FREQUENCY OF SAMPLING
REQUIRED FOR MONITORING
NIAGARA RIVER

by

A. H. El-Shaarawi
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SUMMARY

The Canadian Department of the Environment (DOE) has been routinely measuring the physical and chemical characteristics of water samples collected at the head and mouth of the Niagara River (Fig. 1) since 1983. The basic objectives are to use the generated data to make inferences about the differences between the quality of waters at the head and mouth of the river and to estimate the additional load to the river along its course. The design used to collect the samples and the characteristics of the generated data show that:

(1) water samples were collected at Niagara on the Lake, NOTL, and Fort Erie, FE, on the same day; and

(2) many of the measured concentrations, especially contaminants, are below the level of detection.

Because of (1), a paired comparison statistical test is the most natural approach for evaluating the significance of the difference in the concentration between the two locations, since such a test is not influenced by the variabilities of the concentration from day to day. Also, because of (2), parametric tests such as matched paired t-test are very difficult to compute and interpret. If a portion of the data

at each of the two stations is below the level of detection, then the sign test is the only exact test available for assessing the differences between NOTL and FE.

The aims of this paper are two fold. The first is to provide a technique for estimating the frequency of taking paired samples from the two locations for monitoring the river within a one year period. The second is estimating the number of years required for monitoring the river in order to detect a linear trend in the water quality of the river.

FRÉQUENCE D'ÉCHANTILLONNAGE REQUISE POUR SURVEILLER
LA POLLUTION DE LA RIVIÈRE NIAGARA

par

A.H. El-Shaarawi

SOMMAIRE

Le ministère de l'Environnement du Canada (MDE) mesure de façon régulière les caractéristiques physiques et chimiques des échantillons d'eau recueillis à la source et à l'embouchure de la rivière Niagara (figure 1) depuis 1983. Ces prélèvements ont pour objectifs fondamentaux de produire des données qui permettront de déterminer les différences qui existent entre la qualité des eaux à la source et à l'embouchure de de la rivière et d'estimer la charge additionnelle de polluants qui s'ajoute à la rivière le long de son parcours. Deux précisions s'imposent quant à la procédure utilisée pour recueillir les échantillons et les caractéristiques des données qui en résultent :

(1) les échantillons d'eau ont été recueillis à Niagara on the Lake, NOTL, et à Fort-Érié, FE, le même jour;

(2) bon nombre des concentrations qui ont été mesurées, surtout celles des contaminants, sont inférieures au seuil de détection.

Vu l'énoncé (1), la meilleure manière d'évaluer l'importance de l'écart des concentrations entre la source et l'embouchure de la rivière est de procéder à un test statistique comparatif jumelé puisqu'un test de ce genre n'est pas susceptible à la dispersion des données sur la concentration

d'une journée à l'autre. De plus, en raison de l'énoncé (2), il est très difficile de calculer et d'interpréter les tests paramétriques tels que les tests-T jumelés correspondants. Dans la mesure où une portion des données de chacun des deux points de prélèvement est inférieure au seuil de détection, le test des signes est le seul test qui permette d'évaluer exactement les écarts entre NOTL et FE.

Cette étude vise deux objectifs. Premièrement, elle présente une technique servant à estimer la fréquence à laquelle on doit prélever des échantillons jumelés aux deux emplacements pour surveiller la pollution de la rivière au cours d'une période d'un an. Deuxièmement, elle se propose d'estimer le nombre d'années pendant lesquelles il faudra prélever des échantillons pour établir une tendance linéaire de la qualité de l'eau de la rivière.

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ABSTRACT

Water samples have been collected routinely at the head and mouth of the Niagara River since 1983 and analyzed for their chemical and physical characteristics. Statistical techniques are given for estimating (1) the frequency of sampling the river at two locations within a given year, and (2) the number of years required for monitoring the river in order to detect a linear trend in its water quality.

INTRODUCTION

The Canadian Department of the Environment (DOE) has been routinely measuring the physical and chemical characteristics of water samples collected at the head and mouth of the Niagara River (Fig. 1) since 1983. The basic objectives are to use the generated data to make inferences about the differences between the quality of waters at the head and mouth of the river and to estimate the additional load to the river along its course. The design used to collect the samples and the characteristics of the generated data show that:

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Because of (1), a paired comparison statistical test is the most natural approach for evaluating the significance of the difference in the concentration between the two locations, since such a test is not influenced by the variabilities of the concentration from day to day. Also, because of (2), parametric tests such as matched paired t-test are very difficult to compute and interpret and nonparametric tests which require the calculation of ranks such as Wilcoxon signed rank test are not possible to evaluate due to difficulties in assigning ranks to the differences among the concentrations. It should be stated that if a portion of the data at each of the two stations is below the level of detection, then the sign test is the only exact test available for assessing the differences between NOTL and FE.

The aims of this paper are two fold. The first is to provide a technique for estimating the frequency of taking paired samples from the two locations for monitoring the river within a one year period. The second is estimating the number of years required for monitoring the river in order to detect a linear trend in the water quality of the river. This is done by assuming that the sign test will be used to detect differences between the two locations. The choice of the sign test is based on the following three facts: (1) values below detection are likely to occur when measuring the concentration of contaminants in water samples and the sign test is the only exact test available that deals with this situation; (2) the sign test is very simple to apply and requires very little assumption; and (3) the estimated sample size based on the sign test is conservative in the sense that the more efficient parametric and non-parametric tests will require smaller sample size than that required to achieve the same efficiency by the sign test.

THE SIGN TEST

Let y_i and x_i be the concentrations on day i of substance A at NOTL and FE, respectively, and i=1,2,...,N. The null hypothesis H_0 states that the observed differences

$$d_i = y_i - x_i \qquad (i=1,2,\ldots,N)$$

are the results of chance. In the calculation of the sign test, the random variable z_i is defined by

$$Z_i = \begin{cases} 1 & \text{if } d_i > 0 \\ 0 & \text{if } d_i < 0 \end{cases}$$

and the null hypothesis is

$$H_0: \text{ Prob } \{Z_i=1\} = \text{Prob } \{Z_i = 0\}$$
$$= 0.5$$

The data needed for testing H_0 can be represented in the following contingency table.

	Niagara on the Lake		
	Concentration	Below Detection	Above Detection
Fort Erie	Below detection	N_{00}	N_{01}
	Above detection	N_{10}	N_{11}

where N_{00} = the number of days in which both y_i and x_i are below the detection limit;

N_{01} = the number of days with y_i above detection and x_i below detection;

N_{10} = the number of days with y_i below detection and x_i above detection;

and N_{11} = the number of days with both y_i and x_i are above the level of detection.

In the application of the sign test N_{00} is ignored and the test is conducted conditionally on

$$N = N_{01} + N_{10} + N_{11}$$

Suppose that for r_0 days out of the N days, the concentration at NOTL exceeds that at FE, then the exact significance level α is given as

$$\alpha = 2^{-N} \sum_{r=r_0}^{N} \binom{N}{r} .$$

If α is below a prespecified value (say 0.05), then H_0 is rejected at the significance level α.

ESTIMATION OF THE NUMBER OF SAMPLES WHEN THE SIGN TEST IS USED

In testing H_0, N_{00} was ignored and only N was used. This is reasonable after the data have been collected, but in sample size estimation N_{00} cannot be ignored since it is not known before the data collection. Let M be the number of days that need to be sampled, N the number of usable pairs in the calculation of the sign test and r the number of pairs for which $y_i > x_i$. Then the conditional distribution of r given N is

$$\text{Prob } (r/N) = \binom{N}{r} \theta^r (1 - \theta)^{N-r}$$

where θ is the prob $\{Z_i = 1\}$.

Further, let P_{00} be the probability that y_i and x_i are below the level of detection. Then

$$\text{Prob } (N_{00} | M) = \binom{M}{N_{00}} P_{00}^{N_{00}} (1 - P_{00})^{M-N_{00}}$$

$$= \binom{M}{N} P_{00}^{M-N} (1 - P_{00})^{N}$$

Therefore

$$\text{Prob}\,(r/M) = \sum_{N} \text{Prob}\,(r|N)\,\text{Prob}(N|M)$$

$$= \binom{M}{r} \{(1 - P_{00})\,\theta\}^{r} \{1 - (1 - P_{00})\theta\}^{M-r}$$

Once $\text{Prob}(r|M)$ is derived, it is then possible to estimate M. To do this, it is important to distinguish between two types of errors which are associated with any statistical test. Type I is estimated by the probability α of rejecting H_0 when it is true. This probability is commonly known as the significance level. The second type of error represents the risk of accepting H_0 when H_1 is true. Let β be the probability of committing the second type of error. $(1-\beta)$ is known as the power of the test.

From the above α and β are given by

$$\alpha = 2^{-M} \sum_{r=r_0}^{M} \binom{M}{r} (1 - P_{00})^{r} (1 + P_{00})^{M-r}$$

and
$$\beta = \sum_{r=0}^{r_0-1} \binom{M}{r} \{(1 - P_{00})\theta\}^{r} \{1 - (1 - P_{00})\theta\}^{M-r}$$

M can be determined by solving the above two equations for specified values for α, β, θ and P_{00}. If the binomial distribution is approximated by the normal distribution, then M is given by

$$M = \frac{[\phi^{-1}(\beta)\,\sqrt{\theta\{1 - (1 - P_{00})\theta\}} - 0.5\,\phi^{-1}\,(1 - \alpha)\,\sqrt{1 + P_{00}}\}^{2}}{(1 - P_{00})\,(0.5 - \theta)^{2}}$$

Further, taking $\alpha = \beta$, we get

$$M = \frac{\{\phi^{-1}(1-\alpha)\}^2 \{2\sqrt{\theta}\{1 - (1 - P_{00})\theta + \sqrt{1 + P_{00}}\}^2}{(1 - P_{00})(1 - 2\theta)^2} ,$$

where $\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/z} dx$ and $\phi^{-1}(x)$

is the inverse of $\phi(x)$.

APPLICATIONS

The previous results are used to estimate the minimum number of paired samples M needed from NOTL and FE when $\alpha=\beta=0.05$. Table 1 gives the values of M as a function of θ (the probability of observing a higher value at NOTL than at FE) and P_{00} (the probability that the two paired samples have values below the level of detection). As can be seen from the table, the value of M increases as θ decreases and as P_{00} increases. For example, to detect a change in θ from 0.5 to 0.55 then 1077 and 1330 paired samples are required to be taken from the two stations when $P_{00}=0$ and $P_{00}=0.1$, respectively. It can also be noticed that weekly samples will allow the detection of change in θ from 0.5 to 0.715 while sampling once every two weeks allows the detection of change from 0.5 to 0.79 in the value of θ. The changes in the probability can be transformed into changes in the centre of the distribution (median) in units of standard deviation if a particular form of probability distribution is assumed. This can be done by noting that

$$\theta = 1 - \phi\left(-\frac{\Delta}{\sigma}\right)$$

where ϕ is the cumulative distribution function, Δ and σ are the median and the standard deviation of the concentration differences. Hence

$$\Delta = -\sigma\,\phi^{-1}(1-\theta)$$

For example, if θ=0.95 and the distribution is normal then

$$\Delta = 1.645\ \sigma\ .$$

TREND DETECTION AND ESTIMATION

Let y_{ij} and x_{ij} be the concentrations of substance A at NOTL and FE for the jth ($j=1,2,\ldots,N_i$) sample in the ith year ($i=0,1,2,\ldots,k$) and suppose that

r_i = the number of pairs in the ith year with $y_{ij} > x_{ij}$

and θ_i = the probability that $Y_{ij} > x_{ij}$.

Then the problem is testing

$$H_0: \quad \theta_0 = \theta_1 \ldots = \theta_k$$

against

$$H_1: \quad \theta_0 \neq \ldots \neq \theta_k$$

This can be tested using the statistic

$$D^2 = \sum_{i=0}^{k} (r_i - n_i\hat{\theta})^2 / n_i\hat{\theta} \; (1 - \hat{\theta})$$

where

$$\hat{\theta} = t_1 / n.,$$

$$t_1 = r_0 + r_1 + \ldots r_k \quad \text{and} \quad n. = n_0 + \ldots + n_k \, .$$

Under H_0 the distribution of D^2 is well approximated by the χ^2 distribution with k degrees of freedom.

This test is a general test for the homogeneity of the θ'_is, more specialized tests will perform better if different forms for H_1 are considered. The logistic transform (Cox, 1970) is more likely to perform better for testing for trend. The linear logistic model is given by

$$\theta_i = e^{\alpha+\beta i} \;/\; (1+e^{\alpha+\beta i}) \qquad\qquad i = 0,\ldots,k$$

where α and β represent the intercept and the slope of the linear logistic model. In this case, the interest is testing

$$H_0:\quad \beta = 0$$

against $\quad H_1:\quad \beta \neq 0$

Under the logistic model the likelihood function is

$$L = \binom{n_0}{r_0} \cdots \binom{n_k}{r_k} \frac{e^{\alpha t_1 + \beta t_2}}{\prod_{i=0}^{k} (1+e^{\alpha+\beta i})^{n_i}}$$

where $t_1 = \sum_{i=0}^{k} r_i$ and $t_2 = \sum_{i=0}^{k} i\, r_i$. The statistics t_1 and t_2 are jointly sufficient for α and β respectively. Inference about β can be made conditional on t_1. The conditional likelihood function for β is

$$L_c = \frac{\binom{n_0}{r_0} \cdots \binom{n_k}{r_k} e^{\beta t_2}}{\sum\limits_{\substack{r_0,\ldots,r_k \\ \sum r_i = t_1}} \binom{n_0}{r_0} \cdots \binom{n_k}{r_k} e^{\beta t_2}}$$

Under H_0, L_c becomes

$$L_c\ (H_0) = \frac{\binom{n_0}{r_0} \cdots \binom{n_k}{r_k}}{\binom{n.}{t_1}}$$

The moments of t_2 can then be calculated from $L_c(H_0)=P(r_0,\ldots, r_k/t_1)$. It is easy to verify that the conditional mean and variance of t_2 are

$$E(t_2) = \sum_{i=0}^{k} \frac{in_i}{n.} t_1$$

$$= \hat{\theta} \sum_{i=0}^{n} in_i$$

and $$V(t_2) = \frac{\hat{\theta}(1-\hat{\theta})}{n.-1} \left(n. \sum_{i=0}^{k} i^2 n_i - (\sum in_i)^2\right)$$

$$= \frac{n.\hat{\theta}(1-\hat{\theta})}{n.-1} \left(\sum_{i=0}^{k} i^2 n_i - \frac{(\sum in_i)^2}{n.}\right)$$

respectively.

Then the test for linear trend on the logistic scale is

$$Z = (t_2 - E(t_2)) / \surd V(t_2)$$

which has approximately normal distribution with mean 0 and variance 1. The statistic Z takes a simple form when $n_i = n$, since

$$E(t_2) = \hat{\theta} n \frac{k(k+1)}{2}$$

$$V(t_2) = \frac{n.\hat{\theta}(1-\hat{\theta})}{n.-1} n \left(\sum_{i=0}^{k} i^2 - \frac{(\sum_0^k i)^2}{k+1} \right)$$

$$= \frac{nn.\hat{\theta}(1-\hat{\theta})}{n.-1} \left\{ \frac{k(k+1)(2k+1)}{6} - \frac{k^2(k+1)}{4} \right\}$$

$$= \frac{n^2.k(k+2)}{12(n.-1)} \hat{\theta}(1-\hat{\theta})$$

ESTIMATION OF THE NUMBER OF YEARS NEEDED FOR DETECTING THE EXISTENCE OF A LINEAR TREND

In this section the estimation of the number of years k necessary for monitoring the head and the mouth of the river in order to detect a specific deviation in θ_i is considered. The difference, on the logistic scale, between year 0 and year k (Cox, 1970) is given by

$$\ell n \left\{(\theta_k/(1-\theta_k)) \; / \; (\theta_0/(1-\theta_0))\right\} = \beta k \; .$$

To use the procedure given previously, the conditional distributions of t_2 given t_1 under H_0 and H_1 are required. Although these distributions can be derived in general, the exact expressions for the moments under H_1 are, however, not known. To avoid this difficulty, the score statistics procedure (Harris, 1985 and Plackett, 1974) is used instead. This is done by noting that t_1 and t_2 have asymptotically a bivariate normal distribution with means

$$\mu_1 = E(t_1) = n \sum_{i=0}^{k} \frac{e^{\alpha+\beta i}}{1+e^{\alpha+\beta i}} = n \sum_{i=0}^{k} \theta_i$$

and $\mu_2 = n \sum_{i=0}^{k} i \, \theta_i$.

The element of the variance covariance matrix is

$$\text{Var}\,(t_1) = I_{11} = n \sum_{i=0}^{k} \theta_i \, (1-\theta_i)$$

$$\text{Var}\,(t_2) = I_{22} = n \sum_{i=0}^{k} i^2 \, \theta_i \, (1-\theta_i)$$

and $\text{Cov}\,(t_1,t_2) = I_{12} = n \sum_{i=0}^{k} i \, \theta_i \, (1-\theta_i)$.

The conditional mean and variance of t_2 given t_1 are

$$\mu_{2.1} = E(t_2/t_1) = \mu_2 + (I_{12}/I_{11})(t_1-\mu_1)$$

and $$I_{22.1} = \mathrm{Var}(t_2/t_1) = I_{22} - I^2_{12}/I_{11}$$

respectively. The score statistic is

$$Z = (t_2 - \tilde{\mu}_{2.1}) / \sqrt{\tilde{I}_{22.1}}$$

which has asymptotically a normal distribution with mean 0 and variance 1. The use of ~ over $\mu_{2.1}$ and $I_{22.1}$ indicates that these quantities are evaluated under H_0. In this case

$$H_0 : \beta = 0$$

which implies that $\theta_i = \theta_0$ for all i. Hence

$$\tilde{\theta}_0 = t_1/n.$$

where n. = n(k+1). Similarly

$$\tilde{I}_{22.1} = \frac{n.\tilde{\theta}_0 (1-\tilde{\theta}_0) k(k+2)}{12} .$$

Note that if n.-1 is replaced by n. in the expression for var(t_2) then $\tilde{I}_{22.1}$ is obtained. Approximations to the conditional mean and variance of t_2 can be obtained using a procedure due to Cox (1970). This gives

$$E(t_2/t_1) = \tilde{\mu}_{2.1} + \beta \tilde{I}_{22.1} + \frac{1}{2} \beta^2 \tilde{\mu}_3 (t_2/t_1) + \ldots$$

and $$V(t_2/t_1) = \tilde{I}_{22.1} + \beta \tilde{\mu}_3 (t_2/t_1) + \beta^2 [\tilde{\mu}_4 (t_2/t_1) - 3 \tilde{I}^2_{22.1}] + \ldots$$

where $\tilde{\mu}_3(t_2/t_1)$ and $\tilde{\mu}_4(t_2/t_1)$ are the third and fourth Central conditional moments of t_2 under H_0. It is easy to verify that $\tilde{\mu}_3(t_2/t_1)=0$. Hence, ignoring terms of order β^2 in the above expressions, the number of years k is estimated assuming that the rates of type I and type II errors is equal to 0.05 by solving the equation

$$\beta^2 \tilde{I}_{22.1} = 4(1.645)^2$$

$$\beta^2 = 129.889/n \tilde{\theta}_0 (1-\tilde{\theta}_0) k(k+1) (k+2),$$

which is a cubic equation in k. It should be noted that the function $\tilde{\theta}_0\,(1-\tilde{\theta}_0)$ is symmetric about $\tilde{\theta}_0=0.5$, hence the value of k satisfying the above equation is the same for the two values $\tilde{\theta}_0=.5+\varepsilon$ and $\tilde{\theta}_0=.5-\varepsilon$. Furthermore $\tilde{\theta}_0(1-\tilde{\theta}_0)$ is nearly constant within the range $.3\leq\tilde{\theta}_0\leq.7$ hence vague knowledge about $\tilde{\theta}_0$ when small deviation from H_0 is expected will have small effect on the estimated value of k.

It is simpler to calculate the value of β for each value of k. This is given in Table 2 when $\tilde{\theta}_0=0.5$ and for n=26 and n=52. It is clear that the number of years decreases steadily as β decreases and as n increases. Monitoring for three years will allow the detection of values of $\beta=0.5752$ and 0.4067 when n=26 and 52 respectively.

TABLE 2. The Number of Years Required for Detecting Linear Trend Using the Logistic Model.

No. of Years k	slope, β	
	n=26	n=52
1	1.8196	1.2867
2	0.9096	0.6432
3	0.5752	0.4067
4	0.4068	0.2877
5	0.3075	0.2174
6	0.2431	0.1719
7	0.1985	0.1404
8	0.1661	0.1175
9	0.1416	0.1001
10	0.1227	0.0868
20	0.0464	0.0328
50	0.0122	0.0086

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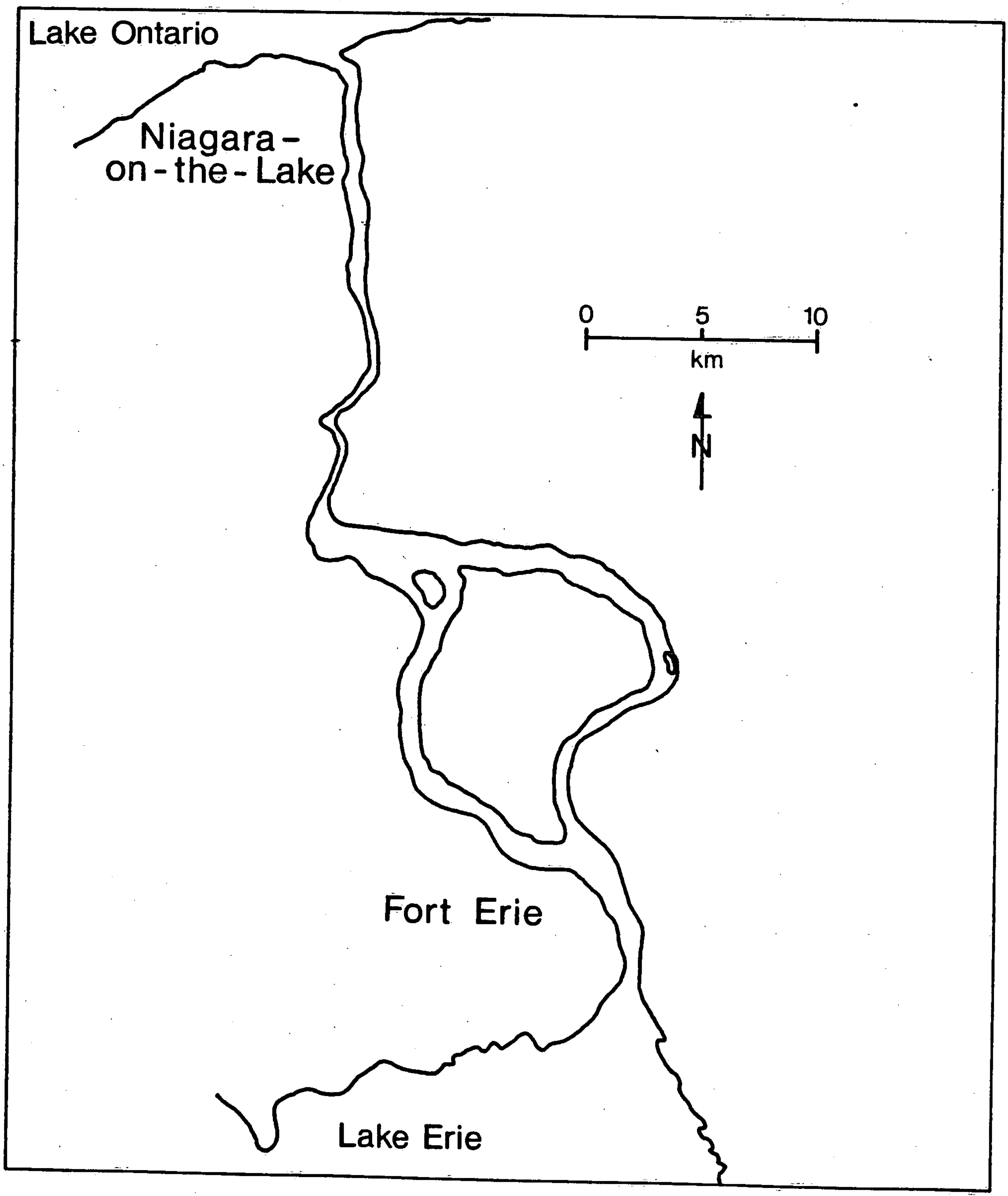
Table 1 The estimated number of paired samples M for monitoring the head and mouth of the Niagara River

Theta	P_{00}					
	0.0	0.1	0.2	0.3	0.4	0.5
.500	∞	∞	∞	∞	∞	∞
.510	27054.837679	33128.338130	40720.160620	50481.028809	63495.477325	81715.664679
.520	6759.649367	8293.046292	10209.737473	12674.006834	15959.655782	20559.522799
.530	3001.279955	3689.466837	4549.643479	5655.535522	7130.013552	9194.240326
.540	1685.849231	2076.712709	2565.233022	3193.279679	4030.629335	5202.875408
.550	1076.991151	1329.552056	1645.191986	2050.962277	2591.942216	3349.269689
.560	746.251749	923.316701	1144.584421	1429.017368	1808.212921	2339.041232
.570	546.824559	678.147003	842.234212	1053.147649	1334.315893	1727.904929
.580	417.346812	518.878159	645.674004	808.639629	1025.876122	1329.959644
.590	328.642829	409.585039	510.691846	640.626868	813.821182	1056.244565
.600	265.162835	331.337455	413.982027	520.177975	661.718776	859.826126
.610	218.192820	273.390523	342.311057	430.859825	548.869592	714.032357
.620	182.466222	229.276980	287.711789	362.774552	462.802808	602.790294
.630	154.660426	194.914846	245.150021	309.670527	395.639554	515.942928
.640	132.595225	167.624423	211.324683	267.440907	342.202622	446.814539
.650	114.791938	145.587446	183.992012	233.297140	298.975740	390.870046
.660	100.218986	127.534859	161.586092	205.291628	263.502396	344.940456
.670	88.138953	112.559088	142.986631	182.030578	234.024126	306.756767
.680	78.013377	99.997128	127.374927	162.495099	209.255120	274.659476
.690	69.441642	89.355492	114.141383	145.926300	188.237423	247.411942
.700	62.120698	80.260720	102.824499	131.749481	170.245399	224.077214
.710	55.817874	72.425963	93.069679	119.522931	154.721180	203.934805
.720	50.352130	65.627940	84.600808	108.902597	141.230126	186.423249
.730	45.580892	59.690745	77.200283	99.617252	129.429505	171.099710
.740	41.390637	54.474272	70.694723	91.450723	119.046087	157.611072
.750	37.690066	49.865805	64.944600	84.228995	109.859867	145.672945
.760	34.405072	45.773834	59.836597	77.810699	101.692071	135.054184
.770	31.474987	42.123434	55.277907	72.080022	94.396204	125.565336
.780	28.849758	38.852784	51.191933	66.941348	87.851287	117.049917
.790	26.487777	35.910503	47.514996	62.315169	81.956698	109.377739
.800	24.34225	33.253604	44.193806	58.134926	76.628182	102.439772
.810	22.419766	30.845895	41.183489	54.344559	71.794755	96.144137
.820	20.659535	28.656725	38.446038	40.896574	67.396268	90.412961
.830	19.052324	26.659997	35.949089	47.750517	63.381471	85.179882
.840	17.579941	24.833373	33.664939	44.871754	59.706482	80.388063
.850	16.226683	23.157642	31.569765	42.230497	56.333544	75.988592
.860	14.978914	21.616203	29.642989	39.801005	53.230027	71.939188
.870	13.824713	20.194645	27.866754	37.560943	50.367610	68.203149
.880	12.753577	18.880411	26.225509	35.490853	47.721616	64.748485
.890	11.756171	17.662508	24.705656	33.573723	45.270462	61.547213
.900	10.824100	16.531275	23.295262	31.794619	42.995204	58.574766
.910	9.949701	15.478191	21.983819	30.140393	40.879155	55.809500
.920	9.125827	14.495707	20.762043	28.599431	38.907571	53.232288
.930	8.345607	13.577105	19.621707	27.161438	37.067387	50.826175
.940	7.602125	12.716390	18.555496	25.817268	35.346986	48.576088
.950	6.887935	11.908175	17.556889	24.558768	33.736018	46.468588
.960	6.194178	11.147604	16.620055	23.378652	32.225230	44.491663
.970	5.508668	10.430269	15.739767	22.270395	30.806331	42.634547
.980	4.810711	9.752139	14.911325	21.228135	29.471878	40.887572
.990	4.050571	9.109497	14.130494	20.246595	28.215166	39.242031
1.000	2.706025	8.498873	13.393443	19.321015	27.030150	37.690066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