

**PROGRAM PARCOH2: PARTIAL AND MULTIPLE  
COHERENCES, TRANSFER FUNCTIONS IN THE  
FREQUENCY DOMAIN**

by

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NWRI Contribution # 86-

*26*

## PROGRAM PARCOH2

### Partial and Multiple Coherences, Transfer Functions in the Frequency Domain

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#### Abstract.

It is hypothesized that bottom currents in Lake Erie are driven by three forces, wind stress (through surface pressure gradients), internal pressure gradients (thermocline tilts), and coriolis force. An individual velocity component is thus a function of 5 variables, two components of wind stress, two components of internal pressure gradient, and the other horizontal velocity component. Ordinary coherence functions between the dependent variable and the independent variables, taken one at a time, are not reliable indicators of this functional dependence because of the correlations among the independent variables. From standard references on time-series analysis (Bendat and Piersol, 1971. Random Data: Analysis and Measurement Procedure. Wiley-Interscience. Otnes and Enochson, 1972. Digital Time Series Analysis. Wiley-Interscience.) and using IMSL and matrix inversion routines, a computer program has been assembled to compute multiple and partial coherences, transfer functions and phase angles, between the dependent (single velocity component) and the independent variables. This report has been written because extensions of this routine may be useful in other, similar applications.

## PROGRAMME PARCOH2

Cohérences partielles et multiples, fonctions de transfert dans le domaine des fréquences.

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### Sommaire

On suppose que les courants de fond du lac Érié sont produits par trois forces : la force d'entraînement du vent (par le truchement des gradients de pression de surface), les gradients de pression interne (l'inclinaison de la thermocline) et la force de Coriolis. Une composante de vitesse particulière est donc fonction de 5 variables : deux composantes de la force d'entraînement du vent, deux composantes du gradient de pression interne et une composante horizontale. Les fonctions de cohérence ordinaires entre une variable dépendante et les variables indépendantes, prises isolément, ne donnent pas une indication fiable de la relation fonctionnelle en raison des corrélations qui existent entre les variables indépendantes. En s'inspirant des documents classiques sur les analyses de série chronologiques (Bendat et Piersol, 1971. Random Data : Analysis and Measurement Procedure. Wiley-Interscience. Otnes et Enochson, 1972. Digital Time Series Analysis. Wiley-Interscience.) et en se servant de routines de l'IMSL et d'inversion de matrices, on a élaboré un programme informatique qui permet de calculer les cohérences multiples et partielles, les fonctions de transfert et les angles de phases entre une variable dépendante (composante de vitesse simple) et la variable indépendante. Nous diffusons les résultats de ces travaux parce que les extensions de cette routine pourraient s'avérer utiles dans d'autres applications semblables.

## Introduction.

The theory underpinning these developments is given in Chapter 5 of Bendat and Piersol (1971) (see Abstract for reference). Let  $Y$  be the dependent variable and the vector  $X = X_1, X_2, \dots, X_p$  be  $p$  independent but inter-correlated variables. In the time domain we seek the optimum (in the least-square-error sense) linear predictor of  $\hat{Y}$  on the  $p$  independent variables. This predictor can be expressed in the form

$$(1) \quad \hat{Y}(t) = \int_0^t \left\{ \sum_{i=1}^p h_i(\tau) X_i(\tau) \right\} d\tau$$

where the weighting functions  $h_i(\tau)$  are to be chosen so as to minimize the error between the observed and predicted values of  $Y$ .

## Transfer Functions.

The Fourier transforms of the optimum weighting functions in the frequency domain are termed transfer functions and can be determined from the  $p+1$  by  $p+1$  cross-spectral matrix formed by the dependent variable and the  $p$  independent variables. Let  $S_{ijk}$  be the complex cross-spectral estimate between independent

variables  $X_i$  and  $X_j$  in the  $k$ th frequency band. Similarly, let  $S_{Yik}$  be the cross-spectral estimate between the dependent variable,  $Y$ , and the  $i$ th independent variable, again for the  $k$ th frequency band. The transfer functions ( $H_{Y1k}, H_{Y2k}, \dots, H_{Ypk}$ ) are defined by  $p$  equations of the form

$$(2) \quad S_{Yik} = H_{Y1k}S_{1ik} + H_{Y2k}S_{2ik} + \dots + H_{Ypk}S_{pik}$$

for each frequency band. Note that the  $p$  by  $p$  matrix formed by the auto and cross-spectra of the independent variable set is "Hermitian", that is to say  $S_{jik}$  is the complex conjugate of  $S_{ijk}$ . The complex transfer functions can be expressed in terms of an amplitude [ $H_{Yik}$ ] and a phase angle  $A_{ik}$  between the component of the dependent variable and the  $i$ th independent variable at the  $k$ th frequency.

#### Multiple Coherence.

In a noise-free linear system where the output variable may be expressed exactly as a linear combination of the  $p$  input variables in the sense of equation (1) above, the relationship

$$(3) \quad S_{yyk} = H_{Y1k}S_{y1k} + H_{Y2k}S_{y2k} + \dots + H_{Ypk}S_{ypk}$$

also obtains. That portion of the observed variance of  $y$  in

frequency band k NOT associated with the optimum linear combination of input variables is defined

$$(4) S_{zzk} = S_{yyk} - \{H_{y1k}S_{y1k} + H_{y2k}S_{y2k} + \dots + H_{yk}S_{yk}\}$$
$$= S_{yyk}(1.0 - COH2M_k)$$

This equation also defines a multiple coherence-squared function COH2k linking the dependent variable with the optimum linear combination of the input variables. COH2k may be defined directly in terms of the elements of the full cross spectral matrix (see Otnes and Enochson, Chapter 9, p 343) and represents the fraction of the variance of y associated with the optimum linear combination of the X's in band k.

#### Partial Coherence Functions.

The ordinary coherence-squared between the dependent variable and one of the independent variables, let us say X1, is defined

$$(5) COH201k = S_{y1k}S_{1yk}/(S_{yyk}S_{11k})$$

Note that  $S_{1yk}$  is the complex conjugate of  $S_{1yk}$ . Each of the auto and cross-spectral estimates on the right hand side of equation

(5) depends not just on variable  $X_1$ , but on all the other independent variables as well. By eliminating the effects of the other independent variables ( $X_2, X_3, \dots, X_p$ ) we arrive at a measure of the coherence between  $Y$  and  $X_1$  alone, with the other variables in effect being held constant. Let us consider the variable  $X_1$  as being dependent on the remaining  $p-1$  independent variables. We then construct transfer functions between  $X_1$  and ( $X_2, X_3, \dots, X_p$ ) and compute in a fashion identical to that of equation (4) the variance of  $X_1$  not accounted for by the optimum linear relation among the  $p-1$  remaining variables:

$$(6) S_{11k'} = S_{11k} - (H_{12k}*S_{12k} + H_{13k}*S_{13k} + \dots + H_{1pk}*S_{1pk})$$

where the  $H_{ijk}$ ,  $j = 2, 3, \dots, p$  are the transfer functions between  $X_1$  and the remaining independent variables. Similarly the variance of  $y$  not accounted for by the linear combination of  $X_2$  through  $X_p$  may be expressed:

$$(7) S_{yyk'} = S_{yyk} - (H_{y2k'}*S_{y2k} + H_{y3k'}*S_{y3k} + \dots + H_{ypk'}*S_{ypk})$$

where  $H_{yjk'}$  are the transfer functions between  $Y$  and the  $p-1$  independent variables,  $X_2$  through  $X_p$ . It can be shown that the expression for  $S_{lyk'}$  is

$$(8) S_{lyk'} = S_{lyk} - (H_{y2k'}*S_{12k} + H_{y3k'}*S_{13k} + \dots + H_{ypk'}*S_{1pk})$$

The partial coherence between Y and X1 is now defined,

$$(9) \text{ COH2P1k} = S_{1yk'} * \text{CONJG}(S_{1yk'}) / (S_{11k'} * S_{yyk'})$$

where  $\text{CONJG}(S_{1yk'})$  is the complex conjugate of  $S_{1yk'}$ . Otnes and Enochson (Chapter 9, p344) give a formula for the partial coherence between Y and X1 in terms of the full  $p+1$  by  $p+1$  cross-spectral matrix. This formulation is used in the computer program PARCOH2. The partial coherences between Y and the other independent variables are computed in the same way (shifting the desired variable into the place of X1 in the algorithms).

#### Confidence Limits for Multiple Coherences, Transfer Functions, and Partial Coherences.

The procedures given by Otnes and Enochson (Chapter 9 , Sections 9.6 and 9.7, pp351 - 357) are adopted. The confidence limits for coherence are defined in terms of the degrees of freedom of the spectral and cross spectral estimates, the number of independent variables, and the required accuracy. Program PARCOH2 constructs a table of limits for both multiple and partial coherence-squared functions for both 65% (1 sd) and 95% (2 sd) confidences. Confidence limkits for the gains and phase angles of the transfer functions are more complicated to compute. They depend upon the degrees of freedom of the spectral estimates, the number of

variables, and the frequency-dependent structure of the cross-spectral matrix. The version of PARCOH2 used to compute Tables 1 through 4 does not compute the confidence limits for the transfer function, but the listing and sample output in the appendix contains this feature.

#### Description of the Program PARCOH2

The first part of the program is devoted to the input of the 6 contemporaneous time-series, appropriately scaled. Hourly samples are assumed in the present case; records are up to 1000 samples long. The matrix algorithm will work with a minimum of 4 time-series, one dependent variable and three independent variables. The case of 3 variables, one dependent and two independent, is treated directly in Bendat and Piersol, Chapter 5. Changes in variable numbers, maximum record lengths, will require redimensioning of the program with consultation of the IMSL manual describing routine LEQ2C. The operator must specify the number of lags or number of frequency bands in the interval 0. to 1./(.4.\*TSAM) (TSAM is the sampling interval) to be used in the spectral analysis routine, LAGS, and the number of frequency bands to be retained in the output, KEEP. The ratio

$$(10) \quad n = 2*INN/LAGS$$

where INN is the number of samples in each record defines the

degrees of freedom of the spectral estimates.  $n > 20$  is considered desirable; the larger  $n$  the more accurate the results.

Depending on the absolute values of the input variables, the resulting cross-spectral matrix and derived equations could be poorly-conditioned and the resulting solutions inaccurate. A loop in the program removes the means and scales the variables so that the variances are all identically equal to 1. This scaling has no effect on the coherence functions but does alter the transfer function amplitudes or gains. The normalized gain is of order 1 when there is a strong functional relationship between the dependent and the independent variable, and becomes very small when the two variables are not connected. Gains can be restored to physical dimensions using the scaling factors developed for variable normalization. This loop could be bypassed in many cases but provision should be made to remove means since the subroutine ROTSPEC assumes that the means are zero.

The next step in the program is to compute the elements of the cross spectral matrix for each of the required frequency bands. The first row and first column of the matrix contains the cross spectra between the dependent variable and the 5 independent variables. The routine ROTSPEC, the heart of the COHO family of spectral analysis programs, is employed for this purpose. Note that the elements of the cross spectral matrices are complex. ROTSPEC uses a Blackman-Tukey algorithm and is efficient for

record lengths of up to 1000 samples. Some care must be taken in smoothing the raw spectral and cross-spectral estimates; certain schemes that produce accurate auto-spectral estimates are less than optimum for cross-spectral estimates. ROTSPEC was modified to include a "Parzen Window" (see Otnes and Enochson, Chapter 6). At this stage the ordinary coherence-squared function is calculated for each pair of variables and each frequency band retained. The program prints selected power spectra and ordinary coherence-squared values.

Confidence limits for selected values of partial and multiple coherences are now calculated and output in tabular form (see description above).

Computation of multiple coherence, transfer functions, and partial coherences are next performed in that order. All of these require partial or complete inversions of portions of the complex cross-spectral matrix. The IMSL routine LEQ2C carries out these operations. The transfer functions are converted to polar form, amplitude and phase angle. The phase angle is the fraction of a cycle that the independent variable lags BEHIND the dependent variable. Partial coherences between the dependent variable and the 5 independent variables are obtained by successive rearrangements of the elements of the cross spectral matrix and reapplication of the algorithm.

The main output of Program PARCOH2 is a table organized by rows and columns, one row for each of the KEEP frequency bands of interest and the columns from left to right containing frequency (cph), multiple coherence-squared, transfer functions (amplitude above, phase angle below) (5 columns) and 5 columns of coherence-squared values, partial above, ordinary below.

In the present version, the two current velocity components are each considered in turn as dependent variables; this necessitates a second pass through parts of the program with the velocity components interchanged.

The annotated code for Program PARCOH2 is listed in Appendix A of this report.

#### Sample Computations.

##### Case 1. Uncorrelated random inputs.

Variables 1 through 6 are uncorrelated random inputs in the range (-1, +1). With variances normalized to 1.0, the spectral density in each band is theoretically 4.0 (spectrum estimated over the frequency interval 0 to 0.25 cph). All coherences are theoretically zero. The computed spectral density values are very close to the theoretical figure (a rms error of 7%).

DEPENDENT VEL = V-COMPONENT VELOCITY

$$\text{DEPENDENT } VRL = \frac{\text{V-COMPONENT VELOCITY}}{\text{U-COMPONENT}}$$

TABLE 1. Uncorrelated random inputs

FREC	COH2M	HAM/ANG1	HAM/ANG2	HAM/ANG3	HAM/ANG4	HAM/ANG5	HAM/ANG6	PCOH21	PCOH22	PCOH23	PCOH24	PCOH25
0.000	0.029	0.004E-01	0.361E-01	0.145E+00	0.452E-01	0.713E-01	0.002	0.001	0.018	0.002	0.005	0.002
0.025	0.024	0.120E-01	0.486E-01	0.110E+00	0.332E-01	0.946E-01	0.160	0.042	0.014	0.01	0.008	0.003
0.050	0.017	0.344E-01	0.335E-01	0.288E-01	0.354E-01	0.121E+00	0.301	0.001	0.001	0.001	0.013	0.006
0.075	0.020	0.609E-01	0.354E-01	0.358E-01	0.771E-01	0.110E+00	0.004	0.001	0.005	0.006	0.010	0.008
0.100	0.024	0.745E-01	0.605E-01	0.244E-01	0.104E+00	0.659E-01	0.006	0.003	0.001	0.010	0.004	0.011
0.125	0.031	0.116E+00	0.493E-01	0.789E-01	0.613E-01	0.333E-01	0.615	0.042	0.007	0.004	0.001	0.004
0.150	0.046	0.166E+00	0.549E-01	0.104E+00	0.408E-01	0.214E-01	0.030	0.003	0.012	0.002	0.001	0.003
0.175	0.042	0.163E+00	0.222E-01	0.854E-01	0.979E-01	0.778E-02	0.021	0.001	0.009	0.011	0.000	0.017
0.200	0.034	0.102E+00	0.806E-01	0.534E-01	0.119E+00	0.297E-01	0.011	0.007	0.003	0.015	0.001	0.002
0.225	0.028	0.102E+00	0.106E+00	0.243E-01	0.734E-01	0.423E-01	0.010	0.012	0.001	0.005	0.002	0.001
0.250	0.025	0.115E+00	0.878E-01	0.822E-02	0.371E-01	0.683E-01	0.012	0.008	0.000	0.001	0.005	0.003

TABLE 1 b.

COH2 CONFIDENCE LIMITS ZALPH = 1.0  
 INN = 1000 LAGS = 10 NDF = 200.0

COH2	MULTIPLE	PARTIAL		
0.000	-1.000	.001	-1.000	.002
.200	.135	.231	.150	.246
.400	.325	.434	.342	.449
.600	.536	.628	.551	.640
.800	.762	.816	.771	.823

COH2 CONFIDENCE LIMITS ZALPH = 2.0  
 INN = 1000 LAGS = 10 NDF = 200.0

COH2	MULTIPLE	PARTIAL		
0.000	-1.000	.006	-1.000	.009
.200	.694	.283	.107	.298
.400	.270	.487	.287	.501
.600	.485	.669	.501	.679
.800	.730	.839	.740	.844

Table 1 contains the output from this example. An average of the multiple coherence values in the 11 reported bands is .03, and a similar average of the partial coherences is .006. From the standard deviations we determine that a multiple coherence greater than .05 is significantly greater than zero at the 95% level, and the corresponding value for a partial coherence is .018. These seem to be less accurate results than those anticipated from the computation of confidence limits by the formulae given by Otnes and Enochsen and are perhaps the consequence of smoothing of the raw spectral estimates.

#### Case 2. Correlated random inputs.

Variables 2 through 6 are random numbers in the range -1 to +1. Variable 1, treated as the dependent variable, is the sum of variables 2, 3, and 4. Table 2 contains the results. The theoretical values of the multiple coherence-squared and the partial coherence-squared for the three inputs summed to yield the output are 1.0. The remaining partial coherences should be zero. The transfer function should show an in-phase relationship with the first three input variables and random phase with the remainder. These theoretical results are obtained in practice. Note that the ordinary coherences between the dependent variable and the inputs in turn show differences between the first three independent variables and the last two but that the partial



coherence computation reveals the true nature of the relationship.

**Case 3. Episode of winds, currents, and internal pressure gradients from the Central Basin of Lake Erie:**

The equations of motion for hypolimnion currents show that local acceleration of one component of the currents will depend on surface pressure gradients caused by the wind set-up of the lake, internal pressure gradients caused in simplest form by the set-up or tilt of the thermocline, and coriolis force. A simple statement of this balance of forces and local acceleration is

$$(11) \quad dU/dt = fV + Px - kU$$

$$dV/dt = -fU + Py - kV$$

where  $P_x$ ,  $P_y$  is the net pressure force vector and  $k$  is a friction factor. By solving this set of equations for forcing functions of variable frequency, ( $P_x = P \sin(\omega t)$ , etc.) a model frequency response can be obtained for comparison against observations. The general solution at forcing frequencies above and below the inertial frequency,  $f$ , comprises a forced response at frequency  $\omega$  and a free, but damped response at frequency  $f$ . The two combine to satisfy the initial conditions. The system becomes resonant as the forcing frequency approaches the local inertial frequency. The phase angle between a forcing component and the response is

the most simply analysed, and may be compared directly to the phase angle associated with the complex transfer function. The relationships would be expected to hold in the bands where the multiple coherence-squared value was relatively large. Table 3 summarizes the phase relationships:

TABLE 3

Forcing Frequency	Dependent Variable	Independent Variable	Phase Angle (cycles)
very low	U	Py	0.0
very low	V	Px	0.5
w < f	U	Px	+0.25
w < f	V	Py	+0.25
w > f	U	Px	-0.25
w > f	V	Py	-0.25
w = f	U	V	+0.25
w = f	V	U	-0.25

The phase angle obeys the same convention as that of the transfer function; it is the angle by which the independent variable lags the dependent variable.

We turn now to the analysis of the observations. In the absence of direct measurements of the surface pressure gradient, and knowing these to be tightly coupled to wind stress, we replace the two components of surface pressure gradient by the two

components of wind stress. The subsurface pressure force due to wind will be 180 degrees or half a cycle out of phase with the stress. The internal pressure gradient is also expected to be a function of wind stress, at least in some integrated sense. The effect of coriolis force is included by retaining the other velocity component as an independent variable. Thus one current component becomes the dependent variable, and the independent variables chosen in the study are the remaining horizontal current component, the two components of wind stress, and the two components of internal pressure gradient. Friction forces are neglected; they are expected to be strongly coherent with the dependent variable. The internal pressure gradient is estimated from the thermistor arrays surrounding the current meter location. The horizontal separation of these arrays is of the order of 10 km so that the estimated gradient may be contaminated by processes occurring at smaller scales and would therefore not be reliable at frequencies greater than say 0.1 cph (scale based on the travel time of an internal wave across the array). Coherences among the individual thermistor arrays are small, especially at frequencies greater than the inertial. Indeed the question approached by this analysis is whether the pressure gradient estimated from the individual readings (in effect a linear combination of these readings) is a significant dynamic variable of the system.

The examination of the relation between the output and input

INGRAD2 PRESSURE SIGNALS, INTERNAL PRESSURE GRADIENT, WIND VECTOR  
AND 3 CURRENT VECTORS (27-10, 27-15, 27-19) FROM MID EASTIN EXPT  
1979. FILE CREATED FEBRUARY 3, 1986.

TA = 4944.00 TB = 5304.00

START 0. 0. 26. 7 1979. STCP 0. 0. 10. 8 1979.

DEPENDENT VBL = V-COMPONENT VELOCITY

DEP VBL 1 = U-COMPONENT

DEF VBL 2 = WXX

DEF VBL 3 = WYY

DEF VBL 4 = PTX

DEF VBL 5 = PIY

FREC COH2M HAM/ANG1 HAM/ANG2 HAM/ANG3 HAM/ANG4 HAM/ANG5 PCOH21 PCOH22 PCOH23 PCOH24 PCOH25

0.000	.548	.728E-01	.451E+00	.338E-01	.138E+00	.747E-01	.005	.411	.003	.043	.023
		.50	0.00	.50	0.00	0.00	.002	.30	.015	.026	.004
.025	.727	.736E+00	.479E+00	.247E+00	.226E+00	.790E+01	.527	.484	.145	.167	.021
		-.26	.01	-.31	.17	.05	.074	.278	.026	.022	.004
.050	.747	.766E+00	.501E+00	.301E+00	.302E+00	.179E+00	.593	.276	.092	.091	.029
		-.29	.02	-.34	.21	.12	.427	.253	.080	.034	.028
.075	.681	.634E+00	.384E+00	.188E+00	.469E+00	.516E+00	.430	.101	.019	.121	.082
		-.28	.11	-.30	.19	.14	.680	.129	.261	.031	.072
.100	.497	.459E+00	.347E+00	.205E+00	.407E+00	.441E+00	.269	.081	.026	.096	.072
		-.30	.21	-.31	.16	.16	.365	.103	.155	.042	.051
.125	.130	.1135E+00	.210E+00	.134E+00	.109E+00	.134E+00	.031	.044	.026	.011	.011
		-.37	.18	-.37	.13	.18	.065	.068	.033	.014	.016
.150	.079	.920E-01	.165E+00	.109E+00	.908E-01	.103E+00	.014	.027	.026	.009	.008
		.10	.13	-.44	.17	.25	.062	.027	.022	.002	.014
.175	.135	.156E+00	.128E+00	.115E+00	.194E+00	.116E+00	.037	.013	.027	.031	.008
		.01	.24	.44	.12	.24	.059	.037	.017	.110	.076
.200	.163	.101E+00	.190E+00	.149E+00	.187E+00	.249E+01	.016	.024	.038	.023	.000
		-.03	.35	.40	.05	.32	.013	.069	.048	.207	.163
.225	.139	.253E-01	.294E+00	.895E-01	.154E+00	.879E-01	.001	.056	.013	.013	.003
		.18	.45	-.06	-.24	.00	.001	.163	.051	.169	.166
.250	.142	.543E-01	.168E+00	.289E-01	.132E+00	.530E-01	.004	.054	.001	.010	.001
		.50	.50	-.50	-.00	-.01	.007	.208	.050	.154	.170

TABLE 4.

variables in the frequency domain assumes that forcing in one frequency band will produce motion in that band; it assumes that the process is linear. Table 4, ~~located in Appendix A~~, contains the results for the component of velocity eastwards along the lake during the interval 26 July to 8 August, 1979. Multiple coherences are large in the lowest five frequency bands that account for the low frequency and resonance band motions. The phase angle between the y-component of velocity and the x-component of wind stress is consistent with the low-frequency geostrophic relationship of Table 3, but the corresponding phase relation between the y-component of current and the x-component of the internal pressure gradient does not fit the model, and furthermore, the partial coherences are barely significant. In the next frequency band, the phase between the y-component of velocity and the y-component of wind stress closely fits the model, but again this is not true for the internal pressure gradient. At frequencies above the resonance band, the multiple and partial coherences are low, suggesting that very little variance is explained by the model. In the resonance band, (0.05 cph) the phase angle between the two velocity components indicates clockwise rotation of the velocity vector.

The PARCOH2 computations have thus led to a negative but useful result. In direct contrast with the surface pressure gradient directly related to the wind stress, the internal pressure gradients computed from the array of thermistor chains have only

marginal statistical significance, and have no clear dynamical interpretation. Therefore they must be considered as a possible consequence of spatial aliasing and/or measurement error.

#### Conclusions.

Test Case 2 demonstrates the value of the computed multiple and partial coherences in detecting a linear relation between a dependent variable and a group of presumed independent variables.

In Test Case 3, both the coherence functions and the transfer functions are employed to examine the apparent relationship between estimated internal pressure gradient and measured currents. The PARCOH2 routine provides a useful diagnostic tool for such multivariate analyses in the frequency domain.

\* PARCOH1 PROGETTO PARCOH1 (SIMBIOTIC PROJECT) - TARGA 27-01-2007

PROGRAM PARCO-11 OPERATES ON SIMULTANEOUS TIME-SERIES OF CURRENT AND WIND TRANSFER FUNCTIONS, MULTIPLE AND PARTIAL COHERENCES (SHARED) IN FREQUENCY DOMAIN BETWEEN U (LAND) AND V (SEA) AS INDEPENDENT VARIABLES WITH THE REMAINING VARIABLES AS INDEPENDENT VARIABLES. INPUT TO PROGRAM IS FILE CREATED BY PROGRAM INGRAD 4.

**NOTE:** PROGRAM IS CONFIGURED TO OPERATE WITH UP TO 7 VARIABLES, ONE DEPENDENT AND 6 INDEPENDENT WITHOUT REQUIRING FORMAT OR DIMENSION CHANGES OTHERWISE.

REFERENCE-6 OTNES AND ENGHØSON-1-972  
DIGITAL TIME SERIES ANALYSIS (CHAPTER 9)

**PROGRAM USES INS-ROUTINE TO SOLVE LINEAR EQUATIONS WITH COMPLEX COEFFICIENTS**

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**G(17,7,21) = COMPLEX CROSS SPECTRAL MATRIX**

IN THIS COMPILATION ARE FREQUENCY BANDS FROM 0-6PH TO 6-25 GPH

FREQ(K) = FREQUENCIES OF SPECTRAL ESTIMATES  
 COHM2(K) = MULTIPLE COHERENCE (SQUARED) BETWEEN DEPENDENT-VARIABLE AND LINEAR COMBINATION OF 3-INDEPENDENT-VARIABLES  
 IN 11 FREQUENCY BANDS  
 HAM(K) = AMPLITUDE OF TRANSFER FUNCTION BETWEEN DEPENDENT VARIABLE AND INDEPENDENT VARIABLE  
 IN BAND K

HANG(K) = PHASE ANGLE OF TRANSFER FUNCTION IN FRACTION OF A CYCLE (-5 TO +5)  
 PCOH2(M,K) = PARTIAL COHERENCE (SQUARED) BETWEEN M-INDEPENDENT-VARIABLE AND K-DEPENDENT-VARIABLE

THE INSTITUTE OF MANAGEMENT

PLX 6177-2114(77), 88(77) WA(63), AR(2,0.5).

W[K(7)] NCHAN[7] H[6, 21] S[2, 7] Z[7] V[7].

MON/DAT/Z(7+1000),FREQ(21),GOSPI(21),QWAD(21),PI(21),PJ(21),

START AND STOP TIMES END SEGMENT OF DATA FILE

A. (NEAN) : = 1. 2. 3. 4. 5. 6. 7. / 9.

## NOSEGATINE PARAMETERS

DEMISSIONS- & JUDICIAL INDEP. VBLS., INDEP. VBLs. - 1  
CIVIL & COMMERCIAL CASES

**CONFIDENCE LIMIT FOR GAIN AND PHASE. CONFIDENCE LIMIT  $\chi = 100 \cdot (1 - \text{ALPHI})$**

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# APPENDIX A

## LISTING OF SAMPLE OUTPUT

# APPENDIX A

## LISTING OF SAMPLE OUTPUT

C THE NEXT GROUP OF PARAMETERS CONDITIONS THE INPUT  
 VARIABLES. THEY ARE PARTICULAR TO THE APPLICATION.

HT=23. JE=.J6/HT  
 FACH=2. JE=.J6/HT  
 FAPE=2.9E-4

RADG=57.296  
 DOX=55.0  
 IA=1.0  
 IB=1.0B6\*

DELT=7.6-IA  
 IF(DELT.GT.999.1)I3=TA+999.

90 C NUMBER OF LAGS IN CORRELATION COMPUTATIONS (LAGS)  
 C NUMBER OF SPECTRAL ESTIMATES RETAINED (KEEP)  
 C KEEP MUST NOT EXCEED LAGS+1

95 C LAGS=11  
 C KEEP=11

100 C READ IN TIME SERIES TEST DATA  
 C LINE TEST GENERATES TEST DATA

105 C CALL INTEST  
 C IPASS=2  
 C DO 333 IT=1,4  
 C DO 333 JT=1,8  
 C AT(IT,JT)=10H\*\*IST\*\*\*  
 C 333 CONTINUE

110 C GO TO 102

DO 555 KRA=1  
 IPASS=1  
 REHIND 1  
 TA=TAN(KR)  
 TA=JOM(KR)

115 C READ TITLE  
 C

DO 100 I=1,4 AT(IT,KR),K=1,8  
 READ(1,100)  
 100 FORMAT(8A10)  
 100 CONTINUE  
 PRINT 599

125 DO 149 I=1,4 IT(I,J), J=1,8  
 149 CONTINUE  
 PRINT 597 TA TB

130 597 FORMAT(5X,5H TA TB,  
 CALL JUL1M1, THA, TMA, TJA, TIV, TA)

CALL JUL1M1, THB, TMB, TJB, TIV, TB)

PRINT 580 THA, TMA, TJA, TIV, THB, TMB, TJB, TIV  
 580 FORMAT(1/,5X,6H START, 12F3.0,1X, F3.0,13,F7.0,8H STOP ,

135 C READ DATA (PARTICULAR TO FILE USED)  
 C TAA=TA-1.  
 C INN=0

140 161 READ(1,4J1) TIME  
 401 FORMAT(F8.2)  
 401 IETIME.LE.TAA GOTO 141

145 163 IF(ETIME.EQ.18) GOT 142  
 INN=INN+1  
 READ(1,402) TIME,Z(1, INN), Z12, INN), MX, MY

C FORMAT FGR-BOTTOM-CURRENTS-FROM-NODEL  
 C 402 FORMAT(F8.2,6GX,4E1.3)

150 C WS=SQRT(MX\*MX+MY\*MY)\*FACH  
 Z(3, INN)=WS\*MX  
 Z(4, INN)=WS\*MY





COMMA<sup>2</sup>(K)=1:-1,/[G(1:1:K)\*B(1:1)]

COMPUTE TRANSFER FUNCTIONS

```

385      CALL LEQ2C (AC, IPP, S, BC, IPP, 5, 0, WA, WK, IER)
380      DO 130 J=1,1IPP
          AC(1,J)=A(I+2,J+2)
          BC(1,J+2)=A(I+2,J+2)
          DO 130 J=1,1IPP
          AC(1,J)=A(I+2,J+2)
130      END

```

SYNTHETIC POLY(URIDYLIC ACID)

200

```

465      IF(J.EQ.I) J=I
        IF(J.EQ.I+1) J=I+1
        DO 151 I=1,IT
        DO 151 J=1,IT
        DO 151 K=1,IT
        G(I,J,K)=A(I,J,K)
1501    CONTINUE
151
151    C(I,J,K)=A(I,J,K)
1501    CONTINUE

```

```

470      IPSS=2
        GO TO 555
475      5555  CONTINUE
        STOP 777
        END

```

```

1      FUNCTION FDIST(EN1,EN2)
        H=2.0/(1.0/EN1+1.0/EN2)-1.
        FDIST=SQR(H-H*EN1*EN2)
        FDIST=EXP(12.0*W)
        RETURN
5

```

```

1      SUBROUTINE INEST
        COMMON/INEST/COH(7,21),FRE(421),C95P(421),P1(421),P3(421),
        COH(7,21)
5      DO 100 I=1,100
        F1=2.0*(5.0-RANF(1))
        F2=2.0*(5.0-RANF(2))
        F3=2.0*(5.0-RANF(3))
        F4=2.0*(5.0-RANF(4))
        F5=2.0*(5.0-RANF(5))
        F6=2.0*(5.0-RANF(6))
        Z(6,1)=F6
        Z(4,1)=F4
100
10      Z(5,1)=F5
        Z(3,1)=F3
        Z(2,1)=F2
20      100  CONTINUE
        RETURN
        END

```



```

    QX = FAC*QX/NN
    QY = FAC*QY/NN
    QXY = FAC*QXY/NN
    QXX = FAC*QXX/NN
    QYY = 0.5*(QX*QY)
    AXY = 0.5*(QX-2*Y)
    F = LAG+GT-0.5*LT-MAX) GO TO 50
  85    QX = 5*QX
    QY = 5*QY
    AXY = 5*AXY
    QXY = 5*QXY
    F = 5*F
  90    PUJ1 = LAG+PUJ
    PUJ2 = LAG+MAX2
    DO 90 K=1,MAX2
    F = GOSR(PUJ2)
    TS = SIN(PI*PUJ2)
    FREX(K) = F*EX(K)+QX*TG
    FREY(K) = F*EY(K)+QY*TG
    GOSR(K) = GOSR(K)+AXY*TG
    QUAD(K) = QUAD(K)+QY*TS
  95    PUJ2 = PUJ2+PUJ1
    COH(IZ,JZ,L)=COH(IZ,JZ,L)+1
    100    QUAD(IZ,JZ,L)=QUAD(IZ,JZ,L)+1
    70  CONTINUE
    IF (NAO.LT.1MN) GO TO 20
    RETURN
    END

```



4634.00 - .171E+01 .113E+01 - .579E+00 .211E+01 .153E+00 .655E-01 .401E+00 .37E-01 -  
 4634.00 - .172E+01 .119E+01 - .863E+00 .153E+01 .141E+01 .606E-01 .965E+00 .982E-02 -  
 4635.00 - .164E+00 .182E+01 .399E+00 .141E+01 .139E+01 .303E+00 .139E+01 .512E-01 -  
 4636.00 .175E+01 .105E+01 .227E+01 - .119E+00 .386E+02 .569E+02 TA = 4634.00 TB = 5423.00

START 2. 0. 13. 7 1979. STOP 23. .. 14. 8 1979.

START & STOP  
Times

VBL 1 2 3 4

RAW MEAN .312E+00 .623E+00 .373E+02 - .433E+02

RAW VARIANCE .177E+02 .213E+02 .672E+03 .309E+03

SCALE FACTOR .230E+00 .217E+00 .386E+02 .569E+02

VBL NO. POWER SPECTRAL DENSITY (SCALED FOR UNIT VARIANCE)  
BAND 1 2 3 4 5 6 7 8 9 10 11

1	.052E+01	.106E+02	.125E+02	.086E+01	.310E+01	.460E+01	.536E+01	.267E+01	.310E+01	.125E+01	.469E+02
2	.117E+02	.123E+02	.117E+02	.744E+01	.246E+01	.349E+00	.476E+01	.236E+01	.237E+01	.765E+02	.224E+02
3	.215E+02	.162E+02	.733E+01	.262E+01	.110E+01	.646E+00	.425E+00	.320E+00	.320E+00	.227E+00	.223E+00
4	.163E+02	.129E+02	.683E+01	.306E+01	.169E+01	.121E+01	.925E+00	.679E+00	.530E+00	.516E+00	.530E+00

1	.100E+01	.119E+01	.102E+00	.292E+00	.119E+01	.321E+01	.100E+01	.237E+01	.237E+01	.765E+02	.224E+02
2	.119E+01	.100E+01	.100E+00	.460E+00	.100E+01	.321E+01	.100E+01	.236E+01	.236E+01	.765E+02	.224E+02
3	.102E+00	.460E+00	.460E+00	.100E+01	.321E+01	.321E+01	.321E+01	.321E+01	.321E+01	.320E+00	.223E+00
4	.292E+00	.119E+01	.119E+01	.306E+01	.169E+01	.121E+01	.925E+00	.679E+00	.530E+00	.516E+00	.530E+00

FREQ = 5.0000 VBL 3 4

1	.100E+01	.119E+01	.102E+00	.292E+00	.119E+01	.321E+01	.100E+01	.237E+01	.237E+01	.765E+02	.224E+02
2	.119E+01	.100E+01	.100E+00	.460E+00	.100E+01	.321E+01	.100E+01	.236E+01	.236E+01	.765E+02	.224E+02
3	.102E+00	.460E+00	.460E+00	.100E+01	.321E+01	.321E+01	.321E+01	.321E+01	.321E+01	.320E+00	.223E+00
4	.292E+00	.119E+01	.119E+01	.306E+01	.169E+01	.121E+01	.925E+00	.679E+00	.530E+00	.516E+00	.530E+00

FREQ = .0500

1	.100E+01	.500E+00	.111E+00	.246E+00	.246E+00	.437E+01	.437E+01	.437E+01	.437E+01	.437E+01	.437E+01
2	.588E+00	.100E+01	.365E+00	.100E+01	.365E+00	.770E-01	.770E-01	.770E-01	.770E-01	.770E-01	.770E-01
3	.111E+00	.365E+00	.100E+01	.365E+00	.100E+01						
4	.246E+00	.437E+01	.770E-01	.770E-01	.770E-01	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01

FREQ = .100

1	.100E+01	.370E+01	.835E+00	.968E+00	.968E+00	.161E+01	.161E+01	.161E+01	.161E+01	.161E+01	.161E+01
2	.715E+00	.100E+01	.109E+01	.109E+01	.109E+01	.247E+00	.247E+00	.247E+00	.247E+00	.247E+00	.247E+00
3	.968E+00	.247E+00	.109E+01	.109E+01	.109E+01	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01
4	.160E+00	.692E+00	.946E+00	.946E+00	.946E+00	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01

FREQ = .150

1	.100E+01	.370E+01	.835E+00	.968E+00	.968E+00	.161E+01	.161E+01	.161E+01	.161E+01	.161E+01	.161E+01
2	.322E+01	.100E+01	.109E+01	.109E+01	.109E+01	.247E+00	.247E+00	.247E+00	.247E+00	.247E+00	.247E+00
3	.635E+00	.247E+00	.109E+01	.109E+01	.109E+01	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01
4	.184E+00	.692E+00	.946E+00	.946E+00	.946E+00	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01	.100E+01

FREQ = .200

.100E+01	.353E+00	.307E-01	.321E-01
.353E+00	*100E+01	*486E-01	*617E-01
.307E-01	*486E-01	*100E+01	*423E-01
.321E-01	.617E-01	.623E-01	.100E+01

FREQ = .250

*100E+01	*137E+00	*323E-02	*623E-01
.137E+00	*100E+01	.145E+01	.157E-01
*323E-02	*145E+00	*100E+01	*335E-02
.623E-01	.157E-01	.335E-02	*100E+01

COH2 CONFIDENCE LIMITS ZALPH = 1.0  
INN = 786 LAGS = 10 NDF = 157.2

COH2	MULTIPLE	PARTIAL
VALUES	VALUES	VALUES
0.000	-1.000	.001
*2.00	*1.34	*242
.400	.323	.446
*6.00	*534	*637
.800	.761	.821

COH2 CONFIDENCE LIMITS ZALPH = 2.0  
INN = 786 LAGS = 10 NDF = 157.2

COH2	MULTIPLE	PARTIAL
VALUES	VALUES	VALUES
0.000	-1.000	.009
*2.00	*0.89	*300
.400	.261	.504
*6.00	*477	*682
.800	.725	.846

```

4633.00 -171E+01 *113E+01 -579E+00 *211E+01 0. -1 0
4634.00 -172E+01 *119E+01 -*863E+00 *153E+01 -655E-01 *401E+00 *137E-01 -
4635.00 -104E+00 *102E+01 *399E+00 *141E+01 *606E-01 *965E+00 *382E-02 -
4636.00 *175E+00 *105E+01 *227E+01 -*119E+00 *303E+00 *139E+01 *512E-01 -
TA = 4634.00 TB = 5423.00

```

**START** 2. 0. 13. 7 1979. STOP 23. 3. 14. 8 2979.

**DEPENDENT VBL = U-COMPONENT VELOCITY** (not defined here)  
**DEP VBL 1 = V-COMPONENT**

FREQ GOM2N HANZANE1 HANZANG2 HAWAII3 PCGH21 EC09H22 EC09H23

PARTIAL COHERENCE SQUARED

Transfer Function Phase (cycles)	Partial Coherence Squared
0.000	0.627
0.025	0.921
0.050	0.985
0.075	0.989
0.100	0.972
0.125	0.928
0.150	0.814
0.175	0.659
0.200	0.454
0.225	0.249
0.250	0.302

ORDINARY COHERENCE SQUARED

TRANSFER FUNCTION PHASE

CONFIDENCE LIMIT  
95%

CONFIDENCE LIMIT  
PHASE (95%)

TRANSFER FUNCTION PHASE (CYCLES)

4633. -0.171E+01 \*113E+01 -0.579E+00 \*211E+01 0. -1  
 4634.00 -0.172E+01 \*115E+01 -0.563E+00 \*153E+01 -0.655E+01 \*461E+00 \*137E+01 -  
 4635.01 -0.104E+00 -0.162E+01 \*399E+00 \*141E+01 -0.636E+01 \*636E+01 -0.965E+01 \*382E+01 -  
 4636.01 \*175E+01 \*105E+01 \*227E+01 -0.119E+00 \*303E+00 \*139E+01 \*512E+01 -  
 TA = 4634.00 TB = 5423.00

START 2. 0. 13. 7 1979. STOP 23. 0. 14. 8 1979.

DEPENDENT VBL = V-COMPONENT VELOCITY ELLIPID

DEP VBL 1 = U-COMPONENT

DEP VBL 2 = WXX

DEP VBL 3 = HYV

FREQ COM2H HAM/ANG1 HAM/ANG2 HAM/ANG3 PCOH21 PCOH22 PCOH23

0.000	0.816	0.029	-0.144	-0.685	-0.110	-0.134	-0.102	-0.091	-0.055	-0.052
0.025	0.946	0.022	0.064	0.594	0.056	0.375	0.053	0.857	0.878	0.621
0.050	0.986	0.913	0.028	0.453	0.27	0.302	0.036	0.979	0.874	0.731
0.075	0.989	0.917	0.025	0.265	0.24	0.178	0.026	0.588	0.365	0.044
0.100	0.971	0.995	0.041	0.186	0.137	0.121	0.056	0.961	0.285	0.219
0.125	0.921	0.853	0.075	-0.147	-0.65	-0.112	-0.046	0.877	0.234	0.316
0.150	0.804	0.764	0.129	0.089	0.117	0.133	0.035	0.623	0.162	0.393
0.175	0.686	0.686	0.128	0.013	0.126	0.153	0.034	0.577	0.006	0.173
0.200	0.479	0.543	0.153	0.028	0.132	0.131	0.047	0.415	0.015	0.037
0.225	0.361	0.326	0.05	0.017	0.041	0.136	0.118	0.032	0.189	0.066
0.250	0.353	0.323	0.172	0.166	0.114	0.19	0.017	0.235	0.211	0.026