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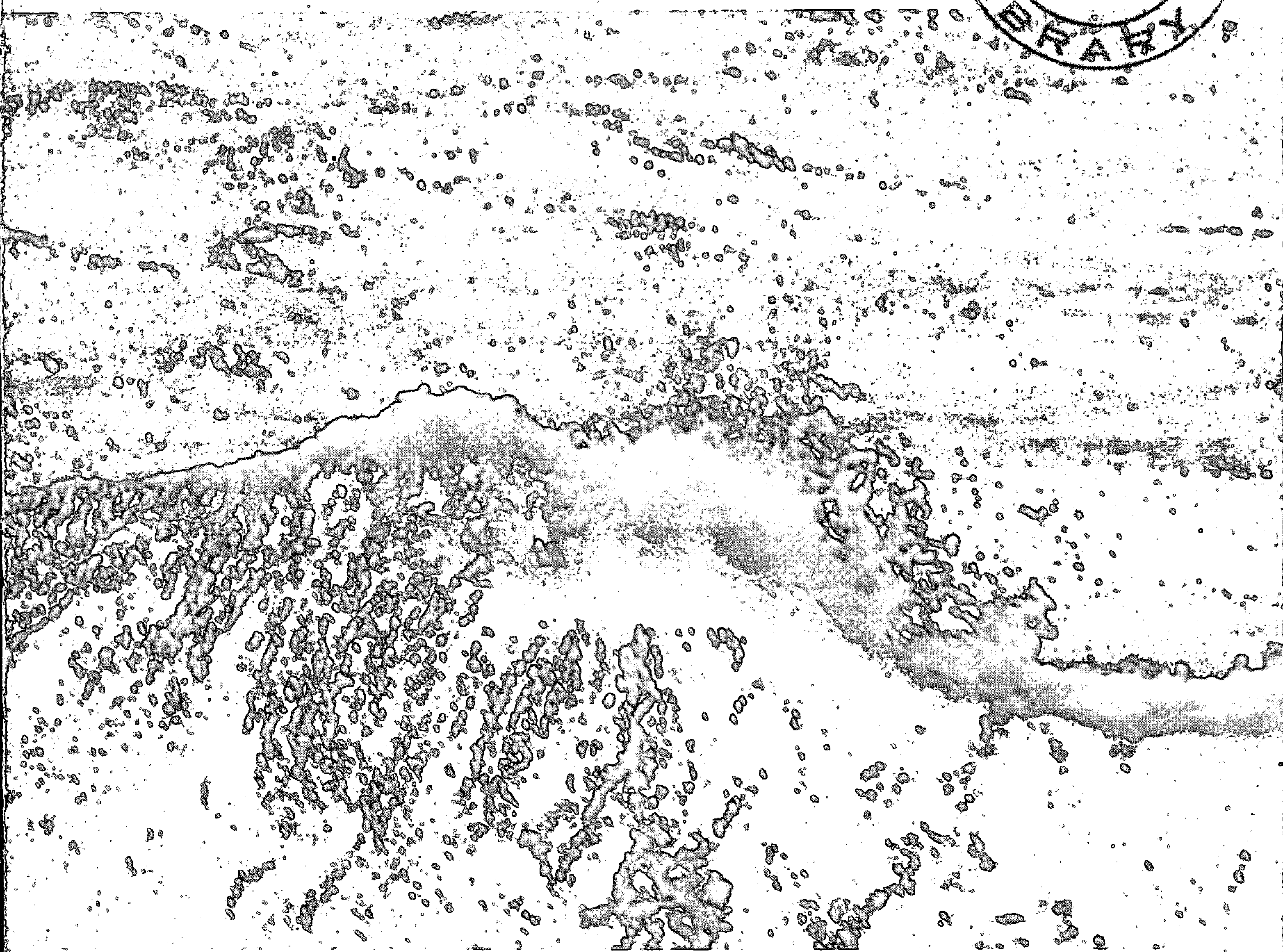
Thermal Wave Trains of Finite Amplitude in Lake Ontario

F. M. Boyce and K. P. B. Thomson



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(Résumé en français)

INLAND WATERS DIRECTORATE,
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Abstract

Temperature measurements made with a towed array of thermistors, during the period of July 7 to 14, 1970, near Oshawa, Ontario reveal thermal structures which are interpreted as trains of internal waves moving shoreward. Banded structures of regular wavelength near the shorelines have been observed from high altitude photographs of the surface of Lake Ontario taken during 1970. The wavelengths of the structures measured with the towed thermistor array are comparable to those observed in the aerial photographs. It is concluded that the structures seen in the photographs are associated with internal wave trains. No satisfactory mechanism has yet been found for the generation of these waves but the possibility of detecting them from aircraft will be useful to their further study.

Résumé

Durant la période s'étalant du 7 au 14 juillet 1970, des mesures de température ont été effectuées à l'aide d'un dispositif de thermistances attachées à un bateau. L'étude a été entreprise près d'Oshawa, Ontario. Elle a révélé des structures thermiques qui ont été interprétés comme étant des trains d'ondes internes se déplaçant en direction de la rive. Des structures en bandes, ayant des longueurs d'ondes régulières ont été décelées sur des photographies aériennes de la surface du lac, prises en 1970, à haute altitude dans les zones proches de la ligne des côtes. Les longueurs d'ondes des structures mesurées avec le dispositif de thermistances ont montré des similitudes avec celles observées sur les photographies aériennes. Il en a été conclu que les structures observées sur les photographies étaient associées aux trains d'ondes internes. Aucun mécanisme satisfaisant n'a été jusqu'à présent trouvé et qui serait à l'origine de pareilles ondes. Cependant, la possibilité de détecter ces ondes à partir des photographies aériennes permettra de mieux les étudier dans l'avenir.

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OBSERVATION OF INTERNAL WAVES OF LAKE ONTARIO FROM HIGH ALTITUDE PHOTOGRAPHS

On July 6, 1970, two high altitude aircraft performed photographic missions over western Lake Ontario. One of these was a Canadian Forces CF-100 aircraft which was carrying out a photographic experiment for the Physical Limnology Section at CCIW. The other, a NASA RB-57 aircraft, was undertaking the first in a series of flights over Lake Ontario for a NASA satellite simulation experiment.

On examination, the photographs obtained from these experiments showed a number of wavelike features along the north shore of Lake Ontario. These features appear as dark streaks, (Figure 1) of between 2-10 m wide in regular arrays parallel to the shoreline. The streaks are more easily observed in the sun glint areas of the photograph. As the illumination falls off towards the edge of the photograph the features become more difficult to observe against the dark background. It is presumed that these features are lines of convergence produced by internal waves in the lake.

Since no ground truth, other than meteorological observations at shore stations on Lake Ontario, was available on the day of the overflights only a qualitative description of the photographs can be given at this time.

Description of the Surface Features

Though the waves were easily visible on both the aerial color, and color infrared films, a color infrared film with a 60% overlap, taken with a 6-inch lens, was found to be the most suitable for the observation of the waves. A mosaic prepared from this film is shown in Figure 2. The area shown extends from Toronto to just east of Oshawa.

Individual groups of waves could not be connected on a continuous east-west basis due to the illumination fall off at the edges of the photographic image. However, there was sufficient coverage to show the east-west continuity of the features. The waves can be divided into two main groups on the basis of their wavelengths. In figure 2 the series A through E have wavelengths between 1000 and 1700 m. A second group indicated by the letters X, Y and Z have shorter wavelengths. The waves denoted by X and Y are closer inshore than the A to E series and have wavelengths of between 700 and 300 m. The group of waves marked Z which occur in deeper water has a wavelength of about 500 m.

The main orientation of all these waves is basically east-west and parallel to the shore. There are, however, some other wave patterns orientated at various angles to this main structure.

Meteorological Conditions

Meteorological data from stations at Toronto Island and Oshawa show that the western basin of the lake was under a southwesterly flow on July 4, two days prior to the flight. On July 5, the southwesterly flow was replaced by a strong northwesterly circulation with winds in the order of 5 - 8 mps throughout the day at both stations. On the day of the flight the region was under the influence of a well developed high pressure system, with the prevailing winds at both stations having a southwesterly direction.

DETECTION OF LARGE AMPLITUDE PROGRESSIVE INTERNAL WAVES IN THE NEARSHORE ZONE OF LAKE ONTARIO FROM TOWED THERMISTOR ARRAY MEASUREMENTS

Data

From July 8, 1970, to July 17, 1970, temperature data was collected in the nearshore zone off Oshawa, Ontario. During the experiment the ship CSS Limnos steamed repeatedly around a 10 km square (see Figure 2) towing a thermistor array and an undulating body. The average speed of the ship was 6 knots; each leg of the path was travelled at approximately four-hour intervals. Analysis of the voluminous digital records has not been completed but some analogue records made of the temperature at 9 m depth in the main thermocline reveal interesting wave train structures which appear to correlate with the photographic observations made from high altitude aircraft. A typical wave train is selected for analysis.

Figure 3 is a facsimile of the temperature variations at 9 m depth on two legs ab and cd (perpendicular to the

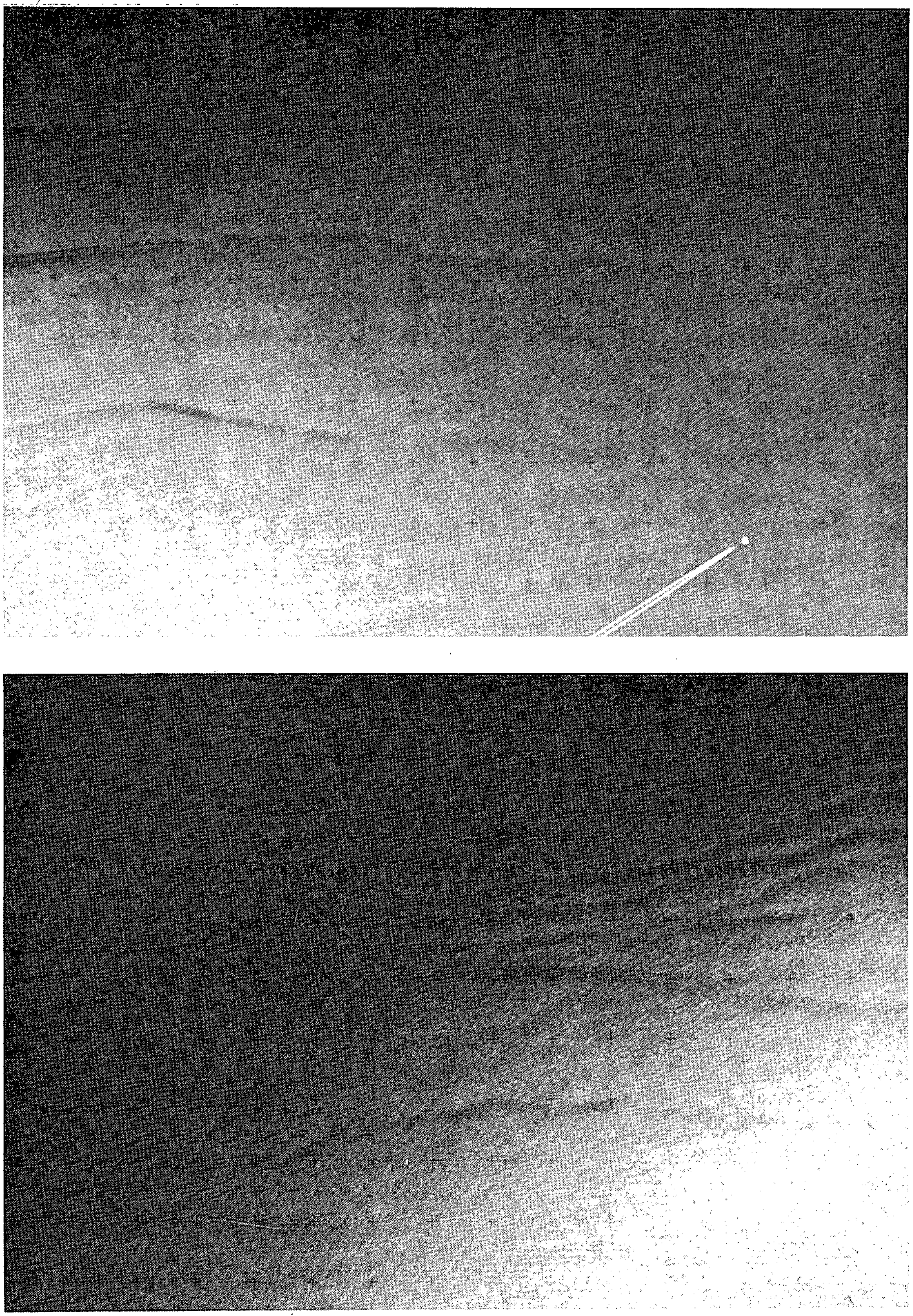

Figure 1. Surface indications of internal waves from NASA high-altitude photographs, taken on July 6, 1970, at an altitude of 20 km; (a) the distance between the surface features is in the order of 1200 m; (b) the distance between the surface features is in the order of 500 m.

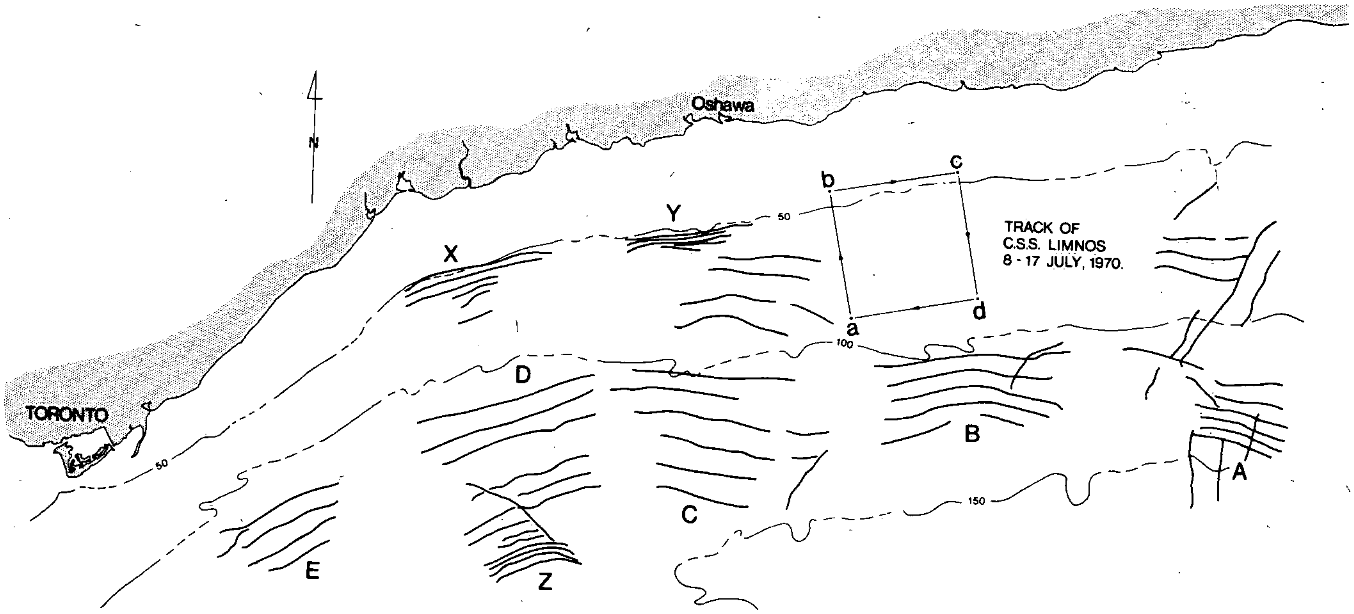

Figure 2. Mosaic produced from NASA high-altitude photographs, showing wave patterns, along the north shore of Lake Ontario on July 6, 1970. The broken lines are the depth contours in metres.

shoreline) made on July 14, 1970. We wish to focus attention on the large amplitude wavelike excursions of the temperature from "baseline" level. The peaks are numbered in order starting with the nearshore peak and labelled with the time at which they were encountered and distance from the nearshore point.

Figure 4 gives the temperature profile near the point b. The corresponding density profile approximates a two layer fluid with a relative density difference of 0.00094. The upper layer is about 10 m thick.

The minimum Brunt-Vaisala period may be estimated from the formula

$$t = 2\pi \left(g\,\alpha \frac{\partial T}{\partial Z} \right)^{-1/2}$$

where g is the acceleration due to gravity, α is the coefficient of thermal expansion of water, and $\partial T/\partial Z$ is the vertical gradient of temperature T, with depth Z, Z being positive in the upward direction. The formula is evaluated at the depth of maximum $\partial T/\partial Z$ and yields a value of 113 seconds at 10m for the conditions depicted in Figure 4.

Interpretation

Assuming that the density profile is a good approximation to a two layered fluid, as far as long internal waves are concerned, we estimate the phase speed of the disturbances to be about 0.3 ms^{-1}. The formula used is

$$C = \left(g \left(\frac{\Delta\rho}{\rho_0}\right) \frac{H_2 H_1}{H_1 + H_2} \right)^{-1/2}$$

where $(\Delta\rho/\rho_0)$ is the non-dimensional density difference between the two layers and the H_2 and H_1 are the thicknesses of the upper and lower layers respectively.

If the train of waves observed on the line ab is interpreted as a train of progressive internal waves with crests parallel to the shoreline, we can estimate the wavelength of the disturbance to be about 1000 m. From the temperature gradient at the 9m level, we calculate the vertical movement (neglecting Doppler shift effects which are about 10% in this case) associated with the first peak to be about 2 m (amplitude 1 m). Now if the crests of this wave train are assumed to be long (10 km) the two peaks (1) and (2) observed on the line cd (Figure 3b) can be correlated with peaks 3 and 4 of the line ab.

The rest of the wave train in Figure 3b is assumed to lie inshore of point c, or to have been modified beyond recognition by the effects of the shoaling bottom (the latter is more probable since the wave length is considerably shorter on line c-d). If then peak (4) along ab and (2) along cd are assumed to be the same wave, the average velocity component perpendicular to the shoreline works out to be 0.28 ms^{-1}.

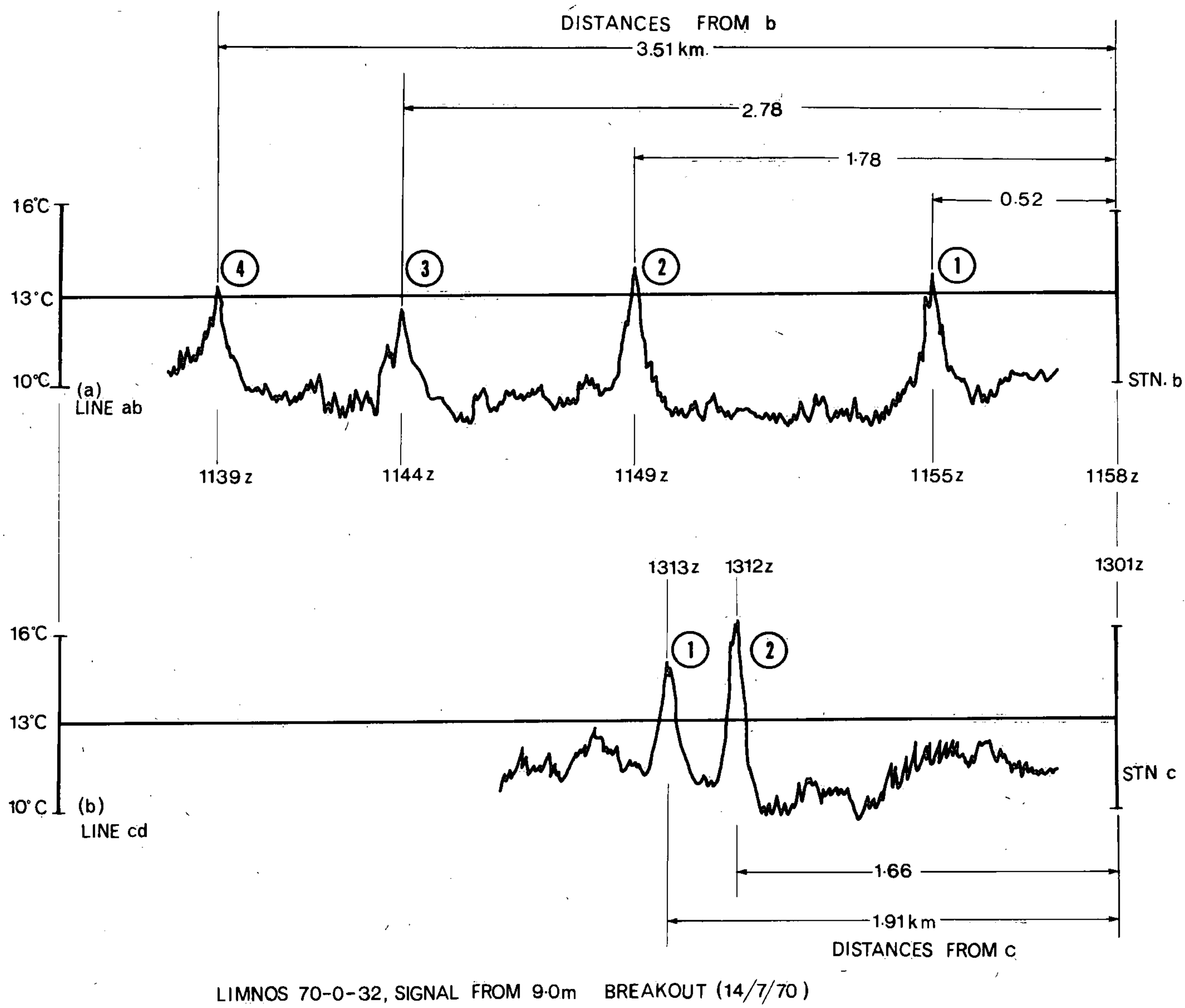

Figure 3. Example of a train of internal waves detected by a thermistor towed at 9.0 m depth. Wave crests are labelled with the times at which they were encountered and their estimated distances from the end points of the run.

The wavelength and crest length of the train as determined from these assumptions are compatible with the high-altitude observations. We assume, in the absence of simultaneous measurements, that the air-photos and the temperature measurements are different observations of the same phenomenon.

With a phase speed of about 0.3 ms^{-1}, a wave would take about 9 h to cross a distance of 10 km. On the crossing of ab, 4 h previous to the 1105-1158 crossing (Figure 3a) no wave train structures were observed. This can be interpreted to mean either:

(i) the wave train was created in the interval between the crossings

or

(ii) the wave train arrived obliquely to the line ab from the southwest (trace velocity on the line ab = 0.3 ms^{-1}) and was subsequently refracted until the crests lay parallel to the shore (bottom contours).

We note further that the orbital velocities of water particles associated with progressive internal wave such as we have described (Phillips 1966) have a maximum value of 3 cms^{-1} at the surface. The temperature gradients in the epilimnion are very small and it is most probable that the distributions of other properties are equally uniform in this shallow layer. It seems most likely that the surface patterns visible in the photographs result from the effects of the

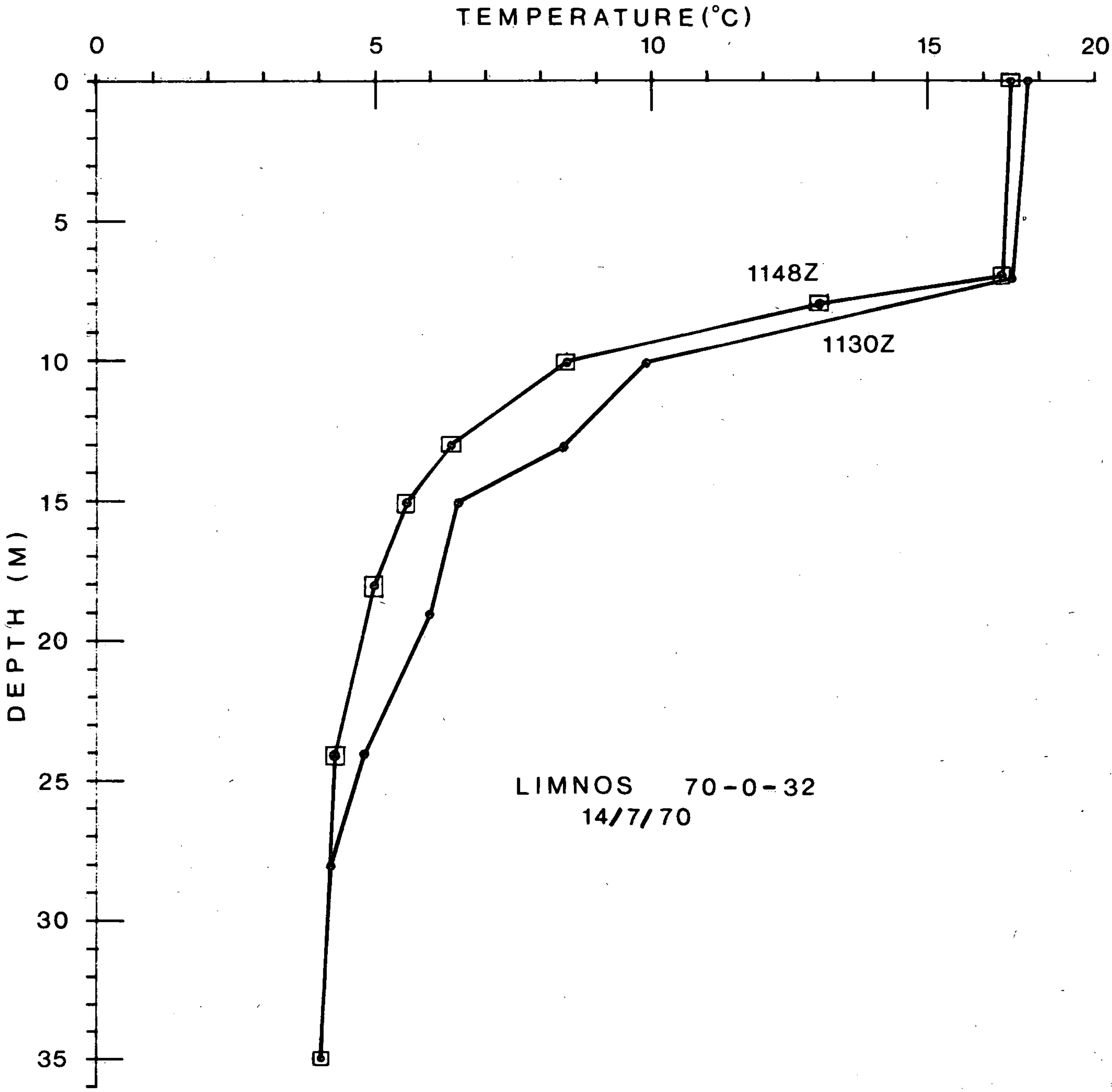

Figure 4. Vertical temperature profiles obtained near Station b with an undulating towed body. Maximum Brunt-Vaisala frequency is 0.0031 rad s^{-1} at 7 m.

internal wave orbital velocities on the surface ripples.

The narrowness of the bands can be associated with the asymmetrical profile of internal waves occurring when the thickness of upper layer is much less than the wave length (Thorpe, 1965). The troughs of the wave are much narrower than the crests. The wave trains of Figure 3 exhibit this property; a rise in temperature at a fixed level is equivalent to a depression of the isothermal surfaces.

It must be mentioned that mechanisms other than internal waves can produce banded patterns visible on the water surface. One such process currently receiving attention is the Langmuir Vortex (Faller, 1971). These appear as foam lines or surface streaks orientated predominantly along the direction of the wind. The distance separating individual streaks seems to be related to and of the same order of magnitude as the depth of the first stable layer or, in the absence of stratification, to the bottom depth of the water. An upper limit to the "wave length" of Langmuir circulations under the observed stratification would not exceed several tens of metres. Thus the Langmuir circulation mechanism does not appear to explain the photographic observations we are now discussing.

DISCUSSION AND CONCLUSIONS

It is interesting to speculate on the generating mechanisms responsible for these well-marked waves. The mechanism of resonant interaction of two surface waves to produce an internal wave (Ball, 1964) does not appear to be a likely source because of the relatively short wavelength and fetch of surface waves in the lake and the long interaction time required to produce significant internal wave amplitudes. The transfer of energy to internal waves from moving squalls, essentially another resonance mechanism, is effective only when the disturbance moves at the phase speed of the internal waves (less than 0.3 ms^{-1}) (Polyanskaya, 1969). Most squalls advance at speeds of 5 ms^{-1} or more.

The thermocline is known to move vertically in direct response to changes in wind stress (upwelling and downwelling) and to oscillate vertically at or near the inertial period (17.5 h) in an internal seiche mode (Mortimer, 1971). When extended to the shoreline, these motions cause the intersection of the thermocline with the bottom to move over considerable distances. The mixing caused by advance and retreat can give rise to short wavelength internal waves (Munk and Wimbush, 1969; Jin Wu, 1969) but these should be characterized by wavelengths of about 100 m and periods in the order of minutes. Moreover, since the zone of contact of the thermocline with the bottom is inshore from point b, such disturbances should appear to be propagating offshore when observed on line ab. There is some indication in the towed array records that this is the case.

The interaction of a long internal wave with a variable bottom topography can also lead to the production of wave trains when the nonlinear terms of the equations of motion become important. Examination of Figure 2 shows two distinct groups of waves; X and Y form an inshore group, C and D form the offshore group. The distance separating the leading edges of these groups is about 8 miles. If the groups emanate from the same source, the time taken for an internal wave to cover the 8 miles is close to 17.5 h, the inertial period. This occurrence may be entirely fortuitous and must be investigated much more completely.

Similar wave-like structures were observed on the onshore, offshore legs on July 9, July 10, and July 17, with one latter group of waves being particularly well marked. When one examines the wind records for the month of July, 1970 one finds that the groups tend to occur when the winds are either SW or NE along the shoreline. The winds preceding the appearance of the groups by several hours have strong but short-lived northerly components.

Again, the relationship between the winds and the wave trains is extremely tentative. We point out, nevertheless, that the periods of the longer waves are about one hour, according to estimates based on observed lengths and calculated phase speeds. The transverse seiching period of Lake Ontario in the fundamental mode is about one hour and this motion would be excited most readily by bursts of strong winds with a component transverse to the axis of the lake. The barotropic seiching motion, through interaction with the sloping bottom could produce the observed waves (Rattray, 1960).

We hope that future experiments will better determine the nature and the source of these motions. We believe that the photographic observations could easily be made from lower altitudes. On the basis of this experiment we expect that remote sensing support with photographs taken from light aircraft will provide a useful input to the internal wave program at CCIW.

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