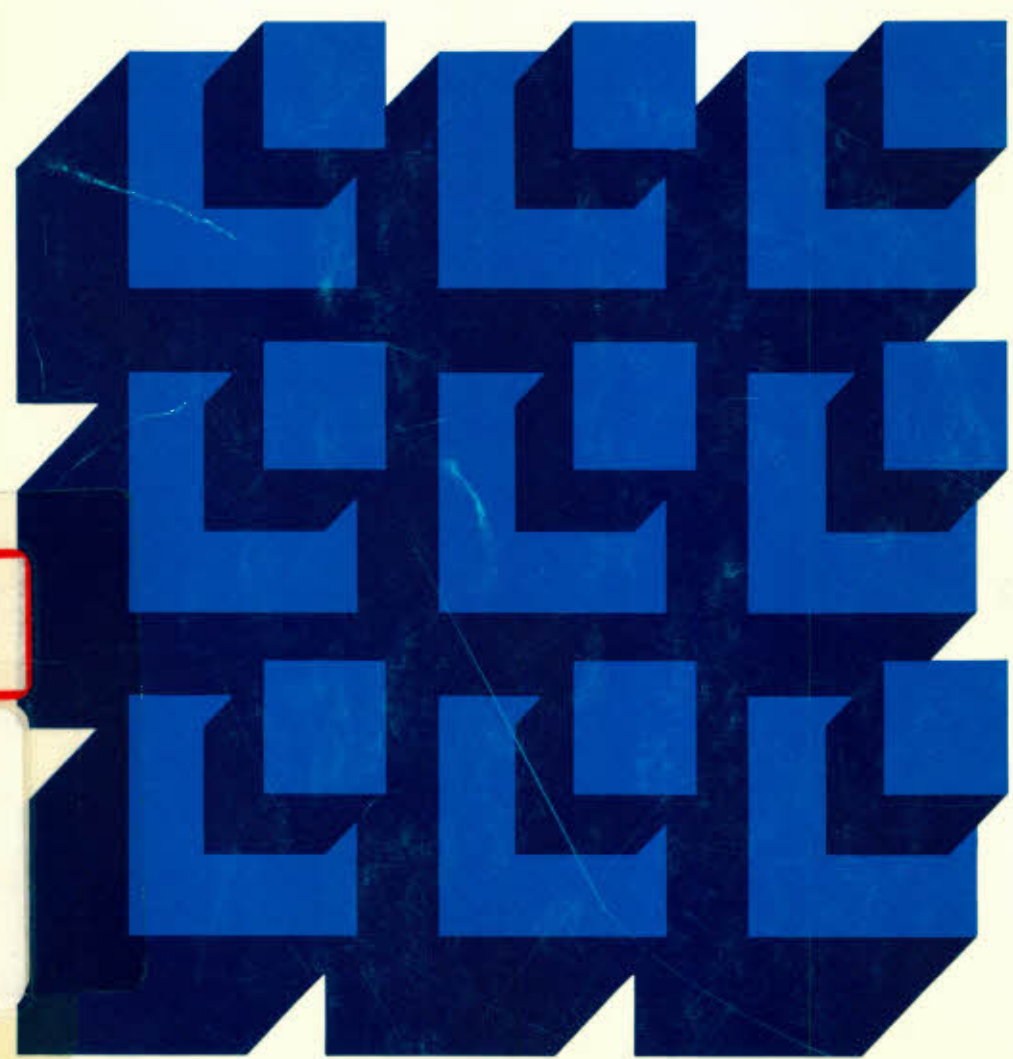




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DISCUSSION PAPER NO. 291

On the Importance of an Endogenous  
Domestic Savings Response for  
Capital Income Taxation in  
Open Economies

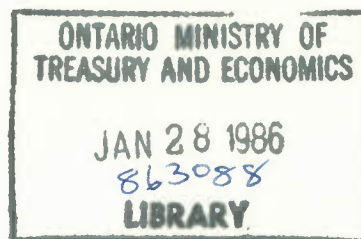
by David F. Burgess

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## RÉSUMÉ

Lorsque l'offre d'épargne intérieure réagit aux variations du taux de rendement après impôts, un impôt sur le revenu du capital s'accompagnera alors d'une perte d'efficacité qu'il faudra comparer aux avantages de réduire le degré de financement externe et son coût. Dans le présent document, l'auteur exprime mathématiquement la deuxième meilleure formulation possible d'un impôt optimal sur le revenu du capital, dans le cas d'une économie qui détient une position de force sur ses marchés d'exportation, mais ne peut en profiter parce qu'elle respecte les principes du libre-échange. Si l'objectif de l'impôt sur le revenu du capital est fixé à un niveau optimal, le rendement national de l'épargne sera le produit marginal intérieur du capital, même si le pays peut emprunter ou prêter, sur le marché mondial, à un taux d'intérêt réel plus faible. Si le revenu du capital peut être imposé à des taux différents pour les résidents et les étrangers, le pays hôte aura intérêt, d'une part, à exempter de l'impôt celui de ses résidents mais, d'autre part, à prélever un impôt positif sur celui des étrangers, s'il dispose sur ses marchés d'exportation d'une position dominante qu'il ne peut exploiter autrement. Enfin, même si le dégrèvement pour impôt étranger compense entièrement l'effet de toute réduction de l'impôt du pays hôte sur le revenu du capital, ce pays aura généralement avantage à fixer son taux d'impôt sur le revenu du capital à un niveau inférieur à celui qui prévaut dans le pays investisseur, lorsque l'offre d'épargne intérieure réagit aux variations du taux de rendement après impôts.

## ABSTRACT

When the supply of domestic savings is responsive to changes in the after-tax rate of return a tax on capital income will involve an efficiency cost that must be weighed against any benefit from reducing the amount of external funding and lowering its cost. This paper derives an expression for the second-best optimum tax on capital income for an economy that has market power in its export sector but is precluded from exercising this power by a commitment to free trade principles. If the capital income tax is set optimally the national return to savings will be the domestic marginal product of capital even though the economy can borrow or lend at a given world real interest rate that is lower than this. If the capital income of domestics and foreigners can be taxed at different rates it will be in the interest of the host country to exempt the capital income of domestics from tax but impose a positive tax on the capital income of foreigners if the host country has market power in its export sector that it cannot otherwise exploit. Finally, even if the foreign tax credit functions to fully offset the impact of any reduction in the host country's tax on capital income, it will generally be in the interest of the host country to set its rate of capital income tax below the rate prevailing in the investing country when the supply of domestic savings is responsive to changes in the after-tax rate of return.

## 1. Introduction

The purpose of this paper is to extend the analysis of capital income taxation in an open economy carried out in Burgess (1985) to cases where the supply of domestic savings is responsive to changes in the after-tax rate of return. In this broader context, a tax on capital income will be a mixed blessing for the host country; while it reduces the economy's reliance on foreign funding (if the price elasticity of demand for exports is less than minus one) and lowers its real cost to the country as a whole, it also creates an efficiency loss by driving a wedge between the marginal rate of time preference of domestic savers and the marginal productivity of capital. Therefore, for a country that is committed to a free trade policy (thereby ruling out the use of tariffs, export taxes, subsidies, quotas, etc.) and also constrained to accord national treatment to foreign investors, there exists a second best optimum tax on capital income where the marginal gain from an improved terms of trade is offset by the marginal loss from creating a wider gap between private and social returns to saving.

Section 2 derives the second best optimum rate of tax on capital income for an economy with unrestricted access to external funding from the international capital market at a given world real interest rate. We find that the second best optimum tax is very sensitive to assumed values of the interest elasticity of supply of domestic savings. For plausible parameter values we are able to show that the second best optimum tax is significantly greater than zero, although it is always less than minus the inverse of the price elasticity of export demand.

Section 3 asks and answers two related questions which have been inspired by the recent debate between Feldstein (1980,1983) and Harberger



(1980) on the functioning and relevance of the international capital market, namely: What is the national return to saving in a small open economy with unrestricted access to external funding from the international capital market at a predetermined world real interest rate when the economy has in place a tax on capital income?; and What proportion of an increase in saving will remain within the economy to finance additional investment there rather than flowing abroad to finance additional investment elsewhere? We show that if the capital income tax is set at its second best optimum value the national return to saving will always be at least as great as the pre-tax marginal product of capital, and for values of the price elasticity of export demand customarily obtained in empirical work most of a wealth induced increase in domestic savings will be invested within the economy rather than flowing abroad.

A commitment to free trade principles may preclude a country from interfering directly in commodity trade via tariffs and export taxes, etc., but it may still be prepared to derogate from the principles of national treatment insofar as the taxation of capital income is concerned. Section 4 derives the second best optimum rate of tax on the capital income earned by foreigners in the host country for various pre-determined rates of tax on the capital income of domestics, assuming that in taxing capital income at different rates the country perceives no risk of retaliatory action by the investing country. It is shown that for a domestic rate of tax on capital income set sufficiently high it will be optimal for the host country to offer a subsidy rather than impose an additional tax on the capital income of foreigners. It is also shown that, provided the compensated interest elasticity of supply of domestic savings is not zero, it will never be optimal

for the host country to neutralize the negative effect on capital formation of a given tax on capital income by offering an equal and offsetting subsidy to foreign investors.

Finally, in Section 5 we examine the costs and benefits of the host country moving in whole or in part towards eliminating its capital income tax when international tax arrangements permit taxes on foreign investment income paid to the host country to be credited against taxes owed in the investing country. It is shown that whether the country stands to gain or lose depends critically upon the workings of the foreign tax credit and upon the responsiveness of domestic savings to changes in the after-tax rate of return. It is also shown that the terms of trade effects that come into play when capital is reallocated as a result of reducing the capital income tax may either strengthen or weaken the case for a consumption tax depending upon the interest responsiveness of domestic saving.

## 2. An Endogenous Domestic Savings Response

Throughout this paper we assume that the private sector strives to maximize a two-period Fisherian intertemporal preference function  $U(C_0, C)$  which takes the additively separable form:

$$U(C_0, C) = U(C_0) + (1+\rho)^{-1} U(C)$$

where  $U'(\cdot) > 0$ ,  $U''(\cdot) < 0$  and where  $\rho \geq 0$  is the pure rate of time preference.<sup>1</sup> Initial resources are given by  $V$  and they must be allocated between current consumption and domestically financed investment  $K$  so that:

$$V = C_0 + K.$$

Following closely the analysis presented in Burgess (1985), future consumption

$C$  is equal to the after-tax earnings from private sector production, less principal plus interest on foreign funding, plus lump-sum transfers. Assuming that the government of the host country imposes a proportional tax at rate  $t$  on future output  $F(\cdot)$  with no deductions for initial resources invested, future consumption can be written in the form:

$$C = (1-t)F(V - C_o + b) - \frac{(1+i^*)}{p} b + a.$$

Since the tax structure,  $t$ , lump-sum transfers,  $a$ , and the foreign real interest rate  $i^*$  are all taken as given by the private sector, and since the foreign exchange price of exports  $p$  is treated as a parameter by the individual agent, the conditions for private sector equilibrium can be derived from the following maximization problem:

$$\underset{\{C_o, b\}}{\text{maximize}} \quad U(C_o, (1-t)F(V - C_o + b) - \frac{(1+i^*)}{p} b + a)$$

Two first-order conditions for an interior maximum are obtained:

$$U_1 - (1-t)F U_K = 0$$

$$(1-t)pF_K - (1+i^*) = 0$$

A third equilibrium condition emerges once it is recognized that the foreign exchange price of exports  $p$  is not exogenously determined but instead must satisfy the balance of payments constraint:

$$p(x)x - (1+i^*)b = 0$$

where  $p'(\cdot) < 0$ , and where  $n = p/xp'$  is assumed to be a constant parameter.

These three equations determine the private sector's optimal choices of  $C_o$ ,  $x$ ,



and  $b$  as functions of  $t$ ,  $a$ , and  $i^*$ , and thus indirectly the private sector's level of well-being.

Now consider the comparative statics effects of a change in the tax rate  $t$  on the private sector's decision variables  $C_o$ ,  $b$ , and  $x$ . The basic matrix equation of change can be written in the form:

$$\begin{bmatrix} U & -(1-t)U F_{2KK} & -(1-t)F U_{K22}/n \\ -(1-t)p F_{KK} & (1-t)p F_{KK} & (1-t)p' F_K \\ 0 & -(1-t)p F_K & p(1 + \frac{1}{n}) \end{bmatrix} \begin{bmatrix} \partial C_o / \partial t \\ \partial b / \partial t \\ \partial x / \partial t \end{bmatrix} = \begin{bmatrix} -F U_{K2} & -(1-t)F F U_{K22} \\ p F_K \\ 0 \end{bmatrix}$$

where  $U = U_{11} + (1-t)^2 F_K U_{22} + (1-t)U_2 F_{KK}$ . This equation can then be solved to yield the following:

$$\partial b / \partial t = (1+1/n)p F_K \{U_{11} + (1-t)^2 U_{22} (F_K - F_{KK})\} / \Delta$$

$$\partial x / \partial t = (1-t)p F_K \{U_{11} + (1-t)^2 U_{22} (F_K - F_{KK})\} / \Delta$$

and

$$\begin{aligned} \partial C_o / \partial t = & -(1-t)^2 p F_K \{F U_{22} [(1-t)p' F_K^2 + p(1 + 1/n)F_{KK}] + \\ & p' F_K^2 U_{22} - p F_K^2 U_{22} / n\} / \Delta \end{aligned}$$

where  $\Delta$  is the determinant of the coefficient matrix which we shall hereafter denote as  $D$ , and where

$$\begin{aligned} \Delta = & [U_{11} + (1-t)^2 F_K U_{22}] [(1-t)p F_K (1 + \frac{1}{n}) + (1-t)^2 p p' F_K^2] \\ & + (1-t)^3 p F_K^2 F_{KK} (p' U_{22} - p U_{22} / n) \end{aligned}$$

Notice that  $\Delta > 0$  is assured whenever  $n < -1$ . In general, however, stability of equilibrium requires that  $\Delta > 0$ .<sup>2</sup> Whereas it follows immediately that  $\partial x/\partial t < 0$  and  $\partial b/\partial t < 0$  if  $n < -1$ , the sign of  $\partial C_0/\partial t$  is not known a priori. The intuitive explanation for this result is familiar. Whenever  $p' < 0$  an increase in  $t$  lowers the after-tax return to saving and therefore raises the price of future consumption measured in units of current consumption foregone. This tends to encourage a switch towards greater current consumption, but the substitution effect may be offset by a negative wealth effect arising from less favourable investment opportunities. Since  $\partial K/\partial t = -\partial C_0/\partial t$ , it follows that domestically financed investment may rise or fall as a consequence of an increase in  $t$  alone.

Any change in the tax rate  $t$  will require a change in lump-sum transfers  $a$  in order to preserve the government's budget constraint. Therefore, consider now the comparative statics effects of a change in lump-sum transfers on the private sector's decision variables. The basic matrix equation of change now takes the form:

$$D \begin{bmatrix} \partial C_0/\partial a \\ \partial b/\partial a \\ \partial x/\partial a \end{bmatrix} = \begin{bmatrix} (1-t)F_{K22} & U_{22} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

from which we obtain:

$$\partial C_0/\partial a = (1-t)^2 p_{K22}^F U_{22} [p_{KK}^F (1 + \frac{1}{n}) + (1-t)p_K'^F] / \Delta$$

$$\partial x/\partial a = (1-t)^3 p_{KK}^F F_{KK}^2 U_{22} / \Delta$$

and

$$\partial b / \partial a = (1-t) p_{KK}^2 F_{KK}^2 U_{22} (1 + \frac{1}{n}) / \Delta$$

Recall that stability of equilibrium requires that  $\Delta > 0$  so that we have  $\partial C_0 / \partial a > 0$ ,  $\partial x / \partial a > 0$ , and  $\partial b / \partial a > 0$ . We conclude that an increase in lump-sum transfers will increase current period consumption (thereby reducing domestic savings), and increase foreign borrowing as well as the amount of future output that must be exported. It is also worth noting that an increase in lump-sum transfers will reduce the sum of domestically financed plus externally financed investment unless  $p' = 0$ . This follows since  $\partial K / \partial a = -\partial C_0 / \partial a$ , and we have:

$$\partial (K+b) / \partial a = -(1-t) p_{KK}^3 F_{KK}^3 U_{22} / \Delta < 0.$$

## 2.1 The Second Best Optimum Capital Income Tax

The aim of the government is to choose  $t$  and  $a$  in such a way that the private sector, taking these parameters as given, succeeds in attaining the highest possible level of well-being. Formally, the government's optimization problem can be written:

$$\begin{aligned} & \underset{\{t, a\}}{\text{maximize}} \quad U(C_0, (1-t)F(V_0 - C_0 + \frac{p(x)x}{1+i^*}) - x + a) \\ & \text{subject to} \quad tF(V_0 - C_0 + \frac{p(x)x}{1+i^*}) - a = 0 \end{aligned}$$

It should be noted that we have substituted the balance of payments constraint directly into the objective function and into the government budget constraint in order to eliminate explicit representation of the amount of foreign borrowing  $b$ .

The Lagrangian function to be maximized with respect to  $t$  and  $a$  then takes the form:

$$L(t, a) = U(C_o, (1-t)F(V - C_o + \frac{p(x)x}{1+i^*}) - x+a) \\ - \lambda [tF(V - C_o + \frac{p(x)x}{1+i^*}) - a]$$

Differentiating this function partially with respect to  $t$  and  $a$  and setting the corresponding expressions equal to zero we have:

$$\partial L / \partial t = U_1 \frac{\partial C_o}{\partial t} + U_2 [(1-t)F \{ - \frac{\partial C_o}{\partial t} + \frac{p(1 + \frac{1}{n})}{1+i^*} \frac{\partial x}{\partial t} \} - \frac{\partial x}{\partial t} - F]$$

$$- \lambda [tF \{ - \frac{\partial C_o}{\partial t} + \frac{p(1 + \frac{1}{n})}{1+i^*} \frac{\partial x}{\partial t} \} + F] = 0$$

$$\partial L / \partial a = U_1 \frac{\partial C_o}{\partial a} + U_2 [(1-t)F \{ - \frac{\partial C_o}{\partial a} + \frac{p(1 + \frac{1}{n})}{1+i^*} \frac{\partial x}{\partial a} \} - \frac{\partial x}{\partial a} + 1]$$

$$- \lambda [tF \{ - \frac{\partial C_o}{\partial a} + \frac{p(1 + \frac{1}{n})}{1+i^*} \frac{\partial x}{\partial a} \} - 1] = 0$$

These expressions can be simplified by making use of the first-order conditions and eliminating  $\lambda$  to obtain a single equation which implicitly defines the second-best optimum tax rate.

$$\left(-\frac{1}{n} \frac{\partial x}{\partial t} - F\right) \left[ tF_K \left\{ -\frac{\partial C_o}{\partial a} + \frac{p(1 + \frac{1}{n})}{1+i^*} \frac{\partial x}{\partial a} \right\} - 1 \right] = \left(-\frac{1}{n} \frac{\partial x}{\partial a} + 1\right)$$

$$\left[ tF_K \left\{ -\frac{\partial C_o}{\partial t} + \frac{p(1 + \frac{1}{n})}{1+i^*} \frac{\partial x}{\partial t} \right\} + F \right]$$

Further simplification is possible by eliminating terms in common on both sides to obtain:

$$\frac{tF_K}{n} \left[ \frac{\partial x}{\partial a} \frac{\partial C_o}{\partial t} - \frac{\partial x}{\partial t} \frac{\partial C_o}{\partial a} \right] = \left[ \frac{t}{1-t} \left( 1 + \frac{1}{n} \right) + \frac{1}{n} \right] \left( \frac{\partial x}{\partial t} + F \frac{\partial x}{\partial a} \right) - tF_K \left( \frac{\partial C_o}{\partial t} + F \frac{\partial C_o}{\partial a} \right) (*)$$

At this point it is helpful to make a specific assumption about the form of the intertemporal preference function. We shall assume that  $U(C_o, C)$  takes the following Cobb-Douglas form:  $\ln C_o + (1 + \rho)^{-1} \ln C$ . It is then possible using various normalizations to write the crucial terms which enter into this expression in the following form:

$$\partial x / \partial a = n / \left\{ \frac{2+\rho}{1+\rho} \left[ n+1+(1-t) \frac{F}{x\beta} \right] - \frac{F}{x} \right\}$$

$$\frac{\partial C_o}{\partial a} = \left\{ \left[ n+1+(1-t) \frac{F}{x\beta} \right] / (1-t) F_K n \right\} \partial x / \partial a$$

$$\frac{\partial x}{\partial t} = \frac{F}{\beta} \left( \frac{2+\rho}{1+\rho} - \beta \right) \partial x / \partial a$$

$$\frac{\partial C_o}{\partial t} = \left\{ -\frac{F}{\beta} \left[ t \frac{F}{x} - \beta(n+1) \right] / (1-t) F_K n \right\} \partial x / \partial a$$

where  $\beta = F_{KK}^2 / F_K$  represents minus the ratio of labour's share to capital's share of GDP divided by the elasticity of substitution between capital and



labour, namely  $-\theta_L/(1-\theta_L)\sigma$ .

Making use of these expressions we are then able to write:

$$\partial x/\partial t + F \partial x/\partial a = [F(2+\rho)/(1+\rho)] \partial x/\partial a$$

$$\frac{\partial C}{\partial t} + F \frac{\partial C}{\partial a} = [F^2/(1-t)F_K n\beta x] \partial x/\partial a$$

$$\left(\frac{\partial C}{\partial t}\right)(\partial x/\partial a) - (\partial x/\partial t)\left(\frac{\partial C}{\partial a}\right) = [-F/(1-t)F_K \beta] \partial x/\partial a$$

Substituting these expressions into the equation (\*) above which implicitly defines the second-best optimum tax rate and solving, we obtain the following explicit solution for  $t_{opt}$ :

$$t_{opt} = \frac{2+\rho}{1+\rho} / \left( \frac{F-x}{x} - \frac{2+\rho}{1+\rho} n \right)$$

According to this expression the second-best optimum tax varies inversely with the absolute value of  $n$ , and  $t_{opt}$  goes to zero as  $n$  approaches minus infinity. Moreover,  $t_{opt}$  varies directly with  $x/F$ , which is a measure of how large are the net export earnings required to repay principal plus interest on foreign borrowing as a proportion of GDP. Finally,  $t_{opt}$  varies directly with  $\rho$ , which is a measure of the degree of impatience of the domestic citizenry.

Table 1 presents implied estimates of the second-best optimum tax rate for various values of  $n$  and assuming three different values for the proportion of net exports to GDP.<sup>3</sup> For simplicity, we assume throughout that  $\rho = 0$  which implies a zero pure rate of time preference by domestics. If we were to assume instead that  $\rho = .1$  (which would seem to be an upper bound for  $\rho$ ) the results would not be altered to any significant degree. In the first column

Table 1

Second-Best Optimum Tax on Capital Income  
for Alternative Values of  $n$  and  $x/F$  when Preferences are Cobb-Douglas

$n \backslash x/F$	1/12	1/9	1/6
- .5	.17	.22	.33
- 1.0	.15	.20	.29
- 2.0	.13	.17	.22
- 3.0	.12	.14	.18
- 5.0	.10	.11	.13
-10.0	.07	.07	.08

net exports constitute one-twelfth of GDP, which would be roughly consistent with foreign funding accounting for 25 percent of total capital formation and capital earnings accounting for one-third of GDP.<sup>4</sup> In the second column net exports constitute one-ninth of GDP, which would imply that foreign funding accounts for one-third of total capital formation. The third column assumes that net exports constitute one-sixth of GDP, so that foreign funding accounts for 50 percent of total capital formation. For example, if the price elasticity of demand for exports were given by  $n = -2.0$  the second-best optimum tax rate would be 13 percent if  $x/F$  is one-twelfth and 22 percent of  $x/F$  is one-sixth.

The entries in Table 1 are substantially below what would be the optimal tax if domestic savings were completely independent of changes in the after-tax rate of return (e.g. in Burgess (1985) it is shown that when the supply of domestic savings is taken to be exogenous the optimum tax rate is 50 percent if  $n = -2.0$ ). Even if the export demand schedule were price inelastic the optimum tax rate would be quite small; in fact, an upper bound for  $t_{opt}$  as  $n$  approaches zero is just 18 percent in the first case, 25 percent in the intermediate case, and 40 percent in the final case. The table therefore serves to emphasize the important role of an endogenous domestic savings response in determining what would be an optimal tax treatment of capital income in an open economy.

## 2.2 Sensitivity Analysis

One reason for the rather modest values for the second-best optimum tax rate as shown in Table 1 is the assumption--heretofore implicit--of a fairly sizeable compensated interest elasticity of supply of domestic savings. The Cobb-Douglas form for the intertemporal preference function implies that the

compensated interest elasticity of supply of savings is equal to the saving rate itself (see e.g., Atkinson and Stiglitz (1980), p. 76). In the two-period analysis of this paper the saving rate will be one-half if the pure rate of time preference is zero and only slightly less if the pure rate of time preference is a small positive value. Cobb-Douglas preferences probably overstate the degree to which intertemporal substitution will occur in response to changes in the rate of return. For this reason, and because econometric estimates of the interest elasticity of supply of savings span a wide range (c.f. Harberger (1974), Boskin (1978), and Summers (1981)), it seems worthwhile to examine the implications of an alternative, more general specification.

The appendix describes the steps involved in the derivation of the second-best optimum tax corresponding to an arbitrary isoelastic intertemporal preference function of the form  $U(C_0, C) = C_0^\gamma + (1 + \rho)^{-1} C^\gamma$ , where  $\gamma-1$  represents the elasticity of the marginal utility of consumption at each date. It can be shown that  $\gamma = (\sigma-1)/\sigma$  where  $\sigma$  is the elasticity of substitution between present and future consumption. Notice that  $\gamma$  can vary from one to minus infinity as  $\sigma$  varies from infinity to zero. The appendix shows that the expression for the second-best optimum tax takes the following form:

$$t_{\text{opt}} = \left\{ \frac{1}{1+\rho} \left( \frac{C}{C_0} \right)^\gamma + 1 \right\} / \left\{ \frac{1}{1-\gamma} \left( \frac{F-x}{x} \right) - \frac{n}{1+\rho} \left( \frac{C}{C_0} \right)^\gamma - n \right\}$$

It should be noted that if  $\gamma = 0$  the formula simplifies to the one obtained earlier for Cobb-Douglas preferences. Negative values for  $\gamma$  will therefore reflect a belief that the degree of intertemporal substitution is weaker than the Cobb-Douglas functional form indicates. Also, as  $\gamma$  approaches minus

infinity the formula shows that  $t_{opt}$  approaches  $-1/n$ , which is the optimum tax on capital income when the compensated interest elasticity of supply of domestic savings approaches zero. Conversely,  $t_{opt}$  approaches zero as  $\gamma$  approaches unity. If the supply of domestic savings is perfectly interest elastic then it is not optimal to tax capital income no matter what the price elasticity of foreign demand for exports happens to be.

Table 2 presents the implied estimates for the second-best optimum tax for three alternative values of the intertemporal elasticity of substitution ( $\sigma = .25, .5, .75$ ), and also for the case where the amount of net exports required to repay principal plus interest on foreign borrowing constitutes one-ninth of future GDP. In arriving at the figures it is also assumed, with little loss in accuracy, that  $\rho = 0$  and that  $C_0 = C$ .

It is apparent from the Table that the degree of intertemporal substitution is a crucial parameter in the determination of the second-best optimum tax; the smaller is  $\sigma$  the larger is  $t_{opt}$  ceteris paribus. For example, if  $n = -2.0$ ,  $x/F = 1/9$ , and  $\sigma = .25$  we are able to rationalize an effective tax on capital income of 33 percent as being consistent with welfare optimizing behaviour on the part of the host country even though it has unlimited access to external funding at a predetermined world real interest rate. To put the matter differently, if the host country were unable or unwilling to impose a heavier tax on the earnings of externally funded capital than on the earnings of capital funded from the savings of its own citizens then it would not be in its interest to reduce the effective rate of tax on capital income below 33 percent when  $n = -2.0$ ,  $x/F = 1/9$ , and  $\sigma = .25$ .



Table 2

Second-Best Optimum Tax on Capital Income  
for Alternative Values of  $n$  and  $\sigma$  and for  $X/F$  equal to  $1/9$ .

$n \backslash \sigma$		.25	.5	.75
- .5		.67	.40	.29
- 1.0		.50	.33	.25
- 2.0		.33	.25	.20
- 3.0		.25	.20	.17
- 5.0		.17	.14	.13
-10.0		.09	.08	.07

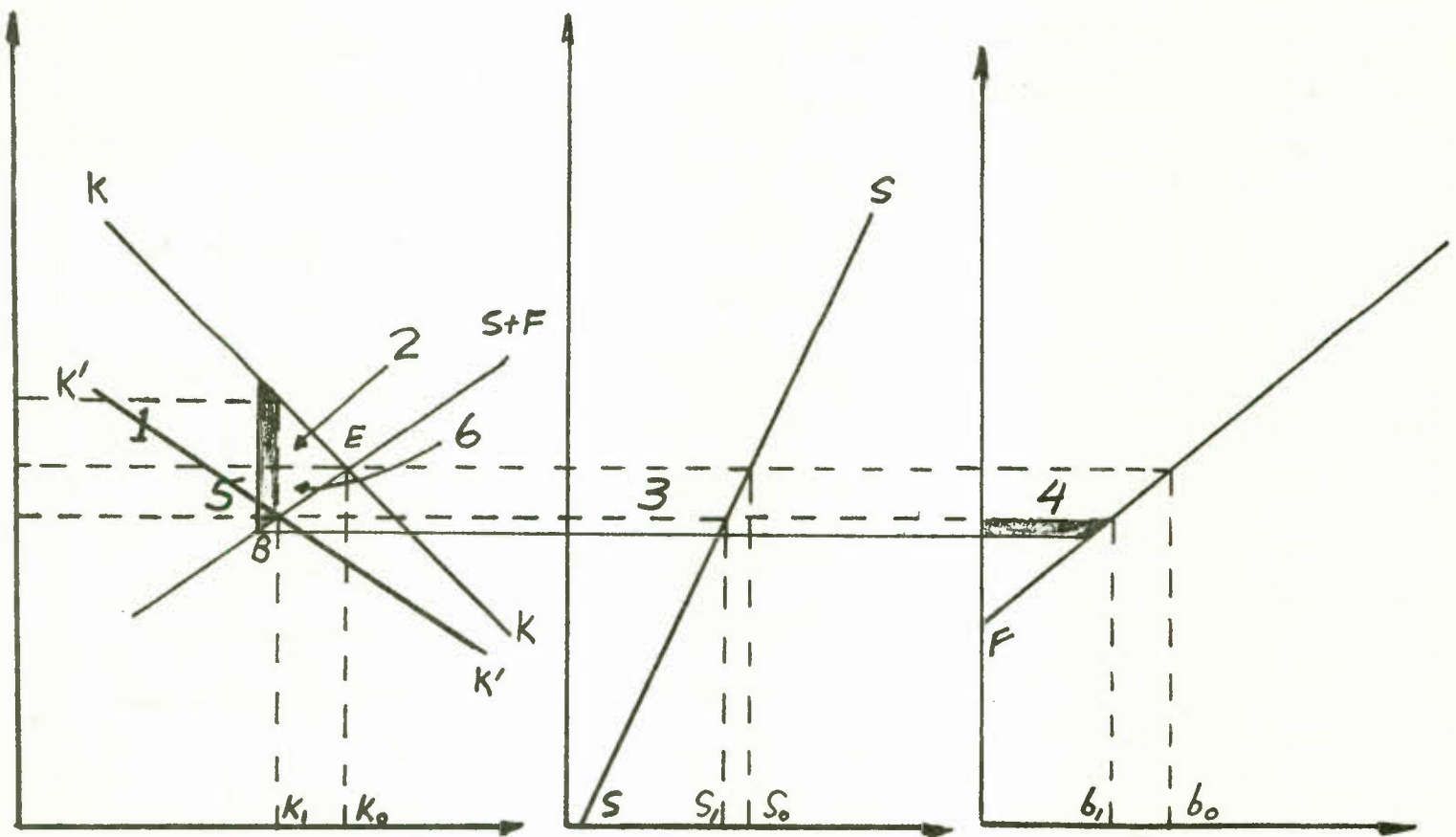
### 2.3 Diagrammatic Treatment

The determinants of the second best optimum tax can be conveniently illustrated using Figure 1. The tax-free competitive equilibrium occurs at E in the left panel where the demand for capital is  $K_0$  and the supplies of domestic and foreign savings (represented by  $S_0$  and  $b_0$  in the middle and right panels respectively) are sufficient to finance this demand. The imposition of the tax shifts the demand for capital schedule downward by the percentage of the tax to  $K'K'$  thereby creating an excess supply of funding at the initial domestic real interest rate. Capital market equilibrium is then restored through a reduction in the domestic real interest rate which partially restores the demand for capital to  $K_1$  while reducing the supplies of domestic and foreign savings to  $S_1$  and  $b_1$  respectively.

Domestic suppliers of complementary factors (labour and land) are made worse off by area 1 plus 2 in the left panel which represents the reduction in their pre-tax earnings. Domestic suppliers of capital are made worse off by area 3 in the middle panel. Foreign suppliers of capital are made worse off by area 4 in the right panel, but this is irrelevant to a national welfare calculation. Finally, the increase in tax revenue which we assume can be lump-sum redistributed to the domestic citizenry is given by areas 1 plus 5. Therefore, the domestic welfare gain from the imposition of the tax is given by  $1+5-(1+2+3) = 5-2-3$ . But since  $5=3+4-6$  the increase in domestic welfare is  $4-(2+6)$ . Area  $2+6$  can be thought of as the efficiency cost of the capital income tax since it represents the cost to the economy of failing to exploit all domestic investment opportunities yielding a return equal to or greater than the market interest rate. Area 4 can be thought of as the amount of real income transferred from foreigners to domestics through the imposition of the

Figure 1

Determinants of the Second Best Optimum Capital Income Tax



capital income tax. In the model of this paper this real income transfer occurs by having foreigners pay a higher real price for exports rather than by having foreign savers receive a lower real after-tax rate of return on their investments in the host country.

It is clear that whenever the supply of foreign savings schedule is upward sloping it will be in the interest of the host country to impose a positive tax on capital income provided that other forms of intervention (e.g., taxes or tariffs or commodity trade, and withholding taxes on foreign repatriations) are unavailable. The upward slope to the foreign savings supply schedule occurs not because foreign savers demand a higher real after-tax rate of return the more funding they supply to the host country, but rather because foreigners are willing to pay less and less for the host country's exports as the volume of its exports increases. For small values of  $t$  the triangular area 2+6 will be small relative to the rectangular area 4. The capital income tax will be optimal when any small increase in the tax expands the efficiency cost triangle 2+6 by the same amount as it expands the foreign real income transfer rectangle 4. Thus, in Figure 1 the tax is set optimally because the two shaded areas are equal. At this point the extent to which area 4 exceeds 2+6 will be at its maximum.

### 3. Domestic Saving, Capital Formation, and the National Return to Saving

Recent discussions about capital income taxation in open economies (c.f. Feldstein (1980, 83) and Harberger (1980)) have focussed on the extent to which an increase in domestic savings will spill over into the international capital market rather than fund additional domestic investment. It has been argued that if capital markets are perfect and the economy is small then any increase in domestic savings will flow abroad leaving domestic capital

formation unchanged. We can shed further light on this issue here by considering whether and to what extent a wealth-induced increase in the supply of domestic savings will result in an increase in the total supply of capital employed in the domestic economy.<sup>5</sup>

If we totally differentiate the conditions for private sector equilibrium and the balance of payments constraint with respect to a small change in initial wealth and solve the resulting system we obtain the following expressions for the implied changes in current consumption, foreign funding and exports:

$$\frac{\partial C_o}{\partial V} = (1-t) p_{K 22}^3 \frac{F U}{K 22} \{ (\beta/F)(1+(F-x)/x(\gamma-1)n) + (1-t)/nx \} / \Delta$$

$$\frac{\partial b}{\partial V} = -(1-t) p_{KK 11}^2 \frac{F U}{K K 11} (1+1/n) / \Delta$$

$$\frac{\partial x}{\partial V} = -(1-t) p_{KK 11}^2 \frac{F F U}{K K K 11} / \Delta$$

Recall that  $\beta = -\theta_L / (1-\theta_L) \sigma < 0$ ,  $\gamma < -1$ , and  $n < 0$ . Not surprisingly,

then, it follows that an increase in initial wealth must increase current consumption. However, the level of foreign funding may fall, remain unchanged, or increase depending upon whether  $n \gtrless -1$ . No matter what happens to the level of foreign funding, the volume of exports will necessarily decline with an increase in  $V$ .

The amount of any increase in initial wealth that is not consumed must add to the supply of domestic savings so that  $\partial S / \partial V = 1 - \partial C_o / \partial V$ . Moreover, given that our model abstracts from risk and uncertainty and therefore ignores the possibility of cross-hauling in capital assets, and given that the economy is assumed to be a net debtor, any increase in domestic savings is used to



acquire claims on capital employed at home rather than abroad so that  $\partial S/\partial V = \partial K/\partial V$ . After some manipulation we obtain the following expression for the wealth-induced change in domestic savings:

$$\partial K/\partial V = (1-t) p_{K1}^2 \frac{F U}{K} \{ (\beta/F)(n+1) + (1-t)/x \} / n \Delta$$

which is necessarily positive for all values of  $n$ .

The change in the total supply of capital employed in the domestic economy is equal to the sum of the changes in the supplies of domestic and foreign savings which is given by:

$$\partial(K+b)/\partial V = (1-t) p_{K1}^2 \frac{F U}{K} / n x \Delta$$

We noted above that an increase in domestic savings will normally cause the supply of foreign funding to fall, but that foreign funding would actually increase if  $n > -1$ . The expression for  $\partial(K+b)/\partial V$  indicates that in the limit as  $n$  approaches minus infinity every dollar increase in the supply of domestic savings will displace an equivalent amount of foreign savings leaving total domestic capital formation unchanged.

It may be of interest to know more precisely what factors determine the proportion of any increase in domestic savings that adds to the total supply of capital employed in the domestic economy rather than simply financing the repatriation of the existing supply of capital. We obtain the following expression for the ratio of the increase in the domestic capital stock to the increase in domestic savings:

$$\frac{\partial(K+b)/\partial V}{\partial K/\partial V} = 1 / \left\{ \frac{\beta(n+1)(x/F)}{1-t} + 1 \right\}$$

It is apparent from this expression that the proportion of incremental domestic savings that adds to domestic capital formation varies inversely with the absolute size of  $n$ . The proportion can vary from zero to unity to a number even greater than unity as  $n$  varies from minus infinity to minus one to zero. As well, for values of  $n$  less than  $-1.0$  the proportion varies inversely with the absolute size of  $\beta$ , and with the magnitudes of  $x/F$  and  $t$ .

Suppose the tax on capital income is set optimally, i.e., such that it enables the private sector to achieve the highest level of well-being subject to the constraint that domestic and foreign savings are taxed at the same rate. We can then substitute the expression for  $t_{opt}$  derived earlier into the expression above and rearrange terms to obtain an alternative expression for the response in domestic capital formation per dollar of domestic savings. For simplicity we focus on the special case where  $\gamma = \rho = 0$  so that

$$t_{opt} = 1/\left\{-\frac{1}{2}\left(\frac{F-x}{x}\right) - n\right\}. \text{ It then follows that}$$

$$\frac{\partial(K+b)/\partial V}{\partial K/\partial V} = \frac{(F-x)/x-2(n+1)}{\beta(n+1)(x-F)((F-x)/x-n) + (F-x)/x-2(n+1)}$$

Table 3 gives the implied estimates of the fraction of incremental domestic savings that adds to domestic capital formation for alternative values of  $n$  and  $x/F$ . To maintain consistency with earlier work we have assumed throughout that  $\sigma = 1.0$  and  $\theta_L = .75$ , which implies that  $\beta = -3.0$ . Thus, suppose that net exports account for one-ninth of GDP and that the price elasticity of export demand is  $-2.0$ . Then provided that the capital income tax is set optimally (which according to Table 1 is 17 percent) 71 percent of any increase in domestic savings will be used to finance additional domestic investment rather than spilling over into the international capital market to

Table 3

The Proportion of Incremental Domestic Savings that  
 Adds to Domestic Capital Formation for  
 Alternative Values of  $n$  and  $x/F$  when the  
 Capital Income Tax is Set at its Second-Best Optimum Value

n \ x/F			
	1/12	1/9	1/6
.5	1.18	1.27	1.60
-1.0	1.0	1.0	1.0
-2.0	.78	.71	.61
-3.0	.65	.56	.45
-5.0	.48	.40	.30
-10.0	.29	.20	.17

finance investment abroad. On the other hand, if the capital income tax were zero the proportion would rise to 75 percent whereas if  $t = 0.5$  the proportion would fall to 60 percent.

The inverse relationship between  $x/F$  and the fraction of incremental domestic savings that adds to domestic capital formation is a feature of Table 3 that warrants further comment. One tends to think of small countries as having high ratios of net indebtedness to GDP, and therefore high values of  $x/F$ . For such countries a large proportion of any increase in indigenous savings will be absorbed in the repatriation of foreign owned claims against the existing capital stock, thereby causing a large absolute reduction in the level of foreign funding for any given value of  $n$ . The table emphasizes this fact, since if the  $x/F$  ratio is reduced from one-ninth to one-twelfth with  $n$  maintained at  $-2.0$  then the fraction of incremental domestic savings that adds to domestic capital formation rises from 71 percent to 78 percent (assuming that the capital income tax is adjusted to its new second-best optimum level of 13 percent (see Table 1)).

Previous authors (e.g., Feldstein and Horioka (1980) and Feldstein (1983)) have inferred that an economy cannot be small in the international capital market unless any increase in its saving spills over completely into the international capital market leaving its capital formation unchanged. Our analysis shows that an economy can have unrestricted access to external funding at a predetermined world real interest rate and still find that a change in its savings rate has a direct impact on its rate of capital formation. Evidence adduced by Feldstein (1983) that increases in domestic savings are largely allocated to domestic capital formation within individual OECD countries is still consistent with the existence of a perfectly

functioning international capital market to which each individual OECD country has full and unrestricted access. The apparent fact that an increase in the savings of one OECD country tends to flow largely into capital formation within that country may indicate that the tradeables produced by each individual OECD country are less than perfect substitutes for the tradeables produced by other OECD countries. If each OECD country faces a less than perfectly elastic external demand for its exports each is forced to suffer a deterioration in its terms of trade in order to expand its export volume.

Before concluding this section it is useful to ask what is the national return on incremental domestic savings, or in other words by how much will future domestic consumption increase as a result of a small increase in domestic savings. Future consumption is just total future output or GDP less the amount that must be exported to repay principal plus interest on external funding. Therefore, the impact on future consumption of a small increase in initial wealth is given by:

$$\frac{\partial C}{\partial V} = F \left( 1 - \frac{\partial C}{\partial V} + \frac{\partial b}{\partial V} \right) - \frac{\partial x}{\partial V}$$

and we can define the national return to saving (hereafter,  $R_{soc}$ ) as just the ratio of  $\partial C/\partial V$  to  $\partial K/\partial V$  which takes the form:

$$R_{soc} = \frac{\partial C/\partial V}{\partial K/\partial V} = \frac{F \frac{\partial (K+b)}{\partial V} - \frac{\partial x}{\partial V}}{\partial K/\partial V}$$

Substituting the expressions derived earlier and simplifying, we obtain

$$R_{soc} = (1-t) \left\{ \frac{1+\beta(x/F)n}{1-t+\beta(x/F)(n+1)} \right\} \frac{F}{K}$$



It is apparent from this expression that if the government sets the tax on capital income equal to minus the inverse of the price elasticity of demand for exports (i.e.,  $t = -1/n$ ) then the national return to saving will be the pre-tax marginal product of capital  $F_K$ . In general, for arbitrary values of  $t$  the national return to saving may be greater than or less than the pre-tax marginal product of capital.<sup>6</sup>

Suppose that the capital income tax is set at its second best optimum value. For simplicity we again focus on the special case where  $\gamma = \rho = 0$  so that  $t_{opt} = 1/\{x(F-x)/x-n\}$ . If we then substitute  $t_{opt}$  for  $t$  in the expression for  $R_{soc}$  derived above and rearrange terms we obtain the following expression for the ratio of  $R_{soc}$  to  $F_K$ :

$$\frac{R_{soc}}{F_K} = (1+\beta \frac{x}{F} - n) / \{1+\beta \frac{x}{F} (n+1) (\frac{F-x}{x} - 2n) / [\frac{F-x}{x} - 2(n+1)]\}$$

It is apparent from this expression that the national return to savings will always exceed the pre-tax marginal product of capital for all finite values of  $n$  and that  $R_{soc}$  will approach  $F_K$  only as the limit as  $n$  approaches minus infinity.

Table 4 presents the ratio of the national return on savings to the pre-tax marginal product of capital in the domestic economy for various pairings of  $t$  and  $n$  as well as for the case where  $t$  is set equal to  $t_{opt}$ . The table is based upon the intermediate case discussed earlier where  $x/F$  is equal to  $1/9$  and where  $\beta = -3.0$ . For example, if the elasticity of demand for exports is  $-2.0$  and the tax on capital income is set at 50 percent then the national return on savings will coincide with the pre-tax marginal product of capital, but if the tax rate is set at 33 percent then the national return

Table 4

The Ratio of the National Return on Savings to the Pre-tax  
 Marginal Product of Capital for Alternative  
 Values of  $n$  and  $t$

$n \backslash t$	.5	.33	.20	$t_{opt}$
-.5	1.75	1.56	1.47	1.48
-1.0	1.33	1.33	1.33	1.33
-2.0	1.00	1.11	1.18	1.19
-3.0	.86	1.00	1.09	1.13
-5.0	.73	.89	1.00	1.07
-10.0	.62	.79	.91	1.02

will actually exceed the pre-tax marginal product by eleven percent. However, if the capital income tax is set at its second best optimum value in this situation (which according to Table 1 is just 17 percent) then the national return to savings will exceed the pre-tax marginal product of capital by 18 percent. The table serves to emphasize the basic point that the national return to saving can be substantially above the prevailing world real interest rate even though the economy has unrestricted access to external funding at the prevailing world real interest rate. To put the matter differently, it would be quite inappropriate to use the prevailing world real interest rate as a measure of the social opportunity cost of capital for an economy that can borrow and lend freely at the world real interest rate because it neglects the terms of trade costs that must be borne by the economy in expanding its exports to repay its international debt. If the capital income tax is set at its second best optimum value the national return to savings will always be at least as great as the pre-tax marginal product of capital.<sup>7</sup>

#### 4. The Second Best Optimum Tax on Foreign Investment Income

Suppose that the host country has complete freedom to set its tax structure in the sense that it is no longer constrained to tax capital income at the same rate whether it accrues to nationals or to foreigners. If lump-sum taxation were feasible, then the first best policy would be to tax the earnings of capital financed by domestic savings at a zero rate and tax the earnings of capital financed by foreign savings at rate  $t = -1/n$ . This assumes, as before, that the host country is precluded from imposing taxes or tariffs on commodity trade. Alternatively and equivalently, the host country could set  $t$  equal to zero and impose a withholding tax on all capital repatriations at rate  $\tau = -1/(n+1)$ . However, suppose that the tax on the capital

income of nationals has been set at a positive rate  $t$  that for some reason cannot be changed. How does the existence of such a tax affect the host country's decision about an optimal rate of tax on the capital income of foreigners? We address ourselves to this issue in this section. For simplicity we assume that the tax rate  $t$  applies to all income from capital and consider whether to impose an additional with-holding tax on capital income accruing to foreigners.

The expression for future consumption must now be re-written to take into account both the existence of a tax at rate  $t$  on future period output and the existence of a withholding tax at rate  $\tau$  on all capital repatriations:

$$C = (1-t) F(K+b) - (1+\tau)(1+i^*)b/p + a$$

The tax base for the withholding tax is assumed to be the value of all capital repatriations--principal plus interest payments to foreigners.<sup>8</sup> It should be stressed that nothing precludes  $\tau$  from being negative; the host country could offer a subsidy to all capital repatriations rather than imposing a tax on them. We shall see that this is in fact the appropriate policy if the rate of capital income tax  $t$  is set high enough and if the elasticity of demand for exports is sufficiently large.

According to the balance of payments constraint, principal plus interest payments to foreigners in the second period must equal the value of (net) exports:

$$p(x)x - (1+i^*)b = 0$$

Therefore, by imposing a withholding tax at rate  $\tau$  the host government

receives tax revenue equal to  $\tau(1+i^*)b/p$  which is necessarily identical to  $\tau x$ . The withholding tax serves the same purpose as an across-the-board export tax. The government's budget constraint can then be written in the form:

$$tF(K+b) + \tau x - a = 0$$

which indicates that if  $t$  is to be held fixed for political or other unspecified non-economic reasons then any change in  $\tau$  must be offset by an appropriate change in lump sum transfers  $a$ .

As before, the private sector strives to maximize an intertemporal preference function:

$$U(C_0, (1-t)F(K+b) - (1+\tau)(1+i^*)b/p + a)$$

subject to the constraint that initial wealth must be either consumed or invested:

$$V = C_0 + K.$$

The private sector takes  $t$ ,  $\tau$ ,  $a$ ,  $i^*$ , and  $p$  as exogenous, and optimizes with respect to  $C_0$  and  $b$ . First order conditions for an interior maximum are then given by:

$$U_1 - (1-t)F U_K = 0$$

$$(1-t)pF_K - (1+\tau)(1+i^*) = 0$$

These two conditions are combined with the balance of payments constraint to give a system of 3 equations which determine 3 endogenous variables  $C_0$ ,  $b$ ,  $x$  as functions of the policy parameters  $\tau$  and  $a$ .



Totally differentiating these three equations with respect to a small change in  $\tau$  (holding  $t$ ,  $a$ , and  $i^*$  fixed, and recognizing that  $p$  will generally vary with  $x$ ), we obtain the following matrix system:

$$\begin{bmatrix} U & -(1-t)F_{KK} U & -(1-t)(1+\tau)F_{KK} U /n \\ -(1-t)pF_{KK} & (1-t)pF_{KK} & (1-t)p'F_K \\ 0 & -(1+i^*) & p(1+1/n) \end{bmatrix} \begin{bmatrix} \partial C / \partial \tau \\ \partial b / \partial \tau \\ \partial x / \partial \tau \end{bmatrix} = \begin{bmatrix} -(1-t)F_{KK} U x \\ 1+i^* \\ 0 \end{bmatrix}$$

which can be solved to yield:

$$\begin{aligned} \partial C / \partial \tau = & \{ -(1-t)F_{KK} U x [(1-t)p^2 F_{KK} (1+1/n) + (1-t)p'F_K (1+i^*)] \\ & + (1+i^*)[(1-t)pF_{KK} U (1+1/n) + (1-t)(1+\tau)(1+i^*)F_{KK} U /n] \} / \Delta' \end{aligned}$$

$$\partial x / \partial \tau = (1+i^*)[(1+i^*)U - (1-t)p^2 F_{KK} F_{KK} U x] / \Delta'$$

$$\text{where } \Delta' = U[(1-t)p^2 F_{KK} (1+1/n) + (1+i^*)(1-t)p'F_K] - (1-t)pF_{KK}$$

$$[(1-t)pF_{KK} U (1+1/n) + (1+i^*)(1-t)(1+\tau)F_{KK} U /n]$$

and where stability of equilibrium ensures that  $\Delta' > 0$ .

Totally differentiating the 3 equilibrium conditions with respect to a small change in lump-sum transfers  $a$  holding  $\tau$ ,  $t$ , and  $i^*$  fixed, and letting the basic matrix equation of change from above be denoted by  $D'$ , we obtain:

$$D' \begin{bmatrix} \partial C / \partial a \\ \partial b / \partial a \\ \partial x / \partial a \end{bmatrix} = \begin{bmatrix} (1-t)F_{KK} U \\ 0 \\ 0 \end{bmatrix}$$

from which it follows that:

$$\frac{\partial C_o}{\partial a} = (1-t)F_{KK} U_{22} [(1-t)p F_{KK}^2 (1+1/n) + (1-t)p' F_K (1+i^*)]/\Delta'$$

$$\frac{\partial x}{\partial a} = (1-t) p F_{KK} F_K U_{22} (1+i^*)/\Delta'$$

The above comparative statics effects can be rewritten in a more compact way by making use of the following normalizations familiar from earlier work:

$F_{KK} = \beta F_K^2/F$ ;  $p' = p/nx$ ;  $U_{11} = (\gamma-1)U_1/C_o$ ;  $U_{22} = (\gamma-1)U_2/C$ ;  $U = (1-t)F_K^2 U_{22} [1-t + C/C_o F_K + (\beta/F)C/(\gamma-1)]$ . We can then rewrite the various comparative statics effects as functions of  $\partial x/\partial a$  as follows:

$$\frac{\partial C_o}{\partial \tau} = -[(n+1)/(1-t)F_K n] [(1+\tau)x - C/(\gamma-1)] \partial x/\partial a$$

$$\frac{\partial x}{\partial \tau} = (F/\beta)\{C/C_o F_K (1+\tau) + (1-t)/(1+\tau) + (\beta/F)[C/(\gamma-1)(1+\tau) - x]\} \partial x/\partial a$$

$$\frac{\partial C_o}{\partial a} = (F/\beta(1-t)F_K n) [(\beta/F)(n+1)(1+\tau) + (1-t)/x] \partial x/\partial a$$

The foregoing comparative statics effects indicate how the private sector responds to arbitrary changes in  $\tau$  and  $a$ . The aim of the government is to choose  $\tau$  and  $a$  optimally given a pre-determined non-zero value for  $t$ . Specifically, the government will want to choose  $\tau$  and  $a$  in such a way as to achieve an interior maximum of the following Lagrangian:

$$L(\tau, a) = U(C_o, (1-t)F(V - C_o + \frac{p(x)x}{1+i^*}) - (1+\tau)x + a)$$

$$-\lambda[tF(V - C_o + \frac{p(x)x}{1+i^*}) + \tau x - a]$$

where the balance of payments constraint has been used to eliminate any explicit reference to the level of foreign funding  $b$ .

Differentiating  $L(\cdot)$  partially with respect to small changes in  $\tau$  and  $a$ , and noting that changes in  $\tau$  and  $a$  imply changes in the private sector's decision variables  $C_0$ ,  $b$ , and  $x$ , we obtain the following two first order conditions for a second best optimum:

$$\partial L / \partial \tau = U_1 \frac{\partial C_0}{\partial \tau} + U_2 [(1-t)F_K (-\frac{\partial C_0}{\partial \tau} + p \frac{(1+1/n)}{1+i^*}) - (1+\tau) \frac{\partial x}{\partial \tau} - x]$$

$$- \lambda [tF_K (-\frac{\partial C_0}{\partial \tau} + p \frac{(1+1/n)}{1+i^*} \frac{\partial x}{\partial \tau}) + \tau \frac{\partial x}{\partial \tau} + x] = 0$$

$$\partial L / \partial a = U_1 \frac{\partial C_0}{\partial a} + U_2 [(1-t)F_K (-\frac{\partial C_0}{\partial a} + p \frac{(1+1/n)}{1+i^*} \frac{\partial x}{\partial a}) - (1+\tau) \frac{\partial x}{\partial a} + 1]$$

$$- \lambda [tF_K (-\frac{\partial C_0}{\partial a} + p \frac{(1+1/n)}{1+i^*} \frac{\partial x}{\partial a}) + \tau \frac{\partial x}{\partial a} - 1] = 0$$

These two equations can be reduced to a single equation by eliminating  $\lambda$ . Making use of the first order conditions we then arrive at the following equation which implicitly determines the second best optimum withholding tax  $\tau$  on all capital repatriations flowing abroad:

$$\begin{aligned} & tF_K [\frac{\partial C_0}{\partial a} \frac{\partial x}{\partial \tau} - \frac{\partial C_0}{\partial \tau} \frac{\partial x}{\partial a}] - \frac{ntF_K}{1+\tau} [\frac{\partial C_0}{\partial \tau} + x \frac{\partial C_0}{\partial a}] \\ & + [\frac{1+tn}{1-t} + \frac{\tau n}{1+\tau}] [\frac{\partial x}{\partial \tau} + x \frac{\partial x}{\partial a}] = 0 \end{aligned}$$

If we then substitute into this expression the comparative statics effects obtained earlier and solve for the second best optimum withholding tax we

obtain the following expression for  $\tau_{opt}$ :

$$\tau_{opt} : -\frac{1}{n+1} \left(1 + \frac{tn}{\alpha}\right),$$

$$\text{where } \alpha = \left(\frac{F-x}{F}\right) \left(\frac{1-s}{\gamma-1}\right) \beta + 1$$

and where  $s = C/(C_0(1-t)F_K + C)$  represents the domestic saving rate, which is just the ratio of expenditure on future consumption to total current plus future consumption (see Appendix).

Several points are worth emphasizing about this expression. First, observe that if  $t = 0$  then  $\tau_{opt} = -1/(n+1)$  implying that even if the host country were to exempt all capital income from tax it would still be in its interest to impose a withholding tax on repatriated earnings whenever the demand for its exports is less than perfectly elastic.<sup>9</sup> Notice that if  $t=0$  the optimum withholding tax will be independent of  $\alpha$ , so that the responsiveness of domestic savings to the rate of return will be irrelevant in this case. The reason for this result is that when  $t$  is zero it makes no difference whether the withholding tax causes less domestic investment or more domestic savings since their social opportunity costs are equal.

Second, observe that as  $n$  approaches minus infinity the optimum withholding tax  $\tau_{opt}$  approaches  $-t/\alpha$ . Moreover, since  $\beta \leq 0$  and  $\gamma \leq 1$  it is necessarily the case that  $\alpha \geq 1$  so the optimum withholding tax is actually a subsidy. Thus, if the host country were small enough to approximate the traditional small open economy (in the sense of being both a price taker for its exports and an interest rate taker in the world capital market) and if it had in place a tax on capital income that could not be removed, then it would be in its interest to subsidize capital repatriations. However, note that the



optimal rate of subsidy on capital repatriations would be less than the rate at which capital income was being taxed so as not to offset completely the adverse effect of the capital income tax on domestic capital formation. The reason for this is that when the domestic real interest rate is determined internationally, any attempt to subsidize the return on foreign savings will reduce the cost of capital and thereby lower the return on domestic savings. Discouraging domestic savings that is available at a social opportunity cost below the world real interest rate entails a domestic welfare loss that must be compared to any welfare gain from encouraging additional domestic investment that has a social marginal productivity in excess of the world real interest rate.

Two final observations are worth making with regard to the determinants of  $\tau_{opt}$ . The first is that for  $n$  finite and such that  $n < -1$ ,  $\tau_{opt} > 0$  as  $tn + \alpha > 0$ . Therefore  $\tau_{opt}$  can be larger or smaller than  $t$  itself, and  $\tau_{opt}$  will be zero only if  $n = -\alpha/t$ . The second point is that for given values of  $t$  and  $n$ ,  $\tau_{opt}$  will vary directly with  $\alpha$ , which, in turn, varies directly with the elasticity of marginal utility parameter  $\gamma$  but inversely with the saving rate  $s$  and the size of the foreign sector as measured by  $x/F$ . In fact, as  $\gamma$  approaches unity  $\alpha$  becomes indefinitely large and  $\tau_{opt}$  approaches  $-1/(n+1)$ . Thus, if the interest elasticity of supply of domestic savings were infinite the optimum withholding tax would depend only on the elasticity of demand for the host country's exports and would be independent of the domestic tax rate on capital income. With a perfectly elastic supply of domestic savings the withholding tax captures inframarginal rents from foreign savers without itself causing any reduction in total domestic investment. On the other hand, as  $\gamma$  approaches minus infinity  $\alpha$  approaches



one and  $\tau_{opt}$  approaches  $-(1+nt)/(n+1)$ . Thus, if the interest elasticity of supply of domestic savings were negligible so that the domestic savings schedule were vertical then the second best optimum withholding tax would be positive or negative depending upon whether  $nt + 1 \begin{matrix} > \\ < \end{matrix} 0$ .

It may seem somewhat paradoxical that  $\tau_{opt} = -1/(n+1)$  both when  $t=0$  and the domestic savings elasticity is of any value and when  $t > 0$  but the domestic savings elasticity is infinite. To be sure, the effect of the tax on domestic capital formation will differ in the two cases, being positive in the first case and zero in the second. But this does not imply that the optimum tax rates should differ in the two cases. Thus, in the case where  $t=0$  it does not matter whether and to what extent the tax on foreign savings will discourage domestic investment rather than encourage domestic savings since they have the same social opportunity costs. The essential point is that the optimal tax on foreign savings must correct any discrepancy between the private and social costs of this source of funding; the tax itself will have no secondary effects on other markets since they are distortion-free by assumption. In the case where  $t > 0$  but where domestic savings is perfectly interest elastic the marginal social benefit of foreign funding is no longer the domestic marginal productivity of capital but instead the lower (and pre-determined) marginal rate of time preference. Thus, no matter what  $t$  is (i.e., no matter whether and to what extent a distortion exists in the domestic economy) the optimum tax on foreign savings will be what it was before, namely  $\tau_{opt} = -1/(n+1)$  because the pre-existing domestic distortion will neither be aggravated nor ameliorated by the tax itself.

Table 5 computes the second best optimum rate of withholding tax for various values of the elasticity of demand for exports  $n$  and the domestic tax on capital income  $t$ . The calculation is based upon the assumptions used in the intermediate case discussed in earlier sections, namely that  $x/F = 1/9$ ,  $s = 0.5$ ,  $\gamma = 0$ , and  $\beta = -3.0$ . This results in an implied value of  $\alpha$  of 2.33. Suppose, for example, that the effective tax on all capital income earned in the host country were 40 percent and that the price elasticity of demand for the exports of the host country were  $-3.0$ . Then the second best optimum withholding tax on repatriated earnings would be 24 percent. The table indicates very clearly that the second best optimum withholding tax declines as the host country tax on capital income increases and as the export demand elasticity increases. Moreover, a subsidy on repatriated earnings would be appropriate for sufficiently large values of  $t$  and  $n$ .

Finally, it is of interest to compare the effective tax on capital income accruing to domestics with the effective tax on capital income accruing to foreigners. The combined effect of a tax at rate  $t$  on capital income together with a withholding tax at rate  $\tau$  on any capital earnings that are repatriated is to drive a wedge between the social marginal product of capital and one plus the world real interest rate of percentage rate  $t' = (t + \tau) / (1 + \tau)$ . This is easily confirmed by noting that the level of foreign funding in the tax distorted competitive equilibrium must satisfy  $(1 - t)pF_K(\cdot) = (1 + i^*)(1 + \tau)$  for any given values of  $t$  and  $\tau$ . Therefore  $t'$  must satisfy the condition  $1 - t' = (1 - t) / (1 + \tau)$ . The bottom panel of Table 5 then shows the implied values for the second best optimum effective tax on capital income accruing to foreigners corresponding to the data in the top panel of Table 5. Thus, if  $t = .4$  and  $n = -3.0$  the second best optimum tax on foreign investment

Table 5

Second Best Optimum Withholding Tax  
for Alternative Values of  $n$  and  $t$

$n$	$t$	0.0	.20	.33	.40	.50
-1.5		2.0	1.74	1.57	1.49	1.36
-2.0		1.0	.83	.71	.66	.57
-3.0		.50	.37	.29	.24	.18
-5.0		.25	.14	.07	.04	-.02
-10.0		.11	.02	-.05	-.08	-.13
$-\infty$		0	-.09	-.14	-.17	-.21

Second Best Optimum Effective Tax on Foreign Investment Income  
for Alternative Values of  $n$  and  $t$

$n$	$t$	0.0	.20	.33	.40	.50
-1.5		.67	.71	.74	.76	.79
-2.0		.50	.56	.61	.64	.68
-3.0		.33	.42	.48	.52	.58
-5.0		.20	.30	.37	.42	.49
-10.0		.10	.22	.29	.35	.43
$-\infty$		0	.12	.22	.28	.37

$x/F$  is assumed to be  $1/9$ ,  $s = 0.5$ ,  $\gamma = 0$ ,  $\beta = -3.0$   
for all entries.

income is 52 percent. It is interesting to note that the effective tax on capital income accruing to foreigners may be higher or lower than the predetermined effective tax on capital income accruing to domestics depending on the size of  $n$ . It is also worth noting that  $t'$  increases with  $t$  for any given value of  $n$ .

## 5. The Importance of the Foreign Tax Credit

The welfare consequences of reducing or eliminating the capital income tax and shifting towards a consumption tax remain an unresolved empirical issue for a small open economy when international tax arrangements permit taxes on foreign investment income paid to the host country to be credited against taxes owed in the investing country. The purpose of this section is to review some of the issues at stake and to expand upon their relevance for an open economy that is small in the international capital market but has some market power in the determination of the world price for its exports.

We begin by assuming that the host country has in place an effective rate of tax on capital income that is equal to the rate prevailing in the investing country and then ask whether any change in this rate will improve national welfare.

Suppose the host country is indeed small in both commodity and capital markets and that the foreign tax credit functions perfectly to preserve capital export neutrality (i.e., if the host country reduces its tax rate the effective rate of tax on foreign investment income earned in the host country remains unchanged) then any small reduction in the host country's tax rate below the foreign rate will impose a welfare loss upon the host country.<sup>10</sup> If only domestic savers are eligible for the capital income tax cut the

domestic welfare loss will be reduced, but it will still be a loss. However, if both domestic and foreign savers are eligible for the capital income tax cut and if the foreign tax credit fails to preserve capital export neutrality (i.e., the host country's tax cut does indeed lower the effective rate of tax on foreign investment income in the host country, perhaps because of the deferral provision, see Brean (1984)) then a small cut in the host country's tax rate could improve national welfare because of a positive effect on domestic capital formation.

Even if there are adverse effects on national welfare when a small reduction in the capital income tax is introduced it may still be in the interest of the host country to eliminate its capital income tax entirely and to permit domestic and foreign savers alike to capture the full pre-tax rate of return on investments in the host country. But in order for such a policy change to improve national welfare the supply of domestic savings must be sufficiently elastic with respect to changes in the after-tax rate of return that the country's reliance on external funding disappears entirely when capital income is exempt from tax. This is a necessary but not a sufficient condition for domestic welfare gain, however, because the source of welfare gain arises from an expansion in domestic capital formation to include all investments yielding rates of return in excess of the marginal rate of time preference of domestic savers. This efficiency gain must be sufficiently large to offset the efficiency cost of substituting foreign savings available at the prevailing world real interest rate with domestic savings available at a higher marginal rate of time preference.

The foregoing points are readily confirmed using Figure 2 where KK represents the social marginal productivity of capital and SS represents the



marginal rate of time preference of domestic savers. Assuming that the host country sets its tax rate equal to the rate prevailing in the investing country the initial equilibrium occurs at  $E$  where the pre-tax marginal productivity of capital is  $(1+i^*)/(1-t^*)$ . Domestic and foreign savers then earn an after-tax rate of return equal to the world real interest rate so that the amount of domestic savings is  $S_0$ , the level of domestic investment is  $K_0$ , and  $AB$  represents the amount of external funding. The host country captures all of the tax revenue from foreign investment in the country; there is no tax revenue transfer to the investing country.

In Figure 2a the host country should maintain its capital income tax rate at  $t=t^*$  unless the foreign tax credit functions so imperfectly that a cut in  $t$  below  $t^*$  raises the after-tax return to the foreign investor and encourages a significant influx of additional foreign funding. Notice also that any increase in  $t$  above  $t^*$  will reduce domestic capital formation to, say  $K'$  and cause a domestic welfare loss equal to  $E'EBD$ . If the foreign tax credit works perfectly to preserve capital export neutrality then a small cut in  $t$  below  $t^*$  will impose a domestic welfare loss of  $AA'B'B$ . If only domestic savers are eligible for the capital income tax reduction the domestic welfare loss will be limited to  $AA'G$ . The complete elimination of the capital income tax will leave domestic capital formation unchanged and simply replace foreign saving with more costly domestic saving since the domestic savings schedule is relatively inelastic. The domestic welfare loss will be equal to  $ACEB$  if both domestic and foreign savers are eligible to receive the full marginal social productivity of their investments but just  $ACF$  if only domestic savers are eligible. However, if foreign savers receive the capital income tax exemption and the foreign tax credit is effectively neutralized by the deferral

Figure 2a

Capital Tax Should be Set at  $t=t^*$  if Domestic Savings is Interest Inelastic

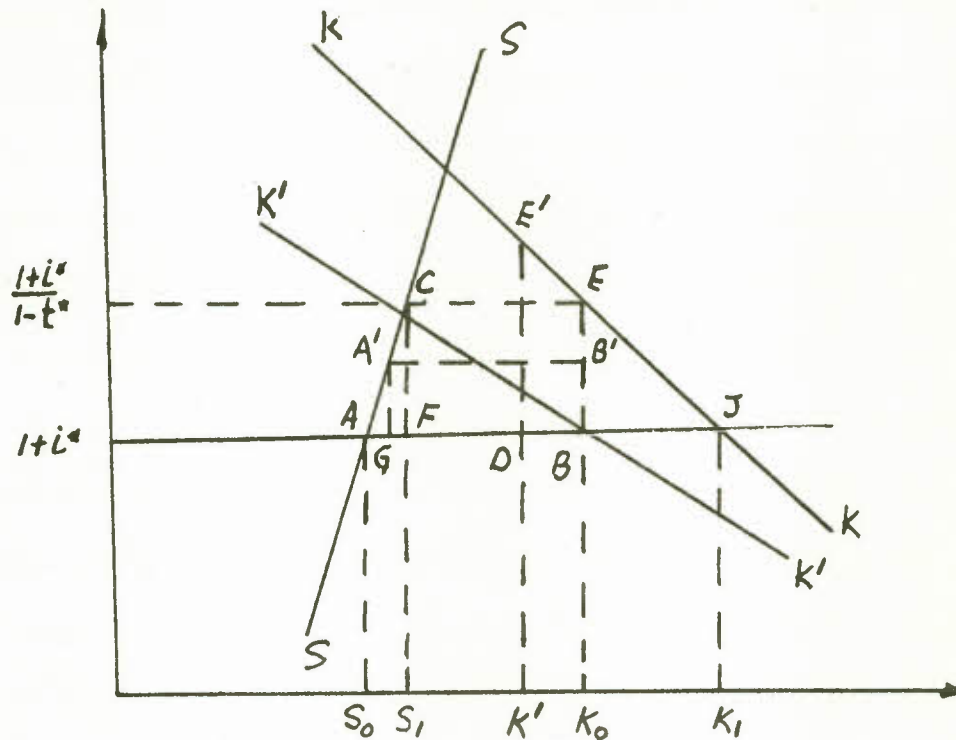
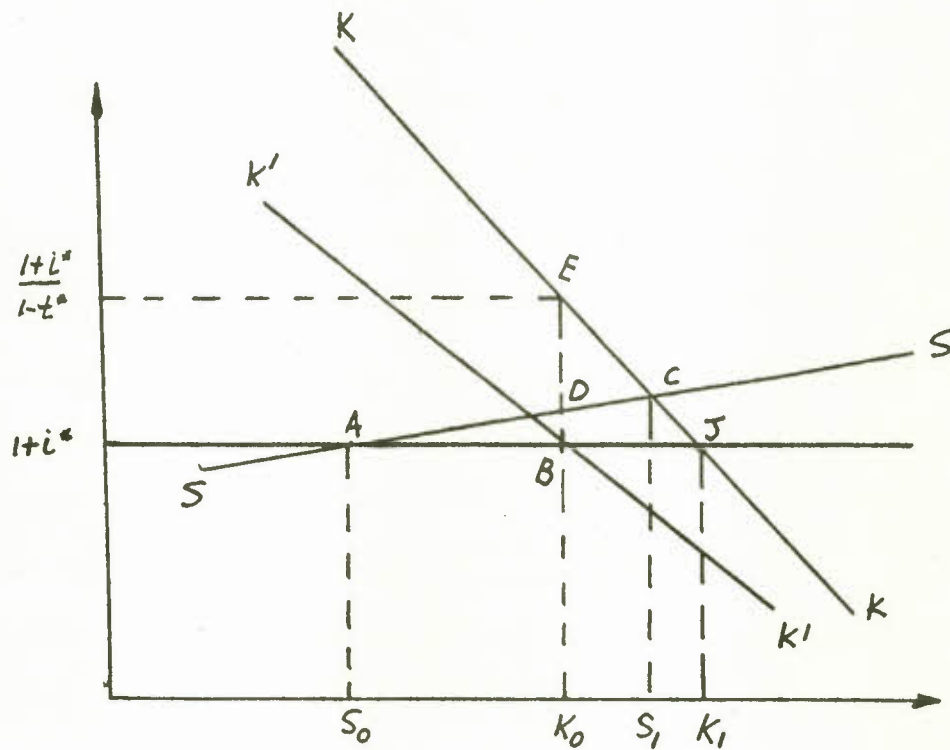


Figure 2b

Capital Tax Should be Set at Zero if Domestic Savings is Highly Interest Elastic



provision then domestic capital formation will increase to  $K_1$  and equilibrium will be restored at J with a domestic welfare gain of EBJ. The shift to consumption taxation will leave domestic savings unaffected in this case and the additional capital formation will be fully funded from abroad.

In Figure 2b the domestic savings schedule is drawn to be sufficiently elastic that elimination of the capital income tax by the host country will cause domestic saving to fully crowd out foreign saving plus fund additional domestic capital formation. Thus, even though a small reduction in the capital income tax may impose a domestic welfare loss (which is assured if the foreign tax credit functions to maintain the after-tax rate of return to foreign investors unchanged in the face of any cut in the host country tax rate), it is conceivable that the elimination of the capital income tax will result in a domestic welfare gain even if the foreign tax credit functions perfectly to preserve capital export neutrality. Area EDC must exceed area ABD for a net welfare gain to occur. Of course, if the foreign tax credit is effectively neutralized by the deferral provision the domestic welfare gain will be area EBJ as it was in the previous case.

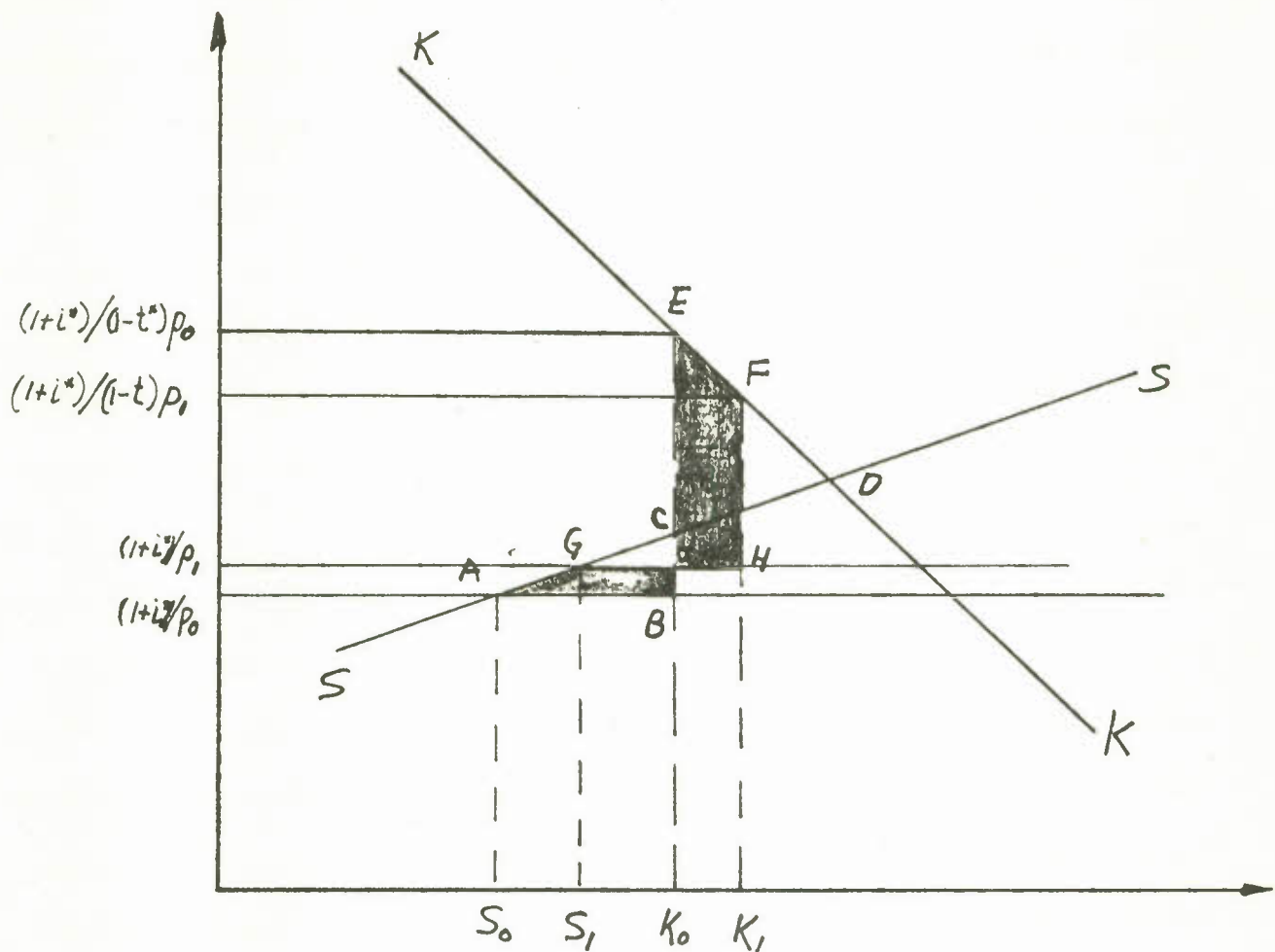
Our brief review of the welfare consequences of reducing the capital income tax in a small open economy has uncovered a potential conflict between the effects of small reductions and the effects of a complete elimination of the tax. If the foreign tax credit works perfectly (or nearly so) to preserve capital export neutrality then the argument for adopting a consumption tax must rest upon questionable and uncertain evidence about a substantial interest elasticity of supply of domestic savings. Unless small reductions in the capital income tax can be shown to increase domestic welfare it seems highly unlikely that large changes will be palatable to

policymakers. However, if there are significant terms of trade effects in play when capital is reallocated internationally these effects in certain cases reinforce the argument in favour of consumption taxation by making small reductions in the capital income tax welfare improving. In other cases the terms of trade effect may operate in the opposite direction, but if so it signals that large reductions in the capital income tax will also be undesirable. The issue of potential conflict between the effects of small and large changes seems important enough to be developed in further detail.

Figure 3 is a redrawing of Figure 2b above but with an extension to permit induced effects on the terms of trade. If the world price for the home country's exports were pre-determined and independent of the volume of its trade then a modest reduction in the capital income tax would cause domestic saving to replace foreign saving and restore equilibrium at C with total capital formation unchanged. The domestic welfare loss would be given by area ABC. However, if the tax were completely eliminated the domestic welfare gain would be given by area EDC minus area ABC. But suppose that any reduction in the economy's reliance on external funding induces an improvement in its terms of trade. Then the same modest reduction in the capital income tax will raise the world price for the host country's exports which will lower the cost of capital from abroad and expand domestic capital formation. When the induced terms of trade effect is factored into the analysis the new equilibrium level of domestic saving occurs at G rather than at C and the new level of domestic investment occurs at F rather than at E. Here a modest cut in the host country's tax rate will involve a net welfare gain for the country provided that area EFHJ exceeds area ABJG. This is, of course, an empirical question but it is comforting to know that the terms of trade effect is operating to

Figure 3

A Reduction in  $t$  below  $t^*$  will Improve the Terms of Trade and Increase Capital Formation if Export Demand is Unitary Elastic and Domestic Savings is Interest Elastic.





raise welfare rather than to lower it. It should be emphasized that the welfare gain will occur even if the foreign tax credit functions perfectly to ensure that any cut in the host country's tax rate leaves the after-tax rate of return to the foreign investor unchanged.

To complete this discussion, and to relate the results to previous work in Burgess (1985) it should be noted that if the supply of domestic savings schedule were highly inelastic (a vertical SS schedule would be an extreme case) then the terms of trade effect would work in the opposite direction to enhance the domestic welfare cost of reducing the capital income tax below the rate prevailing in the investing country. In this situation the phasing in of a consumption tax would not only entail a significant tax revenue transfer to the investing country government under the foreign tax credit but also affect adversely the economy's terms of trade because of the need to finance a higher level of foreign investment earnings.

In sum, if the foreign tax credit serves its intended purpose of preserving capital export neutrality then the responsiveness of domestic savings to changes in the after-tax rate of return is crucial in the assessment of the welfare consequences of eliminating capital income taxation in an open economy. This statement is true whether the economy is small in both the capital market and in the market for its exports or whether it is small in the capital market but still has market power in the determination of the price for its exports.

## 6. Conclusion

In conclusion there are four points worth restating with respect to the principles which should guide the taxation of capital income in an economy that is a small participant in the international capital market but has market power in its export sector yet is precluded from exercising this power through commercial policy. First, if for whatever reason the foreign tax credit is inoperative or irrelevant it will not be in the interest of the home country to eliminate its capital income tax completely, but it may well be in its interest to lower it substantially. The size of the second best optimum tax varies inversely with the price elasticity of demand for exports and the interest elasticity of supply of domestic savings.

Second, even though the home country can borrow or lend freely at a predetermined world real interest rate the national return to savings will be at least equal to the pre-tax marginal product of capital if the capital income tax is set optimally. Therefore, contrary to Feldstein (1983) there are sound reasons why domestic savings should be encouraged via savings incentives because the benefits will be measured in terms of higher real incomes and additional capital formation and not simply a reduced level of foreign ownership of the economy or a smaller net indebtedness.

Third, how much the home country stands to gain by derogating from the principle of national treatment in setting its capital income tax depends, in part, on the pre-existing rate of tax on capital income. If this rate is already low and the price elasticity of export demand is also low there will be substantial potential benefits from imposing a higher tax on foreign investment income--potential benefits that may outweigh the risk of foreign retaliation. However, if the rate of tax on the capital

income of domestics is already high an additional tax on the capital income of foreigners will be ill-advised not only because it risks retaliation but also because even without retaliation domestic real income will fall unless the export demand elasticity is sufficiently low. It should also be emphasized that if the elasticity of supply of domestic savings is small there will be very little to gain from taxing the capital income of domestics at a lower rate than the rate applicable to foreigners. Rather, a uniform rate of capital income tax should be set approximately equal to minus the inverse of the price elasticity of demand for exports.

Finally, if the foreign tax credit functions reasonably well in the sense that it leaves the effective rate of tax on the capital income of foreigners essentially unaffected by reductions in the host country tax rate it may still be in the interest of the host country to reduce its capital income tax rate below the foreign rate. If the host country is genuinely small in both the capital market and in the market for its exports then it will be in its interest to set  $t=t^*$  if the supply of domestic savings is sufficiently interest inelastic but to set  $t$  equal to zero if the supply of domestic savings is sufficiently interest elastic. However, if the host country has significant market power in its export sector it will typically be in its interest to set its capital income tax at some rate below  $t^*$  but above zero because the capital income tax reduces the supply of external funding, reduces the volume of exports required to pay for this funding, and thereby reduces the real cost of this funding to the economy.



## Footnotes

<sup>1</sup>A two-period model is chosen here for analytical simplicity. In a multiperiod context  $C$  could be interpreted either as the increment to consumption in all future periods from current period savings or as the amount of wealth available at the beginning of the next period which can at that time be allocated between consumption and further wealth accumulation.

<sup>2</sup>The two first-order conditions for private sector equilibrium determine the private sector's demand for external funding  $b$  as a function of  $p$  for given values of the truly exogenous variables  $t$ ,  $a$ , and  $i^*$ . If we differentiate these two first-order conditions partially with respect to a small change in  $p$  and solve for the implied change in  $b$  we obtain:

$(\partial b / \partial p)_D = (-F_{KU_2} + (1-t)F_{KK}^2 U_{22} x) / p F_{KK} (U + (1-t)F_{KK} U_2)$  which is necessarily positive. Thus, any increase in  $p$  will result in an increase in the private sector's demand for external funding. Next consider the balance of payments constraint and ask how a change in  $p$  affects the supply of external funding, i.e., the amount of external funding that can be obtained given the projected foreign exchange earnings from exports. Differentiating the balance of payments constraint partially with respect to a small change in  $p$  and solving for the implied change in  $b$  we obtain:  $(\partial b / \partial p)_S = (1+i^*) / (n+1)x$  which will be positive, negative, or zero depending upon whether  $n$  is less than, greater than, or equal to minus one. Stability of equilibrium requires that

$(\partial b / \partial p)_D - (\partial b / \partial p)_S > 0$  which yields the condition

$$(1-t)F_{KU}^2 + [(n+1)U - (1-t)F_{KK}^2 U_{22}] x F_{KK} + (n+1)F_{KK}^2 U_2 x < 0, \text{ which ensures that } \Delta > 0.$$

<sup>3</sup>It should be stressed that since the ratio of net exports to GDP, namely  $x/F$ , is itself dependent upon the tax rate  $t$ , the optimum tax formulae derived in this section and the following are merely statements of the

relationships that must hold between the tax rate  $t$ , parameters representing the price elasticity of demand for exports  $n$  and the interest elasticity of supply of domestic savings  $\gamma$ , and  $x/F$  that must hold at the second best optimum.

<sup>4</sup>For a country like Canada gross exports are roughly one-third of GDP. However, what is relevant for the formulae is the ratio of net exports to GDP which is much smaller. Thus, if Canada's net international indebtedness is of the order of 25 percent of the value of its capital stock and if 33 percent of GDP is a return to capital then one-twelfth of Canada's GDP must constitute net exports. In fact, Canada's net indebtedness is probably somewhat smaller since part of national savings is invested abroad and foreign savings have accounted for roughly 25 percent of capital formation in Canada over the post-war period.

<sup>5</sup>National savings may also increase if government deficits are reduced. Thus, the model might be extended such that the initial wealth constraint becomes  $V = C_0 + K + g$  where  $g$  represents the amount of public sector borrowing to finance current government expenditure which is assumed not to alter private sector decision making. Then any reduction in  $g$  would be allocated by the private sector between increases in  $C_0$  and  $K$  with effects similar to those discussed in the text.

<sup>6</sup>It should be emphasized that our definition of the national return on savings focuses on the contribution made by a dollar's worth of actual (i.e., ex post) savings rather than the effect of a one dollar rightward shift of the planned savings schedule. Whenever the supply of domestic savings is responsive to changes on the after-tax rate of return any increase in the amount of planned savings will be partly spent on current consumption rather



than financing additional capital formation at home or abroad because of the induced negative effect on the after-tax rate of return to saving.

<sup>7</sup>The conclusion arrived at in this section should not be misunderstood. As noted in footnote 5 above we are focusing on the national return per dollar of actual savings rather than the national return per dollar of planned savings. The latter will be a weighted average of the pre-tax and post-tax

marginal product of capital with weights equal to  $\frac{\frac{\partial C}{\partial V}}{\frac{\partial C}{\partial V} + \frac{\partial K}{\partial V}}$  and  $\frac{\frac{\partial K}{\partial V}}{\frac{\partial C}{\partial V} + \frac{\partial K}{\partial V}}$  respectively. It turns out that if  $t$  is set at

the second-best optimum the national return per dollar of planned savings is exactly equal to the pre-tax marginal product of capital.

<sup>8</sup>In the two period model used in this paper the withholding tax cannot be avoided or deferred through the reinvestment of retained earnings. In a multi period framework it would be possible to take into account the deferral provision which tends to erode the effectiveness of the withholding tax. See Brean (1984) for further details.

<sup>9</sup>This assumes that in taxing the capital income of nationals and foreigners at different rates the host country risks no retaliation from the investing country. Our model ignores the possibility that some host country savings may be invested abroad in which case any increase in the withholding tax by the host country would trigger a corresponding increase in the foreign country's withholding tax.

<sup>10</sup>Capital export neutrality is preserved under a perfectly functioning foreign tax credit because the savings of the investing country are being efficiently allocated between investments within the country and in the host country. The pre-tax rates of return are equalized whether or not  $t$  is set equal to or less than  $t^*$ .

## Appendix

The purpose of this appendix is to carry out the derivation of the optimum tax formula shown in the text for the general case of additively separable, isoelastic, intertemporal preferences. The first step is to express the various comparative statics effects as functions of  $\partial x/\partial a$ . Note first that:

$$\frac{\partial C}{\partial a} = (1-t) p \frac{F}{K} U_{22} \left\{ \frac{F}{K} \frac{(n+1)}{n} + (1-t) \frac{F^2}{K} \frac{1}{nx} \right\} / \Delta$$

and if we define  $\beta = \frac{F}{K} \frac{F}{F}$  we can write:

$$\frac{\partial C}{\partial a} = (1-t) p \frac{F}{K} U_{22} \left[ \frac{(n+1)}{n} + (1-t) \frac{F}{x} \beta \right] / \Delta$$

so that,  $\frac{\partial C}{\partial a} = \frac{\partial x}{\partial a} \left\{ \frac{(n+1) + (1-t)F/x\beta}{(1-t)F/n} \right\}$

The next step is to note that our specific functional form enables us to write all second partial derivatives of  $U$  in terms of first partials so that  $U_{11} = (\gamma - 1)U_1/C$  and  $U_{22} = (\gamma - 1)U_2/(F-x)$ . Recalling that  $FF_{KK} = \beta F_K^2$  and that  $U_1 = (1-t)F_K U_2$  we can write:

$$\frac{\partial x}{\partial t} = (1-t) p \frac{F}{K} U_{22} (\gamma-1) \left\{ \frac{1}{C} + (1-t) \frac{F}{K} \frac{(1-\beta)}{(F-x)} \right\} / \Delta$$

and since  $U_2/U_{22} = (F-x)/(\gamma-1)$  we have:

$$\frac{\partial x}{\partial t} = (F/\beta) \left\{ (1-\beta) + \frac{(F-x)}{C} \frac{(1-t)F}{K} \right\}$$

The next step is to use the various normalizations above to write:

$$\frac{\partial C}{\partial t} = - \left[ (1-t) p \frac{F}{K} U_{22} \frac{(F-x)}{n} \right] \left\{ (\gamma-1)F \left[ \frac{(1-t)}{x} + \frac{(n+1)\beta}{F} \right] + \frac{(F-x)}{x} (\gamma-1) \right\}$$

Therefore,  $\frac{\partial C}{\partial t} = - \left( \frac{F}{\beta} \frac{(1-t)F}{K} \frac{n}{n} \right) \left\{ (1-t)F/x + (1+n)\beta - 1 + \frac{(F-x)}{(\gamma-1)x} \right\} \frac{\partial x}{\partial a}$

Given the above expressions it is straightforward to obtain:

$$\frac{\partial C_o}{\partial t} + F \frac{\partial C_o}{\partial a} = \frac{(F/\beta(1-t)F_K)}{K} \{1 - (F-x)/x(\gamma-1)\} \frac{\partial x}{\partial a} \quad (A1)$$

$$\frac{\partial x}{\partial t} + F \frac{\partial x}{\partial a} = \frac{(F/\beta(1-t)F_K)}{K} \left\{ \frac{(F-x)/C_o}{C_o} + (1-t)F_K \right\} \frac{\partial x}{\partial a} \quad (A2)$$

It is also possible to derive the following expression:

$$\frac{\partial C_o}{\partial t} - [(\partial x/\partial t)/(\partial x/\partial a)] \frac{\partial C_o}{\partial a} = \frac{(-F/\beta(1-t)F_K)}{K} \{[(1-t)\beta F/x + (1+n)]$$

$$[1 + (F-x)/C_o (1-t)F_K] - 1 + (F-x)/x(\gamma-1)\} \frac{\partial x}{\partial a}$$

Finally, it is necessary to derive an explicit expression for  $\partial x/\partial a$ .

It turns out that  $n(\partial x/\partial a)^{-1}$  is precisely equal to the expression in braces immediately above. Therefore we have:

$$\frac{\partial C_o}{\partial t} - [(\partial x/\partial t)/(\partial x/\partial a)] \frac{\partial C_o}{\partial a} = -F/\beta(1-t)F_K \quad (A3)$$

We can then substitute (A1), (A2), and (A3) into the expression (\*) of the text which implicitly defines the second-best optimum tax rate and after some simplifications we obtain:

$$t_{opt} = 1/\left\{ \frac{1}{1-\gamma} \frac{F-x}{x} \left( \frac{C_o (1-t)F_K}{C_o + C_o (1-t)F_K} \right) - n \right\} \quad (A4)$$

Now  $(1-t)F_K = 1+i$  where  $i$  is the domestic real interest rate. Also, we can define the domestic saving rate  $s$  as the ratio of expenditure on future consumption to total current plus future expenditure namely

$s = C/(C_o (1-t)F_K + C)$ . It therefore follows that the expression in round

brackets in the formula for  $t_{opt}$  above is just  $1-s$  and  $t_{opt}$  can be written in the alternative form:

$$t_{\text{opt}} = 1/\left\{\frac{1-s}{1-\gamma} \frac{F-x}{x} - n\right\} \quad (\text{A5})$$

This form for  $t_{\text{opt}}$  has the advantage of summarizing in a compact way the relevant parameters and their direction of influence. If intertemporal preferences are Cobb-Douglas then  $\gamma = 0$  and if as well the pure time preference rate  $\rho$  equals zero then  $s$  equals .5. The optimum tax formula then reduces to:

$$t_{\text{opt}} = 1/\left\{\frac{F-x}{x} - n\right\} \quad (\text{A6})$$

More generally, if the pure discount factor  $\rho$  is positive then with Cobb-Douglas preferences the savings rate is given by  $s = (2 + \rho)^{-1}$  and we arrive at the expression shown in the text. Finally, for the calculations shown in Table 2 we assumed that  $s = 0.5$ , which will be its approximate value in the simple two-period model discussed here.



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