table ignores all possible feedback effects. For example, when $\bar{r}_e$ changes this will cause a change in the w^i_{ab}'s through the productive sector. Such feedback effects will be dealt with in the next section when we consider the overall effects of parametic changes.

Note that, in general, the effects of increasing the level of coverage of CPP (as in (b) or (c)) depend upon whether the rate of return on pension contributions is more or less than competitive with private equity holdings (that is, whether $\bar{R}_c/R_c \gtrless 1$), as indicated by Q. If the return on contributions is less than competitive then the effects on type 3 households at ages 1 and 2 will depend (as indicated by S) upon by how much the return is less than competitive. For a type 3 household who is "forced" to save in the form of more contributions, the extra (forced) saving at age 0 will increase his middle-aged wealth, because it represents saving that would not otherwise have occurred. But the extra saving at age 1 will decrease middle-aged wealth because this extra saving is a substitute for private saving that would have occurred at a higher yield.

Other than this, the main source of ambiguity is the effect upon leisure demand (and thus labour-supply) of any change that produces off-setting income and substitution-effects. These affect mainly type 1 households, who are affected on the margin by changes in the CPP provisions. But they may also affect other types, as in the case of a change in wages or in the return to equity. In the latter case the ambiguity affects not just leisure demand but also consumption and liquidity demand.

V. Macroeconomic Effects of Demographic and Pension Program Changes

In this section we are concerned with the overall effects of pension and demographic changes on certain aggregate variables. The variables that we wish to examine are labour supply, N_t^*, output Y_t, total domestic saving, S_t, the balance of payments, ΔE_{gt}, net foreign investment in Canada, ΔE_{et}, and the size of the non-government financial sector, ${}_fB_t$, all for $t = 1,2,3,4$ as well as the net worth of the fund in period $t=4$, V_{c4}. The periods are chosen to cover the economic life of the demographic bulge generation and the period immediately following to see if there will be a residual effect of the bulge or of parameter changes introduced at the beginning of $t=1$. We shall consider the effects of changes in:

1) the demographic bulge
2) the portfolio policy of the fund
3) the defined level of YMPE
4) the level of contributions to the fund
5) the level of benefits from the fund
6) the marginal tax rate.

To facilitate comparisons among alternative pension plan changes the last five experiments will be constructed to imply an increase in the net worth of the fund in $t=4$ under the assumption that $r_c > \bar{r}_e$.

1. <u>The Bulge</u>

The supply of labour in efficiency units is:

$$N_t^* = (1-g)^{-t} \sum_{i=1}^{3} \sum_{a=o}^{2} q_a^i \, PN_{a,t-a}^i (1 - \ell_{a,t-a}^i) \quad (5.1)$$

To find the effect of the bulge on the labour supply differentiate (5.1) w.r.t. $\overline{PN}$:

$$\partial N_t^*/\partial \overline{PN} = (1-g)^{-t} \sum_{i=1}^{3} \gamma^i q_{t-1}^i (1 - \ell_{t-1,1}^i) > 0 \quad (5.1a)$$

$$t = 1,2,3$$

$$\partial N_4^*/\partial \overline{PN} = 0 \quad (5.1b)$$

The effect in this case is what one would expect from the increase in population. A more interesting experiment would be to determine the per capita supply of efficiency units but the results are ambiguous, depending on the specific q_a^i's, α_ℓ^i's and ℓ_a^i's. Because some of our predictions depend upon the per capita supply of efficiency units we should assume that:

$$\text{AI} \qquad N^*_o/PN_o \begin{cases} = N^*_1(1-g)^{-1}/PN_1 \\ < N^*_2(1-g)^{-2}/PN_2 \\ > N^*_3(1-g)^{-3}/PN_3 \end{cases}$$

The equality is a matter of convenience and says that the average supply of efficiency units of the young equals that of the economy as a whole. The rationale of the first inequality is that the middle-aged are more productive than the young and probably will supply roughly the same amount of labour time per agent. The last inequality is valid if the old have roughly the same productivity as the young but agents demand more leisure in their old age than in their youth.

One of the reasons N^*_t is important is that it determines total output (Gross Domestic Product) by the relationship:

$$Y_t = \left[\frac{\xi}{\frac{\bar{r}_e}{1-tx}} \right]^{\xi/1-\xi} N^* \qquad (5.2)$$

Since $\partial\bar{r}_e/\partial\overline{PN} = 0$, it follows immediately that

$$\partial Y_t/\partial\overline{PN} \begin{cases} > 0 & t = 1,2,3 \\ = 0 & t = 4 \end{cases} \qquad (5.2a)$$

Total domestic saving is defined as the change in financial assets held by households plus the change in financial assets held by the CPP less the government deficit:

$$S_t = \sum_{i=1}^{3} \sum_{a=o}^{2} [PN^i_{a,t-a} \; A^i_{a,t-a} - PN^i_{a,t-a-1} \; A^i_{a,t-a-1}] + (F_t - F_{t-1}) - DF_t \qquad (5.3)$$

Differentiating w.r.t. $\overline{PN}$:

$$\partial S_1/\partial\overline{PN} = [\sum_{i=1}^{3} \gamma^i (A^i_{o1} + CN^i_{o1}(1-r_f)^{-1})] - \partial DF_1/\partial\overline{PN} > 0 \quad (5.3a)$$

$$\partial S_2/\partial\overline{PN} = [\sum_{i=1}^{3} \gamma^i (A^i_{11} - A^i_{o1} + CN^i_{11}(1-r_f)^{-1} + r_f CN^i_{o1}(1-r_f)^2)] - \partial DF_2/\partial\overline{PN} > 0 \quad (5.3b)$$

$$\partial S_3/\partial\overline{PN} = -\sum_{i=1}^{3} \gamma^i [A^i_{11} + BN^i_{21} - r_f\ \partial V_{c3}/\partial\overline{PN}] - \partial DF_3/\partial\overline{PN} < 0 \quad (5.3c)$$

$$\partial S_4/\partial\overline{PN} = r_f\ \partial V_{c4}/\partial\overline{PN} + \bar{r}_e\ \partial E_{g4}/\partial\overline{PN} \quad (5.3d)$$

The bracketed terms in (5.3a) and (5.3b) are positive but the terms $\partial DF_t/\partial\overline{PN}$ are ambiguous: From AI the tax base will increase more than expenditure given that OAS payments are not affected until $t = 3$. This causes the per capita deficit to fall, but given that the deficit was initially positive, the increased scale of government activity may cause the deficit to rise further. For simplicity we shall make use of AI to suppose:

$$\partial DF_1/\partial\overline{PN} = 0, \quad \partial DF_2/\partial\overline{PN} < 0, \quad \partial DF_3/\partial\overline{PN} > 0 \ . \quad (5.4)$$

The effect of the population bulge on saving in $t = 4$ and subsequent periods depends upon the residual effect of $\overline{PN}$ on the reserve fund account and the government accounts. The former depends on whether the fund is marginally sustainable. This follows from:

$$\partial V_{c3}/\partial\overline{PN} = R_c^{-1} \sum_{i=1}^{3} \gamma_i \{CN^i_{o1}R_f^{-2} + CN^i_{11}R_f^{-1} - [CN^i_{o1}R_c^{-2} + CN^i_{11}R_c^{-1}]\} \quad (5.5)$$

so that sign $\partial V_{c3}/\partial\overline{PN}$ = sign $(r_f - r_c)$ and from sign $\partial V_{ct}/\partial\overline{PN}$ = sign $\partial V_{c3}/\partial\overline{PN}$ for $t > 3$. Unless otherwise indicated, we shall suppose that:

$$r_f < r_c$$

which appears to be consistent with current CPP policy. We shall further suppose that the residual effect of $\overline{PN}$ on the deficit is zero to reflect the two offsetting forces at work: On the one hand, the operation of OAS payments is similar to the reserve fund account where the rate of return to "contributors" is r_g and the government portfolio rate of return is $\bar{r}_e$ earned on international reserves and assumed to be greater than r_g. On the other hand, because tax revenues generated by an agent over his lifetime do not cover the additional expenditure incurred over his lifetime (i.e., because, on average, the government is running a deficit), $\partial DF_t/\partial\overline{PN}$, $t > 3$ would tend to be positive. We therefore assume:

AII The effect of an additional representative worker on the net value of government operations is zero: i.e., $\partial DF_t/\partial\overline{PN} = 0$ for $t > 3$.

Thus, using AII and (5.5)

$$\text{sign } \partial S_t/\partial\overline{PN} = \text{sign } (r_f - r_c) \qquad t > 3$$

The effect of $\overline{PN}$ on the level of the government's net international reserves will equal:

$$\partial E_{gt}/\partial\overline{PN} = \partial M_t/\partial\overline{PN} + \partial\, {}_gB_t/\partial\overline{PN} - \sum_{j=1}^{t} \partial DF_j/\partial\overline{PN}$$

$$= \sum_{i=1}^{3} \gamma^i \left(M^i_{t-1,1} + \frac{\eta_m}{1-\eta_b} B^i_{t-1,1}\right) + \left\{\frac{{}_gB_o\left(1 - \frac{\eta_m}{1-\eta_b}\right)}{PN_o(1-g)^t} + \left(\frac{\eta_m}{1-\eta_b}\right)(1-\sigma_f)\frac{\partial F_t}{\partial\overline{PN}}\right\} \tag{5.6}$$

$$- \sum_{j=1}^{t} \partial DF_j/\partial\overline{PN}$$

We define the balance of payments as the change in the net international reserve position of the government:

$$\Delta E_{gt} = E_{gt} - E_{g,t-1}$$

Using (5.4), AII and the assumption of $E_{go} = 0$ we get

$$\left.\begin{array}{ll}\partial\Delta E_{gt}/\partial\overline{PN} > 0 & t=1,2\\ \partial\Delta E_{g3}/\partial\overline{PN} \gtrless 0 & \\ \partial\Delta E_{g4}/\partial\overline{PN} < 0 & \\ \partial\Delta E_{gt}/\partial\overline{PN} = 0 & t > 4\end{array}\right\} \quad (5.7)$$

The balance of payments is positive in t=1, because of an increase in money demand and an increase in non-tradable government bonds according to the rule (3.43). The surplus in t=2 is due to the fall in the deficit and the further increase in liquidity demand (at least by 1 and 2 type agents) while the ambiguity in t=3 is due to these two effects working in opposite directions. Given AII and the necessity for $_gB$ and M to fall back to their trend level in t=4 means that $\partial\Delta E_{g4}/\partial\overline{PN} < 0$ and that there is no influence of the bulge subsequently.

The impact of $\overline{PN}$ on the capital account is obtained from (5.8) and is given in (5.8a)-(5.8d).

$$\Delta E_{et} \equiv E_{e,t} - E_{e,t-1} = K_t - (E_{ht}+E_{ft}+E_{ct}) - E_{et-1}$$

$$= \sum_{i=1}^{3}\sum_{a=o}^{2} PN^i_{a,t-a}\{X_1(r_e)(1-g)^{-t}\, q^i_a(1-\ell^i_{at}) + M^i_{a,t-a} + \frac{\eta_m}{1-\eta_b} B^i_{a,t-a} - A^i_{a,t-a}\}$$

$$- (1 - \frac{\eta_m}{1-\eta_b}(1-\sigma_f))F_t + (1-\frac{\eta_m}{1-\eta_b})\, {}_gB_o PN_t/PN_o(1-g)^t - E_{et-1} \quad (5.8)$$

$$\partial\Delta E_{e1}/\partial\overline{PN} = X_1(r_e)\sum_{i=1}^{3}\gamma^i(1-g)^{-1}\, q^i_o(1-\ell^i_{o1})$$

$$- \sum_{i=1}^{3}\gamma^i[A^i_{o1} - M^i_{o1} - \frac{\eta_m}{1-\eta_b} B^i_{o1}] - [1-(1-\sigma_f)\frac{\eta_m}{1-\eta_b}]\,\partial F_1/\partial\overline{PN}$$

$$+ (1-\frac{\eta_m}{1-\eta_b})\, {}_gB_o/PN_o(1-g) \quad (5.8a)$$

$$\partial \Delta E_{e2}/\partial \overline{PN} = X_1(r_e) \sum_{i=1} \gamma^i [(1-g)^{-2} q_1^i(1-\ell_{11}^i) - (1-g)^{-1} q_o^i(1-\ell_{o1}^i)]$$

$$- \sum_i \gamma^i [A_{11}^i - A_{o1}^i - (M_{11}^i - M_{o1}^i) - \frac{\eta_m}{1-\eta_b}(B_{11}^i - B_{o1}^i)]$$

$$-[1-(1-\sigma_f)\frac{\eta_m}{1-\eta_b}][\partial F_2/\partial \overline{PN} - \partial F_1/\partial \overline{PN}$$

$$+(1-\frac{\eta_m}{1-\eta_b})[\frac{g\ {}_gB_o}{PN_o(1-g)^2}] \quad (5.8b)$$

$$\partial \Delta E_{e3}/\partial \overline{PN} = X_1(r_e) \sum_{i=1}^{3} \gamma^i [(1-g)^{-3} q_2^i(1-\ell_{21}^i) - (1-g)^{-2} q_1^i(1-\ell_{11}^i)]$$

$$+ \sum_{i=1}^{3} \gamma^i [A_{11}^i + (M_{21}^i - M_{11}^i) + \frac{\eta_m}{1-\eta_b}(B_{21}^i - B_{11}^i)]$$

$$- [1 - (1-\sigma_f)\frac{\eta_m}{1-\eta_b}][\partial F_3/\partial \overline{PN} - \partial F_2/\partial \overline{PN}]$$

$$+ (1 - \frac{\eta_m}{1-\eta_b})[\frac{g\ {}_gB_o}{PN_o(1-g)^2}] > 0 \quad (5.8c)$$

$$\partial \Delta E_{e4}/\partial \overline{PN} = - \partial E_{e3}/\partial \overline{PN}$$

$$= - \Big\{X_1(r_e) \sum_{i=1}^{3} \gamma^i q_2^i (1-g)^{-3}(1-\ell_{2,1}^i) + \sum_{i=1}^{3} y^i [M_{21}^i + \frac{\eta_m}{1-\eta_b} B_{21}^i]$$

$$-[1-\frac{\eta_m}{1-\eta_b}(1-\sigma_f)]\partial F_3/\partial \overline{PN} + (1-\eta_m/(1-\eta_b))\,{}_gB_o/PN_o(1-g)^3\Big\} < 0 \quad (5.8d)$$

The problem in signing capital movements is that households supply equity (by increasing the desired K_t by working and borrowing for liquidity needs) as well as demand it and we cannot really tell which effect dominates in either $t = 1$ or $t = 2$. For $t = 3$, however, age 2 households are suppliers of equity so that (5.8c) and (5.8d) can be unambiguously signed.

The size of the FI sector responds positively to household and fund demands for bonds and negatively to alternative supplies by the government:

$$ {}_fB_t = \frac{1}{1-\eta_b}[B_{hi} + B_{ct} - {}_gB_t] $$

$$ = \frac{1}{1-\eta_b}[\sum_{i=1}^{3}\sum_{a=0}^{2} PN^i_{a,t-a} B^i_{a,t-a} + (1-\sigma_f)F_t - {}_gB_o PN_t/PN_o(1-g)^t] \quad (5.9) $$

$$ \partial_f B_t/\partial\overline{PN} = \frac{1}{1-\eta_b}[\sum_{i=1}^{3} \gamma_i B^i_{t-1,1} + (1-\sigma_f)\partial F_t/\partial\overline{PN} - {}_gB_o/PN_o(1-g)^t] \quad t = 1,2,3 \quad (5.9a) $$

$$ \partial_f B_4/\partial\overline{PN} = \frac{1}{1-\eta_b}(1-\sigma_f)\ \partial F_4/\partial\overline{PN} \quad (5.9b) $$

It is difficult to sign all but $\partial_f B_4/\partial\overline{PN}$ (sign $(r_f - r_c)$ if $\sigma_f < 1$) because of the increases in both demand and alternative supplies and because $B^i_{a,t-a}$ increases with a for i = 1,2. Thus all the increased demand for bonds by households and the fund may be met by government supplies in t = 1. Since household and fund demand for bonds increases further in t = 2 we can say that $\partial_f B_2/\partial\overline{PN} > \frac{\partial_f B_1}{\partial\overline{PN}}$ but there are still circumstances where $\partial_f B_2/\partial\overline{PN}$ could be negative. In t = 3 there is a further increase in household demands but because $\partial F_3/\partial\overline{PN}$ is negative the ambiguity remains.

The effects of $\overline{PN}$ that have been discussed above are reproduced in Table 2, along with the results of the other parameter changes we are interested in. The other changes are discussed below but are not, in the interest of brevity, mathematically derived as with $\partial\overline{PN}$.

2. The Fund's Portfolio Policy

Next consider a change in portfolio policy of the CPP through a change in σ_f. The effects of this change are given in column 2 of Table 2. The basic effect is to alter the portfolios of the government, the foreign sector and FI's as well as the fund itself. Initially ignore the effect on the deficit and suppose the CPP purchases one unit of equity and reduces its holdings of bonds by one unit in period 1. For the economy as a whole the effect does not change equity holding: the additional government bond in the private sector reduces FI equity holding by $1-\eta_m/(1-\eta_b)$ and, by causing a reduction in required money holdings, leads the government to reduce its international reserves by $\eta_m/(1-\eta_b)$ so that the total effect is zero. There is, however, an additional effect associated with the tax treatment of CPP income: Because CPP interest income is untaxed the government deficit will be increased entailing cumulative additional losses of international reserves that will be exactly offset by cumulative surpluses by the fund.

3. YMPE.

In conducting the next three experiments involving changes in CPP parameters we suppose the policy changes are not retroactive and apply only to generation one and subsequent generations. Column 3 of Table 2 describes the effects of decreasing the YMPE for households with no change in the contribution rate, τ, or the rate of return on contributions, r_c, and given $r_c > \bar{r}_e$. This experiment will only affect types 2 and 3 households, but still has highly uncertain effects despite the absence of the substitution effects associated with type 1 households. The reason for this is that the "wealth" effects move in opposite directions for age 0, types 2 and 3 households: for the type 3 spenders the decrease in contributions in period 1 increases disposable income, so they work less, consume more and hold more liquid assets. But for the type 2's the decrease in benefits with $r_c > \bar{r}_e$ means a negative wealth effect causing just the opposite result. On the other hand, if $r_c < \bar{r}_c$ type 2's will have an increase in wealth, in which case $-\partial N_1^*/\partial \overline{CN}$ is unambiguously negative in all periods! The relative magnitudes of $-\partial N_1^*/\partial \overline{CN}$ listed in Table 2 are strictly valid only if the bulge is small ($\overline{PN} \approx 0$). Under this assumption they occur because the increase in labour supply is spread over the household's lifetime with the decrease in initial wealth. Overall (with the one exception of type 3 age 0 households) the effect on labour supply will be positive if $r_c > \bar{r}_e$ and unambiguously negative if the pension rate of return is less than the best return households can get by investing their own funds (i.e., $r_c < \bar{r}_e$).

The effect of total saving is ambiguous because of type 3's increase in consumption in age 0. If $-\partial S_1/\partial \overline{CN} < 0$ then the amount saving can fall in t = 2 and t = 3 is restricted by the amounts indicated in Table 2 (again assuming a "small" bulge). On the other hand, if $-\partial S_1/\partial \overline{CN} > 0$ then $-\partial S_t/\partial \overline{CN} > 0$ for all $t > 0$ since the decrease in contributions without any immediate change in benefits still increases savings. In the case where $-\partial S_1/\partial \overline{CN} < 0$ one may have a subsequent increase in saving in period 2 because the fund has yet to begin making reduced benefit payments. In the situation where r_c is less than $\bar{r}_e$, $-\partial S_t/\partial \overline{CN}$ will be negative for all t since agents are working less and consuming more over their lifetime.

The effect on the surplus (or deficiency) of the fund of contracting the scope of the CPP by decreasing $\overline{CN}$ is ambiguous. It depends upon whether or not the fund is marginally sustainable; that is, whether or not $r_f \geqq r_c$. If it is, then a decrease in the fund will decrease the surplus (or increase the deficiency). We shall assume, in line with most of the available evidence, that the fund is not marginally sustainable, i.e., that $r_c > r_f$.

Thus the decrease in $\overline{CN}$ will increase the surplus (or reduce the deficiency). Note, however that, in general, whether or not the fund is marginally sustainable cannot be inferred from knowing whether or not it is sustainable <u>overall</u>, as defined by (3.29) above. A marginally sustainable fund may be unsustainable overall because of insufficient initial funding, and a marginally unsustainable fund may be sustainable overall if it has enough initial funding.

The ambiguities in $-\partial\Delta E_g/\partial\overline{CN}$ and $-\partial\Delta E_e/\partial\overline{CN}$ should come as no surprise. If $-\partial N_t^*/\partial\overline{CN} < 0$, the deficit should rise, worsening the balance of payments, but $-\partial M_t/\partial\overline{CN}$ should be positive, causing a rise in international reserves. We cannot say which effect dominates except for the following proviso: If the fund is not marginally sustainable then F_t will rise at an accelerating rate, increasing money demand. Ultimately this effect will swamp the other effects so that $-\partial\Delta E_{gt}/\partial\overline{CN}$ will (ultimately) become positive. With $-\partial\Delta E_{et}/\partial\overline{CN}$ we are more in the dark because it depends negatively on $-\partial N_t^*/\partial\overline{CN}$ and positively on $-\partial S_t/\partial\overline{CN}$ and these two items tend to move in the same direction. If any guess might be hazarded, the marginal non-sustainability of the fund will ultimately lead to decreasing capital inflow.

The size of the financial sector will shrink in periods 1 and 2 but will begin to grow in periods $t \geq 3$ as the fund size grows due to the lower benefit payments to type 2 and 3 agents under the assumption that the fund is not marginally sustainable.

4. The Rate of Return on Contributions: Increasing Contributions

The next two experiments are directly concerned with the problem of moving the Pension Fund to a position of sustainability through changes in the rate of return on contributions, r_c. The first experiment, summarized in column 4 of Table 2, involves increasing the contribution rate with no increase in benefits. We denote such a change as $d\hat{\tau}$. The change is the same as in (d) of Section IV. As is clear from Table 1, there is some ambiguity in the labour supply of type 1's in ages 0 and 1 due to conflicting wealth and substitution effects. To make any headway we will assume that the two effects exactly cancel. That is,

AIII Whenever conflicting wealth and substitution effects exist on the supply of labour result in ambiguity according to Table 1, we assume they just cancel out.

Consequently, using AIII, wealth effects cause a decrease in type 2 and 3's leisure demand and an increase in N_t^* for all $t \geq 1$. Further, as more generations come to be covered by the new contribution plan, per capita labour supply will grow from $t = 1$ to $t = 3$.

The effect on saving will be positive since types 2 and 3 will be working more and all agents will be consuming less. V_{ct}, $t \geq 1$, will increase for the obvious reason that only contributions have increased and benefits have not. There is substantial ambiguity in determining the effect of $d\hat{\tau}$ on both the balance of payments and the capital account. Causing the balance of payments to improve is the reduction in the government deficit due to increased tax revenue levied on the increase in output. However the tax exemption on the pension fund net revenue may more than offset this effect on the deficit and, in addition, the decrease in money demand by households (and perhaps FIs if σ_f is high) leads to a portfolio shift that ceteris paribus will cause the balance of payments to worsen. The ambiguity connected with $d\Delta E_{ct}/d\hat{\tau}$ is because both the supply of and demand for domestic equity have increased. Depending on the portfolio policy of the pension fund the scale of the FI sector will increase or decrease: If the fund holds only equity ($\sigma_f = 1$), then demand for bonds will unambiguously fall since household liquidity demand has decreased. On the other hand, if the assumption of $\sigma_f = 0$ is imposed, $d_fB_t/d\hat{\tau} > 0$ because household disposable income will not fall by the entire amount of the increased contributions ($= \Delta B_c$) and the marginal propensity to hold bonds is less than one. Table 2 describes the latter, more realistic case.

5. The Rate of Return on Contributions: Decreased Benefits

This experiment supposes that the benefit levels for pension fund contributions are reduced given $\overline{CN}_t$ and the payroll tax rate τ. The effects of this change are qualitatively the same as those caused by an increase in contributions described in column 4 of Table 2. Rather than discuss these results, we would like to indicate whether an increase in contributions or decrease in benefits as a method of increasing V_{c4} by a given amount would be preferred by the various types of agents. We suppose AIII so that the contribution of type 1's to the increase in V_{c4} is the same under both experiments.

For type 1 agents the preferred policy depends on whether $r_c \gtrless \bar{r}_e$: if the new r_c remains greater than $\bar{r}_e$ then the increase in contributions is preferred to a decrease in benefits because the pension fund still provides the best vehicle for saving. Hence type 1's can cushion the effects of the fall in wealth by reducing saving for old age in the form of the lower yielding equity holding. On the other hand, if the new $r_c < \bar{r}_e$, personal saving is preferred to pension saving and a reduction of benefits will be preferred. The same reasoning holds true for type 2 agents. For type 3 agents the preferred method is somewhat different. Recall that these agents are income constrained in their youth and would not choose to save even through the pension fund if $\bar{R}_e > R_c > \Omega^3$, (i.e., the rate of return on contributions is less than the rate of time preference). Consequently, for some $r_c > \bar{r}_e$ type 3 agents will still prefer decreased benefits to increased contributions. It should also be noted that for a given decrease in $\hat{F}_t$, and thus increase in V_{ct}, the required reduction in r_c will be less for a decrease in benefits than for an increase in contributions. Also, if the change in contributions or benefits is retroactive, the latter will include the oldest generation whereas the increase in contributions will not. Thus, if it is felt that, in moving to a sustainable fund, there should be a redistribution toward future generations and/or that a sustainable fund would require $r_c \leq \bar{r}_e$, a decrease in benefits is preferred to an increase in contributions.

6. The Marginal Income-Tax Rate

The next experiment involves an increase in tx, the marginal tax rate. We consider it for two reasons: first, if the government decides to increase OAS payments--or if the CPP is absorbed into the government--and this is financed from general revenues, it would be useful to know the consequences of financing any changes from income taxes. Secondly, if the CPP is currently in a non-sustainable position, one alternative is for the government to provide it with adequate capitalization ($\hat{F}_t$) by a once-and-for-all increase in their debt. This then leaves the liability for the initial largess in the lap of the lawmakers who created the CPP. One method of ultimately reducing this liability is through an increase in taxes to pay off the interest and/or principal of that liability.

To remove at least some of the ambiguities, we again invoke A III so that the labour supply is unaffected by the effect of the increase in taxes on the effective real wage.[13] Even with no decrease in labour supply, income (and the pre-tax real wage) will fall because, to maintain the after tax rate of return on equity, the capital stock must decline.

While personal savings falls, it does not fall by the full amount of the increase in tax revenue so that total savings in the economy rises with individuals saving less and both the government and the pension fund saving more. There are two channels causing the fund to obtain an increased net worth. First, for a given portfolio policy, σ_f, the pre-tax rates of return on bonds and equities have increased causing r_f to rise. Secondly, the increase in tax rates lowers the real wage for all agents which lowers contributions and benefits for type 1 agents and, given the current legal definition of YMPE, lower the $\overline{CN}_t$ and $\overline{BN}_t$ facing type 2 and 3 agents. As indicated earlier, this decrease in scale reduces $\hat{F}_t$.

With no change in government spending and bond issue rules, international reserves ΔE_{gt}, will certainly increase although mitigating this somewhat is the decrease in money demand by households and FIs. It should also be noted that if inflation is positive, $d\bar{r}_b/dtx$, is negative causing a shift toward money and away from bonds by households. The effect of increased taxes has an ambiguous effect on foreign ownership. On the one hand the decrease in private saving and increase in government saving (held in the form of foreign equity) would tend to increase capital inflows, while on the other hand the lower level of income would tend to reduce ΔE_{et}. While we cannot sign ΔE_{et} a priori, we do know that income will fall to its new equilibrium path in t = 1 so the decrease in the stock of capital should be greatest in t = 1. We have therefore signed $d\Delta E_{e1}/dtx$ as negative in column 5 of Table 2. Furthermore, if $\bar{r}_e > g + n$, it means that governmental foreign equity holdings will be growing faster than the rest of the economy and so we have supposed $d\Delta E_{e4}/dtx > 0$. The decrease in household liquidity demand should cause ${}_fB_t$ to fall, but offsetting this is the increase in pension fund demand for bonds if $\sigma_f < 1$. Because this latter effect will become progressively more important we have indicated that $d_fB_4/dtx > 0$ and $d_fB_t/dtx < 0$ for $t \leq 3$.

In column 5 of Table 2 it is assumed that only taxes were increased. If on the other hand, OAS payments were increased simultaneously so that the government deficit remained unchanged in t = 1 the following changes would be indicated: First labour supply would fall in t = 1 and t = 2 because of the positive wealth effect to cohorts b = -1 and (to a lesser extent) b = 0. Total savings would be reduced although the increase in savings by the fund may ultimately reverse this effect. The balance of payments would be negative at least for $t \geq 3$ because of the decreased liquidity demand due to decreased wealth. Capital flows will be more positive than if only tx were increased because consumption will not fall as far.

At this point the question arises: Ignoring intra-generational transfers, is it preferable to finance a given total level of payments to the old (OAS plus CPP benefits) from general tax revenue or from payroll taxes? This question is important not only for Canada, where both options are available but also in the United States where there is currently much discussion on alternatives to the payroll tax to finance Social Security. To handle this issue here we can simplify considerably by ignoring the inter- and intra-generational problems of OAS and the pension fund and look only at the effect of tx and τ on the after tax wage for a labour unit,

$$H_t = w_t^*(1-tx)(1-\tau) \tag{5.10}$$

The financial policy that maximizes H subject to the constraint that total payments to the old,

$$Z_t = tx(w_t^* + r_e k_t) + \tau(1-tx)w_t^*$$

is constant represents the preferred policy because it maximizes wealth and minimizes work disincentives to private agents. Carrying out the maximization:

$$\left. dH_t/dtx \right|_{dZ_t = 0} = -(1-\tau)w_t^* - (1-\tau)r_e k_t - (1-tx)w_t^* \, d\tau/dtx$$

$$= tx\, r_e\, \partial k_t/\partial tx < 0 \tag{5.11}$$

This is to say that the optimal general tax rate on capital is zero and, under the assumption of perfect capital mobility and ignoring intra-generational distribution effects, the best method of financing old age payments is through a payroll tax rather than a general income tax.

The reason for this result is quite simple. Given perfect capital flows, the government cannot affect the after-tax rate of return on capital so that labour must bear the full burden of any tax. The government, by attempting to extract some rents from capital owners, causes some to withdraw their services from Canada thereby reducing the tax base while leaving the net return on capital unchanged. Since the tax base is reduced, a given increase in the general tax rate will mean a decrease in the payroll tax by a smaller amount causing the after-tax and before-tax wage rates to fall.[14]

VI. Concluding Remarks

The preceding section described five experiments that bear on the performance of the Canadian Pension Plan and the consequences to macroeconomic aggregates. The model can be used to consider additional experiments which we will leave the reader to work out. We will, however, briefly indicate how to structure a few that might be of interest as well as tie up a few loose ends.

One complication we have not discussed is adding private pension plans. Given that they receive the same tax treatment as the CPP (and competition enforces a zero profit on them), they can be treated as an integral part of the pension fund discussed in this paper. Empirically, this would be handled by requiring a higher σ_f for the "consolidated fund" and would suggest a fund that was more sustainable since, if the private sector component were not, they would ultimately go bankrupt. Furthermore, in practice, private funds collect a payroll tax on income above the YPME of the CPP. This suggests an operationally greater number of our type 1 agents relative to the entire population. In addition, the type 3's welfare is most likely to suffer from the higher contributions made under private funding, except insofar as they can, by changing jobs obtain access to the pension funds at an earlier date. Current provincial legislation that does not permit an agent to spend these accumulated funds hurts these type 3's probably to the gain of types 1 and 2. Vesting and portability regulations also will affect the intra-generational distribution of welfare via a similar mechanism.

Throughout our analysis we have implicitly supposed that the type of pension is of the money purchase sort. If, instead, benefits were not completely tied to contributions, e.g., by basing benefits on second period income as many company pensions do, the effect on type 1's will be altered. In particular, the substitution effects due to $r_c \neq \bar{r}_e$ will not exist for the young worker provided he does not perceive that ultimately the rate of return must be tied to his contributions. This might suggest greater mobility among the young relative to the middle aged especially if there are many liquidity constrained type 1's (i.e., a combined high time preference "low" income type).

One program that has not received any attention in our analysis but is important to the Canadian scene is the Registered Retirement Saving Plan. RRSPs have been touted by some as a cureall for many of the saving, portability, vesting, and flexibility problems associated with public and private pensions. In our model they only provide a mechanism to delay (and avoid some) taxes. This follows immediately from the fact that RRSPs have a ceiling so that they will not affect marginal decisions and, more importantly, because one can borrow using RRSPs as collateral. As such, (and in the absence of transactions costs) it pays everyone to borrow to the limit allowed to obtain the maximum reduction in current taxes. Such "private saving" is more than offset by either an increase in the government deficit, since the wealth effect of such a program will also reduce the labour supply and output, or an increase in taxes if government spending is to be maintained. The effect of the latter is shown in column 5 of Table 2.

One can interpret the analysis we have gone through as refering to an economy on a fixed exchange rate. An alternative interpretation is that it describes a "managed" float where the government intervenes in the foreign exchange market to maintain a given rate of inflation that differs from the world rate of price change. With such an interpretation it would then be possible to conduct the experiment of altering the rate of inflation. In our model this would alter the price of liquidity and the relative yields on money and bonds. It should also be pointed out that if tax policy is not fully indexed--e.g., if capital gains due to inflation are taxed--then the "inflation tax" policy will affect the capital stock in a manner qualitatively the same as a change in tx. We leave it to the reader to work out the implications of such a change. Finally it should be noted that if a $\Delta E_{gt} = 0$ rule were adopted instead of the bond rule in the text, this would correspond to a clean float (or "properly managed" fixed rate if π were set equal to the world rate of inflation).

TABLE 1

Effects of Changes on Individual Demands

	a	b	c	d	e	f	g	h
c^1_{ob}	-	0	Q	-	-	0	-	+
c^1_{1b}	-	0	Q	-	-	0	?	+
c^1_{2b}	-	0	Q	-	-	0	+	+
c^2_{ob}	-	Q	Q	-	-	0	-	+
c^2_{1b}	-	Q	Q	-	-	0	?	+
c^2_{2b}	-	Q	Q	-	-	0	+	+
c^3_{ob}	0	-	-	-	-	0	0	+
c^3_{1b}	-	S	S	-	-	0	-	+
c^3_{2b}	-	S	S	-	-	0	+	+
ℓ^1_{ob}	?	0	?	?	-	0	?	?
ℓ^1_{1b}	?	0	?	?	-	0	?	?
ℓ^1_{2b}	-	0	-	-	-	0	+	?
ℓ^2_{ob}	-	Q	Q	-	-	0	-	?
ℓ^2_{1b}	-	Q	Q	-	-	0	?	?
ℓ^2_{2b}	-	Q	Q	-	-	0	+	?
ℓ^3_{ob}	0	-	-	-	-	0	0	+
ℓ^3_{1b}	-	S	S	-	-	0	-	?
ℓ^3_{2b}	-	S	S	-	-	0	+	?
L^1_{ob}	-	0	Q	-	-	-	-	+
L^1_{1b}	-	0	Q	-	-	-	?	+
L^1_{2b}	-	0	Q	-	-	-	+	+
L^2_{ob}	-	Q	Q	-	-	-	-	+
L^2_{1b}	-	Q	Q	-	-	-	?	+
L^2_{2b}	-	Q	Q	-	-	-	+	+
L^3_{ob}	0	-	-	-	-	-	0	+
L^3_{1b}	-	S	S	-	-	-	-	+
L^3_{2b}	-	S	S	-	-	-	+	+
ϕ^1_{ob}	-	0	Q	-	-	-	?	+
ϕ^2_{ob}	-	Q	Q	-	-	-	?	+
ϕ^3_{ob}	-	?	?	-	-	-	?	+

Q: $\text{sgn}\ (\bar{R}_e/R_c - 1)$

S: $\text{sgn}\ \left[\left(\frac{\bar{R}_e}{R_c}\right)^2 + \frac{\bar{R}_c}{1-g}\left(\frac{\bar{R}_c}{R_c} - 1\right) \right]$

Table 2

	(1) $\overline{PN}$	(2) σ_f	(3) $-\overline{CN}_o$ $(r_c > \bar{r}_e)$	(4) $\hat{\tau}^d$	(5) tx^d
N_1^*	+	0	?	+	0
N_2^*	+	0	$> R_g^{-1}\ \Delta N_1^*$	+	0
N_3^*	+	0	$> R_g^{-1}\ \Delta N_2^*$	+	0
N_4^*	0	0	$= R_g^{-1}\ \Delta N_3^*$	+	0
Y_t	SAME SIGN AS FOR N_t				$< \partial N_t^* / \partial tx$
S_1	+	0	?	+	+
S_2	+	0	$> (R_g + \bar{r}_e)\ \Delta S_1$	$> (R_g + \bar{r}_e)\Delta S_1$	+
S_3	$-^a$	0	$> (R_g + \bar{r}_e)\ \Delta S_2$	$> (R_g + \bar{r}_e)\Delta S_2$	+
S_4	sign $(r_f - r_c)^c$	0	$> R_g\ \Delta S_3 > 0$	$> (R_g + \bar{r}_e)\Delta S_3$	+
V_{c4}	sign $(r_f - r_c)$	+	$+^a$	+	$+^a$
ΔE_{g1}	$+^b$	-	?	?	+
ΔE_{g2}	$+^b$	-	?	+	+
ΔE_{g3}	?	-	$+^a$	+	+
ΔE_{g4}	$-^{bc}$	-	$+^a$	+	+
ΔE_{e1}	?	-	?	?	-
ΔE_{e2}	?	-	?	?	?
ΔE_{e3}	+	-	?	?	?
ΔE_{e4}	-	-	$-^a$	?	+
$f^B{}_1$	?	-	-	$+^e$	-
$f^B{}_2$	?	-	$< R_g^{-1}\ \Delta f^B{}_1$	$+^e$	-
$f^B{}_3$	?	-	?	$+^e$	-
$f^B{}_4$	$-^a$	-	$+^{ae}$	$+^e$	$+^a$

a) assuming $r_c > r_f$ (i.e., that the fund is not marginally sustainable)

b) using AI

c) using AII

d) using AIII

e) assuming $\sigma_f = 0$.

FOOTNOTES

[1] We are treating the QPP as part of the CPP.

[2] Such a scheme has been worked out by Uzawa (18), Gaudet (8), and, for an open economy, by Frenkel and Rodriguez(7).

[3] This follows the work of Chetty (5) whose interest was in measuring the "nearness" of near money.

[4] This is consistent with the observation that most saving occurs among ages 40-60 (our age 1), and with the evidence from cross sectional studies of wage incomes.

[5] $\bar{r}_e$ is given by the after-tax rate of return in the rest of the world.

[6] Note that these are pretax rates of return because of the exemption from taxation of fund income.

[7] Equations (3.29) and (3.32)~(3.35) are derived explicitly in Appendix B. They involve the simplifying assumption that at any date a household of a given type make the same sized contribution regardless of age. This will be exactly true for types 2 and 3 but not 1.

[8] Currently, $\sigma_f = 0$ by statute.

[9] As in the case where a country's capital stock exceeds the golden rule level, every generation can be made better off if any initial funding is dissipated than if it is maintained.

[10] Under a fixed exchange rate, the rate of inflation is given by the rest of the world.

[11] The exact form is derived in Appendix C.

[12] This is done to be consistent with prices which are also evaluated as of age 0.

[13] In fact, due to the unanticipated nature of dtx on cohorts b = 0 and b = -1 it is likely that they will reduce their labour supply by more than that of subsequent generations because their wealth falls by a smaller percentage than does that of generations $b \geq 1$.

[14] Even if capital is less than perfectly elastic it may still be better to use the payroll tax on the general principle of taxing the least elastic factor of production. One qualification is in order if countries tax their nationals' income from overseas investment but permit tax payments to the host country to be deducted from the tax liability of the home country. If this were the case, the optimal income tax rate is no greater than the foreign income tax rate. Further, if labour supply is constant, an increase in tx from zero up the foreign tax rate, tx^*, will leave gross domestic product unchanged but will reduce gross national product--i.e., will increase foreign ownership--supposing, of course, that foreign ownership is positive at tx = 0.

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APPENDIX A

$A^{i}_{a,\,b}$: Real value of financial assets held by an agent of type i, aged a, born at time b.

$A_{h,\,t}$: $\sum_{i=1}^{3} \sum_{a=0}^{2} A^{i}_{a,\,t-a} \; PN^{i}_{a,t-a}$

$_{z}B_{y,\,t}$: Real value of bonds outstanding in period t that are issued by sector z and held by sector y.

B^{i}_{ab} : Real value of bonds held by an aged a, type i, generation b household.

$\hat{B}_{t}, \hat{B}^{r}_{t}$: Real value of bonds that must be held in the pension fund's normal and reserve accounts in period t if the pension program is sustainable.

$\overline{BN}_{t}$: Maximum pension an individual is permitted in period t.

$BN^{i}_{a,\,b}$: pension benefit payment to an (i, a, b) household.

BN_{t} : $\sum_{i=1}^{3} \sum_{a=0}^{2} B^{i}_{a,t-a} \; PN^{i}_{a,t-a}$.

$C^{i}_{a,\,b}$: Consumption of an (i, a, b) household.

$\overline{CN}_{t}$: τ . YMPE, the maximum individual contribution in period t.

$CN^{i}_{a,\,b}$: Pension contribution of an (i, a, b) household.

CN_{t} : $\sum_{i=1}^{3} \sum_{a=0}^{2} CN^{i}_{a,\,t-a} \; PN^{i}_{a,t-a}$.

DF_{t} : The government deficit in period t.

$E^{i}_{a,b}$: Real value of equity held by an (i, a, b) household.

$E_{x,\,t}$: Real value of equity held by sector x in period t.

$\Delta E_{g,\,t}$: $E_{gt} - E_{g,\,t-1}$, the balance of payments in period t.

$\Delta E_{e,\,t}$: $E_{et} - E_{e,\,t-1}$, the capital account in period t.

$\hat{E}_t$, $\hat{E}^r_t$: Real value of equity that must be held in the pension fund's normal and reserve accounts in period t if the pension program is sustainable.

F_t : The real value of financial assets held by the pension fund in period t.

$\hat{F}_t$, $\hat{F}^r_t$: The real value of financial assets that must be held by the pension fund in period t if the pension program is sustainable.

g : Rate of labour augmenting technical change.

G_t : Government purchases of goods in period t.

I_t : Investment in period t.

K_t : The capital stock in period t.

k^*_t : K_t/N^*_t, the capital-labour ratio in period t.

$L^i_{a,b}$: Liquidity demanded by an (i, a, b) household.

$\underline{L}^i_{ab}$: Subsistence liquidity level for an (i, a, b) household.

ℓ^i_{ab} : Leisure demand of an (i, a, b) household.

$\underline{\ell}^i_{ab}$: Subsistence leisure level for an (i, a, b) household.

M^i_{ab} : Money holdings of an (i, a, b) household.

n : The trend rate of growth of population.

N^*_t : The supply of efficiency units of labour in period t.

OAS_t : Total old age security payments in period t.

P^i_{axb} : The effective price of x to an (i, a, b) household.

PN^i_{ab} : Population of (i, a, b) households.

PN_t : Population at time $t = \sum_{i=1}^{3} \sum_{a=o}^{2} PN^i_{a,\, t-a}$

$\overline{PN}$: The excess population of generation 1.

q^i_a : The relative productivity of an (i, a) household in period 0.

R_x : The interest rate factor $(1-r_x)$, $x = b, e, c, f$.

$\bar{R}_x$: $(1-\bar{r}_x)$, $x = b, e$.

r_g : Growth rate of the economy: $(1-r_g) = (1-n)(1-g)$.

r_b : Nominal pre-tax rate of return on bonds.

$\bar{r}_b$: Real after-tax rate of return on bonds.

r_c : Real rate of return on pension plan contributions.

r_e, $\bar{r}_e$: Real pre-tax and after-tax rates of return on equity.

r_f : Real rate of return on the CPP portfolio.

r_k : The marginal product of capital.

S_t : Domestic saving in period t.

tx : The (constant) marginal tax rate on taxable income.

TB_t : The tax base in period t.

TR^i_{ab} : General government transfers (excluding OAS payments) to (i, a, b) households.

TR_t : $\sum_{i=1}^{3} \sum_{a=0}^{2} TR^i_{a,\, t-a} \eta^i_{a,\, t-a}$.

$V_{c,\, t}$: Net worth of the CPP in period t.

w^*_t : Wage rate for an efficiency unit of labour in period t.

w^i_{ab} : Wage rate for a unit of labour-time from an (i, a, b) household.

W^i_{ab} : Present value of lifetime disposable income less the present value of lifetime subsistence requirements of an (i, a, b) household (where everything is discounted back to the household's birth date).

Y_t : Gross domestic product.

Y^i_{ab} : Disposable income less subsistence requirements of an (i, a, b) household.

$YMPE_t$: Maximum pensionable earnings in period t.

α^i_{ax}, β^i_{ax} : Income and wealth coefficients in demand functions.

γ^i : Proportion of households of type i to total population.

δ : Coefficient of the liquidity function.

η_x : Required reserve ratios for FIs, x = m, b.

λ : The price of liquidity.

ξ : A coefficient of the production function, capital's share of output.

π : The (expected) rate of inflation.

ρ : Coefficient of the liquidity function.

σ : $(1 + \rho)^{-1}$

σ_f : Share of CPP portfolio held in equities.

τ : The contribution rate on wage income.

Φ^i_{ab} : The welfare level of an (i, a, b) household.

ψ_t : The level of OAS payments to a household in period t.

Ω^i : The rate of time preference factor of an i type household.

APPENDIX B

In this appendix we first derive the formula (3.29) for the sustainable size of the "normal" account of the CPP. To be sustainable the account must grow at the rate r_g. That is:

$$(B.1)\quad \hat{F}_t = \hat{F}_{t-1} + r_g \hat{F}_t.$$

But the equation for the actual growth of F_t is:

$$(B.2)\quad \hat{F}_t = \hat{F}_{t-1} + CN_t - BN_t + r_f \hat{F}_t.$$

Subtracting (B.1) from (B.2) and rearranging produces:

$$(B.3)\quad \hat{F}_t = (CN_t - BN_t)/(r_g - r_f).$$

To produce (3.29) from (B.3) we need to express BN_t in terms of CN_t and r_c. To do this let χ denote the fraction of CPP beneficiaries in the total population at any date and $(1 - \chi)$ denote the fraction of CPP contributors in the total population. In a steady state with population growing at the rate n:

$$(B.4)\quad \left(\frac{\chi}{1 - \chi}\right) = \frac{(1 - n)^2}{2 - n}.$$

In a steady state the benefit per beneficiary at date t is: $BN_t/\chi\eta_t$. Then the rate of return on contributions must satisfy:

$$(B.5)\quad \frac{BN_t}{\chi\eta_t} = \frac{CN_{t-1} R_c^{-1}}{(1-\chi)PN_{t-1}} + \frac{CN_{t-2} R_c^{-2}}{(1-\chi)PN_{t-2}}$$

(This formula assumes that each contributor makes a contribution at age 1 equal to $(1-g)^{-1}$ times his age 0 contribution, which will be true for types 2 and 3 but not exactly for type 1.

In a steady state, PN_t grows at the rate n and CN_t at the rate r_g. Therefore, from (B.5):

$$(B.6)\quad \frac{BN_t}{CN_t} = \left(\frac{\chi}{1-\chi}\right)\left((1-n)^{-1}(1-r_g)R_c^{-1} + (1-n)^{-2}(1-r_g)^2 R_c^{-2}\right).$$

From (B.3), (B.4), and (B.6):

$$(B.7)\quad \hat{F}_t = \frac{CN_t}{(2-n)}\left\{2-n-(1-r_g)(1-n)R_c^{-1} - (1-r_g)^2 R_c^{-2}\right\} / (r_g - r_f)$$

which is just (3.29).

Next we derive formula (3.32) in the text. Note that the required reserve fund at the end of date 0 is just the present value at that date of the benefits to the surplus members of the bulge generation, minus the present value of their contributions, where benefits and contributions are both disconnected at the rate (r_f) on the fund's portfolio. That is:

$$(B.8)\quad \hat{F}_o^r = BN_3^r (1-r_f)^2 - CN_2^r (1-r_f) - CN_1^r,$$

where the surplus members' total benefits are:

$$(B.9)\quad BN_3^r = BN_3\ \overline{PN}/PN_3\ \chi.$$

and their total contributions are:

$$(B.10)\quad CN_t^r = CN_t\,\overline{PN}/PN_t\ (1-\chi) \qquad t = 1,\ 2.$$

Formula (3.32) follows from (B.8), with the help of (B.4), (B.6), (B.9), and (B.10).

Formulas (3.33) and (3.34) follow from the growth equation:

$$(B.11)\quad \hat{F}_t^r = (1-r_f)^{-1}\left[\hat{F}_{t-1}^r + CN_t^r - BN_t^r\right],$$

with the help of (B.4), (B.6), (B.9), and (B.10).

Equation (3.35) asserts simply that after $t = 3$ the reserve fund will vanish if initially it was just sustainable, because the surplus members are all dead.

APPENDIX C

This appendix derives the exact form of λ, the price of liquidity, as expressed on page 26 in the text following equation (4.5). This price is just the Lagrangean multiplier for the problem;

$$\text{(C.1)}\quad \begin{cases} \text{Min}(\bar{r}_e + \pi)M + (\bar{r}_e - \bar{r}_b)B \\ \text{s.t. } L = [v M^{-\rho} + (1 - v) B^{-\rho}]^{-(1/\rho)}. \end{cases}$$

The first-order conditions for (C.1) are:

$$\text{(C.2)}\quad \begin{cases} (\bar{r}_e + \pi) = \lambda v (L/M)^{1/\sigma} \\ (\bar{r}_e - \bar{r}_b) = \lambda(1 - v)(L/B)^{1/\sigma} \end{cases}$$

where $\sigma \equiv (1 + \rho)^{-1}$. Because this CES function has constant returns to scale, therefore λ is independent of L. So in deriving λ we may set L = 1 in (C.2). Then, multiplying the two equations of (C.2) by M and B respectively produces:

$$\text{(C.3)}\quad \begin{cases} (\bar{r}_e + \pi) M = v \lambda M^{-\rho} \\ (\bar{r}_e - \bar{r}_b)B = (1 - v) \lambda B^{-\rho}. \end{cases}$$

Adding the two equations of (C.3) produces:

$$\text{(C.4)}\quad (\bar{r}_e + \pi)M + (\bar{r}_e - \bar{r}_b)B = \lambda[vM^{-\rho} + (1 - v)B^{-\rho}] = \lambda L^{-\rho} = \lambda.$$

Rearranging (C.3) produces:

$$\text{(C.5)}\quad \begin{cases} M = [v \lambda/(\bar{r}_e + \pi)]^{\sigma} \\ B = [(1 - v) \lambda/(\bar{r}_e - \bar{r}_b)]^{\sigma}. \end{cases}$$

Substituting from (C.5) into (C.4) produces:

$$\text{(C.6)}\quad [v^{\sigma}(\bar{r}_e + \pi)^{1-\sigma} + (1 - v)^{\sigma} (\bar{r}_e - \bar{r}_b)^{1-\sigma}]\lambda^{\sigma} = \lambda.$$

The formula for λ follows directly from (C.6). (Note that $1 - \sigma = -\delta$.)