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OBSERVATIONS OF VELOCITIES BENEATH WIND-DRIVEN WAVES

by

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MANAGEMENT PERSPECTIVE

The hydrodynamics of deep water wave breaking (white-capping) is particularly relevant to the development of Canada's offshore conventional energy resources. New knowledge of it will be applied to engineering design, operational efficiency and safety, and environmental protection through improved wave forecasting and improved understanding of mixing processes. This report, sponsored in part by PERD, presents preliminary results obtained from field experiments in 1985.

PERSPECTIVE DE LA GESTION

L'hydrodynamique du déferlement des vagues en eau profonde (moutons) est particulièrement intéressante pour la mise en valeur des ressources énergétiques classiques dans les eaux canadiennes du large. Le progrès des connaissances dans ce domaine pourra, par le biais d'une amélioration de la prévision des vagues et de la connaissance des processus de mélange, avoir des applications en matière de conception des structures, d'efficacité et de sécurité des opérations et de protection de l'environnement. Ce rapport, parrainée en partie par le CRDE, présente les premiers résultats obtenus sur le terrain en 1985.

Chef, Division de l'hydraulique

RÉSUMÉ

La tour de l'Institut national de recherche sur les eaux, installée dans le lac Ontario, constitue un observatoire idéal pour étudier le comportement des grosses vagues de la mer du vent (limitées dans l'espace par le fetch). Les propriétés statistiques de ces vagues sont les mêmes que celles des grosses ondes de tempête (limitées dans le temps) qui s'appliquent à la conception des structures offshore. Il est clair que l'apparition de moutons sur ces vagues va affecter les mesures des composantes du vecteur vitesse près de la surface. L'étude décrit la mesure de certaines données statistiques sur les composantes horizontales et verticales du vecteur vitesse en fonction des forces qui peuvent s'exercer sur des membres cylindriques des structures.

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ABSTRACT

The National Water Research Institute's tower in Lake Ontario provides an ideal site for observing the behaviour of steep (fetch-limited) wind-generated waves. The statistical properties of these waves are akin to those of the steep (duration-limited) storm waves that are applicable to the design of offshore structures. It is clear that the incidence of whitecapping in these waves will affect the statistics of the near-surface velocity components. This paper describes measurements of some statistics of horizontal and vertical velocity components with reference to the expected forces on cylindrical structural members.

INTRODUCTION

The design of offshore structures depends critically on the expected wave orbital velocities, accelerations and pressures. Very few measurements have been made of actual velocities beneath natural wind-generated waves and the design engineer generally relies on linear wave theory to derive appropriate design forces from a suitable climatology of wave (surface elevation) information. Laboratory tests with regular and irregular (non-breaking) paddle-generated waves generally find that linear theory yields surprisingly good estimates of orbital velocites derived from measured surface elevation (see, for example, Vis, 1980). However, in an actively wind-driven sea, waves periodically break producing whitecaps and injecting an impulse of momentum to the underlying current structure (Donelan, 1978). In addition, the wind-driven current near the surface alters the velocity pattern from that which would be expected based on linear theory. These differences from linear theory might be important in the calculation of forces on offshore structures - all the more so because the forces are related to the square of the velocity and to the acceleration of the fluid. Thus sudden increases in fluid velocity caused by whitecapping at the crest of a wave, where the orbital velocity is a maxiumum, produce disproportionately large increases in the drag force on the structure. The scale of the velocity fluctuations introduced by whitecapping is much smaller than the scale of the wave itself (wavelength), so that these relatively small fluctuations, advected past the structure by the substantial orbital velocities, may produce significantly different local accelerations from those expected by linear theory. Furthermore, rapid variations with Reynolds number of the drag characteristics of bluff bodies emphasize the importance of good velocity statistics in design.

In view of the expected differences between statistics of velocities in laboratory paddle-generated (albeit irregular) waves and those encountered in a natural wind-driven sea, an observational program in Lake Ontario was designed to explore the statistics of velocities beneath wind-driven waves.

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EXPERIMENTAL ARRANGEMENTS

A fixed tower provides the ideal platform for measurements of sub-surface velocities and that of the National Water Research Institute in Lake Ontario is particularly well suited to this purpose. Having been designed expressly for wave measurements, the tower is free of cross-bracing in the vicinity of the water surface. The location of the tower (Figure 1) is indicated in Figure 2. The tower is supplied with power via underwater cables and 48 channels of data, sampled at 20 Hz by computer, are transmitted by cable to shore where another computer accepts the information and writes it to disk for eventual transferal to tape. Further details of the research site are given in Donelan et al. (1985).

The instruments used for measuring both vertical and horizontal components of velocity were "drag spheres", in which the fluid force on a sphere yields a measure of the velocity components (Donelan and Motycka, 1977). There were three drag spheres mounted on a rotatable mast, the "fifth leg" (Figure 1) of the tower at depths of about 1.2 m, 2.0 m and 4.0 m with the axis of symmetry of the instrument aligned horizontally. The mast could be rotated by control from the shore station so that the axes of the drag spheres were aligned normal to the mean wave direction. The instruments thus yielded vertical and horizontal (down-wave) velocity components. The size of the drag spheres (4 mm) was such that they responded essentially to drag and not to inertial effects (Donelan and Motycka, 1977) in the range of wave heights and periods expected. Since the drag response is non-linear (almost precisely a square law in the Reynolds number range used), the instruments were zeroed mechanically before and after each measurement episode by means of pneumatically activated sleeves that shielded the drag spheres from the ambient flows. The drag spheres were carefully calibrated both before and after field exposure. Calibration was accomplished by towing the instruments in the 120 m towing tank of the National Water Research Institute.

BACKGROUND

The incremental horizontal force (per unit length) exerted by a moving fluid on a fixed vertical cylinder is generally estimated from "Morison's equation" (Morison et al., 1950):

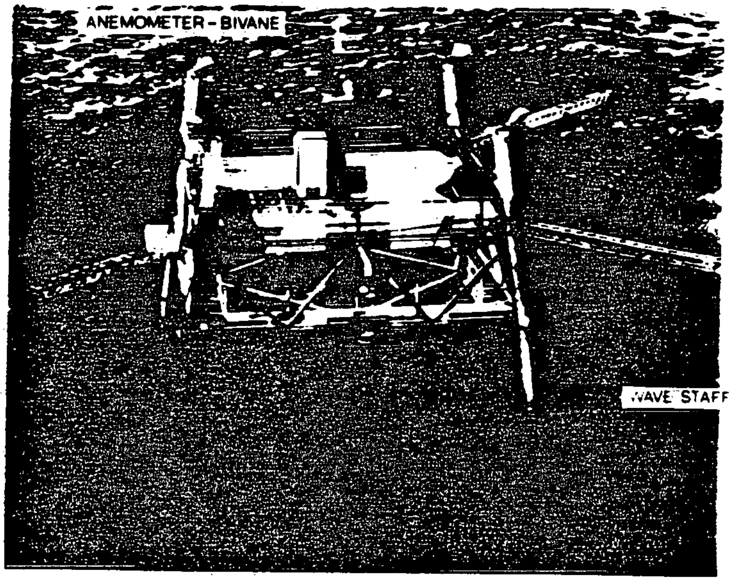


Figure 1. Photograph of the National Water Research Institute's offshore tower showing the rotatable mast on which the drag spheres and wave staffs are mounted. Meteorological instruments are installed at about 11 m on the central mast. The deck area is square with a side of 10 m.

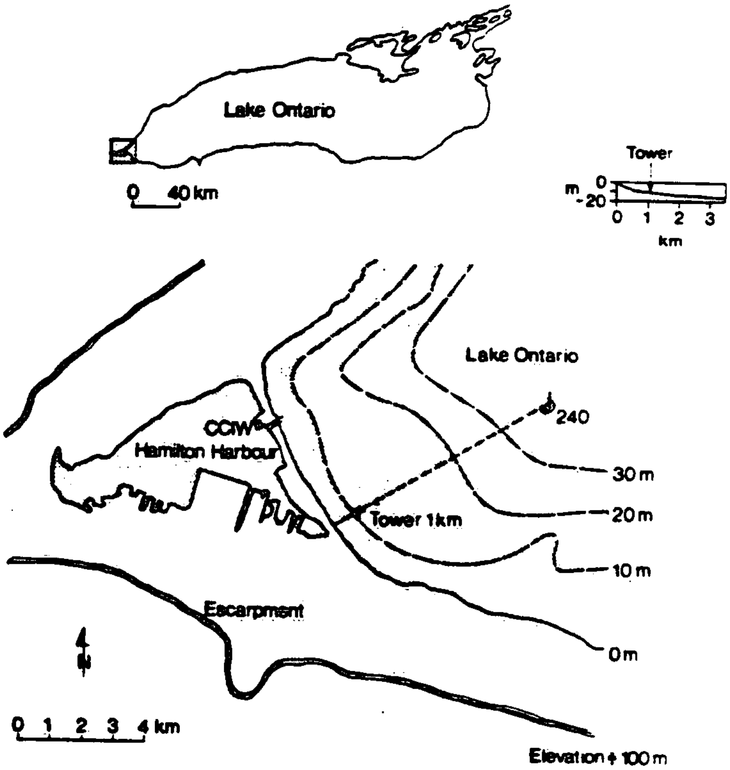


Figure 2. Map showing the location of the tower in Lake Ontario and the shore-normal profile in the vicinity of the tower.

$$F(t) = C_D \rho r\, u(t)\, V(t) + C_M \rho \pi r^2\, \dot{u}\,(t) \qquad (1)$$

where ρ is the fluid density, r is the radius of the cylinder, C_D and C_M are drag and inertia coefficients, $u(t)$ and $\dot{u}(t)$ are horizontal fluid velocity and acceleration and $V(t)$ is the magnitude of the velocity vector. C_D and C_M are functions of the Reynolds number $R_e = 2|u|r/\nu$, the relative surface roughness $(k/2r)$ and the Keulegan-Carpenter number $N_{kc} = AT/2r$, where ν is the fluid kinematic viscosity, k is the average roughness diameter, A is the velocity amplitude of the oscillatory part of the flow, and T is its period.

Morison's equation ignores wave drag, which occurs if the cylinder is at or near density interfaces, and skin drag. Nonetheless, for most engineering applications the form drag and inertial resistance modelled by Morison's equation are the dominant forces. Laboratory measurements of the in-line (with horizontal velocity) force on vertical cylinders seem to agree well with that deduced from Morison's equation (Bearman et al., 1985). As pointed out by Chaplin (1985), Morison's equation is applicable to planar oscillatory flow, i.e., flow in which the instantaneous velocity and acceleration vectors are colinear. In circular oscillatory flow, characteristic of deep water surface gravity waves, the instantaneous acceleration and velocity vectors are mutually perpendicular and additional inertial terms need to be included in Morison's equation.

The behaviour of the drag and inertial coefficients with Reynolds and Keulegan-Carpenter numbers has been the subject of many investigations (see for example, Batchelor, 1967; Mogridge and Jamieson, 1976; Sarpkaya, 1976; Yamamoto and Nath, 1976; Garrison et al., 1977; Holmes and Chaplin, 1978; Koterayama, 1980). Most of these investigations have been done in laboratories under idealized conditions of uni-directional, planar oscillatory or circular oscillatory flows. Strong Reynolds number and Keulegan-Carpenter number dependencies of the drag and inertial coefficients imply that the standard practice of using constant values for these coefficients for force calculations over the entire length of vertical cylinders in irregular waves is fraught with error (Ramberg and Niedzwecki, 1979). An additional source of error arises in the calculation of orbital velocities from observed surface elevations through a theoretical model. It is to this aspect of force calculations on engineering structures that this paper is directed. Furthermore, the strong Reynolds number dependences of the drag and inertia coefficients underscore the need for accurate information on the actual velocities in a wind-driven sea.

RESULTS

Over one hundred hours of data were gathered in episodes of 80 minutes duration. A small sub-set (four episodes) of these data is presented here. Table 1 summarizes the ambient conditions during the four episodes. In each case the analysis was done on the first 27.3 minutes of data.

TABLE 1

Summary of Runs

Run	$\overline{U}$ m/s	WD deg.	Fetch km	$H_{1/3}$ m	T_p s	$\overline{U}/C_p$
85111	1.2	80	300	0.7	4.8	0.2
85105	10.6	87	300	1.9	6.7	1.0
85145	14.1	70	280	2.6	6.8	1.3
85159	17.4	240	1.2	0.6	2.5	4.5

$\overline{U}$ is the average measured wind speed, WD is the wind direction, $H_{1/3}$ is the characteristic wave height and T_p is the period of the spectral peak.

In the following we illustrate some of the temporal and spectral characteristics of the wave height and velocity data of these four runs. The first three runs listed correspond to long fetch (easterly wind) conditions. The parameter $\bar{U}/C_p$ (the ratio of wind speed to the phase speed of the waves at the spectral peak) reflects the state of development of the waves. The first case $(\bar{U}/C_p = 0.2)$ corresponds to "swell", i.e., over-developed waves, produced by an earlier higher wind. The second case is nearly fully developed, while the third is under-developed. The last case, corresponding to strong westerly winds over a short fetch, yields very young (undeveloped) and strongly forced waves with intense whitecapping.

Figure 3a shows a section of the time series of observed surface elevation, η and horizontal (downwave) and vertical velocities, u and w at a depth of 1.25 m beneath the mean water level. The top curve, uV, with C_D assumed constant, illustrates the temporal dependence of the first term of Morison's equation (1). Because of the quadratic nature of the drag force, strong events, such as the group near 50 seconds, are accentuated and the intergroup small waves are relatively unimportant. These waves are over-developed so that there is very little whitecapping and this is reflected in the smooth traces of horizontal (u) and vertical (w) velocity. The high frequency wavelets, evident in the surface elevation (η) traces, are rapidly attenuated with depth and therefore do not appear in the velocity traces.

Figures 3b, c and d illustrate the changes in the observed velocities and uV product as the waves are more and more strongly forced. The surface elevation becomes more

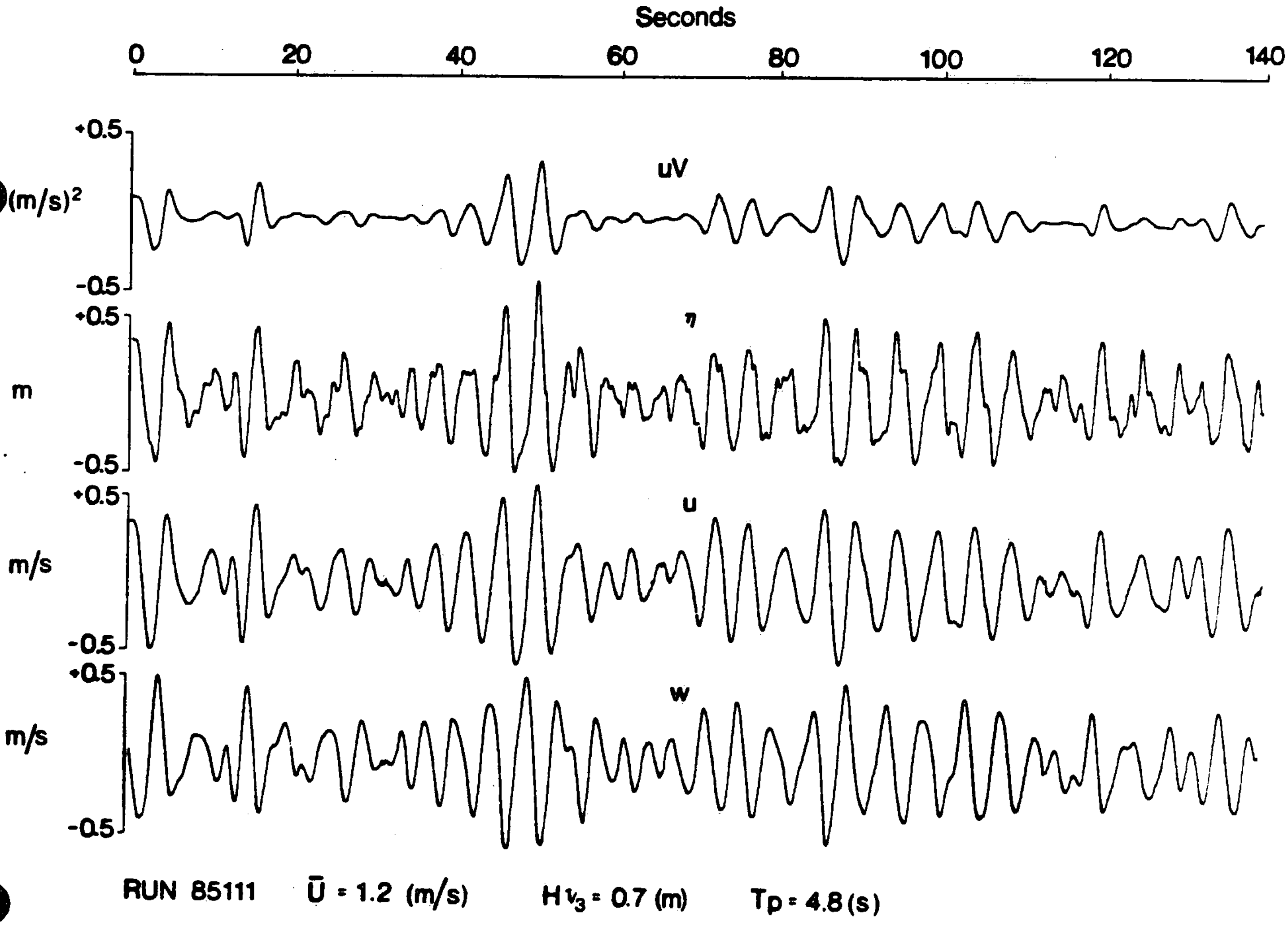
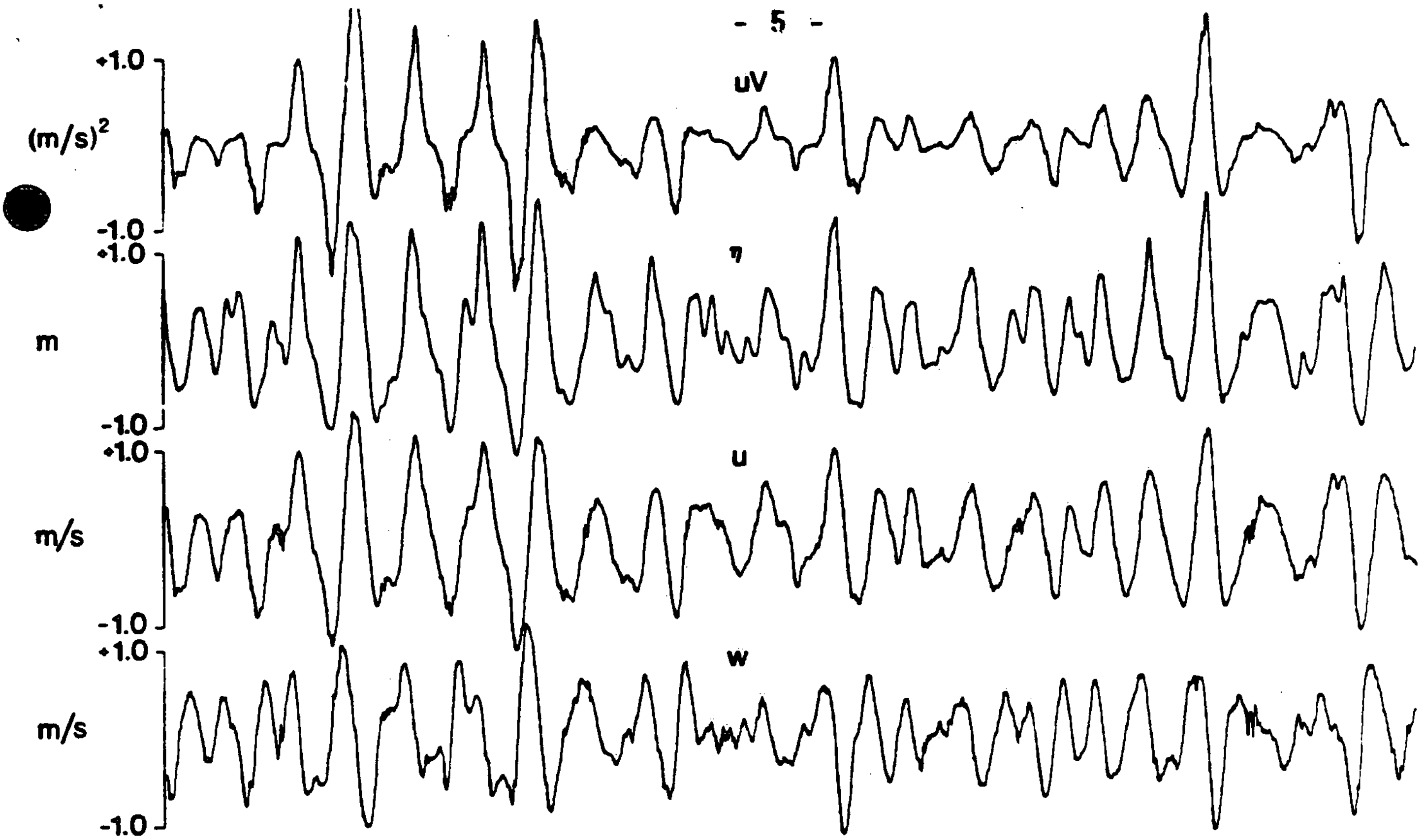
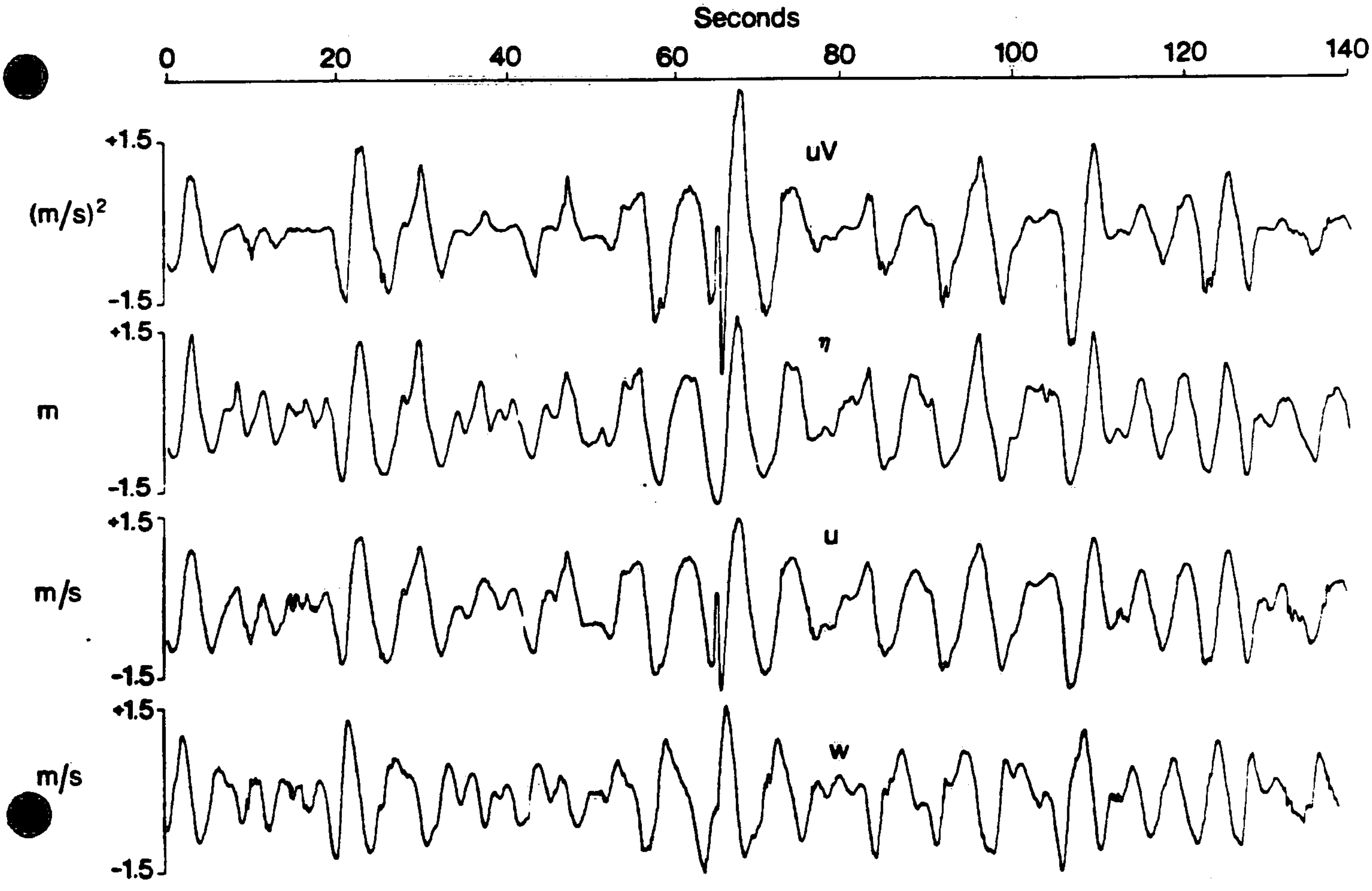


Figure 3a. A section of the time series of the measured surface elevation (η), horizontal (u) and vertical (w) velocity components at 1.25 m depth. The top curve (uV) is the instantaneous product of the horizontal velocity and the magnitude of velocity vector. This figure is drawn from the case of recent swell, run 85111 - over-developed.



RUN 85105 $\bar{u}$ = 10.6 (m/s) $H_{1/3}$ = 1.9 (m) Tp = 6.7 (s)



RUN 85145 $\bar{U}$ = 14.5 (m/s) $H_{1/3}$ = 2.5 (m) Tp = 6.8 (s)

Figure 3b. The same as for Figure 3a but for run 85105 - nearly fully developed.
Figure 3c. The same as for Figure 3a but for run 85145 - under-developed.

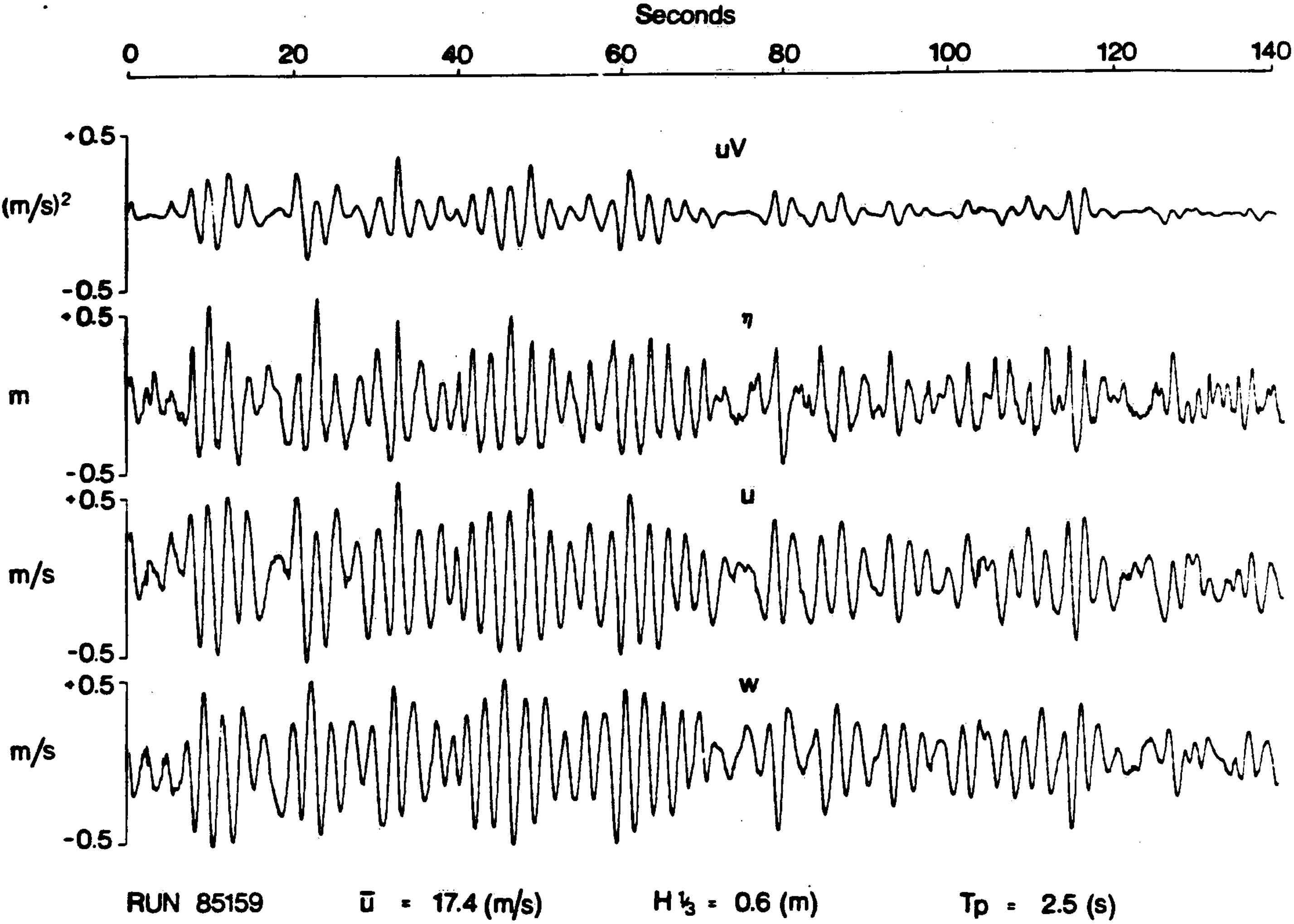


Figure 3d. The same as for Figure 3a but for run 85159 - strongly wind-generated or very underdeveloped.

positively skewed and evidence of small scale turbulence can be seen in the velocity traces. Occasional deep troughs in run 85145 cause the drag sphere to break the surface and the horizontal velocity signal to change abruptly from strongly negative (orbital velocity under the trough) to weakly positive (wind above surface). An example of this is seen around 65 seconds in Figure 3c. Increasing skewness in the uV product is also apparent in the progress through the panels of Figure 3.

For brevity, sample spectra for two runs only are shown in Figures 4 and 5. These are the two extreme cases of Table 1. Each spectrum is computed from 32768 samples in blocks of 1024. The spectra cover up to seven decades in the range of spectral densities. To avoid contamination of the low spectral densities, through window leakage from the peak, a 4-term Blackman-Harris taper (Harris, 1978) was applied to the separate blocks. The spectral estimates are averaged in pairs so that each plotted point has 128 degrees of freedom corresponding to 90% confidence limits of 1.23 and 0.82.

The spectra of surface elevation are graphed in Figure 4 with a straight line of slope -4 (Donelan et al., 1985) added. The line is fitted to the high frequency part of Figure 4b and redrawn on Figure 4a to demonstrate the sensitivity of the rear face (high frequency part) of the spectrum to wind speed (Donelan et al., 1985).

Figure 5 illustrates the spectra of horizontal velocity fluctuations and compares the observed spectra with calculations from the surface elevation spectra using linear long-crested theory. The deviations from linear theory are most obvious at high frequencies in both cases and at low frequencies also in the strongly forced case (Figure 5b). Away from the peak (and several decades lower in spectral density) the deviations are caused by turbulence generated by the wind-driven sheared current and by wave breaking (Donelan, 1978; Kitaigorodskii, et al., 1983) as modified by the advection of the orbital

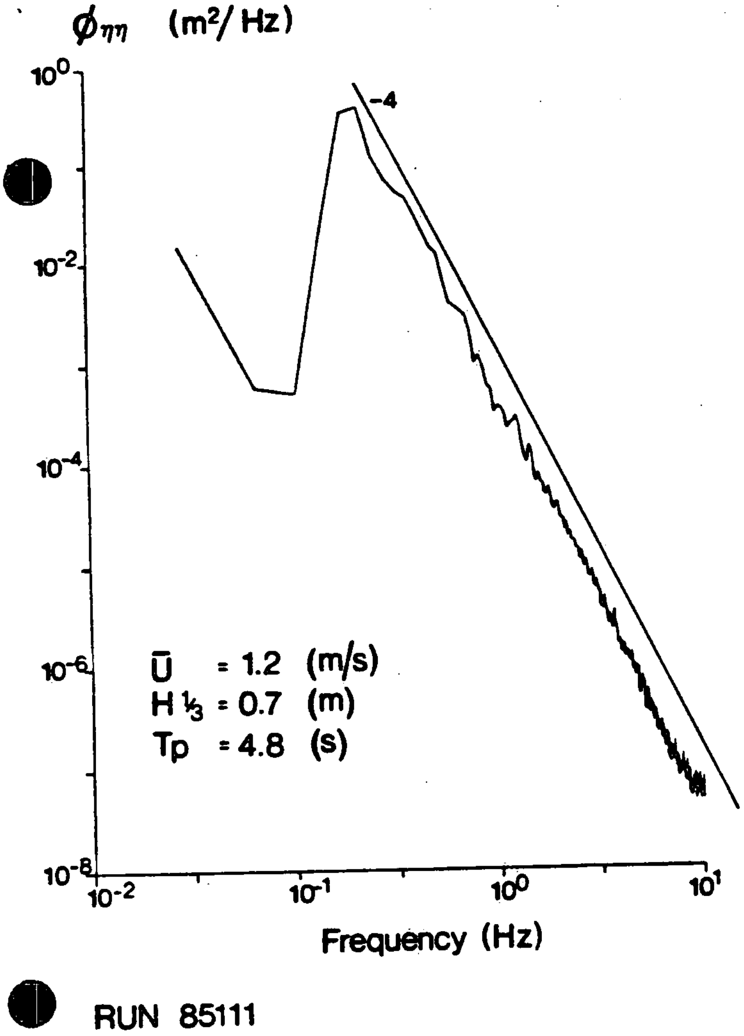


Figure 4a. The spectrum of surface elevation for the case of recent swell. The -4 power law fitted to Figure 4b is also shown here.

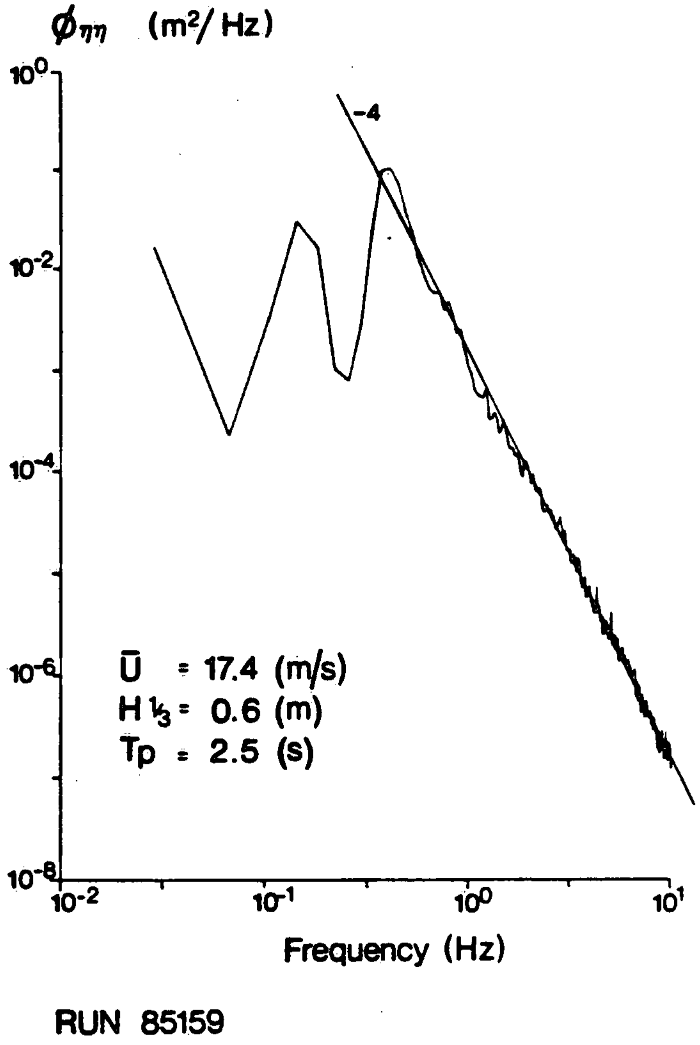


Figure 4b. The spectrum of surface elevation for the strongly wind-generated case. Above the peak, the spectrum conforms to a -4 power law (Donelan et al, 1985).

velocities (Lumley and Terray, 1983). For purposes of calculation of forces on structures these differences are less important than the differences near the peak (especially in Figure 5a) at substantially higher energy levels.

An exploration of the source of these differences is beyond the scope of this preliminary paper. However, it is important to note that differences of this magnitude may have significant effects on the higher order statistics. For example a strictly linear model, in which the wave components are independent (in random phase) and freely propagating, necessarily yields no skewness of either the surface elevation or the underlying velocity field. Figure 6 illustrates the dependence of the skewness $[\overline{x^3}/(\overline{x^2})^{3/2}]$ of the uV product on the skewness of surface elevation η. The skewness of η and the skewness of uV both increase as the waves are more and more strongly forced. However, although the skewness of η is always positive (crests sharper than troughs) as expected, the skewness of uV is positive only for the very strongly forced case. Negative skewness of uV corresponds to generally larger (in magnitude) velocities under the troughs than under the crests. Similar results have been noted by Vis (1980) in the laboratory. Differences (observations versus theory) in skewness and kurtosis $[\overline{x^4}/(\overline{x^2})^2]$ of uV reflect differences in the symmetry and extremes of fluid forces on structures that may have important consequences in establishing engineering design parameters.

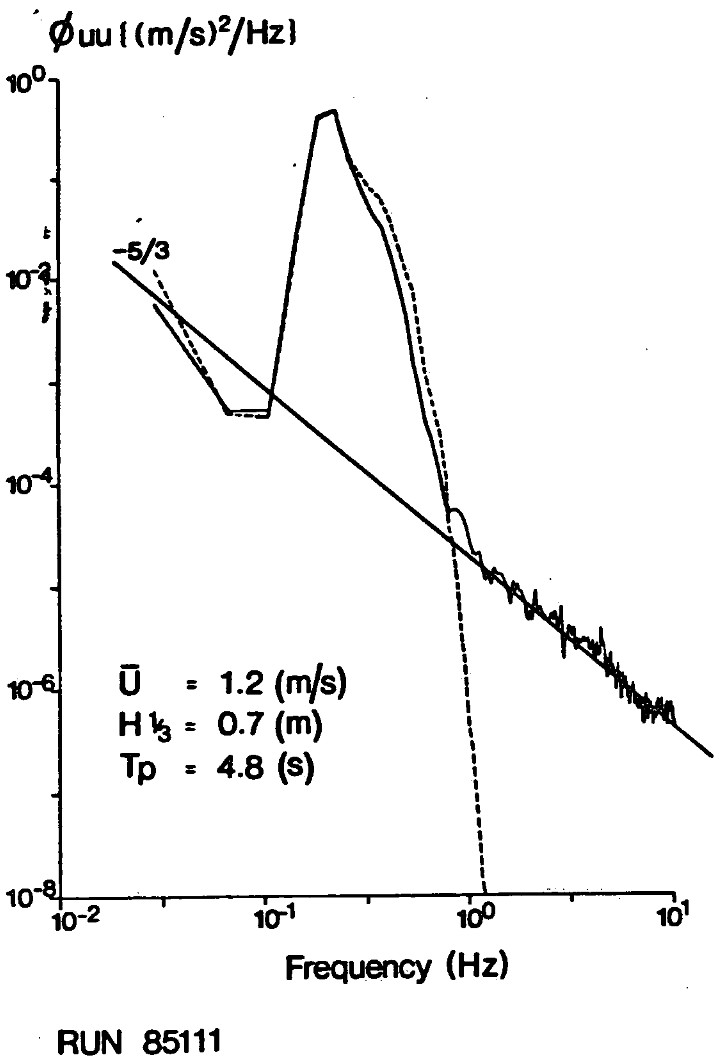


Figure 5a. The spectrum of horizontal velocity for the case of recent swell. The solid line is for the measured horizontal velocity and the dotted line is the calculated spectrum from the spectrum of surface elevation through linear long-crested theory. The -5/3 line shown corresponds to the inertial sub-range of isotropic turbulence (Lumley and Terray, 1983).

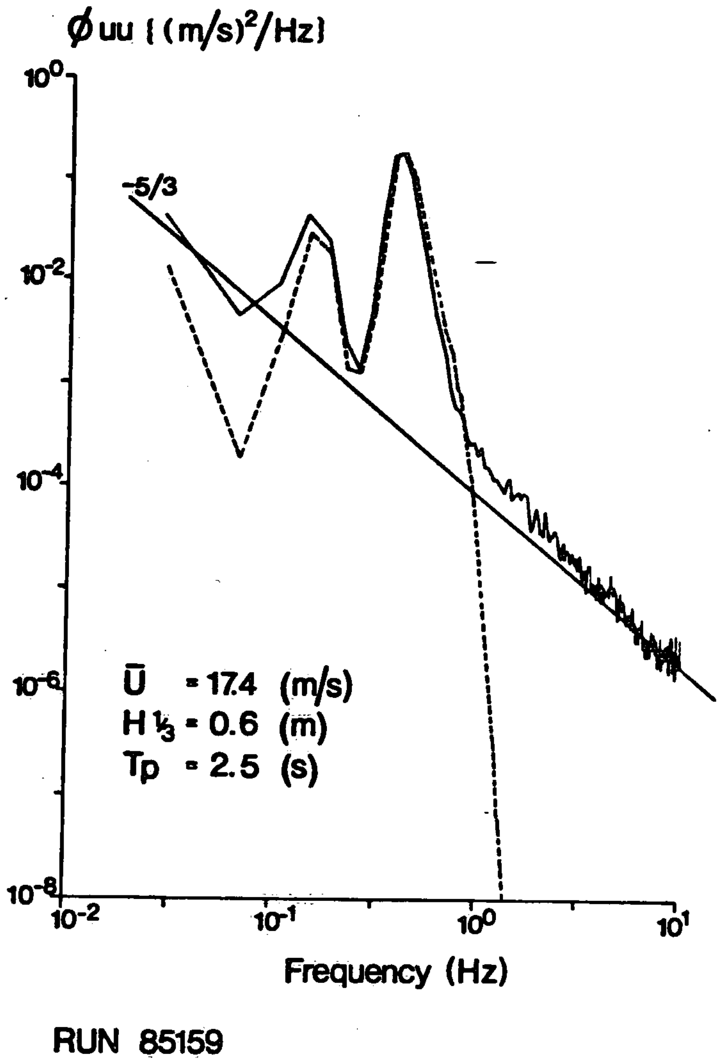


Figure 5b. The same as for Figure 5a but for the case of strongly wind-generated waves.

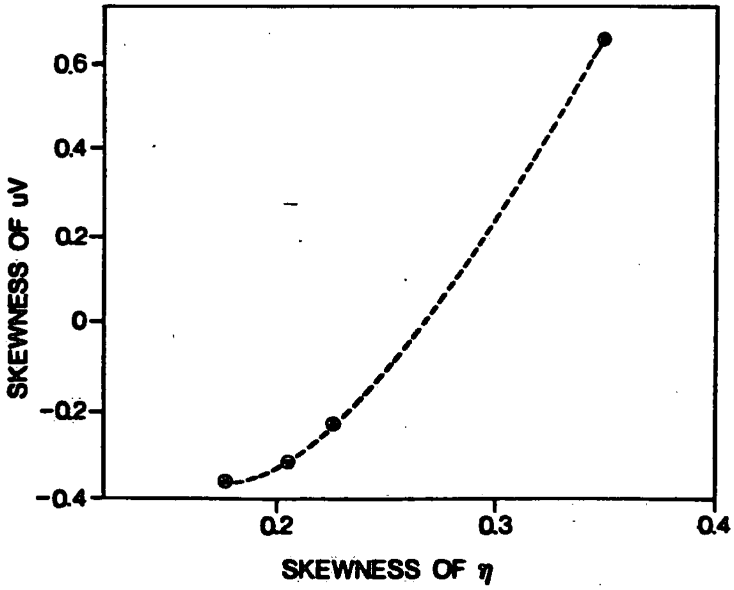


Figure 6. Skewness of uV versus skewness of η.

CONCLUDING REMARKS

These preliminary results have demonstrated the ability of our observational method to explore the velocity field beneath breaking waves. It is clear that calculations from linear theory are unable to account for the observed velocities. However, considerably more analysis must be completed before we can recommend suitable corrections to current design practices for the calculation of wave-induced loadings on structures in a wind-driven sea.

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