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A Mathematical Description of the Effects of Prolonged Water Level Fluctuations on the Areal Extent of Marshlands

**R.P. Bukata,* J.E. Bruton,* J.H. Jerome* and
W.S. Harast†**

* Environment Canada
National Water Research Institute
Rivers Research Branch
Water Quality Modelling/Monitoring
Canada Centre for Inland Waters
Burlington, Ontario
L7R 4A6

† Fisheries and Oceans
Pacific and Freshwater Fisheries
Great Lakes Fisheries Research Branch
Canada Centre for Inland Waters
Burlington, Ontario
L7R 4A6

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Abstract

The chemical, biological, and physical interrelationships dictating the environmental status and evolution of shoreline wetlands are clearly dependent upon water level fluctuations of both short-term (i.e. seasonal) and long-term (i.e. longer than seasonal) time scales. In this report, the impact of prolonged water level changes on the areal extent of shoreline marshlands is discussed. The model presented is a simple conceptual approach based solely upon the geometric variables defining the morphology of the marsh and its confining basin. Two general and contradictory conditions are considered. Both conditions tacitly assume that the marshland in question contains a phreatophytic vegetation canopy, and that the dynamic nature of the plant/atmosphere/sustaining soil relationships, while disregarded, is nonetheless acting in a manner that preserves rather than destroys the marsh proper. The first condition, however, assumes that both the offshore and onshore reaches of the wetland area are capable of supporting and transforming either marshland or onshore vegetation as required, and that, given sufficient regeneration time, a dynamic equilibrium may be established between marsh and onshore conditions. The second condition tacitly

assumes no such vegetative equilibrium may be established on the onshore reaches.

Dividing the principal shoreline marsh configurations into the general geometric categories of linear, concave/convex, and elliptical, mathematical expressions are developed that relate changes in persistent water levels to the disappearance or re-emergence of marshlands in terms of the onshore and offshore slopes of the wetlands, the change in water level, the initial marsh area, and the maximum marsh depth beyond which emergent vegetation becomes indiscernible in a synoptic overview.

A "user-friendly" computer program written in IBM PC BASIC has been included. This computer program is intended to enable workers concerned with particular marshland regions to use the predictive (both for conditions of total vegetative regeneration and total vegetative non-regeneration) capabilities of the conceptual mathematical marsh model to evaluate the impacts of either naturally occurring or anticipated man-made changes in persistent ambient wetland water levels.

Résumé

Les relations d'échanges chimiques, biologiques et physiques qui dictent les conditions de milieu et l'évolution des milieux humides de rives dépendent de façon évidente des fluctuations du niveau de l'eau, sur une courte période (c'est-à-dire fluctuations saisonnières) aussi bien que sur une longue période (c'est-à-dire fluctuations plus longues que saisonnières). Ce document étudie les liens entre les changements prolongés du niveau de l'eau et la superficie des marais riverains. Le modèle utilisé constitue une approche conceptuelle simple basée uniquement sur les variables géométriques qui définissent la morphologie du marais et de son bassin adjacent. Deux conditions générales et contradictoires sont envisagées. Ces deux conditions sous-entendent que le marais comporte un couvert végétal de phréatophytes et que la nature dynamique des relations entre les plantes, l'atmosphère et le sol nourricier, bien qu'on n'en tienne pas compte, exerce une action préservative plutôt que destructive sur le marais. Sous la première condition, toutefois, on suppose que les parties sous l'eau et hors de l'eau de la superficie du terrain humide peuvent alimenter et transformer la végétation du marais ou du terrain hors de l'eau et que, si la période de régénération est suffisante, un équilibre dynamique s'établira entre les conditions du marais et celles de la rive. Sous la deuxième condition, on

sous-entend qu'un tel équilibre du couvert végétal ne peut s'établir sur les rives.

Lorsqu'on distingue les principales configurations du marais riverain à l'aide des grandes catégories géométriques, soit linéaire, concave, convexe et elliptique, on parvient à des expressions mathématiques qui relient les changements persistants dans les niveaux d'eau à la disparition ou l'émergence des marais, en termes de pentes sous l'eau et hors de l'eau des milieux humides, de changement du niveau de l'eau, de superficie du marais initial et de profondeur maximale du marais au-delà de laquelle la végétation émergente ne peut être distinguée dans une vue synoptique.

Ce document comprend un programme informatique simple à utiliser et écrit en BASIC pour un IBM PC. Le but du programme informatique est de permettre aux chercheurs s'intéressant à des marais particuliers d'utiliser les prévisions issues du modèle conceptuel mathématique du marais (autant pour prévoir la condition de régénération totale du couvert végétal que la condition de non-régénération totale du couvert végétal). Les chercheurs évalueront les répercussions des changements provenant de phénomènes naturels ou provoqués par l'homme sur les niveaux d'eau persistants des milieux humides ambiants.

Management Perspective

Coastal wetlands are vibrant, valuable, and vulnerable ecosystems that support a delicately balanced vegetation/fishstock/waterfowl population. Since this delicate population balance necessitates that coastal wetlands act as simultaneous locales of spawning, nursing, and feeding, water levels play an integral role in the status and continual evolution of these wetlands. Equally important to the overall problems of water availability are the level fluctuations characterizing this available water. Marshlands require seasonal or short-term water level fluctuations to maintain the continuum of life cycle activities so essential to their evolution. Nutrients must be imported and waste materials must be flushed away.

Fluctuations of time scales significantly different from seasonal can serve to compound these seasonal effects, and thereby induce effects which may or may not be desirable. Coastal storms, for example, can produce devastating consequences. Long-term (substantially greater than seasonal) water level fluctuations may produce shifts in indigenous plant communities, corresponding shifts in wetland classifications and areal extents, and the possibility of dramatic impacts on the fish and wildlife populations. The relationships between water level fluctuations and the areal extent of coastal marshlands are, therefore, clearly important to any assessment which attempts to understand and/or predict the impact of a natural or man-made adjustment to the ambient water levels. These water level adjustments may be a consequence of climate, land use management, or regulation and/or diversion of river and lake waters. A means of predicting the impact of such water level fluctuations on existing marshland area can provide a valuable input to the sensible management of aquatic resources.

A simple conceptual model is presented for predicting the change in areal extent of marshlands with change in persistent water level in terms of the morphology of the marshes and their confining basins. In particular, such marshland changes are described in terms of the change in water level, the onshore and offshore slopes, the marsh area at zero water level datum, the maximum water depth which allows a marsh to be delineated in synoptic overview, and the basic geometric configuration of the shoreline accommodating the coastal marshland. The model is intended to provide a predictive capability for the impact of persistent water level changes when used in conjunction with aerial photography and/or satellite delineation of coastal marsh areas.

Two general conditions are considered within the current model. The first condition assumes that although the soil/water/air chemical and biological processes are, in essence, disregarded, these processes are acting in such a manner as to attempt complete restoration of the marshland under increased water levels. This implies that onshore reaches previously not inundated with well-defined persistent standing water will, subsequent to an appropriate response time, establish a vegetative equilibrium between onshore and marshland growth canopies, i.e., the metamorphosis of one wetland classification to another is allowed to proceed in a natural, uninterrupted manner designed to maximize the restorative capabilities of impacted wetlands. The second condition, which is contradictory, assumes that no such delayed regeneration is possible, i.e., no vegetative equilibrium may be established, and the restorative capabilities of metamorphosis are completely curtailed. It is reasonable to consider that reality is to be found somewhere between these two extremes.

Introduction

Freshwater wetlands are a dynamically complex natural resource. Wetland classification types include swamps, meadows, marshes, bogs and fens of glacial, prairie or lake origin. Further, depending upon both the ability of wetlands to respond and adapt to changing environmental conditions, and the capacity of the environmental parameters to allow such response and adaptation, the wetlands themselves may metamorphose from one classification to another.

Irrespective of the marked variations in wetland type, origin, and past history, however, detailed analyses of wetland behaviour require careful consideration of the interrelated dependencies existing among the flora, fauna, sustaining aquatic and soil regimes, and climatic conditions pertinent to the wetland area under scrutiny. This is particularly true when the impacts of both transient and prolonged fluctuations in environmental parameters (such as water levels, climate, contaminants, terrain dynamics) on wetland definition and vigor of both its resident biota and vegetative canopies are sought.

The predominant wetlands comprising the Great Lakes basin are both lacustrine and riverine shoreline marshes, meadows and swamps. The general distinctions made between such wetlands are the following:

- (a) Marshes contain persistent standing water throughout their region of definition, whereas meadows and swamps are characterized by water-saturated sediments with usually little or no standing water.
- (b) Marshes are generally recognizable by emergent bottom-anchored vegetation of the bulrush, reed, and cattail variety, while meadow and swamp vegetation types include grasses, shrubs, thickets and trees.
- (c) Meadows and swamps are normally located upland from the shoreline marshes.

In this report Great Lakes basin shoreline marshes are discussed. In particular, the impact of basin-wide fluctuations in water level upon the areal extent of such lacustrine and/or riverine marshlands is assessed. The fluctuations considered are further restricted to those

changes in water level that persist over a longer time period than the cyclical annual variation associated with the region under consideration.

The importance of the interrelationships among biota, vegetation, aquatic and climatic parameters in governing the behaviour (both destructive and regenerative) of marshlands is rapidly becoming more appreciated, and although the interrelationships themselves are far from the desired state of robust mathematical expression and vindication, they are, nonetheless, becoming much more intuitively understandable. This is clearly evident in the evolution of much of the scientific literature. Detailed discussions of wetlands classifications, the interdependence of internal and external wetland parameters, and the impact of water level fluctuations on wetland dynamics have been excellently presented from a wide variety of sources and scientific perspectives (see, for example, Chapman and Putnam, 1966; Greeson *et al.*, 1979; Gosselink and Turner, 1978; Geis and Kee, 1977; Geis, 1979; Lands Directorate, 1981, 1983; Simpson *et al.*, 1983; Jaworski *et al.*, 1979; Lyon, 1981; Burton, 1985; Whillans, 1982). While the directives, theories, and inferences drawn from such literature may display real and/or perceived variances, the activities, methodologies, and analyses presented in such studies are clearly required to advance the multidisciplinary scientific thought so vital to proper assessment of wetland dynamics.

It is singularly apparent that water level fluctuations are of integral importance to wetland development and status. Seasonal or short-term fluctuations provide a natural opportunity for the uninterrupted continuance of the growth and life cycles of the fish and wildlife inhabiting regions such as the Great Lakes basin. Shallow water environments such as marshes are essential to the preservation of fish stocks, since marshes provide appropriate spawning, nursing and feeding locales. Consequently, marshlands require short-term water level fluctuations to enhance and protect their productivity. Periodic short-term floodings are required to simultaneously provide nutrient inputs and flush away waste materials, thereby enabling the marsh to rigorously maintain the spectrum of vegetative communities essential to the health and vigor of its wildlife and fish populations.

Extended periods of high or low water levels can compound these short-term effects of fluctuating water levels, and thus induce effects that may or may not be desirable. Long-term lake level fluctuations may produce shifts in indigenous plant communities (Harris and Marshall, 1963; van der Valk and Davis, 1978; Keddy and Reznicek, 1982, 1985; Pederson and van der Valk, 1984; Hutchinson, 1975). Low water conditions generally result in an associated displacement of emergent vegetation by sedge/meadow plants and shrubs coupled with an obvious reduction in open water and aquatic communities. High water conditions generally result in increased open water communities at the expense of sedge/meadow communities. Dramatic impacts on fish and wildlife may clearly ensue (Jaworski and Raphael, 1978).

Consequently, an understanding of the impact of both natural (i.e. climatic) and artificially created (e.g. flood diversion, fabricated drainage systems, etc.) long-term water level fluctuations on the wetlands which prominently occupy basins of the magnitude and importance of the Great Lakes basin plays a significant role in the sensible management and regulation of natural water bodies (see International Great Lakes Levels Board, 1973; Great Lakes Basin Commission, 1975; International Joint Commission, 1976; 1978; International Great Lakes Diversion and Consumptive Uses Study Board, 1981).

It is therefore clear that persistent fluctuations in water levels may influence the areal extent of coastal marshlands. One potentially advantageous method of investigating the relationship between water levels and marshland areal extent involves the use of aerial photography or satellite imagery (Carter, 1978; Klemas *et al.*, 1978; Hardisky and Klemas, 1983; Sasser *et al.*, 1986; Lyon, 1979; Lyon and Drobney, 1984; Bukata *et al.*, 1978; Butera, 1985; Civco *et al.*, 1986; Gross and Klemas, 1985; Ridd *et al.*, 1981; Shima *et al.*, 1976). This method basically requires that first, the remotely sensed images at known water levels be used to delineate the areal extent of the marshlands in question, and secondly, some workable model be developed which could allow for the predictions of impact on marshland area of an anticipated or planned persistent change in water level.

A means of obtaining a predictive model that readily suggests itself is the simple point-by-point regression of water levels and their corresponding marshland areas. Yet such a method (as properly pointed out by Reznicek and Keddy, 1984) takes into account neither the interdependence of the myriad parameters influencing the marshland's character and composition nor the temporal and spatial variabilities that so obstinately complicate those parametric

interdependencies. In fact, it has long been the bane of the remote sensing community (Bukata *et al.*, 1982) that a casual interchange of the roles of dependent and independent variables in the cause/effect relationships of environmental phenomena must be avoided in all research areas that rely upon regression analyses. The impact of lake water fluctuations on shoreline marshes is a convoluted consequence of the nature of the vegetative canopies comprising the marshes, the ability of this vegetation to respond to changing aquatic environments, the nutrient characteristics of the sustaining soils, the general climatic conditions indigenous to the area, the bathymetry of the standing water region of the marsh, and the topography of the surrounding basin. Consequently, a single unified predictive marsh model satisfying the physical, chemical, and biological aspects of the system is, at best, a highly elusive concept, especially since the governing parameters, as well as their interdependencies, will undoubtedly vary from marsh to marsh. However, this variance from marsh to marsh somewhat paradoxically serves as the strongest argument in support of the simple point-by-point regression of water levels with their corresponding marshland areas. This argument presumes that whereas such regressions could possess a certain restricted appropriateness to the region under consideration, these regressions would not be used to attempt an explanation of the behaviour of wetlands unrelated to that region. Consequently, within the confines resulting from an awareness of the limitations inherent to such regression techniques, valuable information and predictive modelling could result from such activities.

Despite the fact, however, that the marsh is a highly dynamic ecological system variably responsive to a spectrum of environmental changes, this report presents an attempt to relate marshland areas as determined from aerial photographs (using the presence or absence of identifiable plant species as a means of indicating the hydrological characteristics of the regions being remotely sensed) taken during periods of high and low water levels to long-term water level fluctuations, using a simple conceptual mathematical marsh model based solely upon the geometric variables defining the morphology of the marsh and its confining basin. Two general and contradictory conditions will be considered. Both conditions tacitly assume that the marshland in question possesses a phreatophytic vegetation canopy and that the dynamic nature of the plant/sustaining soil relationships, while disregarded, are, nonetheless, present and acting in a manner which attempts to preserve rather than destroy the marsh proper. The first condition, however, assumes that both the onshore and offshore reaches of the wetland area are capable of sustaining either marshland vegetation (when appropriately inundated) or onshore vegetation (when appropriately de-inundated). This assumes that given sufficient regeneration time, a dynamic

equilibrium may be readily established between marsh and onshore conditions, and that this dynamic equilibrium, although lagging, is nevertheless responding to persistent water level changes. The second condition tacitly assumes no such dynamic equilibrium may be readily established. In this situation, only the marsh region defined at zero water level datum is considered capable of sustaining marshland vegetation. The onshore region defined at zero water level datum is considered incapable of sustaining marshland vegetation for a number of reasons such as steepness of shore from the strand line, absence of suitable sustaining soils, large depositions of rocks and gravel, and restrictive wave activity.

It is logical to regard the first condition as a utopian situation in which the maximum amount of marshland, despite a time delay, will re-emerge subsequent to a persistent increase in water level, while the second condition represents the situation in which the minimum amount of marshland exists subsequent to a persistent increased water level, but a possible maximum amount of re-emerged marshland subsequent to a persistent decreased water level. It is equally logical to assume that reality is located somewhere between these extremes. Knowledge of the terrain in question is an obvious aid to a possible preference that should be shown to either of these two conditions for a particular marshland.

Geometric Marsh Model: Linear Shorelines

Marshes located along lake and river shorelines quite naturally assume geometric shapes determined by both the configuration of the shoreline and the onshore and offshore slopes. It is this consistent feature of marshland formation that is the basis of the marsh model considered, in this report. Three basic geometric shapes are considered, namely linear, concave/convex, and elliptical. While these shapes certainly do not completely exhaust the spectrum of possible marsh configurations, they do, nonetheless, conform to a large percentage of marshes observed via synoptic overviews of the Great Lakes basin. For the purpose of this work, a marsh is taken to have persistent standing water across its vegetation, and would be located offshore of meadow/swamp regions which, although water-saturated, contain no significant observable persistent standing water.

Let us consider Figure 1a, which is a simplified diagram of a rectangular marsh along a linear shoreline as seen in plan view. The total marsh area is taken to be made up of a basic marsh area B (offshore portion of the marsh, the maximum extent of which is determined by the limit of observable emergent vegetation) and a fringe marsh area F (offshore extension of the basic marsh to accommodate the submerged vegetation that cannot be observed directly). Only the basic marsh area B is considered in this model.

Figure 1b illustrates a vertical cross section of the basic marsh configuration under two distinct water level conditions. The initial condition assumes that the water level is such that the basic marsh area originates at the strand line, and that the strand line separates an aquatic regime of offshore slope α and onshore slope β . This latter assumption is satisfied only at zero water level datum (International Great Lakes Datum, 1955). The initial length, b_0 , of the basic marsh at zero water level datum is taken as the offshore distance to the water depth d (corresponding to the depth beyond which there is no further emergent vegetation). The second water level (dashed line) represents the condition subsequent to an increase in depth to a level R_n above the zero water level. The offshore length of the basic marsh (again taken to the depth d which is assumed invariant to the fluctuating water levels) associated with this new water level is taken to be b_n .

If $R_n < d$, it may be seen that

$$b_n = x + y$$

$$= \frac{d - R_n}{\tan \alpha} + \frac{R_n}{\tan \beta} \quad (1)$$

and

$$\frac{b_n}{b_0} = 1 - \frac{R_n}{d} \left(1 + \frac{\tan \alpha}{\tan \beta} \right) \quad (2)$$

If the alongshore extent of the marsh is L , then the plan view areas of the new and initial marshlands are $b_n L$ and $b_0 L$, respectively. Equation (2) thus expresses the ratio of new basic marsh area (at water level R_n above zero water level datum) to initial basic marsh area (at zero water level datum) in terms of the offshore and onshore slopes of the

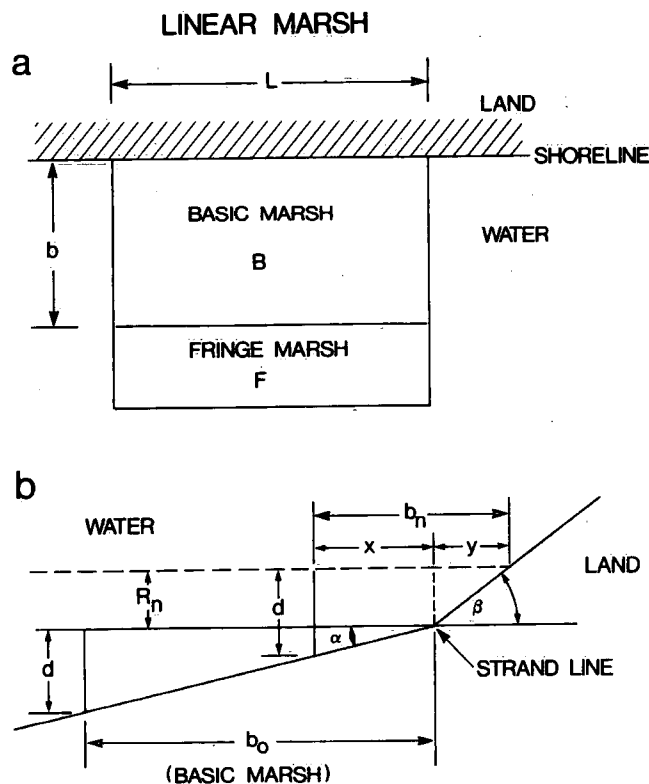


Figure 1. Linear shoreline marsh configuration (a) plan view and (b) vertical cross section.

marsh region, the water depth beyond which there is no observable emergent vegetation and the lake level R_N . In the rest of this report, the individual definitions of b_N and b_0 as both linear and areal measurements will be considered as completely interchangeable for linear shoreline marshes.

Equation (2) suggests the following:

- (a) For a positive R_N (i.e., an increase in water level above the zero water level datum), basic marsh area will be reduced if α is a smaller angle than β (i.e., the slope of the lake bottom is less than the slope of the shore). The basic marsh area will be increased if α is a larger angle than β (provided, of course, that excessive flooding does not occur which would suffocate vegetation).
- (b) For a drop in water level from $R_1 > 0$ to $R_2 > 0$ with $R_2 < R_1$ (i.e., a decrease in water level but not to a value below the zero water level datum), basic marsh area will be increased if $\alpha < \beta$ and decreased if $\alpha > \beta$.
- (c) For $\alpha = \beta$ (i.e., identical slopes for the offshore and onshore regions or, equivalently, for those water level increases or decreases that occur solely within the offshore region of Fig. 1), the basic marsh area will remain unchanged (i.e., $b_N/b_0 = 1$).
- (d) For large values of R_N ($R_N > d$), the x term of equation (1) vanishes and equation (2) reduces to

$$\frac{b_N}{b_0} = \frac{\tan \alpha}{\tan \beta} \quad (3)$$

which is a constant for a particular marshland geometry. From equation (3) b_N/b_0 approaches zero (i.e. total destructive flooding) as $\tan \alpha / \tan \beta$ approaches zero. This would occur if the marsh were contained within steep banks (i.e. $\beta \gg \alpha$).

In reality, of course, total destructive flooding may occur at intermediate values of α and β , since there is undoubtedly some limiting value of β beyond which the marshland vegetation cannot be sustained. Further, equations (1), (2) and (3) assume that α and β are constants over the flood plain domain. Clearly, these slopes are not maintained indefinitely. In fact, many basins often display quite marked departures from such constancy.

The principal assumption of equations (1), (2) and (3), however, is that an equilibrium may be established between marsh and non-marsh regions. This implies that other parameters are not adversely affected and that the maximum

amount of marshland possible will adapt to the new set of circumstances. Logically, marshland may be destroyed more easily than created, since destruction may be a relatively instantaneous event (flooding, for example), while creation requires time to modify and/or establish growth cycles. Jaworski *et al.* (1979) indicate that field investigations suggest that two- or three- year lags between water level fluctuations and dieback or recolonization are normally encountered. Consequently, for the situation of positive R_N (i.e., increased water level above zero water level datum) resultant marshland areas may be easily underestimated from aerial photography.

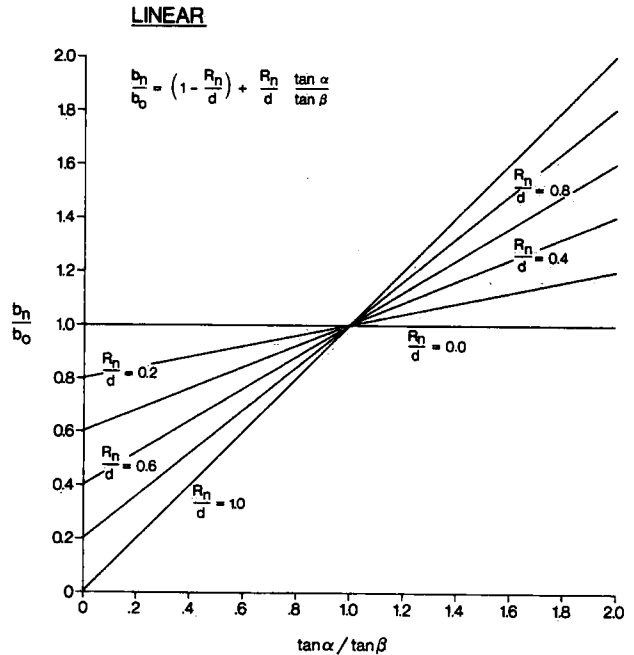


Figure 2. Relationships between b_N/b_0 (the ratio of linear shoreline marsh area associated with water level condition R_N above datum to the linear shoreline marsh area associated with zero water level datum) and $\tan \alpha / \tan \beta$ (ratio of offshore to onshore slopes) for a variety of R_N/d values.

Figure 2 illustrates the linear relationship (expressed in equation (2)) that exists between b_N/b_0 (the ratio of basic marsh area associated with a linear shoreline under a water level condition R_N above datum to the basic marsh area that would be observed at zero water level datum) and $\tan \alpha / \tan \beta$ (the ratio of offshore to onshore slopes) for a family of R_N/d values ≤ 1.0 (ratio of water level R_N above zero water level datum to the maximum depth d at which emergent vegetation may be synoptically observed). Clearly, the ordinate intercept of each linear relationship occurs at

$$\frac{b_N}{b_0} = 1 - \frac{R_N}{d}$$

and the slope of each linear curve has the value R_n/d . Further, the point (1.0, 1.0) satisfies each curve, irrespective of the R_n/d value ≤ 1.0 . For values of $R_n/d > 1.0$, equation (2) becomes invalid and equation (3)

$$\left[\frac{b_n}{b_0} = \frac{\tan \alpha}{\tan \beta} \right]$$

becomes operative. The plot of equation (3) is identical with the linear relationship expressed by equation (2) for the case of $R_n/d = 1.0$. Consequently, the $R_n/d = 1.0$ curve of Figure 2 also applies for all values of $R_n/d > 1.0$. This suggests that on the basis of the linear marsh model discussed herein, scatter plots of b_n/b_0 against $\tan \alpha/\tan \beta$ would not display data points located to the right of the $R_n/d = 1.0$ line for marshland reductions (i.e. for values of $\tan \alpha/\tan \beta < 1.0$) nor to the left of the $R_n/d = 1.0$ line for marshland increases (i.e. for values of $\tan \alpha/\tan \beta > 1.0$). Thus, for the condition of complete dynamic equilibrium being established among wetlands subsequent to a persistent change in water level (thereby resulting in the maximum areal extent of marshland under the new aquatic condition), scatter plots of b_n/b_0 are expected to be contained within the cones bounded by the $R_n/d = 0.0$ and $R_n/d = 1.0$ lines.

Figure 2 has been generated considering $R_n > 0$ (i.e., for an increase in water level to a value R_n above the low water datum). The case of $R_n < 0$ (i.e., for a drop in water level from the low water level datum to a value R_n below the low water level datum) may also be readily considered, since in this instance, the value of b_n/b_0 will remain at the value 1.0 (a consequence of the onshore and offshore slopes becoming the same). This condition of invariant marshland areal extent assumes that the water level drop does not completely remove the water from the marsh, in which case the marsh could totally disappear, or that the water level drop does not produce an extended region of standing water of depth $< d$, in which case the total basic marsh area could substantially increase.

Several possible uses of Figure 2 become immediately evident. Clearly, if α and β are known for a marsh region, then b_n/b_0 (i.e., the areal extent impact of persistent water level changes R_n above zero water level datum) may be easily predicted for wetlands not inhibited by drastic departures from abilities to establish dynamic equilibria. If α , β , b_n and b_0 are known or can be suitably estimated, R_n/d may be determined. Under some conditions, estimates of the slopes of the terrain may be calculated. Determinations from synoptic overviews of the areal extent of basic marshland under two water level conditions play an integral role in such applications of Figure 2. However, Figure 2 has been generated assuming knowledge of b_0 (i.e., the offshore

extent of the basic marsh at zero water level). Very rarely do historical aerial records contain such data, and equally rarely is it convenient to wait for zero water level conditions to collect such data. It is considerably more convenient to locate or obtain two synoptic data sets over a marshland area under study at two distinct water levels, neither of which is at zero water level datum. Consequently, to benefit from Figure 2, these two synoptic data sets must be used somehow to estimate marsh conditions at zero water level.

Let b_0 , b_1 and b_2 represent the basic marsh linear extents corresponding to water levels R_0 (zero water level datum), R_1 and R_2 (both above zero water level datum), respectively. It may readily be shown that the parameters d (maximum water depth at which emergent vegetation may be aurally observed) and b_0 (basic marsh linear extent at zero water level) are given by:

$$d = \frac{R_2 b_1 - R_1 b_2}{b_1 - b_2} \left(1 - \frac{\tan \alpha}{\tan \beta} \right) \quad (4)$$

and

$$b_0 = \frac{b_1 R_2 - b_2 R_1}{R_2 - R_1} \quad (5)$$

It is of interest to note that equation (5) is independent of the marshland slopes. Thus, if the appropriate marshland areal extent may be determined corresponding to two known water levels above zero water level datum, then the expected marshland area corresponding to zero water level datum may be readily calculated without precise knowledge of the topography. The determination of the maximum basic marsh depth d from such information (equation (4)), however, does require precise topographical knowledge.

It should be further noted that the water levels R_1 and R_2 are the water levels that exist concurrently with the measured marsh areas. Since there is a significant lag time involved (Jaworski *et al.*, 1979; Keddy and Reznicek, 1986) for the equilibrium to become fully established, it is not unreasonable for the appropriate water levels R_1 and R_2 to be taken as the water levels that were present two or three years prior to the aerial photography or environmental satellite overpass. The values of b_0 obtained from equation (5) can then be used in the applications of equations (2) and (3) and Figure 2 to linear marshland conditions under two distinct values of $R_n > 0$.

To this point, the linear marsh model has assumed that the environmental interrelationships dictating wet-

lands behaviour are such as to optimize the adaptability of the marshlands to persistent changes in water levels. In particular, this implies that the onshore reaches of the marshes (defined by the angle β) readily accommodate the vegetative equilibrium required to maintain marshland definition. Very often, however, the observation is made of the apparent elimination under high water conditions of a large percentage of marshland which had existed under low water conditions, even when sufficient time has elapsed for equilibrium establishment. Such a condition of minimal regeneration of marshland could arise from a variety of sources, but two very common reasons are that the marsh is characterized by onshore slope $\beta \gg$ offshore slope α or the onshore reaches of the marsh are not conducive to vegetative growth and/or transformation.

Let us consider such a marsh in which $0 < R_n < d$ and, for whatever reason, no new marsh is created on the previously onshore region. This is equivalent to the situation in which distance y of Figure 1 is taken to be zero (due to the absence of vegetative growth or due to such conditions as the presence of a bedrock or very steep shoreline), whereas distance x retains its original marshland definition (due to the presence of vegetative growth). For this situation b_n is given by $(d - R_n)/\tan \alpha$ and b_0 is again given by $d/\tan \alpha$. Therefore, equation (2) becomes

$$\frac{b_n}{b_0} = 1 - \frac{R_n}{d} \quad (6)$$

In this case, it may easily be shown that if two water level conditions R_1 and R_2 are considered, both of which are above the zero water level datum, and correspond to the marshland linear extents b_1 and b_2 , respectively, the value of b_0 (linear extent of marshland corresponding to zero water level datum) may once again be calculated from equation (5), viz.

$$b_0 = \frac{b_1 R_2 - b_2 R_1}{R_2 - R_1} \quad (5)$$

The value of d (maximum water depth at which emergent vegetation may be detected), however, may be determined from

$$d = \frac{R_2 - R_1 \frac{b_2}{b_1}}{1 - \frac{b_2}{b_1}} \quad \text{or} \quad \frac{R_2 b_1 - R_1 b_2}{b_1 - b_2} \quad (7)$$

Hence, for the case of total non-regeneration of marshland subsequent to an increase in water level, equa-

tions (5) and (7) indicate that both the parameters b_0 and d may be estimated from two sets of remotely sensed data without precise knowledge of the terrain slopes.

Equation (6) indicates that b_n approaches zero as R_n approaches d . Hence for $R_n > d$, all marshland area will disappear. This is indicated in Figure 3, which illustrates the linear relationship existing between b_n/b_0 and R_n/d for values of $R_n \geq 0$. Such a relationship is totally independent of $\tan \alpha/\tan \beta$ and decreases linearly from $b_n = b_0$ at $R_n = 0$ to $b_n = 0$ at $R_n = d$. Values of $R_n < 0$ are not considered in Figure 3, since equation (6) has been based on the premise that the reason a vegetative equilibrium is not established is the inability of the onshore (at zero water level datum) slope to adapt to marsh vegetation growth. No such inability characterizes the offshore (at zero water level datum) slope. Consequently, since only the original offshore slope is involved in the consideration of negative values of R_n , equation (6) does not apply and the value b_n/b_0 remains constant at 1.0.

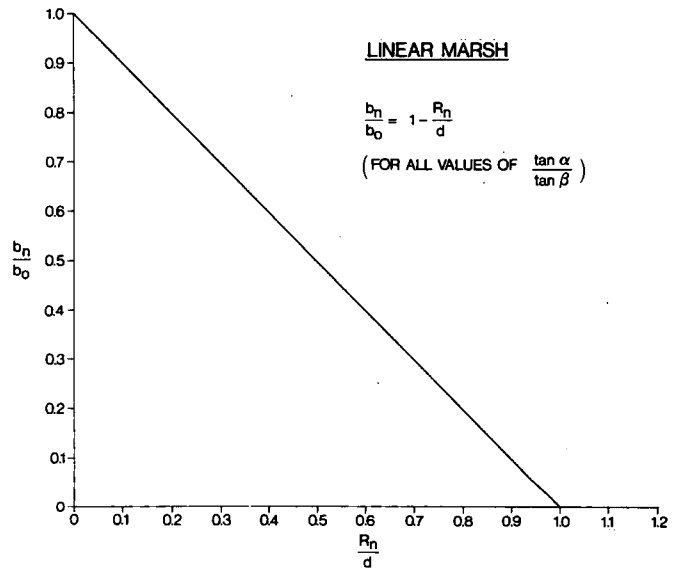
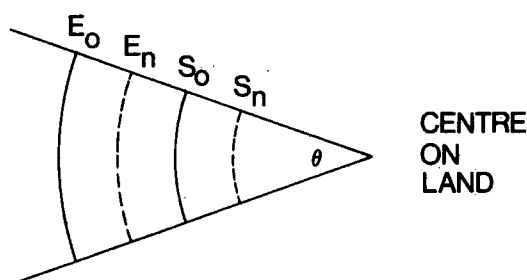


Figure 3. Relationship between b_n/b_0 and R_n/d for a linear shoreline marsh.

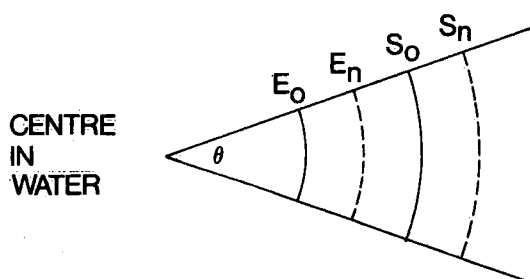
Figures 2 and 3 therefore represent the impact of persistent water level changes on linear marshlands characterized by offshore slopes α and onshore slopes β for two extremes of onshore vegetative regeneration capability. Figure 2 represents the maximum areal extent of a linear shoreline that can re-emerge subsequent to a persistent water level change, while Figure 3 represents the minimum such areal extent that could emerge.

Geometric Marsh Model: Convex and Concave Shorelines

Figure 4 illustrates the situation for marshes located along convex or concave shorelines. A convex shoreline may be typified by some islands and headlands, while a concave shoreline may be typified by some bays and bights.



a) CONVEX SHORELINE MARSH



b) CONCAVE SHORELINE MARSH

Figure 4. Angular sector of (a) convex shoreline marsh and (b) concave shoreline marsh.

An angular sector of a convex marshland is sketched in Figure 4a, the centre of curvature of the sector being considered to lie inland. E_o and S_o are taken to be the physical distances from the centre of curvature to the offshore extent of the convex marsh, and from the centre of curvature to the convex shoreline, respectively, for the initial water level condition (viz. zero water level datum). Similarly, E_n and S_n represent these physical distances corresponding to a water level R_n above zero water level

datum. If A_o and A_n represent the basic marsh areas corresponding to these two distinct water level conditions, then

$$\begin{aligned} \frac{A_n}{A_o} &= \frac{\frac{1}{2} \theta E_n^2 - \frac{1}{2} \theta S_n^2}{\frac{1}{2} \theta E_o^2 - \frac{1}{2} \theta S_o^2} \\ &= \left(\frac{E_n + S_n}{E_o + S_o} \right) \cdot \left(\frac{E_n - S_n}{E_o - S_o} \right) \end{aligned} \quad (8)$$

However, $\frac{E_n - S_n}{E_o - S_o} = \frac{b_n}{b_o}$, i.e., the equivalent of the basic marsh areal ratios for a linear shoreline. Therefore

$$\frac{A_n}{A_o} = \left(\frac{E_n + S_n}{E_o + S_o} \right) \cdot \frac{b_n}{b_o} \quad (9)$$

where b_n/b_o is as given by equation (2) for the case of $R_n \leq d$ and by equation (3) for the case of $R_n > d$.

Equation (9) thus relates the areal impact on a convex marshland of a long-term increase in water level in terms of the geometric parameters of the marsh, the on-shore and offshore slopes, the water level R_n and the water depth d . It is stressed that equation (9) represents the condition of maximum regenerated vegetation, i.e., a condition in which the establishment of vegetative equilibrium is totally favoured, i.e., onshore slope β defines a regime of sufficient fertility to accommodate, subsequent to a lag time, a full marshland vegetative canopy.

An angular sector of a concave marsh area is sketched in Figure 4b, with the centre of curvature of the sector, in this case, however, being considered to lie offshore. It may be readily shown that for concave shorelines, the ratio of areal extent of associated marshlands for these two water level conditions (initially at zero water level datum and finally at R_n above zero water level datum) may once again be expressed by equation (9), the governing equation for convex shorelines. The only qualifier on the use of this single equation for convex and concave shorelines is that for

convex shorelines $E_n < E_0$ and $S_n < S_0$, while for concave shorelines $E_n > E_0$ and $S_n > S_0$. Consequently, the ratio A_n/A_0 for convex shorelines will be less than the corresponding ratio for linear shorelines, while the ratio A_n/A_0 for concave shorelines will be greater than the corresponding ratio for linear shorelines.

Thus, for a given rise in water level, and assuming that other factors are comparable, marshes characterized by offshore slopes α less than the onshore slope β and located around convex shorelines should lose a greater area than would their linear counterparts. That is, marshes located around headlands and islands are most vulnerable to destructive flooding.

If equation (9) is rewritten as

$$\frac{b_n}{b_0} = \frac{E_0 + S_0}{E_n + S_n} \cdot \frac{A_n}{A_0} \quad (10)$$

then Figure 2 may be considered applicable to both convex and concave shoreline marshes as well as linear shoreline marshes, since equation (10) represents the linear equivalent of convex/concave marshes subject to increased water levels. For a decrease in water level (i.e., from zero water level datum to a water level $-R_n$ below this datum), it is clear that the offshore retreat of the sectors considered in Figure 4 would result in an increased marsh area for convex shorelines and a decreased marsh area for concave shorelines because the locations of E_n and S_n are interchanged with E_0 and S_0 . That is, for convex shorelines $E_n > E_0$ and $S_n > S_0$, while for concave shorelines $E_n < E_0$ and $S_n < S_0$. Consequently, the ratio A_n/A_0 for convex shorelines will be greater than the corresponding ratio for linear shorelines, while the ratio A_n/A_0 for concave shorelines will be less than the corresponding ratio for linear shorelines. Since the linear shoreline ratio b_n/b_0 is equal to 1 (see equation (2) with $\alpha = \beta$), then the $A_n/A_0 > 1$ for convex shoreline marshes is indicative of an increased marsh area subsequent to a drop in water level, whereas the $A_n/A_0 < 1$ for concave shoreline marshes is indicative of a decreased marsh area subsequent to a drop in water level. In fact, the minimum decreased marsh area for concave shoreline marshes suggested by Figure 4 occurs when E_n retreats to the centre of curvature, i.e. $E_n = 0$. This corresponds to a sectorial area $A_n = \theta S_n^2/2$ implying the complete domination of the standing water portion of the bay by emergent marsh vegetation. For a semicircular bay, the marsh area A_n would be $\pi S_n^2/2$ or, equivalently, $\pi b_n^2/2$ or $\pi b_0^2/2$.

While it is clear that reduced water levels (along offshore slopes α) will result in a decreased marsh area for a concave shoreline and an increased marsh area for a convex

shoreline, and equally clear that an increased water level (along onshore slope β) will result in a decreased marsh area for a convex shoreline (provided, of course, that vegetative equilibrium may be established), it is not as immediately evident what impact such an increased water level would have on a marsh area around a concave shoreline.

From equation (9) it may be seen that since

$$\frac{E_n + S_n}{E_0 + S_0} > 1$$

for a concave marsh subject to an increase in water level R_n where $0 < R_n < d$, the ratio A_n/A_0 for this marsh is greater than the ratio b_n/b_0 for its linear marsh equivalent. However, since $b_n < b_0$, the value of A_n/A_0 cannot be immediately determined as being > 1 or < 1 .

Let us consider the concave shoreline marsh (Fig. 4b) in terms of the onshore/offshore and subsurface parameters of Figure 1. It is seen that

$$E_0 = S_0 - \frac{d}{\tan \alpha}$$

$$E_n = S_0 - \frac{d - R_n}{\tan \alpha}$$

$$\text{and } S_n = S_0 + \frac{R_n}{\tan \beta}$$

from which

$$\frac{E_n + S_n}{E_0 + S_0} = 1 + \frac{R_n}{2S_0 - \frac{d}{\tan \alpha}} \left(\frac{1}{\tan \alpha} + \frac{1}{\tan \beta} \right) \quad (11)$$

Substituting equation (11) into equation (9) yields:

$$\frac{A_n}{A_0} = \left[1 + \frac{R_n}{2S_0 - \frac{d}{\tan \alpha}} \left(\frac{1}{\tan \alpha} + \frac{1}{\tan \beta} \right) \right] \times \left[1 - \frac{R_n}{d} + \frac{R_n}{d} \frac{\tan \alpha}{\tan \beta} \right] \quad (12)$$

Clearly, the relative magnitudes of α , β and S_0 (the radius of the concave shoreline at zero water level datum) will play integral roles in whether A_n/A_0 is greater or less than 1.

Consider the first derivative of equation (12) with respect to R_n . A positive value of

$$\frac{d}{dR_n} \left(\frac{A_n}{A_0} \right)$$

would indicate an increase in concave marsh areal extent with a rise in water level, while a negative value of

$$\frac{d}{dR_n} \left(\frac{A_n}{A_0} \right)$$

would indicate a decrease in concave marsh areal extent with a rise in water level.

$$\begin{aligned} \frac{d}{dR_n} \left(\frac{A_n}{A_0} \right) &= \text{first term} \frac{d}{dR_n} (\text{second term}) \\ &\quad + \text{second term} \frac{d}{dR_n} (\text{first term}) \\ &= \left(\frac{E_n + S_n}{E_0 + S_0} \right) \frac{d}{dR_n} \left(\frac{b_n}{b_0} \right) \\ &\quad + \frac{b_n}{b_0} \frac{d}{dR_n} \left(\frac{E_n + S_n}{E_0 + S_0} \right) \\ &= \left(\frac{E_n + S_n}{E_0 + S_0} \right) \left(-\frac{1}{d} + \frac{1}{d} \frac{\tan \alpha}{\tan \beta} \right) \\ &\quad + \left[\left(1 - \frac{R_n}{d} \right) + \frac{R_n}{d} \frac{\tan \alpha}{\tan \beta} \right] \times \\ &\quad \left[\left(\frac{1}{\tan \alpha} + \frac{1}{\tan \beta} \right) \cdot \frac{1}{2S_0 - \frac{d}{\tan \alpha}} \right] \end{aligned} \quad (13)$$

For a concave marsh the term

$$\frac{E_n + S_n}{E_0 + S_0}$$

is positive, the term

$$-\frac{1}{d} + \frac{1}{d} \frac{\tan \alpha}{\tan \beta}$$

is negative (for $\alpha < \beta$), the term

$$1 - \frac{R_n}{d} + \frac{R_n}{d} \frac{\tan \alpha}{\tan \beta}$$

is positive (for $\alpha < \beta$), and the term

$$\left(\frac{1}{\tan \alpha} + \frac{1}{\tan \beta} \right) \cdot \frac{1}{2S_0 - \frac{d}{\tan \alpha}}$$

is positive. Consequently

$$\frac{d}{dR_n} \left(\frac{A_n}{A_0} \right)$$

is given as

$$\begin{aligned} \frac{d}{dR_n} \left(\frac{A_n}{A_0} \right) &= (\text{positive term} \times \text{negative term}) \\ &\quad + (\text{positive term} \times \text{positive term}) \\ &= \text{negative term} + \text{positive term} \end{aligned}$$

Clearly, therefore

$$\frac{d}{dR_n} \left(\frac{A_n}{A_0} \right)$$

may be either positive or negative, depending upon the relative magnitudes of these two terms. Thus, for an increase in persistent water level ($R_n > 0$) and $\alpha < \beta$, a concave marsh may lose or gain areal extent in a totally regenerative vegetation system. Further, this gain or loss is dependent upon the geometric characteristics of the marshland in question, viz. the parameters S_0 , R_n , d , α , and β (see equation (12)).

To illustrate the dependency of the impact of increased water levels on the areal extent of concave marshlands, equations (11) and (12) were used to determine A_n/A_0 for a wide variety of combinations of S_0 , α and β for fixed values of R_n and d . Figure 5 illustrates the family of curves representing the concave marsh factor

$$\frac{E_n + S_n}{E_0 + S_0}$$

(from equation (11)) for a fixed increased water level $R_n = 1.0$ m, a fixed emergent vegetation limit $d = 1.25$ m,

and a fixed onshore slope $\beta = 1^\circ$. The offshore slope α is allowed to vary between 0.1° and 10° and the zero water concave marsh radius S_0 is allowed to vary between 30 m and 500 m. The minimum theoretically allowable value of α for each marsh is clearly given from equation (11) as $\alpha > \tan^{-1}(d/2S_0)$. However, in practical terms, the minimum α permissible in the model occurs for $\alpha = \tan^{-1}(d/S_0)$, since the location of d will then be at the centre of curvature. It is readily seen from Figure 5 that

- (a) The geometric factor for concave marshlands subjected to an increase in persistent water level change is greater than unity, indicating that provided regenerative vegetation equilibrium may be established, the ratio of resulting concave marshland to zero water level concave marshland (A_n/A_0) will be greater than the equivalent ratio (b_n/b_0) for a linear shoreline marsh. However, since $b_n/b_0 < 1.0$ for a persistent water level rise, the positive values of the concave marsh factor indicated by the family of curves in Figure 5 cannot guarantee consistent values of A_n/A_0 greater than unity.
- (b) The smaller the zero water level concave marshland

S_0 , the larger the geometric factor and therefore the greater the A_n/A_0 ratio compared to its corresponding b_n/b_0 ratio, i.e., the smaller the concave marsh, the less severe will be the impact of a positive persistent water level rise.

- (c) The smaller the marsh parameter S_0 , the larger the required offshore slope α to apply this current model.
- (d) While the range of S_0 considered in Figure 5 appropriately considers the bulk of the marshlands encountered in the Great Lakes basin, it is seen from both Figure 5 and equation (11) that the limit of the concave marsh factor

$$\frac{E_n + S_n}{E_0 + S_0}$$

as S_0 becomes large is 1. Thus, Figure 5 clearly indicates that the larger the concave marsh (i.e. the larger the S_0), the more nearly it approximates the behaviour of a linear marsh, and the smaller will have to be the offshore slope α to emphasize a departure from linear behaviour.

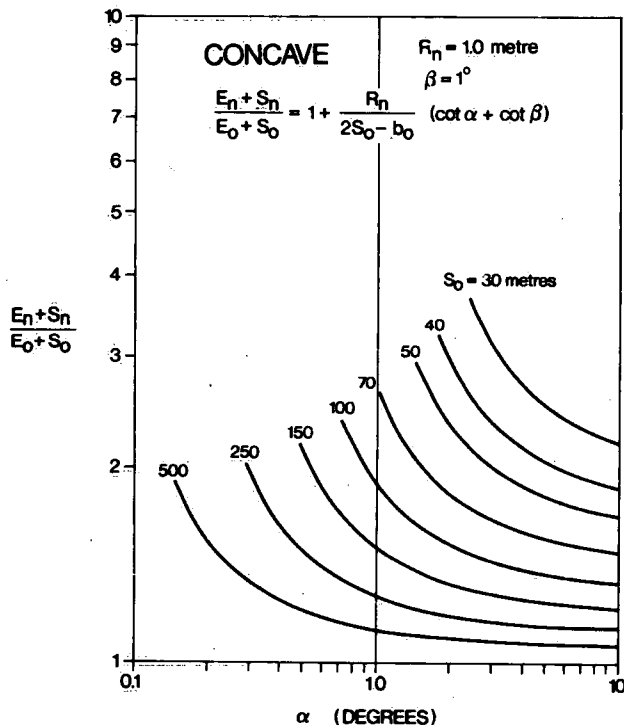


Figure 5. Relationships between the concave shoreline marsh factor $(E_n + S_n)/(E_0 + S_0)$ and offshore slope α for a variety of S_0 values.

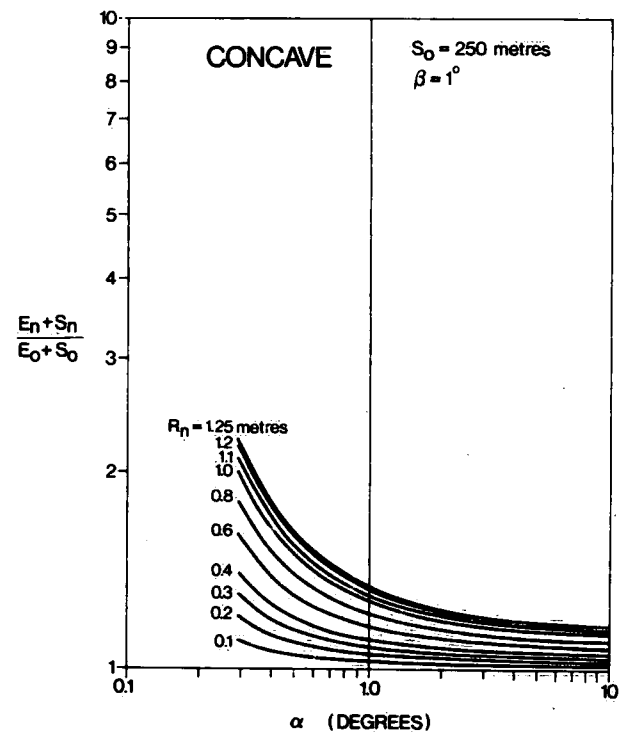


Figure 6. Relationships between the concave shoreline marsh factor $(E_n + S_n)/(E_0 + S_0)$ and offshore slope α for a variety of increased water level values R_n .

Figure 6 illustrates the dependency of the concave marsh factor

$$\frac{E_n + S_n}{E_o + S_o}$$

on offshore slope α for a fixed value of $\beta = 1^\circ$, a fixed inundated concave marsh radius $S_o = 250$ m, and a variety of R_n values ranging between 0.1 and 1.25 m, the latter value being equivalent to the maximum water depth d at which emergent vegetation may be synoptically observed. From Figure 6, it is clear that the maximum departure from equivalent linear marshland behaviour is experienced at large values of R_n and minimal values of α . The minimal value of α (as seen from equation 11) is once again given by $\alpha = \tan^{-1} (d/2S_o)$ and is, of course, independent of R_n .

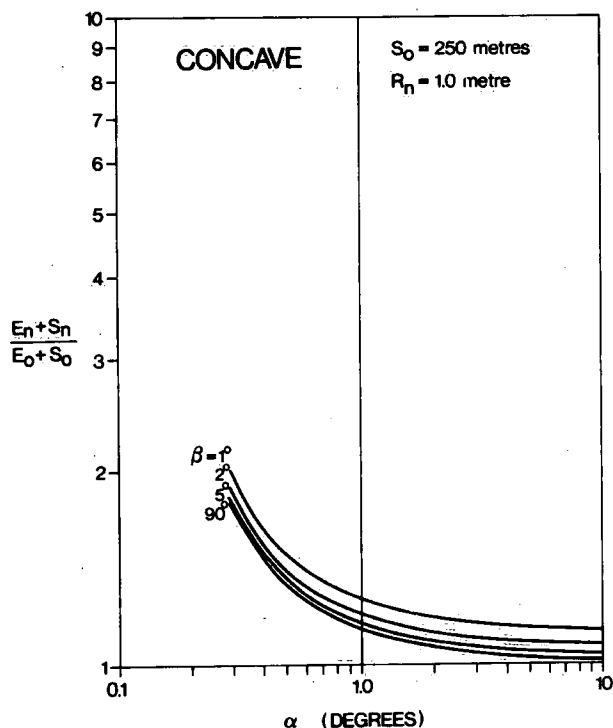


Figure 7. Relationships between the concave shoreline marsh factor $(E_n + S_n)/(E_o + S_o)$ and offshore slope α for a variety of onshore slopes β and initial marsh size of 250 m.

Figure 7 illustrates the dependency of the concave marsh factor

$$\frac{E_n + S_n}{E_o + S_o}$$

on α for a fixed $R_n = 1$ m, a fixed $S_o = 250$ m, and a variety of onshore slope angle values varying between 1° and 90° . Figure 7 shows that the lower the value of α and/or β , the greater the departure of a concave marsh from linear behaviour. The situation for a small marsh ($S_o = 50$ m) is shown in Figure 8. The effect of β is clearly more pronounced for a small marsh than for a larger marsh. However, a much larger value of offshore slope α is also required.

The situation for $\beta = 90^\circ$ is of importance to this discussion, since in essence, this is equivalent to the situation in which distance y of Figure 1 is taken to be zero (i.e., no new marshland being created due either to a very steep shoreline or the inability of the onshore reaches to sustain marshland vegetation on account of rocks, gravel, soil infertility, etc.). Consequently, $\beta = 90^\circ$ represents the situation in which no vegetation equilibrium can be established.

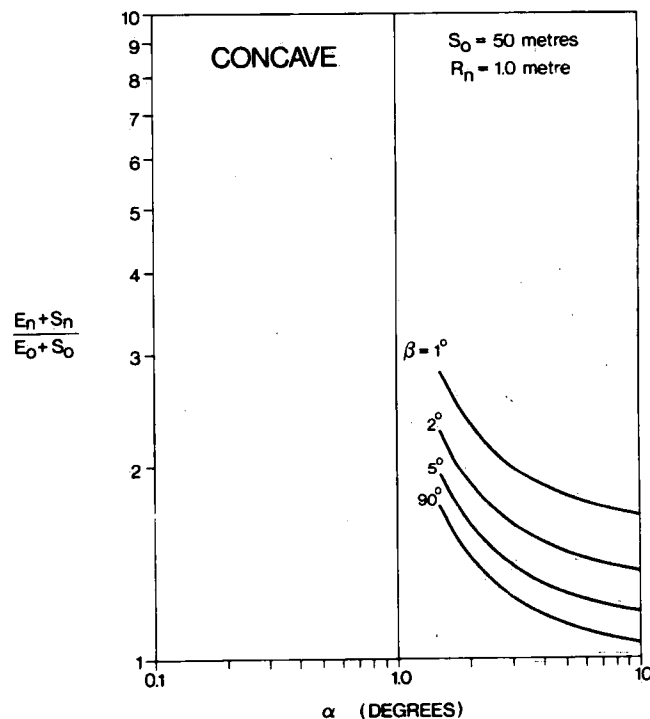


Figure 8. Relationships between the concave shoreline marsh factor $(E_n + S_n)/(E_o + S_o)$ and offshore slope α for a variety of onshore slopes β and initial marsh size of 50 m.

As such, $\beta = 90^\circ$ defines the condition for which there is theoretically zero (or realistically minimal) onshore marsh regeneration subject to a persistent increase in water level.

Further, as seen by Figures 7 and 8, maximum impacts on

$$\frac{E_n + S_n}{E_o + S_o}$$

occur up to $\beta \sim 5^\circ$. Beyond this value of β the impact of β becomes dramatically reduced.

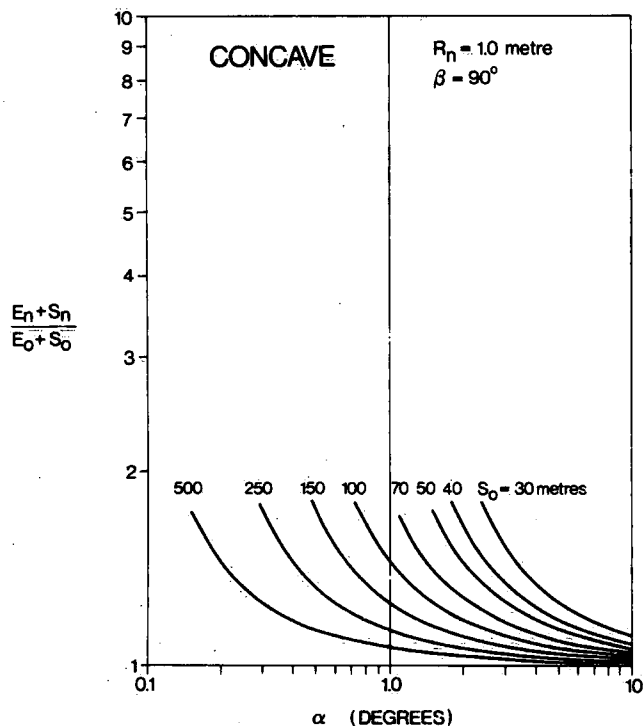


Figure 9. Relationships between the concave shoreline marsh factor $(E_n + S_n)/(E_o + S_o)$ and offshore slope α for a variety of initial marsh sizes and onshore slope $\beta = 90^\circ$ (i.e. a non-regenerative marsh) for $R_n = 1.0$ m.

Figure 9 illustrates the role of original concave marsh radius S_o on the factor

$$\frac{E_n + S_n}{E_o + S_o}$$

for $R_n = 1.0$ m and absence of a vegetation equilibrium (i.e. equivalent of $\beta = 90^\circ$) and a range of S_o values. Figure 10 considers the corresponding family of curves for $R_n = d = 1.25$ m. It is seen that for $\beta = 90^\circ$, the concave marsh factor is represented by a curve which decreases with increasing α from a value which is independent of marsh size S_o to a limit of 1 (i.e. for large α , $A_n/A_o = b_n/b_o$). Clearly, from equation (11), the conditions for $\beta = 90^\circ$ and $R_n = d$ and any value of S_o result in a maximum factor value of 2.0.

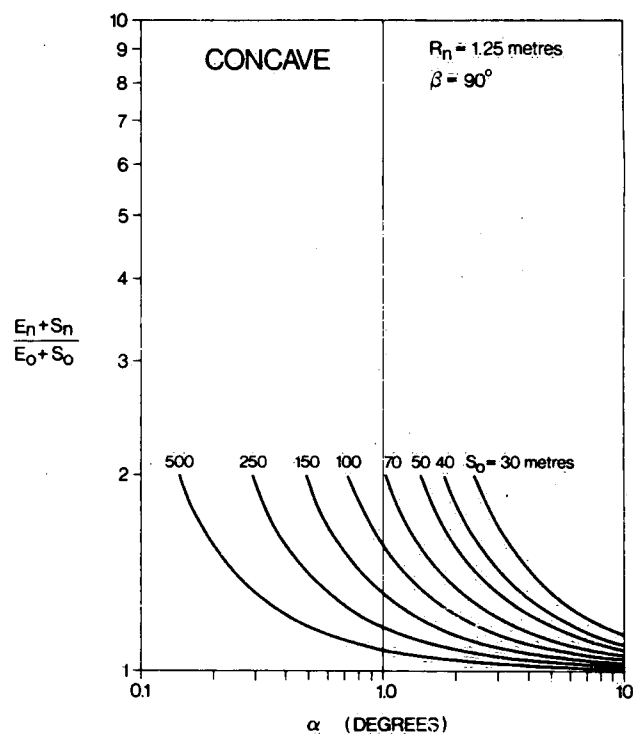


Figure 10. Relationships between the concave shoreline marsh factor $(E_n + S_n)/(E_o + S_o)$ and offshore slope α for a variety of initial marsh sizes and onshore slope $\beta = 90^\circ$ (i.e. a non-regenerative marsh) for $R_n = 1.25$ m.

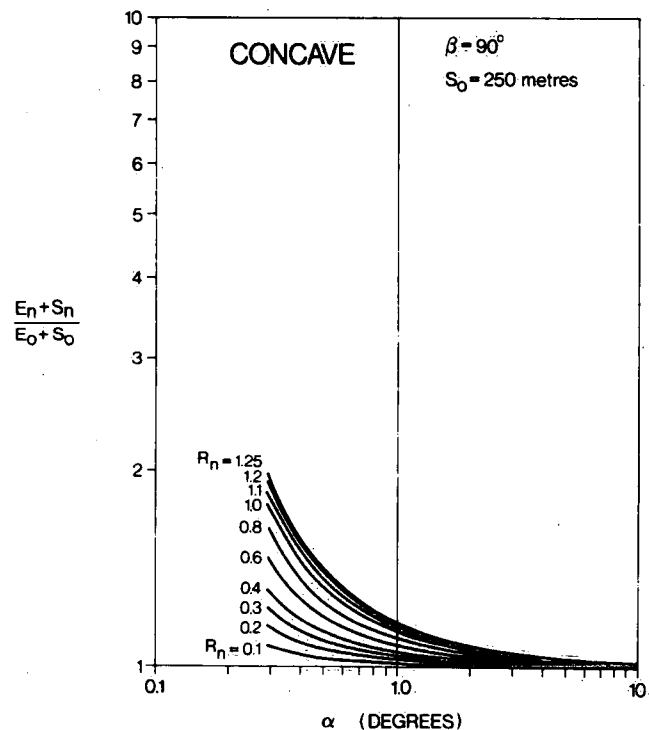


Figure 11. Relationships between the concave shoreline marsh factor $(E_n + S_n)/(E_o + S_o)$ and offshore slope α for a variety of increased water level values R_n and onshore slope $\beta = 90^\circ$ (i.e. a non-regenerative marsh).

Figure 11, in a manner analogous to Figure 6, illustrates the dependency of the concave marsh factor

$$\frac{E_n + S_n}{E_o + S_o}$$

on the offshore slope α for a fixed inundated concave marsh radius $S_o = 250$ m and a fixed value of $\beta = 90^\circ$ (representing the extreme case of no marshland vegetative regeneration) and a variety of R_n values. Once again, the minimum value of α is independent of R_n and maximum departure from linear shoreline marshland behaviour is exhibited at larger values of R_n . The departure from linear shoreline behaviour is, however, considerably reduced from the corresponding departures for small values of onshore slopes β .

Figures 5 to 11 have considered the concave marsh factor

$$\frac{E_n + S_n}{E_o + S_o}$$

as the ordinate. To convert this ordinate to the ratio A_n/A_o , the marsh factor must be multiplied by the concave marsh's associated linear shoreline marsh counterpart b_n/b_o , as obtained from equation (12). Taking the linear shoreline counterpart ratio b_n/b_o into account, Figure 12 illustrates the dependency of A_n/A_o upon offshore slope α for a concave marsh subjected to a persistent water level increase $R_n = 1$ m above zero water level datum and a fixed onshore slope $\beta = 1^\circ$ for a variety of S_o (radius of curvature of concave marshland at zero water level datum). Figure 12 may immediately be considered in conjunction with Figure 5. While the multiplicative factors of Figure 5 are always greater than unity for all S_o and α values, the values of A_n/A_o may be greater or less than unity and are dependent upon both zero level marsh size S_o and offshore slope α . Clearly, the case of a linear marsh (represented in Figure 12 as the curve for a concave marsh of infinite radius, S_o) indicates, as expected, a reduced marsh area resulting from increased water levels for basins characterized by $\alpha < \beta$ and increased marsh area for non-excessively flooded basins characterized by $\alpha > \beta$.

From equation (12) it is seen that b_n/b_o is a function of R_n , d , α and β , but, understandably, independent of S_o . Figure 13 illustrates the family of curves defining the linear marsh ratio b_n/b_o as a function of α for a fixed $\beta = 1^\circ$ and a variety of R_n values $0 < R_n \leq d$. A very apparent hinge point, through which every curve passes, is seen at the

location $b_n/b_o = 1$ and $\alpha = \beta$. For values of $\alpha < \beta$, reduced linear marsh areas result from increased water levels, while at values of $\alpha > \beta$, increased linear marsh areas ensue.

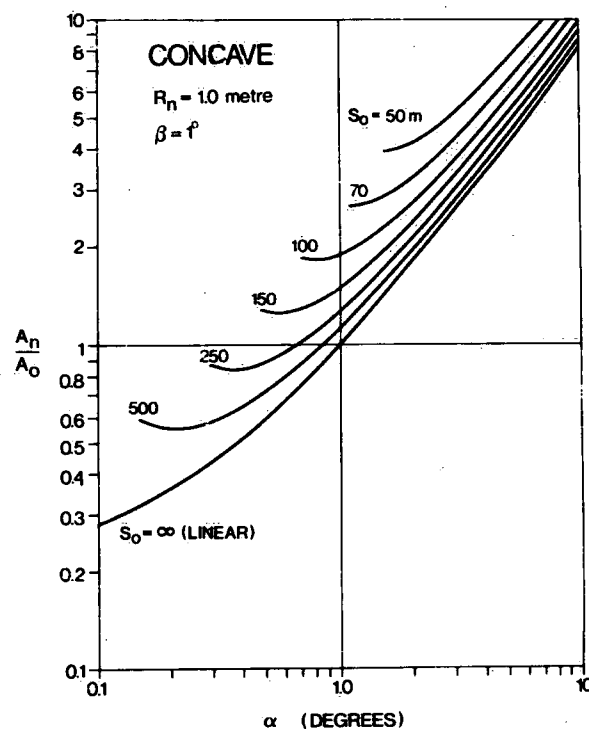


Figure 12. Relationships between A_n/A_o (the ratio of concave shoreline marsh area associated with water level condition R_n above datum to the concave shoreline marsh area associated with zero water level datum) and offshore slope α for a variety of S_o values.

When Figure 13 is multiplied by Figure 6 for a fixed value $S_o = 250$ m, Figure 14 results. The hinge point in Figure 14 clearly occurs at an offshore slope $\alpha < \beta$. Both Figure 13 for linear shoreline marshes and Figure 14 for concave shoreline marshes indicate that as R_n increases to a maximum value of d , so also does the impact on marshland areal extent. This increased impact with R_n occurs on both sides of the hinge point, i.e., for α values small enough to be associated with a reduction in marsh area and for α values large enough to be associated with an increase in marsh area. Much larger impacts, however, are associated with reductions of linear marsh area than with reductions of concave marsh area. For larger values of α (corresponding to increases in marsh area), the impact of increasing R_n is comparable (at fixed β values) for both linear and concave marshes. The effect of varying β values on the areal extent of concave marshlands subject to persistent water level increases is shown in Figure 15. Fixed values of $S_o = 250$ m

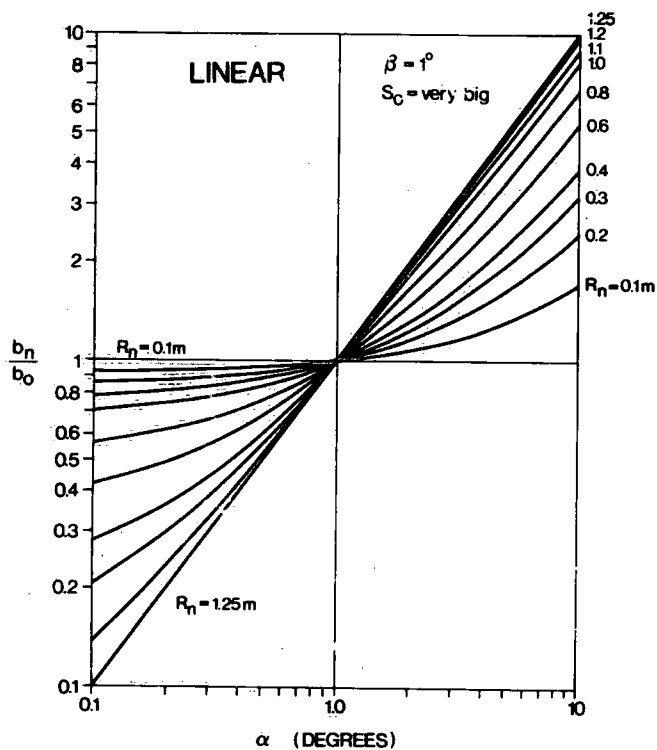


Figure 13. Relationships between the linear marsh ratio b_n/b_0 and offshore slope α for a variety of increased water levels R_n .

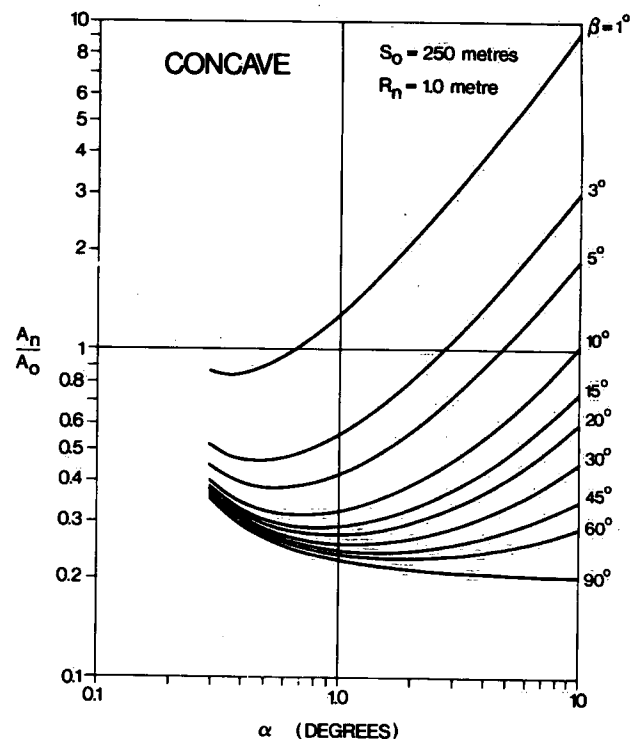


Figure 15. Relationships between A_n/A_0 for concave shoreline marshes and offshore slope α for a variety of onshore slopes β .

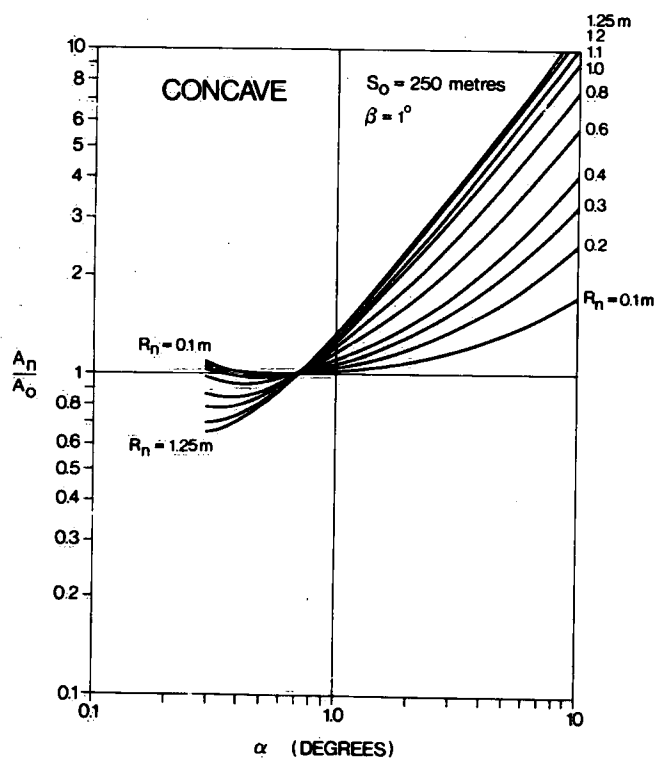


Figure 14. Relationships between A_n/A_0 for concave shoreline marshes and offshore slope α for a variety of R_n values.

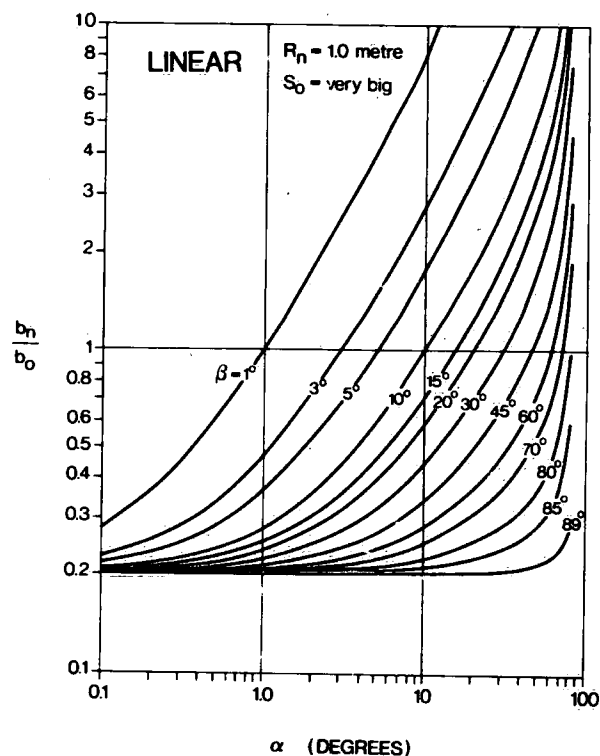


Figure 16. Relationships between b_n/b_0 for linear shoreline marshes and offshore slope α for a variety of onshore slopes β .

and $R_n = 1.0$ m are taken, and a family of curves depicting A_n/A_0 as a function of α are shown for $1^\circ \leq \beta < 90^\circ$. As expected, it is seen that for the range of α values $0.1^\circ \leq \alpha \leq 10^\circ$, only values of $\beta < 10^\circ$ result in situations that may be accompanied by increased marshland areal extent. It is also evident that the value of α beyond which the concave marsh areal extent increases (i.e. $A_n/A_0 > 1$) occurs at some value $\alpha < \beta$. This may be compared to the corresponding condition for linear shoreline marshes (Fig. 16) in which this transition from a reduction in areal extent to an increase occurs at $\alpha = \beta$.

Figures 12 to 16 have assumed the ideal condition of ready establishment of vegetative equilibrium. The condition for which no such equilibrium may be established is represented by $\beta = 90^\circ$. Figure 17 illustrates A_n/A_0 for a concave shoreline marsh in which $\beta = 90^\circ$ and $R_n = 1.0$ m for a variety of S_0 values. For $S_0 \rightarrow \infty$ the linear marsh curve becomes a horizontal line independent of α and equal to the limit of A_n/A_0 for a concave marsh of any S_0 value

as $\alpha \rightarrow 90^\circ$. For the parameters of Figure 17, this limiting value of A_n/A_0 as seen from equation (12) is $1 - R_n/d = 0.2$.

Figure 18 illustrates this non-equilibrium ($\beta = 90^\circ$) concave marsh situation and the response of A_n/A_0 vs. α to changing R_n values (S_0 is fixed at 250 m). Clearly, such a situation may only result in a decrease in marsh area with increase in persistent water level. For comparison, Figure 19 illustrates the non-equilibrium situation for a linear shoreline marsh (S_0 very large) and once again the near-linear behaviour of concave marshes (Fig. 18) at large values of α is distinctly evident, the departure from near-linear behaviour dramatically increasing as α drops below 1° or 2° .

While, for $R_n > 0$, the geometric marsh factor

$$\frac{E_n + S_n}{E_0 + S_0}$$

for a concave shoreline marsh is always > 1 , this same factor for a convex shoreline marsh is always < 1 . Quite simply

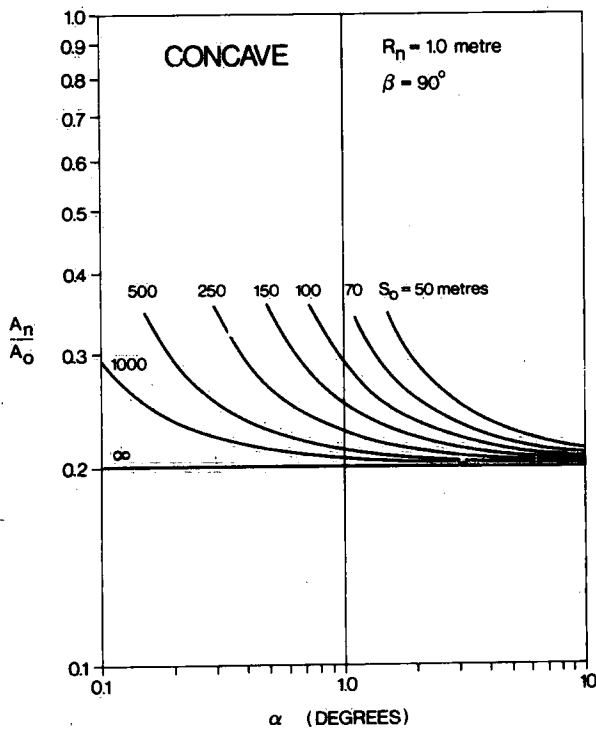


Figure 17. Relationships between A_n/A_0 for concave shoreline marshes and offshore slope α for a variety of S_0 values and onshore slopes $\beta = 90^\circ$ (i.e. a non-regenerative marsh).

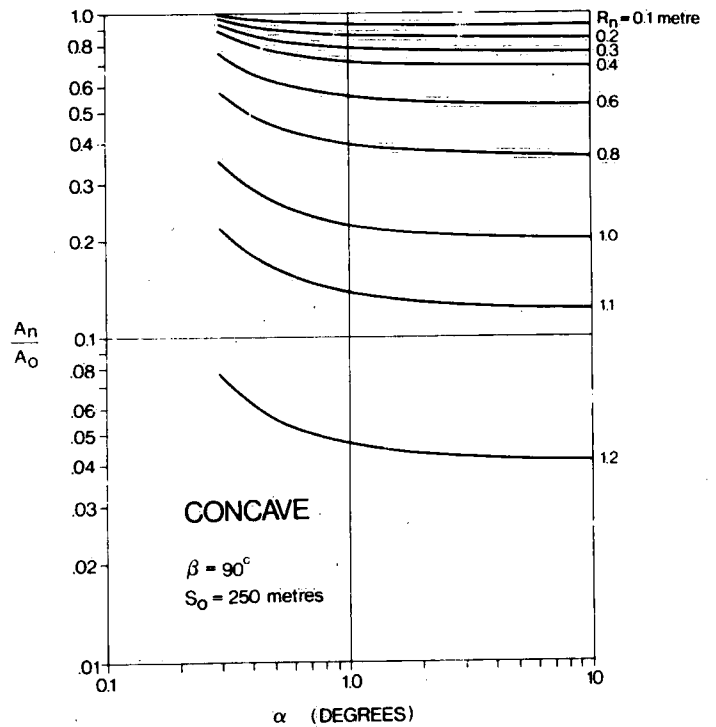


Figure 18. Relationships between A_n/A_0 for concave shoreline marshes and offshore slope α for a variety of R_n values and onshore slope $\beta = 90^\circ$ (i.e. a non-regenerative marsh).

expressed in terms of S_0 , R_n , b_0 , α and β , this factor becomes:

$$\left(\frac{E_n + S_n}{E_0 + S_0} \right)_{\text{concave}} = 1 + \frac{R_n}{2S_0 - b_0} (\cot \alpha + \cot \beta) \quad (11a)$$

$$\left(\frac{E_n + S_n}{E_0 + S_0} \right)_{\text{convex}} = 1 - \frac{R_n}{2S_0 + b_0} (\cot \alpha + \cot \beta) \quad (11b)$$

Figure 20 illustrates the convex shoreline marsh factor (< 1) for a fixed $R_n = 1$ m, and a fixed $\beta = 1^\circ$ depicted as a function of α for a family of S_0 values. Clearly, each S_0 value has its own associated geometric factor which is almost independent of α for all but the smallest and largest of marshes. When the geometric factors of Figure 20 are multiplied by the corresponding b_n/b_0 ratios for a fixed $R_n = 1$ m and $\beta = 1^\circ$, the family of curves shown in Figure 21 results. For comparison, the linear shoreline marsh (S_0 very large) is also shown. Figure 21 illustrates

that the A_n/A_0 values for a convex shoreline marsh are always less than the b_n/b_0 values for a linear shoreline marsh; the larger the convex marsh, the more closely it resembles a linear marsh; and the areal extent of the convex marsh may be either reduced or increased subsequent to a persistent water level elevation followed by vegetative equilibrium. The transition from areal extent reduction to areal extent expansion, as seen previously, occurs at $\alpha = \beta$ for linear shoreline marshes. However, such transition occurs at $\alpha > \beta$ for convex shoreline marshes (just the opposite of the situation for concave shoreline marshes wherein such transition occurs at $\alpha < \beta$). The extreme case for which no vegetation equilibrium is established ($\beta = 90^\circ$) for the convex shoreline marsh is shown in Figure 22, again for $R_n = 1$ m. The linear marsh situation (again, as in Figs. 17 and 19) is represented by a fixed value

$$\frac{A_n}{A_0} = \frac{b_n}{b_0} = 0.2$$

with the family of curves representing the various convex marshes lying below this linear value.

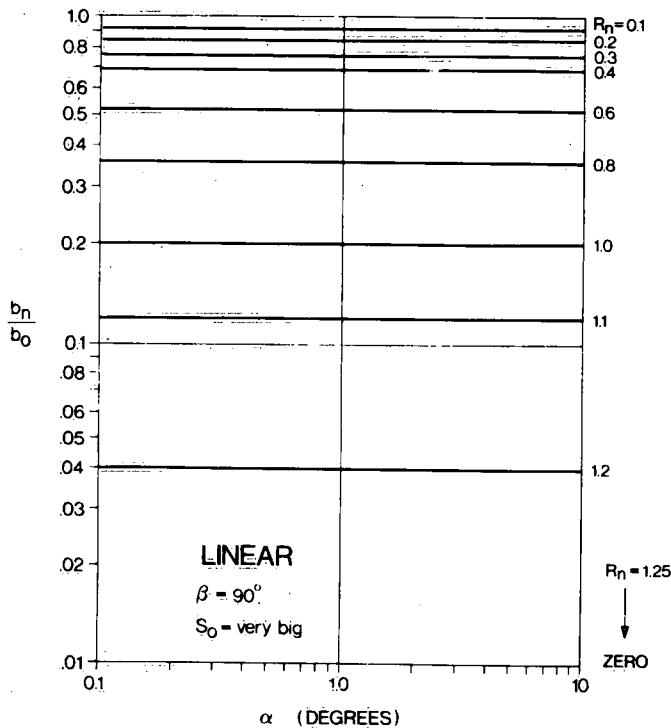


Figure 19. Relationships between b_n/b_0 for linear shoreline marshes and offshore slope α for a variety of R_n values and onshore slope $\beta = 90^\circ$ (i.e. a non-regenerative marsh).

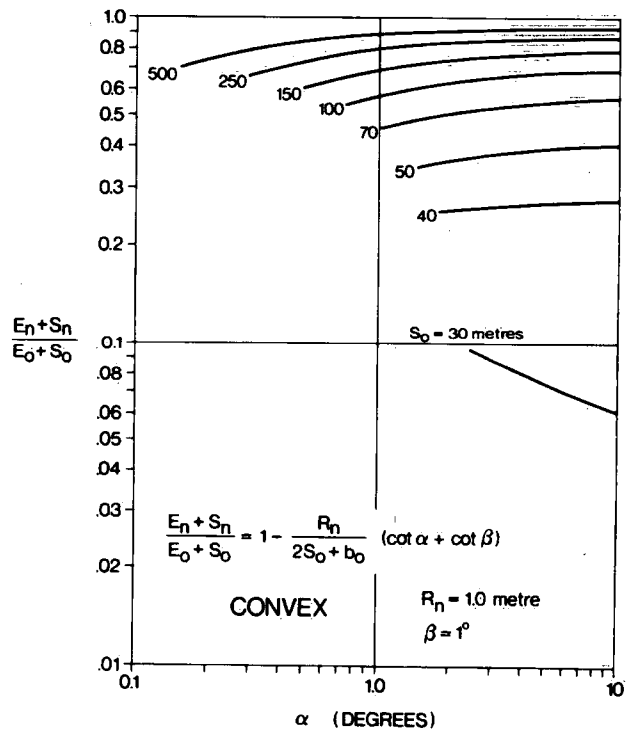


Figure 20. Relationships between the convex shoreline marsh factor $(E_n + S_n)/(E_0 + S_0)$ and offshore slope α for a variety of S_0 values.

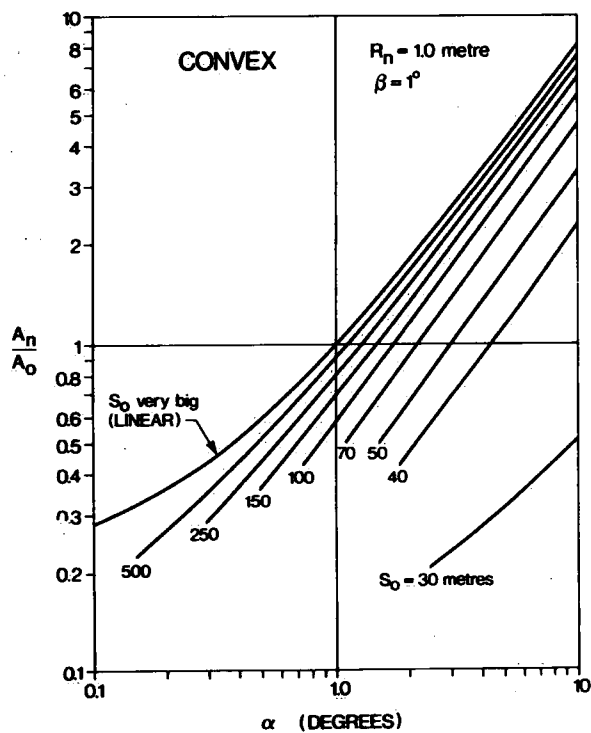


Figure 21. Relationships between A_n/A_0 for convex shoreline marshes and offshore slope α for a variety of S_0 values.

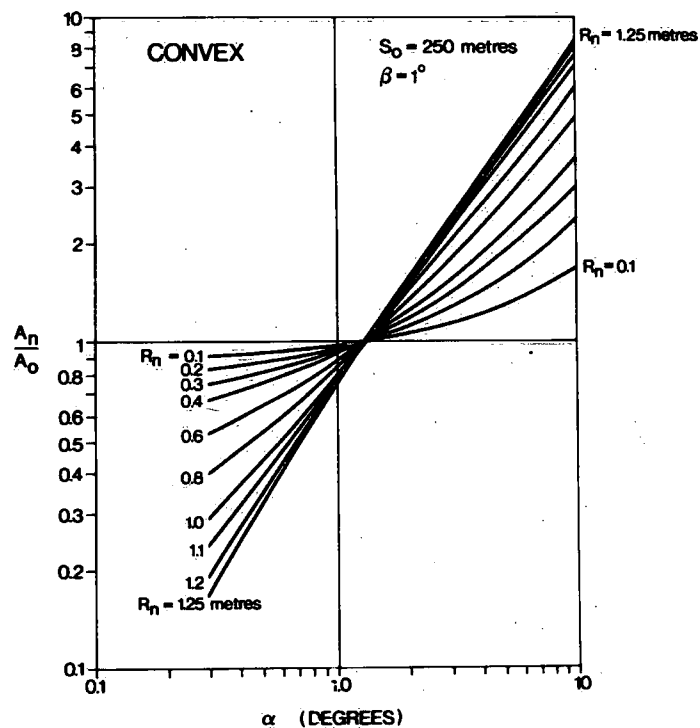


Figure 23. Relationships between A_n/A_0 for convex shoreline marshes and offshore slope α for a variety of R_n values.

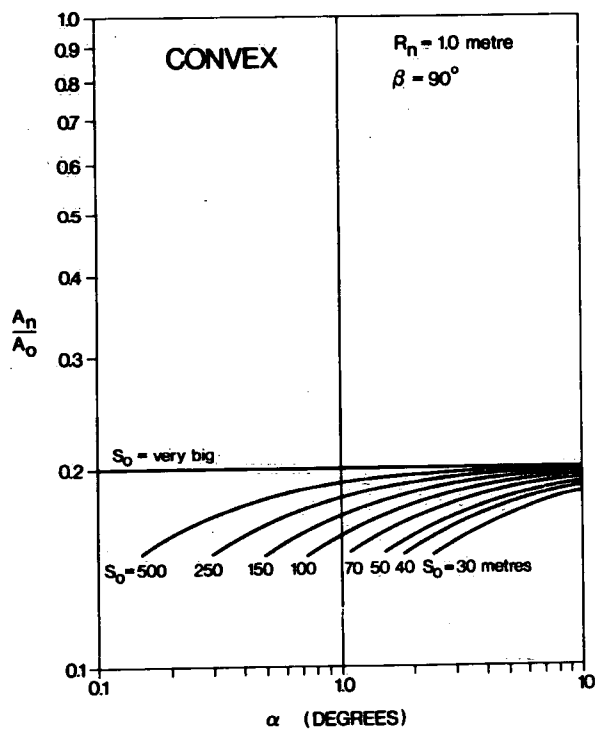


Figure 22. Relationships between A_n/A_0 for convex shoreline marshes and offshore slope α for a variety of S_0 values and onshore slope $\beta = 90^\circ$ (i.e. a non-regenerative marsh).

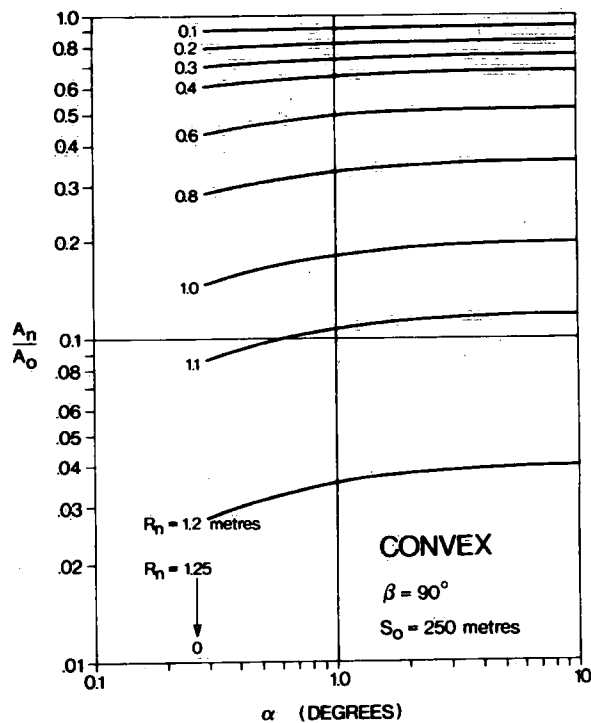


Figure 24. Relationships between A_n/A_0 for convex shoreline marshes and offshore slope α for a variety of R_n values and onshore slope $\beta = 90^\circ$ (i.e. a non-regenerative marsh).

Figure 23 (which may be directly compared with its concave shoreline counterpart in Fig. 14) indicates the family of curves representing A_n/A_0 as a function of α for convex shoreline marshes with a fixed $S_0 = 250$ m, a fixed $\beta = 1^\circ$, and a variety of persistent water levels R_n above zero water level datum where $0 < R_n \leq d$. The similarity of Figure 23 to Figure 14 is very apparent, a principal difference being that the hinge point at which $A_n = A_0$ for concave shoreline marshes occurs at a value of $\alpha < \beta$ (Fig. 14), while the hinge point at which $A_n = A_0$ for convex shoreline marshes occurs at a value of $\alpha > \beta$ (Fig. 23). The hinge point at which $A_n = A_0$ for linear shoreline marshes occurs, of course, at $\alpha = \beta$ (Fig. 13).

Figure 24 (comparable to its concave shoreline marsh counterpart of Fig. 18) represents the non-equilibrium ($\beta = 90^\circ$) convex marsh situation for $S_0 = 250$ m and the variable R_n values of Figure 23. At large values of α , both concave and convex marshes approach linear behaviour (Fig. 19). At lower values of α , the departure from linear behaviour becomes more pronounced with decreasing α , the concave marshland (Fig. 18) increasing above the linear value, and the convex marshland (Fig. 24) decreasing below the linear value. All three shoreline marshland types (linear, concave, and convex), however, manifest as reduced areal extent when subject to prolonged increases in water level for the case of total inability to establish a vegetation equilibrium (i.e. the case defined by $\beta = 90^\circ$).

Geometric Marsh Model: Elliptical Shorelines

Figure 25a illustrates a general elliptical shoreline marsh condition that might typify the convex nature of some islands and headlands. The ellipse is defined by axes of lengths u and v . The marsh parameters (consistent with the E and S distance parameters of the convex shoreline situation of Fig. 4) associated with these axes are taken as E_u , S_u , E_v and S_v , respectively. For the two distinct conditions of water level R_n (above zero water level datum) and R_0 (zero water level datum), the eight geometric marsh parameters become E_{ou} , E_{nu} , S_{ou} , S_{nu} , E_{ov} , E_{nv} , S_{ov} and S_{nv} .

For a completely elliptical convex marsh (e.g. an elongated island), A_n/A_0 may be approximated as:

$$\frac{A_n}{A_0} = \frac{\pi E_{nu} E_{nv} - \pi S_{nu} S_{nv}}{\pi E_{ou} E_{ov} - \pi S_{ou} S_{ov}} \quad (14)$$

$$= \frac{(E_{nu} + S_{nu}) \frac{b_{nv}}{b_{ou}} + (E_{nv} + S_{nv}) \frac{b_{nu}}{b_{ou}}}{E_{ov} + S_{ov} + (E_{ou} + S_{ou}) \frac{b_{ov}}{b_{ou}}}$$

where b_{nv}/b_{ou} is the ratio of the new marsh linear distance ($E_{nv} - S_{nv}$) along the v -axis at water level R_n to the original marsh linear distance ($E_{ou} - S_{ou}$) along the u -axis at zero water level R_0 ; b_{nu}/b_{ou} is the ratio of the new marsh linear distance ($E_{nu} - S_{nu}$) along the u -axis at water level R_n to the original marsh linear distance ($E_{ou} - S_{ou}$) along the u -axis at zero water level; b_{ov}/b_{ou} is the ratio of the original marsh linear distance ($E_{ov} - S_{ov}$) along the v -axis to the original marsh linear distance ($E_{ou} - S_{ou}$) along the u -axis.

Obtaining a linear shoreline equivalent for an elliptical configuration poses certain problems, since associated with every marsh dimension between u and v is a specific

(α, β) set. Consequently, each point on the circumference of the ellipse is defined by a distinct pair of slopes, and therefore a spectrum of b_n values emerges. It is this variation in slopes that necessitates the cross-axial ratios of equation (14). If it is assumed that the angle α is a constant (i.e. the offshore slope remains invariant along the elliptical shoreline) and that β varies to accommodate the marsh area physically, then the values of b_n/b_0 can be seen to vary from a minimum value b_{nv}/b_{ov} along the v -axis to a maximum value b_{nu}/b_{ou} along the u -axis.

Similarly, the general elliptical shoreline condition that might typify the concave nature of some bays and bights (shown in Fig. 25b) can also be expressed by the governing equation (14). A qualifier analogous to the use of this equation for both convex and concave elliptical shorelines applies as for the use of equation (9) for both convex and concave shoreline marshes. For convex shorelines $E_n < E_0$ and $S_n < S_0$ for both the u - and v -axes and for concave shorelines $E_n > E_0$ and $S_n > S_0$ for both axes.

To display graphically the roles of α , β , R_n , and ellipticity on the effects of persistent water level fluctuations on convex and/or concave elliptical shoreline marshes, it is convenient to expand equation (14) in terms of the geometric parameters of Figure 25c. Herein is depicted an ellipse with axes u and v and an ellipticity factor γ defined as the ratio v/u . Since u may be $>v$, $<v$, or $=v$ (i.e., u may be a major or minor axis), γ may be <1 , >1 or $=1$, respectively. Even though each point on the circumference of the ellipse is defined by an independent (α, β) set, only two input (α, β) sets require consideration, namely α_u and β_u associated with the u -axis, and α_v and β_v associated with the v -axis. To generate families of curves which may be compared with the families of curves already generated for linear, concave and convex shoreline marshes, A_n/A_0 values for convex and concave elliptical shoreline marshes will be calculated and displayed as a function of α_u , viz. the offshore slope associated with the u -axis of the elliptical configuration of Figure 25c. Consistent with this manner of interrelationship presentation is the consideration of (b_n/b_0) along the u -axis as the linear

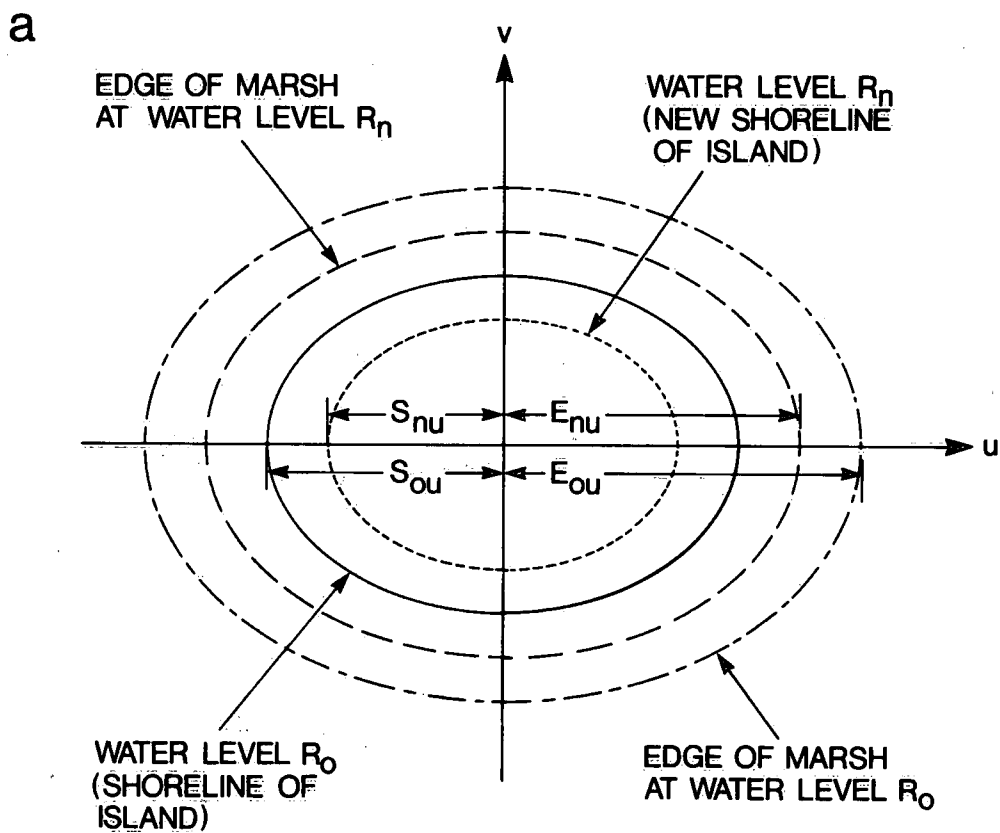


Figure 25a. General elliptical shoreline marsh condition typifying the convex nature of islands and headlands.

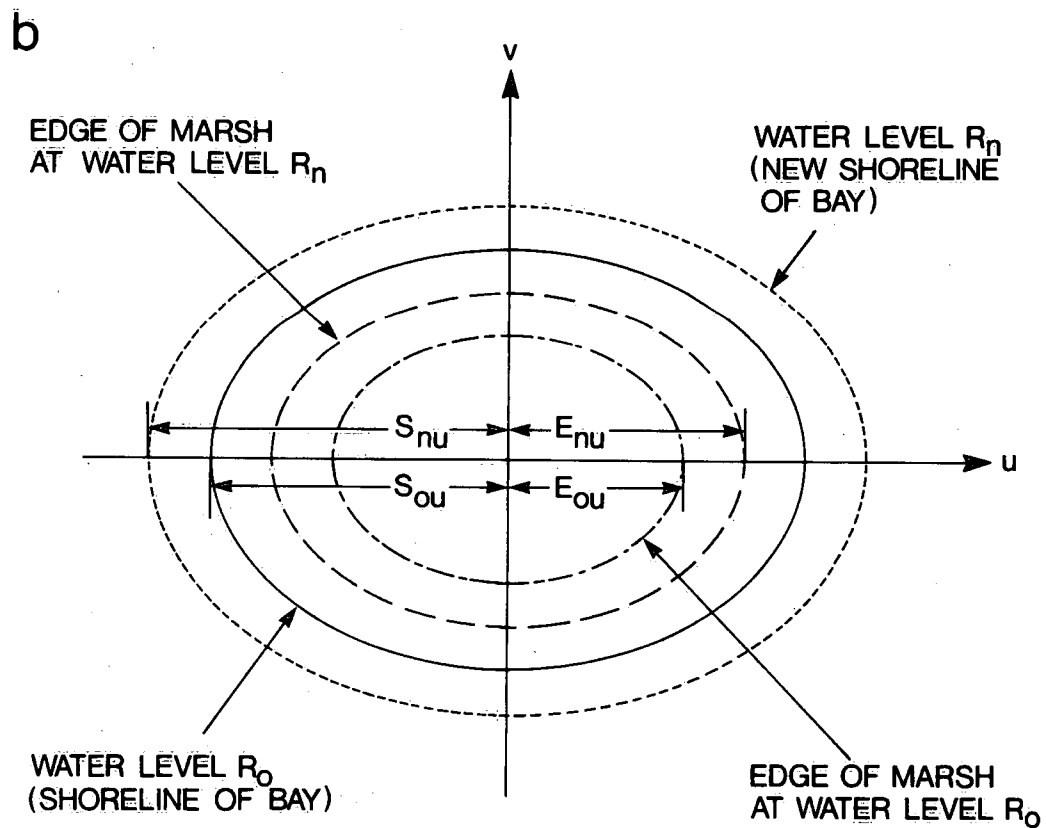


Figure 25b. General elliptical shoreline marsh condition typifying the concave nature of bays and bights.

C

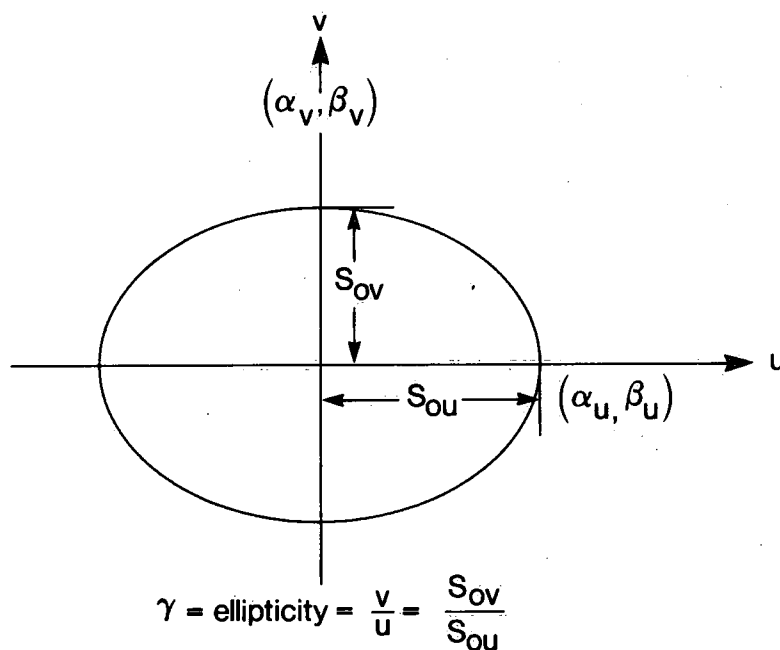


Figure 25c. Geometric parameters for elliptical shoreline marshes.

d

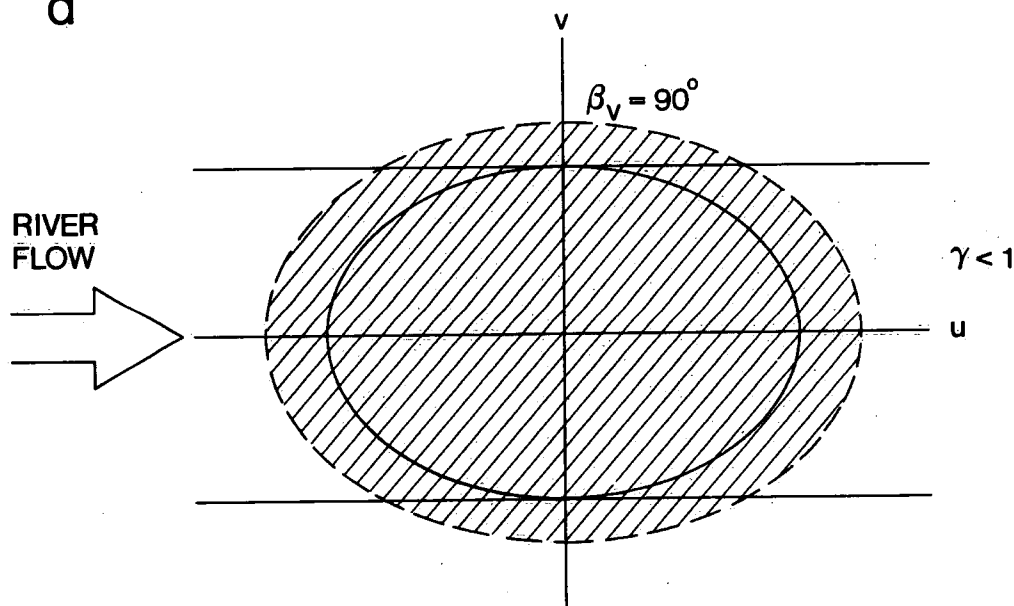


Figure 25d. Establishment of elliptical shoreline marshes at river mouths or coves.

shoreline marsh equivalent, i.e., the linear equivalent (b_n/b_o) is taken to be (b_{nu}/b_{ou}). This is reflected below

in the A_n/A_o equations for convex and concave elliptical shoreline marshes.

Convex

$$\frac{A_n}{A_o} = \left[\frac{b_{nv} + (S_{ov} - R_n \cot \beta_v) + (S_{ou} - R_n \cot \beta_u) \left(\frac{b_{nv}}{b_{nu}} \right)}{b_{ov} + S_{ov} + S_{ou} \left(\frac{b_{ov}}{b_{ou}} \right)} \right] \times \left(\frac{b_{nu}}{b_{ou}} \right) \quad (15)$$

Concave

$$\frac{A_n}{A_o} = \left[\frac{-b_{nv} + (S_{ov} + R_n \cot \beta_v) + (S_{ou} + R_n \cot \beta_u) \left(\frac{b_{nv}}{b_{nu}} \right)}{-b_{ov} + S_{ov} + S_{ou} \left(\frac{b_{ov}}{b_{ou}} \right)} \right] \times \left(\frac{b_{nu}}{b_{ou}} \right) \quad (16)$$

where the terms are as previously defined.

In an analog identical with equations (8) and (9), the square-bracketed portions of equations (15) and (16) represent the convex and concave elliptical marsh factors, factors which, when divided into measured (A_n/A_o) ratios for elliptical marshes subjected to persistent water level changes, will yield related linear marsh ratios (b_n/b_o) along the u-axis.

Figure 26 illustrates the family of curves of (A_n/A_o) as a function of α_u for a concave elliptical marsh of fixed $\alpha_v = 1^\circ$, fixed $\beta_u = 2^\circ$, fixed $\beta_v = 2^\circ$, and fixed persistent water level increase $R_n = 1$ m. The ellipse factor $\gamma = \frac{v}{u}$ is taken as 0.5, and S_{ou} is taken to vary between 150 m and ∞ (i.e. very large). Figures 27 and 28 illustrate the comparable families of curves for $\gamma = 1$ (i.e. circular concave marsh) and $\gamma = 10$, respectively. Figures 26, 27 and 28 represent the condition in which maximum vegetative equilibrium may be readily established.

Figures 29, 30 and 31 illustrate the A_n/A_o versus α_u families of curves for fixed values $\alpha_v = 1^\circ$, $\beta_u = 2^\circ$, $\beta_v = 90^\circ$ (i.e., rocky or steep shorelines perpendicular to the v-axis) and $R_n = 1$ m for $\gamma = 0.5$, $\gamma = 1$ and $\gamma = 10$, respectively.

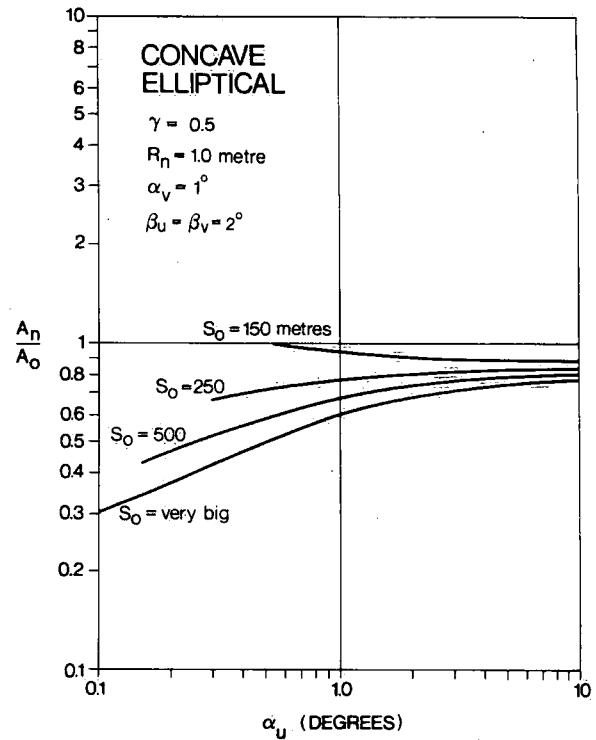


Figure 26. Relationships between A_n/A_o for concave elliptical shoreline marshes and principal axis offshore slope α_u and ellipticity factor $\gamma = 0.5$ for a variety of S_o values.

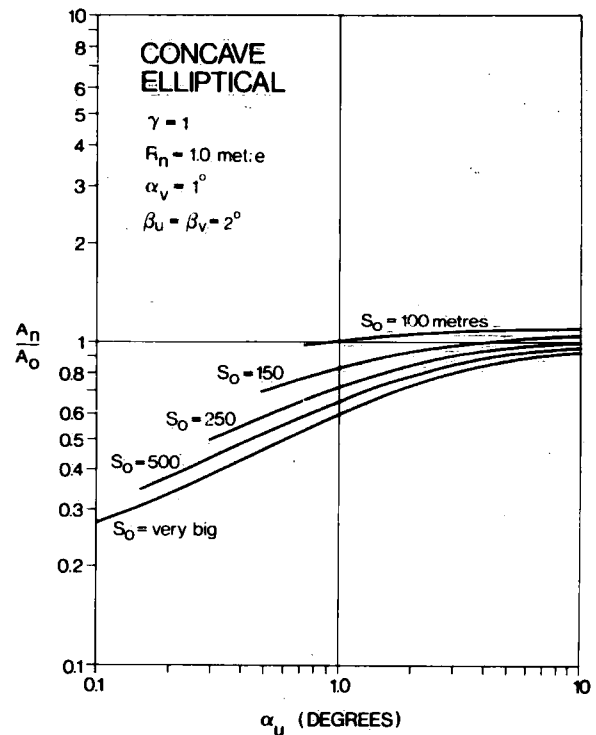


Figure 27. Relationships between A_n/A_o for concave elliptical shoreline marshes and principal axis offshore slope α_u and ellipticity factor $\gamma = 1.0$ for a variety of S_o values.

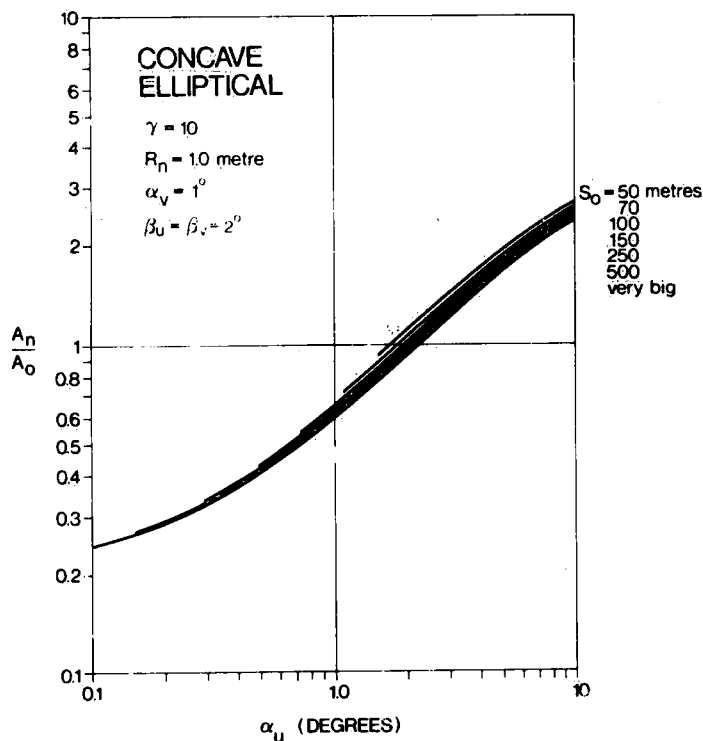


Figure 28. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 10$ for a variety of S_0 values.

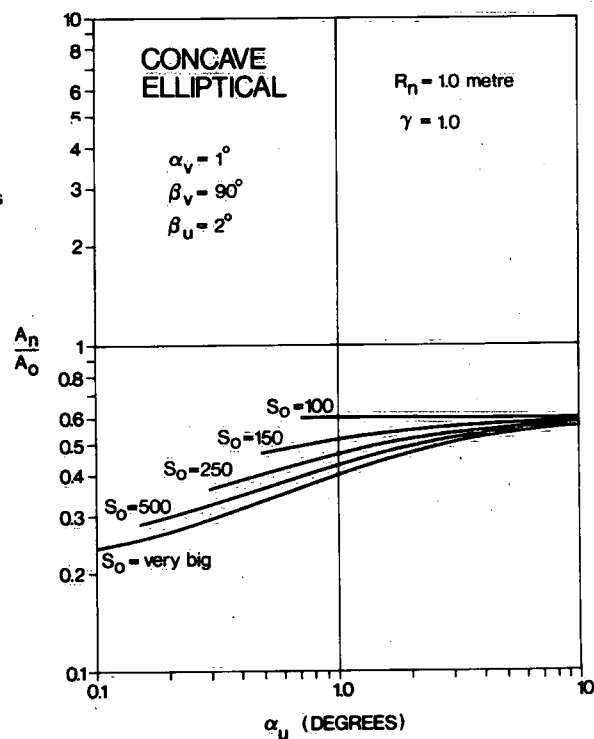


Figure 30. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 1.0$ for a variety of S_0 values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

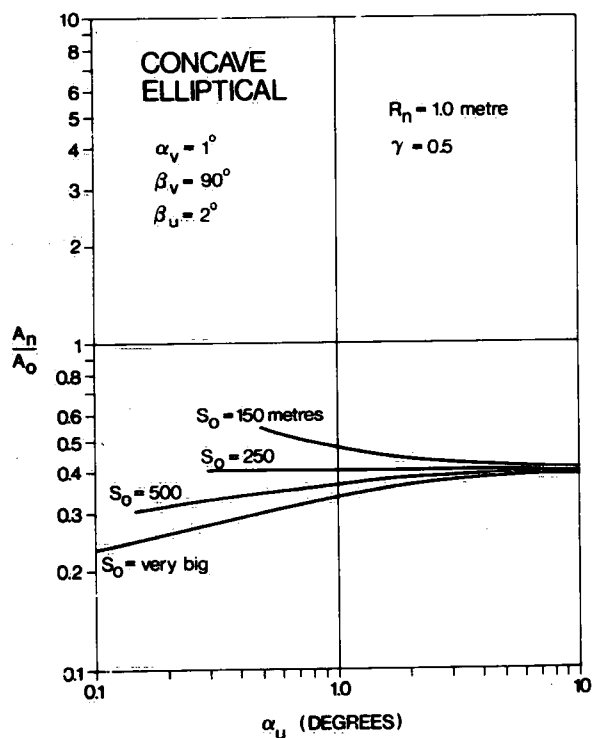


Figure 29. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 0.5$ for a variety of S_0 values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

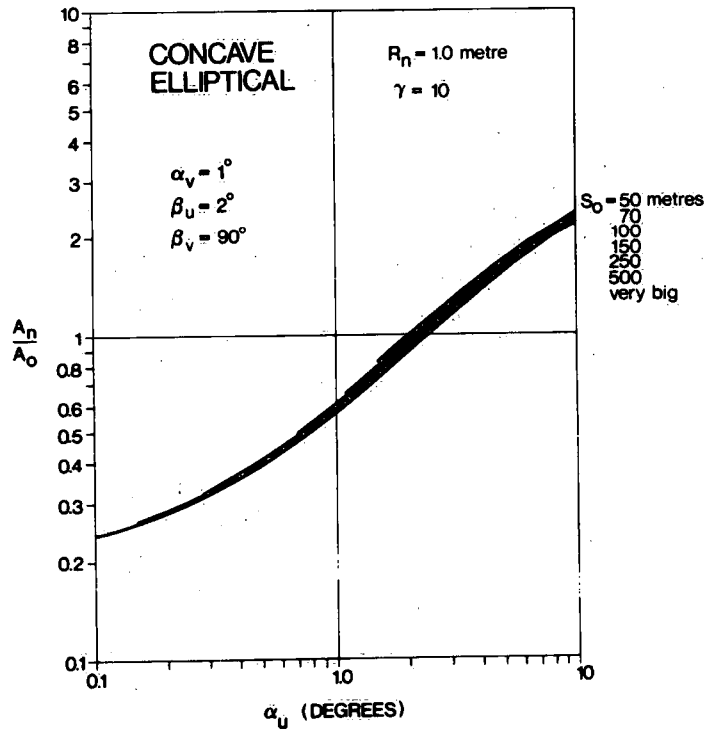


Figure 31. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 10$ for a variety of S_0 values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

Clearly, such restraints on vegetative regeneration result in marshland areas considerably smaller than would be expected under conditions favouring vegetative equilibrium (as shown in Figs. 26, 27 and 28). The higher the ellipticity factor γ for the case of $\beta_V = 90^\circ$, however, the closer the family of curves is to the case of regenerative vegetation (compare Figs. 28 and 31). This is a direct consequence of the considerably reduced impact of β_V upon the total elliptical areal extent for high γ values. Such a situation of high γ coupled with $\beta_V = 90^\circ$ is, however, not frequently encountered in wetland studies. Far more frequently encountered is the situation in which $\beta_V = 90^\circ$ and γ is significantly < 1 , as sketched in Figure 25d. This represents the establishment of marshes either in coves or in river mouths, and assuming either concave elliptical or concave semi-elliptical configurations surrounded by high bluffs physically oriented roughly parallel to the principal direction of water flow.

The effect of varying β_U for a concave elliptical marshland capable of sustaining vegetative equilibrium is illustrated in Figures 32, 33 and 34. Herein are depicted the families of curves of A_n/A_0 as a function of α_U for a fixed $\alpha_V = 1^\circ$, $\beta_V = 2^\circ$, $S_{OU} = 250$ m and $R_n = 1$ m for $\gamma = 0.5$, $\gamma = 1.0$, and $\gamma = 10$, respectively. The values of β_U range from 1° to 90° . Clearly, for each value of ellipticity, increasing the values of β_U results in further depressed values of A_n/A_0 .

The corresponding situations resulting from varying β_U for a concave elliptical marsh that cannot sustain vegetative equilibrium along the v-axis (i.e. $\beta_V = 90^\circ$) are illustrated in Figures 35, 36 and 37, which refer to $\gamma = 0.5$, $\gamma = 1$ and $\gamma = 10$, respectively, all other parameters being identical with those used to generate Figures 32, 33 and 34. Once again the effect of restricting vegetation regeneration results in reductions in A_n/A_0 from what would be anticipated in the absence of such regeneration restrictions, the departures from such unfettered A_n/A_0 values becoming much less significant with increasing values of γ (compare Figs. 34 and 37).

In a similar manner, Figures 38, 39 and 40 illustrate the families of curves resulting from varying the value of the persistent water level increase R_n on a totally regenerative marshland region defined by fixed values $\alpha_V = 1^\circ$, $\beta_V = 2^\circ$, $\beta_U = 2^\circ$, and $S_{OU} = 250$ m. The figures represent γ 's of 0.5, 1.0 and 10, respectively. The comparable situations for complete restriction of vegetative regeneration (i.e. $\beta_V = 90^\circ$) are depicted in Figures 41, 42 and 43.

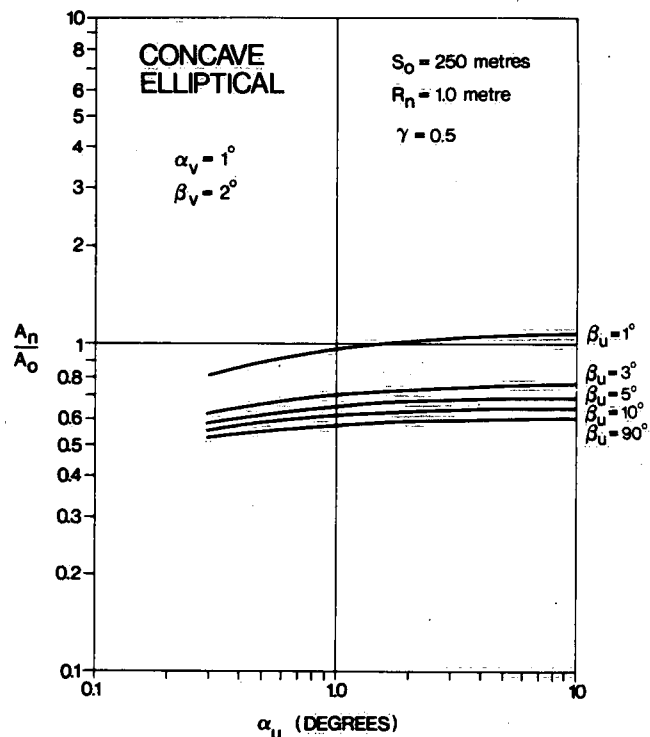


Figure 32. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 0.5$ for a variety of β_U values.

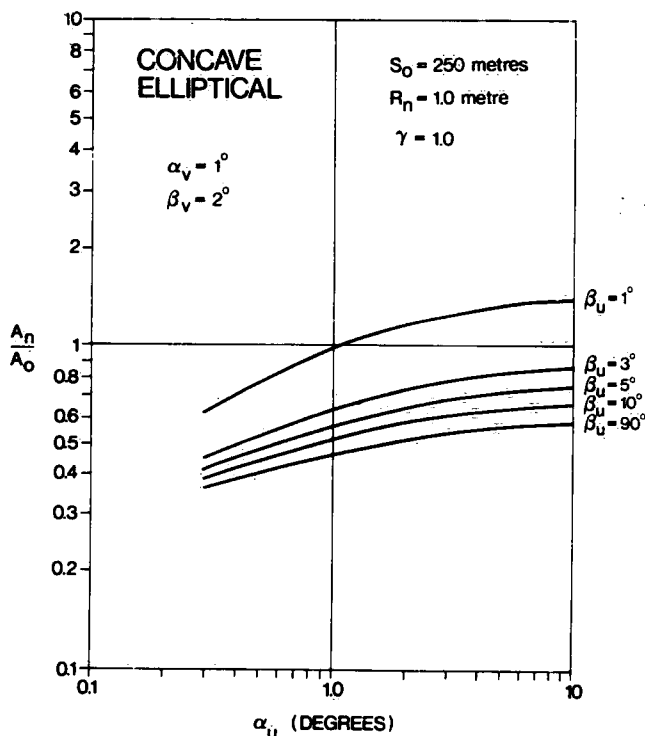


Figure 33. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 1.0$ for a variety of β_U values.

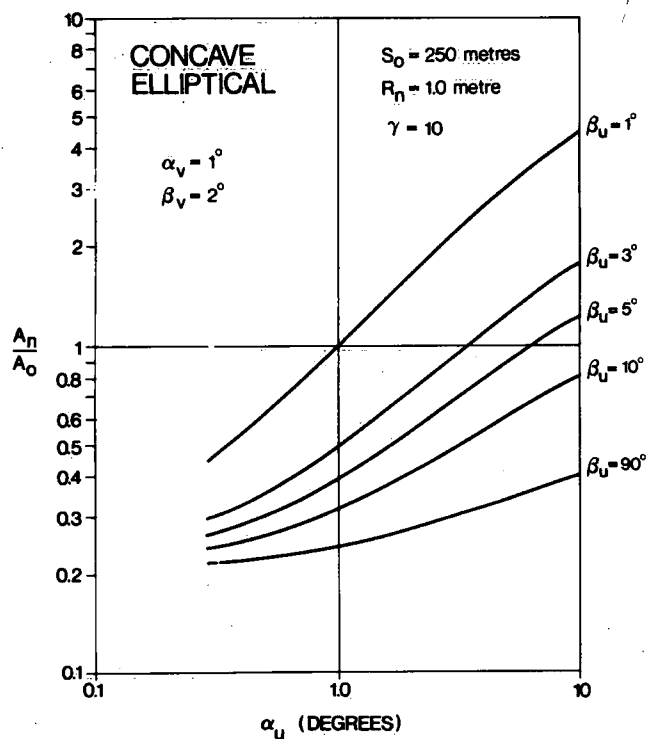


Figure 34. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 10$ for a variety of β_U values.

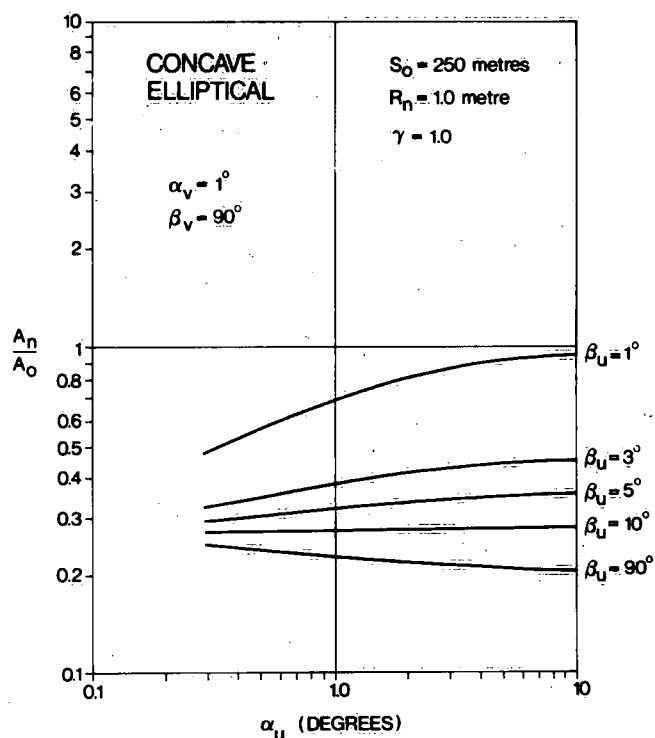


Figure 36. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 1.0$ for a variety of β_U values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

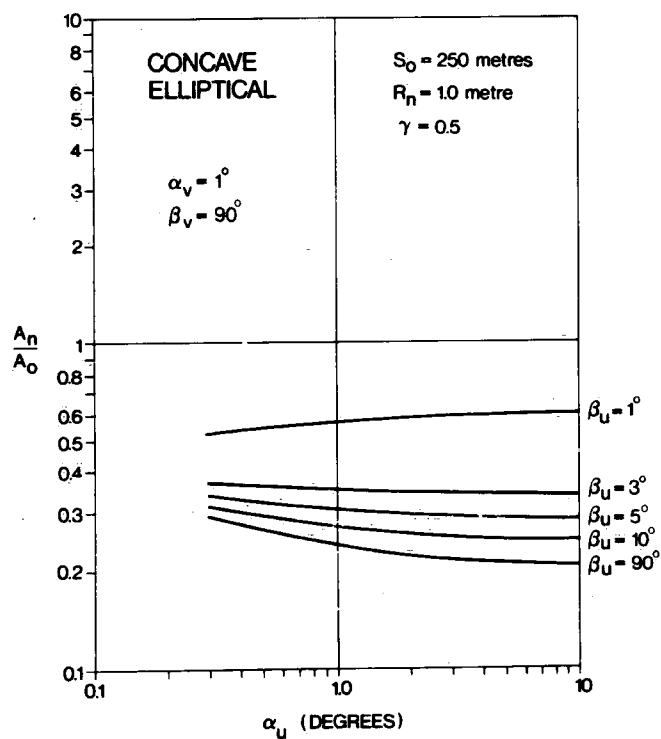


Figure 35. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 0.5$ for a variety of β_U values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

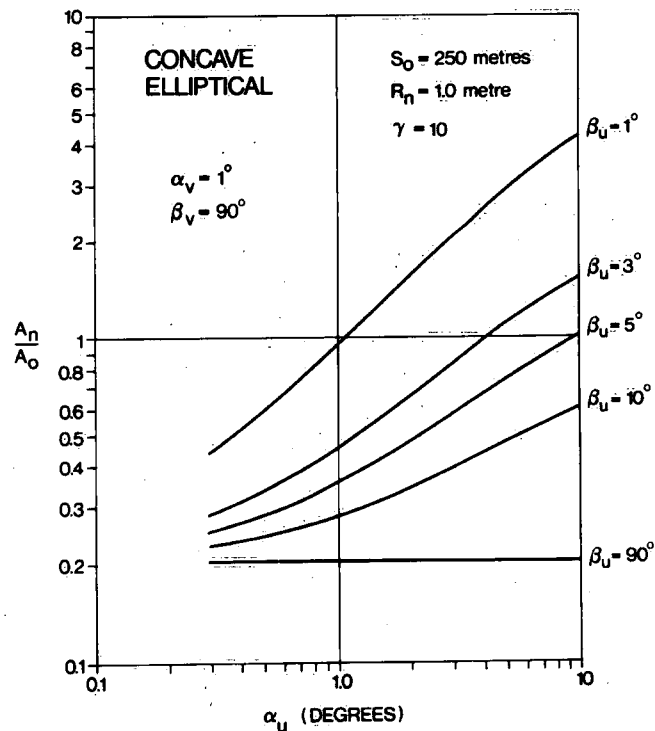


Figure 37. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 10$ for a variety of β_U values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

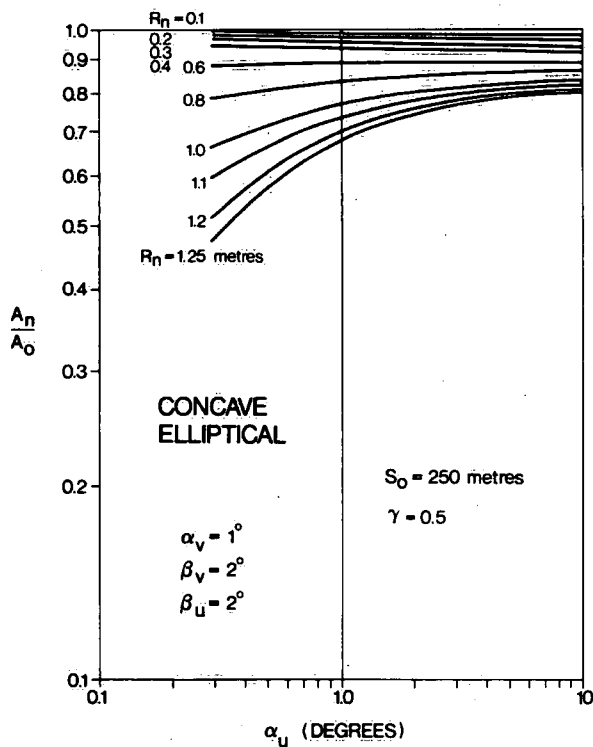


Figure 38. Relationships between A_n/A_o for concave elliptical shoreline marshes and principal axis offshore slope α_u and ellipticity factor $\gamma = 0.5$ for a variety of R_n values.

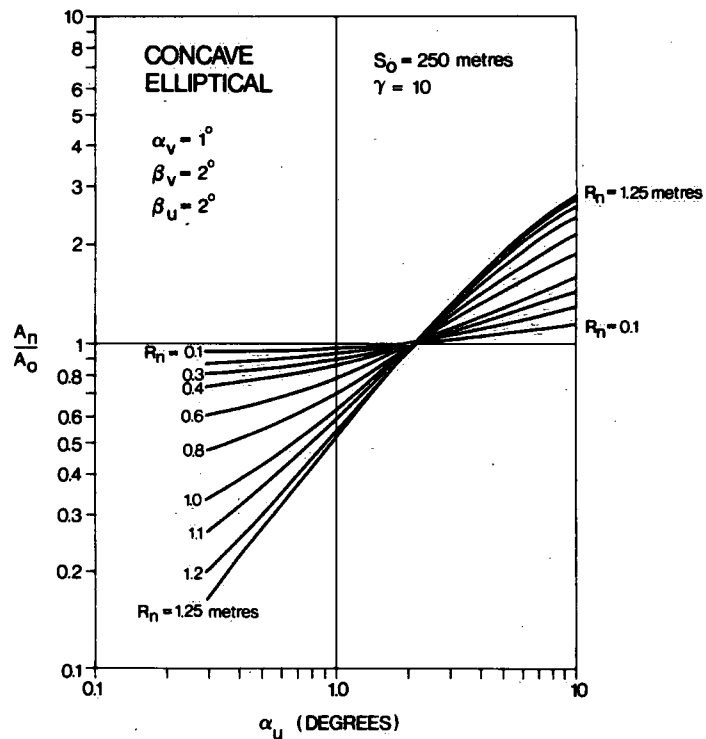


Figure 40. Relationships between A_n/A_o for concave elliptical shoreline marshes and principal axis offshore slope α_u and ellipticity factor $\gamma = 10$ for a variety of R_n values.

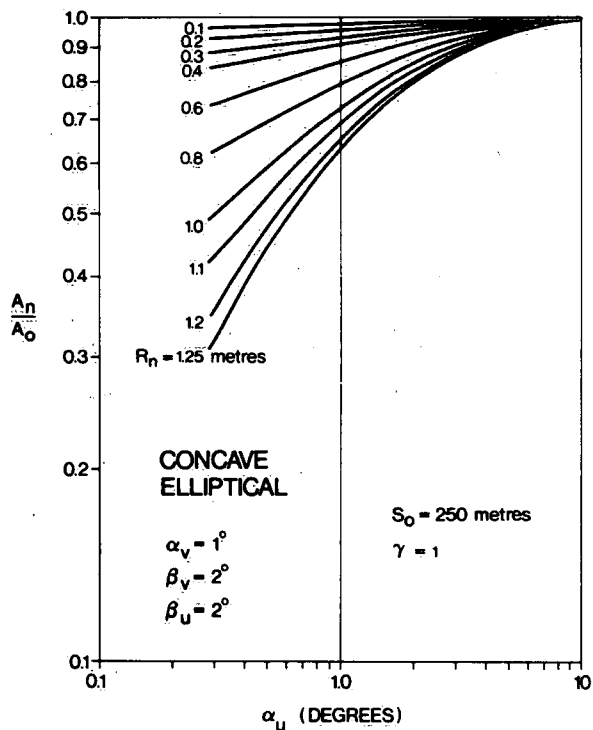


Figure 39. Relationships between A_n/A_o for concave elliptical shoreline marshes and principal axis offshore slope α_u and ellipticity factor $\gamma = 1.0$ for a variety of R_n values.

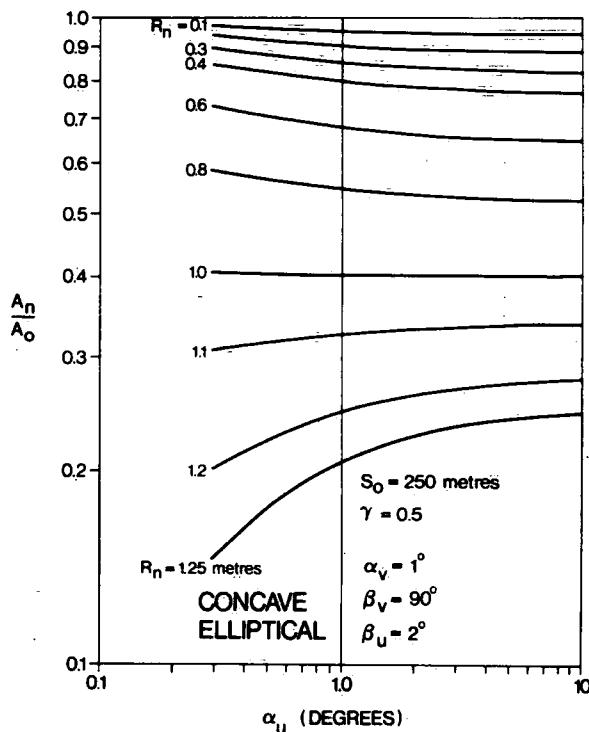


Figure 41. Relationships between A_n/A_o for concave elliptical shoreline marshes and principal axis offshore slope α_u and ellipticity factor $\gamma = 0.5$ for a variety of R_n values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

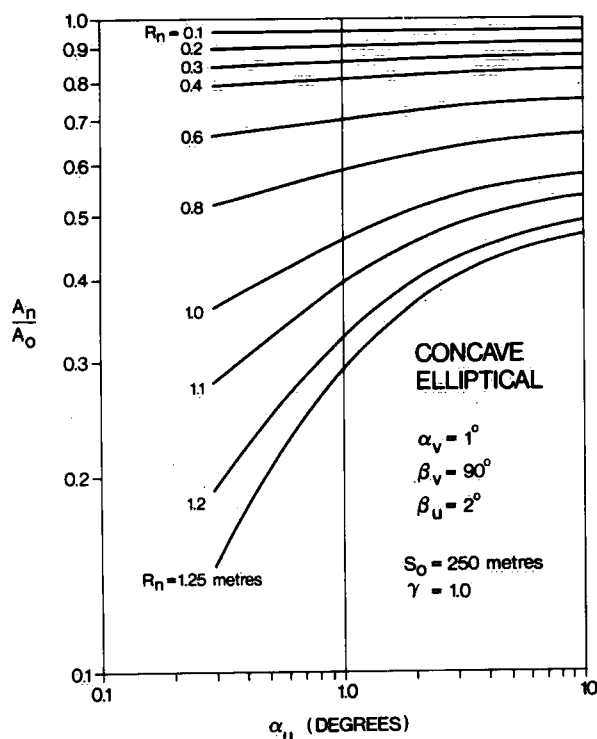


Figure 42. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 1.0$ for a variety of R_n values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

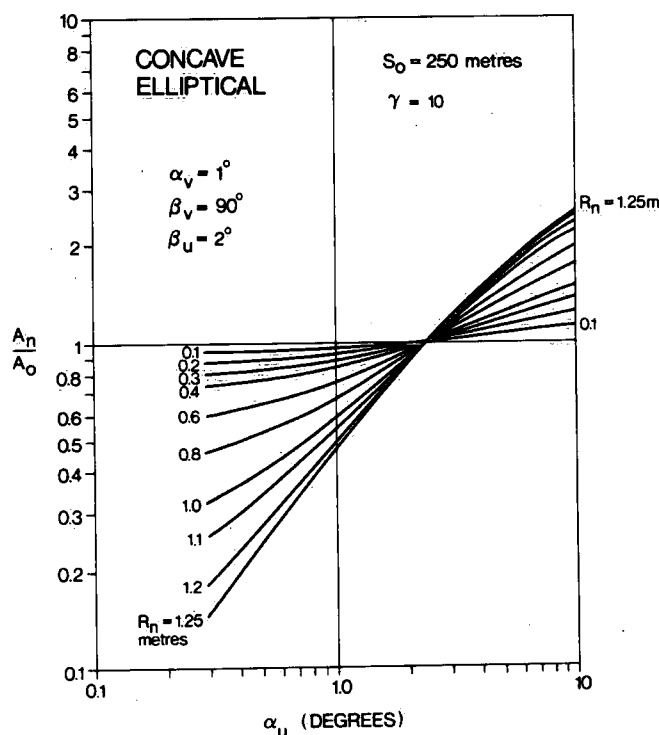


Figure 43. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U and ellipticity factor $\gamma = 10$ for a variety of R_n values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

It is singularly apparent that an infinite set of combinations of the multiplicity of parameters $[(\alpha_U, \beta_U), (\alpha_V, \beta_V), S_{OU}, S_{OV}, R_n, d, \gamma]$ involved in both the concave elliptical and the convex elliptical shoreline marshes may be used to generate families of curves such as those represented in Figures 26 through 43. The interested reader may, indeed, wish to generate such curves pertinent to particular regions of interest. No more of these curves will be generated herein. Rather, the next four figures in this section will illustrate briefly the effect of γ (i.e. the ellipticity) on the A_n/A_0 versus α_U relationships.

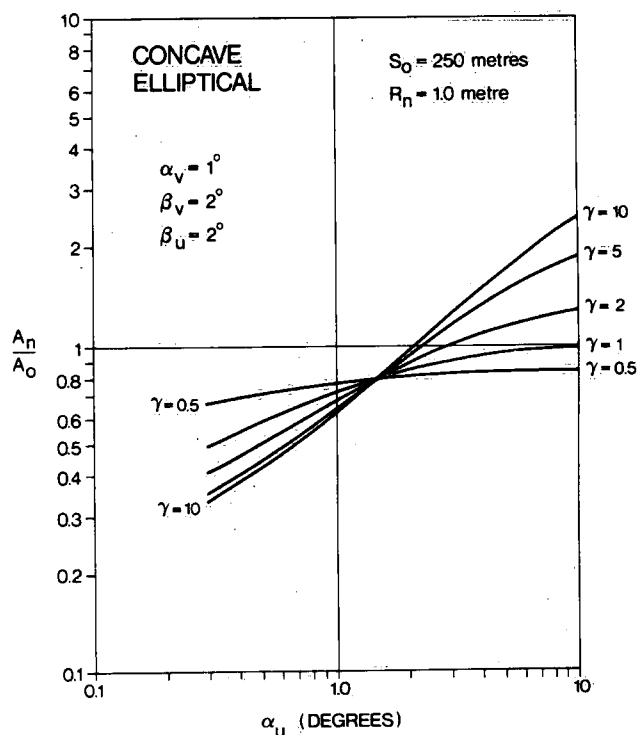


Figure 44. Relationships between A_n/A_0 for concave elliptical shoreline marshes and principal axis offshore slope α_U for a variety of ellipticity factor values.

Figure 44 illustrates the A_n/A_0 versus α_U curves appropriate to a concave elliptical shoreline marsh defined by $\alpha_v = 1^\circ, \beta_u = 2^\circ, \beta_v = 2^\circ, S_{OU} = 250 \text{ m}, R_n = 1.0 \text{ m}, d = 1.25 \text{ m}$, and possessing the capability of establishing a totally effective vegetation equilibrium, for a variety of ellipticities γ ranging from 0.5 (describing a marsh foreshortened in the v-axis) to 10 (describing a marsh foreshortened in the u-axis). The salient features of Figure 44 are the following:

- A distinct hinge point is evident in the family of curves, indicating that for concave elliptical marshes, a value of α_U exists at which the value A_n/A_0 is

independent of γ , i.e., independent of the ellipticity of the shoreline marsh.

- (b) For values of α_U less than the hinge-point value of α_U , the value of A_N/A_O decreases with increasing γ . For values of α_U more than the hinge-point value of α_U , the value of A_N/A_O increases with increasing γ .
- (c) Depending upon the geometric slopes of the confining marsh basin, the areal extent of a totally regenerative concave elliptical shoreline marsh may either increase $A_N/A_O > 1$ or decrease $A_N/A_O < 1$ with an increase in persistent water level.

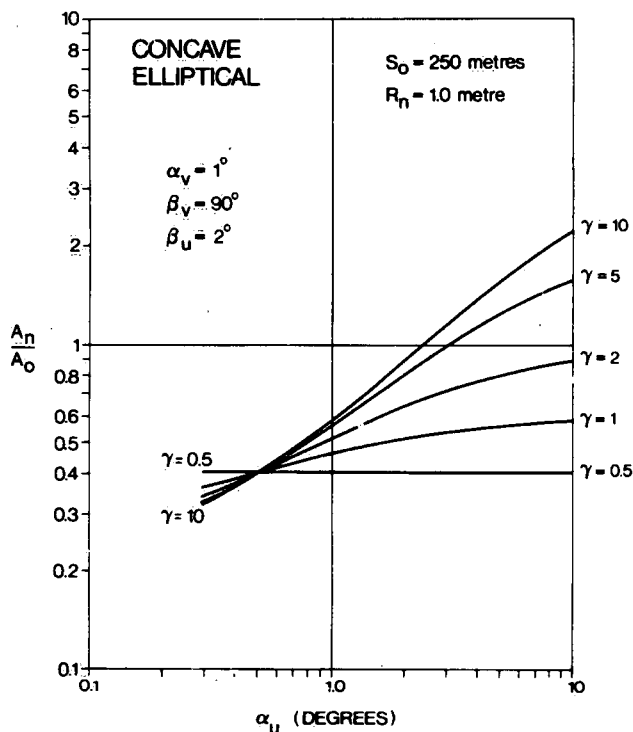


Figure 45. Relationships between A_N/A_O for concave elliptical shoreline marshes and principal axis offshore slope α_U for a variety of ellipticity factor values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

Figure 45 illustrates the family of curves that is a counterpart to that of Figure 44, with the imposed restriction that the concave elliptical marshland be totally unable to establish vegetative regeneration equilibrium along the v-axis (i.e., $\beta_v = 90^\circ$ representing either steep or rocky shorelines encountered at the strand line). Comparing Figure 45 (concave elliptical marsh incapable of sustaining

vegetative equilibrium) with Figure 44 (concave elliptical marsh capable of readily establishing total vegetative equilibrium), it may be seen that:

- (a) The distinct hinge point, defining that value of α_U at which the value A_N/A_O is independent of the ellipticity γ , is found at a lower value of α_U for marshes not displaying vegetative regeneration than the value of α_U appropriate for marshes (of otherwise comparable geometric configurations) which do display such regenerative capability. For the former situation, the hinge-point value of $\alpha_U < \alpha_v$, while for the latter $\alpha_U > \alpha_v$.
- (b) For values of α_U less than the hinge-point value of α_U , A_N/A_O decreases with increasing γ with the values of A_N/A_O for the non-equilibrium concave elliptical marsh at a particular γ being lower than the value of A_N/A_O at that γ for the concave elliptical marsh capable of establishing restorative equilibrium. For values of α_U more than the hinge-point value α_U , A_N/A_O increases with increasing γ , with the values of A_N/A_O for the non-equilibrium concave elliptical marsh at a particular γ once again being lower than the value of A_N/A_O at that γ for the concave elliptical marsh capable of establishing restorative equilibrium. It is seen from Figures 44 and 45 that as γ becomes larger the difference between the behaviour of concave elliptical marshes capable and incapable of establishing vegetative equilibrium along the v-axis becomes increasingly smaller as both shoreline marsh types approach the condition of ideal linear marshland response.

In a comparable manner, the effects of γ on convex elliptical shoreline marshes are briefly indicated in Figures 46 and 47. Figure 46 illustrates the effect of γ on the A_N/A_O vs. α_U relationship for a convex elliptical marsh of fixed parameters $\alpha_v = 1^\circ$, $\beta_u = \beta_v = 2^\circ$, $S_{OU} = 250$ m, and $R_n = 1.0$ m. The shoreline marsh is taken to possess complete vegetative equilibrium capabilities. The corresponding situation for this convex elliptical marsh incapable of establishing v-axis vegetative equilibrium (i.e. $\beta_v = 90^\circ$) is shown in Figure 47. The similarities and differences between these two convex elliptical shoreline marshes and their concave counterparts (Figs. 44 and 45) are clearly seen.

Obviously a plethora of curves may be constructed for convex elliptical shorelines, illustrating the impact on A_N/A_O resulting from changes in R_n , S_{OU} , (α_U, β_u) , (α_v, β_v) and R_n . The consideration of the spectrum of intermediate vegetative equilibrium capabilities inclusive from total regeneration to zero regeneration further compounds this plethora. It is felt, however, that the large albeit extremely limited and restrictive, examples illustrated in this section

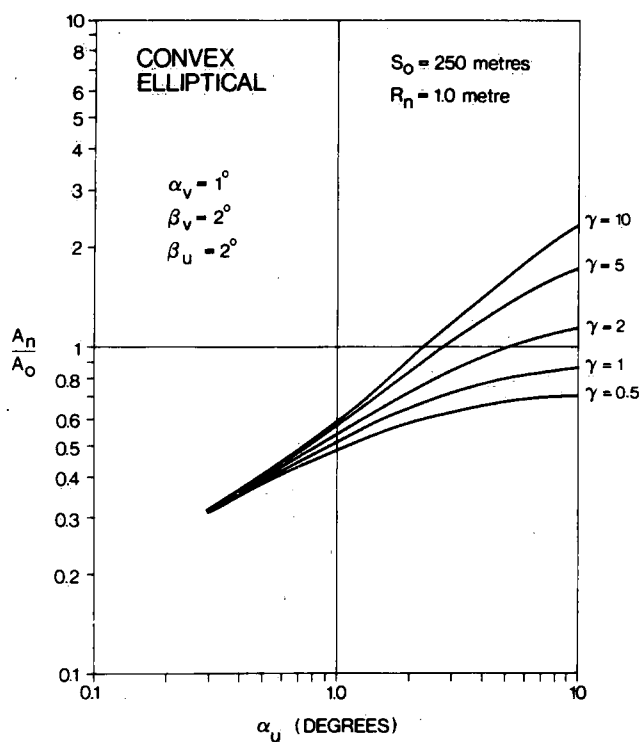


Figure 46. Relationships between A_n/A_o for convex elliptical shoreline marshes and principal axis offshore slope α_u for a variety of ellipticity factor values.

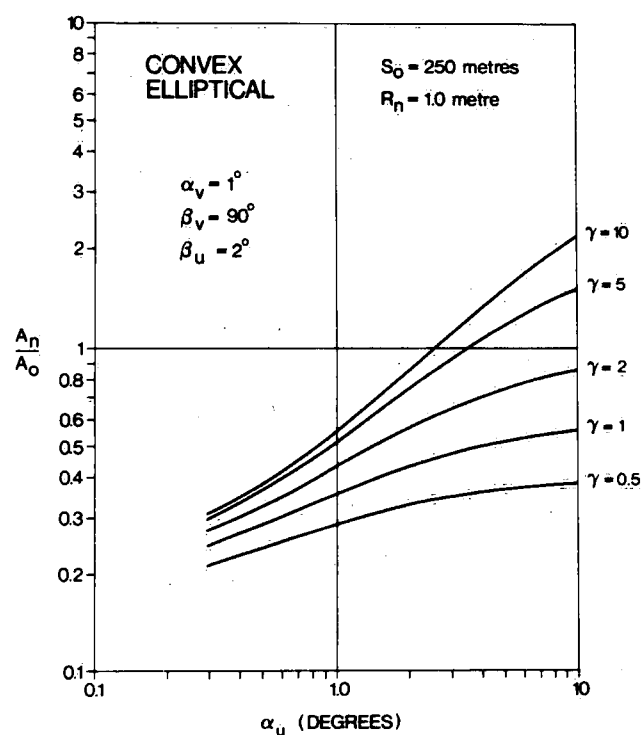


Figure 47. Relationships between A_n/A_o for convex elliptical shoreline marshes and principal axis offshore slope α_u for a variety of ellipticity factor values and $\beta_v = 90^\circ$ (i.e. partially non-regenerative marsh).

(coupled with the equally limited examples of linear and convex/concave shoreline marshes presented in earlier sections) will serve to illustrate adequately the nature and

impact of the geometric physical basin parameters on the vulnerability of shoreline marshes to prolonged changes in water levels.

Marsh Model Computer Program

Throughout this report have appeared a large, but certainly not exhaustive, number of figures displaying the impact on geometrically describable marshlands of persistent changes in ambient water levels. To be of maximum use to interested researchers and/or environmental managers, some provision should be made to allow application of this conceptual mathematical model to local marsh areas of specific interest and concern. Therefore a "user-friendly" interactive computer program, MARSHMODEL, was devised. It is written in IBM PC BASIC language (Appendix).

Up to now it has been explicitly assumed that all values of persistent ambient water levels R_N (reckoned from the zero water level datum) are positive but do not exceed an increase greater than the value of d pertinent to the marsh in question, i.e. $0 \leq R_N \leq d$. Although the cases of R_N lying outside this range were considered in detail for linear shoreline marshes, no such detail was considered for concave/convex or elliptical shoreline marshes. Indeed, most ambient water level conditions in North America, particularly within the past couple of decades (which have been largely characterized by persistent water levels above the zero water level datum), are satisfied by the R_N range $0 \leq R_N \leq d$. Further, the conceptual geometric marsh model derived in this report is adequately described without the need to complicate further its presentation by considering, in detail, the cases for $R_N < 0$ and $R_N > d$. Nevertheless, the situations for which R_N may become negative (i.e., below the strand line which is taken to define zero water level datum in this model) or substantive enough to inundate the emergent marsh vegetation (i.e. $R_N > d$) have been historically encountered and will again be encountered in some instances. Consequently MARSHMODEL has been written to include the possibility of R_N exceeding, in either direction, the limits $0 \leq R_N \leq d$. The governing equations for these situations, assuming that vegetative equilibrium may be ultimately established between marsh and onshore terrains, may be shown to be as follows:

For $R_N < 0$

- (a) Linear Shoreline

$$\frac{A_N}{A_0} = 1 \quad (17)$$

- (b) Concave Shoreline

$$\frac{A_N}{A_0} = 1 + \frac{2R_N \cot \alpha}{2S_0 - d \cot \alpha} \quad (18)$$

- (c) Convex Shoreline

$$\frac{A_N}{A_0} = 1 - \frac{2R_N \cot \alpha}{2S_0 + d \cot \alpha} \quad (19)$$

- (d) Concave Elliptical Shoreline (eq. (20) below)

$$\frac{A_N}{A_0} = 1 + \frac{2R_N \cot \alpha_U \cot \alpha_V}{S_{0U} \cot \alpha_V + S_{0V} \cot \alpha_U - d \cot \alpha_U \cot \alpha_V}$$

- (e) Convex Elliptical Shoreline (eq. (21) below)

$$\frac{A_N}{A_0} = 1 - \frac{2R_N \cot \alpha_U \cot \alpha_V}{S_{0U} \cot \alpha_V + S_{0V} \cot \alpha_U + d \cot \alpha_U \cot \alpha_V}$$

For $R_N > d$

- (a) Linear Shoreline

$$\frac{A_N}{A_0} = \frac{\cot \beta}{\cot \alpha} \quad (22)$$

- (b) Concave Shoreline

$$\frac{A_N}{A_0} = \left[\frac{2S_0 + (2R_N - d) \cot \beta}{2S_0 - d \cot \alpha} \right] \left[\frac{\cot \beta}{\cot \alpha} \right] \quad (23)$$

- (c) Convex Shoreline

$$\frac{A_N}{A_0} = \left[\frac{2S_0 - (2R_N - d) \cot \beta}{2S_0 + d \cot \alpha} \right] \left[\frac{\cot \beta}{\cot \alpha} \right] \quad (24)$$

- (d) Concave Elliptical Shoreline (eq. (25) below)

$$\frac{A_N}{A_0} = \frac{S_{0U} \cot \beta_V + S_{0V} \cot \beta_U + (2R_N - d) \cot \beta_U \cot \beta_V}{S_{0U} \cot \alpha_V + S_{0V} \cot \alpha_U - d \cot \alpha_U \cot \alpha_V}$$

(e) Convex Elliptical Shoreline (eq. (26) below)

$$\frac{A_n}{A_o} = \frac{S_{ou} \cot \beta_v + S_{ov} \cot \beta_u - (2R_n - d) \cot \beta_u \cot \beta_v}{S_{ou} \cot \alpha_v + S_{ov} \cot \alpha_u + d \cot \alpha_u \cot \alpha_v}$$

where all terms are as previously defined.

Note that equations (17) to (21) for $R_n < 0$ are independent of the onshore slope angle β . Equations (22) to

(26), however, are not independent of the onshore slope angle β . For the case of $R_n > d$, therefore, the governing equations for the situation in which no vegetative equilibrium may be established are given by substituting $\beta = 90^\circ$.

The governing equations for the situations $0 \leq R_n \leq d$ are as given in the text, and these equations are also incorporated within MARSHMODEL. The computer program automatically selects the appropriate methodology from the parameters directly supplied to it by the user.

Conclusion

Both short-term (i.e. seasonal) and long-term (persisting for periods of time significantly longer than seasonal) water level fluctuations are vitally important to the establishment and continuing and/or evolving status of shoreline wetland domains. Both natural and artificial activities which directly impact on such water levels must, therefore, be evaluated and considered in terms of their effect on the amount of wetlands that would survive or be transformed as a consequence of such water level changes. This report has considered the effects of prolonged water level fluctuations on shoreline marshes of the kinds found in the Great Lakes (or comparable fresh water) basins.

Despite the full realization that the marsh is a complex dynamic consequence of the interplay among a cluster of physical, chemical and biological parameters defining and dictating the behaviour of flora, fauna, and myriad air, water, and land interactions, the authors have restricted the focus of the report to the geometric parameters defining the basin terrain. A conceptual, simplified mathematical model has been presented, which attempts to relate persistent water level fluctuations to the areal extent of shoreline marshes. The fundamental treatise of this model is the acceptance that knowledge of terrain slope angles both offshore and onshore will enable a calculation of the amount of land subjected to inundation and/or water level recession. Such a mathematical calculation, however, does not enable a precise estimate of permanently destroyed or

totally/partially regenerated marshland. Rather, two opposite extremes of such marshland re-emergence subsequent to a persistent water level fluctuation are considered. These two cases are taken to represent maximum marshland re-emergence (assuming a vegetative community equilibrium may be established between shoreline marsh and meadow/swamp regimes) and minimum marshland re-emergence (assuming no such vegetative equilibrium may be established).

It is indeed intended that the conceptual mathematical marsh model presented and discussed herein may find direct application to the use of synoptic overviews (both aerial and satellite) of marsh regimes associated with various water levels. Such mathematical descriptions of the impact on areal extent of marshlands (of both classes of regeneration capabilities) brought about by persistent water level changes may be of consequence to water managers and planners, particularly when large-scale water diversion schemes are being considered.

The model presented in this report, along with the restrictions which must be adhered to when attempting to use its predictive and interpretive capabilities, is currently being evaluated in a consideration of historical airborne data acquired over shoreline marsh areas in the Georgian Bay/North Channel region. The results of this investigation should soon be available.

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Appendix
Computer Program Listing

Computer Program Listing

The "user-friendly" interactive computer program MARSHMODEL is designed to determine the areal extent of marshland that will result from a given change in persistent ambient water level. While the use of this computer model is facilitated by the liberal appearances of menus and user-prompts for inputs, a very brief description of the program here precedes its actual line-by-line inclusion.

MARSHMODEL presents as its output the resulting marshland areal extent (subsequent to a time lag which either results in total vegetative regeneration or in total vegetative non-regeneration) following an increase or decrease in persistent water level. This output is also presented as a percentage increase or decrease in areal extent from the initial area. To operate the program, it is anticipated that the following information could be supplied to the user prompts:

- (a) Marsh Geometry: To be selected from a choice of linear, concave, convex, concave elliptical, or convex elliptical.
- (b) Ellipse Shape Input Specification: Required if elliptical geometry is selected. The choice is provided of specifying S_{OU} and S_{OV} , the semi-axial lengths along the u- and v-axes of the elliptical marsh at zero water level datum, or alternatively, a format using S_{OU} and the ellipticity factor γ ($\gamma = S_{OV}/S_{OU}$) to provide consistency with the illustrations present in this report.
- (c) Slope Input Format: To be selected from a choice of angles (in degrees) or rise/run ratios (expressed as 1 in x). This latter ratio terminology would perhaps be more convenient when actual collected field data are used, and circumvents the necessity of determining angular values for the terrain slopes.
- (d) Emergent Vegetation Limit: This report has assumed,

for the most part, an emergent vegetation limit of 1.25 m. However, actual field measurements or intimate marsh knowledge may cause this figure to vary.

- (e) Offshore and Onshore Slopes or Slope Angles: For an elliptical geometry, both u-axis and v-axis parameters are required.
- (f) S_O or (S_{OU}, S_{OV}) or (S_{OU}, γ) : These parameters are required for nonlinear geometries.
- (g) Initial Water Level: Expressed as an offset from zero water level datum, either as positive or negative.
- (h) Subsequent Water Level: Again expressed as a positive or negative offset from zero water level datum.
- (i) Initial Marsh Area: To predict changes in marsh area resulting from persistent ambient water level changes occurring for conditions of maximum wetland regeneration, simply operate the program as guided by the "user-friendly" instructions. To predict changes in marsh area resulting from persistent ambient water level changes occurring for conditions of zero wetland regeneration, substitute onshore slope angles of 90° in place of the true onshore slope angles. (This entails a direct substitution of 90° if the "angles" slope input format is selected, or a substitution of 0 if the "1 in x ratio" slope input format is chosen.)

Warning messages are flashed to the user if parameter inconsistencies are encountered. An example would be the relative magnitudes of the terms $d \cot \alpha$ (the magnitude of the linear offshore extent of the marsh) and S_O (the distance from the centre of curvature of a curved shoreline marsh to the strand line). For concave geometries, the situation for which $d \cot \alpha > S_O$ defines an impossible geometric configuration, and the user is informed should the input data contain such incompatibilities.


```

100 '                                     M A R S H M O D E L
101 '
102 '
103 '   THIS PROGRAM IS DESIGNED TO ESTIMATE ON AN INDIVIDUAL MARSH BASIS,
104 '   THE IMPACT ON THE EXISTING SHORELINE MARSH ACREAGE RESULTING FROM A
105 '   LONG-TERM CHANGE IN THE AMBIENT BASIN WATER LEVEL.
106 '
107 '
108 '   FROM:
109 '
110 '   A MATHEMATICAL DESCRIPTION OF THE EFFECTS OF PROLONGED WATER LEVEL
111 '   FLUCTUATIONS ON THE AREAL EXTENT OF MARSHLANDS
112 '
113 '   BY:
114 '
115 '   R.P. BUKATA, J.E. BRUTON, J.H. JEROME, AND W.S. HARAS
116 '
117 '
118 OPTION BASE 0
119 DEFDBL A-F,L,O-Z
120 DEFINT G-K,M-N
121 DIM A(2),R(2),IR(2)
122 KEY OFF
123 BLANK$=SPACE$(77)
124 BORDER$=STRING$(77,205)
125 FMT$="#####.###"
126 LF$=CHR$(10)
127 DEGTORAD=ATN(1#)/45#
128 ON ERROR GOTO 3000
129 CLS
130 GOSUB 1000
131 PRINT "ENTER DESCRIPTIVE TITLE ";
132 GOSUB 5000
133 GOSUB 1000
134 LINE INPUT;TITLE$
135 CLS
136 PRINT TITLE$+LF$
137 PRINT "MARSH GEOMETRIES"+LF$
138 PRINT "LINEAR          1"+LF$
139 PRINT "CONCAVE         2"+LF$
140 PRINT "CONVEX          3"+LF$
141 PRINT "CONCAVE ELLIPTICAL 4"+LF$
142 PRINT "CONVEX ELLIPTICAL 5"
143 GOSUB 1000
144 INPUT;"SELECT MARSH GEOMETRY AND ENTER CORRESPONDING NUMBER ",GEOM
145 IF NOT (1<=GEOM AND GEOM<=5) THEN ERROR=255
146 IF GEOM<=3 THEN GOTO 155
147 CLS
148 PRINT TITLE$+LF$
149 PRINT "ELLIPSE SHAPE SPECIFICATION"+LF$
150 PRINT "SPECIFY Sou AND Sov      1"+LF$
151 PRINT "SPECIFY Sou AND GAMMA     2"
152 GOSUB 1000
153 INPUT;"SELECT ELLIPSE SHAPE SPECIFICATION ",IE

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154 IF IE<>1 AND IE<>2 THEN ERROR=255
155 CLS
156 PRINT TITLE$+LF$
157 PRINT "SLOPE FORMATS"+LF$
158 PRINT "SLOPE INPUT AS RATIO (1/X)          1"+LF$
159 PRINT "SLOPE INPUT AS ANGLE (DEGREES)      2"
160 GOSUB 1000
161 INPUT;"SELECT SLOPE INPUT FORMAT ",IS
162 IF IS<>1 AND IS<>2 THEN ERROR=255
163 CLS
164 PRINT TITLE$+LF$
165 ON GEOM GOTO 166,168,170,172,174
166 GEOM$="LINEAR"
167 GOTO 175
168 GEOM$="CONCAVE"
169 GOTO 175
170 GEOM$="CONVEX"
171 GOTO 175
172 GEOM$="CONCAVE ELLIPTICAL"
173 GOTO 175
174 GEOM$="CONVEX ELLIPTICAL"
175 PRINT "MARSH GEOMETRY IS "+GEOM$+LF$
176 PRINT "EMERGENT VEGETATION LIMIT d"+LF$
177 ON GEOM GOTO 178,178,178,186,186
178 ON IS GOTO 179,182
179 PRINT "OFFSHORE SLOPE ... ONE IN"+LF$
180 PRINT "ONSHORE SLOPE ... ONE IN"+LF$
181 GOTO 184
182 PRINT "OFFSHORE SLOPE ANGLE "+CHR$(224)+LF$
183 PRINT "ONSHORE SLOPE ANGLE "+CHR$(225)+LF$
184 IF GEOM<>1 THEN PRINT "RADIUS OF CURVATURE So"+LF$
185 GOTO 199
186 ON IS GOTO 187,192
187 PRINT "U AXIS OFFSHORE SLOPE "+CHR$(224)+"u ... ONE IN"
188 PRINT "U AXIS ONSHORE SLOPE "+CHR$(225)+"u ... ONE IN"+LF$
189 PRINT "V AXIS OFFSHORE SLOPE "+CHR$(224)+"v ... ONE IN"
190 PRINT "V AXIS ONSHORE SLOPE "+CHR$(225)+"v ... ONE IN"+LF$
191 GOTO 196
192 PRINT "U AXIS OFFSHORE SLOPE ANGLE "+CHR$(224)+"u"
193 PRINT "U AXIS ONSHORE SLOPE ANGLE "+CHR$(225)+"u"+LF$
194 PRINT "V AXIS OFFSHORE SLOPE ANGLE "+CHR$(224)+"v"
195 PRINT "V AXIS ONSHORE SLOPE ANGLE "+CHR$(225)+"v"+LF$
196 PRINT "U AXIS SEMI-AXIAL DISTANCE Sou"
197 IF IE=1 THEN PRINT "V AXIS SEMI-AXIAL DISTANCE Sov"+LF$
198 IF IE=2 THEN PRINT "ELLIPSE SHAPE SPECIFICATION GAMMA"+LF$
199 PRINT "INITIAL WATER LEVEL RELATIVE TO DATUM"
200 PRINT "SUBSEQUENT WATER LEVEL RELATIVE TO DATUM"+LF$
201 PRINT "INITIAL MARSH AREA"
202 PRINT "PREDICTED MARSH AREA"
203 PRINT "PER CENT CHANGE"
204 GOSUB 1000
205 INPUT;"ENTER EMERGENT VEGETATION LIMIT d ",D
206 IF D<0 THEN ERROR=255
207 LOCATE 5,55

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208 PRINT USING FMT$;D;
209 ON GEOM GOTO 210,210,210,251,251
210 GOSUB 1000
211 ON IS GOTO 212,218
212 INPUT;"ENTER OFFSHORE SLOPE ... ONE IN ",SLA1
213 IF SLA1<0 THEN ERROR=255
214 SLA=SLA1
215 LOCATE 7,55
216 PRINT USING FMT$;SLA1;
217 GOTO 226
218 PRINT "ENTER OFFSHORE SLOPE ANGLE "+CHR$(224);
219 INPUT;" ",SLA2
220 IF NOT (0<=SLA2 AND SLA2<=90) THEN ERROR=255
221 IF SLA2>89.999 THEN SLA2=89.999
222 IF SLA2<.001 THEN SLA2=.001
223 SLA=1/TAN(SLA2*DEGTORAD)
224 LOCATE 7,55
225 PRINT USING FMT$;SLA2;
226 GOSUB 1000
227 ON IS GOTO 228,234
228 INPUT;"ENTER ONSHORE SLOPE ... ONE IN ",SLB1
229 IF SLB1<0 THEN ERROR=255
230 SLB=SLB1
231 LOCATE 9,55
232 PRINT USING FMT$;SLB1;
233 GOTO 242
234 PRINT "ENTER ONSHORE SLOPE ANGLE "+CHR$(225);
235 INPUT;" ",SLB2
236 IF NOT (0<=SLB2 AND SLB2<=90) THEN ERROR=255
237 IF SLB2>89.999 THEN SLB2=89.999
238 IF SLB2<.001 THEN SLB2=.001
239 SLB=1/TAN(SLB2*DEGTORAD)
240 LOCATE 9,55
241 PRINT USING FMT$;SLB2;
242 K=9
243 IF GEOM=1 THEN GOTO 332
244 GOSUB 1000
245 INPUT;"ENTER RADIUS OF CURVATURE So ",S0U
246 IF S0U<0 THEN ERROR=255
247 LOCATE 11,55
248 K=11
249 PRINT USING FMT$;S0U;
250 GOTO 332
251 GOSUB 1000
252 ON IS GOTO 253,259
253 INPUT;"ENTER U AXIS OFFSHORE SLOPE ... ONE IN ",SLUA1
254 IF SLUA1<0 THEN ERROR=255
255 SLUA=SLUA1
256 LOCATE 7,55
257 PRINT USING FMT$;SLUA1;
258 GOTO 267
259 PRINT "ENTER U AXIS OFFSHORE SLOPE ANGLE "+CHR$(224)+"u";
260 INPUT;" ",SLUA2
261 IF NOT (0<=SLUA2 AND SLUA2<=90) THEN ERROR=255

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262 IF SLUA2>89.999 THEN SLUA2=89.999
263 IF SLUA2<.001 THEN SLUA2=.001
264 SLUA=1/TAN(SLUA2*DEGTORAD)
265 LOCATE 7,55
266 PRINT USING FMT$;SLUA2;
267 GOSUB 1000
268 ON IS GOTO 269,275
269 INPUT;"ENTER U AXIS ONSHORE SLOPE ... ONE IN ",SLUB1
270 IF SLUB1<0 THEN ERROR=255
271 SLUB=SLUB1
272 LOCATE 8,55
273 PRINT USING FMT$;SLUB1;
274 GOTO 283
275 PRINT "ENTER U AXIS ONSHORE SLOPE ANGLE "+CHR$(225)+"u";
276 INPUT;" ",SLUB2
277 IF NOT (0<=SLUB2 AND SLUB2<=90) THEN ERROR=255
278 IF SLUB2>89.999 THEN SLUB2=89.999
279 IF SLUB2<.001 THEN SLUB2=.001
280 SLUB=1/TAN(SLUB2*DEGTORAD)
281 LOCATE 8,55
282 PRINT USING FMT$;SLUB2;
283 K=8
284 GOSUB 1000
285 ON IS GOTO 286,292
286 INPUT;"ENTER V AXIS OFFSHORE SLOPE ... ONE IN ",SLVA1
287 IF SLVA1<0 THEN ERROR=255
288 SLVA=SLVA1
289 LOCATE 10,55
290 PRINT USING FMT$;SLVA1;
291 GOTO 300
292 PRINT "ENTER V AXIS OFFSHORE SLOPE ANGLE "+CHR$(224)+"v";
293 INPUT;" ",SLVA2
294 IF NOT (0<=SLVA2 AND SLVA2<=90) THEN ERROR=255
295 IF SLVA2>89.999 THEN SLVA2=89.999
296 IF SLVA2<.001 THEN SLVA2=.001
297 SLVA=1/TAN(SLVA2*DEGTORAD)
298 LOCATE 10,55
299 PRINT USING FMT$;SLVA2;
300 GOSUB 1000
301 ON IS GOTO 302,308
302 INPUT;"ENTER V AXIS ONSHORE SLOPE ... ONE IN ",SLVB1
303 IF SLVB1<0 THEN ERROR=255
304 SLVB=SLVB1
305 LOCATE 11,55
306 PRINT USING FMT$;SLVB1;
307 GOTO 316
308 PRINT "ENTER V AXIS ONSHORE SLOPE ANGLE "+CHR$(225)+"v";
309 INPUT;" ",SLVB2
310 IF NOT (0<=SLVB2 AND SLVB2<=90) THEN ERROR=255
311 IF SLVB2>89.999 THEN SLVB2=89.999
312 IF SLVB2<.001 THEN SLVB2=.001
313 SLVB=1/TAN(SLVB2*DEGTORAD)
314 LOCATE 11,55
315 PRINT USING FMT$;SLVB2;

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```

316 K=11
317 GOSUB 1000
318 INPUT;"ENTER U AXIS SEMI-AXIAL DISTANCE Sou ",SOU
319 IF SOU<0 THEN ERROR=255
320 LOCATE 13,55
321 PRINT USING FMT$;SOU;
322 GOSUB 1000
323 IF IE=1 THEN INPUT;"ENTER V AXIS SEMI-AXIAL DISTANCE Sov ",SOV
324 IF IE=2 THEN INPUT;"ENTER GAMMA ",GAMMA#
325 IF IE=2 THEN SOV=SOU*GAMMA#
326 IF SOV<0 THEN ERROR=255
327 LOCATE 14,55
328 K=14
329 IF IE=1 THEN PRINT USING FMT$;SOV;
330 IF IE=2 THEN PRINT USING FMT$;GAMMA#
331 R(0)=0
332 GOSUB 1000
333 INPUT;"ENTER INITIAL WATER LEVEL RELATIVE TO DATUM ",R(1)
334 IF R(1)<0 THEN IR(1)=1
335 IF 0<=R(1) AND R(1)<=D THEN IR(1)=2
336 IF R(1)>D THEN IR(1)=3
337 LOCATE K+2,55
338 PRINT USING FMT$;R(1);
339 GOSUB 1000
340 INPUT;"ENTER SUBSEQUENT WATER LEVEL RELATIVE TO DATUM ",R(2)
341 IF R(2)<0 THEN IR(2)=1
342 IF 0<=R(2) AND R(2)<=D THEN IR(2)=2
343 IF R(2)>D THEN IR(2)=3
344 LOCATE K+3,55
345 PRINT USING FMT$;R(2);
346 GOSUB 1000
347 INPUT;"ENTER INITIAL MARSH AREA ",A(1)
348 IF A(1)<0 THEN ERROR=255
349 ON ERROR GOTO 0
350 LOCATE K+5,55
351 PRINT USING FMT$;A(1);
352 LOCATE 23,1
353 PRINT SPACE$(79);
354 LOCATE 24,1
355 PRINT SPACE$(79);
356 LOCATE 25,1
357 PRINT SPACE$(79);
358 ON GEOM GOTO 359,359,359,362,362
359 COTALPHAU=SLA
360 COTBETAU=SLB
361 GOTO 366
362 COTALPHAU=SLUA
363 COTBETAU=SLUB
364 COTALPHAV=SLVA
365 COTBETAV=SLVB
366 N=1
367 GOSUB 7000
368 A(0)=A(1)/Q
369 N=2

```

```

370 GOSUB 7000
371 A(2)=A(0)*Q
372 LOCATE K+6,55
373 PRINT USING FMT$;A(2);
374 PCC=100#*(A(2)-A(1))/A(1)
375 LOCATE K+7,55
376 PRINT USING FMT$;PCC;
377 RHI=-1000
378 RLO=1000
379 FOR I=0 TO 2
380 IF R(I)<RLO THEN RLO=R(I)
381 IF R(I)>RHI THEN RHI=R(I)
382 NEXT I
383 IWARN=0
384 ON GEOM GOTO 401,385,387,389,392
385 IF (S0U+(RLO-D)*COTALPHAU)<0 THEN IWARN=IWARN+1
386 GOTO 394
387 IF (S0U-RHI*COTBETAU)<0 THEN IWARN=IWARN+1
388 GOTO 394
389 IF (S0U+(RLO-D)*COTALPHAU)<0 THEN IWARN=IWARN+1
390 IF (S0V+(RLO-D)*COTALPHAV)<0 THEN IWARN=IWARN+2
391 GOTO 394
392 IF (S0U-RHI*COTBETAU)<0 THEN IWARN=IWARN+1
393 IF (S0V-RHI*COTBETAV)<0 THEN IWARN=IWARN+2
394 IF IWARN=0 THEN GOTO 401
395 LOCATE 22,1
396 PRINT "WARNING...PARAMETER INCONSISTENCY DETECTED FOR ";
397 IF IWARN=1 THEN PRINT "U AXIS"
398 IF IWARN=2 THEN PRINT "V AXIS"
399 IF IWARN=3 THEN PRINT "BOTH AXES"
400 BEEP
401 END
402 '
403 '
1000 LOCATE 23,1
1001 PRINT CHR$(201)+BORDER$+CHR$(187);
1002 LOCATE 24,1
1003 PRINT CHR$(186)+BLANK$+CHR$(186);
1004 LOCATE 25,1
1005 PRINT CHR$(200)+BORDER$+CHR$(188);
1006 LOCATE 24,2
1007 RETURN
1008 '
1009 '
3000 GOSUB 1000
3001 BEEP
3002 PRINT "INPUT OUT OF RANGE ... PLEASE RETRY";
3003 GOSUB 5000
3004 BEEP
3005 IF ERL=145 THEN RESUME 143
3006 IF ERL=154 THEN RESUME 152
3007 IF ERL=162 THEN RESUME 160
3008 IF ERL=206 THEN RESUME 204
3009 IF ERL=213 THEN RESUME 210

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3010 IF ERL=220 THEN RESUME 210
3011 IF ERL=229 THEN RESUME 226
3012 IF ERL=236 THEN RESUME 226
3013 IF ERL=246 THEN RESUME 244
3014 IF ERL=254 THEN RESUME 251
3015 IF ERL=261 THEN RESUME 251
3016 IF ERL=270 THEN RESUME 267
3017 IF ERL=277 THEN RESUME 267
3018 IF ERL=287 THEN RESUME 284
3019 IF ERL=294 THEN RESUME 284
3020 IF ERL=303 THEN RESUME 300
3021 IF ERL=310 THEN RESUME 300
3022 IF ERL=319 THEN RESUME 317
3023 IF ERL=326 THEN RESUME 322
3024 IF ERL=348 THEN RESUME 346
3025 STOP
3026 '
3027 '
5000 T=TIMER
5001 WHILE (TIMER-T)<1
5002 WEND
5003 RETURN
5004 '
5005 '
7000 ON IR(N) GOTO 7001,7017,7039
7001 LINEAR=1
7002 ON GEOM GOTO 7003,7005,7007,7009,7013
7003 Q=LINEAR
7004 RETURN
7005  $Q=1+(2*R(N)*COTALPHAU)/(2*S0U-D*COTALPHAU)$ 
7006 RETURN
7007  $Q=1-(2*R(N)*COTALPHAU)/(2*S0U+D*COTALPHAU)$ 
7008 RETURN
7009  $QN=2*R(N)*COTALPHAU*COTALPHAV$ 
7010  $QD=S0U*COTALPHAV+S0V*COTALPHAU-D*COTALPHAU*COTALPHAV$ 
7011  $Q=1+QN/QD$ 
7012 RETURN
7013  $QN=2*R(N)*COTALPHAU*COTALPHAV$ 
7014  $QD=S0U*COTALPHAV+S0V*COTALPHAU+D*COTALPHAU*COTALPHAV$ 
7015  $Q=1-QN/QD$ 
7016 RETURN
7017  $LINEAR=1-R(N)/D+(R(N)/D)*(COTBETAU/COTALPHAU)$ 
7018 FACTOR=1
7019 IF GEOM=1 THEN GOTO 7037
7020  $B0U=D*COTALPHAU$ 
7021  $BNU=(D-R(N))*COTALPHAU+R(N)*COTBETAU$ 
7022  $YNU=R(N)*COTBETAU$ 
7023 IF GEOM=2 THEN  $FACTOR=1+(R(N)/(2*S0U-B0U))*(COTALPHAU+COTBETAU)$ 
7024 IF GEOM=3 THEN  $FACTOR=1-(R(N)/(2*S0U+B0U))*(COTALPHAU+COTBETAU)$ 
7025 IF GEOM=2 OR GEOM=3 THEN GOTO 7037
7026  $B0V=D*COTALPHAV$ 
7027  $BNV=(D-R(N))*COTALPHAV+R(N)*COTBETAV$ 
7028  $YNV=R(N)*COTBETAV$ 
7029 IF GEOM=5 THEN GOTO 7034

```

```

7030 FACTORN=-BNV+S0V+YNV+(S0U+YNU)*(BNV/BNU)
7031 FACTORD=-B0V+S0V+S0U*(B0V/B0U)
7032 FACTOR=FACTORN/FACTORD
7033 GOTO 7037
7034 FACTORN=+BNV+S0V-YNV+(S0U-YNU)*(BNV/BNU)
7035 FACTORD=+B0V+S0V+S0U*(B0V/B0U)
7036 FACTOR=FACTORN/FACTORD
7037 Q=FACTOR*LINEAR
7038 RETURN
7039 LINEAR=COTBETAU/COTALPHAU
7040 ON GEOM GOTO 7041,7043,7045,7047,7051
7041 Q=LINEAR
7042 RETURN
7043 Q=(2*S0U+(2*R(N)-D)*COTBETAU)/(2*S0U-D*COTALPHAU)*LINEAR
7044 RETURN
7045 Q=(2*S0U-(2*R(N)-D)*COTBETAU)/(2*S0U+D*COTALPHAU)*LINEAR
7046 RETURN
7047 QN=S0U*COTBETAV+S0V*COTBETAU+(2*R(N)-D)*COTBETAU*COTBETAV
7048 QD=S0U*COTALPHAV+S0V*COTALPHAU-D*COTALPHAU*COTALPHAV
7049 Q=QN/QD
7050 RETURN
7051 QN=S0U*COTBETAV+S0V*COTBETAU-(2*R(N)-D)*COTBETAU*COTBETAV
7052 QD=S0U*COTALPHAV+S0V*COTALPHAU+D*COTALPHAU*COTALPHAV
7053 Q=QN/QD
7054 RETURN
7055 '
7056 '
7057 'END

```


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