

**A PROGRAM FOR SOLVING THE UNSTEADY
FLOW EQUATIONS BY IMPLICIT FOUR-POINT
METHOD: APPLICATION TO NIAGARA RIVER**

by

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ABSTRACT

The implicit four-point method is presented for computing water level elevations and vertically integrated velocities in the open channel. The theory and FORTRAN program listing of the model are given. Two examples of applications (Vistula River, Poland and Niagara River) have also been included. In particular, the importance of incorporating the flow dynamics in the accurate estimation of contaminant loadings from Niagara River into Lake Ontario is demonstrated. Such a model synthesis approach is essential for the prediction of the transport and fate of the toxic contaminants in the coastal zone.

ACKNOWLEDGEMENT

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INTRODUCTION

In this report the numerical method for solution of the one-dimensional unsteady flow problem in a river channel has been developed. The method may be applied for a number of problems connected with transport of the water in the river channel, and also to the problem of dispersion of sediment and contaminants. In particular, the model can be used to investigate river-lake or river-sea interactions at the river mouth. In such cases, the solution of the one-dimensional diffusion equation should be performed (Fread, 1977). In the method a simple rectangular geometry of the river channel was applied; but the computer program may be easily adjusted to the real geometry if the data on depth-flow area, and depth-width of the channel are available from field observations.

COMPUTATION ALGORITHM

The most popular form of the Saint-Venant equations, describing the unsteady flow in an open channel is as follows, Amein, 1968:

$$y(x,t) [\partial v(x,t)/\partial x] + v(x,t) [\partial y(x,t)/\partial x] + [\partial y(x,t)/\partial t] = 0 \quad (1)$$

$$[\partial v(x,t)/\partial t] + v(x,t)[\partial v(x,t)/\partial x] + g [\partial y(x,t)/\partial x] + g(S_t - S_0) = 0 \quad (2)$$

In the equations (1),(2), x is the distance along the channel; t is the time; $v(x,t)$ is the vertically integrated velocity; $y(x,t)$ is the depth; S_0 is the bottom slope; S_f is the friction slope; and g is the acceleration due to gravity. Exact derivation of these equations may be found in Baltzer (1968), Abbott (1979). The additional assumptions concerning the flow conditions are:

- one-dimensional flow,
- width of the channel is constant,
- no lateral inflow or outflow,
- friction slope is given by the Manning formula

$$S_f = M^2 Q^2 / A^2 R^{4/3}, \text{ where}$$

A is the cross section of the flow, R is the hydraulic radius and M is the Manning's coefficient.

Boundary conditions for equations (1) and (2) often require discharges or stages to be known at the first and last cross section of the river. Alternatively, the rating curve for the last cross section may be employed. Initial conditions require the starting velocity and depth to be given along the channel. In the implicit method reported here, equations (1) and (2) are first approximated by a set of finite difference equations, which are then solved by numerical algebraic techniques.

In the finite difference approximation, the average values of the functions and derivatives in equations (1) and (2) are taken in the four-point scheme (cf. the box scheme, Lam and Simpson, 1976) as follows:

$$y = \frac{1}{4} (y_{i,j} + y_{i,j+1} + y_{i+1,j} + y_{i+1,j+1}) \quad (3)$$

$$\partial y / \partial x = \frac{1}{2} (y_{i+1,j} + y_{i+1,j+1} - y_{i,j} - y_{i,j+1}) / \Delta x \quad (4)$$

$$\partial y / \partial t = \frac{1}{2} (y_{i,j+1} + y_{i+1,j+1} - y_{i,j} - y_{i+1,j}) / \Delta t \quad (5)$$

where the (i,j) indices designate x-position, and t-position, respectively. Substituting equations (3)-(5) into equations (1) and (2), we obtain:

$$\begin{aligned} & y_{i,j+1} + y_{i+1,j+1} + \frac{1}{2} (\Delta t / \Delta x) [y_{i+1,j+1} (v_{i+1,j} + v_{i+1,j+1}) \\ & - y_{i,j+1} (v_{i,j+1} + v_{i,j}) + y_{i+1,j} v_{i+1,j+1} - y_{i,j} v_{i,j+1}] - y_{i,j} - y_{i+1,j} \\ & - \frac{1}{2} (\Delta t / \Delta x) (v_{i,j} y_{i,j} - y_{i+1,j} v_{i+1,j}) = 0 \end{aligned} \quad (6)$$

$$\begin{aligned} & v_{i+1,j+1} + v_{i,j+1} + \frac{1}{4} (\Delta t / \Delta x) (v_{i+1,j+1}^2 - v_{i,j+1}^2 + 2v_{i+1,j} v_{i+1,j+1} \\ & - 2v_{i,j} v_{i,j+1}) + g(\Delta t / \Delta x) (y_{i+1,j+1} - y_{i,j+1}) + g(\Delta t / 2) (S_{f_{i,j}} + S_{f_{i+1,j}} \\ & + S_{f_{i,j+1}} + S_{f_{i+1,j+1}}) - v_{i,j} - v_{i+1,j} - 2g\Delta t S_0 + g(\Delta t / \Delta x) (y_{i+1,j} - y_{i,j}) \\ & - \frac{1}{4} (\Delta t / \Delta x) (v_{i,j}^2 - v_{i+1,j}^2) = 0 \end{aligned} \quad (7)$$

The unknown values in equations (6) and (7) are $y_{i,j+1}$, $y_{i+1,j+1}$ and $v_{i,j+1}$, $v_{i+1,j+1}$. $S_{f_{i,j+1}}$ and $S_{f_{i+1,j+1}}$ are functions of these unknown values. If N is the total number of discrete points along x-axis, $2N-2$ equations are available in the form of equations (6) and (7). Two additional equations are obtained from the boundary conditions to make up a total of $2N$ equations for the $2N$ unknowns. Since the terms with subscript j are known at the current time level, the set of equations (6)-(7) may be rewritten as:

$$\eta_i = y_i + y_{i+1} + \frac{1}{2} (\Delta t / \Delta x) [y_{i+1} (a_i + v_{i+1}) - y_i (b_i + v_i) + c_i v_{i+1} - d_i v_i] - e_i = 0, \quad \text{for } i = 1, 2, \dots, N-1 \quad (8)$$

$$\begin{aligned} \xi_i &= v_i + v_{i+1} + \frac{1}{4} (\Delta t / \Delta x) (v_{i+1}^2 - v_i^2 + 2f_i v_{i+1} - 2g_i v_i) \\ &\quad + g(\Delta t / \Delta x) (y_{i+1} - y_i) + g(\Delta t / 2) (S_{t_i} + S_{t_{i+1}}) + h_i = 0, \\ &\quad \text{for } i = 1, 2, \dots, N-1 \end{aligned} \quad (9)$$

The coefficients $a_i, b_i, c_i, d_i, e_i, f_i, g_i, h_i$ are all known quantities and their explicit form is given below:

$$a_i = v_{i+1,j}, \quad d_i = y_{i,j} \quad (10)$$

$$b_i = v_{i,j}, \quad f_i = v_{i+1,j} \quad (11)$$

$$c_i = v_{i+1,j}, \quad g_i = v_{i,j} \quad (12)$$

$$e_i = y_{i,j} + y_{i+1,j} + \frac{1}{2} (\Delta t / \Delta x) (y_{i,j} v_{i,j} - y_{i+1,j} v_{i+1,j}) \quad (13)$$

$$\begin{aligned} h_i &= -2g\Delta t S_0 - v_{i,j} - v_{i+1,j} + g(\Delta t / \Delta x) (y_{i+1,j} - y_{i,j}) \\ &\quad - \frac{1}{4} (\Delta t / \Delta x) (v_{i,j}^2 - v_{i+1,j}^2) + \frac{1}{2} g \Delta t (v_{i,j}^2/y_{i,j}^{4/3} + v_{i+1,j}^2/y_{i+1,j}^{4/3}) M^2 \end{aligned} \quad (14)$$

To simplify equations (8) and (9) further, we use the following definition and rearrange the indices:

$$\left. \begin{array}{l} y_i = x_{2i-1}, \quad v_i = x_{2i} \\ y_{i+1} = x_{2i+1}, \quad v_{i+1} = x_{2i+2} \\ \eta_i = F_{2i-1}, \quad \xi_i = F_{2i} \end{array} \right\} \text{for } i = 1, 2, \dots, N-1 \quad (15)$$

So that equations (8) and (9) now become:

$$\begin{aligned}
 F_{2i-1} &= x_{2i-1} + x_{2i+1} + \frac{1}{2} (\Delta t / \Delta x) [x_{2i+1} (x_{2i+2} + a_i) \\
 &\quad - x_{2i-1} (x_{2i} + b_i) + c_i x_{2i+2} - d_i x_{2i}] - e_i \\
 &= 0
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 F_{2i} &= x_{2i} + x_{2i+2} + \frac{1}{4} (\Delta t / \Delta x) (x_{2i+2}^2 - x_{2i}^2 + 2f_i x_{2i+2} - 2g_i x_{2i}) \\
 &\quad + g(\Delta t / \Delta x) (x_{2i+1} - x_{2i-1}) + g(\Delta t M^2 / 2) (x_{2i+2}^2 / x_{2i+1}^{4/3} \\
 &\quad + x_{2i}^2 / x_{2i-1}^{4/3}) + h_i \\
 &= 0
 \end{aligned} \tag{17}$$

$$F_{2N-1} = Q_{Lj} - x_1 x_2 = 0 \tag{18}$$

$$F_{2N} = x_{2N} - \varphi(x_{2N-1}) = 0 \tag{19}$$

The last two equations are furnished by the boundary conditions. For example, in F_{2N-1} , the discharge Q_{Lj} as a function of time (j) is given, whereas F_{2N} uses the rating curve at the last cross section. The solution of equations (16)-(19) requires the Newton-Raphson iterative scheme. The computation starts from an initial approximation of the solution vector \bar{x} , $\bar{x}_0 = (x_1^0, x_2^0, \dots, x_{2N}^0)$. After substituting the initial vector \bar{x}_0 into equations (16)-(19), a vector of residuals R_0 is obtained:

$$\left| \begin{array}{c} R_1^0 \\ R_2^0 \\ \vdots \\ R_{2N}^0 \end{array} \right| = \left| \begin{array}{c} F_1(x_1^0, x_2^0, \dots, x_{2N}^0) \\ F_2(x_1^0, x_2^0, \dots, x_{2N}^0) \\ \vdots \\ F_{2N}(x_1^0, x_2^0, \dots, x_{2N}^0) \end{array} \right| \tag{20}$$

The first-order corrections p_1^0, \dots, p_{2N}^0 for components x_1, x_2, \dots, x_{2N} may be found by solving the set of equations:

$$\begin{vmatrix} P_1^0 \\ \vdots \\ P_{2N}^0 \end{vmatrix} = \begin{vmatrix} \frac{\partial F_1}{\partial x_1}(x_1^0, \dots, x_{2N}^0), \dots, \frac{\partial F_1}{\partial x_{2N}}(x_1^0, \dots, x_{2N}^0) \\ \vdots \\ \frac{\partial F_{2N}}{\partial x_1}(x_1^0, \dots, x_{2N}^0), \dots, \frac{\partial F_{2N}}{\partial x_{2N}}(x_1^0, \dots, x_{2N}^0) \end{vmatrix}^{-1} \times \begin{vmatrix} R_1^0 \\ \vdots \\ R_{2N}^0 \end{vmatrix} \quad (21)$$

Then, the first-order approximation of the vector \bar{x} is

$$\begin{vmatrix} x_1^1 \\ \vdots \\ x_{2N}^1 \end{vmatrix} = \begin{vmatrix} \frac{\partial F_1}{\partial x_1}(x_1^0, \dots, x_{2N}^0), \dots, \frac{\partial F_1}{\partial x_{2N}}(x_1^0, \dots, x_{2N}^0) \\ \vdots \\ \frac{\partial F_{2N}}{\partial x_1}(x_1^0, \dots, x_{2N}^0), \dots, \frac{\partial F_{2N}}{\partial x_{2N}}(x_1^0, \dots, x_{2N}^0) \end{vmatrix}^{-1} \times \begin{vmatrix} F_1(x_1^0, \dots, x_{2N}^0) \\ \vdots \\ F_{2N}(x_1^0, \dots, x_{2N}^0) \end{vmatrix} + \begin{vmatrix} x_1^0 \\ \vdots \\ x_{2N}^0 \end{vmatrix} \quad (22)$$

In general, the k-th approximation of the vector \bar{x} is,

$$\begin{vmatrix} x_1^k \\ \vdots \\ x_{2N}^k \end{vmatrix} = \begin{vmatrix} \frac{\partial F_1}{\partial x_1}(x_1^{k-1}, \dots, x_{2N}^{k-1}), \dots, \frac{\partial F_1}{\partial x_{2N}}(x_1^{k-1}, \dots, x_{2N}^{k-1}) \\ \vdots \\ \frac{\partial F_{2N}}{\partial x_1}(x_1^{k-1}, \dots, x_{2N}^{k-1}), \dots, \frac{\partial F_{2N}}{\partial x_{2N}}(x_1^{k-1}, \dots, x_{2N}^{k-1}) \end{vmatrix}^{-1} \\ \times \begin{vmatrix} F_1(x_1^{k-1}, \dots, x_{2N}^{k-1}) \\ \vdots \\ F_{2N}(x_1^{k-1}, \dots, x_{2N}^{k-1}) \end{vmatrix} + \begin{vmatrix} x_1^{k-1} \\ \vdots \\ x_{2N}^{k-1} \end{vmatrix} \quad (23)$$

The iteration procedure will be repeated until $\|\bar{R}\| \leq \epsilon$, where $\|\bar{R}\| = 2N$

$\sqrt{\sum_{i=1}^n R_i}$, and ϵ is the tolerance error. Elements of Jacobian matrix in equations (21)-(23) are computed from equations (16)-(19), i.e.,

$$(\partial F_{2i-1}/\partial x_{2i-1}) = 1 - \frac{1}{2}(\Delta t/\Delta x)(b_i + x_{2i}) \quad (24)$$

$$(\partial F_{2i-1}/\partial x_{2i}) = -\frac{1}{2}(\Delta t/\Delta x)(d_i + x_{2i-1})$$

$$(\partial F_{2i-1}/\partial x_{2i+1}) = 1 + \frac{1}{2}(\Delta t/\Delta x)(a_i + x_{2i+2})$$

$$(\partial F_{2i-1}/\partial x_{2i+2}) = \frac{1}{2}(\Delta t/\Delta x)(c_i + x_{2i+1})$$

$$(\partial F_{2i}/\partial x_{2i-1}) = -g(\Delta t/\Delta x) - \frac{2}{3}g\Delta tM^2(x_{2i}^2/x_{2i-1}^{7/3}) \quad (26)$$

$$(\partial F_{2i}/\partial x_{2i}) = 1 - \frac{1}{2}(\Delta t/\Delta x)(g_i + x_{2i}) + g\Delta tM^2(x_{2i}/x_{2i-1}^{4/3}) \quad (27)$$

$$(\partial F_{2i}/\partial x_{2i+1}) = g(\Delta t/\Delta x) - \frac{2}{3}g\Delta tM^2(x_{2i+2}^2/x_{2i+1}^{7/3}) \quad (28)$$

$$(\partial F_{2i}/\partial x_{2i+2}) = 1 + \frac{1}{2}(\Delta t/\Delta x)(f_i + x_{2i+2}) + g\Delta tM^2(x_{2i+2}/x_{2i+1}^{4/3}) \quad (29)$$

PROGRAM DESCRIPTION

The program for solving the unsteady flow equations was written in FORTRAN IV language and was run on the CDC CYBER 172 computer. Organization of the program corresponds closely to the algorithm described. On the each time step of computations, the following operational subroutines are employed:

- COEFF, subroutine for computing coefficients in the equations (10)-(14).
- PRSTR, subroutine for computing the right-side of the equations (16)-(19).
- JAKVKL, subroutine for computing the Jacobian matrix of equations (21)-(22).
- LINVIF, library subroutine (IMSL library for computing matrix inversion.
- DRVK, subroutine for output data processing.

Listing of the program and examples of input and output data are given in the Appendix.

APPLICATIONS

The proposed method is supported by two examples of applications. The first example refers to the recent paper by Stepien, 1984. In that paper, the time-spatial evolution of the flow on the 60 km long reach of the Vistula River, Poland, was tested.

Flow conditions on that reach of the river are controlled by hydro-power station Wtoctawek. Figure 1 shows the diagram of the river segment and location of the control depth gauges. The water level and discharge records in the 679.4 km of the river (cross section Wtoctawek) are shown on Fig. 2. These records were taken as a left boundary condition (eq. 18).

Depths of flow computed for two control gauges and comparison with the observed data are given in Fig. 3. It is easily seen that amplitude, phase and shape of the wave are reproduced correctly.

In the second example, conditions of flow on the lower reach of the Niagara River were simulated. The length of the reach (10 km) is much shorter than in the former case, but problem is similar. In particular, the flow undergoes the cyclic fluctuations corresponding to the emptying and refilling of the two reservoirs used to generate hydroelectricity at the hydro-power stations, located in the middle part of the Niagara River (Fig. 4). The daily records of discharge through Canadian (I) and U.S.A.(II) hydro-power stations and discharge of the Niagara River in the Ashland Avenue gauge (III) for 10-12 May 1983 are shown on Fig. 5 (Falkenirk and Yee, 1985). As the first computational cross section (segment 1, Fig. 4) is just located below the reservoirs, the total discharge there is the sum of the above mentioned records (upper curve on Fig. 5). Since the total length of the modelled reach is 10 km only, it is assumed that the flow is kinematic, i.e. the waves are moving without significant modification

of the shape. The results of computations confirm this assumption. Figure 6 shows the computed values of discharge at three computational cross sections, located at 8, 5, 2 km, respectively, upstream of the mouth of Niagara River. The discharge varies periodically over a range of from 5400 to 8200 m³/s. These variations of discharge will certainly influence the variation of contaminant concentrations in the river. In particular, we are interested to find out if the concentration of toxic contaminants (e.g. 1,2,3,4-TeCB) observed in the Niagara Bar area in Lake Ontario are affected in turn by the concentration in the Niagara River.

In the simplest case, we can assume that the concentration at the river mouth is inversely proportional to the river discharge (Turk, 1980). Based on this hypothesis, we obtained the estimated 1,2,3,4-TeCB concentration as a function of the discharge at the river mouth (Fig. 7). This estimated concentration was then used by a computer model, POLTRA (Lam and Simons, 1982) as the input concentration to Lake Ontario. The computed two-dimensional distributions of 1,2,3,4-TeCB in the Niagara Bar area at 15, 27 and 42 hours after the release are shown in Fig. 8.

In general, the model reconstructed reasonably well the isolated patch of high concentrations in the north-east part of the Niagara Bar as shown in the observed data at 15 hours after the release (Carey and Fox, 1984; Fig. 8). This patch apparently results from the alternating low-flow and high-flow condition in the river. In other words, at low flow, the river concentration is high and

creates the patch which moves along with the river plume in the lake; at high flow the river concentration is immediately diluted and the patch in the lake appears to be isolated. After some time (e.g. 27 hours after the release), the patch has moved out of the Niagara Bar area, until at a much later time (e.g. 42 hours after the release), a new patch is formed near the river mouth when the flow slows down again (Fig. 8). The quantitative comparison of computed and observed data, however, should be made with caution. Such a comparison is limited by the following factors:

- Only several measurement points in the area are covered by the POLTRA computational grid.
- No hourly record of concentrations was available at the river mouth during the simulation period to offer more accurate boundary conditions for the model POLTRA. The assumed average value of concentration ($10 \mu\text{g/L}$) at release time was interpolated from weekly data (Oliver and Nicol, 1983).
- The rectangular shape of the channel of the lower Niagara River was taken for convenience. A more accurate channel shape and a more sophisticated method of computing the discharge in the first cross section should have been used.
- The simple, hypothetical relationship between discharge and concentration according to Turk (1980) is usually valid in the low-to-moderate range of river discharges. Further

confirmation with high flow measurements, however, is needed.

CONCLUSIONS

Based on the results of the two applications, we concluded that the model FRMT is appropriate for analyzing flow dynamics and can be integrated with coastal zone hydrodynamic models such as POLTRA to study river-lake or estuary-sea interactions. Such a model synthesis approach is important for the accurate estimation of toxic contaminant loadings into Lake Ontario and the prediction of transport and fate of the contaminants in the coastal zone.

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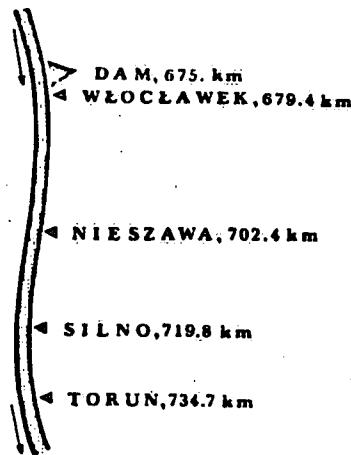


Fig. 1 The diagram of the Vistula River, Poland; location of the control depth gauges.

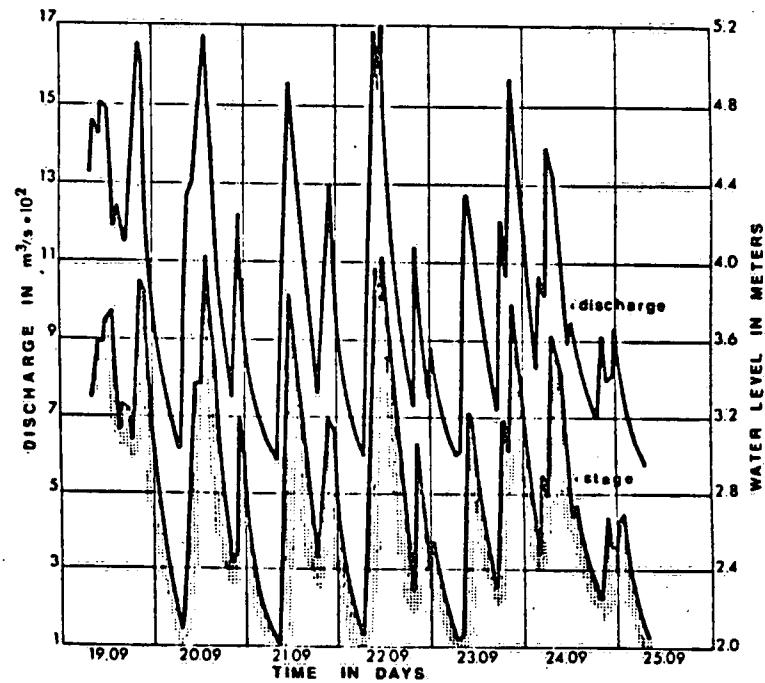


Fig. 2 The water level and discharge records, gauge Włocławek, Vistula River, Poland, 19-25 September 1977.

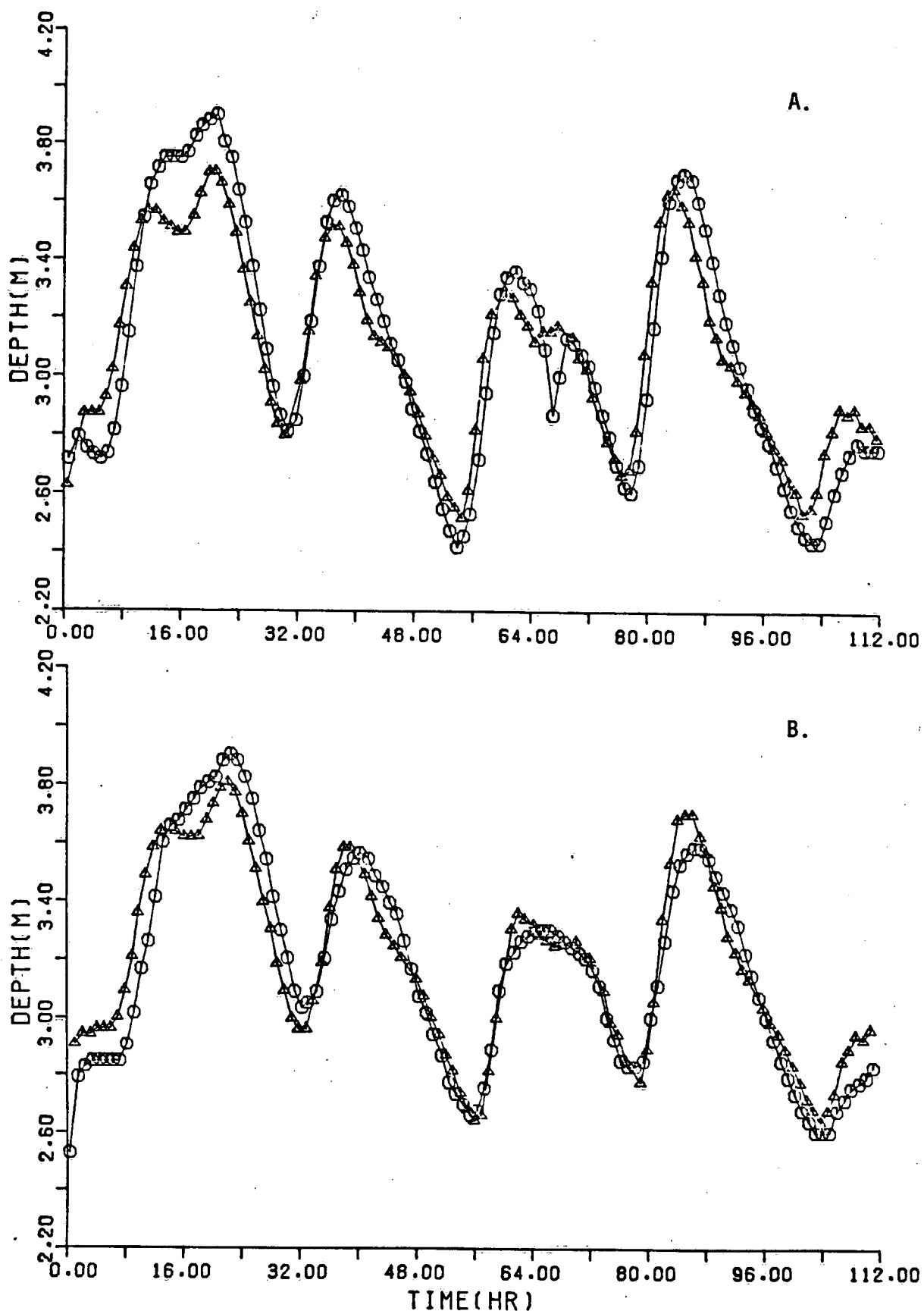


Fig.3 Computed (Δ) and observed (O) depths of flow in the control gauges Silno (A.) and Torun (B.), Vistula River, Poland, 19-25 Sept. 1977.

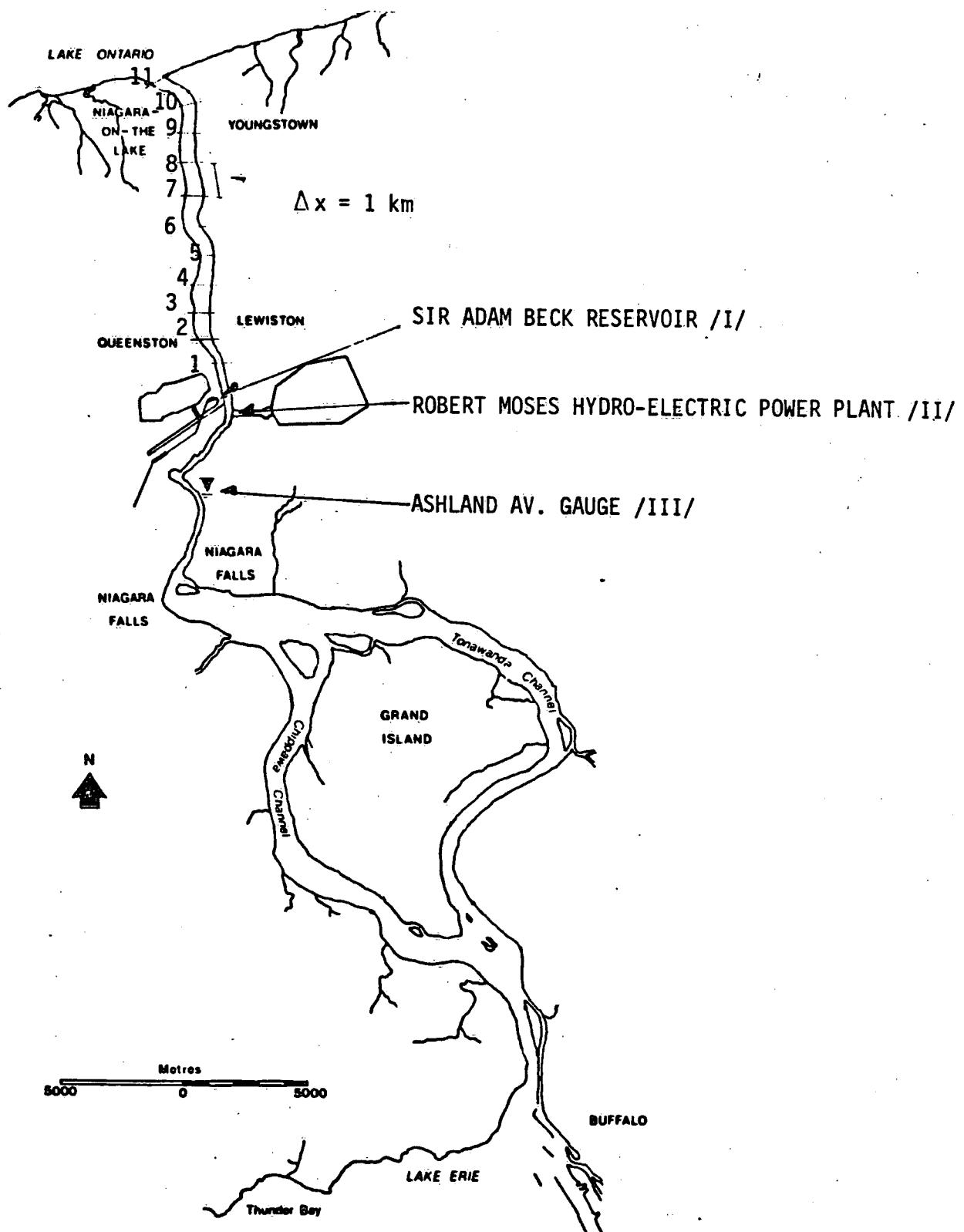


Fig.4 Diagram of Niagara River; location of the computational cross-sections.

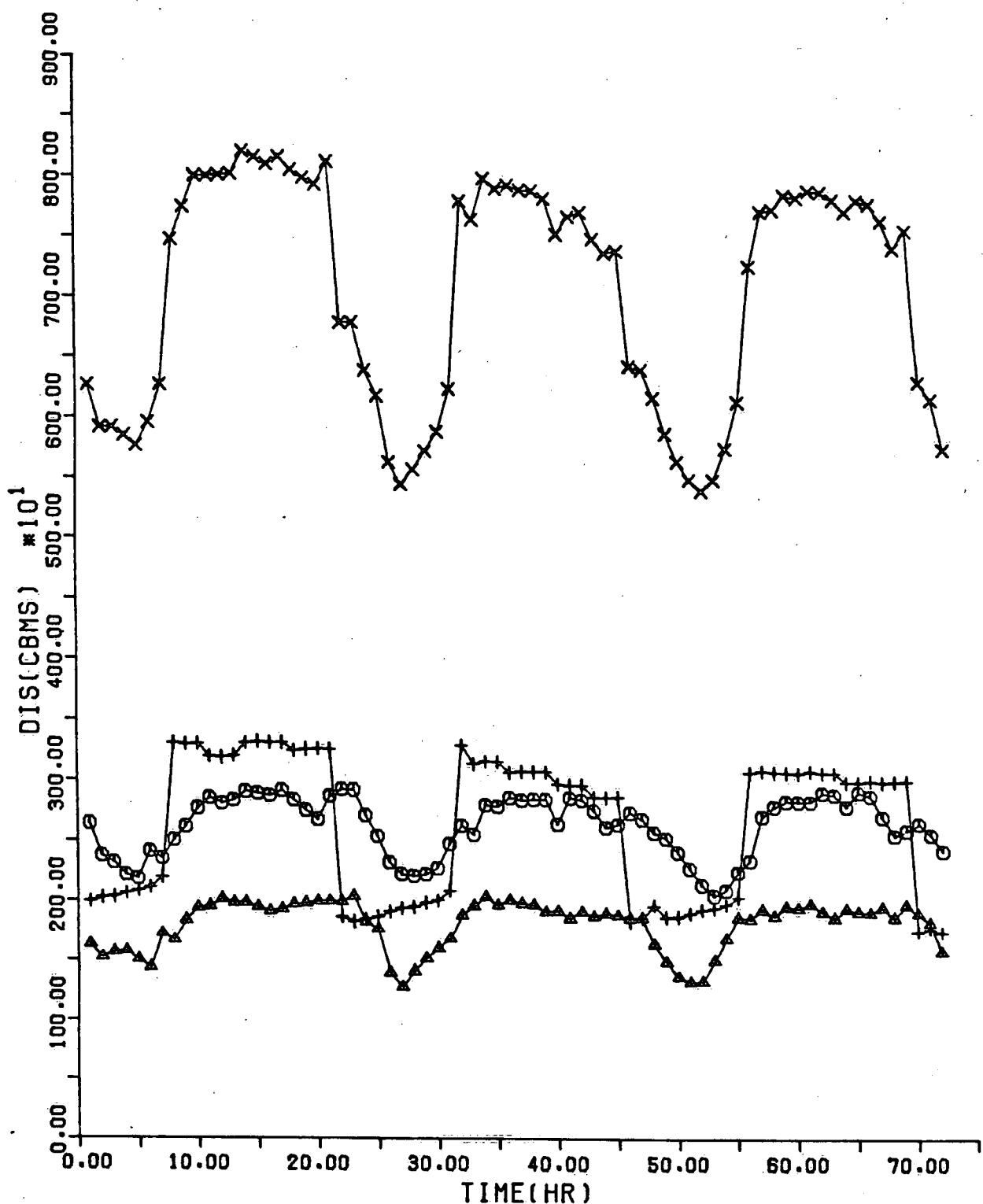


Fig. 5 Discharge through hydropowerstations : Canada(I)-/o/, U.S.A.(II)-/△/, Niagara River discharge, Ashland Av. gauge (III)-/+/.
Upper diagram-discharge in the first boundary cross-section (see Fig.4).

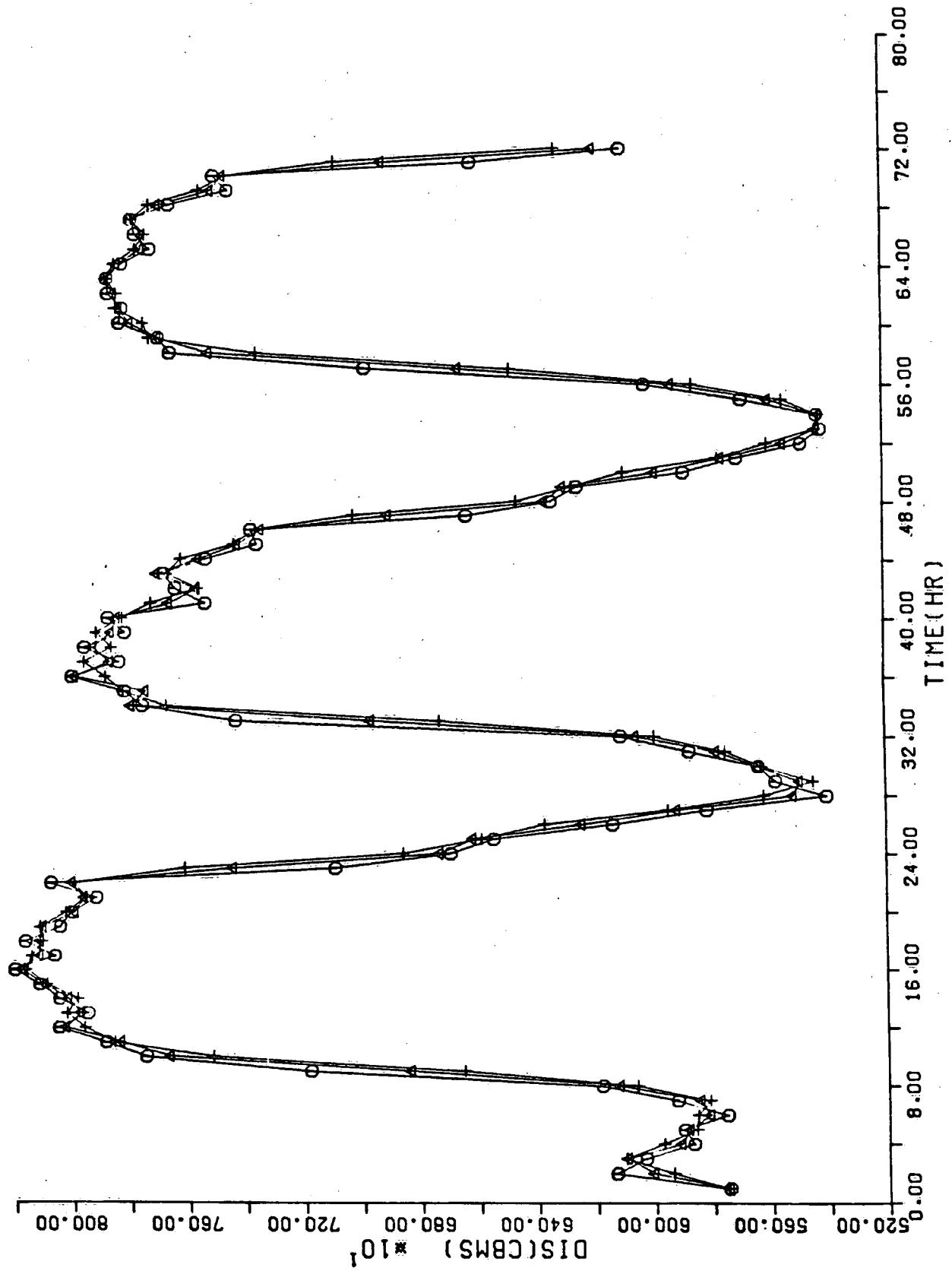


Fig. 6 Computed discharge in cross-sections located 2 (+), 5 (Δ) and 8 (\circ) km upstream of Niagara River mouth.

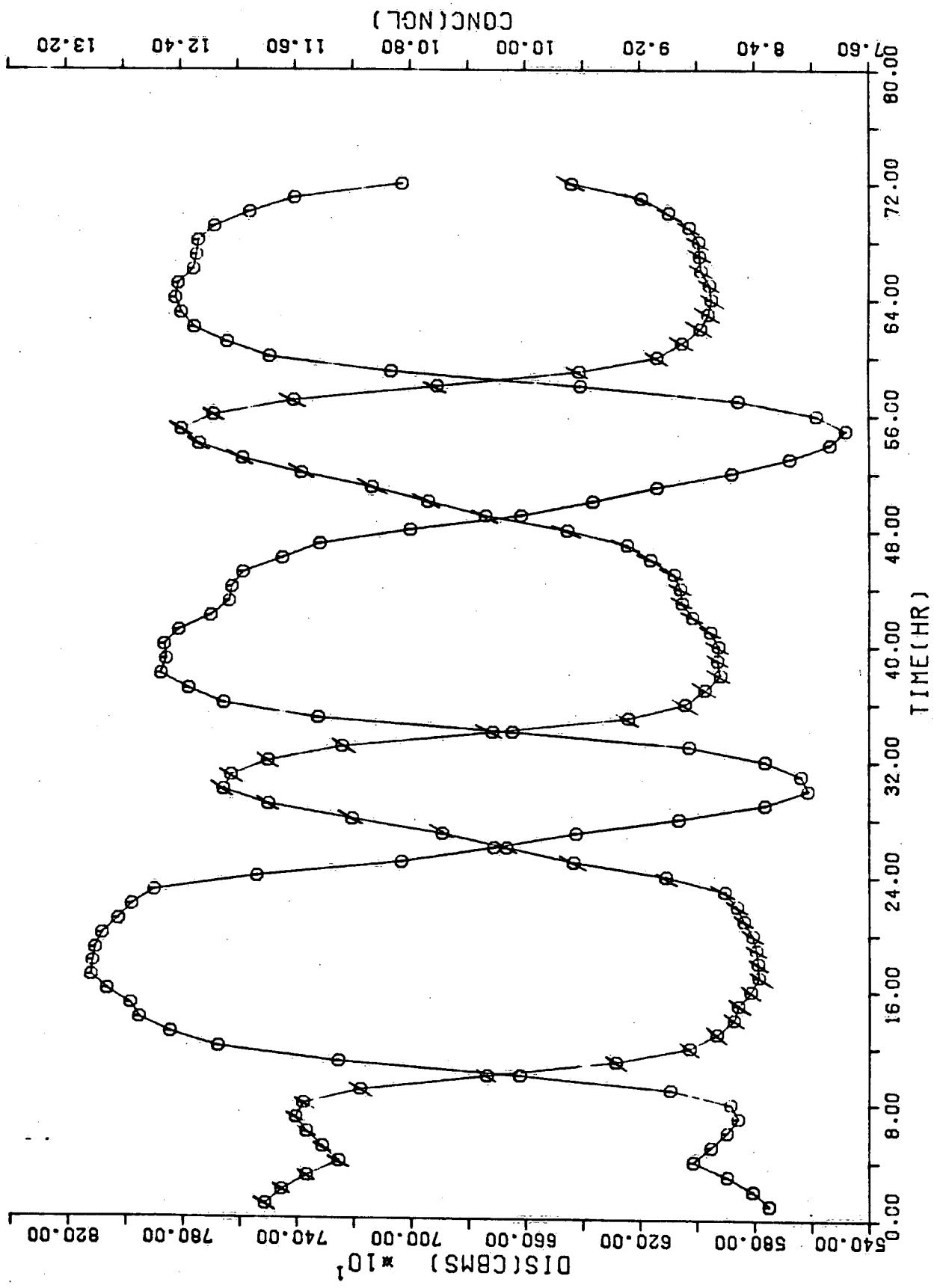


Fig.7 Computed discharge at the eleventh boundary cross-section (Fig.4) at the river mouth (○) and assumed concentration of 1,2,3,4-TeCB (ϕ), 10-12 May 1983.

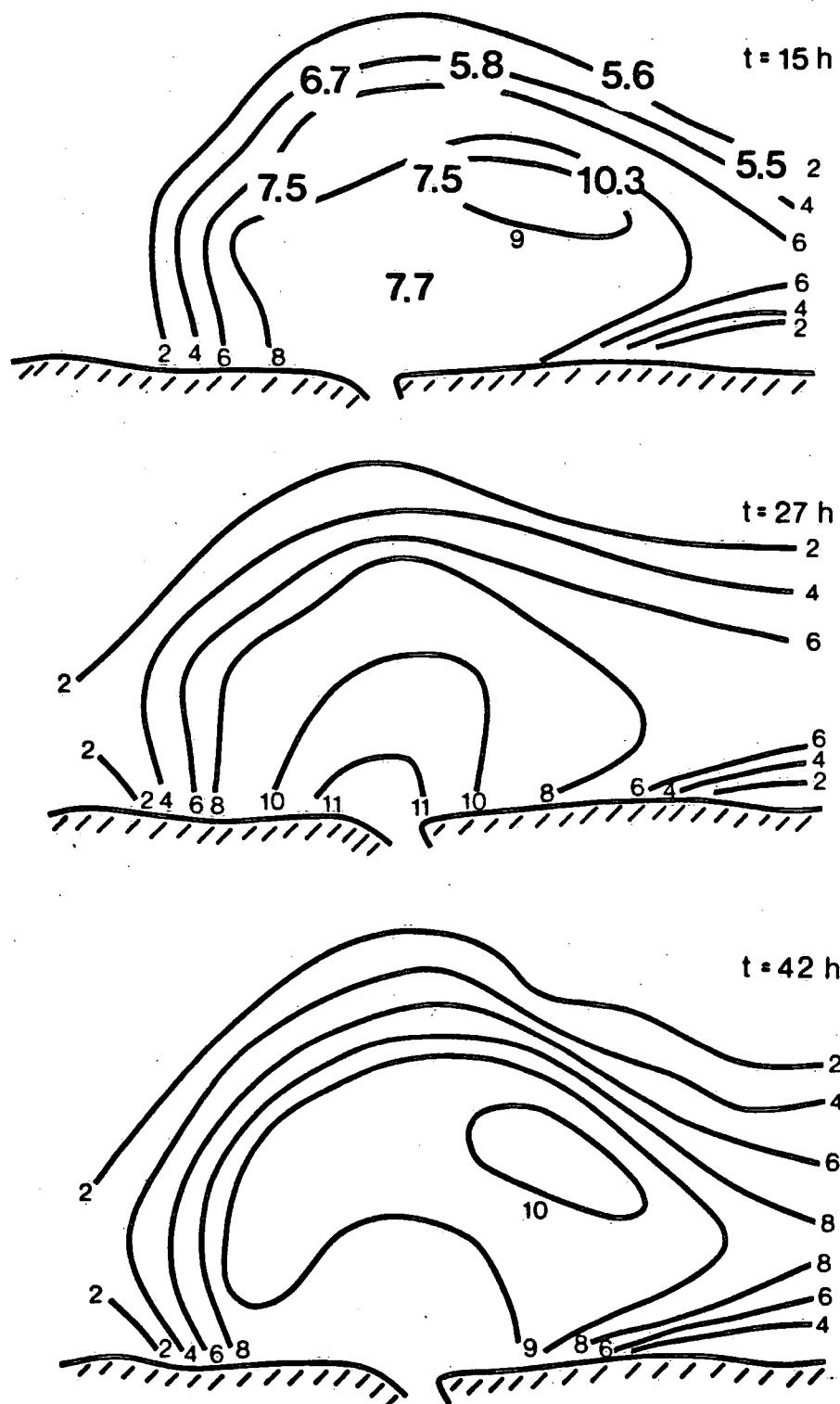


Fig.8 Computed concentrations of 1,2,3,4-TeCB at 15,27,42 hours after release.
The observed values for May 11, 1983 are superimposed in the top figure.

APPENDIX

Program FRMT

Programmed by I. Stepien

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PROGRAM FRMT(INPUT,OUTPUT,TAPF5=INPUT,TAPF2=OUTPUT,TAPF1)
DIMENSION QMX(11),TOMX(11)
REAL L,MAN
DIMENSION WT(9)
DIMENSION FT(9),XT(3)
DIMENSION YM(11),TYMX(11)
DIMENSION VMX(11),TVMX(11)
DIMENSION YPR(11),VPR(11)
DIMENSION QLL(144)
DIMENSION DX(11)
DIMENSION QLL(144)
DIMENSION HH(36)
DIMENSION QZ(11),QZZ(11)
DIMENSION TX(144),QY(144,11)
DIMENSION HY(144,11),VY(144,11)
DIMENSION TAB(2,144)
DIMENSION FF(22),UU(484)
DIMENSION AT(11),BT(11),LT(11),UT(11),ET(11),HT(11),LT(11),PT(11)
DIMENSION TX(146),YARR(146)
DIMENSION TT(144,11)
DIMENSION Z(22),ZZ(22),F(22),U(22,22)
DIMENSION UINV(22,22),WKAREA(44)
DIMENSION BB(22),IRR(44),ICC(44)
COMMON Z,ZZ,F,U,A,B,C,D,E,H,L,P
COMMON NT,DX,G,S0,BJ,MAN,NT,N,QL,OP
COMMON /B1/HU,VU,BETO,PSI,VNN
C
C-----THIS PROGRAM PERFORMS SOLUTION OF THE SAINT-VENANT EQUATIONS
C---FOR ONE-DIMENSIONAL CASE
C---REFERENCE: AMEIN,M., 1968, AN IMPLICIT METHOD FOR NUMERICAL FLOOD
C---ROUTING, WATER RES. RESEARCH, PP 719-726, STEPPIEN,I.,1984, ON THE
C---NUMERICAL SOLUTION OF THE SAINT-VENANT EQUATIONS, J. OF HYDROL.
C---NO.67, P. 1-11
C-----OPIS PARAMETROW /PARAMETERS DESCRIPTION/
C-----DT-KROK CZASOWY /TIME STEP IN SECCNDS/
C-----DX-KROK PRZEST. /SPACE STEP IN METERS/
C-----SD-SPADEK DNA /BOTTOM SLOPE, NONDIMENSIONAL/
C-----BD-SZEROKOSC KORYTA /WIDTH OF THE CHANNEL IN METERS/
C-----DLGK-DLUGOSC KANALU /LENGTH OF THE CHANNEL IN METERS/
C-----N-NUMBER OF SPACE STEPS /LICZBA KROKOW PRZESTRZENNCH/
C-----NT- NUMBER OF ELEMENTS OF QLL, QPP ARRAYS
C-----MAN-WSP. MANINGA /MANNING'S COEFFICIENT/
C-----G-ACCELERATION DUE TO GRAVITY IN M/SEC*2
C-----EPS-ITERATION TOLERANCE ERROR
C-----IKDR /CONTROL PARAMETER FOR OPTIONAL OUTPUT DATA/
C-----ZZ-TABL. WAR. POCZ. /INITIAL DATA, DEPTH AND VELOCITY ALONG THE CHANNEL/
C-----QLL,QPP-TABL. WAR. BRZEG. /BOUNDARY CONDITION DATA/
C-----HPR. PARAM. /PARAMETER,S VALUE / DT,DX,G,S0,MAN,NT,N
C
      IAA=22
      JAA=22
      IDGT=3
      NKKC=1
      NKK=0
      NKK=NKK+1
      N=11
      NT=5
      IKDR=0
      QPP0=0.0
      EPS=0.01
      DT=3600.0
      BD=371.0
      SD=0.00022762265
      DLGK=50000.0
      MAN=0.037
      CZKUV=3600000.0
      G=9.81
      H0=3.22
      V0=0.707
      BET0=0.3284
      PSI=-0.0072
      SKLT=0.25
C
C-----CZYT. TABL. KR. PRZESTRZ. /READ SPACE STEP,S ARRAY, UX(I),I=1,N/
C
      READ(1,*) (DX(I),I=1,N)
      WRITE(2,1000)(DX(I),I=1,N)
1000 FORMAT(1X,5F6.1)
115 FORMAT(11F5.1)
C
C-----CZYT. WAR. POCZ. /READ INITIAL CONDITION, DEPTH, VELOCITY
C-----H(I),V(I), I=1,N/
C
      READ(1,*)(ZZ(I),I=1,2*N)
      WRITE(2,1000)(ZZ(I),I=1,2*N)

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C-----CZYT. WAR. BRZEG. /READ LEFT BOUNDARY CONDITION,
C-----QLL(I), I=I,NT/
C
C      READ(1,*)(QLL(I),I=1,NT)
C      WRITE(2,1E00)(QLL(I),I=1,NT)
C      WRITE(2,141)
C-----WYPR. DANYCH WEJSC. /WRITE INPUT DATA/
C
C      WRITE(2,105) DT
C      WRITE(2,108) N
C      WRITE(2,109) NT
C      WRITE(2,110) SO
C      WRITE(2,104) BO
C      WRITE(2,102) DLGK
C      WRITE(2,107) G
C      WRITE(2,103) CZKON
C      JK=0
C      DO 7 I=1,N
C      QZZ(I)=ZZ(2*I-1)*ZZ(2*I)*BO
C 7      CONTINUE
C      DO 37 I=1,N
C      QMX(I)=QZZ(I)
C      TQMX(I)=0.0
C 37      CONTINUE
C-----PRZYG. TABL. DLYVA CALCOMPA / DATA FOR GRAPH. PLOT. /
C
C      DO 13 I=1,N
C      VV(JK+1,I)=ZZ(2*I)
C      HY(JK+1,I)=ZZ(2*I-1)
C 13      QY(JK+1,I)=QZZ(I)
C      T(JK+1)=DT*JK
C-----WYPR. WYNIKOW DLA T=0.0 /OUTPUT DATA FOR T=0.0/
C
C      CALL DRUK(N,ZZ,QZZ,DX,DT,JK)
C      JK=1
C-----PETLA PO CZASIE / TIME LOOP /
C
C 1      QL=QLL(JK)
C-----PIERWSZE PRZYBL. WEKTORA Z / FIRST APROX. OF Z-VECTOR /
C
C      DO 2 I=1,2*N
C      Z(I)=ZZ(I)
C 2      CONTINUE
C-----LICZ. WSPOLCZ. /COMPUTE COEF./
C
C      CALL COEF
C-----PIERWSZA ITERACJA /FIRST ITERATION/
C
C      JJ=1
C-----LICZ. PR. STRON /COMPUTE RIGHT-SIDE OF EQUATIONS/
C
C 10      CALL PRSTR
C-----LICZ. MAC. JAK. /COMPUTE JACOBIAN MATRIX/
C
C      CALL JAKUKL
C-----WYPR. POSR. MAC. / OPTIONAL OUTPUT DATA /
C
C      IF (IKDR.EQ.0) GO TO 17
C      WRITE(2,140)((U(I,J),J=1,2*N),Z(I),F(I),I=1,2*N)
C      WRITE(2,141)
C-----ODWRACANIE MACIERZY /MATRIX INVERSION/
C
C 17      CALL LINV1F(U,IAA,IAA,UINV,IDGT,WKARE,A,IER)
C-----MN. MAC. ODWR. INV. MATRIX * RIGHT-SIDE VECTOR/
C
C      CALL MNMAC(2*N,UINV,F,FF)
C-----WYPR. POSR. MAC. ODWR. /OPTIONAL DATA OUTPUT/
C
C      IF (IKDR.EQ.0) GO TO 18
C      WRITE(2,140)((U(I,J),J=1,2*N),F(I),FF(I),I=1,2*N)
C      WRITE(2,141)

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C-----BADANIE WEKT. RESZT. /INVESTIG. OF RESIDUALS/
C
18 DO 21 I=1,2*N
IF (ABS(FF(I)).GE.EPS) GO TO 23
21 CONTINUE
DO 5 I=1,2*N
5 Z(I)=Z(I)+FF(I)
DO 6 I=1,2*N
6 DO 6 J=1,2*N
U(I,J)=0.0
CONTINUE
C-----WYPR. WEKTORA Z /WRITE Z-VECTOR/
C
DO 25 I=1,N
25 QZ(I)=Z(2*I-1)*Z(2*I)+BD
CALL DRUK(N,Z,QZ,DX,DT,JK)
C-----LICZ. WART. MAX. /COMPUTE MAX. VALUES/
C
DO 29 I=1,N
CALL MAXIM(I,JK,QZ,QMX,TQMX,DT)
29 CONTINUE
DO 43 I=1,N
YPR(I)=Z(2*I-1)
VPR(I)=Z(2*I)
43 CONTINUE
C-----LICZ. MAX. GLEB. I PR./COMP. MAX. VEL. AND DEPTHS/
C
DO 45 I=1,N
CALL MAXIM(I,JK,YPR,YMX,TYMX,DT)
CALL MAXIM(I,JK,VPR,VMX,TVMX,DT)
45 CONTINUE
C-----PRZYG. DAN. DLA CALCOMPA /DATA FOR GRAPH PLOT./
C
DO 14 I=1,N
HY(JK+1,I)=Z(2*I-1)
14 VV(JK+1,I)=Z(2*I)
QY(JK+1,I)=QZ(I)
CONTINUE
TX(JK+1)=DT*JK
DO 22 I=1,2*N
22 ZZ(I)=Z(I)
JK=JK+1
WRITE(2,111) JK
IF (JK.LT.N) GO TO 1
GO TO 9
23 JJ=JJ+1
WRITE(2,112) JJ
19 DO 3 I=1,2*N
3 Z(I)=Z(I)+FF(I)
DO 4 I=1,2*N
4 DO 4 J=1,2*N
U(I,J)=0.0
4 CONTINUE
GO TO 10
112 FORMAT(1X,12HITERATION NR,I2)
111 FORMAT(1X,16HTIME STEP NR ,I3)
110 FORMAT(1X,15HBOTTOM SLOPE =,F8.3)
109 FORMAT(1X,15HTIME STEP, S NR=,I3)
108 FORMAT(1X,17HSPACE STEP,S NR=,I3)
107 FORMAT(1X,15HGRAVITY(M/S2) =,F7.2)
106 FORMAT(1X,15HDX-SP. STEP (M)=,F7.2)
105 FORMAT(1X,15HDT-TM. STEP (S)=,F7.1)
104 FORMAT(1X,15HCHANNEL WIDTH(M)=,F7.1)
103 FORMAT(1X,15HFFINAL TIME (S)=,F9.1)
141 FORMAT(/)
142 FORMAT(10(2X,F8.4))
102 FORMAT(1X,18HCHANNEL LENGTH(M)=,F7.1)
101 FORMAT(24F5.1)
100 FORMAT(22F5.3)
120 FORMAT(11(F6.3,2X))
131 FORMAT(2F5.3)
130 FORMAT(F8.1)
140 FORMAT(11(1X,F7.4)/5X,11(1X,F7.4),2X,F6.3,2X,F6.2)
150 FORMAT(3F6.3)
151 FORMAT(4(1F5.2,2X))
155 FORMAT(3(2X,F6.3),2X)
113 FORMAT(8F6.3)
114 FORMAT(3F6.3)

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201  FORMAT(11(1X,F7.3)/11(1X,F7.3)//)
250  FORMAT(2X,9HODLEGLOSC,3X,8HGLB,4AX.,3X,4HCZAS
251  1,5X,9HPREDK.MX.,3X,4HCZAS,5X,9HPPZEP,MX.,3X,4HCZAS/)
252  1,3H(S),7X,7H(M+S),5X,3H(S)/)
252  FORMAT(2X,F8.1,5X,F6.3,3X,F8.1,3X,
9  1F6.3,3X,F8.1,3X,F7.1,3X,F8.1)
9  CONTINUE
C-----WYPR. WART. MAX. / WRITE MAX. VALUES/
C
      WRITE(2,250)
      WRITE(2,251)
      DO 51 I=1,N
      ODL=(I-1)*DX(I)
      WRITE(2,252) ODL,YMX(I),TVMX(I),
1 VMX(I),TVMX(I),QMX(I),TQMX(I)
51  CONTINUE
      IF(NKK.NE.NKKC) GO TO 55
      STOP
      END

      SUBROUTINE MAXIM(I,J,A,B,C,R)
      DIMENSION A(11),B(11),C(11)
      IF (A(I)-B(I)) 1,2,2
2  B(I)=A(I)
1  C(I)=J*R/3600.0
      CONTINUE
      RETURN
      END

      SUBROUTINE DRUK(M,A,B,R,S,II)
      DIMENSION A(22),B(11)
      DIMENSION R(11)
      T=II*S
      WRITE(2,199) T
      WRITE(2,200)
      WRITE(2,253)
      DO 1 I=1,M
      X=(I-1)*R(I)
      WRITE(2,202) X,A(2*I-1),A(2*I),B(I)
1  CONTINUE
199  FORMAT(1X,7HWARUNKI,1X,3HDLA,1X,2HT=,F8.1/)
200  FORMAT(2X,44HODLEGLOSC GLEBCKOSC PREDKOSC PRZEPLYW/)
202  FORMAT(2X,F8.1,5X,F6.3,6X,F6.3,5X,F7.1/)
253  FORMAT(5X,3H(M),9X,3H(M),8X,5H(M/S))
1,6X,7H(M+S)/)
      RETURN
      END

      SUBROUTINE MNMAC(M,A,B,C)
      DIMENSION A(22,22),B(22),C(22)
      DD 1 I=1,M
      S=0.0
      DO 2 J=1,M
      S=S+A(I,J)*B(J)
2  CONTINUE
      C(1)=S
1  CONTINUE
      RETURN
      END

      SUBROUTINE COEF
      REAL L,MAN
      DIMENSION A(11),B(11),C(11),D(11),E(11),H(11),L(11),P(11)
      DIMENSION Z(22),ZZ(22),F(22),U(22,22)
      DIMENSION SFZ(22)
      DIMENSION DX(11)
      COMMON Z,ZZ,F,U,A,B,C,D,E,H,L,P
      COMMON D,DX,G,S0,B0,MAN,NT,N,QL,OP
      DO 1 I=1,N-1
      A(I)=ZZ(2*I+2)
      B(I)=ZZ(2*I+1)
      C(I)=ZZ(2*I+1)
      D(I)=ZZ(2*I+1)
      E(I)=ZZ(2*I-1)+ZZ(2*I+1)+1*(ZZ(2*I-1)*ZZ(2*I)-ZZ(2*I+1)*ZZ(2*I+2))*0.5*DT/DX(I+1)
      H(I)=ZZ(2*I+2)
      L(I)=ZZ(2*I)
      SFZ(I)=(MAN**2)*(ZZ(2*I)**2/ZZ(2*I-1)**(1.333))
      SFZ(I+1)=(MAN**2)*(ZZ(2*I+2)**2/ZZ(2*I+1)**(1.333))
      P(I)=-2*G*U*S0
      1*(ZZ(2*I)+ZZ(2*I+2))+2G*DT*0.5*(ZZ(2*I+1)-ZZ(2*I-1))/DX(I+1)-
      30.25*DT*(ZZ(2*I+2)-ZZ(2*I+1))/DX(I+1)+
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40.5*G*DT*(SFZ(I)+SFZ(I+1))
1 CONTINUE
RETURN
END

SUBROUTINE JAKUKL
REAL L,MAN
DIMENSION A(11),B(11),C(11),D(11),E(11),H(11),L(11),P(11)
DIMENSION Z(22),ZZ(22),F(22),U(22,22)
DIMENSION DX(11)
COMMON Z,ZZ,F,U,A,B,C,D,E,H,L,P
COMMON DT,DX,G,SC,B0,MAN,NT,N,QL,OP
COMMON /B1/H0,V0,BET0,PSI,VWN
DO 1 I=1,N-1
U(2*I-1,2*I-1)=1-0.5*DT*(Z(2*I)+B(I))/DX(I+1)
U(2*I-1,2*I)=DT*(Z(2*I-1)+D(I))/DX(I+1)
U(2*I-1,2*I+1)=1+0.5*DT*(A(I)+Z(2*I+2))/DX(I+1)
U(2*I-1,2*I+2)=0.5*DT*(C(I)+Z(2*I+1))/DX(I+1)
U(2*I,2*I-1)=-G*DT/DX(I+1)-
10.6666*G*DT*(MAN**2)*(Z(2*I)**2)*(Z(2*I-1)**(-2.333))
U(2*I,2*I)=1-DT*(Z(2*I)+L(I))*0.5/DX(I+1)-
1G*DT*(MAN**2)*Z(2*I)*(Z(2*I-1)**(-1.333))
U(2*I,2*I+1)=G*DT/DX(I+1)-
10.6666*G*DT*(MAN**2)*(Z(2*I+2)**2)*(Z(2*I+1)**(-2.333))
U(2*I,2*I+2)=1+0.5*DT*(H(I)+Z(2*I+2))/DX(I+1)-
1G*DT*(MAN**2)*Z(2*I+2)*(Z(2*I+1)**(-1.333))
1 CONTINUE
U(2*N-1,1)=Z(2)
U(2*N-1,2)=Z(1)
U(2*N,2*N-1)=-0.229
U(2*N,2*N)=1.0
RETURN
END

SUBROUTINE PRSTR
REAL L,MAN
DIMENSION A(11),B(11),C(11),D(11),E(11),H(11),L(11),P(11)
DIMENSION Z(22),ZZ(22),F(22),U(22,22)
DIMENSION DX(11)
DIMENSION SF(22)
COMMON Z,ZZ,F,U,A,B,C,D,E,H,L,P
COMMON DT,DX,G,SC,B0,MAN,NT,N,QL,OP
COMMON /B1/H0,V0,BET0,PSI,VWN
DO 1 I=1,N-1
F(2*I-1)=Z(2*I-1)+Z(2*I+1)-
10.5*DT*(Z(2*I+1)*(A(I)+Z(2*I+2))-2*Z(2*I-1)*(Z(2*I)+B(I))-
3C(I)*Z(2*I+2)-
4U(I)*Z(2*I))/DX(I+1)-E(I)
SF(I+1)=(MAN**2)*(Z(2*I+2)**2)/(Z(2*I+1)**(1.333))
SF(I)=(MAN**2)*(Z(2*I)**2)/(Z(2*I-1)**(1.333))
F(2*I)=Z(2*I)+Z(2*I+2)+G*DT*(Z(2*I+1)-2*Z(2*I-1))/DX(I+1)-
10.25*DT*(Z(2*I+2)**2-Z(2*I)**2+2*H(I)*Z(2*I+2)-2*L(I)*Z(2*I))/
2DX(I+1)+G*DT*0.5*(SF(I)+SF(I+1))+P(I)
1 CONTINUE
F(2*N-1)=QL/B0-Z(1)*Z(2)
F(2*N)=Z(2*N)-0.176-0.229*Z(2*N-1)
DO 2 I=1,2*N
F(I)=F(I)
RETURN
END

SUBROUTINE BLANK(ITAB,M)
DIMENSION ITAB(132)
DO 1 I=1,M
ITAB(I)=1H
1 CONTINUE
RETURN
END

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6000.06000.06000.06000.06000.0 — space steps array
 6000.06000.06000.06000.06000.0
 6000.0
 2.6 .8 2.6 .8 2.6 — initial condition /depth,velocity for
 .8 2.6 .8 2.6 .8 successive cross-section along the river/
 2.6 .8 2.6 .8 2.6
 .8 2.6 .8 2.6 .8
 2.6 .8
 696.0 692.0 688.0 684.0 680.0 — left boundary condition /discharge for
 676.0 successive time steps in the first
 computational cross-section/

DT-TM-STEP (S)= 3600.0
 SPACE STEP,S NR.= 11
 TIME STEP,S NR= 6
 BOTTOM SLOPE = .000
 CHANN.WIDTH(M)= 371.0
 CHANNEL LENGTH(M)=60000.0
 GRAVITY(M/S2) = 9.81
 FINAL TIME (S)= 36000.0
 WARUNKI DLA T= 0.0

ODLEGOSC	GLEBOKOSC	PREDKOSC	PRZEPLYW	distance	depth	velocity	discharge
(M)	(M)	(M/S)	(M ³ /S)				
0.0	2.600	.771	743.7				
6000.0	2.600	.771	743.7				
12000.0	2.600	.771	743.7				
18000.0	2.600	.771	743.7				
24000.0	2.600	.771	743.7				
30000.0	2.600	.771	743.7				
36000.0	2.600	.771	743.7				
42000.0	2.600	.771	743.7				
48000.0	2.600	.771	743.7				
54000.0	2.600	.771	743.7				
60000.0	2.600	.771	743.7				

ITERATION NR 2
WARUNKI DLA T= 3600.0

ODLEGLOSC (M)	GLEBOKOSC (M)	PREDKOSC (M/S)	PRZEPLYW (M*3/S)
0.0	2.543	.738	696.0
6000.0	2.591	.766	736.3
12000.0	2.599	.770	742.4
18000.0	2.600	.771	743.3
24000.0	2.600	.771	743.5
30000.0	2.600	.771	743.5
36000.0	2.600	.771	743.5
42000.0	2.600	.771	743.5
48000.0	2.600	.771	743.5
54000.0	2.600	.771	743.4
60000.0	2.599	.771	743.7

TIME STEP NR 2
ITERATION NR 2
WARUNKI DLA T= 7200.0

ODLEGLOSC (M)	GLEBOKOSC (M)	PREDKOSC (M/S)	PRZEPLYW (M*3/S)
0.0	2.506	.744	692.0
6000.0	2.546	.744	702.7
12000.0	2.586	.763	732.4
18000.0	2.597	.769	741.2
24000.0	2.599	.771	743.2
30000.0	2.600	.771	743.6
36000.0	2.600	.771	743.6
42000.0	2.600	.771	743.7
48000.0	2.600	.771	743.6
54000.0	2.600	.771	743.8
60000.0	2.599	.771	743.7

TIME STEP NR 3
ITERATION NR 2
WARUNKI DLA T= 10800.0

ODLEGŁOSC (M)	GLEBOKOSC (M)	PREDKOSC (M/S)	PRZEPFLYW (M*3/S)
0.0	2.495	.743	688.0
6000.0	2.520	.749	700.3
12000.0	2.550	.749	708.5
18000.0	2.583	.762	730.4
24000.0	2.595	.768	739.8
30000.0	2.599	.770	742.6
36000.0	2.600	.771	743.3
42000.0	2.600	.771	743.4
48000.0	2.600	.771	743.6
54000.0	2.600	.771	743.5
60000.0	2.599	.771	743.6

TIME STEP NR 4
ITERATION NR 2
WARUNKI DLA T= 14400.0

ODLEGŁOSC (M)	GLEBOKOSC (M)	PREDKOSC (M/S)	PRZEPFLYW (M*3/S)
0.0	2.482	.743	684.0
6000.0	2.503	.743	690.4
12000.0	2.530	.752	705.4
18000.0	2.555	.753	713.5
24000.0	2.581	.762	729.9
30000.0	2.594	.768	738.9
36000.0	2.598	.770	742.3
42000.0	2.600	.771	743.3
48000.0	2.600	.771	743.5
54000.0	2.600	.771	743.7
60000.0	2.599	.771	743.6

TIME STEP NR 5
 ITERATION NR 2
 WARUNKI DLA T= 18000.0

ODLEGŁOSC	GLEBOKOSC	PREDKOSC	PRZEPLYW
(M)	(M)	(M/S)	(M^3/S)
0.0	2.473	.741	680.0
6000.0	2.489	.745	687.8
12000.0	2.510	.745	693.4
18000.0	2.537	.753	708.6
24000.0	2.559	.755	717.1
30000.0	2.581	.762	729.9
36000.0	2.593	.767	738.2
42000.0	2.598	.770	741.7
48000.0	2.599	.771	743.1
54000.0	2.600	.771	743.4
60000.0	2.599	.771	743.6

TIME STEP NR	6	ODLEGŁOSC	GLB.MAX.	CZAS	PREDK.MX.	CZAS	PRZEP.MX.	CZAS
		(M)	(M)	(S)	(M/S)	(S)	(M^3/S)	(S)
0.0		2.543		1.0	.744	2.0	743.7	0.0
6000.0		2.591		1.0	.766	1.0	743.7	0.0
12000.0		2.599		1.0	.770	1.0	743.7	0.0
18000.0		2.600		1.0	.771	1.0	743.7	0.0
24000.0		2.600		1.0	.771	1.0	743.7	0.0
30000.0		2.600		1.0	.771	2.0	743.7	0.0
36000.0		2.600		1.0	.771	2.0	743.7	0.0
42000.0		2.600		1.0	.771	2.0	743.7	0.0
48000.0		2.600		2.0	.771	2.0	743.7	0.0
54000.0		2.600		1.0	.771	2.0	743.8	2.0
60000.0		2.599		1.0	.771	1.0	743.7	1.0

4.978 OF SECONDS EXECUTION TIME.

/ IDLE

IDLE
 bye

084 LOG OFF 16.30.21.
 084 SRU 26.234 UNTS.

IAF CONNECT TIME 00.13.34.
 LOGGED OUT.