NWRI CONTRIBUTION 86-124

Bishop (30)

Donelan (44)

This manuscript has been submitted to Coastal Engineering
for publication and the contents are subject to change

MEASURING WAVES WITH PRESSURE TRANSDUCERS

by

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August 1986

MANAGEMENT PERSPECTIVE

Measurement of waves in lakes, especially during the freeze up period, is impractical with surface sensors. Indirect measurements using subsurface pressure fluctuations offer a simple way to beat the environment and the thieves and vandals in one stroke.

Although wave pressure systems have been used for some time, the conversion to surface waves is subject to error. This paper establishes that linear theory is adequate to convert from pressure measurement to surface waves under certain conditions. Environmental research for northern development where waves must be measured in cold air masses will benefit from this study which shows clearly that there is a need to establish both methods of analysis and its concomitant error from the truth.

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PERSPECTIVE-GESTION

Les détecteurs de surface se prêtent mal à la mesure des vagues de surface des lacs surtout pendant la période de gel. Les mesures indirectes des variations de pression sous la surface permettent de prélever des données en tout temps et rendent les instruments moins vulnérables au vol et au vandalisme.

Bien que les systèmes de mesure de la pression des vagues soient en usage depuis longtemps, la transposition des données piézométriques en terme de vagues de surface est entachée d'erreurs. La présente étude établit qu'on peut se fonder sur la théorie linéaire pour passer des mesures piézométriques à des mesures de vagues de surface dans certaines conditions. Cette étude sera particulièrement utile pour les recherches environnementales entreprises dans le cadre des projets d'aménagement dans le Grand Nord puisque la surface des eaux est en contact avec des masses d'air froides. L'étude démontre en outre qu'il faut établir les deux méthodes d'analyse et déterminer l'erreur concomitante.

Le chef,

Division de l'hydraulique

T. Milne Dick

SOMMAIRE

Bien qu'on ait commencé à mesurer le vagues au moyen de transducteurs de pression dès 1947, les résultats obtenus sont trop divergents pour qu'on puisse savoir si la théorie des ondes linéaires permet de compenser adéquatement les données piézométriques. Dans cette étude, on compare les mesures de l'élévation des vagues formées par gravité au-dessus de la surface avec les mesures simultanées de la pression sous la surface. On passe également en revue la documentation existante à ce sujet avant de présenter les résultats des essais qui ont été effectués en laboratoire dans un canal de 103 m servant à l'étude des vagues, à l'Institut national de recherche sur les eaux. Des transducteurs de pression et des capteurs capacitifs installés à fleur d'eau ont servi à recueillir des données sur les vagues monochromatiques et irrégulières à des profondeurs de 0,9 m et de 1,2 m. Les résultats indiquent que la théorie linéaire permet de compenser adéquatement les données piézométriques de sorte que la hauteur des vagues peut être estimée à 5 p. 100 près. On examine dans cette étude les raisons des écarts plus importants signalés dans les études précédentes.

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ABSTRACT

Although the measurement of waves with pressure transducers has been practised since around 1947, there still remains a considerable difference in findings on the adequacy of linear wave theory to compensate the pressure records. Measurements of surface elevation in gravity waves are compared with corresponding estimates from simultaneous subsurface pressure measurements. A review of previous work precedes a description of laboratory tests in a 103 m long wave flume at Canada's National Water Research Institute. Pressure transducers and surface-piercing capacitance wave probes are used to collect data in water depths of 0.9 and 1.2 m with monochromatic and irregular waves. Results indicate that linear theory is adequate to compensate pressure records to give surface wave heights to within five percent. Reasons for greater discrepancies in previous studies are discussed.

KEYWORDS: coastal engineering; pressure gauges; pressure response factor; waves; pressure transducer; wave measurement

INTRODUCTION

The measurement of surface gravity waves with pressure transducers has been practised since 1947 (Folsom, 1947; Seiwall, 1947). However, a controversy exists over the adequacy of the transfer function from subsurface pressure to surface wave height.

The expression for pressure beneath a progressive wave may be obtained from Bernoulli's theorem as (Lamb 1932)

$$p = -\rho g z + \rho\frac{\partial\phi}{\partial t} - \frac{1}{2}\rho(u^2 + w^2) + p_s + \rho\gamma(t) \tag{1}$$

where
- p = total pressure
- ρ = density of liquid
- g = gravitational acceleration
- z = depth of submergence (of the pressure transducer), measured positive upward from the still water level.
- ϕ = velocity potential
- u = horizontal wave orbital velocity
- w = vertical wave orbital velocity
- p_s = atmospheric pressure at the surface
- $\gamma(t)$ = a function of time only.

The first three terms on the right hand side of Equation 1 are the hydrostatic pressure, the pressure due to the passage of the wave form, and the pressure due to the local kinetic energy. One is normally interested in the departure from hydrostatic pressure

$$\frac{p'}{\rho} = \frac{\partial \phi}{\partial t} - \frac{1}{2}(u^2 + w^2) + \gamma(t) \qquad (2)$$

The magnitudes of the kinetic energy term and of $\gamma(t)$ are of second order in the wave slope, giving a first order expression

$$\frac{p'}{\rho} = \frac{\partial \phi}{\partial t} \qquad (3)$$

From linear wave theory for a progressive gravity wave

$$\frac{\partial \phi}{\partial t} = \frac{\eta \omega^2}{k} \frac{\cosh k(d+z)}{\sinh kd} \qquad (4)$$

where η = $\eta(x,t)$ = water surface position at distance x and time t

ω = $2\pi/T$ and T = wave period

d = water depth

k = $2\pi/L$ and L = wavelength

A subsurface wave pressure head fluctuation, $H_p = p'/\rho g$, can be related to the surface wave height, H, by

$$H = \frac{H_p}{K_p} \qquad (5)$$

where K_p is the pressure response factor, which, from linear theory, is

$$K_p = \frac{\cosh k(d+z)}{\cosh kd} \qquad (6)$$

To account for the differences between theory and observation, an empirical correction factor, N, has been introduced to equation (5) by many investigators:

$$H = N \frac{H_p}{K_p} \tag{7}$$

Typical of some engineering studies is the choice of N = 1.25 as an "instrument factor" by Shahul Hameed and Baba (1985). The Shore Protection Manual (1984) states "In general, N decreases with decreasing period, being greater than 1.0 for long-period waves and less than 1.0 for short-period waves." However, a review of pertinent literature reveals a considerable difference of opinion on this issue:

- Lee and Wang (1984): "In terms of wave energy spectrum, the linear transfer function is found to be good for intermediate [and deep] water depth application. The bulk of the spectral components can be faithfully recovered [by linear wave theory] except in the high frequency range. As water becomes shallower, nonlinearity effect and current influence may also become more prominent. In this case, the linear transfer function should be modified to account for these effects."
- Biesel (1982): "Simultaneous measurements of pressure and surface levels [of irregular sea waves] have shown that this first

approximation [equation (5)] is not always satisfactory... This poor agreement was rather surprising because laboratory tests, made with regular waves, have consistently shown that first order wave theory gives a very reliable estimate of the ratio between wave pressure and wave height."

- Forristall (1982): "The weight of the evidence suggests that equation (5) may be used to convert pressure measurements to wave heights, but in view of the careful work by Cavaleri et al. (1978), some doubt must remain."
- Cavaleri (1980): "It was found [in Cavaleri et al. (1978)] that waves are more attenuated than it is foreseen by the linear theory, the difference being up to 10%."
- Grace (1970): "That this theoretical prediction [equation (5)] may be in error has been demonstrated conclusively in the literature."
- Esteva and Harris (1970): "The agreement [of equation (5)] reported here is much better than most of those cited in the review paper by Grace (1970)."
- Hom-ma, Horikawa and Komari (1966): "The above equation [5] has been recognized for many years to be inaccurate to correlate [H_p] with [H] even in the case of regular wave condition [sic]."

In spite of the foregoing, some investigators still measure waves with pressure transducers giving little or no thought to the adequacy of the transfer function from pressure to wave height. The purpose of this note is to summarize existing information on the transfer function and to report on the results of some recent laboratory tests.

PREVIOUS RESULTS

Grace (1970) summarizes the data on N through 1969 and provides an extensive bibliography. The results of several investigators are shown in Table 1 and Figure 1; those of Cavaleri et al. (1978) are averages of the mean trends of their results. The variation in empirical results is surprisingly large.

The curves of Glukhovsky (1961) and of Tubman and Suhayda (1976) are averaged results determined for varying values of relative submergence z/d. The curve of Hom-ma et al. (1966) is substantially different from the results of other investigators. By omitting these three studies and plotting N versus $|z|/L$ (Figure 2), much of the variation in results (Figure 1) is removed.

From laboratory tests with monochromatic waves, Hom-ma et al. (1966) concluded that the relative depth of submergence of the pressure transducer has a negligible effect on N for $0.375 \leq |z|/d < 1$. However, from tests in the Adriatic Sea, Cavaleri et al. (1978) showed a definite relationship between N and $|z|/d$. The results of Hom-ma et al. (1966) and of Cavaleri et al. (1978) have been replotted in Figures 3 and 4 as N versus $|z|/L$. In both data sets, N tends to increase with $|z|/d$.

The laboratory and field data of Hom-ma et al. (1966), that were used to derive their relation for N plotted in Figure 1 are markedly different from their other data plotted in Figure 3. The plotted points in Figure 3 are from different laboratory tests and are in fair agreement with the results of other investigators. Reasons for the differences between the two data sets are not adequately explained in their paper. The relation for N of Hom-ma et al. (1966) shown in Figure 1 may not be applicable to other sites.

Forristall (1982) has pointed out three potential reasons for the deviation of the measured pressure from that given by linear theory.

1. The second order pressure term, due to the kinetic energy of the wave orbital motion, has been neglected.
2. Nonlinear or higher order wave effects are not considered.
3. For random waves, spectral analysis should be used rather than a wave-by-wave analysis. The latter method ignores the fact that individual waves contain energy at frequencies other than that of the inverse of the zero-crossing period.

Cavaleri (1980) used linear theory for deep water to estimate the relative contribution of the kinetic energy term to the measured pressure. He found that it could easily be of several percent. Pressure measurements from Cavaleri et al. (1978) were made with a system specifically designed to exclude dynamic pressures due to flow past the pressure probe. Nevertheless, results from Cavaleri et al. (1978) for N are comparable to those of several other investigators (Figure 2) in showing significant deviations from linear theory.

Kinsman (1965) also addressed the question of the relative importance of the kinetic energy term. In a scale analysis he found it to be generally negligible except in shallow water.

Biesel (1982) and Lee and Wang (1984) show that non-linear wave effects account for part of the discrepancy between N and unity for random waves. The pressure to wave height transfer function from second order random wave theory is shown to be slightly smaller than the linear transfer function K_p for the subharmonics and slightly greater for the superharmonics. This has the same trend as the experimental data showing N to be greater than unity at small values of $|z|/L$ and less than unity at large values of $|z|/L$.

Lee and Wang (1984) show that a current in the same direction as the waves causes an underestimation of the pressure head in the lower frequency range and an overestimation in the higher frequency range. For some cases, this could help to explain the observed empirical trend for N to exceed unity at small values of $|z|/L$ and to be less than unity at large values of $|z|/L$.

One record of surface wave and subsurface pressure data obtained simultaneously during the ARSLOE experiment was analyzed using linear wave theory, as in equation 5, by Lee and Wang (1984). The surface gauge was a Baylor type gauge. Results were similar to those of other investigators in that N exceeded unity at low frequencies and was less than unity for high frequencies. Non-linear wave and current effects were shown to account for part of the discrepancy from linear theory. Meniscus error on the Baylor gauge may have also contributed to the discrepancy.

Bergan et al. (1968) measured surface wave heights and subsurface pressures at ports in a flume wall using regular waves. Comparisons were made of pressures calculated from the wave heights with those measured by the pressure transducer (Figure 5). In general, the measured pressure was greater than the calculated pressure. Discrepancies were far less when Stokes fifth order wave theory was used instead of linear theory. Discrepancies increased with increasing wave steepness as would be expected for a finite amplitude effect. These results agree with the findings of Biesel (1982) and Lee and Wang (1984) that nonlinear effects are important in calculating subsurface pressure.

Forristall (1982) used simulated wave height and pressure data to show that N equalled unity if a spectral analysis was performed but that N varied in a manner similar to the results of Grace (1978) if a wave-by-wave analysis was done.

Forristall (1982) also performed spectral analysis on wave data collected in the Gulf of Mexico in water 20.7 m deep, with a pressure transducer mounted 3.7 m above the seafloor. Forristall found that equation (5) was satisfactory for the larger wave height records (where the signficant pressure variation exceeded 0.3 m), for frequencies such that $|z|/L$ is less than 0.5. However, for low wave height records, N exceeded unity. The latter behaviour may have been due to the recording system's poor response at low signal levels (electronic and/or numerical noise).

Grace (1978) performed spectral analysis on one ocean wave record obtained in water 11.3 m deep, with a pressure transducer on the seafloor. Equation (5) was found to be valid for frequencies such that $|z|/L$ is less than 0.3.

Simpson (1969) carried out analysis on measurements made in 6.1 m of water at high tide with $|z| = 3.7$ m. Results were in good agreement with equation (5) for frequencies such that $|z|/L$ is less than 0.4.

Esteva and Harris (1970) carried out spectral analysis on measurements made in 4.7 m of water at mean low tide. They found good agreement with equation (5) for frequencies such that d/L is less than 0.3. In their paper, the parameter n(f) is identical to $N^2(f)$ as used in this paper (D. Esteva, pers. commun., 1984). After taking the square root of n(f), agreement with equation (5) is even better than indicated in their paper.

LABORATORY TESTS

To attempt to resolve discrepancies in past results for N, tests were conducted in the wind-wave flume of the Hydraulics Laboratory at Canada's National Water Research Institute. The flume is 103 m long, 4.6 m wide and 1.5 m deep. Simultaneous measurements of surface wave heights and subsurface pressure were made using capacitance wave probes and pressure transducers. The capacitance wave probes (outside diameter 1.14 mm) consisted of teflon-coated, 24 gauge, stranded, copper wire connected to Robert Shaw Level-Tel transmitters, Model 157 with modified filters (changed from 1 to 10 Hz). The pressure transducers were Viatran Corporation Model 218 M14-15 with a 0 to 5 volt DC output signal. The transducers and wave probes were calibrated in a static mode by raising and lowering the water level in the flume. Typically this task took about two hours. Barometric pressure compensation can be important over this length of time. Accordingly, barometric pressure was measured and used to compensate the measured pressures.

Two completely separate sets of tests were conducted. Series 1 had five wave probes and four pressure transducers installed in the flume as shown in Figure 6. The transducers had a protective covering over their sheaths and were mounted on 6 cm diameter aluminum pipe (Figure 7). The pressure sensing elevation, taken to be the middle of the lower hole in the protective covering, was 0.345 m above the flume floor. The transducers were connected by Amergraph cable to a custom-built control unit (Valdmanis and Savile 1984), through Neff instrumentation amplifiers Model 126 with modified filters to a PDP 11/40 minicomputer.

The still water depth for all Series 1 tests was 1.20 m. Waves were generated by an hydraulically-powered piston-type wave board. Wave energy was dissipated at the end of the flume on a beach of fibrous matting at a slope of 1:8. Tests were run with monochromatic and pseudo-random waves. For the latter, a JONSWAP spectrum was used. Tests were conducted for peak frequencies from 0.286 to 0.50 Hz with corresponding characteristic* wave heights from 0.23 to 0.17 m

Wave and pressure data were collected for 1024 scans at 2 Hz. Wave and pressure spectra were computed using fast Fourier transform techniques after applying a cosine taper.

Series 2 tests were conducted more than one year after Series 1. They involved 2 or 3 wave probes and one pressure transducer in the same flume as in Series 1. In order to avoid spurious pressure fluctuations due to flow around the pressure transducer, the transducer was located outside the flume and was connected to ports in the flume wall at four different levels in turn. For these tests two wave probes were located 3 and 10 cm in from the flume wall on a line perpendicular to the location of the ports (close to cross-section A in Figure 6). The ports were 0.635 mm diameter holes drilled through a 9.5 mm thick steel "window". To avoid rust buildup the holes were re-pierced before each set of tests. Care was taken to purge any air in the plastic tubing from the port to the transducer.

Series 2 also included tests with the pressure transducer in the flume, mounted on the same pipe as shown in Figure 7. The transducer's oil-filled sheath had an aluminum protective covering over

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* Characteristic wave height $H_{m_0} = 4\sigma$ where σ^2 is the variance of water surface elevation.

it (Figure 8). Tests were conducted with water depths of 90 and 120 cm. The top of the protective covering was 43.75 cm above the flume floor.

Similar tests were conducted with a special sphere, 57.2 mm outer diameter, attached to a threaded pipe over the pressure transducer (Figure 8). The sphere had 16 evenly spaced 6.4 mm diameter holes drilled radially into the centre. The centre of the sphere, where the pressure is measured, was 58.6 cm above the flume floor.

For the tests with the pressure transducer in the flume, an additional wave probe was installed at the flume centreline in line with the other two probes. The transducer was located 25 cm out from the wall, in line with the wave probes.

For the Series 2 tests, the pressure transducer was connected by Electro-Oceanics cable to Neff amplifiers and then to the PDP 11/40 minicomputer. Tests were run with monochromatic waves at frequencies from 0.495 to 1.11 Hz with regular wave heights of 0.21 to 0.06 m. Wave and pressure data were collected for 1024 scans at a sampling rate which varied from 5 to 10 Hz, depending on the regular wave frequency, in order to give a 100-wave sample.

Each test began with calm water conditions, the wave board was activated, and wave conditions at the test area were given time to reach steady conditions (as observed on a strip chart recorder). Accordingly, the measured waves included waves reflected from the beach, but the reflection coefficient was less than six percent. Reflection from the beach is an additional source of error in laboratory measurements and it can be a problem at some field installations. The degree to which reflections have contaminated our measurements is assessed in the next section.

TEST RESULTS

For test Series 1, spectral estimates were formed from record lengths of 512 seconds, using a frequency resolution of 0.02 Hz. The resultant spectral estimates have 20 degrees of freedom, with the expected spectral value within 0.64 and 1.84 of the sample value at 90 percent confidence limits. It goes without saying that the large sampling errors associated with small degrees of freedom do not detract from a comparison of coincident and simultaneous surface elevation and pressure data.

The power spectral component, $S_s(f)$, of the water surface elevation can be related to the power spectral component of the subsurface pressure fluctuation, $S_p(f)$ as

$$S_s(f) = \left[\frac{N(f)}{K_p(f)}\right]^2 S_p(f) \qquad (8)$$

Values of $N^2(f)$ were determined using equation (8) by averaging the spectral estimates of the two wave probes closest to each transducer[1].

An example of the cross-spectral analysis for one test is shown in Figure 9. The top panel shows the coherence squared between one pair of wave probes and pressure transducer spectral estimates. For the frequency range over which the coherence is high, N is close to unity. This behaviour is very similar to that found by Forristall (1982), Simpson (1969), Esteva and Harris (1970) and the one spectral case in Grace (1978).

Results of N versus $|z|/L$ for Series 1 tests using linear wave theory are plotted in Figure 10. Only those values of frequency for which the wave probe spectral estimate, $S_w(f)$, is greater than or equal to 10 percent of the wave probe spectral estimate at the peak frequency, $S_w(f_p)$, are included. For these values the coherence squared is also greater than 0.95. It can be seen that N varies from 1.07 to 0.90.

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[1] Wave probe 1 stopped working soon after installation. Therefore, transducers 1 and 2 were compared to the average of wave probes 2 and 3, transducer 3 to wave probes 3 and 4, and transducer 4 to wave probes 4 and 5.

Spectral Leakage

To check for spectral leakage, the raw pressure and wave data were pre-whitened (by differentiating twice) before tapering and FFT analysis. Spectral leakage affects the high frequency part of the pressure spectra more than that of the surface elevation spectra and thus tends to make the values of N smaller at high frequencies in the untreated spectra. Results showed that spectral leakage accounted for less than one percent decrease in values of N in the untreated spectra.

Hydrodynamic Noise

For test Series 2 results for N versus $|z|/L$ from root-mean-square estimates of surface elevation and pressure are plotted in Figure 11. Also plotted in Figure 11 are the results for monochromatic tests from Series 1. The wave probe located 3 cm from the wall was used to compare with pressure measurements at the ports. Similarly, the wave probe located 10 cm from the wall was used to compare with pressure measurements made 25 cm from the wall. As seen in Figure 11 there is a moderately defined trend for N to exceed unity for small values of $|z|/L$ and to be less than unity for large values of $|z|/L$. Agreement with linear theory is much better than found by Bergan et al. (1968) under similar conditions (pressure measurements made at ports in the flume wall). A comparison of results for N between the pressure measurements at the ports with those taken in the water column shows the latter to be 2 to 3 percent less. This suggests that the effects of disturbing the flow around the transducer have a small, but noticeable, impact on the measurement of pressure. These effects will be called hydrodynamic noise.

Results for N using the special sphere can be seen in Figure 11 to be distinctly higher than the other results. Use of the sphere causes the surface wave height to be underestimated by approximately 4 percent. This contrasts with the other curves which all tend to overestimate the surface wave height. This suggests that the shape of a pressure transducer and the resulting hydrodynamic noise can influence the predicted wave heights by several percent.

Meniscus Error

A known source of error in measuring waves with surface-piercing gauges is the meniscus effect. Surface tension and contamination cause the water surface fluctuations on the probe to be somewhat less than the surrounding water body. From tests with specially cleaned probes and a laser slope gauge (K.K. Kahma, personal communication), it was determined that the meniscus error for the wave probes used in these tests could be up to -3 mm. For low frequency tests with the largest wave heights (around 20 cm), the meniscus error could contribute -1.5 percent to the calculated pressures. For high frequency tests with the smallest wave heights (around 6 cm), the corresponding error could be up to -5.0 percent. Thus, the meniscus error may account for a considerable part of N's discrepancy from unity at larger values of $|z|/L$. This error is associated with measurement of the water surface fluctuation and thus affects the evaluation of N, both with immersed and wall (port) pressure measurements. It does not, however, affect wave estimates from pressure records.

Calibration Error

Wave probe and pressure transducer calibrations are also potential sources of error. Their accuracies are approximately ±2 and ±3 percent respectively. Most of the plotted discrepancies from unity in Figures 10 and 11 are within five percent. Therefore, calibration errors could be responsible for a considerable part of the discrepancies.

White Noise

Due to the nature of the pressure response factor, the high frequency components of the pressure spectrum may be very small or may be completely lost due to instrument limitations. Lee and Wang (1984) point out that electronic and/or numerical noise can mask the true pressure signal at high frequencies. Therefore, before compensating the pressure record to get the surface wave spectrum $S_s(f)$, they suggest subtracting (assumed white) noise from the pressure spectral densities $S_p(f)$ as in

$$S_s(f) = (S_p(f) - C)/(K_p(f))^2 \quad (9)$$

where C is the average noise level. Alternately, a high frequency cutoff may be used such that

$$S_p(f) = S_p(f) \text{ for } f \leq f_c \quad (10)$$

$$S_p(f) = 0 \quad \text{for } f > f_c \quad (11)$$

where f_c is the cutoff frequency. If one of these procedures is not used, the high frequency end of the compensated pressure spectrum may bear little ressemblance to the true surface wave spectrum. At high frequencies, the value of K_p can be large, so that a small amount of noise can significantly distort the compensated pressure spectrum. Noise in the pressure signal will tend to make N less than unity at higher frequencies. In the following we illustrate the importance of this effect.

Let us assume that there is a component of noise in both the surface and pressure spectral densities so that

$$S_s(f) = S_s'(f) + n_s(f) \quad (12)$$

and

$$S_p(f) = S_p'(f) + n_p(f) \quad (13)$$

where $S(f)$ = measured spectral density

$n(f)$ = noise component

and the primes denote the true values of spectral density (uncontaminated by noise). Furthermore, let us assume that the noise is "white" so that $n_s(f) = C_s$ and $n_p(f) = C_p$, and also that $S_p'(f)/K_p = S_s'(f)$.

The expression for N becomes

$$N = \left[\frac{1 + C_s/S_s'(f)}{1 + C_p/(K_p S_s'(f))} \right]^{1/2} \quad (14)$$

Using the expression for the DHH spectrum (Donelan et al. 1985) to calculate $S_s'(f)$ equation 14 can be evaluated for selected values of C_s and C_p. If $C_s = C_p = 0$, then $N = 1$. The spectrum chosen is appropriate to the very short fetches encountered in the laboratory. The wind speed U and fetch x used were 25 m/s and 200 m respectively leading to non-dimensional fetch gx/U^2 of 3.14 and wave age (phase speed of peak of spectrum/wind speed) W of 0.112.

Results for various combinations of $C_s/S_s'(f_m)$ and $C_p/K_pS_s'(f_m)$, where f_m is the frequency at the peak of the spectrum, are shown in Figure 12. In general, if the noise ratios are both set equal to one percent, the resulting values of N exceed unity for values of f generally less than f_m, and are less than unity for vales of f generally greater than f_m. These results agree qualitatively with the trends in Figure 2. The minimization of noise levels is an important step in realizing accurate wave height estimates from pressure records.

From Figure 11, looking at the port measurements, one can see very little variation in N as a function of $|z|/d$ for $0.31 < |z|/d < 0.81$. This contrasts with the results shown in Figures 3 and 4, and with the trends predicted in Figure 12, which show the curve of N versus $|z|/L$ shifting to the right for increasing values of $|z|/d$. This suggests that the hydrodynamic noise, associated with immersing the probe in the flow, may be at least part of the reason for the observed trends in N.

Wave Reflections

The presence of reflected waves in the flume may contribute to N's discrepancy from unity. Miche (1944) discovered that the mean pressure on the bottom beneath a train of standing waves is not constant, as in a progressive wave, but fluctuates with an amplitude independent of the depth and proportional to the square of the wave height. Furthermore, the frequency of this pressure variation is twice the fundamental frequency of the waves. For two progressive waves of the same frequency travelling in opposite directions, Longuet-Higgins (1950) derived the expression

$$\frac{\bar{p}_d}{\rho} = -2a_1 a_2 \omega^2 \cos 2\omega t - gd + p_s \qquad (15)$$

where $\bar{p}_d$ = mean pressure on bottom over one wavelength

a_1 = amplitude of the incident wave

a_2 = amplitude of the reflected wave

ω = $2\pi/T$

From equation 1, using linear wave theory, the fluctuating pressure may be obtained to second order:

$$\frac{p'}{\rho} = \frac{A\omega^2}{k}[(a_1 - a_2)\cos(kx - \omega t) + 2a_2\cos kx \cos \omega t]$$

$$- \frac{\omega^2}{4\sinh^2 kd}[a_1{}^2\cos 2(kx - \omega t) + a_2{}^2\cos 2(kx + \omega t)$$

$$- 2a_1a_2\cos 2kx]$$

$$+ B^2\omega^2(a_1a_2\cos 2\omega t) - 2a_1a_2\,\omega^2\cos 2\omega t \qquad (16)$$

where $A = \dfrac{\cosh k(z+d)}{\sinh kd}$, $B = \dfrac{\sinh k(z+d)}{\sinh kd}$

From equation 16 it is clear that reflections contribute several additonal terms and in particular, the last term, which is independent of depth. However, only the first term appears at the fundamental frequency of the incident wave. This term is simply the linear supersition of the linear pressure response of two wave trains of amplitude a_1 and a_2 travelling in opposite directions.

Consequently, the value of N should not be affected by reflection if N is evaluated from the appropriate spectral estimates rather than the complete variance of the records. In the present tests, for regular wave trains, 99% of the variance was due to the input frequency of the incident waves and therefore the total variance could be used without introducing significant error.

The second and fourth terms in equation 16 are independent of the depth of measurement and therefore contribute relatively more to the variance of measured pressure as the measuring point is lowered. In deep water all terms but the fourth vanish and thus the variance of surface elevation estimated from the pressure variance will be arbitrarily large as K_p decreases indefinitely. In the absence of reflection the second term retains its first part and in water of moderate depth this term will contribute correspondingly larger amounts to the variance of pressure as the probe depth is increased. This term arises through the variation of kinetic energy that occurs as the orbital path is distorted and it increases with the eccentricity of the path.

CONCLUSIONS

There is now considerable information affirming that linear wave theory is adequate to compensate pressure data and give reliable estimates of surface wave heights (Simpson 1969, Esteva and Harris 1970, Grace 1978 (spectral analysis case), Forristall 1982, Lee and

Wang 1984, this study). A well-designed pressure transducer system with proper analysis techniques should give estimates of surface wave heights accurate to within ±5 percent. In designing such a system, the pressure signal to noise ratio must be given careful consideration, and, if necessary, be compensated. Only spectral analysis of the data, not a wave-by-wave analysis, will give adequate results. When measuring waves with pressure transducers in shallow water the linear theory pressure response factor may require modification to account for currents or wave non-linearity (Lee and Wang 1984).

Previous results, which indicate that linear theory requires a substantial correction factor N to adequately compensate pressure records, probably suffer from one or more of the following:

- inaccurate measurement of surface wave heights (including meniscus errors);
- instrument limitations (including signal to noise ratio, calibration error or drifting, hydrodynamic noise);
- analysis methods (including wave-by-wave method, spectral leakage).

ACKNOWLEDGEMENTS

Helpful comments were provided by Mr. T. Judge and Drs. D.G. Aubrey, T.M. Dick, K. Kahma, and M.G. Skafel. The assistance of Messrs. T. Nudds and G. Duncan in the laboratory tests is appreciated. The programming assistance of Ms. J. Dowell is gratefully acknowledged.

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TABLE 1. Summary of Other Investigations with Results Plotted in Fig. 1

Reference	Location	Water Depth (m)	$\frac{\lvert z \rvert}{d}$	Method of Comparison
Draper (1957)	English Channel	6.1	1.0	Spectral
Glukhovsky (1961)	Caspian Sea	10.7-11.6	.09-.90	Spectral
Hom-ma et al. (1966)	Sea of Japan and Lab	7.6 < 0.9	.87 ?	Spectral
Shooter and Ellis (1967)	Buzzard's Bay, Mass.	20.1	1.0	Spectral
Tubman and Suhayda (1976)	East Bay, Louisiana	19.2 and 5.3	? ?	Wave by wave, some spectral
Grace (1978)	Pacific Ocean near Honolulu and Lab	11.3 and 3.51 2.90	1.0 .95 .94	Wave by wave
Cavaleri et al. (1978)	Adriatic Sea near Venice	16	.24-.71	Spectral

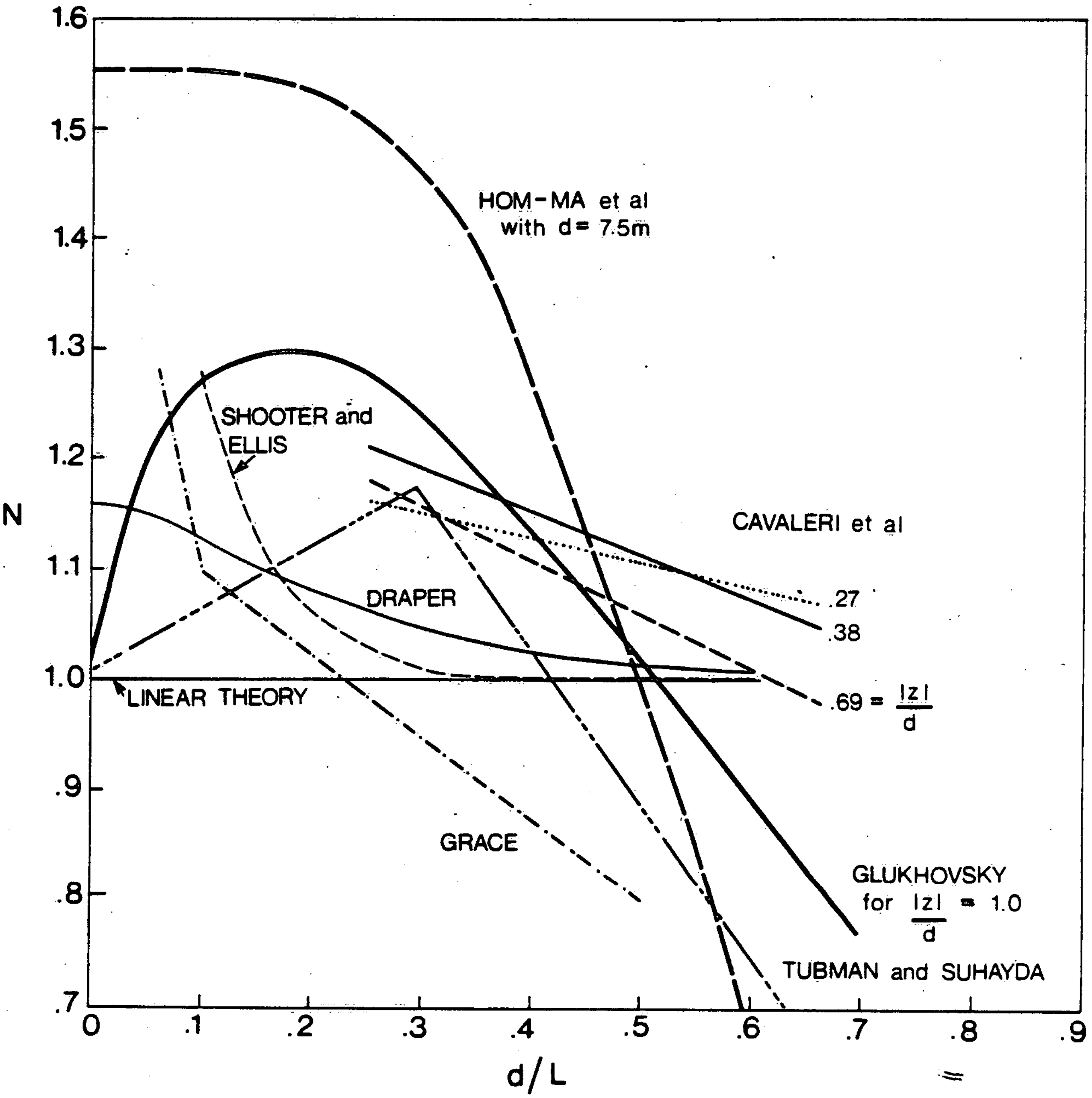


Figure 1. Relationship between correction factor N and relative water depth (d/L) from previous work.

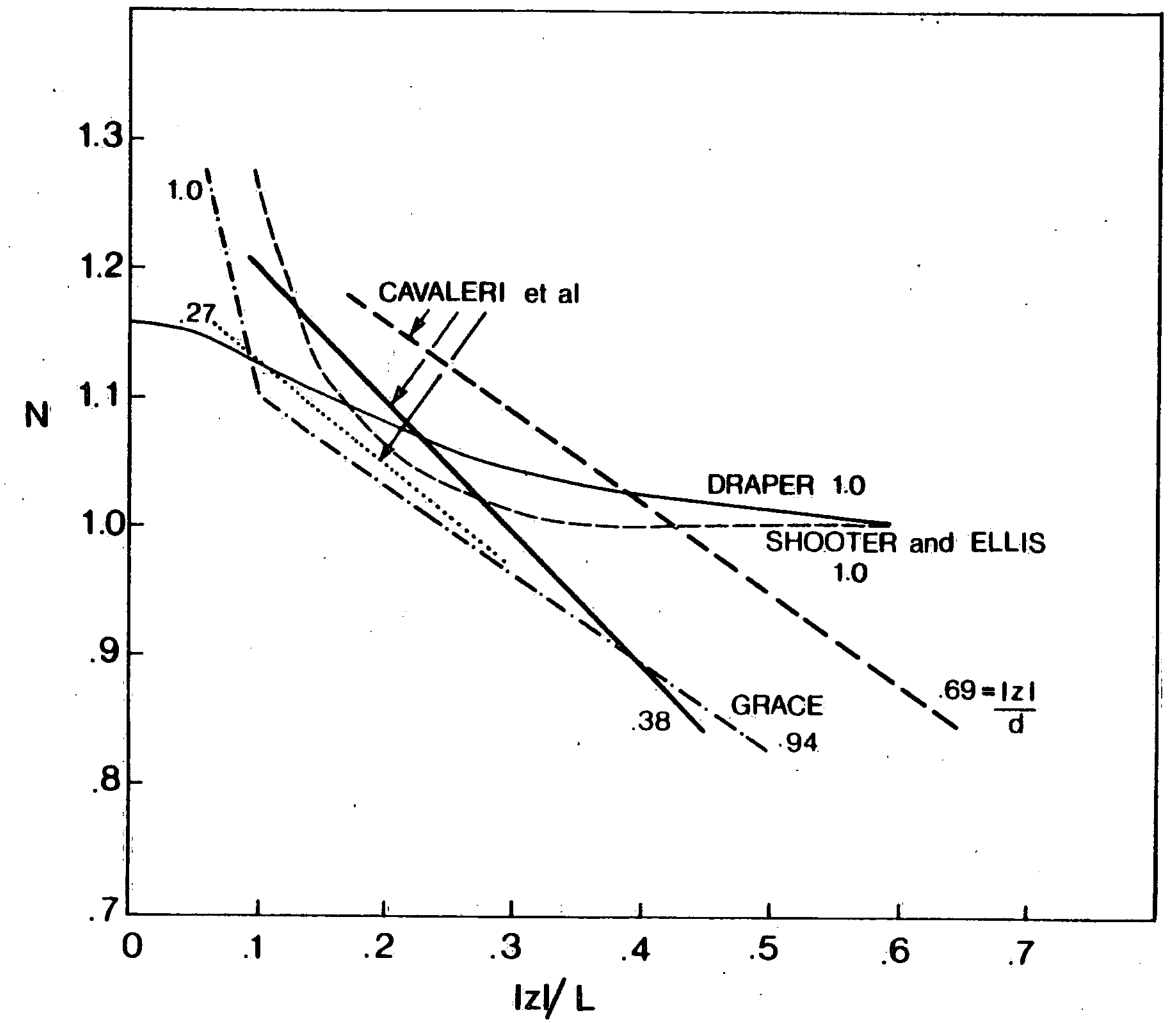


Figure 2. Relationship between correction factor N and ratio of depth of submergence to wavelength ($|z|/L$) from previous work.

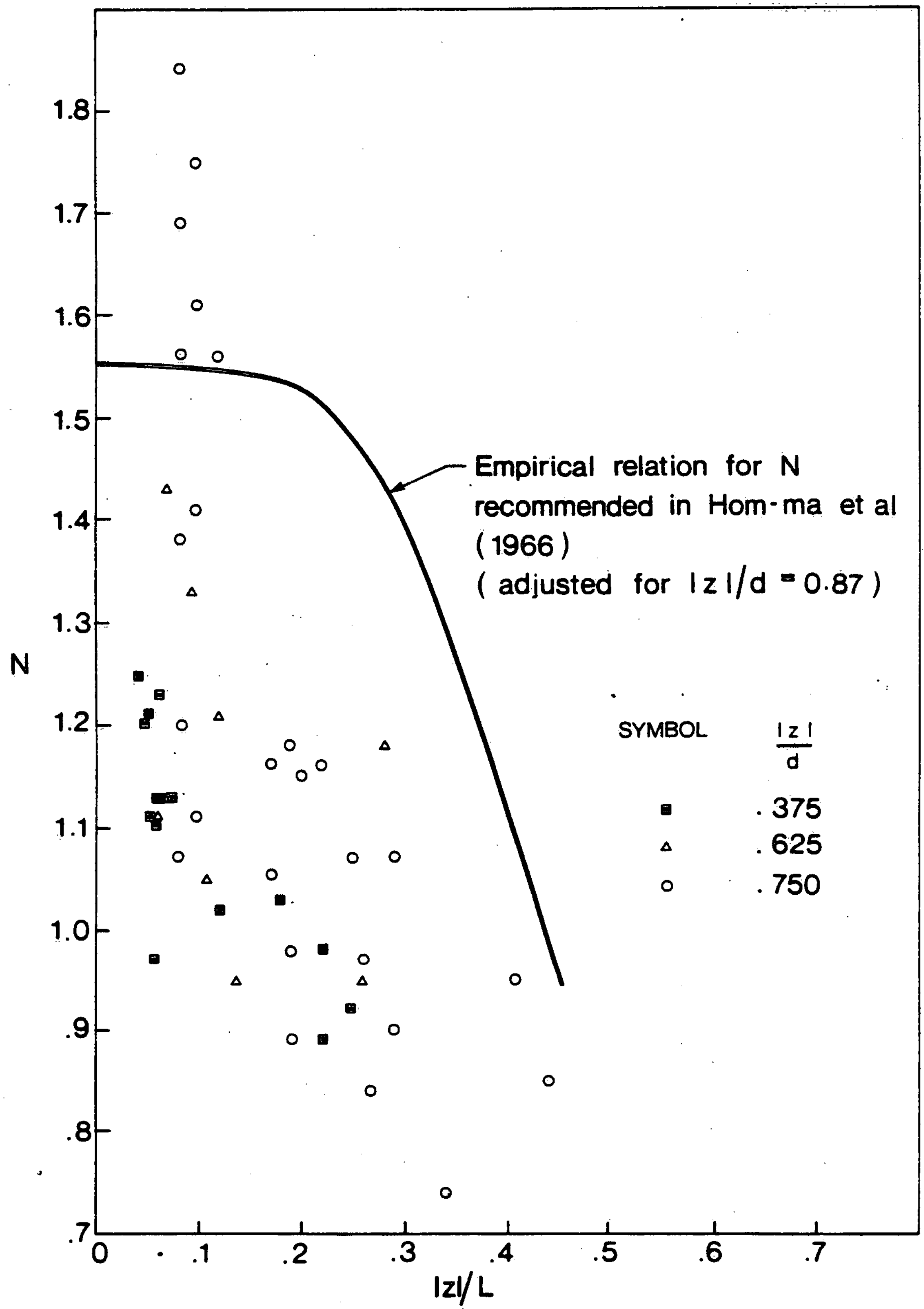


Figure 3. Relationship between correction factor N and relative submergence ($|z|/d$) from Hom-ma et al. (1966).

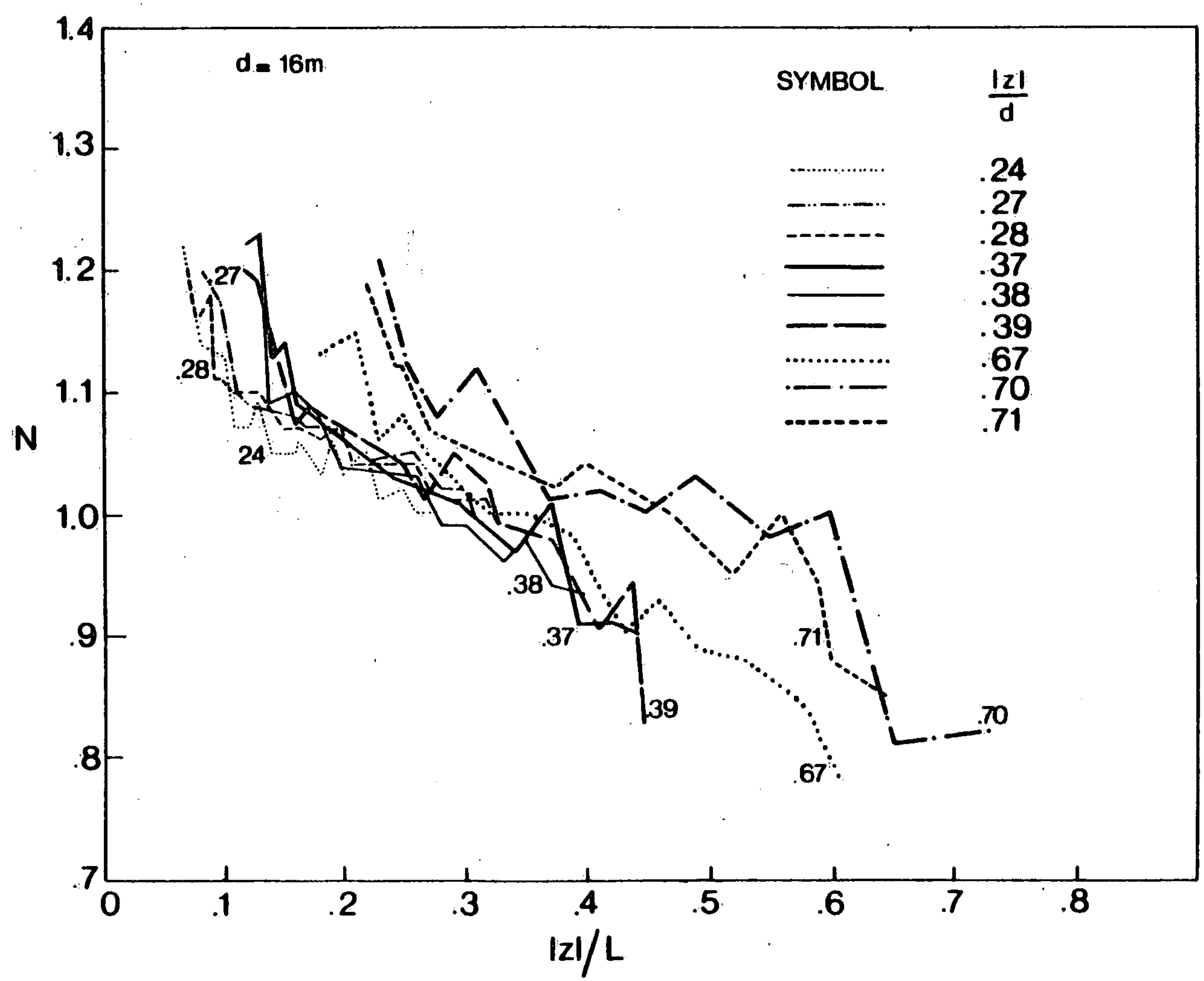
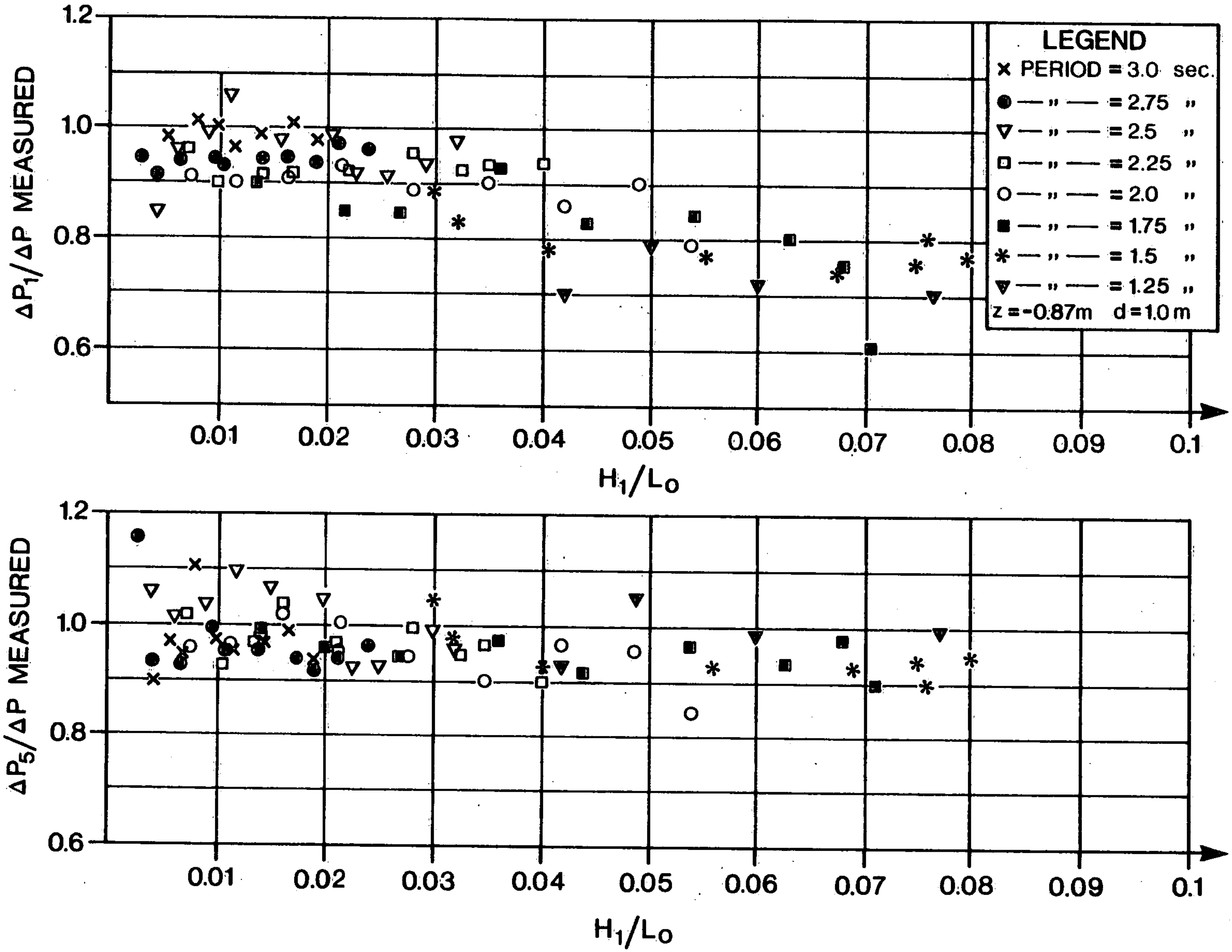


Figure 4. Relationship between correction factor N and relative submergence ($|z|/d$) from Cavaleri et al. (1978).



ΔP_1 = PRESSURE VARIATION ACCORDING TO 1st ORDER WAVE THEORY

ΔP_5 = " " " " 5th " " "

FIGURE 5. RESULTS FOR N FROM BERGAN ET AL. (1968)

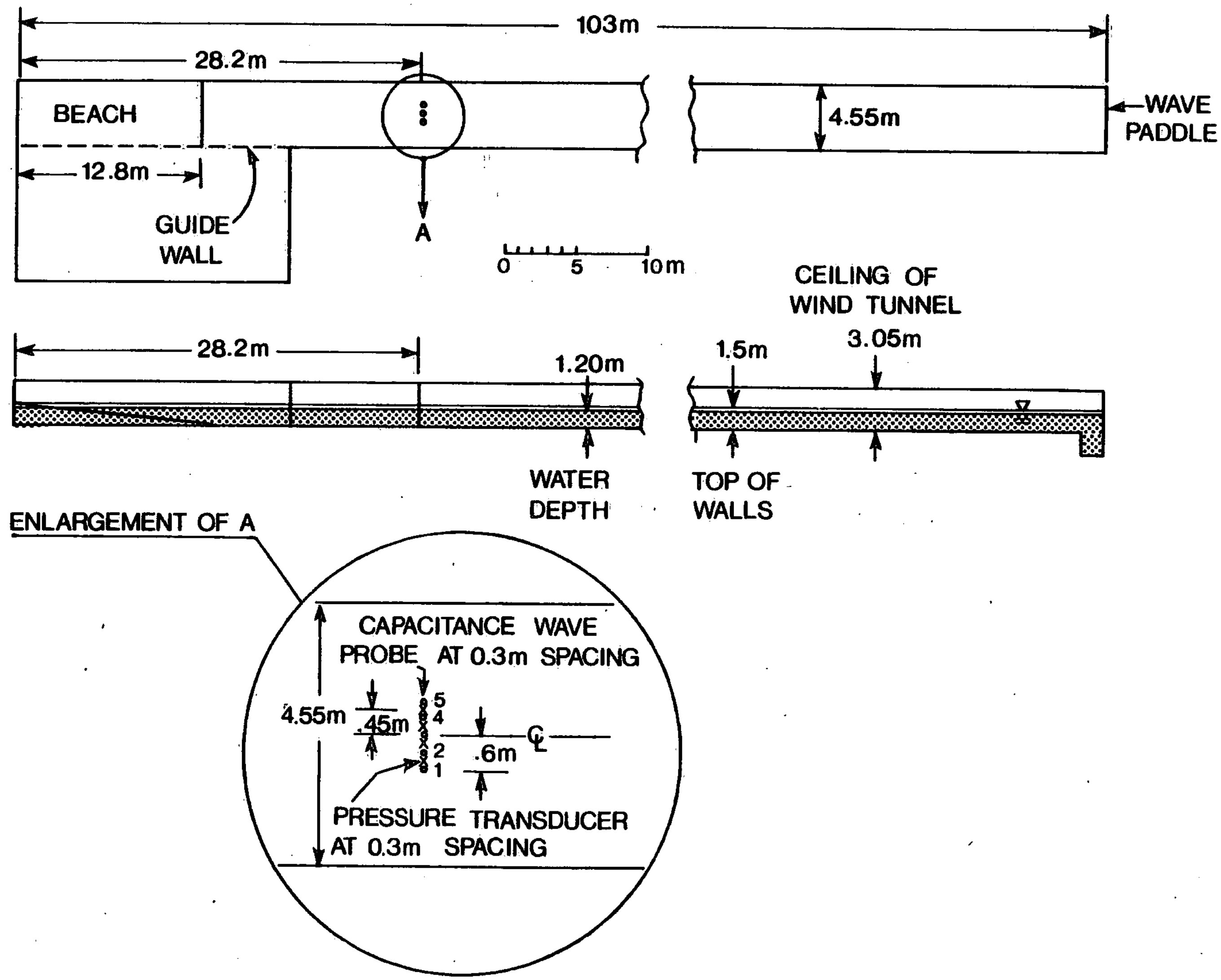
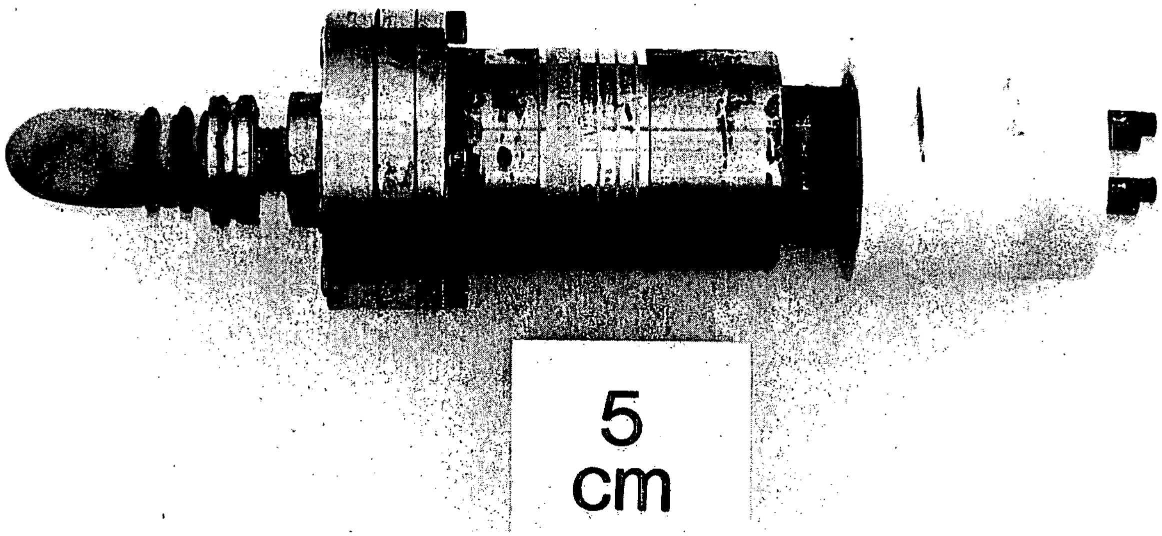


Figure 6. Experimental arrangement in NWRI's wind-wave flume.



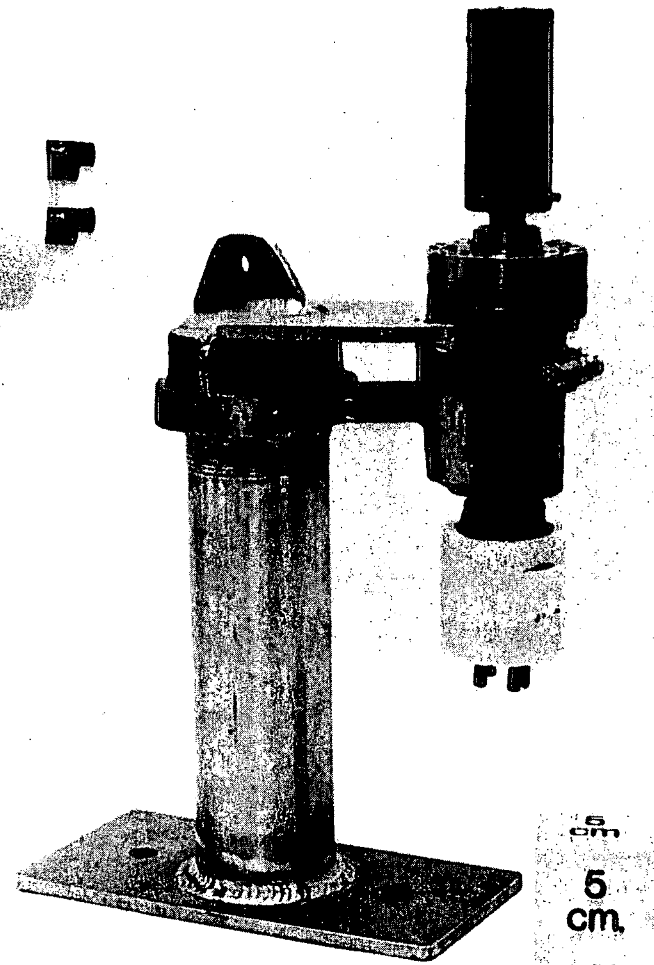
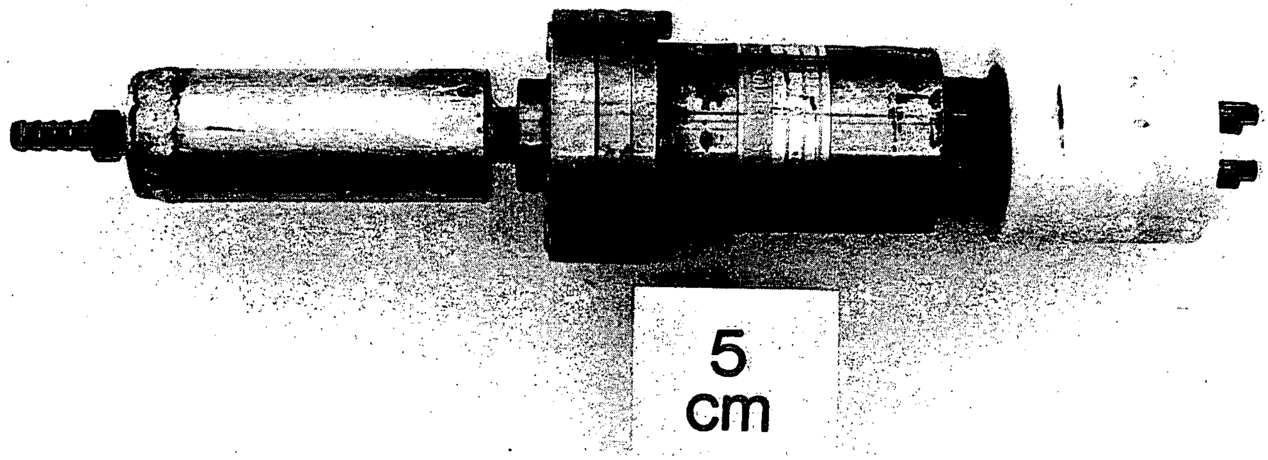


FIGURE 7. VIEWS OF PRESSURE TRANSDUCER, OIL-FILLED SHEATH, PIPE BRACKET AND PROTECTIVE COVERING.



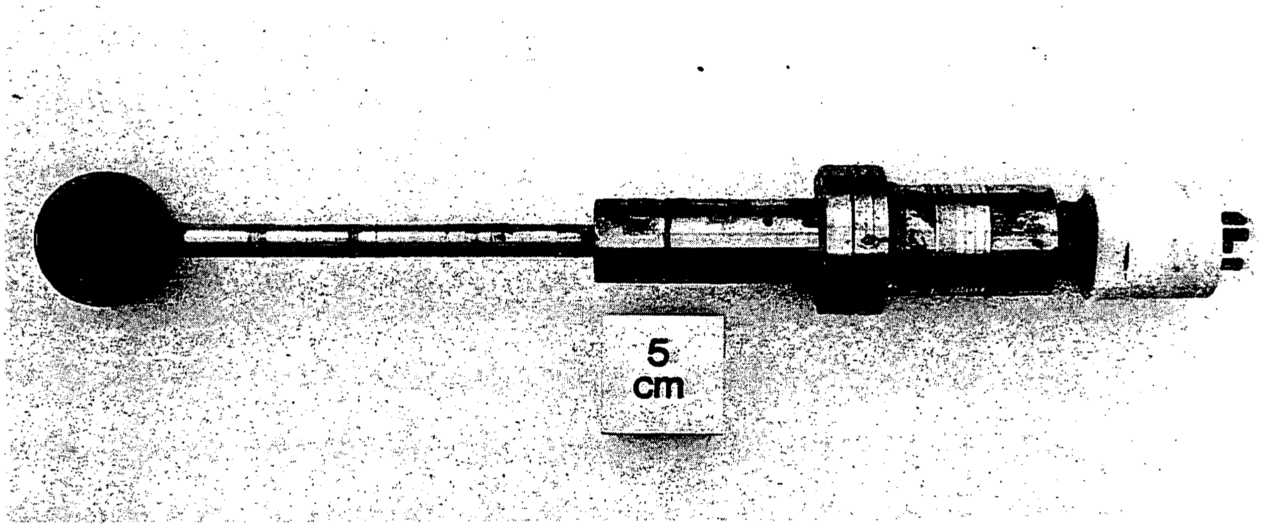


FIGURE 8. VIEWS OF PRESSURE TRANSDUCER WITH PROTECTIVE COVERING (Series 2) AND SPECIAL SPHERE

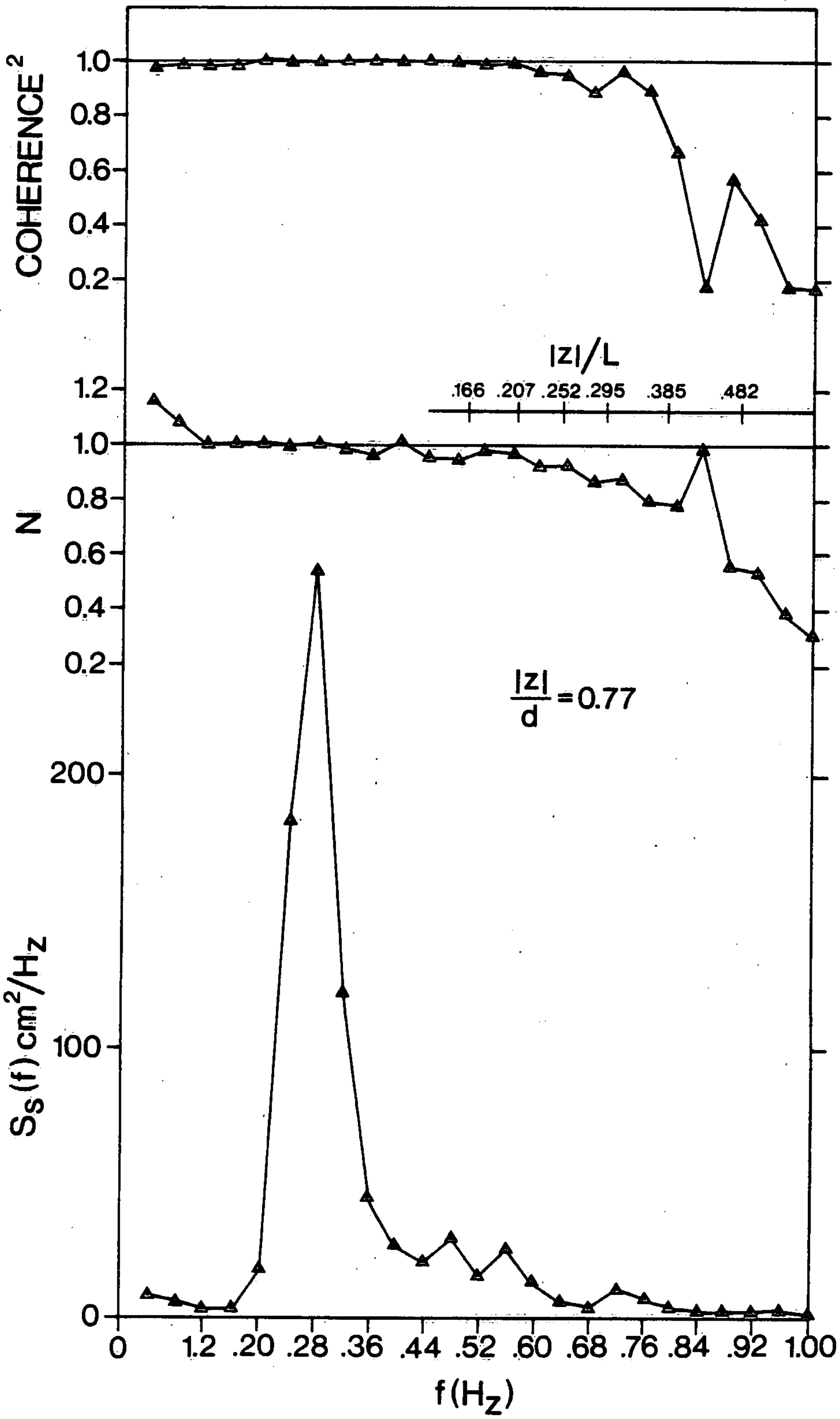


FIGURE 9. CROSS-SPECTRAL ANALYSIS OF WAVE PROBE AND PRESSURE TRANSDUCER SPECTRAL ESTIMATES

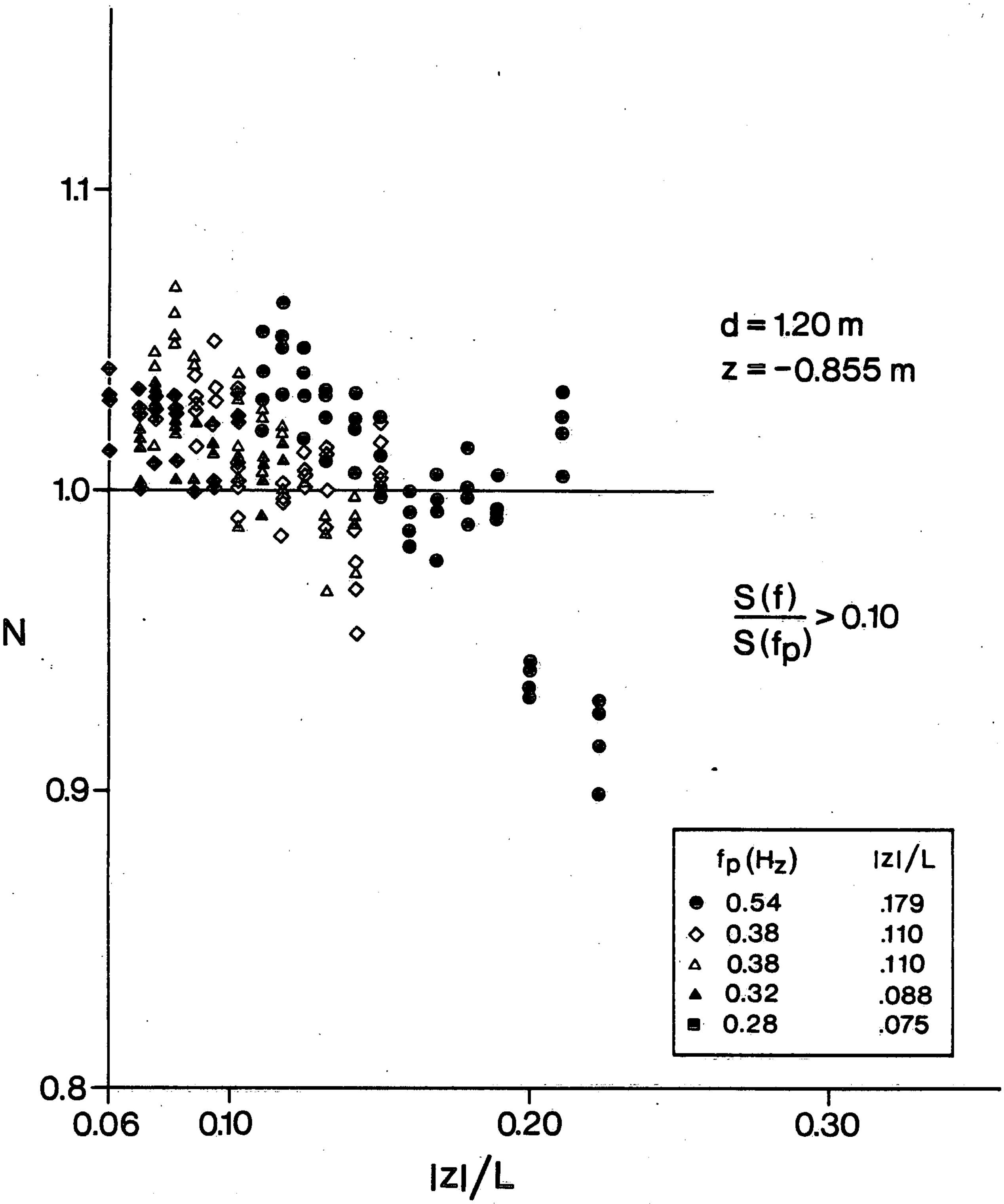


FIGURE 10. EXPERIMENTAL RESULTS FROM IRREGULAR WAVE TESTS OF THIS STUDY

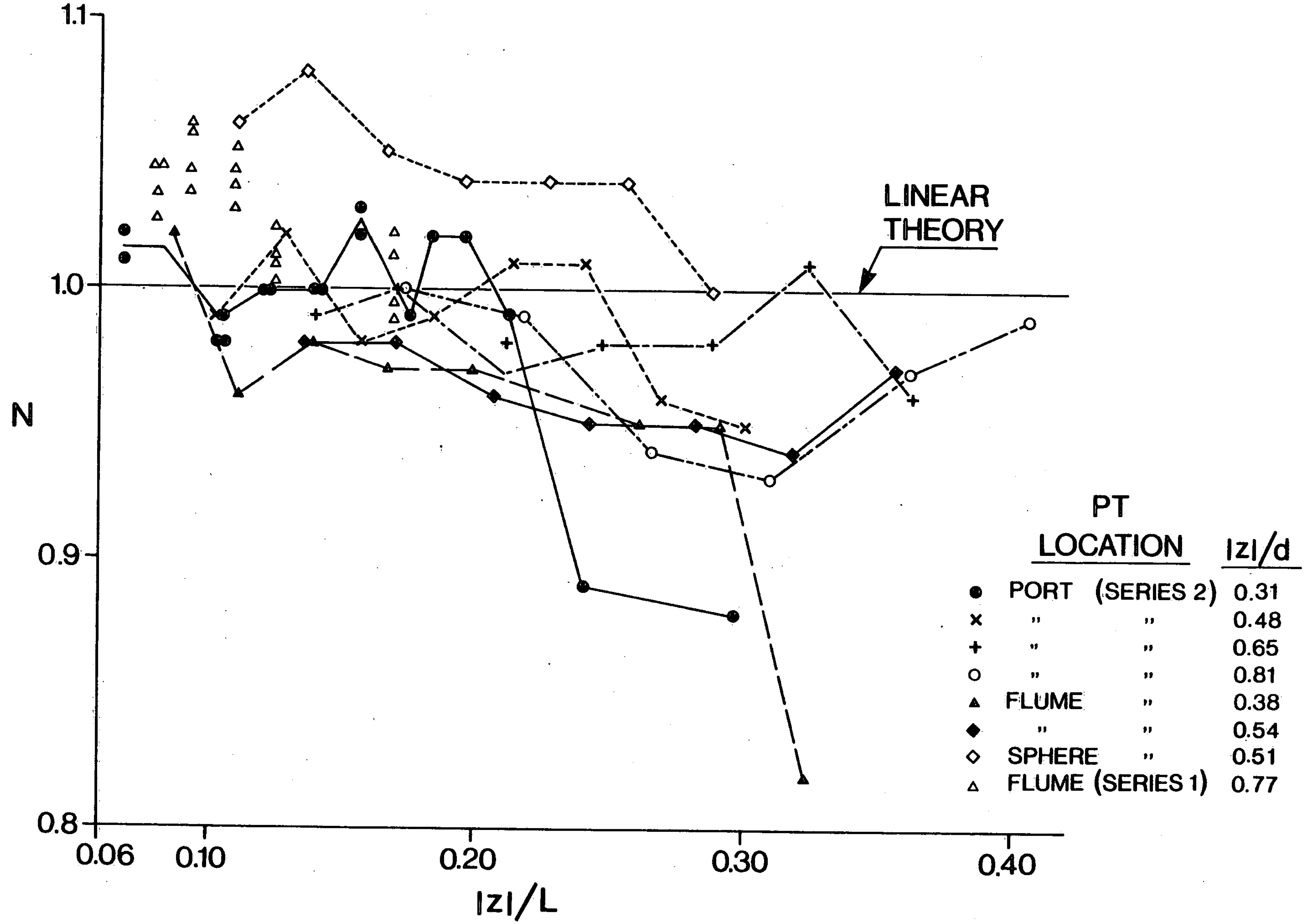


FIGURE 11. EXPERIMENTAL RESULTS FOR REGULAR WAVE TESTS OF THIS STUDY

	$\frac{\lvert z \rvert}{d}$	$\frac{C_s}{S'_s(f_m)}$	$\frac{C_p}{S'_p(f_m)}$
a	.167	.01	.01
b	.333	.001	.001
c	.333	.01	.01
d	.333	.01	.001
e	.500	.01	.01
f	.333	.001	.01

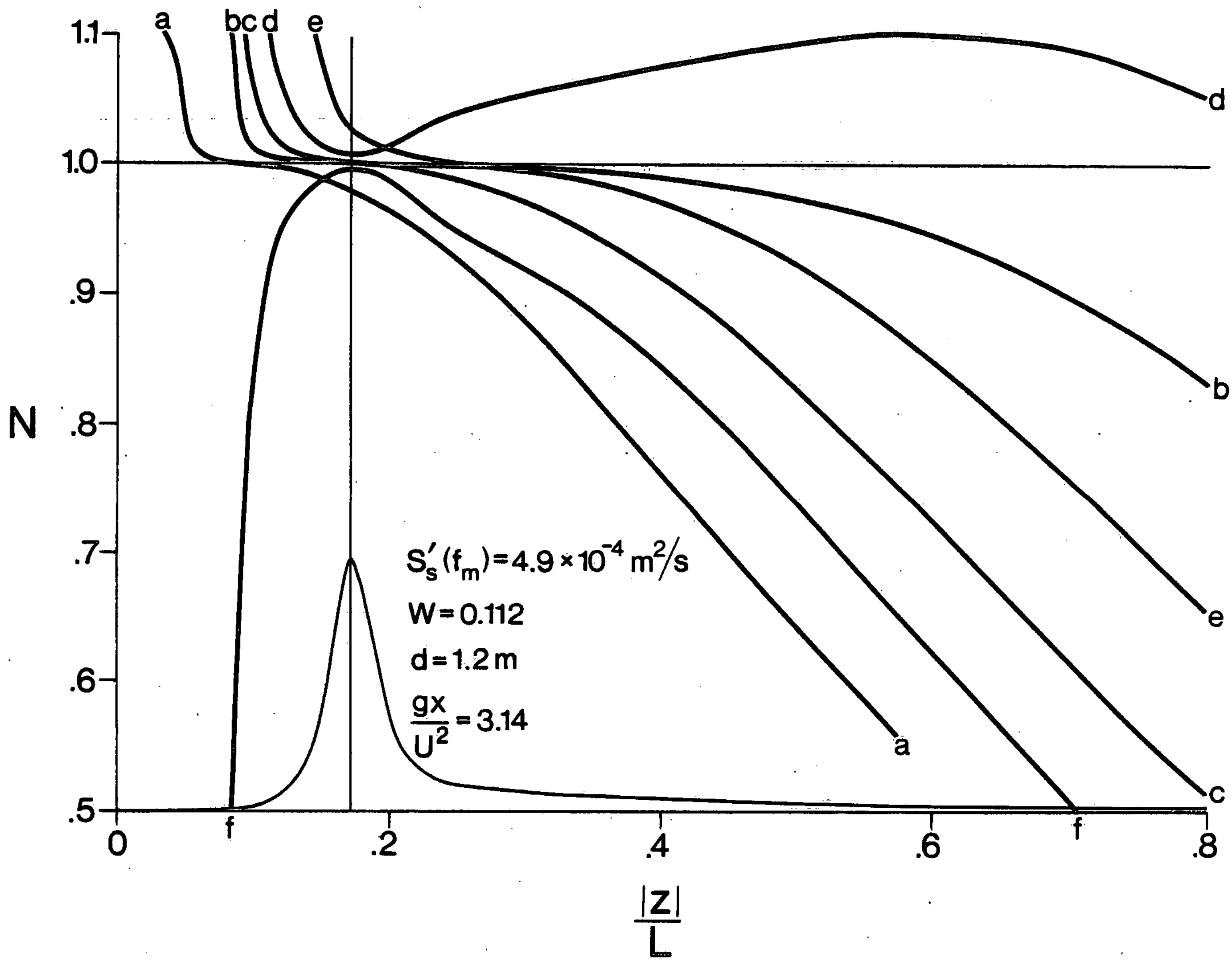


Figure 12. Simulated results for N when signals are contaminated by white noise.