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P.O. Box 527
Ottawa, Ontario
K1P 5V6

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Productivity Trends and their Causes in the Canadian Mining Industry, 1957-79

By: Kenneth R. Stollery

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RÉSUMÉ

L'auteur de cette étude calcule les tendances de la productivité plurifactorielle dans sept industries minières canadiennes, et il établit une relation entre les variations de la productivité et divers facteurs, y compris les variations dans la production et les prix des facteurs, ainsi que la baisse de qualité des minerais d'extraction. Il constate que le déclin de la productivité dans l'industrie minière a été très prononcé et qu'il a eu tendance à précéder de plusieurs années le ralentissement de la productivité dans le secteur de la fabrication. Il semble que les plus importants facteurs de ce déclin ont été la forte baisse de la qualité des minerais, la réduction de la production, la hausse des taux d'intérêts et un ralentissement apparent du progrès technologique. Comme facteurs négatifs, soulignons le peu d'importance apparente des grèves et des coûts de la main-d'oeuvre et de l'énergie. Selon d'autres travaux de recherche, le ralentissement de l'innovation technologique peut être attribué à la forte proportion de propriété étrangère dans l'industrie minière et à l'absence d'une puissante industrie d'extraction canadienne.

ABSTRACT

This study calculates multifactor productivity trends in seven Canadian mining industries and relates changes in productivity to various factors including changes in output and factor prices and the decline in the quality of the ore being mined. It is found that productivity decline in mining has been pronounced and tended to predate that of manufacturing by several years. There are indications that the most important factors in the decline have been the fall in mineral grade, contraction of output, increases in interest rates, and an apparent decline in the rate of technical innovation. Important negative findings are the apparent unimportance of strikes, labour and energy costs in productivity decline. Other research suggests that the decline in the rate of technical innovation may be related to the prevelance of foreign ownership in mining and to the lack of a significant Canadian-owned mineral supply industry.

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INTRODUCTION

Since the early 1970's, an apparent slowdown in the growth of industrial productivity has occurred throughout the industrialized countries. From an average annual growth rate of total factor productivity in Canada of 2.2% from 1967-73, the rate fell to zero from 1974-79. This productivity decline is very serious, for it implies the possibility of increased social conflict within increasingly economically constrained societies in the developed world, and increased international tension between developed and developing countries.

As the focus of this study, the performance of the mining industry in Canada has been one of the worst in terms of productivity. As we shall see, the start of the productivity decline in mining also tended to predate that of other industries, beginning in the middle 1960's instead of post-1973.

The possible causes of productivity decline have been extensively analysed but by and large remain a mystery. In the most comprehensive study to date, for the U.S. case, Edward Denison examined seventeen alternative hypotheses and concluded that all seventeen could explain no more than a fraction of the slowdown.

As the most dramatic economic event of the 1970's, the sharp increases in energy prices in 1974 and 1979 have been widely blamed. Apart from the timing of productivity decline in mining, this hypothesis should be an especially strong one in this industry, since it is relatively energy as well as capital-intensive. Even in this industry the energy share does not exceed 13 percent of costs, however, and thus it would be difficult to attribute the entire decline in productivity to this factor.[1] In testing the hypothesis for the different Canadian mining industries in this study, we in fact find that no more than a small percentage of the decline can be attributed directly to energy price increases.

A useful explanation for productivity change in a highly capital-intensive industry such as mining is the effect of under-utilized capacity in the short term or economies of scale over longer periods. We shall see that much of the productivity growth in mining in the late 1950's and early 1960's was associated with rapid increases in output, and that the decline in productivity in this industry can be strongly linked with the contracting or more slowly growing markets in more recent years.

The other obvious explanation for productivity decline in depletable resource industries is decline in the quality or grade of the natural resource itself. The average mineral yield in Canadian copper-zinc mines, for example, fell from 1.8% in 1956 to .535% in 1979. In the past, the effects of falling grades have been fully offset in most industries by regular if discontinuous cost-reducing technical change. We shall review some of the major innovations that have taken place in Canadian mining in the 1950's and 1960's. However, there seems now to be evidence of a lack in the 1970's of the type of major innovation that had occurred regularly since the beginning of the 1950's.

Finally, it is likely that environmental and other regulatory changes occurring in the early 1970's contributed to the decline in measured productivity by diverting capital from generating output toward the abatement of pollutants. Unfortunately, little direct evidence is available on this point; we shall be able in this study to obtain only indirect evidence by treating such factors as a residual.

The approach used in the study is to calculate trends in labour and total factor productivity and then attempt to explain these trends by means of the hypotheses just discussed. This is accomplished by estimating a general cost (production) function for the Canadian mining industry as a whole and for several sub-industries. The degree of significance of the measures for the effects of factor price, grade and scale changes on unit costs reflect the importance of these explanations in the various industries. Finally, a residual productivity measure is calculated for each industry that nets out the effects of these factors and reflects the effects of technical change and other excluded factors.

The contents of the report follow this general plan. The first chapter outlines the general model of mineral extraction used throughout the study. The definitions and measures of productivity used are then summarized in the next chapter. Chapter 3 provides a brief summary of the main technical changes that have occurred in the Canadian mining industry as well as computing and discussing the trends in factor shares and single factor productivities during our 1957-1979 sample period. Chapter 4 employs an econometric model to calculate measures of total factor productivity in the different mining industries and to assess the contributions of factor price changes, grade mining industry as well as computing and discussing the trends in factor shares and single factor productivities during our 1957-1979 sample period. Chapter 4 employs an econometric model to calculate measures of total factor productivity in the different mining industries and to assess the contributions of factor price changes, grade decline, and scale economies to productivity change. Chapter 5 then deals with the general policy implications of these results. Two appendices respectively list the detailed estimation results of our model and outline the methods of data collection.

Footnotes

1. In a study based on the 1973-76 period, George Perry [1978] concluded that energy could not have been the major cause of the aggregate productivity decline in the U.S., because its cost is too small a fraction of GNP, and because not enough energy had been saved since the energy crisis to justify the sacrifice of output implied by the fall in productivity.

1. A Model of Mineral Extraction

Unlike most other industries, that at least in the long run can be represented in a static framework employing replaceable inputs, the mining industry is inherently dynamic. The most well-known model of mineral extraction derives from Hotelling [1931], who represented a mineral firm as depleting a fixed stock of homogenous ore that was exhausted in finite time. Recently, it has been realized that mineral ore is almost never physically exhausted, but simply declines in quality or grade, and the Hotelling model has been appropriately modified by Puu [1977], Stollery [1979] and others.

The Hotelling model and its variants typically predicts a slowly declining output rate and concomitant increase in the price of the mineral as the stock of ore is exhausted. This derives from an assumption of increasing marginal extraction cost and an increasing or U-shaped average cost curve. It is now also being realized, however, that the traditional Hotelling model seems to describe poorly the behaviour of many mineral industries in which the planned output rate remains constant for most of the lifetime of the mineral deposit.[1] This constant extraction rate can be predicted by employing a model of constrained capacity.

For a given mine, the constraint on capacity is typically given by the large initial capital investment required for development of the deposit. In fact, this point was originally raised by Hotelling [1931] who noted that the "capital investment in developing the mine... is a source of a need for steady production." A model assuming capital-caused capacity-constrained extraction with no depreciation and irreversible investment was derived by Campbell [1980] and employed by Olewiler [1980] and Stollery [1981]. This model predicts extraction remaining constant until a certain date sometime before exhaustion, then declining gradually to zero. Over the long term and an entire industry, however, the capital stock will not remain constant, as new mines are developed and exhausted mines close. The model with output constrained by a fixed stock of capital (and a fixed capital/output ratio) is therefore inappropriate in modelling productivity change over the fairly long term.

To retain the appealing features of capacity-constrained extraction and yet allow the capital stock to vary, we employ the fact that the extraction rate will be ultimately limited, especially in underground mines, by the physical characteristics of the orebody itself, instead of by the extent of the capital infrastructure built to accommodate the deposit. An underground mine typically consists of one or more vertical shafts from which run horizontal passageways or drifts. These tunnels extend a very long way, and through them must travel all the extracted ore as well as ventilation, heating and water pipes, and all the workers and equipment back and forth to the actual digging face. Of course the decision to drill a second shaft or extend the horizontal drifts is an investment decision, but this investment is itself constrained by the fact that the veins of ore may be close together or far apart, may run for miles, or be concentrated in a single area.

If we accept a limitation on the extraction rate created by the physical characteristics of the mineral orebody, a commonplace observation among mining engineers,[2] then over the entire industry the output will be constrained at each moment of time, but can change through time as new mines are brought into production and old mines close. While the process of exploration and development is *ex ante* itself an investment process, there is so much randomness associated with mineral discoveries as to allow us to treat them as exogenous. An exogenous constraint on capacity is also consistent with the possibility of long run economies of scale in mining coexisting with a Canadian industry facing exogenous international mineral prices.

To outline the features of this model, let us denote the exogenous maximum extraction rate as $\bar{Q}$, and the actual mineral output (which may be equal or less than $\bar{Q}$) as $Q(t)$. Q is assumed to vary with the employed services of labour, capital and energy through a traditional neoclassical production function, as well as being affected by technological change and the quality or grade of the mineral.[3] Mathematically, this production function is

$$(1) \qquad Q = F(L,K,E,A,g) \qquad F_i' \geq 0 \qquad F_i'' < 0$$

where A is an index of the state of technology and g is the grade of the ore currently hoisted. As we shall discuss in more detail in the following chapter, for every well-behaved production function there is an associated dual cost function, which we will denote

$$(2) \qquad C = G(Q,P_L,P_K,P_e,A,g) \qquad G_i' \geq 0$$

Here P_L, P_K and P_e denote the input prices of labour, capital and energy, respectively. In what follows it will be convenient to work with the cost function.

The mineral grade plays a crucial role in the model, as it, rather than physical exhaustion of the ore, is assumed to provide the limit to extraction in a given mine. It does this through a functional relationship with cumulative extraction, X $(g(X), g'(X) < 0)$ provided by a grade distribution within the mineral ore and the fact that the best ore will be used first.[4] As extraction proceeds, $X(t)$ rises, grade (g) falls, and this increases total extraction cost, since $\partial C/\partial g > 0$.

To derive the predictions of the model the firm is assumed to maximize the present value of cash flow over the lifetime of the deposit, or

$$(3) \qquad J = \int_0^T e^{-rt} [PQ - C(Q,g(x))] \, dt$$

subject to the constraints

$$(4) \qquad \partial X/\partial t = Q(t) \quad 0 \le X(T) \le \bar{S} \quad \text{and} \quad 0 \le Q \le \bar{Q}$$

where $\bar{S}$ represents the initial stock of mineral (within the ore) before extraction begins.[5] As the input prices (P_L, P_K etc.) are assumed constant for the present, for notational simplicity they are suppressed in the cost function. Equations (3) and (4) form a standard problem in optimal control theory. The present value Hamiltonian for the problem is

$$(5) \qquad H(Q,X,\lambda,\mu) = e^{-rt} \{ [PQ - C(Q,g(x))] - \lambda Q - \mu(Q - \bar{Q}) \}.$$

where λ and μ represent shadow prices associated with the constraints (4). Necessary conditions for the maximum of (5) are given by

(6a) $$P = MC(Q,g) + \lambda(t) + \mu(t)$$

(6b) $$\partial\lambda/\partial t = r\lambda(t) - \overset{(-)}{\frac{\partial C}{\partial g}} \; \overset{(-)}{\frac{\partial g}{\partial X}}$$

(6c) $$\mu \geq 0 \qquad \mu(\bar{Q} - Q) = 0 \qquad \mu(t^*) = 0$$

(6d) $$t^*\bar{Q} + \int_{t^*}^{T} Q(S,T)ds \leq \bar{S}$$

(6e) $$H_T = [\ PQ(T) - C(Q(T),\ g(T)) - (\lambda(T) + \mu(T)\ Q(T)\ ]\ e^{-rT} = 0$$

(6a) shows that the profit-maximizing condition for an extractive firm is different from the usual static condition of price equal to direct marginal cost. "Full marginal cost" is here equal to direct marginal cost plus the shadow value of the mineral, $\lambda(t)$, and the shadow price of capacity, $\mu(t)$. From (6c) this shadow price of capacity is positive when the capacity constraint is binding, and zero otherwise. When the capacity constraint does bind depends on the time path of the mineral shadow price. For illustrative purposes let us first assume that either $\partial C/\partial g$ or $\partial g/\partial X = 0$, so cumulative extraction does not affect costs. In this case, the shadow price simply rises at the rate of discount, r, and, without scale economies, if the capacity constraint is to bind at all, it will do so at the beginning of the extraction sequence. Since during this period $MC(\bar{Q})$ is constant, then $\lambda + \mu$ must be constant and μ must decline at the rate r until some time, t^*, after which $\mu = 0$ and $Q < \bar{Q}$. During this second extraction phase, if $\partial MC/\partial Q > 0$ and there are no long run scale economies, then $Q(t)$ will decline as $\lambda(t)$ rises until (6d) is satisfied with equality and the mineral is exhausted at $t = T$.[6] This two-sequenced time path for $Q(t)$ is illustrated in figure 1.

The effect of significant scale economies on the extraction sequence is to extend the period of capacity-constrained extraction over the entire extraction sequence. This can be shown by contradiction. If $Q = \bar{Q}$ up to t^*, as before, then after t^* $\dot{Q} = -\dot{\lambda}/MC'_Q > 0$ (if $MC'_Q < 0$), which is impossible because $Q \leq \bar{Q}$. On the other hand, if $Q < \bar{Q}$ first, so $\mu = 0$ and Q is rising up to $\bar{Q}$, μ must be declining over this interval, which is also impossible, if $\mu = 0$ and $\mu \geq 0$. The conclusion is that $Q = \bar{Q}$ for the entire extraction period, so $t^* = T$.

When the decline in grade is allowed to affect costs the situation becomes more complex, and we must make an assumption about the effect of grade decline on costs in order to obtain the standard results. The problem is that we can no longer determine from (6b) the exact time path of $\lambda(t)$. However, if cost is to provide the extraction limit instead of physical exhaustion ((6d) is an inequality) then at the moment when extraction ceases $\lambda(T) = \mu(T) = 0$ and price must equal average cost (and marginal cost) at the minimum or cutoff grade from (6e).

$\lambda(t)$ must therefore decline at the latter stage of extraction.

Let us again assume away scale economies by constraining $MC'_Q > 0$. Suppose that $Q < \bar{Q}$ and the capacity constraint is not binding, so $\mu = 0$. In that case, differentiating (6a) with respect to time and substituting (6b) determines the change in extraction as

$$(7) \qquad \dot{Q} = \frac{1}{MC'_Q} \left[-r(p-MC) + \overset{(-)}{Qg'(x)} \left(\overset{(-)}{\frac{\partial AC}{\partial g}} - \overset{(-)}{\frac{\partial MC}{\partial g}} \right) \right]$$

where the dot denotes a time derivative. This is unambiguously negative if $MC'_Q > 0$ and if the decline in grade affects marginal more than average cost. Q will therefore remain less than $\bar{Q}$ until extraction ceases. If instead $Q = \bar{Q}$ and $\mu > 0$, then the same technique determines the change in μ as

$$(8) \qquad \dot{\mu} = \overset{(-)}{Qg'(X)} \overset{(+)}{\left(\frac{\partial AC}{\partial g} - \frac{\partial MC}{\partial g} \right)} - r\lambda < 0$$

and μ will decline to zero, after which time $Q < \bar{Q}$. The extraction sequence thus follows the path of figure 1.

As long as the above assumption holds, the effect of scale economies will also be the same as formerly, in extending the period of capacity constraint. Of course in general there may be several intervals of constrained extraction. In the following chapters we shall assume that output is at its constrained level, and can thus be treated as exogenous. The assumptions justifying this, that grade decline affects marginal more than average costs, and the existence of scale economies in mining, are econometrically testable and will be tested in the subsequent chapters.

The final prediction of the model concerns the relationship of price with marginal cost during periods of capacity constraint. From (6a), it follows that if grade is constant and $Q = \bar{Q}$, then the sum of the resource and capacity shadow prices are also constant, and price is in effect a fixed markup over marginal cost.

If only grade declines while product and factor prices are constant and technology is static, then $\emptyset(t) = \lambda + \mu$ will decline slowly with the falling grade. In general, the markup over marginal cost will remain constant if mineral price increases and cost-reducing productivity change keeps up with increasing factor prices and the decline in grade.

Footnotes

1. A sample of mines studied by Bucovetsky [1971] indicated that for mines that milled their own ore the planned extraction rate was constant over the anticipated mine lifetime.
2. For a very readable description of mining practices and technology, see Northern Miner, Mining Explained (Northern Miner press, Ottawa, 1968).
3. Q is defined as the output of concentrate rather than tons of raw ore hoisted.

4. The grade distribution within a mineral orebody was first discussed by Lasky [1952]. A description of the mathematical relationships is given by Musgrove [1976]. For an empirical application of the effect of declining grades on cost, see Stollery, "Mineral Depletion with Cost as the Extraction Limit", J. Env. Econ. Manag. 10, No. 2 (June, 1983).

5. It might have been more realistic to assume a constraint on ore hoisted from the mine rather than concentrate produced, which is hoisted ore times grade. It can be shown that the two approaches are equivalent with an appropriate normalization of the cost function, however, and for notational simplicity the present formulation was retained.

6. Whether, in fact, $Q(T) = 0$ depends, through (6e), on the shape of the average cost curve. If $\partial AC/\partial Q > 0$ for all Q and $MC > AC$, then (6e) can only be satisfied if $Q(T) = 0$. For the traditional u-shaped cost curve $Q(T) > 0$ and is determined at the point of minimum average cost, where $MC(Q(T)) = AC(Q(T))$.

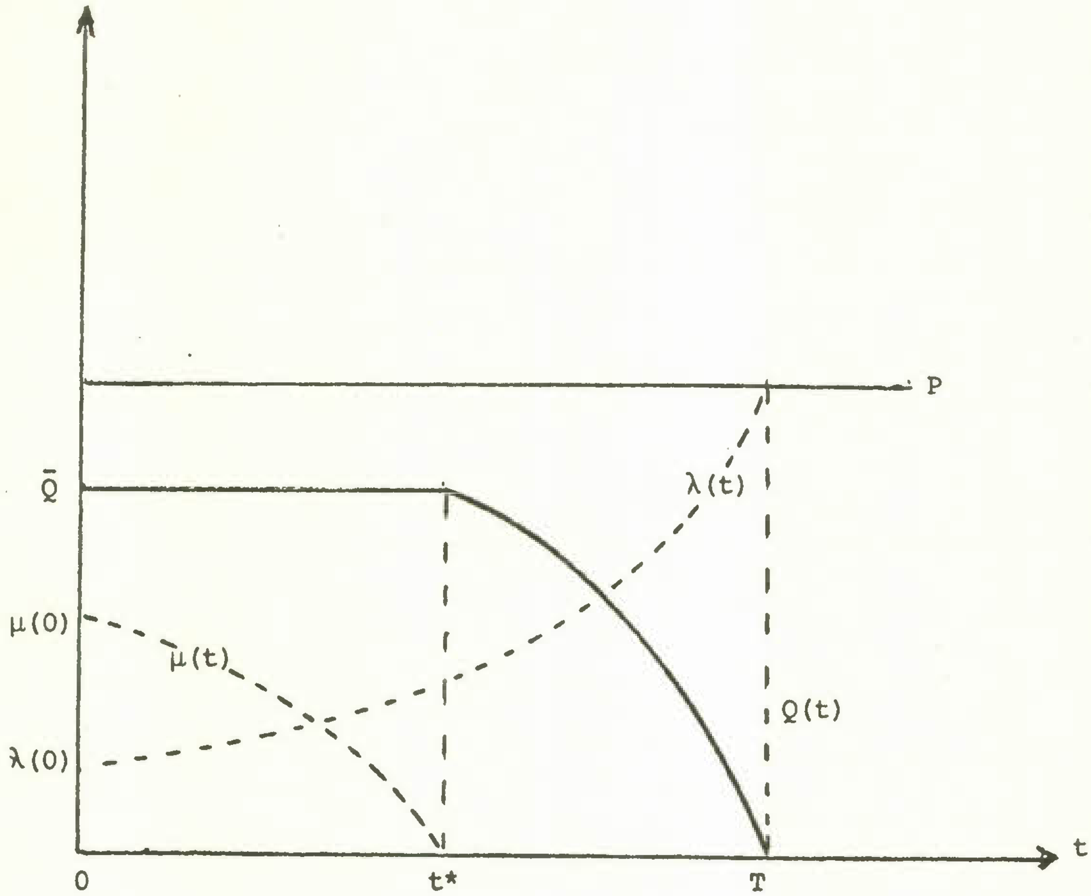

FIGURE 1: Extraction Path and Shadow Prices in Capacity Constrained Model without Grade Decline

2. Productivity Measurement in Mineral Industries

There are two common definitions of productivity. The definition most often used in the popular press concerns labour productivity -- output per person or per labour/hour. It is important in that it forms the basis of changes in real wages. This is because in the absence of labour productivity gains, any increase in wages must inflate unit labour costs, and if prices are increased to preserve profit margins, the result will be no change in real compensation. Labour productivity is, however, a poor approximation to the productivity of all factor inputs.

The reason for this is that labour productivity often changes, not as the result of changes in the efficiency of labour or through technical improvement or regress, but simply through changes in the use of other inputs. In fact, improvement in labour productivity typically occurs through capital investment increasing the ratio of capital to labour, allowing each worker to utilize more or better quality machines. To ascertain if the investment is justified requires the concept of multifactor productivity, reflecting the efficiency of all inputs employed in production. With just labour (L) and capital (K) as inputs, then defining I(L,K) as an index of total factor use, overall factor productivity would be

(1) $$B(t) = Q(t)/I(L,K).$$

The problem with measuring the level of productivity is the determination of the factor weights in the index I. The studies of mining industry productivity by Dawson [1971], Anton [1973] and Pye [1981] employed as weights the input coefficients in the Canadian input-output matrix for 1961. This procedure is necessarily inaccurate because of course these coefficients change over time, and the input-output matrix is usually several years out of date. If we are willing to restrict technological change to be neutral with respect to the factor shares (Hick's neutral), then the production function $Q = F(L,K,A)$ can be written in the separable form

(2) $Q = A(t)F(L(t)K(t))$

with $A(t)$ the index of technology. In this case the proper candidate for the $I(L,K)$ index would be $Q/A = F^*(L,K)$ and $B(t) = A(t)$, with the productivity index exactly equal to the technology index.

In general, although the technology index is included separately in the production function, it is not separable from it, so productivity change may be non-neutral with respect to the usage of labour and capital. To allow for this possible factor bias the analysis in this study shall be restricted to the measurement of the change rather than the level of multifactor productivity.

The change in productivity is defined by taking the logarithmic time derivative of the production function in its general form. Since we no longer require the explicit technology index, rewrite the general production function as

(3) $Q(t) = F(L(t),K(t),t)$

with the $A(t)$ index subsumed within a general function (not the same as the original). Denoting percentage changes $(\frac{dX}{dT}\frac{1}{X})$ by a circumflex over the variable, multifactor productivity change is defined as

$$(4) \qquad \hat{B}(t) = \frac{\partial \ln F}{\partial t} = \hat{Q}(t) - \frac{\partial \ln F}{\partial \ln L}\hat{L} - \frac{\partial \ln F}{\partial \ln K}\hat{K}$$

or the growth in output less the weighted growth in the inputs.

The factor elasticities in (4) are related to the degree of scale economies embodied in the production function. Let the factor inputs be multiplied by a factor, α. The degree of returns to scale is then defined as

$$(5) \qquad RTS = \lim_{\alpha \to 1} \frac{\partial \ln F(\alpha L, \alpha K)}{\partial \ln \alpha} = \frac{\partial \ln F}{\partial \ln L} + \frac{\partial \ln F}{\partial \ln K}$$

Constant returns to scale implies the sum of factor elasticities or RTS equal to unity.

The analysis so far is derived solely from the technical relationship of output with inputs, and is independent of any institutional structure or assumption concerning profit maximization. The empirical measurement of productivity change through equation (4), however, depends on such assumptions. If $S_L = P_L L/PQ$ and $S_K = P_K K/PQ$ are the shares of labour and capital in the value of output, then short-run profit maximization with perfect competition and constant returns to scale predicts $S_L = \partial \ln F/\partial \ln L$, $S_K = \partial \ln F/\partial \ln K$ and $S_L + S_K = 1$. In this case, simply replacing the output elasticities with factor shares in (4) leads to the Divisia productivity index, which can be easily calculated.[1] However, any situation in which the assumption of short-run profit maximization in inappropriate invalidates this procedure.

This is the case with monopoly or with the dynamic extractive industry model of the previous chapter.[2] In general, $\partial \ln F/\partial \ln L = P\, S_L/MC(Q)$ etc.,

(6) $$S_L + S_K = RTS\ (MC(Q))/P\ ,$$

and

(7) $$\hat{B} = \hat{Q} - \frac{P}{MC}\,[S_L\,\hat{L} + S_K\,\hat{K}]\ ,$$

or, written in terms of returns to scale and the weighted changes in productivity of the particular factors,

(8) $$\hat{B} = (1 - RTS)\,\hat{Q} + \frac{P}{MC}\,[S_K\,(\hat{Q} - \hat{K}) + S_L\,(\hat{Q} - \hat{L})]\ .$$

In the case of constant returns to scale and short-run profit maximization, this reduces to simply the sum of the share-weighted average factor productivities. Labour productivity change in general is

(9) $$\hat{Q} - \hat{L} = \hat{B} + \frac{PS_K}{MC}\,(\hat{K} - \hat{L}) + (RTS - 1)\,\hat{L}$$

depending on total factor productivity change, changes in the capital/labour ratio, and, to the extent of scale economies, in the overall growth of the labour force.

Because the measures of productivity change in (7) to (9) contain P/MC , they cannot be properly calculated without an estimate of the relationship of price with marginal cost. Since in our extractive industry model $P/MC = 1 + P/\theta$, where $\theta = \lambda + \mu$, the sum of the shadow prices of the resource stock and extractive capacity, calculation of the simple Divisia index assuming $P = MC$ will provide an upper bound but will tend to overstate productivity gains.

The Dual Cost Function Approach

Because of the difficulties in productivity measurement with the standard Divisia index, we employ an alternative method, relying on the dual relationship of cost and production functions. It is easily shown that maximizing the value of output with fixed product and factor prices leads to the same factor use as choosing the input combination of factors that minimizes total production cost for a given level of output. The theory of duality then states that there corresponds to the primal production function a dual cost function relating minimum total cost of production, given factor use at optimal levels, with output, factor prices, and the state of technology. Generalizing the production function for concentrate to include energy and ore quality or grade, so $A = F(L,K,E,g,t)$, the dual cost function is [3]

$$(10) \qquad C = G(Q,P_L,P_K,P_e,g,t) .$$

Because output is fixed, the optimal factor proportions are those chosen along a given production isoquant and are unrelated to both the final output price and the overall resource shadow price. In fact, Shephard's Lemma states that the demand for any input X_i is simply the partial derivative of the cost function with respect to the factor price of that input; i.e.,

$$(11) \qquad X_i = \frac{\partial G(Q,P_L,P_K,P_e,g,t)}{\partial P_i} \qquad i = L,K,E$$

Using the dual cost function Ohta [1974] has shown that the primal degree of returns to scale (5) is equal to the dual rate,

(12) $$RTS = 1/(\partial lnC/\partial lnQ) = AC/MC ,$$

the reciprocal of the elasticity of cost with respect to output or the ratio of average to marginal cost.

When mineral grade and energy are included explicitly in the production function, the primal rate of multifactor productivity change gross of grade decline is

(13) $$\varepsilon ft = \frac{\partial lnF}{\partial t} = \hat{Q} - \frac{P}{MC} [S_L\hat{L} + S_K\hat{K} + S_E\hat{E}] - \frac{\partial lnF}{\partial lng} \overset{(-)}{\hat{g}}$$

meaning the growth in productivity must be larger to counteract the effects of the decline in grade. The dual measure of multifactor productivity is the effect of technological change (proxied by time) in reducing total costs, or

(14) $$\varepsilon_{Ct} = - \frac{\partial lnG\ (Q,P_L,P_K,P_e,g,t)}{\partial t}$$

holding input prices and output quantity constant. The relationship in general between primal and dual multifactor productivity is

(15) $$\varepsilon_{ft} = RTS\ \varepsilon_{Ct}$$

i.e., they are related by the degree of returns to scale. Since this holds independently of the assumption of short-run profit maximization, it follows that an alternative to the calculation of multifactor productivity change through the Divisia index approach is to calculate it parametrically by estimating a general cost function including a time trend as an argument. This is the approach taken in the present study.

The change in the productivity of a specific factor can also be defined in terms of either the production or the cost function. Since the average productivity of any factor is simply output divided by the factor demand (Q/X_i), labour productivity change from (11) is simply $\hat{B}_L = \varepsilon_{ft} - \hat{L}$, for example. From (11) the factor productivity changes can also be calculated from the cost function as

$$(16) \qquad \hat{B}_L = \hat{Q} - \frac{\partial \ln}{\partial t} (\partial C/\partial P_L)$$

$$\hat{B}_K = \hat{Q} - \frac{\partial \ln}{\partial t} (\partial C/\partial P_K)$$

$$\hat{B}_E = \hat{Q} - \frac{\partial \ln}{\partial t} (\partial C/\partial P_E)$$

Both single and multifactor productivity will in general be affected by changes in input prices, because of the resulting changes in factor proportions. Define the factor demand elasticities

$$(17) \qquad \varepsilon_{ij} = \frac{\partial \ln X_i}{\partial \ln P_j} \qquad i,j = L,K,E \quad .$$

Since output is exogenous in the cost function, it follows that the effect on a given factor's average productivity level (Q/X_i) of a change in any factor price is simply the negative of the demand elasticity. For example, since the own price elasticity (ε_{ii}) is constrained to be negative, an increase in the price of any factor will raise that factor's productivity. Similarly, the effect of energy price changes on labour productivity, for example, depends on the cross price elasticity between energy and labour demand. This cross price elasticity will be positive if energy and labour are substitutes, in which case energy price increases, ceteris paribus, will reduce the productivity of labour and increase energy productivity by encouraging conservation.

The effect of input price changes on the level of multifactor productivity is a generalization of their effect on the average productivity levels of individual factors. Differentiate totally the production function $Q = F(X_1, X_2 \ldots B)$ with respect to the input prices and constrain $dQ = 0$. The effect of an increase in input price P_i on total factor productivity is then

$$(18) \qquad \frac{d\ln B}{d\ln P_j} = - \sum_i \frac{\partial \ln F}{\partial \ln X_i} \frac{\partial \ln X_i}{\partial \ln P_j}$$

because factor demands depend only on prices and total output, from (11). Substituting factor output elasticities from (6) and writing $\partial \ln X_i / \partial \ln P_j$ as the demand elasticity, this can be rewritten as

$$(19) \qquad \frac{\partial \ln B}{\partial \ln P_j} = - \frac{P}{MC} \sum_i S_i \, \varepsilon_{ij}$$

with S_i the factor share of the ith input. However, an even simpler interpretation is possible. From the cost side, a natural measure of total factor productivity is the level of average costs. With this interpretation, by Shephard's Lemma the effect of an input price change on productivity is simply the cost share of the relevant factor, i.e.,

$$(20) \qquad \frac{\partial \ln AC}{\partial \ln P_j} = S_j^* = \frac{P}{AC} S_j$$

where S_j is the jth factor's share in the total value of output.

Finally, as long as there are non-constant returns to scale, both individual factor and total factor productivity depends on the output level, Q. Returns to scale (from (12)) determines if the average cost curve is horizontal, upward or downward sloping. Suppose there are increasing returns, and the production function is homothetic, so the rays on an isoquant diagram joining the tangencies of isocost and isoquant curves are straight lines. In this case, increased output would increase the use of all factors in proportion, but proportionally less than the output increase. Since total costs are homogeneous of degree one in factor use, average cost would fall as the result of a proportional increase in each factor's (and total factor) productivity. A nonhomothetic production function changes this result by biasing factor demand in one direction or another as output increases.

The above refers to the effect of factor price changes on the levels of productivity holding technology constant. In essence, productivity changed because factor price changes induced shifts along and between isoquants. However, the level of relative factor prices may affect the implementation of new techniques as well, and thus productivity changes may also depend on factor prices through (14). The contention of Baily [1981] and others that the technology presently employed may be inefficient because it was designed for use with much lower energy prices, essentially employs the static concept and implicity assumes a low elasticity of substitution between factors. A dynamic version of this might be that the range of employable new techniques is still limited to relatively energy-intensive ones, and therefore the pace of their implementation has been slowed by higher energy prices. We could estimate this effect by the derivative of (14) with respect to the price of energy.

Empirical Implementation using the Translog Cost Function

We have been specifying productivity change so far in terms of general production and cost functions. The empirical calculation of productivity change allowing for nonconstant returns to scale and price not equal to short-run marginal cost, however, requires estimation of the parameters of a specific cost function. Clearly, the specified function must be sufficiently general to allow for productivity growth that varies through time, for example, and yet require as few estimated parameters as possible. While there are more general functional forms such as the generalized Box-Cox function that require more estimated parameters (see Berndt and Khaled [1979]), we shall employ the popular translog form (Christensen, Jorgenson and Lau [1973]) as being sufficiently general, easily estimated, and relatively parsimonious in parameters.[4] The translog places no a priori restrictions on the substitution possibilities among factors of production, and it allows scale economies to vary with the level of output. It can also be made to allow the calculated rate of cost diminution, ε_{Ct}, to vary over time and with the prices of the factors.

The translog cost function for (10) can be written

(21) $$\begin{aligned} \ln C = {} & \theta_0 + \theta_Q \ln Q + \frac{1}{2}\theta_{QQ}(\ln Q)^2 + \theta_g \ln g + \frac{1}{2}\theta_{gg}(\ln g)^2 \\ & + \theta_{Qg}\ln Q \ln g + \theta_t t + \theta_{tQ} t \ln Q + \sum_i \delta_i \ln P_i \\ & + \frac{1}{2}\sum_{ij}\sum \gamma_{ij}\ln P_i \ln P_j + \sum_i \gamma_{ti} t \ln P_i + \sum_i \gamma_{Qi}\ln Q \ln P_i \\ & + \sum_i \gamma_{gi}\ln g \ln P_i \end{aligned}$$

where $\gamma_{ij} = \gamma_{ji}$. The restriction that cost must be homogeneous of degree one in prices (to equal $C = P_L L + P_K K + P_e E$) implies the following parameter restrictions:

$$\sum_i \delta_i = 1 \qquad\qquad i,j, = L,K,E \tag{22}$$

$$\sum_i \gamma_{Qi} = \sum_i \gamma_{ti} = \sum_i \gamma_{gi} = 0$$

$$\sum_i \gamma_{ij} = \sum_i \gamma_{ji} = \sum_i \sum_i \gamma_{ij} = 0 \, .$$

The shares of the factors in total cost (not value of output) are defined (from (11)) as

$$S_i{}^* = \delta_i + \gamma_{ti} t + \gamma_{Qi} \ln Q + \gamma_{gi} \ln g + \sum_j \gamma_{ij} \ln P_j \tag{23}$$

Unlike output shares, the cost shares can be determined independently of assumptions about profit maximization, because they are determined only by cost minimization for a given output level. The elasticities of demand for factor i is determined from the translog cost function as

$$\varepsilon_{ij} = (\gamma_{ij} + S_i{}^* S_j{}^*)/S_j{}^* \qquad\qquad \text{(cross-price)} \tag{24}$$

$$\varepsilon_{ii} = [\gamma_{ii} + S_i{}^* (S_i{}^* - 1)]/S_i{}^* \quad i \neq j \quad \text{(own-price)}$$

These are not constrained to be constant but vary over time. The elasticities of cost with respect to output (Q) and mineral grade (g) are respectively

$$\partial \ln C/\partial \ln Q = \theta_Q + \theta_{QQ} \ln Q + \theta_{Qg} \ln g + \theta_{tQ} \ln t + \sum_i \gamma_{Qi} \ln P_i \tag{25}$$

$$\partial \ln C/\partial \ln g = \theta_g + \theta_{gg} \ln g + \theta_{Qg} \ln Q + \sum_i \gamma_{gi} \ln P_i \tag{26}$$

and productivity change net of grade decline is

$$(27) \qquad \partial \ln C/\partial t = \theta_t + \theta_{tQ} \ln Q + \Sigma\, \gamma_{ti} \ln P_i \;.$$

The production structure is homothetic if $\gamma_{Qi} = 0$ and the degree of scale economies is independent of factor prices (as well as the factor shares being independent of output). Net productivity change from the cost side (27) has been allowed to depend on the levels of output and prices, so determining whether energy price changes have affected productivity growth, for example, amounts to a statistical test of the coefficient γ_{tE} associated with the log of the energy price. The effect of changes in mineral grade on cost-side productivity is defined by (26). Finally, overall productivity change from the production function can be separated (by (13) and (15)) into that due to technological change, economies of scale, or the decline in grade. After a review of mineral industry trends in the following chapter, this model will be applied in chapter 4 of the study.

Footnotes

1. For a comparison of the Divisia approach and the parametric cost function approach to productivity measurement that will be used in this study see Diewert [1979]. The Divisia approach has been used exclusively by Kendrick [1973], Denison [1979] and in most other productivity studies.
2. For a derivation and application to the monopoly case see Nadiri and Schankerman [1981].

3. Two important omissions from the cost function are the price of material inputs and an index of capacity utilization. While unfortunately the omission of materials can significantly affect the results, it was impossible to obtain a meaningful price for inclusion in the function. The reason for this lies in the mineral industry model of Chapter 1. The bulk of the material used in mining is of course ore. However, mining companies do not pay a rental price for the ore to the Crown. The lease rights for mineral land in Canada are traditionally nominal, the collection of mineral rents by governments taking the form of royalties and other taxes. The price of ore to a company is therefore an internal transfer price, and does not appear in mineral industry statistics, the cost of materials inputed by Statistics Canada including only the cost of other purchased inputs. An appropriate proxy for the price of ore might be the provincial mineral royalty rate, and this, in fact, was the approach taken by Smithson et.al. [1977] in a recent study. This approach has not been followed here because royalty rates are available only for Ontario and not for the whole of Canada, and only there for a highly aggregated industry level. A similar problem occurred with the lack of an index of capacity utilization.

4. Other recent productivity studies employing the translog cost function are Berndt and Watkins [1981], Caves, Christensen and Swanson [1981], and Chirstensen and Greene [1976], among others.

3. Historical Trends in Canadian Mineral Industries

In the preceding chapter, equations were developed to describe changes in labour and multifactor productivity in the mineral industry allowing for economies of scale, long-run instead of short-run profit maximization, and changes in ore quality or grade. The translog cost function outlined in the final section of that chapter will be applied in Chapter 4 to the various Canadian mineral industry sectors in an effort to calculate and explain productivity trends. In this chapter we will outline the major changes in output, factor shares, mineral grade, and technology that have occurred in Canadian mining since 1957, the start of our sample period.

The mineral industries employed in this study were selected on the basis of data availability and the nature of the production process. Of the industry series available in our major data source, (the General Review of the Mineral Industries, from Statistics Canada) we have selected nine: asbestos and total nonmetals, copper-gold-silver, gold, silver-lead-zinc, nickel-copper, iron, total metal mines, and total mines, quarries and oil wells.[1] As we shall see, data on a disaggregated industry basis is necessary in mining because there have been significant differences in factor use and productivity between industries. However, several individual industries were omitted from the study and only included in the general category. Quarries were omitted because of their unimportance in total mining. Petroleum, coal and total fuels were also not included as separate industries because, unlike mines, there is no concept of "ore grade", and the production process is, by nature, different. In any case, investment data were not available for coal or total fuels. Finally, although smelting and refining is closely related to mining, being simply the next stage in the production process, investment data were unavailable here also, and this industry was omitted.

Technical Change and Productivity

The major activities of the mining industry could typically be described as the hoisting of metal or non-metal ores, either from open-pit or underground mines, and the subsequent milling of these ores to form a powder of higher metal content called a concentrate, which then goes to the smelting and refining stage (although quite a lot of concentrate is exported directly from Canada, mostly to Japan). The nature of the mining process depends on the characteristics of the mineral deposit. Open-pit mining is the cheapest method, because once the ore is exposed by stripping off overburdon it can be simply dug out with power shovels and trucks. There are significant economies of scale in this method, and, as outlined later in the chapter, increasing equipment size appears to be important in keeping down costs. Underground mining is more complex, involving the sinking of shafts to reach an orebody, the provision for drainage and ventilation, the preparation of the ore face for drilling and blasting, and the hauling of the ore to the surface. As described in Chapter 1, this method implies many more constraints on the rate of production, and probably more limited scale economies. Both methods are employed in Canada even within the same industry.

Primary mineral processing usually occurs at the mine site, because of the significant costs of ore transport. The crushing and grinding of the original ore may be done in several stages, with the final grinding done in rod and ball mills, where ore the size of gravel is rotated with steel rods or balls in large cylinders. Finally, a variety of methods may be used to separate the concentrate from the waste product including flotation, magnetic, or gravity separation. In flotation, for example, air bubbles are blown through the slurry and the mineral adheres to them. The different minerals often found together can be separated this way as well. The concentrate generally comprises 25 to 75 per cent mineral.

Although there has been no major change in the basic technology of mining in the post-war period, there has been persistent, if discontinuous technical advance. Most industry analysts agree that the most important technical changes occurred from the 1950's up to the late 1960's, and no major breakthroughs have taken place in recent years.

In blasting, one of the most important innovations was the introduction of ANFO explosives (ammonium nitrate and fuel oil) which were introduced about 1960 and rapidly adopted.

Drilling technology changed in underground mining through the introduction of first the jackleg drilling machine and then the self-propelled drill jumbo in the 1960's. The drill jumbo is suitable only in large mines, however, and illustrates the important relationship between technological change and scale economies. Increases in the size of drillholes, both in open pit and underground mines, represent the most recent trend. A new hydraulic drill, just patented by CIL, will drill 7-8" blast holes.[2]

The most important innovations, however, have occurred in ore handling. According to the literature, the major innovation in underground mining was the introduction in the 1960's of the load-haul-dump (LHD) machine to replace underground railways. In the early 1960's about fifteen mines in the world were using LHD's; by 1972 they were in use in 120 mines. In Inco's Sudbury mines alone 171 units were introduced from 1966 to 1972.[3] Since that time, the bucket size of the LHD has steadily increased from 2-4 yard capacity when first introduced to 11 yards by the middle 1970's.

In open pit mining, technical change in the 1960's went hand in hand with the opening of larger mines. The major advance here was the increase in size of dump trucks, from 22 to 34 tons in the 1950's, to 45 to 100 tons by 1974. At present, some trucks of 200 tons are in use, although according to an industry analyst, 170 tons is the norm, the scale economies having been exhausted. Bulldozer and dragline sizes have also been rising, with D10's and 120 cu. yard draglines the current state of the art. There is also current interest in overland conveyers for ore as a substitute for trucks. These large conveyers, of up to 20km in length, compensate for high initial capital costs by considerably lower operating costs relative to truck transport.

In milling technology, the size of flotation cells has increased steadily until now 34 feet diameter is common, and mill throughput is generally 20,000 to 30,000 tons. A very recent innovation in milling is the LAROX system of pressure filtration to telescope filtering and drying of the ore into a single operation. It is interesting to note that while many of the early innovations were motivated by efforts to reap economies of scale, recent technical change is equally motivated by the need to cut energy costs. The major advantage of the LAROX system, according to a recent proponent, is a "78% average reduction in energy demand for dewatering and a 10.3% reduction of energy use per tone of ore", compared with current vacuum filtering.[4] Inco is currently experimenting with many energy-saving techniques, including employing the waste heat from compressors to run central heating plants.

Some techniques that originally looked promising have proved less so with experience. The idea of using hydraulic jets for breaking rock has not taken hold in the industry, nor has the innovation of autogenous grinding, in which the ore itself is used for crushing. The efficiency of autogenous grinding was found to be very sensitive to the types of ore, and most operations have had to reintroduce steel balls.[5] Even the LHD machines, the great breakthrough of the 1960's, have recently begun to inspire second thoughts because of the increased pollution and reduced mine safety associated with diesel-power non-tracked vehicles underground. Some industry analysts are contemplating a return to electric underground trams.[6]

Finally, a good deal of recent technical change seems aimed at ameliorating working conditions in the industry and abating environmental damage in conformity with noise, vibration, air, and water pollution standards. There have been dramatic improvements in the measurement of noise in mining over the last ten years, for example. With a new generation of noise dosimeters it is now possible to measure noise pollution with a high degree of accuracy, and many techniques have been instituted for its abatement.[7] While such efforts are not directly measured in productivity terms, they represent an investment in improved worker health and job satisfaction, and long-term productivity improvement in terms of reduced accidents and better employee relations.

Trends in Factor Use and Substitution

The major trends in factor use and factor shares for the selected industries are shown in Table 3.1. The most dramatic of these trends has been the marked decline in the share of labour in total costs across all mining industries, including gold which was experiencing disinvestments and a decline in output over much of the sample period. The fall in labour share resulted partially from a decline in labour costs relative to the calculated rental rate on capital (P_L/P_K) but primarily from the increase in capital per employed worker (K/L) in these industries, an increase that apparently did not occur in response to changes in factor prices. This process of substitution began even before 1957, Dawson [1971] documenting it from 1947. Indeed, while showing some modest growth in the 1966-1970 period, the table shows that since 1971 average yearly growth in employment has been virtually nil in all industries.[8]

If it did not occur as the result of increases in relative labour costs, the increase in capital-labour ratios must have occurred through complementarity with another factor of production (energy), through factor biased technical change, or through a decline in mineral grade requiring

TABLE 3.1 : Factor Use and Substitution in Canadian Minerals: 1957-1979

Industry

		Asbestos	Nonmetals	Copper	Gold	Silver	Nickel	Iron	Metals	Mining
S_K^*	1957-65	.211	.406	.306	.112	.344	.241	.461	.327	.458
	66-70	.356	.603	.441	.222	.483	.299	.559	.439	.651
	71-79	.450	.670	.588	.337	.502	.360	.556	.518	.726
S_L^*	57-65	.659	.488	.616	.802	.580	.713	.456	.601	.467
	66-70	.536	.322	.491	.701	.444	.666	.333	.488	.284
	71-79	.447	.260	.347	.593	.417	.591	.324	.403	.207
S_E^*	57-65	.129	.105	.078	.086	.076	.045	.083	.072	.074
	66-70	.108	.075	.068	.077	.073	.035	.108	.073	.064
	71-79	.103	.070	.065	.070	.081	.048	.120	.078	.066
K/L	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	1.631	1.564	1.482	1.534	1.473	1.074	1.322	1.322	1.773
	71-79	2.622	2.631	2.944	3.172	1.751	1.442	1.401	2.034	2.212
$\hat{K}$	57-65	.079	.079	.023	-.016	.037	.039	.140	.052	.082
	66-70	.081	.154	.171	.023	.107	.121	.047	.057	.065
	71-79	.061	.046	.088	.031	.017	-.002	-.001	.063	.063
$\hat{L}$	57-65	-.013	.006	.018	-.015	-.007	.026	.045	-.003	.007
	66-70	.032	.045	.044	-.119	.038	.067	-.007	.018	.006
	71-79	.006	.013	.019	-.047	.007	-.079	.040	.001	.005
$\hat{E}$	57-65	.058	.066	.067	-.002	.109	.020	.202	.081	.063
	66-70	.063	.071	.137	-.078	.100	.161	.070	.082	.081
	71-79	.023	.034	.042	-.028	.016	.023	.045	.039	.056
P_L/P_K	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	.794	.814	.818	.675	.801	.606	.830	.807	1.766
	71-79	.834	.846	.858	.759	.865	.790	.849	.850	.604
P_L/P_E	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	1.397	1.289	1.415	1.723	1.713	1.386	1.507	1.515	.295
	71-79	1.378	1.311	1.847	1.308	1.762	1.469	1.501	1.655	1.304
P_E/P_K	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	.570	.630	.587	.604	.438	.584	.557	.538	.511
	71-79	.600	.657	.465	.591	.482	.558	.527	.517	.464

Note: Copper refers to Copper-Gold-Silver, Silver to Silver-Lead-Zinc and Nickel to Nickel-Copper mines. Data sources are described in Appendix B.

more capital-intensive benificiation of the ore. While the relative importance of these factors is sorted out in the following chapter, the data itself can indicate the pattern of factor substitution. Table 3.1 shows that the growth in energy use has in most cases more than kept pace with that of capital, with energy also being substituted for labour. This energy-labour substitution does have a basis in factor prices, since for most of the period energy prices fell relative to wages. If energy and capital are complementary factors, as in fact is found in Chapter 4, this will explain the pattern of factor substitution.

The extent of decline in mineral yield or grade is illustrated in Table 3.2 which also shows changes in single factor productivities and average costs. The decline in yields is shown directly by the fall in the calculated indexes of average mineral content in each industry and indirectly by the fact that ore tonnage hoisted (Qore) grew faster than the indexes of real domestic product (Q). On the whole, one would expect the top, mineable grade to decline unambiguously in the absence of new, high grade mines being put into production. As discussed in Chapter 1, this normally will be true even if the average grade of ore within a mine rises due to a fall in mineral prices increasing the minimum or cutoff grade.[9] Any decline in grade will increase energy use in both mining and milling, although not in a simple fashion.

In open pit mining, which is the less energy intensive method, energy use primarily depends upon the total amount of material handled,

TABLE 3.2: Output Growth and Single Factor Productivities

		Industry								
		Asbestos	Nonmetals	Copper	Gold	Silver	Nickel	Iron	Metals	Mining
$\hat{Q}$	57-65	.043	.082	.037	-.044	.089	.084	.163	.023	.057
	66-70	.029	.074	.087	-.081	.046	-.006	-.061	.060	.067
	71-79	-.005	.029	.011	-.066	-.019	-.057	.045	-.022	-.007
$\hat{Q}ORE$	57-65	.099	.084	.153	-.023	.052	.029	1.52	.085	.085
	66-70	.107	.040	.102	-.087	.089	.069	.039	.069	.055
	71-79	-.106	.056	.115	-.022	.012	-.127	-.003	.049	.052
Mineral Yield	57-65	3.274	3.274	1.573	.259	8.599	1.924	.664	.959	1.463
	66-70	2.020	2.020	1.206	.239	8.538	1.615	.522	.725	.693
(Grade) (%)	71-79	1.884	1.817	.628	.203	6.896	1.831	.467	.694	.656
Q/K	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	.745	.753	.876	.643	1.034	.717	.908	.887	.906
	71-79	.442	.617	.435	.316	.783	.427	.696	.598	.717
Q/L	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	1.241	1.362	1.255	.963	1.482	.744	1.244	1.211	1.605
	71-79	1.154	1.614	1.283	.973	1.344	.573	1.001	1.263	1.594
Q/E	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	.865	1.001	.825	.853	.617	.654	.474	.631	.780
	71-79	.718	1.044	.481	.702	.499	.312	.342	.474	.593
AC = TC/Q	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	1.363	1.612	1.492	1.411	1.263	2.154	1.663	1.454	1.332
	71-79	4.565	3.063	3.834	3.763	3.092	5.214	3.803	3.182	3.031
AC_Y = TC/QORE	57-65	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	66-70	1.405	1.601	1.491	1.435	2.657	2.128	1.659	1.469	1.331
	71-79	3.689	2.890	3.827	3.416	5.347	5.167	3.628	2.972	2.392

Notes: Gold grade is in oz./ton.
The grade for asbestos was taken as a proxy for that of total non-metals. The grade for metal mining is an index (1961 = 1.0) of the grades in the different industries. The grade index (1961 = 1) for total mining is an unweighted average of the metal mining grade index and a similar index of grade for nonmetals. For methods of grade calculation and other data see Appendix B.

both crude ore and overburdon. If the ratio of wast rock to ore remains constant, then an increase in raw ore handling necessitated by a fall in grade will increase energy consumption. While data on this appears unavailable in Canada, a recent study of energy use in mining for the U.S. indicates that there has been no pronounced change in the proportion of waste rock (overburdon) to crude ore handled in that country's open pit mines.[10]

Underground mining should also exhibit a relationship between energy use and ore handled, although here the correlation is probably less strong, due to the complicating factors created by following a vein of ore underground. There are also several methods of underground mining, the most important of which are cut-and-fill, open stoping, and sublevel caving, and each of these mining techniques probably employs different amounts of energy per ton of ore.

On the other hand, whatever the mining method, low grade ores require more energy to concentrate than do those of higher quality. It is here, in fact that most energy is consumed.[11] The energy-grade relationship in concentration is not so much that more ore handling is required, but that low grade ores require different, more energy-intensive processes.

Finally, there appear to be exogenous trends in energy use independent of grade or technological change. While Canada at the start of the 1970's had a larger proportion of underground mines than the world total (107 underground mines versus 43 open pit) there has been a trend in metal mining toward surface operations, which, ceteris paribus, will reduce the use of energy.[12] In opposition to this trend is the increase in energy use in the iron ore industry caused by the shift in product mix toward agglomerated and direct reduced ore. The production of agglomerates consumes eight to ten times more energy per ton of product than either concentrates or direct shipping ores, although the former are 15 - 20 per cent higher in iron content [Gelb et. al., p. 35]. There also does not seem to be much dispute that environmental protection measures mean additional consumption of energy. In iron mining, emission controls entail more electricity use per ton of output. Stricter effluent standards increase energy and other costs where large volumes of water are involved, as in washing and flotation processes in concentration. Regulations mandating land reclamation increase the handling of waste rock in open pit mines. Major federal legislation affecting mining in this way was the Water Act of 1969-70, the Clean Air Act of 1971, and the Environmental Contaminants Act of 1974-76. There was also much provincial legislation at that time.

Any of these reasons for increased energy use without a proportional increase in mine output will clearly cause energy productivity to fall, and Table 3.2 shows that this has in fact happened in all industries except nonmetals. Capital productivity has similarly fallen with the rise in capital's share. In constrast, the productivity of labour has increased in some industries (even since 1971) as the result of the substitution of energy and capital for labour (see equation (9) of Chapter 2). Except for the nonmetals sector that includes the productive oil and gas industry, the increase in output per worker has been slight where it has occurred.

The contribution of scale economies to productivity change will require the regression analysis of the following chapter to attempt to sort out. We have stressed the importance of scale economies in mining, and the increase in their importance during the 1950's and 1960's as the size of equipment increased. The increased equipment size has, however, been associated with both technical improvements and increased energy use. To the extent that these are perfectly correlated, even multiple regression will not discern their separate contribution. It is true that increases in average costs relative to the 1957-65 period were highest in nickel and asbestos, where output declined, on average, since 1971. Cost increases were also lowest in total nonmetals, which exhibited the greatest growth in output. However, technical change and grade decline were simultaneously at work, and the industries started from different productivity bases. In 1961, for example, asbestos was the highest productivity industry in terms of ore hoisted per man-shift with 21.1 tons per shift compared with 7.2 tons in nickel and only 3.9 tons in gold-quartz mines [Pye, 1981, table 20]. Indeed, the low gold mining productivity may well have been caused by the historically small scale of operations as well as the innefficiency engendered by the Emergency Gold Mining Assistance subsidy.

Finally, it is necessary to put Canadian mining cost increases in international perspective, since Table 3.2 only relates per-unit costs to their domestic base-levels. A wider comparison of international costs made by Pye [1981] indicates that to the beginning of the 1970's Canada had succeeded in keeping underground mining costs in line with those for other mines around the world. The direct costs of cut-and-fill stoping circa 1973 were $C10.14 per ton compared to a reange of 3.50 - 15.00 (US$) for foreign mines.[13] Sublevel caving, another important mining method, had Canadian costs of $5.07/ton which were toward the lower end of the $4.00-$8.00 world range. Open stoping costs would be expected to bear a similar relationship. The present international cost relationship of course depends on whether the productivity changes in Canadian mines during the 1970's have paralleled those abroad.

Footnotes

1. Data sources and methods of data calculation are provided in appendix B of the study.
2. Conversation with Stan Hodson, senior engineer for Bethlehem Copper in Vancouver on October 29, 1982.
3. C.H. Pye, "Productivity and Profitability", p. 86.
4. J.E. Nosset, "Dewatering Brunswick Concentrates by Pressure Filtration", CIM Bulletin, 75, #843 (July 1982, 103-112).
5. Conversation with Dr. Allen, head of research and technical support for Cominco in Vancouver, on October 29, 1982.
6. Marilyn Scales, "Why Interest in Electric Tracked Haulage Systems is Growing", Canadian Mining Journal (Sept. 1982, pp. 41-50).
7. See M.U. Sarich, "Abatement of Noise and Vibration in the Canadian Mining Industry", Can. Mining Journal (August 1982, pp. 31-38).

8. There may have been some shift within total mining employment from production to salaried workers. MacMillan, Gislason and Lyon [1971, p. 16] reported that starting from 10% in 1948, salaried workers comprised 26% of total mining employment by 1973, compared with 17 and 27% respectively for manufacturing over the same period. On the other hand, for a sample of individual mines surveyed by the Canadian Mining Journal we found no evidence of such a systematic shift in the proportion of supervisory personnel (see appendix C). We could not seperate production and salaried workers in the main analysis of the study because of lack of data for a sufficient time period (the CMJ series could be derived only from 1967) and our labour force series refers to total employment.

9. There seems to be some very recent evidence from South African gold mines that the grade of ore mined sometimes does fall with increased mineral prices, indicating that producers may be saving their higher quality ore for better market conditions. This might results from expectational behaviour, or from an additional constraint that cash flow always be positive. I am indebted to Professor Theo Beukes of the Dept. of Mineral Economics, Rand Afrikaans University in Johannesburg for this observation.

10. B. Gelb, J. Pliskin and M. Wehle, Energy Use in Mining: Patterns Prospects, Ballinger, Cambridge, 1979.

11. Recall that the highly energy-intensive smelting and refining processes are excluded from consideration.

12. R.P. Douglas, "Mining Practices in the Canadian Mineral Industry by the Year 1999". CIM Bulletin (July 1974). The shift to underground mining has been most pronounced in copper, where most new mines have been B.C. open-pit.

13. These comparisons can be made because the Canadian dollar was approximately on par with that of the U.S. at that time.

4. Productivity Change from Estimated Cost Functions

The preceeding chapter summarized various postwar trends in the Canadian mineral industry, the most striking of which are the apparent slowdown in major technical innovations after the mid-1960's, the secular decline in mineral yields, and the rapid substitution of capital and energy for labour. We now employ an econometric model to relate these factors to changes in mineral industry costs and to calculate measures of productivity change.

The estimated cost function is the translog approximation to a general cost function described in the final section of Chapter 2. Because of the large number of parameters necessary to allow for economies of scale, biased technical change, and nonhomothetic effects of grade decline the model was estimated as a three-equation system of factor share equations as well as the overall cost equation. The estimated equations and detailed statistical results for all sectors are listed in Appendix A. Here we first summarize the resulting factor demand elasticities and estimated productivity measures, then employ simulations to attempt to relate productivity change to changes in income, energy prices, mineral grades, and technology.

Factor Demands and Productivity Measures

Table 4.1 summarizes the estimated elasticities of demand for the factors of production in the model. With the exception of silver where the estimated own-price elasticity of capital demand is wrong-signed, the regularity conditions for the estimated cost function are satisfied in all sectors.[1] As one would expect, demand elasticities

TABEL 4.1: Selected Input Demand Elasticities

(Effects of Prices on Factor Productivity)

Industry
(t values in parenthesis)

Elasticities at Means	Asbestos	Nonmetals	Copper	Nickel	Gold	Iron	Silver	Metals	Mining
Own-Price Elasticity									
Labour	-.260 (-.317)	-.332 (-.3.8)	-.130 (-1.4)	-.153 (-1.6)	.010 (.04)	-.611 (-45.5)	-.003 (-.04)	-.211 (-3.4)	-.335 (-4.9)
Capital	-.151 (-2.1)	-.051 (-.4)	-.022 (-.3)	-.287 (-1.7)	-.003 (-.04)	-.135 (-2.4)	.293 (2.8)	-.042 (-.5)	-.135 (-3.7)
Energy	-.394 (-.6)	-.822 (-1.1)	-.263 (-.2)	-.162 (-.1)	-.140 (-.1)	-1.299 (-.17)	-.607 (-.7)	-.540 (-.7)	-.668 (-1.3)
Cross Elasticities									
Labour on Capital	.161 (2.5)	.100 (1.2)	.063 (.7)	.140 (1.6)	-.062 (.4)	.215 (2.4)	-.178 (-2.4)	.087 (1.5)	.200 (2.3)
Energy on Capital	-.039 (-.4)	-.158 (-1.3)	-.208 (-1.9)	-.008 (-.03)	-.116 (-1.8)	-.197 (-.9)	-.507 (-2.9)	-.281 (-1.5)	.086 (1.0)
Energy on Labour	.433 (2.8)	.980 (5.3)	.471 (3.7)	.170 (.6)	.213 (1.4)	1.497 (4.9)	1.073 (8.4)	.821 (3.8)	.612 (8.4)
Labour on Energy	.099 (2.8)	.181 (5.3)	.061 (3.7)	.013 (.6)	.051 (1.4)	.395 (4.9)	.187 (8.4)	.123 (3.8)	.152 (8.4)

for capital are lower, in general, than for either labour or energy, reflecting the difficulty of rapidly adjusting the capital stock.[2] That energy demand has generally insignificant elasticity is not surprising, given little change in energy prices over most of the sample (up to 1974). The significant elasticities of demand for labour in many industries implies that large gains achieved by unions are achieved at the cost of lost employment (assuming competitive labour markets), although the magnitudes of the coefficients shown (all less than unity in absolute value) ensure that the total wage bill would increase.

A generally consistent pattern of interaction between the factors of production also emerges from inspection of table 4.1. Capital and energy tend to be relatively weakly complementary factors, while labour is quite a strong substitute for both energy and capital.[3] This result is quite as expected from the discussion of mineral industry technology in Chapter 3, the major energy use in mining taking place in the heavily capital-intensive process of concentration of the mineral ores.

The table illustrates as well the effects of factor price changes on the productivities of the factors in the model, since it was shown in Chapter 2 that these productivity relationships are identically equal to the negative of the factor demand elasticities. Thus, an increase in the price of labour by 1 percent in asbestos, for example, is predicted to raise labour productivity by .26% in that industry, as long as the other factor prices remain constant. A 1% increase in energy prices, on the other hand, would reduce labour productivity by .433%, reflecting the higher value of the cross elasticity. Remaining with

TABLE 4.2: Calculated Productivity Measures 1957-1979

(t values in parenthesis)

Industry

Statistic		Asbestos	Nonmetal	Copper	Nickel	Gold	Iron	Silver	Metals	Mining
SCE	57-65	1.034	.025	-1.513	.974	.329	.771	.204	.961	1.183
		(10.0)	(.26)	(-3.6)	(5.0)	(2.0)	(9.1)	(3.1)	(4.5)	(2.1)
	66-70	.916	.168	-.279	.999	.270	.855	.496	1.093	.779
		(4.8)	(.9)	(-1.3)	(4.8)	(1.8)	(13.0)	(7.2)	(5.7)	(1.3)
	71-79	.637	-.143	-.691	-.592	.182	1.051	.476	.726	.933
		(5.5)	(-.7)	(-2.8)	(-2.2)	(1.0)	(12.2)	(3.6)	(4.0)	(1.7)
DCDG	57-65	-.034	-.458	.382	.410	.465	-.289	-1.597	-.094	.756
		(-.1)	(-8.7)	(1.7)	(.3)	(1.0)	(-1.8)	(-7.8)	(-.03)	(.9)
	66-70	-.067	.002	-.476	-.449	-.377	-.090	-.619	-.435	1.244
		(-.3)	(.07)	(-2.4)	(-.3)	(-.8)	(-.4)	(-5.2)	(-.1)	(1.3)
	71-79	-.064	-.109	-.732	-.328	-1.257	-.021	-.931	-.266	.863
		(-.3)	(-6.4)	(-5.5)	(-.2)	(-2.6)	(-.1)	(-8.6)	(-.03)	(1.1)
DCDT	57-65	.026	-.043	.014	.054	-.003	.050	.002	.018	.078
		(6.0)	(-4.3)	(.9)	(10.0)	(-.3)	(7.5)	(.01)	(1.4)	(4.0)
	66-70	.044	.013	.030	.060	-.004	.038	.013	.032	.063
		(7.4)	(1.3)	(2.0)	(9.8)	(-.4)	(9.8)	(.07)	(3.0)	(3.6)
	71-79	.043	.040	.036	.042	-.002	.038	.009	.035	.058
		(10.6)	(3.6)	(2.4)	(12.8)	(-.2)	(9.6)	(.05)	(3.2)	(3.6)
PROD1	57-65	.027	.034	.013	.024	-.030	.047	.070	.006	.012
	66-70	-.024	-.037	.002	-.095	.002	-.076	-.029	-.019	.035
	71-79	-.039	-.003	-.053	-.015	-.053	.005	-.030	-.044	-.056
PROD2	57-65	.263	.038	-.006	.184	.005	-.234	-.002	-.033	.344
	66-70	.103	-.016	-.039	.070	.005	-.144	-.028	-.070	-.226
	71-79	-.105	-.031	-.019	.009	.002	.256	-.053	-.013	-.312

the example of asbestos, a glance back at tables 3.1 and 3.2 of the previous chapter indicates that labour productivity in that industry first rose and then fell, while relative to capital both labour and energy prices first fell and then rose. The path of labour productivity in this instance is thus explainable by the higher elasticity of productivity associated with energy. A similar analysis can be performed for the other industries.

Of similar interest are the trends in the productivities of energy in each industry. Again from table 3.2, energy productivity trends are downward with the exception of nonmetals. This of course is the consequence of the rapid increase in energy use (Table 3.1). From Table 4.1, it appears that this increased energy use was not the consequence of the decline in relative energy prices before 1974, as the own-price demand elasticities for energy are insignificant. Instead, it was the result of the substitution of both capital and energy for labour in an industry-wide effort to economize on the wage bill.

The trends in estimated multifactor productivity are shown in Table 4.2. The calculated measure of economies of scale is the one employed by Christensen and Green [1976] and is a transformation of the returns to scale measure of Chapter 2 $(RTS = 1/\partial \ln C/\partial \ln Q))$ defined as

$$SCE = 1 - \partial \ln C/\partial \ln Q = 1 - 1/RTS \qquad (1)$$

This measure is zero if there are constant returns to scale, making it easy to employ standard t statistics to test for the significance of scale economies. According to the model, there are large and highly significant scale economies in most mineral industries. Although this is according to expectations, the magnitudes of these calculated measures are in some instances too large to be believable. A level of SCE equal to unity, for example, implies that total costs are independent of output and the long-run average cost curve is a rectangular hyperbola. There are several possible sources of bias for SCE. In the first place, due to the lack of a capacity utilization measure (see Chapter 2, footnote 3) it is almost certain that the effects of underutilized capacity rather than scale economies are being attributed to SCE, especially in the post-1971 period of output decline. The omission of materials may be a source of bias, although the study by Anders et.al. [1977] indicated that weak separability of ore from other inputs could not be rejected. Another problem may be our specification of the model in static form instead of allowing a gradual change in the capital stock (see footnote 2), a possible misspecification made more likely by the presence of autocorrelation as described in Appendix A.[4] On the other hand, a recent study by Rao and Preston [1982] confirms the presence of strong scale economies in mining. It is interesting to note that in some industries the estimated scale economies appear to have been exhausted by the 1970's, SCE even changing sign in nickel-copper, for example. This is a cause of decline in primal multifactor productivity in such industries, as discussed shortly.

It appears that mineral industry costs have also been affected by the decline in yields over the sample period. The logarithmic derivative of costs with respect to mineral grade (Chapter 2, equation

26) is denoted by DCDG in Table 4.2, and a significantly negative value implies that costs have been increased (and multifactor productivity reduced) by grade decline. DCDG appears significant in nonmetals, copper-gold-silver, gold, and silver-lead-zinc, while remaining insignificant in asbestos, nickel-copper, iron, metal and total mining.[5] This factor is particularly important in industries such as copper where, from Table 3.2, average grade in 1971-79 was only 60% of that in the 57-65 period. This implies, ceteris paribus, a resulting cost increase of 43%.

The calculations of multifactor productivity from the cost and the production functions (dual and primal measures) are shown respectively as DCDT and PROD2. These are as defined in Chapter 2, equations (13) & (14). PROD1 for comparison is the simple Divisia index calculated on the assumption of constant returns to scale, perfect competition, and short-run profit maximization. The rate of cost diminution (DCDT) shows productivity decline by the 1970's in all industries except gold and silver, even after the effect of the decline in grade has been accounted for. This decline must therefore have been due to the slowdown in technical innovation, increased environmental protection legislation, or other factors discussed in Chapter 3. Somewhat disturbing is the indication that in some industries productivity apparently declined during even the 1957-65 period, contrary to expectations. In effect, cost reductions in these years were attributed by the model to scale economies rather than disembodied technical change. This may, in fact, be true in the mining industry, since we observed in Chapter 3 the close association between these phenomenon. The result obtained is, however, probably due to the constraint imposed on the model that the rate of cost diminution depends only on output and relative prices and cannot vary independently with time, i.e.,

$$DCDT = \theta_t + \theta_{tQ} \ln Q + \Sigma\gamma_{ti} \ln P_i \qquad (27, \text{Chapter } 2)$$

In most industries the coefficient representing the disembodied effects of technology, θ_t, was in fact insignificant, the significantly positive value of DCDT resulting only from the combination of changes in income, factor prices, and technology.[7]

In any case, most industries showed the expected pattern of primal productivity change (PROD2), because the very high estimated scale economies counteracted the dual productivity decline in the calculation of the productivity index. As

$$PROD2 = -DCDT/(1-SCE), \qquad (2)$$

the large values of estimated scale economies tended, however, to magnify any change in DCDT, resulting in irregular changes in PROD2 for individual years. Because there have been long and bitter strikes in many mineral industries, notably the INCO strikes in 1969 and 1979 and iron ore industry strikes in 1969, 1972 and 1978, it was thought that short-term variations in productivity would be related to this factor. The model was therefore re-estimated in appendix B to allow strikes to affect both short-term productivity levels and long-term productivity change. While strikes temporarily affect the factor shares in some industries, only in the asbestos and silver-lead-zinc industries is there an estimated negative effect on long-term productivity change. Thus while strikes may affect productivity levels in the year they take place, there is generally no strong econometric evidence to suggest a significant relationship with multifactor productivity growth.[8]

The statistical significance of the measure of primal productivity change could not easily be determined, as it is a ratio of two random variables. For this reason, t values for PROD2 are not shown in Table 4.2. However, a necessary but not sufficient condition for significant PROD2 is surely that DCDT be significant; hence the designation "possibly significant" in Table 4.3, where we summarize the results for each industry.

TABLE 4.3: Summary of Productivity Measures Significance

Industry	SCE	DCDG	DCDT	PROD2
Asbestos	sig.	insig.	sig. > 0	possibly sig.
Nonmetals	insig.	sig. < 0	sig. $\gtrless 0$	possibly sig.
Copper	sig.	sig. < 0	sig. > 0	possibly sig.
Nickel	sig. $\gtrless 0$	insig.	sig. > 0	possibly sig.
Gold	partially sig.	sig. after 1971	insig.	insig.
Iron	sig.	insig.	sig. > 0	possibly sig.
Silver	sig.	sig. < 0	insig.	insig.
Metals	sig.	insig.	sig. > 0	possibly sig.
Mining	partially sig.	insig.	sig. > 0	possibly sig.

Simulated Effects of Prices and Output on Productivity Change

As well as their effects on the productivities of individual factors, changes in factor prices can affect the growth in total factor productivity, according to the model. Table 4.4 shows these predicted effects. Because technical change in all industries has been capital-using, increases in interest rates or other components of capital costs tend to reduce the rate of cost decline (productivity increase). In contrast, wage increases, by stimulating the substitution of capital for labour, tend indirectly to enhance the growth in productivity. Because of the complementarity of energy and capital, energy price increases have an effect similar to that of capital costs in retarding productivity growth. In general, however, the effects of energy prices tend to be less important than those of either wage or capital costs. The magnitudes of the elasticities imply that in overall metal mining, for example, a 1% increase in capital costs reduce the rate of decline of costs by 3% of the original rate, as from 3.5% to 3.4%.

Of special interest is the relationship of output with productivity changes. Except apparently in iron, productivity growth is highly dependent on the growth of output in each industry, with an elasticity as high as 2.5 in nonmetals. This, of course, is a manifestation of scale economies and their relationship with technological change previously discussed. There has at least been a high correlation between periods of high rates of technical change and those of high output growth, implying that a significant part of recent productivity decline can be attributed to the post-1970 decline in demand for minerals shown in Table 3.2.

TABLE 4.4: Effects of Prices and Output on Rate of Cost Diminution

Industry

	Asbestos	Nonmetals	Copper	Nickel	Gold	Iron	Silver	Metals	Mining
$\frac{\partial \ln \hat{C}}{\partial \ln P_K}$	.465	.250	.278	.238	~0	~0	~0	.031	.172
$\frac{\partial \ln \hat{C}}{\partial \ln P_L}$	-.233	-.250	-.280	-.237	~0	-.078	-1.11	-.034	-.126
$\frac{\partial \ln \hat{C}}{\partial \ln P_e}$	.232	.012	.014	.029	~0	.078	1.11	.085	.793
$\frac{\partial \ln \hat{C}}{\partial \ln Q}$	1.163	2.50	.833	2.291	~0	-.768	-2.006	1.143	~0
Mean ($\hat{C}$)	.043	.040	.036	.042	-.002	.038	.009	.035	.058

Note: $\frac{\partial \ln \hat{C}}{\partial \ln P_i} = \gamma_{ti}/\hat{C}$

$\frac{\partial \ln \hat{C}}{\partial \ln Q} = \theta_{tQ}/\hat{C}$

Negative values imply enhancement of productivity change by encouraging the reduction of costs. The elasticities are evaluated at the means of 1971-1979 values.

To attempt to assess the relative contributions of these factors to the post 1970 productivity decline, the primal rate of productivity change has been simulated alternatively holding technology (proxied by time), output, and energy prices constant at their 1971 values. The results, shown in Table 4.5, indicate that for the most part energy price changes have had little effect on productivity, the productivity decline remaining in effect even with 1971 energy prices. This is not difficult to understand given energy costs as generally less than 10% of the cost of production, and parallels the results for a similar study of overall mining (Cf. Smithson et.al. [1971]). Output growth is more important, especially since 1974. The freezing of output at 1971 levels resulted in both increased and reduced productivity decline, depending on the industry. In general, where productivity decline was greater with fixed output, in asbestos, copper, nickel, metal and total mining, the level of output in 1974 was higher than in 1971, even though there may have been an average decline over the whole decade. In iron, gold, and silver, the 1974 output was lower, so holding at the 1971 level provides an improvement. Nonmetals was an anomaly, with actual output higher but productivity decline larger than in the controlled case. This seems to be the result of scale economies becoming negative (see Table 4.2) at the 1971 output level.

Finally, the residual effects on productivity were isolated by holding the time trend at $t = 1971$. In all cases "technological advance" (proxied by time) was still contributing to productivity growth in the sense that productivity declined at a faster rate when it was arrested.

TABLE 4.5: Simulated Changes in Total Factor Productivity 1971-79

	$t = t(1971)$		$Q = Q(1971)$		$P_E = P_E(1971)$		Actual Outcome	
Industry	1971-73	1974-79	1971-73	1974-79	1971-73	1974-79	1971-73	1974-79
Asbestos	-.180	-.169	-.151	-.091	-.153	-.082	-.155	-.081
Nonmetal	-.032	-.060	-.024	-.019	-.026	-.034	-.027	-.034
Copper	-.286	-.081	-.114	-.031	-.239	-.063	-.029	-.019
Nickel	-.040	-.025	-.013	-.071	.043	-.019	.043	-.019
Gold	.003	.002	.004	.007	.003	.004	.003	.002
Iron	-.559	-.314	-.549	.496	.321	.416	.518	.232
Silver	-.044	-.003	-.046	-.104	-.040	-.061	-.039	-.061
Metals	-.607	-.500	.063	-.141	.116	-.088	.116	-.091
Mining	-.131	-.269	-.293	-.388	-.265	-.279	-.269	-.342

Growth Accounting: Gross and Net Multifactor Productivity

The productivity growth measure we have been discussing (PROD2 or ε_{ft}) is a measure of gross multifactor productivity in that is must be greater than otherwise in order to offset the effects of changes in mineral grade.

The corresponding net multifactor productivity measure from (7) and (13) in chapter 2 is

$$\hat{B} = \text{PROD2} - \overset{(-)}{(\text{RTS})}\overset{(-)}{(\text{DCDG})}\,\hat{g} = \hat{Q} - \text{RTS}\ \Sigma S_i^* \hat{X}_i, \qquad (3)$$

where the X_i represent factors K, E, L. $\hat{B}$ calculated from the RHS of (3) is shown in Table 4.6, in a form modified (through (9) of chapter 2) to display changes in labour productivity. The table reiterates that both B and LPROD fell over the period, despite the continued growth in both capital/labour and energy/labour ratios, at least in part as the result of output decline associated with scale economies. The rather wildly inflated estimates of scale economies in some industries (possible causes of bias have been discussed previously) however translate into biased $\hat{B}$ estimates, especially in nickel and iron ore. These estimates can be compared with the division index PROD1 in Table 4.2, the measure of $\hat{B}$ that would result a given constant returns to scale (RTS = 1) and short-run profit maximization. Some indication of the bias is also provided by the fact, shown in Tables 4.2 and 4.3, that only in nonmetals, copper, and silver-lead-zinc is DCDG significantly different from zero. Roughly speaking (ignoring cross correlations) $\hat{B}$ should therefore equal PROD2 in all but these industries.[9]

Table 4.6: Changes in Output, Returns to Scale, Labour and Net Multifactor Productivity Change

Industry	Year	$\hat{Q}$	= [$\hat{LPROD}$	= $\hat{B}$	+ RTS	x (S^*_K	x $(\hat{K/L})$	+ S^*_E	x $(\hat{E/L})$)	+ (RTS − 1)$\hat{L}$]	+ $\hat{L}$
	1957-65	.043	.056	.502	-29.41	.211	.092	.129	.071	.395	-.013
Asbestos	66-70	.029	-.003	-.601	11.91	.356	.049	.108	.031	.349	.032
	71-79	-.005	-.018	-.110	2.75	.450	.055	.103	.017	.011	.006
	1957-65	.082	.058	.022	1.03	.406	.073	.105	.060	.002	.006
Nonmetals	66-70	.072	.029	-.061	1.20	.603	.109	.075	.026	.009	.045
	71-79	.029	.022	.003	.87	.670	.033	.070	.021	-.002	.013
	1957-65	.037	.019	.028	.39	.306	.005	.078	.049	-.011	.018
Copper	66-70	.087	.043	-.002	.78	.441	.127	.068	.093	-.004	.004
	71-79	.011	-.008	-.017	.59	.588	.069	.065	.023	-.018	.019
	1957-65	.084	.010	-1.08	38.46	.241	.013	.045	-.006	.974	.026
Nickel	66-70	-.006	-.073	-86.25	1000.00	.299	.054	.035	.094	.603	.067
	71-79	-.057	.022	-.028	.63	.360	.077	.048	.102	.029	-.079
	1957-65	-.044	-.019	-.013	1.49	.112	-.001	.086	.013	-.007	-.015
Gold	66-70	-.081	.037	.033	1.37	.222	.142	.077	.041	-.044	.110
	71-79	-.066	-.014	-.032	1.22	.337	.078	.070	.019	-.016	-.047
	1957-65	.163	.071	-.338	4.37	.461	.095	.082	.157	.168	.045
Iron	66-70	-.061	-.054	-.162	6.89	.559	.054	.108	.077	-.041	-.007
	71-79	.045	-.005	1.30	-19.61	.556	-.041	.120	.005	-.809	.040
	1957-65	.089	.096	.068	1.26	.344	.044	.026	.116	-.002	-.007
Silver	66-70	.046	.008	-.104	1.98	.483	.069	.073	.062	.037	.038
	71-79	-.019	-.022	-.039	1.91	.503	.010	.081	.009	.006	.007
	1957-65	.023	.039	-.493	25.64	.327	.055	.072	.084	-.074	-.003
Metals	66-70	.060	.042	.489	-10.75	.439	.039	.073	.064	-.212	.018
	71-79	-.022	-.002	-.159	3.65	.518	.062	.078	.038	.003	.001
	1957-65	.057	.046	.300	-5.46	.458	.075	.074	.056	-.047	.007
	66-70	.067	.062	-.157	4.52	.651	.059	.064	.075	.021	.006
	71-79	-.007	.006	-.931	14.93	.726	.058	.066	.051	.069	.005

NOTE: RTS = 1/(1 - SLE)

Footnotes

1. For the estimated cost function to represent the dual to a well-behaved neoclassical production function, it is necessary and sufficient that all estimated factor shares be positive for every observation and that own-price elasticities of factor demand be nonpositive. This was true with the aforementioned exception of silver (the labour demand elasticity for gold being statistically zero).

2. A dynamic model allowing the capital stock to adjust slowly was employed by Berndt and Watkins [1981], who however imposed long-run constant returns to scale and other restrictions which reduced the number of estimated parameters. Because of unwillingness to restrict the model to constant returns, the estimation of such a dynamic model was made impractical by a shortage of data observations. This is unfortunate, as the assumption of immediate capital adjustment is surely unrealistic and probably an important cause of the autocorrelation apparent in the reported equations.

3. Two exceptions to the general pattern are the significantly negative cross elasticity for labour on capital in silver-lead-zinc and the apparent substitutability of capital and energy in total mining in contrast with its sub-industries. The first observation can be discounted since, as previously mentioned, silver did not satisfy the regularity conditions for the cost function. The weak positive capital-energy substitution in total mining (also found by Smithson et.al. [1977]) is surprising although

not inconsistent with the sub-industry results, since this coefficient is not a weighted average of those for sub-industries but was estimated from aggregated data. It may result, for example, from changes in sub-industry shares.

4. A much better estimate of scale economies would, of course, be obtained by a combination of time series analysis with a cross-section of different sized mines. Cost data for a large number of mines are available yearly in the Canadian Mining Journal, Reference Manual and Buyer's Guide, but there are no corresponding capital stock estimates at this level of disaggregation.

5. The results in nonmetals, metal mining, and total mining may have been affected by the proxies used for mineral grade in these aggregative industries. See the notes for Table 3.2.

6. The averages shown in Table 4.2 do not identify the beginning of productivity decline in each industry. For the estimated primal productivity measure PROD2 this was 1967 in asbestos, 1967 in nonmetals, 1958 in copper, 1957 in iron, 1964 in silver, 1959 in metals, and 1966 in total mining.

7. Constraints of time, money and degrees of freedom in the model prevented the addition of another coefficient to deal with the problem.

8. For a discussion of the effects of strikes on productivity change see D. Maki, "The Effects of Unions and Strikes on the Rate of Growth of Total Factor Productivity in Canada", Paper given at Canadian Economic Association meetings in Ottawa, June 1982.

9 For the industries where grade does affect $\hat{B}$ the following are approximate proportions for grade versus PROD2:

Years	Nonmetals	Copper	Silver
1957-65	∿60% grade	grade only sig.	grade only sig.
66-70	neither sig.	∿50% grade	grade only sig.
71-79	∿50% grade	∿60% grade	grade only sig.

5. Conclusions and Policy Implications

This study has analysed productivity trends in the Canadian mining industry and its components from 1957 to 1979. Evidence has been presented to indicate a declining trend in multifactor productivity that in the case of several mineral industries began well before the 1970's. Attention has been focussed on the roles of increased energy prices, declining ore grade, and lower output accompanied by increasing returns to scale in the productivity decline. The model also provided estimates of factor demands and substitution possibilities among factors in response to factor price changes.

With reference to the factor relationships in mining, labour was found in all industries to be a strong substitute for energy, and to be weakly substitutable for capital in all industries except silver and gold, a finding that parallels that of the studies by Smithson et. al. and Rao and Preston (labour-capital substitutes). With the exception of total mining (see Chapter 4, footnote 3) capital and energy were found to be weak complements. Energy price increases apparently result in a significant substitution of labour for energy, and wage increases have the opposite result. An increase in the price of any factor increases the productivity of that factor, although the relationship is weak for energy price changes. The most important trend throughout the sample period has been the substitution of capital and energy for labour, tending to raise labour productivity, although not markedly, and to lower the productivities of both capital and energy.

Factor prices were also found to affect the changes in total factor productivity through time. Because technical change was found to be capital and energy-using and labour-saving, increases in interest rates or other capital costs apparently have had an inhibiting effect on productivity growth, while wage increases have enhanced it. The capital-using bias for technical change (confirmed by both Smithson et.al. and Rao and Preston) however, could not have been primarily in response to relative factor price changes because, as Table 3.1 shows, the general trend of the relative price of labour to capital has been downward. A more likely explanation for the bias is not wages per se but the abysmal postwar record of labour relations in Canadian mining.

The relationship of energy prices with productivity change appears to be weak. This result parallels that of the only similar study of Canadian mining (Smithson et.al.) but contrasts with the results of Rao and Preston [1982] and Daly and Rao [1983] who find technical change to be quite strongly fuel-saving and fuel-using in manufacturing and electric utilities, respectively. The implication of our result is that monetary policy has at least a greater short-term impact on the mining industry than energy policy. As well, wage increases cannot be blamed for the recent productivity decline.

Related to the effects of factor price changes are those of the decline in mineral yields. Declining yields have had a significant effect in reducing productivity in some of the industries studied. The cost effects of the decline in grades have tended to intensify recently because lower mineral yields require more capital and energy-intensive processing, and this has been inhibited by recent increases in energy and capital costs.

There seem to have been significant economies of scale in most mineral industries, and a strong correlation between technical improvements and increases in the scale of operations, especially in the 1960's. Although, as previously noted, the model unfortunately could not distinguish between scale economies and the effects of the degree of capacity utilization, it seems likely that one reason for the productivity slump in mining has been the decline in output in several industries in the 1970's, resulting in lower capacity utilization in mines whose scale of operation was much expanded in the 1950's and 1960's. To retard productivity decline, it is therefore important for the industry to maintain and expand its markets. Unfortunately for policy, these are largely export markets, highly dependent on economic growth in the U.S. and Europe, and facing growing competition from newer mining areas in Asia and Latin America. A step that might be considered, however, would be the promotion of the sort of market research normally conducted by Inco, for example, on new uses of nickel in stainless steels.

Finally, after taking the factors above into account, there remains a significant unexplained residual decline in productivity which may be attributed either to an exogenous slowdown in technical innovation, which does seem to have occurred, changes in taxation, or to the effects of the important health, safety, and environmental regulations introduced in mining in the early 1970's. There is very little independent evidence concerning the importance of environmental regulations for productivity. Papanicolaou and McKenzie (1981) estimated current environmental regulations to have reduced base metal mine investment by 3-10% in Canada. An effect of similar magnitude was predicted by Smithson et. al. (1977), who also estimated a 20 to 26% reduction as the result of post-Carter-Commission tax changes. Since technical change is capital-using, a slowdown in capital formation indirectly affects measured productivity growth. Evidence from U.S. manufacturing (Christainsen, and Haveman [1981]) suggests that in that country between 8-12 percent of the post-1973 slowdown in the growth rate of measured labour productivity is attributable to environmental regulations. Of course, the effects on true productivity are unknown, since in principle this should include the benefits of the regulations to workers and to the public, as well as the intangible but important benefit to the industry of enhanced worker health and safety in improving the industry's poor record of labour relations.

With reference to the technical innovation slowdown, there is independent evidence (Richardson et. al., 1976) to indicate an inadequate level of research and development in Canadian mining and mining supply industries, related to the small size of many mining and mining supply firms and to the high degree of foreign control. (There was only a single Canadian-owned mining supply company when Richardson et. al. did their study.) This hypothesis explains the mining productivity boom of the 1950's by the technology imported under foreign license during that period of large net capital inflows, and subsequent slump by the reversal of such capital movements. If this is correct, the policy choice is either to re-induce foreign investment in Canadian mining or to encourage a greater degree of research and development by Canadian mining equipment companies.

APPENDIX A: Estimated Cost Functions Excluding Strikes

This appendix lists statistical results for the estimated cost functions in the different mineral industries. Because the capital, labour and energy shares sum to unity, only two share equations were necessary to estimate, as well as the overall cost function. The estimated equations with the homogeneity restrictions included and errors added are:

$$S_K^* = \delta_K + \gamma_{tK}t + \gamma_{QK}\ln Q + \gamma_{gK}\ln g + \gamma_{KK}\ln(P_K/P_E) + \gamma_{KL}\ln(P_L/P_E) + u_K$$

$$S_L^* = \delta_L + \gamma_{tL}t + \gamma_{QL}\ln Q + \gamma_{gL}\ln g + \gamma_{KL}\ln(P_K/P_E) + \gamma_{LL}\ln(P_L/P_E) + u_L$$

$$\begin{aligned}\ln(C/P_E) = {} & \theta_0 + \theta_Q\ln Q + \frac{1}{2}\theta_{QQ}(\ln Q)^2 + \theta_g\ln g + \frac{1}{2}\theta_{gg}(\ln g)^2 \\ & + \theta_{Qg}\ln Q\ln g + \theta_t t + \theta_{tQ}t\ln Q + \sum_i^{K,L}\delta_i\ln(P_i/P_E) \\ & + \frac{1}{2}\sum_i^{K,L}\sum_j\gamma_{ij}\ln(P_i/P_E)\ln(P_j/P_E) + \sum_i^{K,L}\gamma_{ti}t\ln(P_i/P_E) \\ & + \sum_i^{K,L}\gamma_{Qi}\ln Q\ln(P_i/P_E) + \sum_i^{K,L}\gamma_{gi}\ln g\ln(P_i/P_E) + u_c\end{aligned}$$

These equations are interdependent; in particular the error terms for the share equations would be expected to be related to that of the cost equation. The estimation method used was consequently that of full-information maximum liklihood which takes the error correlation into account. The estimation was performed with the TROLL econometrics package from MIT, using the Davidon-Fletcher-Powell computational algorithm for non-linear estimation.

Empirical Results

The results of the estimation are shown in Table A-1. Multicollinearity in general was not a problem, the correlations between independent variables seldom exceeding .50. In general, output growth was not homothetic, but labour using, as evidenced by the positive γ_{QL} and negative γ_{QK} coefficients. Grade decline had the opposite effect, increasing the use of capital (and energy) by requiring more processing. The positive γ_{tK} and negative γ_{tL} coefficients indicate technological change that is capital-using, and θ_{tQ} shows the positive relationship of output growth with technical change.

One statistical problem was the significant positive autocorrelation present in several industries. In OLS estimation it is well known that autocorrelation does not bias the coefficient estimates but tends to increase their standard errors, as well as biasing the adjusted $\bar{R}^2$ statistic. With nonlinear FIML estimation the effects are not known. To the extent that the autocorrelation is an indication of misspecification, however, for example the use of a static rather than a dynamic model of capital stock adjustment (see Chapter 4, Footnote 2) then of course this will cause bias even in OLS estimation. Unfortunately, it was not possible to employ the standard generalized-least-squares correction techniques for autocorrelation in conjunction with the simultaneous nonlinear maximum-liklihood technique employed in the estimation.

The standard errors for the SCE, DCDG, and DCDT statistics shown in Chapter 4 were calculated assuming the variables Q, g, t etc. constant at their mean values. The standard errors were calculated

TABLE A-1: Estimated Translog Cost Function Parameters 1957-79

(t statistics in parenthesis)

Parameter	Asbestos	Nonmetals	Copper	Gold	Silver	Nickel	Iron	Metals	Mining
δ_K	3.11 (7.2)	2.70 (9.8)	3.07 (4.2)	2.39 (26.2)	4.65 (7.4)	1.77 (3.4)	2.73 (6.7)	3.23 (6.5)	2.42 (3.3)
γ_{KK}	.18 (5.4)	.21 (7.1)	.22 (4.7)	.08 (2.9)	.36 (8.0)	.12 (2.3)	.18 (6.2)	.21 (6.5)	.15 (7.6)
γ_{KL}	-.13 (-4.4)	-.15 (-6.4)	-.17 (-3.7)	-.07 (-2.4)	-.29 (-8.1)	-.11 (-1.9)	-.11 (-3.1)	-.16 (-5.7)	-.11 (-6.1)
γ_{QK}	-.13 (-4.7)	.08 (.9)	-.01 (-.3)	-.27 (-5.1)	.01 (.1)	-.10 (-3.1)	.02 (.6)	-.04 (-1.4)	-.0006 (-.003)
γ_{gK}	-.04 (-3.2)	-.03 (-3.2)	-.09 (-2.4)	.07 (1.0)	-.02 (-.3)	-.004 (-.1)	-.11 (-2.2)	.05 (1.0)	-.36 (-1.5)
δ_L	-1.12 (-2.9)	-1.12 (-4.4)	-1.28 (-1.8)	-.93 (-8.3)	-2.72 (-5.7)	-.44 (-.8)	-1.26 (-4.4)	-1.49 (-3.8)	-.95 (-1.5)
θ_t	-.02 (-.2)	-.35 (-5.1)	-.08 (-.4)	-.04 (-1.0)	-.04 (-.8)	-.03 (-1.2)	.18 (3.2)	-.05 (-0.4)	.28 (2.4)
γ_{LL}	.13 (4.3)	.10 (3.0)	.17 (3.8)	.12 (3.4)	.24 (7.8)	.13 (2.0)	.001 (.2)	.14 (4.4)	.09 (5.2)
γ_{QL}	.09 (3.4)	-.08 (-1.1)	.01 (.1)	.24 (4.6)	-.02 (-.8)	.12 (3.4)	-.03 (-1.2)	.05 (1.8)	.007 (.1)
γ_{gL}	.04 (3.9)	.01 (2.5)	.09 (2.5)	-.07 (-1.0)	.01 (.2)	.001 (.02)	.09 (2.7)	-.004 (-.1)	.36 (1.8)
θ_0	27.21 (2.1)	11.94 (3.1)	-43.66 (-1.1)	8.10 (50.1)	45.44 (8.0)	23.81 (6.3)	29.00 (6.1)	70.60 (2.5)	28.21 (1.6)
θ_Q	1.39 (.3)	6.99 (6.4)	18.83 (1.8)	4.02 (27.3)	-.15 (-.1)	.78 (.3)	-.78 (-.5)	-16.32 (-1.3)	.35 (.04)
θ_{QQ}	-.73 (-.6)	-1.25 (-4.1)	-2.41 (-1.8)	-.80 (-6.0)	-.60 (-3.2)	-3.05 (-2.5)	.27 (.8)	3.45 (1.3)	-.05 (-.02)
θ_g	-.002 (-.003)	-1.13 (-4.5)	8.50 (1.9)	-9.35 (-57.8)	-5.96 (-3.6)	-8.62 (-2.5)	-2.23 (.8)	-22.69 (-2.4)	4.33 (.4)
θ_{gg}	-.0006 (-.003)	-.65 (-7.2)	-.93 (-1.2)	-.87 (-5.4)	-.96 (-2.0)	6.16 (2.0)	-.53 (.8)	6.74 (3.0)	-2.98 (-.6)
θ_{Qg}	-.09 (-.6)	.24 (5.8)	-1.25 (-2.2)	2.07 (15.1)	1.51 (5.3)	3.40 (2.2)	.10 (.2)	5.19 (2.5)	-1.64 (-.7)
γ_{tL}	-.01 (-21.1)	-.01 (-3.0)	-.01 (-3.1)	.002 (.6)	-.01 (-2.9)	-.01 (-6.5)	-.003 (-2.5)	-.01 (-19.3)	-.01 (-2.0)
γ_{tK}	.02 (20.2)	.01 (2.7)	.01 (2.6)	-.004 (-1.1)	.004 (1.5)	.01 (5.6)	.0008 (.5)	.01 (14.8)	.01 (1.3)
θ_{tQ}	.05 (2.6)	.10 (8.7)	.03 (.9)	-.003 (-.4)	.02 (2.3)	.10 (4.4)	-.03 (-2.9)	.04 (1.4)	-.03 (-1.6)
K-share equation									
$\bar{R}^2$	.979	.884	.907	.956	.897	.859	.778	.986	.786
D.W.	1.23	.25	.54	.47	1.18	1.70	1.40	1.42	2.08
L-share equation									
$\bar{R}^2$	.979	.895	.910	.947	.943	.844	.926	.992	.837
D.W.	1.37	.24	.52	.47	1.34	1.56	1.44	2.04	2.02
TCOST equation									
$\bar{R}^2$	.979	.892	.943	.432	.992	.791	.969	.974	.409
D.W.	1.44	.73	1.02	1.00	1.72	1.51	1.21	1.00	2.07

using the fact that, given a linear relation

$$Y = b_1\bar{X}_1 + b_2\bar{X}_2 + \ldots\ldots$$

with $\bar{X}_1$ and $\bar{X}_2$ the constant means, and b_i the estimated coefficients,

$$V(Y) = \bar{X}_1^{\,2}\, V(b_1) + \bar{X}_2^{\,2}\, V(b_2) + 2\bar{X}_1 X_2 \,\text{Cor}\,(b_{1,} b_2) + \ldots\ldots$$

where $V(b_i)$ etc. are the variances and covariances from the variance-covariance matrix of coefficients.

Finally, one of the testable assumptions in the model of Chapter 1 to guarantee that output could be treated as exogenous was that grade decline affects marginal more than average cost, or $|dMC/dg| > |dAC/dg|$ With the translog cost function this condition is

$$\frac{dMC}{dg} - \frac{dAC}{dg} = AC\ [\theta_{Qg} - \frac{SCE*DCDG}{g}] < 0$$

If this condition is violated, it is theoretically possible that output with cost-constrained extraction may not remain at the capacity level in all periods. Calculating the condition for each industry resulted in it being unambiguously negative in asbestos, copper, and total mining, sometimes negative in iron, and positive in the remaining industries. Consequently, in some industries there may be some simultaneous bias introduced in the estimation in not treating output as an endogenous variable in the empirical model.

There is one industry, nickel, where prior to the middle 1970's the assumption of Canada being a price-taker in world markets is certainly not justified. The Canadian operations of the International Nickel Corporation in 1957 had nearly 75% of the world market (although its current share is under 40%) and there is ample evidence that it operated as a price leader (Stollery [1979]). The exogenous output assumption will therefore introduce a simultaneous bias in the estimation for this industry, and the results should be interpreted in this light.

APPENDIX B

Effects of Strikes on Total Factor Productivity in Mining

We here describe the results of re-estimating the mining industry cost functions including strikes as an exogenous variable. Because of limitations in degrees of freedom and consequent problems of convergence in the nonlinear estimation technique described in Appendix A, it was not possible to include the strike variable in the most general formulation of the cost function; thus the seperate listing of results. To allow the inclusion of strikes we have therefore imposed the additional restriction of homotheticity with respect to output on the estimated cost functions.

A priori, the theoretical effects of strikes on total factor productivity are not one-signed. During the strike, as both labour and capital service flows cease, there should be no impact effect on productivity if these services are properly measured. The expected rise in the measured capital share results from mismeasurement of capital as a stock rather than a flow and should disappear with the inclusion of a proper capacity utilization variable (unfortunately unavailable in this case). There may, however, be true longer-term effects. Some of these might be inefficiency resulting from such strike-induced precautionary measures as keeping large numbers of "supervisory" workers on the payroll to take over in case of a strike or carrying larger finished goods inventories than would otherwise to optimal. (Casual empiricism suggests this latter practice to be highly prevelant in Canadian mining.) In a longer-term context, strike effects are really a subset of the effects of unionization, strike activity being a possible indicator of union militancy.

Unions may in fact be productivity-enhancing in an industry. To the extent that the wage-rental ratio is raised above what it would have been, some substitution of capital for labour may follow, although this is unlikely in mining given the fall in P_L/P_K in most industries. Unions may reduce voluntary turnover by giving workers a "voice" to obtain redress for grievances, thereby increasing efficiency. Offsetting this are the effects of various union-imposed work rules, ranging from "featherbedding" and resistence to the introduction of new technology to simple seniority systems which may both inhibit the productivity of younger workers and retain less productive older ones. While levels of union activity will affect productivity levels and thus only changes in union or strike activity will be reflected in productivity growth or decline, the presence of lags suggests that strikes will have both impact and dynamic effects on an estimated cost function.

To allow for these impact and dynamic effects, we have specified strikes to raise or lower costs through temporarily changing measured factor shares as well as interacting with the technology (trend) variable in affecting long term productivity growth or decline. As previously noted, an additional homotheticity condition was imposed on the function to restrict factor shares to be independent of output (and the degree of scale economies to be unaffected by factor prices). In the context of the previous notation this means that $\gamma_{QK} = \gamma_{QL} = 0$. From Table A-1 this restriction is not binding in non-metals, copper, silver, iron, metals or total mining but may bind in the asbestos, gold and nickel industries.

The revised equations to be estimated are:

$$S_K^* = \delta_K + s_K STRIKE + \gamma_{tK} t + \gamma_{gK} \ln g + \gamma_{KK} \ln(P_K/P_E) + \gamma_{KL} \ln(P_L/P_E) + u_K$$

$$S_L^* = \delta_L + s_L STRIKE + \gamma_{tL} t + \gamma_{gL} \ln g + \gamma_{KL} \ln(P_K/P_E) + \gamma_{LL} \ln(P_L/P_E) + u_L$$

$$\ln(C/P_E) = \theta_0 + \theta_Q \ln Q + \frac{1}{2}\theta_{QQ}(\ln Q)^2 + \theta_g \ln g + \frac{1}{2}\theta_{gg}(\ln g)^2 + \theta_{Qg}\ln Q \ln g$$
$$+ (\theta_t + \theta_{ts}\text{STRIKE})t + \theta_{tQ}t\ln Q + \sum_i^{K,L} (\delta_i + s_i\text{STRIKE})\ln(P_i/P_E)$$
$$+ \frac{1}{2}\sum_i^{K,L}\sum_j \gamma_{ij}\ln(P_i/P_E)\ln(P_j/P_E) + \sum_i^{K,L} \gamma_{ti}t\ln(P_i/P_E)$$
$$+ \sum_i^{K,L} \gamma_{gi}\ln g\ln(P_i/P_E) + u_c$$

In the equations above the strike variable may affect cost both through changing measured factor shares of (with coefficients s_i) and by interacting with the technology index. The measure of cost diminution with strikes thus becomes:

$$DCDT = \theta_t + \theta_{ts}\text{STRIKE} + \theta_{tQ}\ln Q + \Sigma\gamma t_i \ln(P_i/P_E).$$

Since returns to scale are assumed unaffected, primal total factor productivity changes in the same proportion as DCDT. A positive estimated value of θ_{ts} thus indicates, ceteris paribus, that strikes have had a deleterious effect on long-term productivity growth.

The results of the re-estimated model equations are shown in Table B-1. Strikes appear to have increased the share of measured capital in asbestos, silver and nickel and reduced it in iron and overall mining. Labour's share appears to fall significantly only in asbestos and nickel, however; the energy share apparently taking up the slack in the other industries. There is surprisingly no apparent impact effect in copper, gold, or overall metal mining. In terms of long-term productivity change the estimated θ_{ts} coefficient was significantly positive only in the asbestos and silver industries, indicating a negative long-term productivity effect, while it was significantly negative in iron and total mining. On the face of it, there therefore appears to be no strong econometric evidence to support the contention that the deplorable record of labout relations in Canadian mining has materially reduced rate of growth of measured productivity. As noted above, this does not measure the fixed cost of the level of bitterness and general bad feeling in mining industry labour relations on the level of cost or productivity. As this bad record has remained more or less constant over our sample period its effects will be absorbed in the constant term.

TABLE B-1

Estimated Homothetic Translog Cost Functions with Strikes
(t Statistics in parenthesis)

Parameter	Asbestos		Non-Metals		Copper		Gold		Silver		Nickel		Iron		Metals		Mining	
δ_K	2.51	(7.4)	3.81	(9.3)	3.16	(4.4)	1.05	(3.4)	4.79	(9.7)	1.61	(3.3)	2.32	(5.8)	2.05	(2.6)	2.77	(8.0)
a_K	1.9 E-7	(6.6)	1.3 E-5	(0.3)	-5.5 E-9	(-.1)	1.5 E-8	(.02)	4.3 E-7	(2.1)	3.7 E-8	(4.0)	-4.8 E-8	(-2.0)	7.1 E-7	(.4)	-6.1 E-6	(-2.7)
γ_{KK}	.19	(6.8)	.28	(8.4)	.24	(3.7)	.09	(3.1)	.37	(10.2)	.12	(2.6)	.12	(4.2)	.12	(1.9)	.17	(6.9)
γ_{KL}	-.14	(-5.8)	-.22	(-8.4)	-.20	(-3.3)	-.08	(-2.8)	-.31	(-10.3)	-.10	(-1.9)	-.04	(-1.5)	-.05	(-1.2)	-.13	(-5.9)
γ_{gK}	-.005	(-.5)	-.04	(-3.1)	-.10	(-2.6)	-.07	(-.6)	-.02	(-.3)	.09	(2.0)	-.11	(-2.7)	-.9 E-4	(-.01)	-.28	(-1.7)
δ_L	-.85	(-2.8)	-2.27	(-7.1)	-1.52	(-2.2)	.37	(1.2)	-3.00	(-7.9)	-.17	(-.3)	-.89	(-3.1)	-.02	(-.03)	-1.09	(-4.0)
a_L	-1.4 E-7	(-5.6)	-1.3 E-5	(-.3)	1.4 E-8	(.2)	-6.7 E-8	(-.1)	-2.2 E-7	(-1.2)	-4.0 E-8	(-3.6)	1.3 E-8	(.03)	-1.0 E-6	(-.8)	3.0 E-6	(1.0)
θ_t	-.12	(-1.2)	-1.72	(-2.9)	-.36	(-.9)	.07	(1.1)	-.05	(-1.5)	-.01	(-.3)	.22	(3.5)	.07	(.8)	.25	(2.8)
γ_{LL}	.14	(5.1)	.17	(7.3)	.19	(3.4)	.13	(3.9)	.26	(9.4)	.13	(2.0)	-.09	(-2.1)	.04	(1.1)	.10	(5.4)
γ_{gL}	.02	(1.8)	.03	(2.8)	.10	(2.7)	.05	(.4)	-.01	(-.3)	-.10	(-2.0)	.12	(4.1)	.06	(.9)	.29	(2.1)
θ_o	11.97	(.9)	23.38	(2.6)	-15.37	(-.3)	27.59	(1.4)	46.70	(10.0)	31.11	(7.5)	26.37	(7.7)	98.13	(5.8)	18.35	(1.4)
θ_Q	5.74	(1.0)	4.59	(1.4)	11.96	(.7)	-1.64	(-.3)	-.69	(-.7)	-6.68	(-2.1)	-.75	(-.8)	-30.61	(-4.1)	5.84	(1.0)
θ_{QQ}	-1.24	(-1.0)	-.92	(-1.3)	-1.56	(-.7)	.19	(.3)	-.37	(-1.8)	.86	(.5)	.25	(1.2)	6.70	(4.1)	-1.26	(-1.0)
θ_g	1.03	(.9)	-1.72	(-2.9)	6.28	(1.1)	10.89	(.8)	-5.83	(-4.3)	-15.41	(-3.9)	-1.40	(-.7)	-30.54	(-5.6)	12.77	(1.5)
θ_{gg}	-.10	(-.5)	.004	(.02)	-1.24	(-1.3)	6.34	(1.2)	-.63	(-1.7)	7.93	(2.6)	-.41	(-1.0)	8.01	(6.2)	-6.86	(-1.7)
θ_{Qg}	-.19	(-1.0)	.18	(1.8)	-.96	(-1.3)	-.74	(-.4)	1.32	(4.7)	7.53	(4.1)	-.02	(-.1)	6.85	(6.1)	-3.34	(-1.7)
γ_{tL}	-.01	(-22.7)	-.01	(-24.2)	-.01	(-3.4)	-.01	(-5.5)	-.01	(-6.6)	-.01	(-8.3)	.003	(-3.1)	-.01	(-9.5)	-.01	(-2.5)
γ_{tK}	.01	(23.5)	.01	(20.7)	.01	(2.5)	.01	(5.1)	.004	(3.2)	.01	(7.1)	.002	(1.1)	.01	(5.5)	.01	(1.7)
θ_{tQ}	.06	(2.9)	.06	(2.1)	.06	(1.2)	.004	(.3)	.02	(3.0)	.08	(2.6)	.03	(-3.2)	.01	(.7)	-.03	(-1.8)
θ_{ts}	1.4 E-7	(6.0)	2.0 E-5	(.7)	2.9 E-8	(.5)	1.1 E-7	(.5)	3.7 E-7	(2.3)	1.0 E-8	(1.4)	-4.2 E-8	(-3.6)	1.1 E-7	(.1)	-4.3 E-6	(-3.1)
K-Share Equation																		
$\bar{R}^2$	.986		.978		.909		.933		.904		.907		.770		.976		.803	
D.W.	1.16		1.15		.63		.27		1.29		1.20		1.52		1.33		2.08	
L-Share Equation																		
$\bar{R}^2$	.937		.983		.917		.926		.944		.874		.936		.985		.860	
D.W.	1.20		1.20		.61		.26		1.44		1.04		1.30		1.92		2.01	
TCOST Equation																		
$\bar{R}^2$	.976		.927		.970		.883		.993		.887		.964		.952		.888	
D.W.	1.43		.87		1.38		.43		1.80		1.57		1.13		1.00		2.06	

APPENDIX C

Trends in Detailed Mineral Industry Labour Force Characteristics

The productivity calculations performed in the body of this report do not differentiate different levels of labour skill because such information is not published at the industry level by Statistics Canada. To attempt to ascertain the bias introduced into our results by this simplification we have utilized individual mine data collected as the result of an annual survey by the Canadian Mining Journal and published in the CMJ Reference Manual and Buyers Guide. Although publication of the Reference Manual goes back many years, the Labour Force Survey is fairly recent, and as compliance is voluntary, a continuous report by a given mine for a long period of time is relatively rare. The survey was not conducted for the entire 1957-79 sample period employed above and we were able to find only 16 mines that reported for the period 1967-82.

In order to test in an informal way whether labour force composition changed significantly over the period of data availability two ratios were calculated for each mine; the proportion of staff or salaried workers to hourly paid employees, and the number of geologists and engineers relative to staff on the payroll.[1/] These ratios were related to the output of the industry to which the mine was assigned (a proxy for mine output) and a trend variable. The purpose of including output in the regressions is to account for the fact that salaried staff will be a relatively fixed factor compared with hourly rated employees and thus the staff/hourly ratio would be likely to fall in booms and rise in slumps independently of any longer-term trend. In contrast, as geologists and engineers, are relatively less essential than direct supervisory personnel to the daily operation of the mine, their numbers would be expected to rise in boom periods in proportion to total staff.

The regression equations shown in Tables C-1 and C-2 tend fairly broadly to reject the hypothesis of an overall trend in either staff/hourly or geologist/staff ratios from 1967-82. In Table C-1, only for Brunswick Mining and Smelting, Pamour Porcupine and Sigma mines were the coefficients of the trend variable significant at 5% confidence in a 2-tailed test, and for both Porcupine and Sigma the coefficient of the output variable was wrong-signed. A trend of geologists and engineers to total staff is only apparent for Sherritt Gordon Nickel Mines from Table C-2, and the coefficient of the output proxy was wrong-signed in this case as well. It is of course unclear to what degree this 16-mine sample is representative of the industry as a whole. While only 29 mines responded to the survey in 1967, the total was over 90 by 1982 in both underground and open-pit categories. The sample does represent both large and small mines, however, and a fair cross section of products, notwithstanding somewhat of a bias toward uranium and gold-silver mines and away from iron and nickel-copper.

TABLE C-1

Regressions of Mine-Specific Staff-Hourly Worker Ratios
On Output and Trend Variables, 1967-82

Mine	Products	Regression Coefficients of: Output	(t-statistics in parenthesis)	Trend Variable		$\bar{R}^2$	D.W.	S.E.E.
Agnico Eagle (Silver)	A_u	1.4 E-4	(6.8)	-6.1 E-4	(-.5)	.804	1.77	.030
Algoma Steel (ore division)	F_e	8.2 E-5	(1.4)	.004	(1.0)	.033	2.75	.013
Brunswick M. and S. (underground)	P_b, Z_n	-2.7 E-5	(-1.4)	-.005	(-2.6)	.342	1.57	.025
Brunswick M. and S. (open pit)	P_b, Z_n	-.002	(-1.0)	-5.5 E-4	(-.1)	-.129	1.60	.044
Camflo Mines	A_u	.001	(.7)	-.005	(-.9)	.101	2.56	.019
Eldorado Nucleur	U_3, O_8	.004	(2.3)	.002	(.2)	.214	1.60	.094
Giant Yellowknife	A_u, A_g	.004	(1.4)	.003	(.4)	.079	1.96	.040
Heath Steele	P_b, A_n, C_u	-4.0 E-5	(-1.1)	.003	(.2)	-.040	1.95	.041
Kerr Addison	A_u, A_g	-3.0 E-5	(-1.1)	.004	(1.7)	.111	1.98	.018
Mattagami Lake	Z_n, C_u, A_g, A_u	-1.5 E-4	(-2.5)	.003	(.2)	.254	1.60	.055
Noranda Bell	C_u, A_u	-2.4 E-5	(-.6)	-.016	(-1.8)	.227	1.73	.035
Orchan Mines	C_u, Z_n, A_u, A_g	3.4 E-5	(2.3)	-8.3 E-4	(-.5)	.216	1.27	.017
Pamour Porcupine	A_u, A_g	.004	(3.1)	.007	(2.2)	.591	2.63	.011
Rio Algom	U_3, O_8	-1.0 E-5	(-.01)	-.005	(-1.8)		1.08	
Sherritt Gordon	Ni, C_u, C_o	-.015	(-.5)	-.003	(-.9)	-.149	1.72	.074
Sigma Mines	A_u	.003	(4.9)	.006	(4.0)	.079	1.96	.041

TABLE C-2

Regressions of Geologist-Staff Ratios on Output and Trend Variables, 1967-82

Mine	Regression Coefficient of Output		Regression Coefficient of Trend		$\bar{R}^2$	D.W.	S.E.E.
	(t-statistics in parenthesis)						
Agnico Eagle	.001	(.2)	-.004	(-.2)	-.068	1.77	.091
Algoma Steel	3.2 E-4	(2.5)	.001	(.9)	.342	2.46	.027
Brunswick M. and S.	-8.8 E-4	(-.3)	-.007	(-1.3)	-.032	1.88	.102
Brunswick (open pit)	-.015	(-3.8)	-.024	(-1.8)	.580	1.87	.110
Camflo Mines	-1.4 E-5	(-.002)	-.040	(-1.4)	.026	1.93	.093
Eldorado Nucleur	-.006	(-.7)	-.026	(-1.2)	.079	1.69	.136
Giant Yellowknife	.013	(1.9)	.018	(1.0)	.152	1.63	.112
Heath Steele	1.6 E-4	(1.0)	.024	(1.4)	.082	1.86	.230
Kerr Addison	.001	(1.0)	-.004	(-.7)	.038	1.98	.014
Mattagami Lake	.002	(1.1)	.002	(.5)	.106	2.77	.020
Noranda Bell	-7.7 E-8	(-.001)	-2.5 E-5	(-.002)	-.285	1.93	.045
Orchan Mines	.002	(1.3)	-.002	(-.6)	.907	2.84	.026
Pamour Porcupine	-1.0 E-5	(-.3)	-7.9 E-4	(-.1)	-.211	1.82	.045
Rio Algom	2.2 E-4	(.1)	.013	(1.7)	.716	1.54	.046
Sherritt Gordon	-9.1 E-5	(-2.3)	-.016	(-5.0)	.715	1.77	.058
Sigma Mines	-2.5 E-4	(-3.2)	-.009	(-1.0)	.391	1.75	.109

FOOTNOTES

1. The staff designation is given to geologists, engineers, captains, foreman, samplers, safety men (usually only one), and the mine superintendent. Hourly rated employees in the survey were subdivided finely by trade (drillers, timbermen, etc.) within the broad functional headings of stoping, development, and hauling and hoisting workers.

APPENDIX D: Data Definitions and Sources

The main source of data for this study was the Statistics Canada publication, General Review of the Mineral Industries (26-201). This publication contains yearly series on value of production, value added, cost of fuel and electricity and employees and salaries paid. It and its companion publications for the individual mining industries also include ore hoisted and concentrates of specific metals produced, as well as purchased fuel and electricity. Although the data in these publications go back beyond 1957, there was a change in the concept of a mining establishment in 1961 which makes early and later data nonconformable. The data under the new establishment concept was back-dated only to 1957.

These publications do not contain data for real output, capital stocks, the price of capital services, ore grade, or energy, prices. The problem with simply using tonnage of concentrates for output (which in any case is what we want to measure) is that each industry usually produces more than one metal. We employed real domestic product indexes (Statistics Canada, Indexes of Real Domestic Product by Industry of Origin (61-505)) which are base-weighted Laspeyres indexes of concentrate output using prices as weights.

Capital stocks were created from investment data in Fixed Capital Flows and Stocks (13-211) and Private and Public Investment in Canada, Outlook (61-205). The method employed was to calculate gross capital stocks by the perpetual inventory method. This assumes a capital good to be a "one hoss shay" in the sense of remaining with undiminished productivity for a fixed lifetime, then suddenly disappearing. This is the method used by Statistics Canada and most other government agencies. It has obvious deficiencies. Net stocks could not be calculated, however, because this method would require longer data series, and these were not available. We were able to revise Dawson's [1971] gross capital stock estimates for early years and extend them over our sample period. The assumption made of only ten years until depreciation of the stock is a step in the right direction, and incidently embodies Baily's hypothesis of more rapid obsolescence due to energy price changes.

To calculate capital cost shares required a series for the price of capital services. This is theoretically the product of an index of prices of investment goods times a discount rate (gross capital stocks imply sudden-death depreciation) adjusted for the effects of taxes. This was calculated as

$$P_K = r_C P_I C_K \, (1 - \text{taxcr} - r_C {*}\, \text{CCDM})/(1 - r_C)$$

Here P_I is an implicit price index for construction, machinery and equipment for primary metals from Fixed Capital Flows and Stocks, C_K is the imputed cost of capital equipment in the 1961 base year, r_C is the average yield on long-term corporate bonds, taxcr is the investment tax credit, and CCDM is tax depreciation of the capital stock. The latter three variables were obtained courtesy of Jamex Brox from the Statistics Canada quarterly economic forecasting model, which he has running at the University of Waterloo. The formula above is the one employed in that model.

There are no published data on ore grades in Canada through to the 1970's. Ore grade could be measured in several ways, for example on the basis of either reserves or production. Our assumptions imply using production grades because we assume mining firms to produce the best ore first and move down a grade distribution within the ore. Production data in any case are easily accessible and much more reliable than estimates of average grade within the ore. The problem remains of finding a method of properly expressing mineral content in industries such as nickel-copper producing more than one major metal. The need is for a way of weighting the produced concentrates of different metals in a single index. The procedure used by Wedge [1973] was simply to divide the index of real domestic product by an index of Qore, the total tonnage of ore hoisted in the industry. The problem with this, as we have seen, is that the RDP indexes are price-weighted and such a procedure would appear to introduce a bias in the grade index. We have instead followed Pye [1981] in simply summing the total mineral content of nickel-copper concentrates, for example, in effect giving each metal equal weight. Where there is a major difference in the weights and values of the metals, as in nickel-copper mines that produce precious metals as byproducts, we have simply taken the dominant metals only. The grade shown in Table 3.2 for nickel-copper thus reflects the total of nickel and copper concentrates. Of course for single-mineral industries such as gold, iron and asbestos the whole problem disappears.

Because mineral industries use several different types of energy, a similar index problem arose in calculation of energy consumption and energy price indexes. The procedure here was to calculate a Laspeyres quantity index of deisel oil and electricity (the major energy sources) using their cost shares in 1961 as weights. An energy price index was then calculated by dividing the cost of fuel and electricity by this quantity index of energy consumption. Thus energy prices are different in each industry, and being different weighted averages of fuels, can have different growth rates, as shown in Table 3.1.

Finally, admittedly imprecise estimates of employee-years lost to strikes were made using data on strikes by major mining industry firms from Strikes and Lockouts in Canada and assigning these firms to the different mineral industries in our sample on the basis of knowledge of the major metals they produce. This was in fact not so difficult, as a very few firms produce a high proportion of Canadian mineral output.

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