J. Gauvin
A. Reed

A simulation model for the Greater Snow Goose population

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## Abstract

## Introduction

The Greater Snow Goose population has recently undergone a rapid increase from 50000 to more than 200000 . A mathematical model was developed to under-
stand and simulate the growth of that population. Three parameters were used: the size of the population in spring parameters were used: the size of the population in spring,
the percentage of juvenile geese in the fall flight, and the numbers of geese killed by hunters in the USA and Canada over the past 20 years. Those survey data were used to estimate a rate of non-hunting mortality as well as probabilities for rates of reproduction and hunting mortality. The stochastic discrete model reproduces well the recent history
of the population and can be used to simulate scenarios for the future of the population. The model could be refined by developing and introducing other factors such as the carrying capacity of the range, annual variations in nonhunting mortality, and a function relating kill rates to population size and age structure. The model is available on diskette for interactive use on IBM PC
microcomputers.

The Greater Snow Goose (Anser caerulescens atlanticus) migrates between the northeastern Canadian Arctic and he mid-Atlantic coastal states of the United States and makes a major stopover on the St. Lawrence estuary nea Quebec City. The history of the growth of the Greater management plan 1981). From 1860 to 1930 , the population was barely maintained at a level of a few thousand. A yearround prohibition on iiunting in the United States and an open season limited to the fall in Canada allowed the popu-
lation to increase to 50000 by 1967 and to more than 190000 in the spring of 1978 . Resumption of the hunt the United States from 1975 seems to have slowed down the population increase.

The Snow Goose population is well suited to model simulations because of the tendency of Greater Snow Geese to gather in a limited area of the St. Lawrence estu ary for several weeks each spring, thus permitting complete photographic censuses. Another characteristic of be distinguished from older birds in the fall; we can thus determine the percentage of juveniles among the total fall population and obtain an indication of reproductive success. These two parameters, spring population size and fall juvenile percentage, have been measured regularly for nearly two decades by the Canadian Wildlife Service (CWS) and the United States Fish and Wildlife Service
(USFWS). A hird parameter, the number of geese killed by sport hunters, has been estimated annually in Canada since 1967 and in the United States since 1975. We used hese three parameters, which can be measured easily and relatively cheaply, to develop a stochastic discrete mathe matical model to simulate the dynamics of the Greater Snow Goose population. Our aim was not to produce ophisticated model, but ruer to examine whether a imple, hree-pase in a known population All thre parameters, being pestablis.
are subject to biases and errors. Although we discuss the possible sources of error for each type of survey, we have not undertaken the complex (perhaps impossible) task of establishing the magnitude of any such inaccuracies. Ultimately, the success or failure of the model to produce the data.

The model is based on the change in the population from spring 1965 to spring 1984. The model's dependent variable is the spring population; random variables are the juvenile percentages and the Canadian and American hunting kill rates. An estimated rate of non-hunting mor
tality is also applied. The random variables are generated from probability distributions constructed from data published in A Greater Snow Goose management plan (1981) and Reed et al. (1981) and, for recent years, from unpublished CWS and USFWS data. By adjusting the non-hunting the model to simulate population trends under various scenarios. The model is available as a program suitable for interactive use on an IBM PC microcomputer equipped with a graphics card and colour monitor.
Although a mathematical model does not generally have medium- or long-term predictive value (Levin 1984), it can help us to understand the dara and identify features associated with population trends. We can thus anticipate the effects of actions that could modify these trends. The
creation of models for animal populations is an iterative search for an approximate, realistic, and intelligible representation of a living entity that by its nature is too complex to be wholly described by a limited number of mathematical formulas.



## Development of the model

1. Population surveys

Since 1950, aerial surveys have been conducted in midwinter on the wintering grounds in the US and, since 1965, in the spring and most falls in the St. Lawrence used to improve accuracy of flock counts since 1969 in the St. Lawrence (Heyland 1972; A Greater Snow Goose management plan 1981) and on the Atlantic coast since 1978. The spring census in the St. Lawrence since 1969 (Table 1) is the most accurate of the surveys because it involves almost complete photographic coverage at a time when the entire population is present within a relatively small, well-defined
area. In early May a single flight is made over the staging area, and all flocks larger than $200-300$ geese are photographed on $70 \times 70-\mathrm{mm}$ black and white film; for smaller flocks, visual estimates are made and a sample is photographed to establish correction factors (P. Dupuis, pers commun.). The geese are counted directly from enlarged prints using a stereomicroscope, acetate overlay grids, and an automatic point counter. All geese are counted on each photograph, and the results are summed and added to th

Up to 1980-81, few, if any, geese were
spring survey because the areas used were well circumscribed. Since then, a longer stretch of the St. Lawrence has been used by the geese, and inland foraging flights have become more extensive. Those changes have increased the likelihood that some flocks were undetected, but this source of error is minor. Although it is not possible to quantify the accuracy of th
purpose it is a total count.

Before 1968, neither the US nor Quebec surveys benefited from aerial photography; we have used the more complete US winter data for 1964-68 (Appendix 1), along with the 1969-84 St. Lawrence spring counts, to construct a graph showing population growth (Fig. 1). The substantial increase between 1969 and 1978 is noteworthy
2. Recruitment-percentage of juveniles

The goslings, born during summer in the Arctic, are grey when they take part in the fall migration. It is therefore possible to measure the percentage of juveniles in the population and thus estimate reproductive success (Lynch and Singleton 1964). However, several factors make it difficult to obtain an unbiased estimate of recruitment. First, the juvenile birds are not distributed uniflocks are composed entirely of non-breeding adults (sub-
adults and failed breeders), whereas others are groupings of individual families (containing juvenile birds and their parents) and still others contain a mixture of non-breeders and family units. There is considerable variation in the flock) and temporal distribution (migration schedules, claily activity patterns) of family units in comparison with non-breeders (H. Boyd and A. R., personal observations). Second, juvenile geese, being more vulnerable, are shot at a greater rate than adult birds by hunters who are active throughout the fall survey period. Thus, the proportion of young birds in the population is decreasing while the

| Table 1 <br> Numbers of Greater Snow Geese counted in the St Lawrence Valley. Qucbec during spring |  |
| :---: | :---: |
| Year | Number of gesse |
| 1969 | 68800 |
| 1970 | 89600 |
| 1971 | 123300 |
| 1972 | 134800 |
| 1973 | 143000 |
| 1974 | 165000 |
| 1975 | 153800 165600 |
| 1976 1977 | 165600 160000 |
| 1978 | 192600 |
| 1979 | 170100 |
| 1980 | 180000 |
| ${ }_{1}^{1981}$ | 170800 163000 |
| 1983 | 185000 |
| 1984 | 225400 |


| Table 2 <br> Estimated percentages of juveniles in fall flights of Greater Snow Geese |  |  |
| :---: | :---: | :---: |
| Year | Canada | USA |
| 1965 | $11.2{ }^{\circ}$ | 2.8 |
| 1966 | 38.4 : | 37.0 |
| 1967 | $18.8{ }^{\text {* }}$ | 12.4 |
| 1968 | $18.9{ }^{*}$ | 12.5 |
| 1969 | 30.0 | 24.3 |
| 1970 | 45.6 |  |
| 1971 |  | 11.3 |
| 1972 | ${ }_{46.6}^{0.0}$ | ${ }_{41.1}^{0.4}$ |
| 1974 | 6.4 | 2.0 |
| 1975 | 32.7 | 37.3 |
| 1976 | 12.6 | 9.8 |
| 1977 | 23.9 | 23.7 |
| 1978 | 20.1 | 14.7 |
| 1979 | 28.2 | 23.2 |
| 1980 | 40.1 | 36.4 |
| 1981 | 16.8 | 17.0 |
| 1982 | 25.1 | 23.8 |
| 1983 | ${ }_{31.6}$ | 48.9 |
| 1984 | 37.6 | 27.4 |



Figure 2
Percentages of juveniles in fall fights of Greater Snow Geese in Quebec

counts are being conducted. No satisfactory way of adjust ing the data to account for the many sources of bias has been found. However, considerable effort has been made to reduce bias by collecting large samples distributed throughof habitat. ates and Canada since 1965 are listed in Appendix 1 Two annual estimates of juvenile percentages are available for Quebec, one based on counts from aerial photographs, the other from ground counts. For modelling we have retained only the higher of the two annual Quebec value (Table 2) because the surveys appear to yield low estimates. She US values, generally derived from smaller years 1965-68, from the linear regression:

$$
\% \text { juvenile Quebec }=0.795(\% \text { juvenile US })+8.95 .
$$

The Quebec values are plotted in Fig. 2, which shows that reproductive success fluctuates considerabl from year to year and in an apparently random (in the mathematical sense) fachion.

## 3. Hunting kills

Hunting kills of Snow Geese are estimated annually in both countries in the course of national surveys designed for all migratory waterfowl species (see Boyd and Finney 1978). Greater Snow Goose kill areas are geographically restricted in both countries, which renders kill estimates
less accurate than for other species that are hunted more widely. Some field biologists believe the national surveys overestimate the Greater Snow Goose kill (A Greater Snow Goose management plan 1981), but a special survey conducted in Quebec from 1978 to 1980 (Hyslop and Wendt 1982) suggested an underestimation. In earlier population modeling exercises (A Greater Snow Goose management plan 1981; Reed et al. 1981) it was judged that the national surveys provided acceptable estimates. For the sake of reported by the national survey for all Snow Geese (Anser caerulescens, $A$. $c$. atlanticus) in southern Quebec (zone 1) and the American kill as that of all Snow Geese for the Atlantic Flyway States (A Greater Snow Goose management plan 1981 and more recent CWS and USFWS unpublished data). Table 3 gives estimates of hunting kills in Canada since 1967 and in the United States since resumption of the

## Table 3

stimated numbers of Greater Snow Geese killed by sport hunters in Canad $\frac{\text { and the US }}{}$

| Year | Canada | USA | Total |
| :---: | :---: | :---: | :---: |
| 1967 | 16800 | - | 16800 |
| 1968 | 2700 |  | 2700 |
| 1969 | 3300 |  | 3300 |
| 1970 | 25300 |  | 25300 |
| 1971 | 13300 |  | 13300 |
| 1972 | 6100 |  | 6100 |
| 1973 | 26200 |  | 26200 |
| 1974 | 9000 |  | 9000 |
| 1975 | 31400 | 8500 | 39900 |
| 1976 | 25100 | 12300 | 37400 |
| 1977 | 20100 | 28200 | ${ }^{48} 300$ |
| 1978 | 41200 | 21500 | 62800 |
| 1979 | 23400 |  | 48400 |
| 1980 | 54400 | 27300 | 81700 43000 |
| 1981 | 29500 | 13500 | ${ }^{43} 000$ |
| 1982 | $\begin{array}{r}40 \\ 4500 \\ \hline 500\end{array}$ | 21700 | 62400 85700 |
| 1983 | 45300 | 40400 | 85700 |

hunt in 1975. The total hunting kill has fluctuated considerably over the 18 -year period, showing an increasing trend, especially since 1975. After a ban of more than 40 years, the US hunt was not great at the start, but built hunt Greater Snow Geese. American kill figures appear to be independent of juvenile percentages, perhaps because the young birds, having experienced the Canadian hunt, are less vulnerable to the gun when reaching the United States. The relationship between Canadian hunting kills and juvenile percentages is difficult to quantify, because the hunting kill is influenced by weather, the length of th flocks' stay, the birds' social behaviour, and other factors that would help to explain the numbers of hunting kills, we assume in this study that hunting success is random.

## 4. Annual population balance

Taking the spring population $P(k)^{1}$ as a reference and assuming a fixed mean annual natural survival rate $m$ (i.e., accounting only for non-hunting mortality), we have The fall population can be written as:

$$
P A(k)=\sqrt{m} P(k)+J(k)
$$

where $J(k)$, the number of juveniles in the fall population, is calculated from the juvenile percentage $R(k)$ measured in be the $\sqrt{m}$ is the semi-annual survival rate, assumed to be the same in both halves of the year:

$$
R(k)=\frac{100 J(k)}{\sqrt{m} P(k)+J(k)}
$$

from which we obtain the number of juveniles

$$
J(k)=\frac{R(k) \sqrt{m} P(k)}{100-R(k)}
$$

and then the fall population:

$$
P A(k)=\frac{100 \sqrt{m} P(k)}{100-R(k)}
$$

in terms of the juvenile percentage and the spring popula tion. By subtracting Canadian and American hunting kills, $C(k)$ and $D(k)$, we obtain the US winter population

$$
P H(k)=\frac{100 \sqrt{m} P(k)}{100-R(k)}-C(k)-D(k)
$$

and finally the population for the following spring:

$$
\begin{equation*}
P(k+1)=\frac{100 m P(k)}{100-R(k)}-\sqrt{m}[C(k)+D(k)] \tag{1}
\end{equation*}
$$

Formula [1] gives the population balance from one spring to the next taking into account reproduct
hunting) mortality, and hunting kills.

Pdesignates the year: e.g., $($ ( $)=$ popplation, sprimg 1980 $P(k+1)=$ population, autumn 1989; $P H(k)=$ population, winter 1980/81;
$\underset{\substack{\text { Figure } 3 \\ \text { Annual p }}}{ }$


## 5. Estimation of natural survival rate

The literature on Greater Snow Geese gives little information on individual longevity or the natural survival rate. The annual natural survival rate can be estimated by determining the value $m$ that minimizes the sum of the standard deviations between the measured values for the spring population and those calculated using formula [1]:

$$
\begin{aligned}
& \underset{m}{\operatorname{minimize}} \quad \sum_{k}^{1980}\left[{ }_{1967}[P(k+1)-\right. \\
& \left.\frac{100 m^{2} P(k)}{100-R(k)}+\sqrt{m} C(k)+\sqrt{m} D(k)\right]^{2}
\end{aligned}
$$

where $P(k)$ is the spring populations from Table $1, R(k)$ is the Canadian juvenile percentages from Table 2 , and $C(k)$ and $D(k)$ are the Canadian and American hunting kills from Table 3. The problem is formulated only for the period starting in 1967, the first year for which hunting kill data are available. The minimization calculation, made
using the SAS software PROC NLIN procedure, gives a result of $m=0.895$ with a confidence interval of $[0.812$ 0.982 . This yields an annual mortality rate of $10.5 \%$,
with a confidence interval of $[2.8 \%, 19.8 \%]$. In an excepronally detailed study involving resightings of individuall marked Barnacle Geese (Branta leucopsis), on which there was no open huncing season, O aduls and $16.8 \%$ fo uveniles. Our estimate therefore seems plausible. Our large confidence intervals are not surprising in view of the imprecise nature of some of the raw data used in calculation of the mortality rate and the likelihood that the rate varies somewhat from year to year (Owen 1982). For the remainder of the study we assume that the population has a fixed annual survival coefficient of $m=0.895$.

## 6. Correction of data

The population balance formula [1] can be used to detect anomalies in the data and make certain corrections First, it is necessary to correct the spring population estimates for the years 1965-68, which are incompatible with he corresponding fall estimates. Because the American winter population estimates for the same years are consis-
tent, the figures for the subsequent springs can be obtained by multiplying the US figures by the semi-annual survival coefficient $\sqrt{m}=0.946$. This gives

| year | spring population | hunting kills |
| :--- | ---: | ---: |
| $k$ | $P(k)$ | $C(k)$ |
| 1965 | 44000 | 5100 |
| 1966 | 41000 | 20100 |
| 1967 | 56600 |  |
| 1968 | 47800 |  |

where the hunting kills for 1965-66 are obtained by subracting American from Canadian fall population values for the corresponding years.

Assuming that the spring population figures are fairly accurate, we must still verify the consistency of the juvenile percentages and hunting kills. Formula [1] can be juvenile percentages; this yields hunting kill figures that in comparison with the established estimates, are doubtfu In particular, a number of negative values are obtained, suggesting that some juvenile percentages have been under estimated. On the other hand, using the same formula to calculate juvenile percentages from the other parameters, we obtain values that are fairly consistent with the observa ions from Quebec that were sometimes based on small fall uvenile percentages are plotted in Fig. 4. The two chrono ogical series are similar in appearance and except for 1968, vary only in magnitude. The adjusted juvenile per centages $R(k)$ can be used to estimate the fall population in Canada with the formula:

$$
P A(k)=\frac{100 \sqrt{m} P(k)}{100-R(k)}
$$

[2]

| Table 4Hunting rates for Greater Snow Geese and adjusted juverile percentag |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Hunting rates (\% of fall population) |  |  |  | $\begin{gathered} \text { Adjusted } \\ \text { juvenile } \\ \text { percentage } \\ R(k) \end{gathered}$ |
| $\mathrm{Y}_{k}^{\mathrm{car}}$ | $\underset{S(k)}{\substack{\text { Canada }}}$ | $\operatorname{USA}_{T(k)}$ | total |  |
| 1965 | 10.5 | - | 10.5 | 14.2 |
| 1966 | 25.2 |  | 25.2 | 51.4 |
| 1967 | 24.9 |  | 24.9 | 20.5 |
| 1968 | ${ }^{3.6}$ | - | 3.6 | ${ }^{40.0}$ |
| - 1969 1970 | 3.4 |  | 3.4 | ${ }^{33.6}$ |
| 1971 | 16.3 | - | 16.3 | 45.5 <br> 25 <br> 15 |
| 1972 | 3.9 | - | 3.9 | 18.9 |
| 1973 | 13.1 |  | 13.1 | 32.6 |
| 1974 | 5.2 |  | 5.2 | 9.0 |
| 1975 | 14.6 | 4.6 | 19.2 | 32.3 |
| 1976 | 12.2 | 6.8 | 19.0 | 24.1 |
| 1977 | 8.0 | 12.2 | 20.2 | 39.9 |
| 1978 | 17.0 | 10.7 | 27.7 | 24.9 |
|  |  | 11.6 | 21.4 | 32.6 |
| 1980 | 20.7 | 13.1 | 33.8 | 35.1 |
| 1981 | 13.7 | 7.3 | 21.0 | 24.9 |
| 1982 | 15.8 | 10.0 | ${ }^{25.8}$ | 40.2 |
| ${ }_{1984}^{1983}$ | ${ }^{13.8}$ | 13.3 | 29.1 | 46.4 37.6 |
| 1984 | - | - | - | 37.6 |

where $P(k)$ represents the spring populations. The estimated populations with those for the preceding springs are plotted in Fig. 5. The spring population net growth rates, calculated using the formula:

$$
100[P(k+1)-P(k)] / P(k)
$$

are plotted in Fig. 6

Figure 4
Juvenile percentages of Greater Snow Gecse, observed and adjusted


Figure 5
Adjusted estimates of Greater Snow Goose populations during spring and fall


## Figure 6 Greater Snow Goose spring to-spring growth rates



## 7. Hunting rates

Because recruitment is expressed as a percentage, hunting kills must be represented in the same manner.
The Canadian hunting rate is calculated using the formula The Canadian hunting rate is calculated using the formula:

$$
\begin{equation*}
S(k)=100 \frac{C(k)}{P A(k)} \tag{3}
\end{equation*}
$$

where $C(k)$ is the size of the Canadian hunting kill and $P A(k)$ is the fall population in Canada derived from formula [2]. The American hunting rate, which must take into account the earlier Canadian hunt, is calculated from
the formula:

$$
\begin{equation*}
T(k)=100 \frac{D(k)}{P A(k)-C(k)} \tag{4}
\end{equation*}
$$

where $D(k)$ is the size of the American hunting kill. The calculated hunting kill rates appear in Table 4, which also shows the corrected juvenile percentages. The hunting shows the corrected juvenile percentages. The hunting
rates are plotted in Fig. 7; they too vary, essentially at random. In 1968 and 1969 the juvenile percentages were very high and the hunting rates very low. In addition to contributing to an immediate population gain, the many juveniles from those years that did not fall victim to the hunt went on to form a large group of young breeders in 1971 and 1973. They thus ensured high juvenile percentages in subsequent years, accompanied in 1971, 1972, and
1974 by very low hunting rates. This explains the spectacular population increase between 1968 and 1975 and shows that a series of favourable chance occurrences can lead to

Figure 7
Canadian and American hunting rates for Greater Snow Geese

rapid population growth. It also suggests that a series of unfavourable circumstances could cause a correspondingly steep decline in the population.

## 8. Model equations

From formulas [2] and [3] we can derive an expres sion for the Canadian hunting kill:

$$
\begin{equation*}
C(k)=\frac{S(k) \sqrt{m} P(k)}{100-R(k)} \tag{5}
\end{equation*}
$$

From this formula, with [2] and [4], we can also represent American hunting kill

$$
\begin{equation*}
D(k)=\frac{\sqrt{m} P(k)}{100-R(k)}\left[T(k)-\frac{S(k) T(k)}{100}\right] \tag{6}
\end{equation*}
$$

Insertion of these formulas for $C(k)$ and $D(k)$ into balance equation [1], after simplification, yields the relation:

$$
\begin{align*}
& P(k+1)=\frac{m P(k)}{100-R(k)} \\
& {\left[100-S(k)-T(k)+\frac{S(k) T(k)}{100}\right]} \tag{7}
\end{align*}
$$

which gives the spring population in year $k+1$ based on he natural survival coefficient $m$, the juvenile percentage $S(k)$ and $T(k)$ for year $k$. This recurrent relationship can be properties. To improve it would require the addition of capacity of the range, would link hunting rates to juveni percentages and population levels while also compensating for hunting effort. However, Clark (1976, Ch. 7) showed that for models of this type any population $P(k)$ is at equilibrium, but the equilibrium is neither stable nor unstable. That is undesirable in a predictive model.
9. Probability distributions for percentages of juveniles and hunting rates
The corrected juvenile percentages listed in Table 4 and plotted in Fig. 4 are highly random from one fall to the next. For the purpose of the simulation, a probability distribution is estimated to reproduce this phenomenon Examination of the juvenile percentages suggests that a Beta distribution may be suitable. If $x$ is defined as the proportion of juveniles ( $\% / 100$ ), $0 \leqslant x \leqslant 1$, a Beta distribu tion may be written as follows:

$$
f(x ; p, q)=\frac{1}{B(p, q)} x^{p-1}(1-x)^{q-1}
$$

where $p$ and $q$ are parameters that can be determined by the method of moments:

$$
\begin{gathered}
p /(p+q)=\bar{x}=0.314 \\
-)^{2}(p+a+1)=s^{2}=
\end{gathered}
$$

$$
p q /(p+q)^{2}(p+q+1)=s^{2}=0.0124
$$

where $\bar{x}$ is the mean of the observed values and $s^{2}$ is the variance. Resolution of these two equations yields:

$$
p=5.15, q=11.23
$$

To ensure that this probability distribution is acceptable, we conducted a Kolmogorov significance test, which consists of comparing the experimental distribution function:

$$
F_{n}(x)=\frac{i}{n} \text { if } x(i) \leqslant x \leqslant x(i+1)
$$

$i=1, \ldots, 19, x(0)=0, x(20)=1$, where $x(i)$ are obser $i=1, \ldots, 19, x(0)=0, x(20)=1$, where
vations, and the Beta distribution function:

$$
F(x ; p, q)=\int_{0}^{x} \frac{1}{B(p, q)} y^{p-1}(1-y)^{q-1} d y
$$

by measuring the maximum deviation:

$$
D=\max \left|F_{n}(x)-F(x ; p, q)\right|, x=0.00,0.01, \ldots, 0.79
$$

The calculated maximum deviation was $D=0.145$; the $5 \%$ rejection criterion is $D \geqslant 0.301$. The Beta distribution

Figure 9
Probability distribution for Canadian hunting rates for Greater Snow Geese

function $F(x ; p, q)$ and experimental distribution function $n_{n}(x)$ are plotted in Fig. 8

The Canadian hunting rates shown in Fig. 7 also appear to be highly random. Since the mean and variance of $S(k) / 100$ are:

$$
\bar{x}=0.123, s^{2}=0.0048
$$

a Beta function can be estimated with parameters:

$$
p=3.12 \text { and } q=21.5
$$

The Kolmogorov test yields a maximum deviation of $D=0.107$ between the Beta distribution function and the
experimental distribution function in Fig. 9, making the probability distribution acceptable

While we have few data on the American hunting with a Beta distribution. The mean and variance of $T(k) / 100$ being

$$
\bar{x}=0.102, s^{2}=0.0011
$$

the parameters are:

$$
p=8.369 \text { and } q=73.76 \text {. }
$$

The maximum deviation between the Beta distribution function and the experimental distribution function in Fig. 10 is $D=0.076$, meaning that this probability distribution is also acceptable

Figure 10 distibution for American hunting rates for Greater Snow Gcesc
Probability dist




## Results

## 1. Simulation and scenarios

Formula [7] yields a stochastic model for simulating population growth when juvenile percentages $R(k)$ and
hunting rates $S(k)$ and $T(k)$ are considered random varia hunting rates $S(k)$ and $T(k)$ are considered random variales distribimated from the data to probability distribes, different random numbers are produced bes, dinerent random numbers are produced
$(k)$ and $T(k)$ are calculated by applying the which $R(k)$ tion of each of the corresponding probability distributions The model is available in the form of a program for IBM PC microcomputers equipped with a graphics card and colour monitor. Figures 11-21 illustrate the results of a number of experiments conducted with the program to
show the relative influence of the survival parameter and random variables on population growth.

Figure 11 shows the plots of 40 simulations for 20 years starting in 1984 with the estimated natural survival rate $m=0.895(10.5 \%$ natural mortality); populaand range of standard deviations shown in Fig. 12 indicate a tendency toward positive population growth. This is to a tendency toward positive population growth. Whis is
be expected, because the probability functions were derived from data for a period when the population exhibited strong growth.

Replacing the natural survival rate with a lower value $m=0.82$ ( $18 \%$ natural mortality) results in a tendency toward negative population growth (Fig. 13). This add to the credibility of the higher rate estimated from the data.


Figure 12
Preicted Greater Snow Goose population growth, mean and standard
deviations for 40 simulations, $m=0.895$


## ${ }^{\text {Figure }} 13$

deviatiod Greater Snow Goose population growth, mean and standard
dimulations, with reduced natural survival $(m=0.82)$


## Figure 14 Predicted

deviations for 40 simulations, with reduced reproductive success $(j$ juv. $\% \times 0.75)$


## igure 15

deviations for 40 simulations, with increased reproductive success $(\mathrm{juv} \% \times 1.25$ )


Figure 14 indicates that consistently reducing the enerated juvenile percentages by one-quarter leads to rapid population decline. On the other hand, increasing the juvenile percentages by one-quarter produces an incredible population explosion (Fig. 15). This demonstrates the very high sensitivity of the population (or the model) to fluctuations in juvenile percentages.

Reducing he generated hunting rates by one
suggesting that hunting kill is slowing down population increase. Raising hunting rates by one-quarter results in a slow decline (Fig. 17).

Five successive years of poor reproductive success (juvenile percentage $=10 \%$ ) would cause the population to drop quickly to its 1965 level (Fig. 18); the success rate is then allowed to return to normal, and the population slowly starts to grow again. A constant juvenile percentage of Figure 20 suggests that without the American hunt the population might have exhibited even more rapid growth population since 1975.

Figure 21 shows the mean values and range of standard deviations of 40 simulations for 1964-84, as well as the observed population values. With the exception of one year the observed curve fell within the limits of the model's curve, which gives credibility to the model. On a more refined scale, however, the real population grew mo
rapidly from 1969 to 1974 than the average growth predicted by the model. From then on the two curves move predicted by the model. From then on the two curves move the chance occurrence of abnormally favourable combinations of low hunting kills and high juvenile percentages between 1968 and 1974

## Figure 16

Figure 16
Predicted Greater Snow Goose population growth, mean and standard
deviations for 40 simulations, with reduced huming rate (hunt $\% \times 0.75$ )
deviations for 40 simmainens, with



Figure 18
Predictadreater Snow Goose population growth, mean and standard devia-
tions for 40 simulations, with low $(10 \%$ ) reproductive sucess over the first 5 year

igure 19
Medicted Greater Snow Goose population growth, mean and standard devia-
ions for 40 simulations, with fixed annual reproductive rate (juv $\%=314$ )


Figure 20
Predicted Greater Snow Goose population growth, with and without the


Figure 21
Cigure 21
Comparison of 40 model simulations with the observed growth of he Greatc
Snow C Coosc population


## During fall migration the juvenile geese, then about trree months old are easily disisinguished by their gree plumage, which contrasts with the white plumage of the adult.



## Conclusions

## Literature cited

This paper describes a stochastic model capable of convincingly re-creating recent trends in the Greater Snow Goose population and permitting simulation of future population trends under various scenarios. In designing the model we assumed that the population figures measured in the spring were fairly accurate, that hunting kills measured in the fall might have been generally underestimated. The model also assumes that hunting kills are random, an assumption that is clearly not totally accurate. This very basic model does not take into account the carrying capacity of the range or other ecological conditions. Moreover, the model assumes a uniform natural survival coefficient that does not account for annual variations in natural mortality caused by the age composition of th

A ood knowledse of the spring
Henile percentage, and hunting kills is indispensable for effective monitoring of population trends. Accurate juvenile percentages permit an accurate a posteriori calculation of hunting kills. We cannot stress too much the importance of good estimates of fall juvenile percentages based on larger samples than those available for certain past years (see Johnson et al. 1985 for further discussion on th populations). Accurate measurements of fall and winter population size would appear not to be very importantthe model can generate figures for these populations, which are difficult to survey.
Several factors warrant consideration in the design of models for the Greater Snow Goose population. One important inclusion would be a function to account for the carrying capacity of the range and other ecological condikill rates to population levels and juvenile percentages. Although kill rates contain a random component, they a undoubtedly linked to those parameters. At the same time, it would be desirable to inclucle a relation quantifying hunting effort. Once these components have been added to the model, we will seek quantifiable objectives for which it should be possible to determine optimum hunting or management policies.

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