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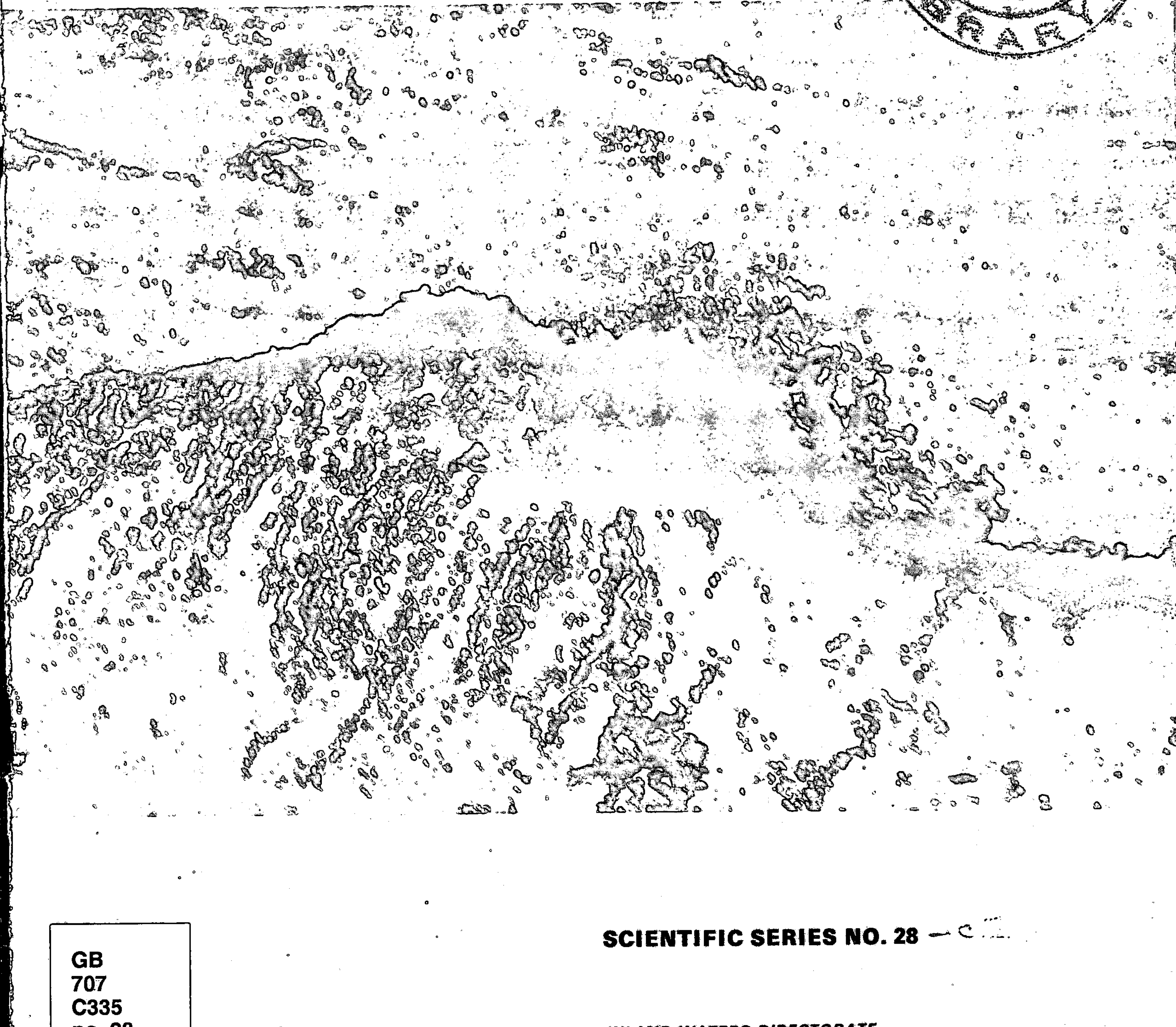
The Physical Effects of Waste Heat Input to the Great Lakes

Bernard C. Kenney



SCIENTIFIC SERIES NO. 28

INLAND WATERS DIRECTORATE,
CANADA CENTRE FOR INLAND WATERS,
BURLINGTON, ONTARIO, 1973.

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Acknowledgments

The author gratefully acknowledges the cooperation of Ontario Hydro in collecting on-site supporting data for this study and for permission to use their unpublished thermal data at Lakeview; also appreciated is the assistance of W. McColl in making the temperature measurements and J. Bond and D. Jordan for abstracting and analyzing the data.

Summary

Surface temperatures were measured with an airborne radiometer in the thermal effluents of two electric generating stations. These measurements are presented directly as isotherm plots and as results of several statistical analyses. It was seen that the reduction of temperature was dominated by mixing of the effluent with ambient lakewater, direct transfer to the atmosphere averaging seven percent. There was no evidence of accelerated shoreline mixing due to turbulence generated by breaking waves.

The measurements were grouped with similar data from other sources to estimate the physical effects of the thermal effluent load predicted for the year 2000. Of the Canadian shoreline of the Great Lakes, the north shore of Lake Ontario will be most heavily exposed to waste heat in the year 2000. It was estimated that on an average day, 25 miles (or approximately 12%) of the north shore of Lake Ontario will be exposed to temperatures greater than one degree centigrade above ambient. The bimodal characteristic of near shore lake currents ensures that twice this amount of shoreline is regularly exposed to such temperatures.

A recommendation is made for comprehensive study of the dynamics of fluctuating currents responsible for diffusion in the near shore zone. Some understanding of these currents is necessary to explain large day to day variation in thermal (and other) effluents.

It is recommended that the thermal effluents of new generating stations with installed capacities larger than 3000 Mw be measured as they become available. Further physical studies on such effluents should be deferred until an ecological assessment of possible detrimental effects of waste heat is completed.

CHAPTER 1

Introduction

The Laurentian Great Lakes represent a major portion of the freshwater reserves for all of North America. Since they are relatively cold lakes they are ideally suited for industrial cooling purposes. This fact is reflected in the results of a recent study on the magnitude and spatial distribution of industrial waste-heat input to the Great Lakes (Acres, H.G. Co. Ltd., 1970).

It was estimated that 10^{11} Btu/hr was presently rejected to the Great Lakes as low grade waste heat and that this would increase elevenfold by the year 2000. Furthermore, this waste heat load is concentrated in certain segments of shoreline rather than being uniformly distributed along the lakeshore. Naturally, the largest concentrations occur in and around the large population centres.

The largest single source of waste heat is the electric power generating industry. The lake water is used for once-through cooling of the steam condensers to improve the thermodynamic efficiency of power generation. The water is pumped from the lake through large heat exchangers and discharged back into the lake, most often by an open channel outfall. The "excess" temperature acquired by the lake water in this process is then reduced by both direct heat transfer to the atmosphere and mixing of the warm water with ambient lake water. Mixing reduces the "excess" temperature in the plume by distributing the heat over a larger mass of lake water. Ultimately, all the waste heat is removed from the lake by transfer to atmosphere. In addition, the "excess" temperature usually makes the outfall plume buoyant relative to the ambient lake water. Positive buoyancy tends to make the outfall plume overrun the more dense lakewater. An indication of the magnitude of this buoyant spread may be seen in Figure 1 (Hayashi, 1967); the larger the density difference, the larger the measureable surface plume.

Negatively buoyant plumes tend to sink until the density difference between the plume and the ambient lake is zero; or to the lake bottom if the ambient lake is isothermal. There may be no measureable surface plume.

In order to assess adequately the total impact of waste heat disposal on the ecology of the lakes, as well as on other present and future uses of lake water, detailed information is first required on the physical effects of waste heat addition. The magnitude and extend of measureable "excess" temperatures in both the horizontal and vertical directions must be known. In addition to absolute values of temperature, temporal and spatial temperature gradients are also important. However, when this study commenced there were little temperature data available for conditions similar to those on the Great Lakes on which an assessment of the physical effects of waste heat could be based.

In the present report, a series of surface temperature measurements made at two electric power generating stations on the Great Lakes are described. The electrical output capacity of the stations chosen differ by an order of magnitude. Using a simple regression analysis, these data are combined with all additional data which have recently been published (either on the Great Lakes or other relatively large lakes) to estimate some of the lake-wide physical effects of the waste heat load projected for the year 2000.

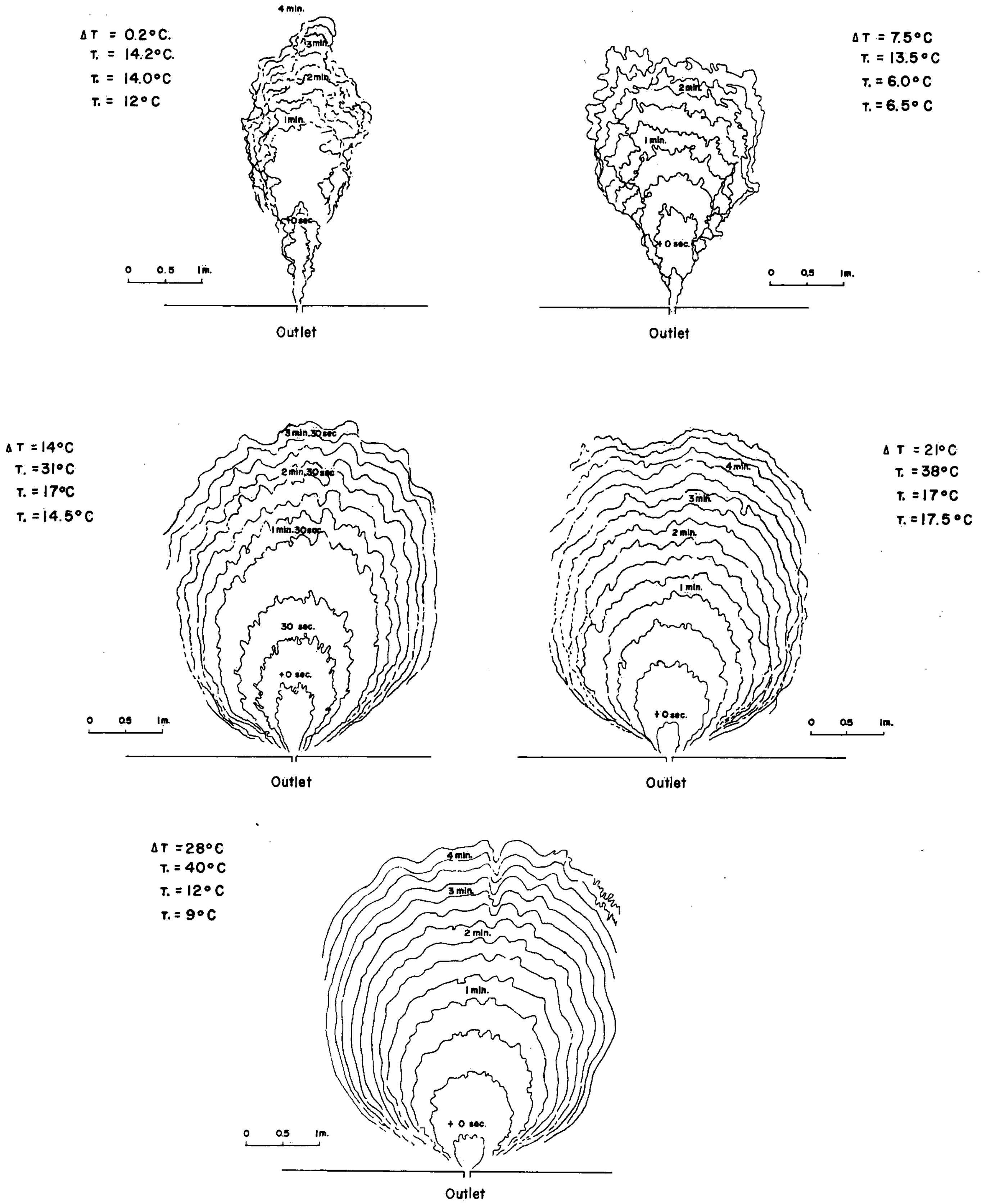


Figure 1. Diffusion of a buoyant jet (after Hayashi).

CHAPTER 2

Thermal Effluents at Two Electric Power Generating Stations

GENERAL

Surface temperature measurements were made at two electric power generating stations, Lakeview and Douglas Point. Both stations are operated by the Hydro-Electric Power Commission of Ontario, hereafter referred to as Ontario Hydro. Lakeview is one of the largest operating fossil-fuel plants with a maximum electric energy output of 2,400 Mw. It is situated on the northwest shore of Lake Ontario near Toronto. The Douglas Point nuclear plant is midway along the east shore of Lake Huron. It has an output capacity of 200 Mw.

The surveys covered the fall and winter seasons from September 1970 to April 1971. A detailed description of the airborne infrared sampling technique is given in Appendix A. Since the temperature survey required only one man and minimum equipment set-up time, the decision to conduct a survey on a particular day was made two hours prior to the start of the actual temperature measurements. Each survey took approximately 30 min to complete. This flexibility enabled the thermal plumes to be sampled over a wide range of environmental conditions. Since the measurements were not adversely affected by moderate to high wind speeds, the surveys were not biased to the low wind speeds as are most measurements made from boats. Supporting data on the plant's operating conditions at the time of each survey, as well as on-site meteorological data, were collected by Ontario Hydro, on request, for each sampling day.

The Lakeview discharge was surveyed on eight occasions, the one at Douglas Point five times.

Shortly after the start of the temperature surveys, Ontario Hydro made available a comprehensive series of temperature measurements made at the Lakeview site. These unpublished data consisted of 40 sets of isotherm plots from measurements at one and five foot depths. The data were collected from April 1969 to September 1970. These data will not be described here in detail; however, they were used with the permission of Ontario Hydro to extend the confidence of statistical analyses presented in this report. These data have since been published in summary form by Ontario Hydro (Bryce et al., 1971).

DESCRIPTION OF THERMAL EFFLUENTS

The results of temperature measurements made during this study are included in Appendix A as surface isotherm maps. Surface areas were determined by planimeter from these maps and these areas were used to calculate the averages and standard deviations presented in Table 1. The averages given in Table 1 for the Lakeview generating station are dominated by data provided by Ontario Hydro.

Table 1. Statistics of thermal plumes

	N	Mean	Standard Deviation
Area within 1°C "excess" isotherm		ft^2	ft^2
Lakeview			
All plumes	37	19.3×10^6	8.76×10^6
Westerly plumes	19	22.6×10^6	7.99×10^6
Easterly plumes	17	15.3×10^6	8.37×10^6
Onshore wind	13	21.1×10^6	7.44×10^6
Offshore wind	15	16.2×10^6	7.66×10^6
Douglas Point			
All plumes	5	2.18×10^6	3.04×10^5
Lakeview Data			
Area within 2°C	29	13.65×10^6	6.03×10^6
Area within 3°C	29	8.73×10^6	4.57×10^6
Area within 4°C	29	5.35×10^6	4.18×10^6
Area within 5°C	29	3.50×10^6	3.02×10^6
Source Strength		Btu/s	Btu/s
Lakeview	29	1.62×10^6	3.03×10^5
Douglas Point	5	3.86×10^5	5.23×10^3
1°C "excess" isotherm at 1 ft depth	29	1.37	0.361
1°C "excess isotherm at 5 ft depth			
Heat transfer out of immediate plume		%	%
Lakeview	34	7.4	3.5
Douglas Point	5	8.4	2.4

A dominant characteristic of effluents discharged into large lakes from open channel outfalls is their very rapid turn parallel to the local shoreline (Csanady et al., 1967;

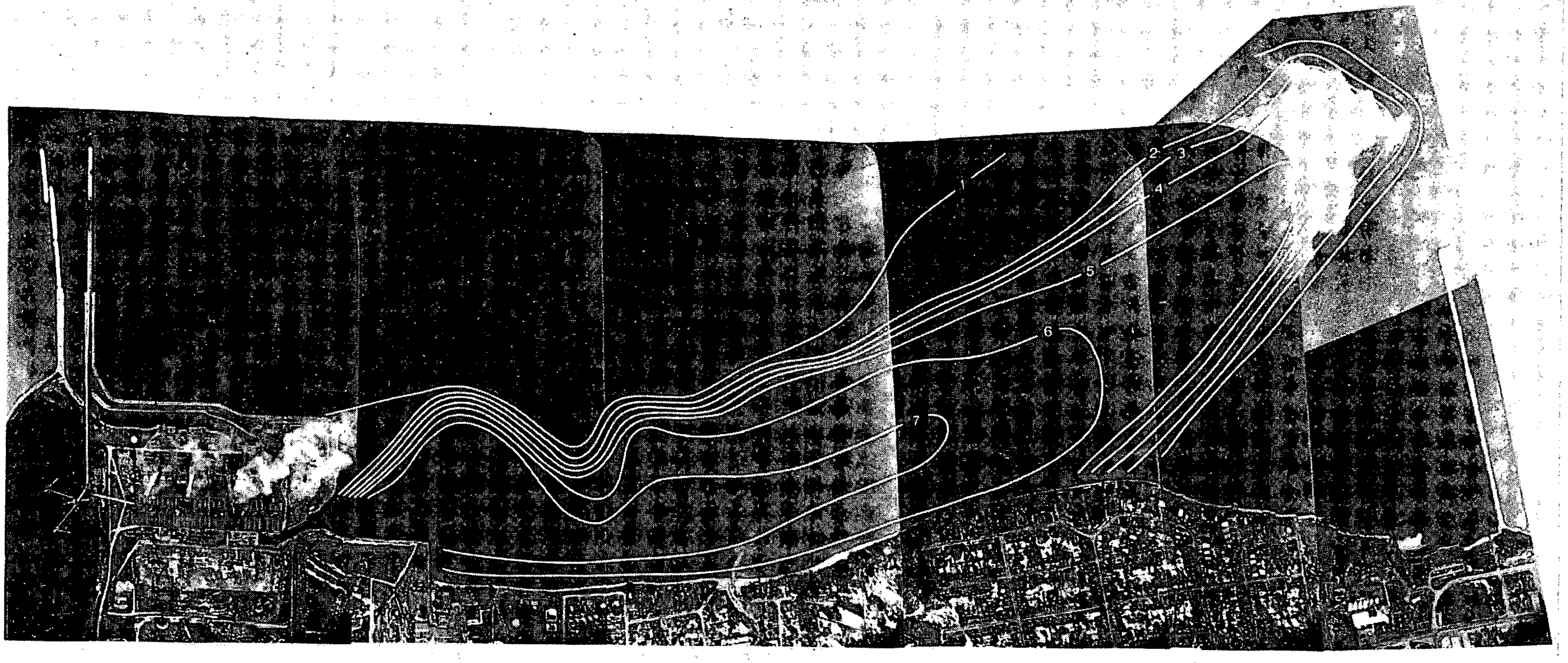


Figure 2. Lakeview generating station, Lake Ontario. Temperature contours, one degree centigrade interval. March 2nd, 1971, 12:00 EST, flight altitude – 2700 ft.

Hamblin and Rodgers, 1967; Jones, 1968). Furthermore, for all the surveys conducted during this study, the alongshore direction of the effluent was the same as the component of the wind direction alongshore. This was generally true of the Lakeview data provided by Ontario Hydro although there were exceptions.

The thermal plume at Lakeview is influenced to some degree by local restrictions near the outfall. Long piers projecting out into the lake east of the outfall cause a large curvature in the centreline of the eastward moving plumes. These piers may also have the effect of reducing the average surface area of the eastward plumes as may be seen in Table 1. However, since the data provided by Ontario Hydro was collected only as far east as the piers, the exact influence of this restriction on surface area is unknown.

The day-to-day variation in waste-heat output (source strength) was less than a factor of two. It may be seen from the isotherm plots that the daily variation in size of the effluent plumes was much larger. A quantitative estimate of this variation is given in Table 1 by the large value of variance when compared with the mean area.

The data collected on one day when a strong turbidity plume was observed at Lakeview are shown in Figure 2 with the surface isotherms overplotted. The boundary of the turbidity plume and the smallest "excess" isotherm agree so well that for most of the figure the turbidity contours are masked by the plot of the isotherm. On the several days when a turbidity plume was present, it was always a good indicator of the maximum extent of "excess" temperature.

Data were collected at the Douglas Point site during the winter season when solid ice cover extended from shore to approximately three miles offshore. The ice ridge visible in Figures 3, 4 and 5 was estimated from the survey aircraft to be approximately 50 ft high and probably extended to the

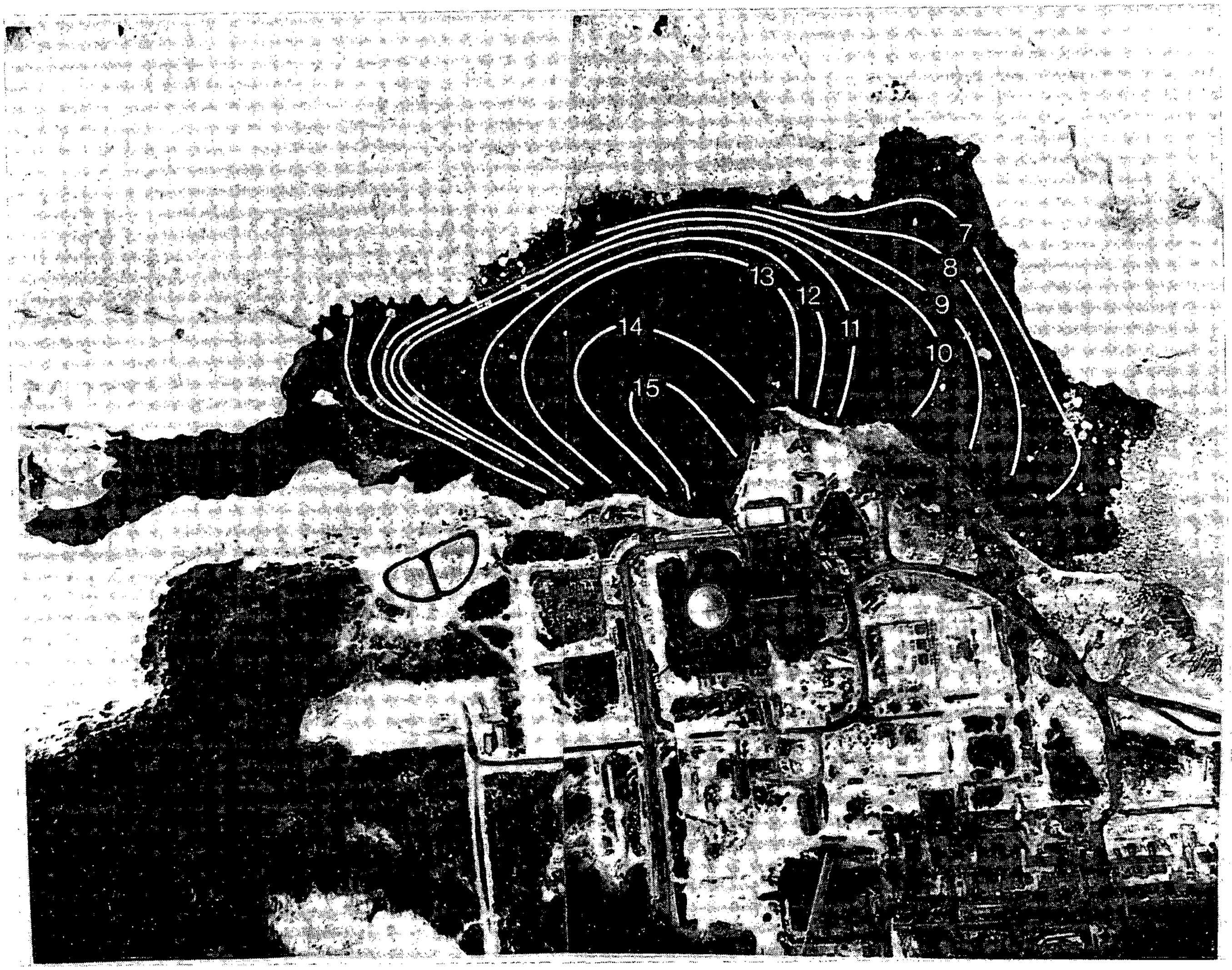
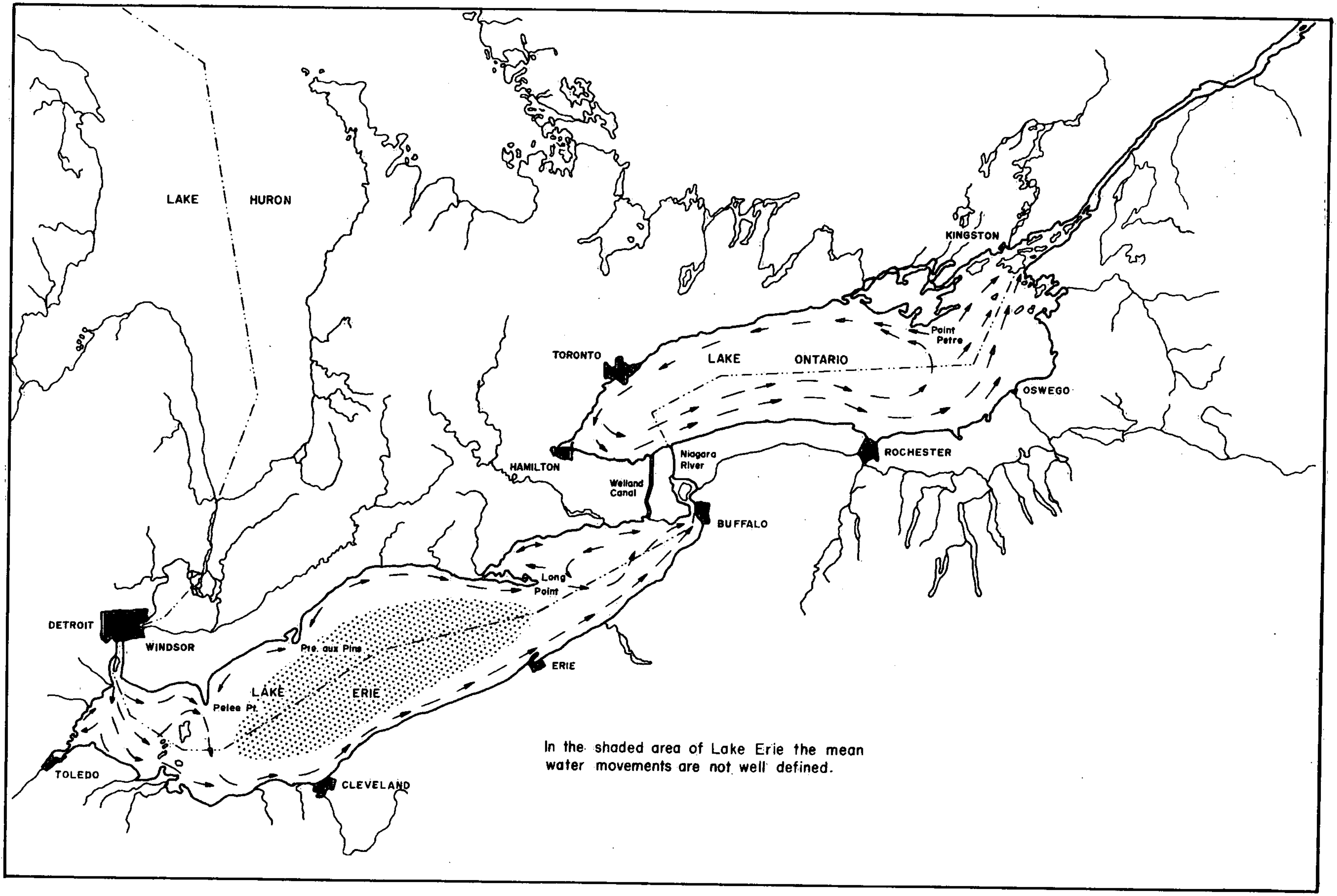


Figure 3. Douglas Point generating station, Lake Huron. Temperature contours, one degree centigrade interval. March 1st, 1971, 14:00 EST, flight altitude – 1500 ft.

Figure 4. Douglas Point generating station, Lake Huron. Temperature contours, one degree centigrade interval. March 2nd, 1971, 14:30 EST, flight altitude – 1500 ft.

Figure 5. Douglas Point generating station, Lake Huron. Temperature contours, one degree centigrade interval. March 9th, 1971, 14:00 EST, flight altitude – 1500 ft.



In the shaded area of Lake Erie the mean water movements are not well defined.

Figure 6. Mean water movement in Lake Erie and Lake Ontario.

lake bottom. The entire open-water area near the outfall contained water with measureable excess temperature. The effluent plume on March 9, 1971 (Figure 5) appeared to coincide with the alongshore component of the local wind direction (NW) although it is unlikely that any alongshore currents were present because of the ice ridge. In these three cases it is likely that the plant was recirculating some of the warm water back through its intake.

ANALYSIS AND RESULTS

Statistical

The pattern of mean water movements in Lake Erie and Lake Ontario is reproduced in Figure 6 from the 1969 report to the International Joint Commission (Weiss, 1970). The patterns shown in Figure 6 differ little in resolution or in the details of the circulation from those presented by Harrington in 1894 for these lakes. In the remaining lakes the situation is less clear. Even though lakewide circulation patterns are known only vaguely, the existence of a high degree of correlation between water current direction near shore and the direction of the local wind velocity is well established (Hamblin and Rodgers, 1967; Jones, 1968). As previously mentioned, this fact is illustrated in the isotherm maps by an alongshore plume usually in the same direction as the alongshore component of the wind. The direction alongshore changes about every 12 hrs on the average (for the north shore of Lake Ontario), but establishes a "mean"

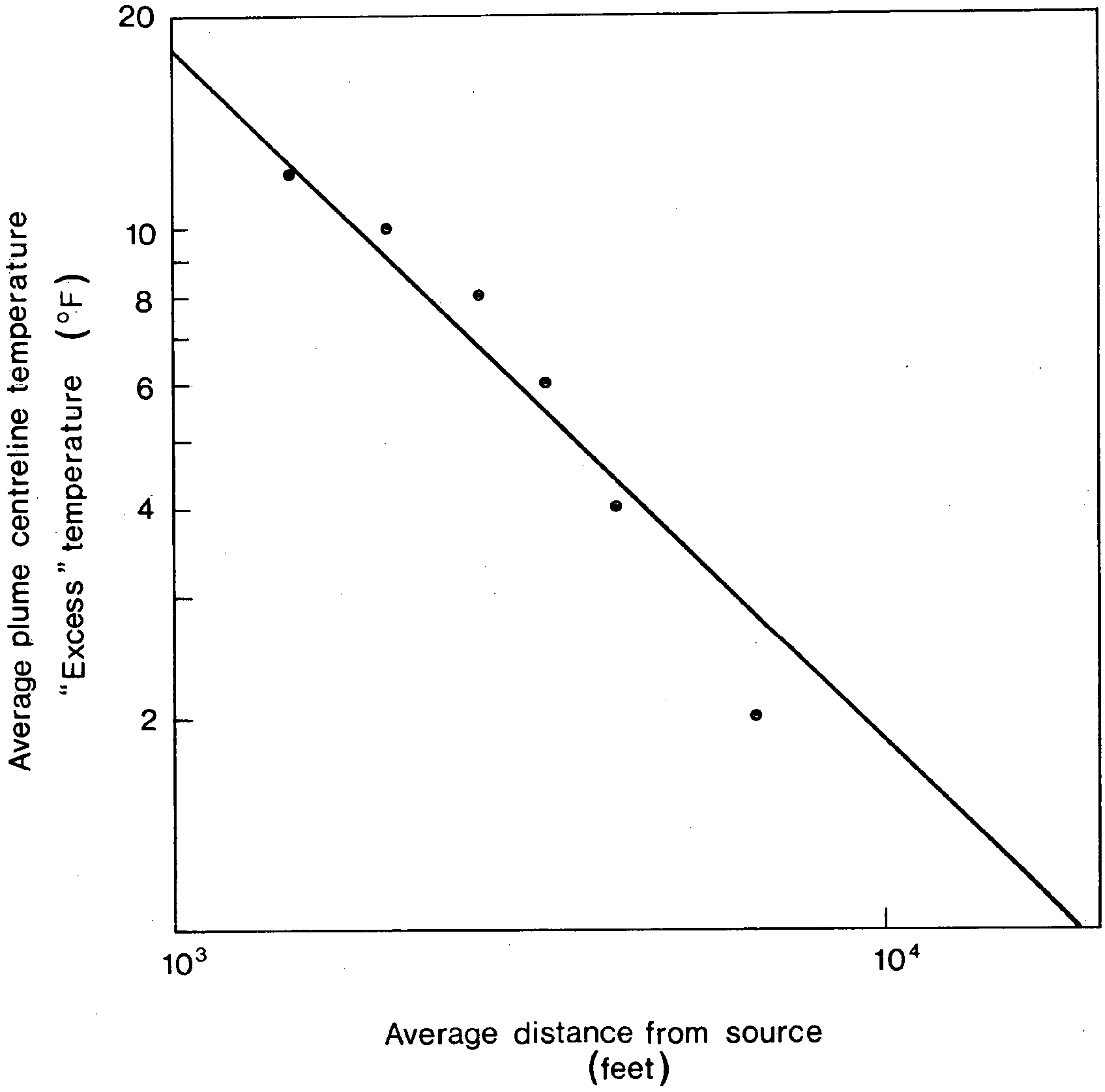


Figure 7. Decay of plume centreline temperature.

direction for the thermal plume for times much shorter than this (Kenney, unpublished manuscript). If the instantaneous velocity vector as measured continuously at a point were resolved into a (short-term) "mean" velocity and a fluctuating or turbulent velocity, then it is the direction of this "mean" velocity which is well correlated with the local wind direction. Although very little is known about the generation of turbulent velocities in the main body of a lake, the generation of turbulence by breaking waves near shore is an obvious mechanism. The thermal plume data were divided into two groups, depending upon the component of the local wind normal to shore at the time the measurements were made. There was, however, no significant difference between the average areas within the 1°C "excess" isotherm for the two groups, even though in many of the surveys taken during high winds blowing toward the shore, there was a profusion of breaking waves. This results in marked contrast to dye diffusion experiments previously conducted at the Douglas Point generating station (Csanady et al., 1967). During periods of high onshore winds, the dye plumes were completely contained within the surf zone. This discrepancy may be caused by the positive buoyancy of the thermal effluent.

The linear regression of source strength (rate of heat discharged into the lake, Btu/s) upon the area contained within the 1°C "excess" isotherm was calculated for all of the Lakeview data. The correlation coefficient indicated there was no significant correlation between source strength and plume size over the range of variation of source strength at Lakeview (a factor of 2). This was anticipated, however, because of the large day-to-day variation in plume size.

The average decrease in "excess" temperature along the centreline of the Lakeview plumes was calculated as a function of distance from the outfall. The plumes were grouped into two classes; those moving alongshore in the easterly direction and those moving westerly. The average temperature decay with increasing distance from the outfall is shown in Figure 7 for the easterly plumes. Also shown in Figure 7 is a line with a minus one slope. This result compares closely to the decay of concentration with distance as determined from large-scale dye diffusion experiments at Douglas Point (Csanady et al., 1967). The westerly flowing plumes exhibited an anomalous behaviour, the decay curve being roughly linear.

Direct Heat Transfer to Atmosphere

The direct heat transfer from the measureable thermal plume to the atmosphere was estimated for the Lakeview data by the method shown in Appendix B. The results of these calculations show that, on the average, only 7% of the waste heat is rejected within the measurable plume. The maximum direct heat transfer was 15%.

An average of the plumes at Douglas Point indicates that 8% of the waste heat is transferred directly to the atmosphere during the winter months. This compares with 33% calculated recently using an energy balance method from data collected at Douglas Point during the summer months (Csanady and Crawford, 1971).

DISCUSSION OF PHYSICAL EFFECTS OF WASTE HEAT

The local physical effects of two existing thermal effluents were described in the previous sections in terms of statistics, such as the mean area contained within the 1°C isotherm. Spatial temperature gradients may be estimated from the isotherm plots and compared with naturally occurring temperature gradients. Since time series data were not collected, information is lacking on temporal temperature gradients but it is likely that the largest temporal gradients are associated with changes in source strength due to varying plant loads. As more data become available, the confidence level of such statistics may be improved. It would be possible to compile accurate statistical data on all existing thermal effluents not only for surface temperature but in three dimensions. If the cost could be justified, this would completely define the local physical effects of the existing waste heat inputs.

CHAPTER 3

Physical Effect of Waste Heat in Year 2000

GENERAL

It is not possible, at present, to "predict" the excess temperature at a point in a lake near an existing generating station for some future date. Even with the advantage of hindsight, "prediction" of actual temperatures which have occurred in the past has not been achieved. The reason for this is clear. Each instantaneous plume is an integrated result of complex fluctuating currents, the dynamics of which are not understood. Even in terms of some gross characteristic (such as surface area) these plumes vary by an order of magnitude from day to day. "Prediction" of characteristics of future effluents depends on these characteristics being somehow similar to those of existing plumes. However, a quick examination of isotherm maps shows that exact similarity does not exist between the thermal structure on two different days at the same location. It is highly unlikely, therefore, that two generating stations widely separated in space and on different lakes would have effluent plumes similar in detail. However, if one is to avoid making measurements in each and every effluent plume, it is imperative to find some similarity among plumes which can then be used to scale existing measurements to new or unmeasured effluents.

Economics ensure considerable similarity among electric power generating stations. The temperature rise of the cooling water in the condensors is of the order of 10°C; the water is usually discharged through an open-channel outfall. Although the actual geometry of each outfall varies somewhat, similarity in the "excess" temperature at the outfall and the associated buoyant spread would tend to equalize differences in depth of the outfall channel.

Based on limited available data, near shore currents are bimodal in all of the Great Lakes, flowing to and fro parallel to the local shoreline with a typical (or median) value of 20 cm/s.

The calculations presented in this section are based on the hypothesis that two identical generating stations with open-channel outfalls on large lakes produce effluents with the same average area contained with the one degree centigrade "excess" isotherm. A second hypothesis required to assess future physical effects is that this average area may be scaled by the source strength of the effluents.

DESCRIPTION OF DATA

All available data from measurements of thermal plumes either on the Great Lakes or where conditions are similar to the Great Lakes were used to estimate the total area exposed to excess temperatures greater than 1°C for the north shore of Lake Ontario in the year 2000. The data were obtained (either from the literature or private communication) in the form of areal plots of surface isotherms in the vicinity of various generating stations. The sources of data are listed under References.

All these generating stations have, as a common mode of discharge, open-channel outfalls with excess temperature at the source in the order of 8°C above ambient lake-surface temperature. The thermal plumes were all buoyant and had a relatively low exit momentum. However, the quantity and quality of the data varied considerably among the references. Since the estimates of future physical effects was based on interpolation and extrapolation from these existing data, some factors which affect measurements of thermal plumes are detailed below. In cases where specific data were not presented by one or more of the references, the assumptions required to complete the analysis are stated.

(1) Vertical gradient of temperature at the cooling water intake

Vertical temperature gradients across the intake have the effect of reducing the average intake temperature relative to the ambient surface temperature. Hence for a given temperature rise in the condensers, the outfall temperature will also be lower relative to ambient. In the extreme case of the intake mounted below the summer thermocline, there could be no measurable thermal plume even though the generating station is operating at peak capacity. For the purpose of studying buoyant thermal plumes, this effect may be taken into account by calculating the actual thermal source strength.

(2) Measurement of ambient temperature

Since thermal plumes are generally defined by "excess" temperature above ambient, an accurate estimate of the ambient is important. However, marked horizontal gradients near shore often make an accurate measurement impossible. Moreover, variations in the ambient gradients may frequently be interpreted as variations in the thermal plume. Such variation in plume size would be large in the case of vertical gradients at the intake discussed above. Internal waves in a strongly stratified ambient temperature field near the cooling water intake would tend to make the size of the thermal plume periodic even though the generating station output remained constant.

For the purpose of this analysis, the ambient temperature quoted by each reference was taken at face value. For the data collected during this study, the ambient surface temperature was defined as the average of temperature outside the "measurable" plume.

(3) Accuracy of temperature measurements

The area between successive (equal temperature interval) isotherms increases at the smaller values of "excess" temperature. The smaller the values of "excess" temperature used to define the plume size, the more accurate the temperature measuring instrument must be. It must be noted, however, that the limiting factor is not the absolute accuracy of the thermometer but the reduction of the "excess" temperature in the plume to a value comparable with natural ambient fluctuations in temperature.

Equally important to the accuracy of thermal plume measurements is the accuracy of positioning at the instant the measurement is made. Although not usually a problem with boat surveys, the accuracy of positioning has been poor in most aerial infrared surveys.

(4) Survey time

If the time required for measurements is long compared with the time scales of lake currents which most affect thermal effluents, the plume may change substantially during the temperature survey. It must be noted that thermal plumes can reverse direction in the three hours required for many boat surveys.

(5) Near source mixing

Spread of the plume near the outfall is influenced by mixing caused by the momentum of the outfall jet and by buoyant spread due to the temperature difference of the warmed effluent. Since no data were available to evaluate these effects, it was assumed they were similar for all the open-channel outfalls.

ANALYSIS

The surface areas within the 1°C "excess" temperature isotherm were calculated from the isotherm maps presented in the various data references. The source strength, Q, of the thermal effluent was calculated from the data presented as

$$Q = \dot{m} C_p \Delta t$$

where $\dot{m}$ is the mass flow

C_p is the specific heat at constant pressure

Δt is the temperature rise

When sufficient data were not presented, the assumption was made that the generating station was operating at its rated capacity and the source strength calculated as by Acres Co. Ltd. (1970a). The average values of the data used in the analysis are given in Table 2. Data were only used if the effluent at a given location had been sampled more than once.

Table 2. Average thermal plume data

Generating Station	N	Area with 1°C "excess" isotherm (ft^2)	Source strength (Btu/s)
Lakeview	37	1.93×10^7	1.62×10^6
Douglas Point			
winter	5	2.2×10^6	3.86×10^5
summer	2	5.35×10^6	3.57×10^5
Waukegan	19	9.8×10^6	9.09×10^5
J.H. Campbell	2	4.6×10^6	7.04×10^5
Milliken	5	1.85×10^6	4.4×10^5
Big Rock Pt.	2	1.77×10^6	9.05×10^4
Allen S. King	3	5.57×10^6	5.16×10^5

A log-log plot of plume surface area versus source strength of the generating station is shown in Figure 8. The least squares fit to a power curve was calculated and is shown in Figure 8. The arrows around one point in Figure 8 indicate plus and minus one standard deviation from the mean value for the Lakeview data. For a Gaussian distribution, 67% of the data would be between these limits. The correlation coefficient which is indicative of the "goodness" of fit of the data to the power curve was calculated to be 0.81.

RESULTS

The equation of the power curve was used to estimate the average surface plume areas for the Canadian shoreline of the Great Lakes in the year 2000. It was assumed that all generating stations would operate at rated capacity and that the efficiency of electric power generation would not substantially improve over the next thirty years. The source strengths used in the calculations were taken from the estimate for 2000 A.D. given by Acres Co. Ltd. (1970a).

Although most generating stations planned for 2000 A.D. are not yet sited, it is assumed that the separation between stations would be large enough that the shoreline affected by the thermal effluents would not overlap. The generating stations along the Detroit and St. Clair Rivers were not considered. The total (average) surface area exposed to "excess" temperature greater than 1°C is given in Table 3 for each of the Great Lakes.

In order to provide some estimate of the distribution of area, the thermal effluent was assumed to form a rectangle parallel to shore and downstream from the outfall. The maximum distance of the 1°C "excess" isotherm was determined for each of the Lakeview plumes. The average distance was taken as the length of the rectangle alongshore. An average length to width ratio of 4.5 was then calculated from the average surface area of the Lakeview

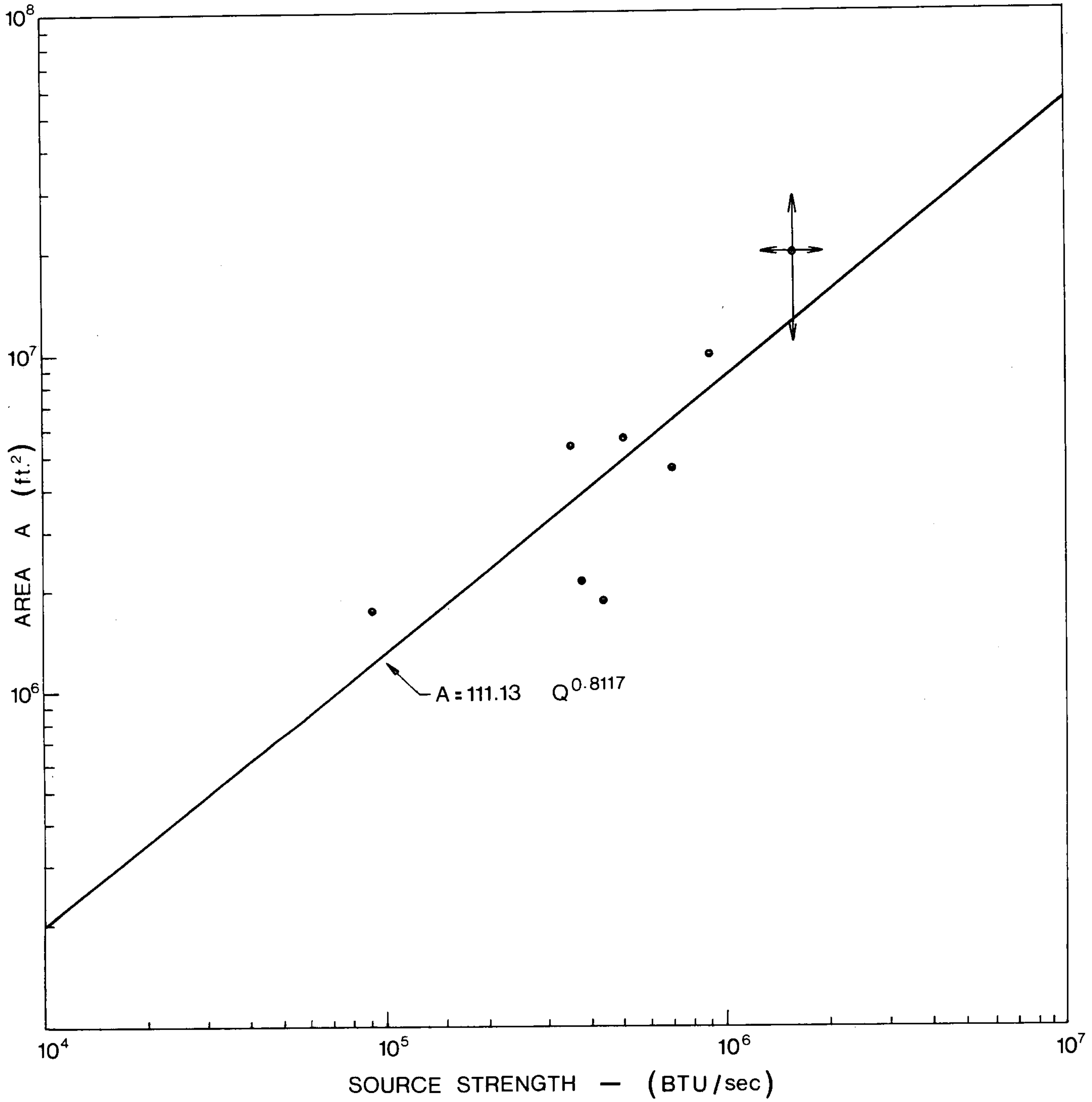


Figure 8. Average thermal effluent data on large lakes.

Table 3. Estimated physical effects of thermal effluents input to the Canadian Great Lakes 2000 A.D.

			Lake Ontario	Lake Erie	Lake Huron	Georgian Bay	Lake Superior
Average surface area contained within 1°C excess temperature 2000 A.D.		sq. ft.	50×10^7	18×10^7	9.5×10^7	16×10^7	4.8×10^7
		sq. mi	17.7	6.3	3.4	7.0	4.2
		hectares	4.6×10^3	1.6×10^3	0.8×10^3	1.8×10^3	1.1×10^3
Length of shoreline exposed to "excess" temp. greater than:	1°C	nautical mi.	25	11	5.4	7.6	6.6
		Km.	46	20	10	14	12
	3°C	nautical mi.	8.3	3.7	1.8	2.5	2.2
		Km.	15	6.8	3.3	4.6	4.1
	5°C	nautical mi.	5	2.2	1.1	1.5	1.3
		Km.	9.3	4.1	2	2.8	2.4

plumes. This length to width ratio was assumed to be typical of all thermal effluents.

A first order of estimate of the length of shoreline exposed to excess temperatures greater than 3°C and 5°C was obtained by assuming a χ^{-1} dependence between "excess" temperature and distance (χ) from the outfall. Such dependence is roughly demonstrated by the Lakeview data in Figure 7 and by dye diffusion data reported by Csanady et al. (1967).

The total length of shoreline which (on the average) is exposed to "excess" temperatures is given in Table 3 for each of the Great Lakes. It may be seen from Table 3 that Lake Ontario is the most heavily loaded of the Canadian portion of the Great Lakes. Twenty-five nautical miles or approximately 12% of the north shore of Lake Ontario will be exposed to "excess" surface temperatures greater than 1°C at any one time. Because of the bimodal current distribution, twice this amount will frequently be exposed (approximately 50% of the time).

These results are based on average data. Assuming that conditions which produce a large plume at one location occur simultaneously over the entire lake, then up to 2.5 times this length of shoreline could be exposed at any one time.

DISCUSSION OF FUTURE PHYSICAL EFFECTS

The estimate of future physical effects presented were based on the hypotheses that the size of a thermal plume is predominantly a function of the rate of waste heat addition and that dynamics of lake currents were similar from one large lake to another. An indication of the accuracy of these hypotheses may be gathered from the magnitude of the scatter of the data about the line of best fit shown in Figure 8. This scatter is caused by a multiplicity of factors which influence the dispersion of an effluent but which were not explicitly taken into account in the analysis. Some of these factors include the geometry of the outfall, the bottom topography in the vicinity of the generating station, basic differences in the fluctuating currents from one lake to another, and an insufficient number of data for a stable mean value. However, the largest uncertainty in the average values "predicted", results from the fact that most future generating stations are planned to be much larger than any station now existing. Until data are available for an effluent from a generating station of this large size, future predictions are at best an extrapolation from existing small scale data.

Conclusions and Recommendations

CONCLUSIONS

(1) The reduction of temperature within the Lakeview effluent is primarily a result of mixing of the warm effluent with ambient lake water.

(2) The turbulent mixing process near shore is complex. There was no systematic variation of thermal plume size with the component of the wind normal to the local shoreline at the Lakeview generating station.

(3) First order estimates of some average physical effects have been presented for present and future waste heat inputs to the Great Lakes. The most heavily loaded portion of the Canadian shoreline of the Great Lakes in the year 2000 will be the north shore of Lake Ontario. It is estimated that on the average 25 miles of shoreline (or approximately 12% of the north shore of Lake Ontario) will be exposed to temperatures in excess of one degree centigrade.

RECOMMENDATIONS

(1) It is recommended that an ecological assessment be made of the possible consequences of disposing of waste heat in the Great Lakes. Definition of possible harmful ecological effects is required to warrant any further refinement of the analysis of physical effects.

Included in this assessment should be an evaluation of the relative biological importance of various shoreline segments for all the shoreline of the Great Lakes. Such an evaluation could provide criteria for establishing the sites of future power plants.

(2) Comprehensive study is required on the dynamics of lake currents, particularly fluctuating lake currents near shore which cause the diffusion of thermal (and other) effluents. Virtually nothing is known about the generation of these current fluctuations which can cause an order of magnitude change in the rate of diffusion under (apparently) identical external conditions.

(3) The thermal effluents of electric generating stations with outputs larger than 3,000 Mw should be surveyed as soon as their operation begins. Such temperature measurements would allow an assessment of the validity of the extrapolation presented in this report and provide a means for adjusting the regression if necessary.

References

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An Infrared System for Measuring Thermal Plumes

General

The measuring system described in this appendix is a synthesis of available components in a manner particularly well suited for the measurement of thermal plumes. It was assembled to provide sufficient data on thermal effluents to allow an assessment of possible physical effects of the large increase in waste heat predicted for the year 2000. It was designed to keep the cost and time per survey low, using as many components already on hand as possible.

Some of the advantages and disadvantages of various measuring techniques are given in Table A1.

Table A1. A comparison of three data collection methods

	Advantages	Disadvantages
Boat survey	– may provide information at various depths	– cost is proportional to number of depths measured. – slow to conduct (3-4 hrs) – affected by weather (high winds)
Fixed thermometer array	– may provide information at various depths – may provide continuous all-weather coverage	– very costly to provide adequate coverage in either time or space – data difficult to analyze because of large volume
Aerial infrared survey	– fast area coverage – may be operated in high winds	– infrared scanners very costly – affected by weather (low cloud ceilings) – surface measurements only

Description of Instrumentation System

Temperature sensor

Surface temperatures were measured using a Barnes PRT-5 radiometer. This instrument which responds in the range of 8-12 microns was incorporated as supplied by the manufacturer with no modification. Details of its operation may be found in Weiss (1970). This instrument has been used for lakewide surveys of surface temperature in the Great Lakes (Richards and Irbe, 1969).

It has also been used to measure surface temperatures in thermal effluents. The instrument was mounted in an aircraft which was then flown back and forth across the plume perpendicular to its centreline. The repeated plume crossings were time consuming and the lack of accuracy in positioning above the ground made data reduction difficult. In both of the above applications, the extremely rapid time response of the radiometer was not utilized, the output simply being recorded on strip chart recorders with time constants of about 0.5 seconds. If the aircraft flew too low or too fast so that it exceeded a speed-altitude envelope, the data recorded on the strip chart was smoothed by the poor response of the strip chart recorder.

Mechanical Scanning

To reduce the aircraft time required to survey a thermal effluent as well as to utilize the rapid response characteristics of the radiometer a system of mechanical scanning of the PRT-5 sensor was employed. This allowed the aircraft to fly along the centreline of the thermal plume while the radiometer was laterally traversing back and forth across the flight line.

The PRT-5 sensor was mounted on a pendulum which was driven by a standard automobile windshield-wiper motor and mechanism. The motor speed was adjusted to provide a sweep frequency of 0.5 cycles/s. The angle of sweep in the vertical plane was ±22° from vertical. The apparatus was mounted on the floor of a single engine "Cherokee 140" aircraft. The hole in the floor allowed an effective sweep angle of ±20° from vertical; for the balance of the sweep cycle, the sensor was measuring the temperature of the floor of the aircraft.

Data Recording

The fast response of the radiometer was utilized by inputing the temperature signal into a portable oscilloscope. The signal was then photographed with a Pentax 35 mm camera for the first half of each cycle only. The time base on the scope was matched to half the sweep rate of the mechanical scanning device (1 second). The trace on the scope was initiated each cycle by a sync pulse generated

Table A2. Lakeview generating station – meteorological and plant data, December 2, 1970

	Eastern Standard Time					
	7.00	8.00	9.00	10.00	11.00	12.00
Plant load, MW	845	1410	1495	1530	1528	1440
Cooling water flow, ft^3/s	–	–	–	2044	2044	2045
Average temperature of cooling-water at inlet, °F	45.5	45.2	45.2	45.5	46.0	47.0
Average temperature of cooling-water at outlet, °F	–	–	–	60.7	61.1	61.1
Wind, direction	W	WSW	WSW	WSW	WSW	W
Wind speed, mph	10	13	16	17	20	21
Air temperature, °F	46	45	45	47	50	52
Humidity, %	70	72	74	60	50	45
Net radiation, Langleys/h	–8	–5	–5	22	30	35

Table A3. Lakeview generating station – meteorological and plant data, March 2, 1971

	Eastern Standard Time						
	08.00	09.00	10.00	11.00	12.00	13.00	14.00
Plant load, MW	1797	1751	1743	1745	1742	1788	1838
Cooling water flow, ft^3/s				2146	2144	2145	2146
Average temperature of cooling-water at inlet, °F				36.5	36.0	36.0	36.0
Average temperature of cooling-water at outlet, °F				50.4	50.4	50.6	50.3
Wind direction	NW	NW	NW	NW	N	NW	
Wind speed, mph	6	5	6	6	4	8	
Air temperature, °F	21	22	26	28	29	31	
Relative humidity, %	84	81	73	68	63	63	
Solar radiation Langleys/h			No Records				

Table A4. Lakeview generating station – meteorological and plant data, March 9, 1971

	Eastern Standard Time						
	08.00	09.00	10.00	11.00	12.00	13.00	14.00
Plant load, MW	1888	1882	1885	1889	1888	1891	1877
Cooling water flow, ft^3/s				2166	2167	2167	2167
Average temperature of cooling-water at inlet, °F				40.8	40.8	40.8	40.8
Average temperature of cooling-water at outlet, °F				56.6	56.6	56.5	56.7
Wind direction	W	W	W	W	WNW	W	W
Wind speed, mph	12	13	12	12	13	15	17
Air temperature, °F					25.5	27.0	28.5
Relative humidity, %					50	51	54
Solar radiation, Langleys/h					65	64	58

Table A5. Lakeview generating station – meteorological and plant data, April 14, 1971

	Eastern Standard Time						
	08.00	09.00	10.00	11.00	12.00	13.00	14.00
Plant load, MW	1626	1588	1546	1542	1543	1552	1547
Cooling water flow, ft^3/s					1880	1880	1880
Average temperature of cooling-water at inlet, °F					42	42	42
Average temperature of cooling-water at outlet, °F					54	54	54
Wind direction,				WNW	WNW	W	WNW
Wind speed, mph				13	14	12	15
Air temperature, °F				35	36	37	39
Relative humidity, %				45	43	38	37
Solar radiation, Langleys/h				66	69	70	66

Table A6. Douglas Point generating station – meteorological and plant data, December 2, 1970

	Eastern Standard Time				
	11.00	12.00	13.00	14.00	15.00
Plant load, MW	201	204	205	205	205
Cooling water flow, ft^3/s	400	400	400	400	400
Average temperature of cooling-water at inlet, °F	47.5	47.3	47.0	47.5	48.0
Average temperature of cooling-water outlet, °F	63.0	66.2	61.3	63.0	64.0
Wind direction,	W	W	W	WSW	WSW
Wind speed, mph	25	25	27	25	26
Air temperature, °F	42	42	42	43	42

Table A7. Douglas Point generating station – meteorological and plant data, March 1, 1971

	Eastern Standard Time					
	11.00	12.00	13.00	14.00	15.00	16.00
Plant load, MW	201	206	205	205	205	205
Cooling water flow, ft^3/s*	–	–	–	–	–	–
Average temperature of cooling-water at inlet, °F**	44	44	44	44	43	44
Average temperature of cooling-water at outlet, °F	72	74	73	73	73	73
Wind direction	W	W	W	W	W	W
Wind speed, mph	15	14	12	14	14	12
Air temperature, °F	33	33	32	32	30	35

*Due to recirculation, actual cooling water inflow and outflow is not known.

**Water temperature measured in the intake forebay.

mechanically by a microswitch which was closed by the pendulum at the start of each cycle. The sync pulse also opened the shutter of the 35 mm camera. The camera was adjusted for a 1 second time exposure. Hence, for the first half of each cycle, the camera was photographing the trace of temperature as the time base moved the dot across the scope in phase with the mechanical sweep of the temperature sensor.

Table A8. Douglas Point generating station – meteorological and plant data, March 2, 1971

	Eastern Standard Time 12.00 to 16.00
Plant load, MS	208
Cooling water flow, ft^3/s*	–
Average temperature of cooling-water at inlet, °F**	44
Average temperature of cooling-water at outlet, °F	74
Wind direction	NW
Wind speed, mph	10
Air temperature, °F	27

* Due to recirculation actual cooling water in– and outflow is not known.

**Water temperature measured in the intake forebay.

The film was manually advanced during the return half of each cycle (one second).

Ground Positioning

A DeHavilland automatic 35 mm aerial camera with a 90° Zeiss lens was used for ground positioning. This camera was fired once each sweep by the sync signal. The wide angle lens provided approximately 50% overlap on either side of the sweep path. Since thermal effluents in large lakes usually flow parallel to shore, the aerial photographs usually contained a shoreline segment and provided a positive ground fix at the start of each sweep cycle.

Table A9. Douglas Point generating station – meteorological and plant data, March 9, 1971

	Eastern Standard Time		
	13.00	14.00	15.00
Plant load, MW	211	211	210
Cooling water flow, ft^3/s*	–	–	–
Average temperature of cooling-water at inlet, °F**	41.5	42.7	45.5
Average temperature of cooling-water at outlet, °F	63.9	65.2	68.0
Wind direction	NW	NW	NW
Wind speed, mph	15	18	18
Air temperature, °F	24	24	26

* Due to recirculation actual cooling water in– and outflow is not known.

**Water temperature measured in the intake forebay.

Data Processing

The sweep track and aircraft flight path for each survey was plotted on a large scale map from the aerial photographs. Temperatures were obtained at equally spaced intervals from alternate frames of the 35 mm data film. The time base of the data film was converted to distance over the ground and the temperatures were plotted on the map. Isotherms of the thermal effluent were estimated at 1°C intervals by linear interpolation from these plotted temperatures.

Results

A photograph of the temperature measuring system is shown in Figure A1 as installed in the aircraft. Isotherms as measured on eight days at Lakeview are shown in Figure A2 – A9. Five sets of data for the Douglas Point generating station are shown in Figures A10 – A14. Ground data as provided by Ontario Hydro and included as Tables A2 – A9 and Figures A15 – A17.

Each survey was comprised of three flights along the centreline of the plume and required approximately 20 min air time over the site.

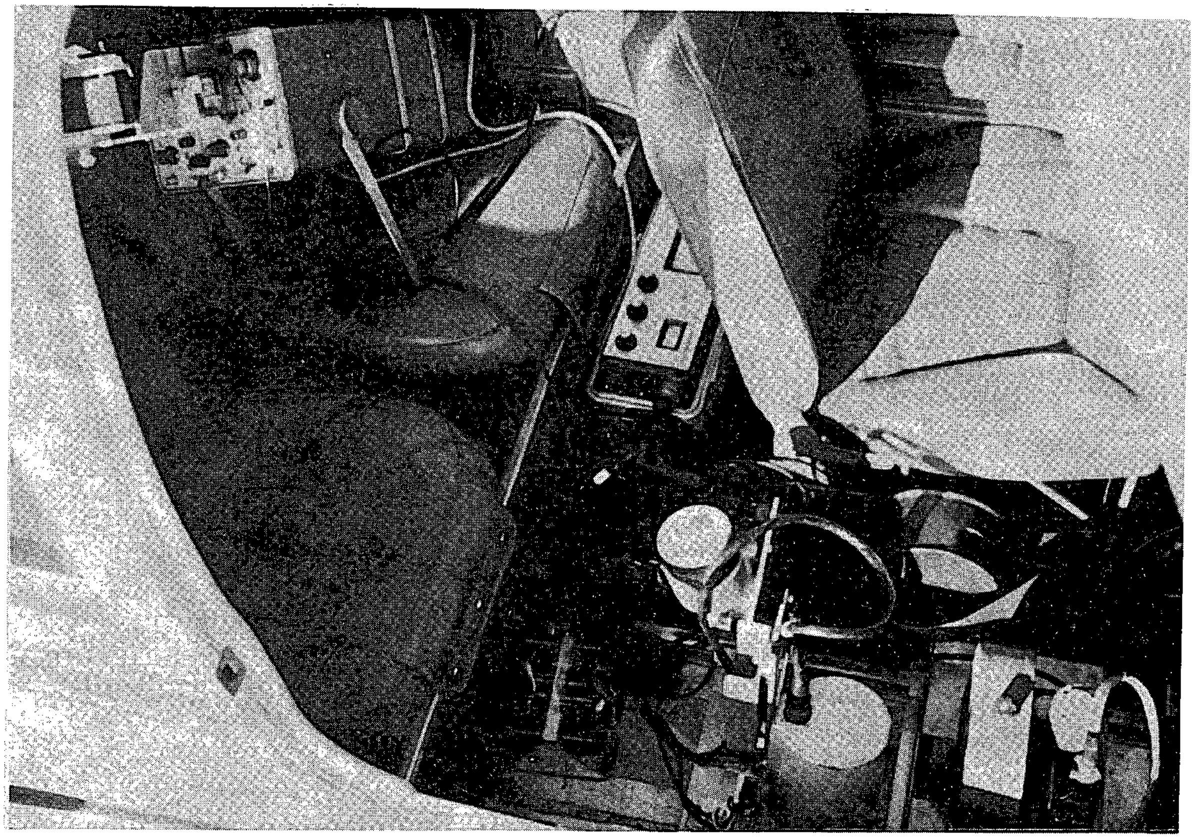


Figure A1. Temperature measuring system installed in aircraft.

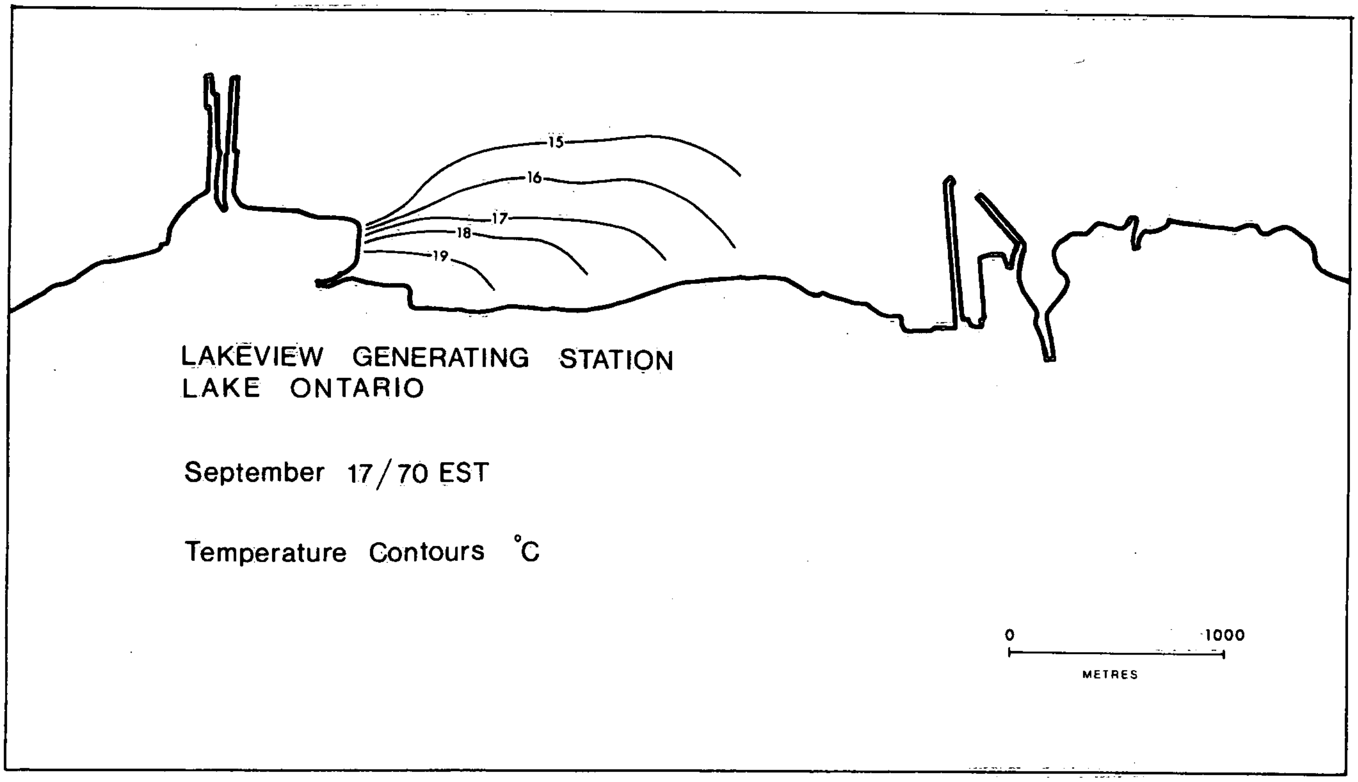


Figure A2. Lakeview generating station, Lake Ontario. Temperature contours, °C – September 17, 1970.

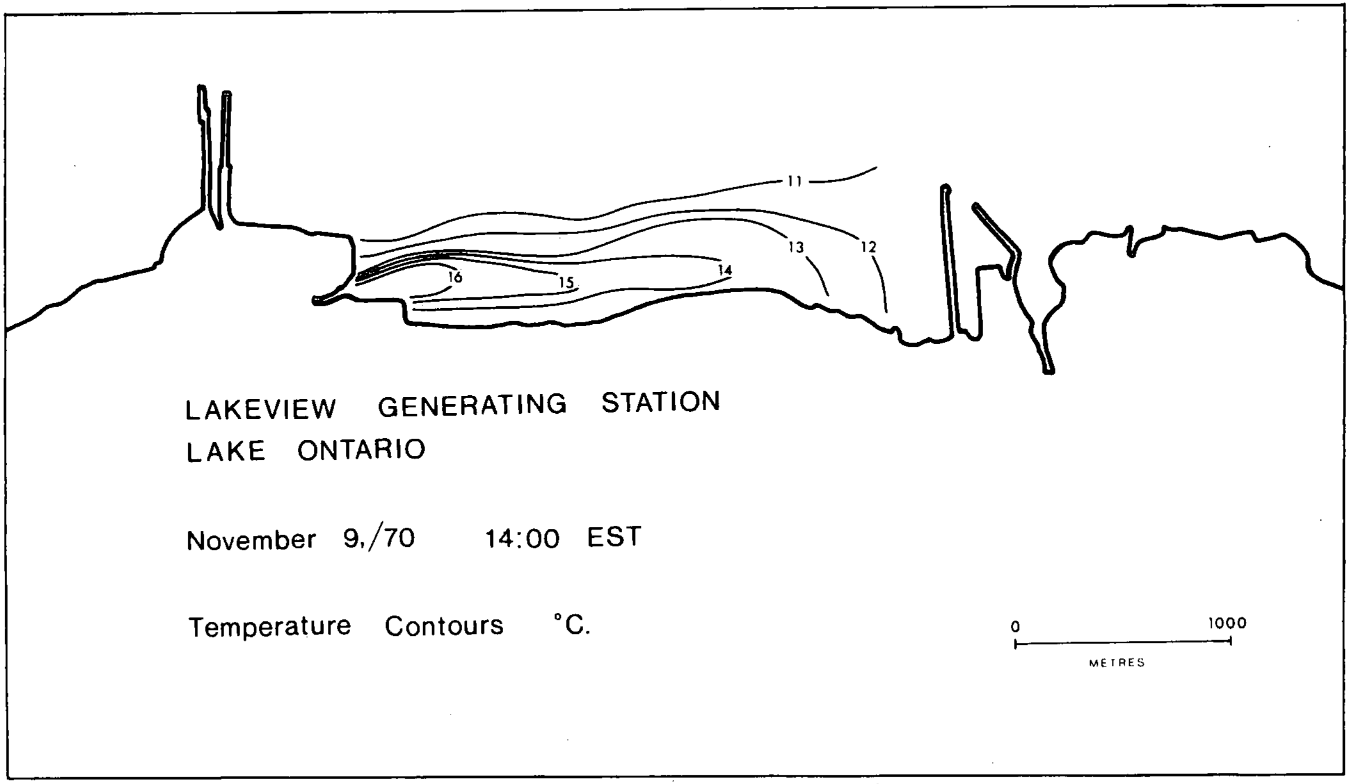


Figure A3. Lakeview generating station, Lake Ontario. Temperature contours, °C – November 9, 1970, 14:00 EST.

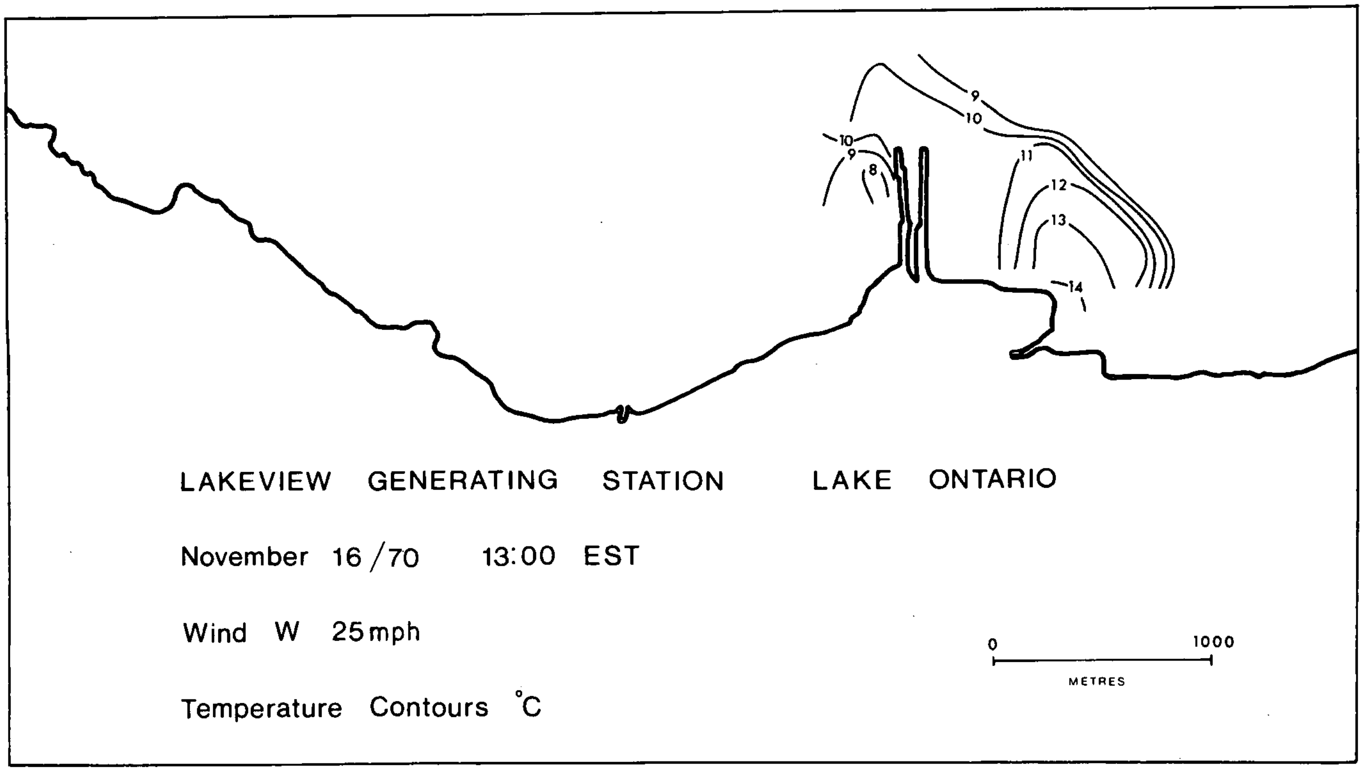
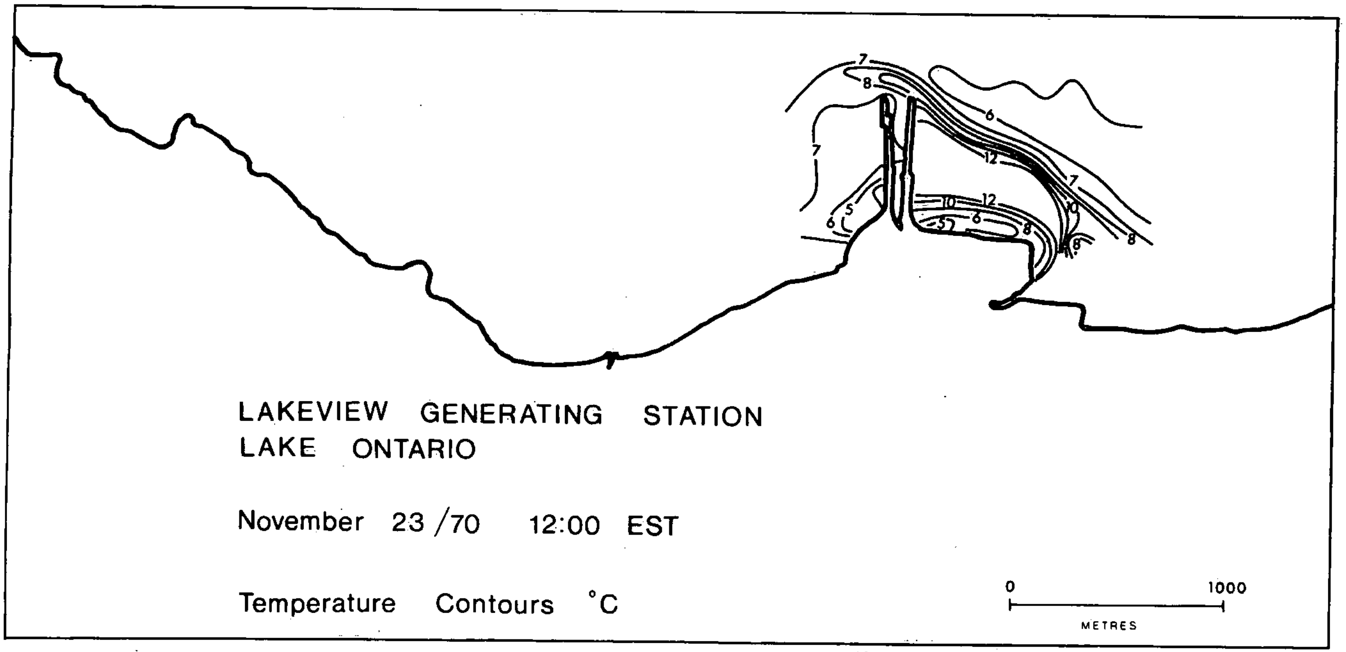


Figure A4. Lakeview generating station, Lake Ontario. Temperature contours, °C – November 16, 1970, 13:00 EST, wind W., 25 mph.

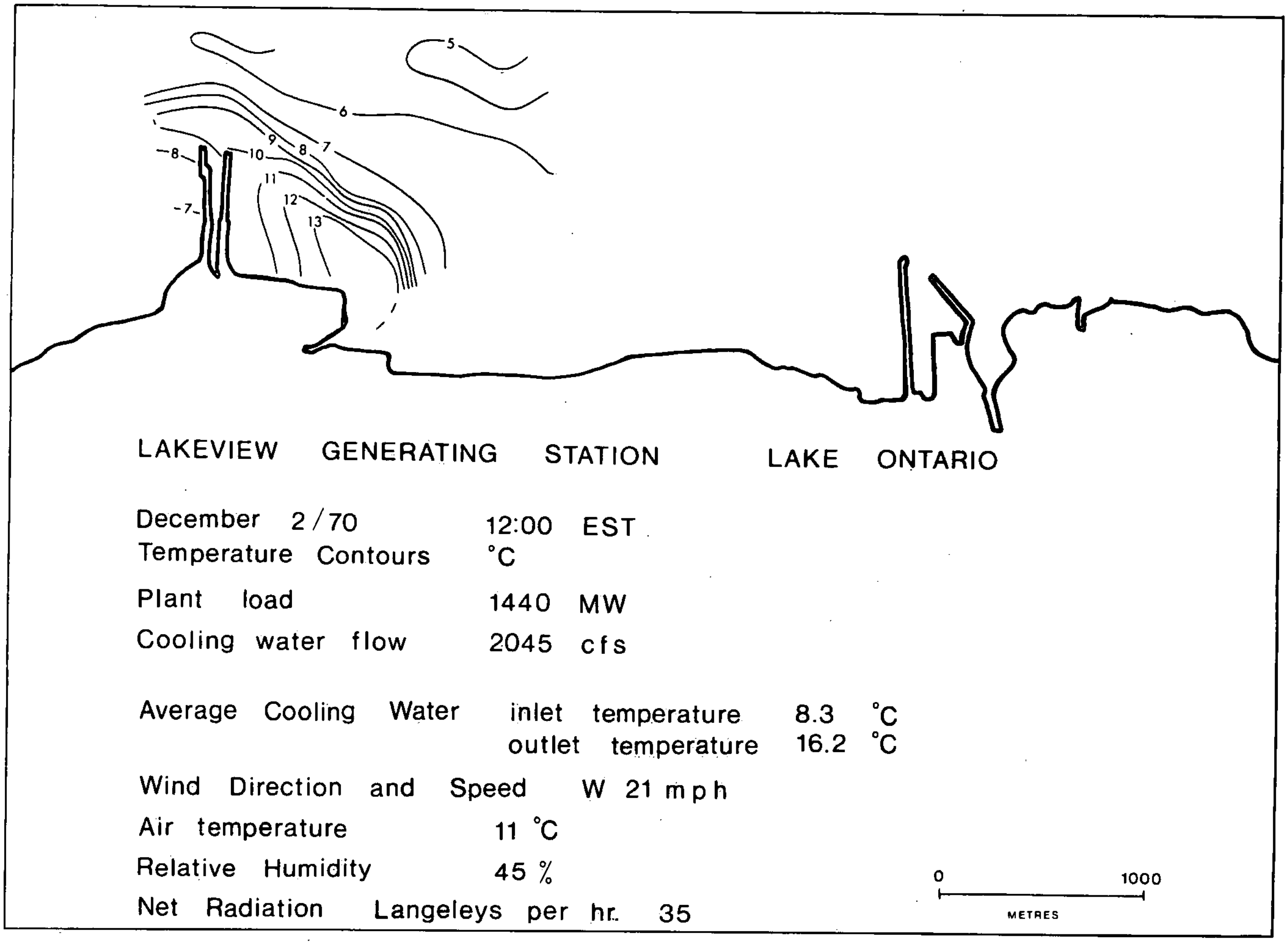


LAKEVIEW GENERATING STATION
LAKE ONTARIO

November 23 /70 12:00 EST

Temperature Contours °C

Figure A5. Lakeview generating station, Lake Ontario. Temperature contours, °C – November 23, 1970, 12:00 EST.

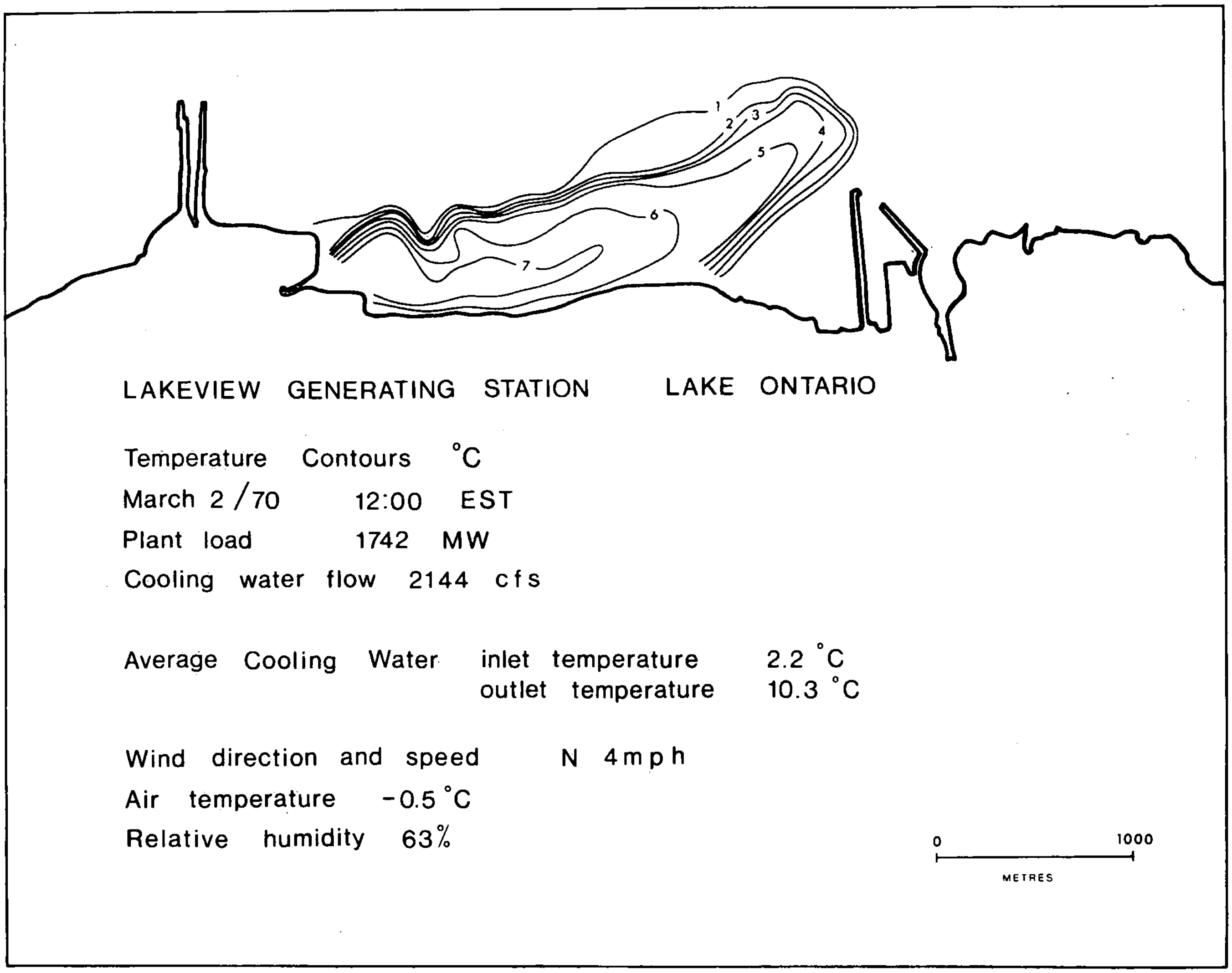


LAKEVIEW GENERATING STATION LAKE ONTARIO

December 2/70 12:00 EST
Temperature Contours °C
Plant load 1440 MW
Cooling water flow 2045 cfs

Average Cooling Water inlet temperature 8.3 °C
outlet temperature 16.2 °C
Wind Direction and Speed W 21 mph
Air temperature 11 °C
Relative Humidity 45 %
Net Radiation Langeleys per hr. 35

Figure A6. Lakeview generating station, Lake Ontario. Temperature contours, °C – December 2, 1970, 12:00 EST.



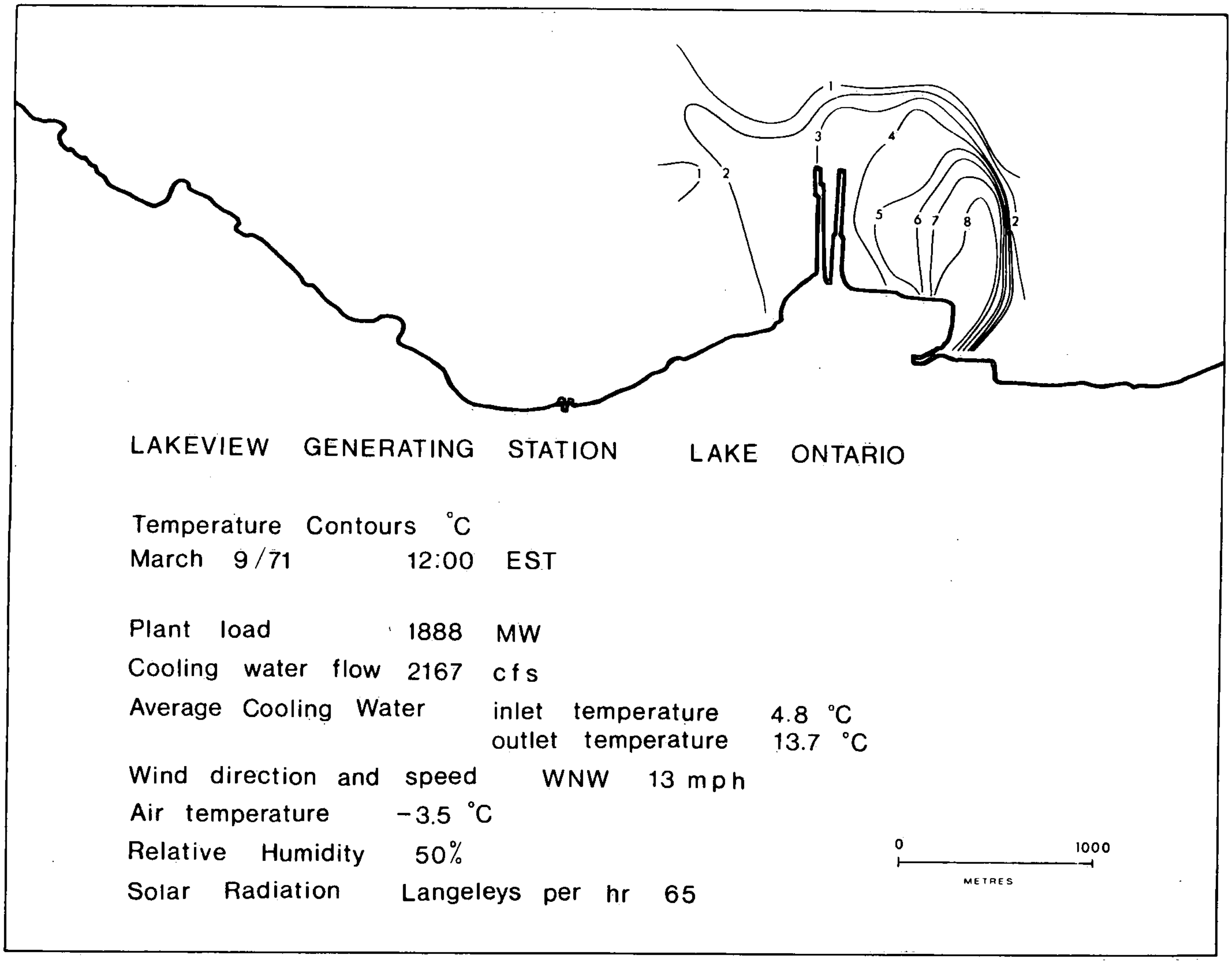
LAKEVIEW GENERATING STATION LAKE ONTARIO

Temperature Contours °C
March 2 /70 12:00 EST
Plant load 1742 MW
Cooling water flow 2144 cfs

Average Cooling Water inlet temperature 2.2 °C
outlet temperature 10.3 °C

Wind direction and speed N 4mph
Air temperature -0.5 °C
Relative humidity 63%

Figure A7. Lakeview generating station, Lake Ontario. Temperature contours, °C – March 2, 1970, 12:00 EST.



LAKEVIEW GENERATING STATION LAKE ONTARIO

Temperature Contours °C
March 9/71 12:00 EST

Plant load 1888 MW
Cooling water flow 2167 cfs
Average Cooling Water inlet temperature 4.8 °C
outlet temperature 13.7 °C
Wind direction and speed WNW 13 mph
Air temperature −3.5 °C
Relative Humidity 50%
Solar Radiation Langeleys per hr 65

Figure A8. Lakeview generating station, Lake Ontario. Temperature contours, °C – March 9, 1971, 12:00 EST.

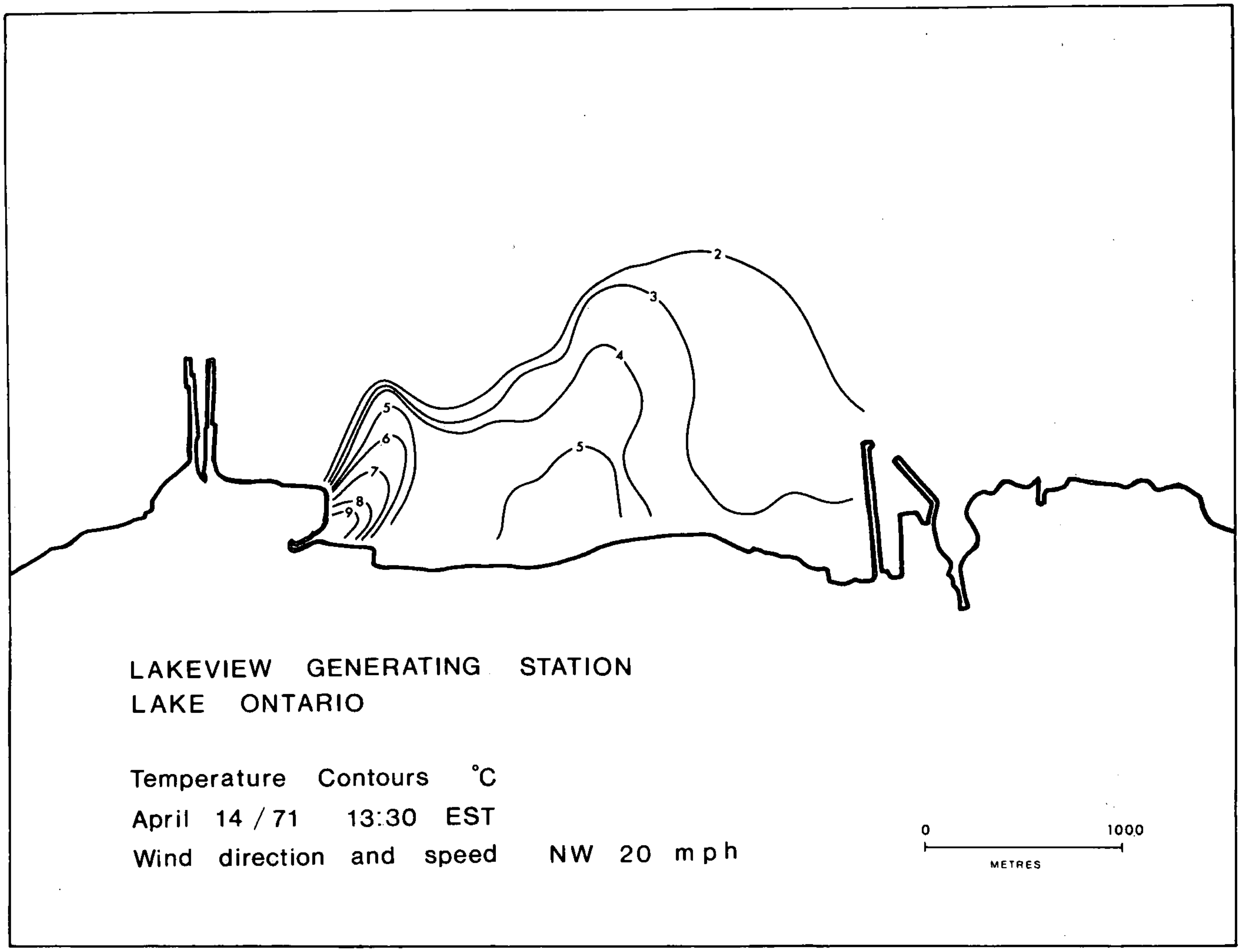


Figure A9. Lakeview generating station, Lake Ontario. Temperature contours, °C – April 14, 1971, 13:30 EST.

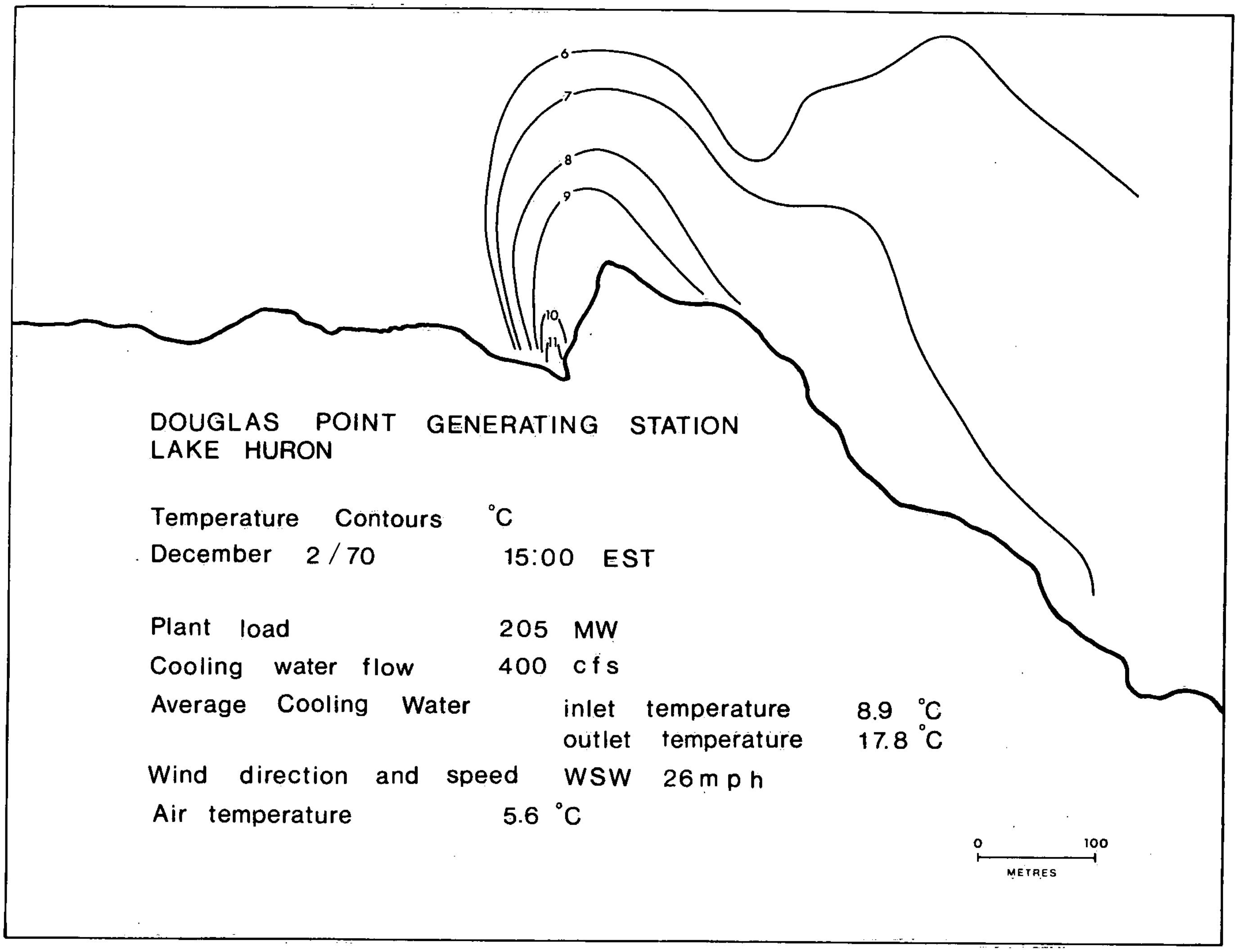


Figure A10. Douglas Point generating station, Lake Huron. Temperature contours, °C – December 2, 1970, 15:00 EST.

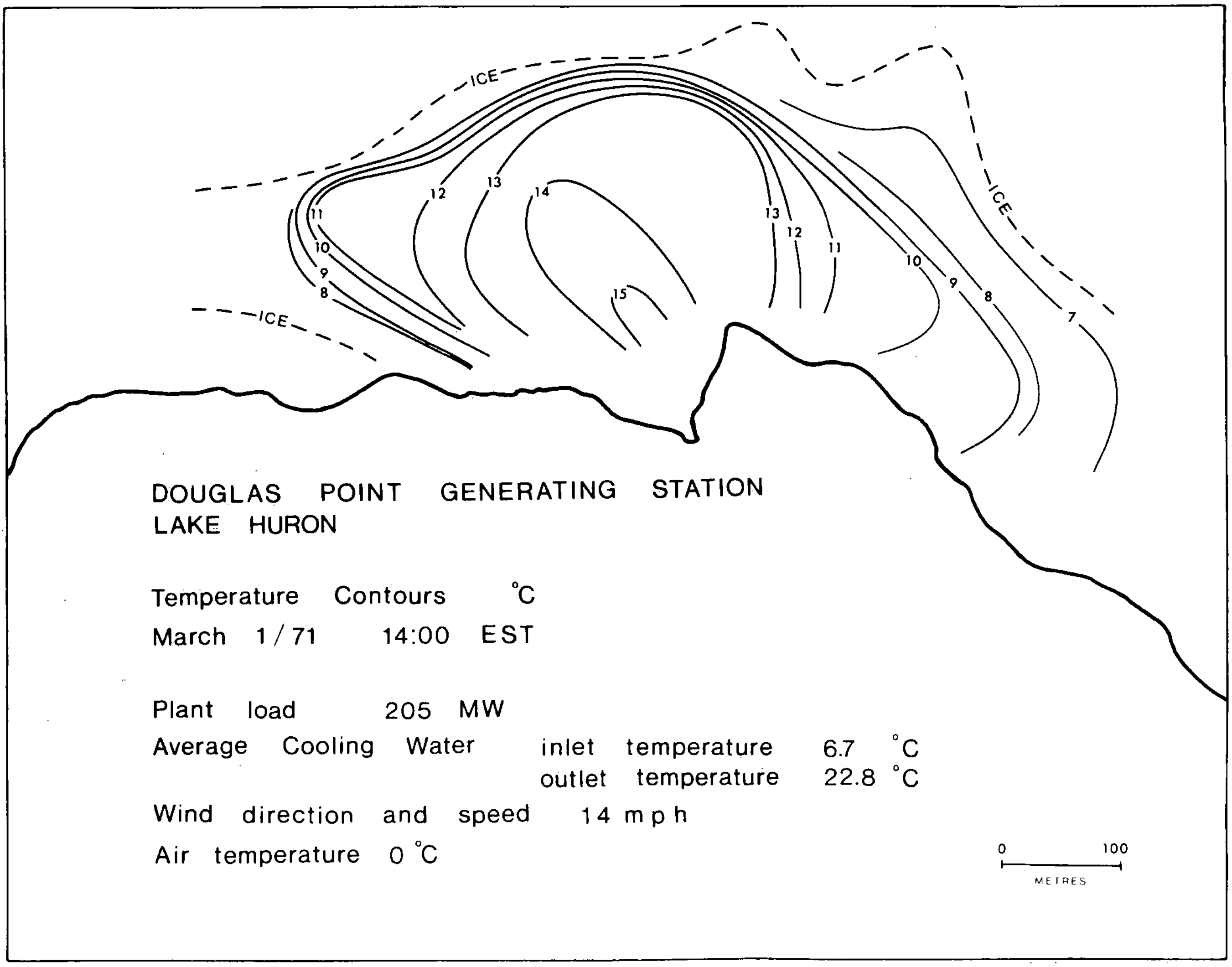


Figure A11. Douglas Point generating station, Lake Huron. Temperature contours, °C – March 1, 1971, 14:00 EST.

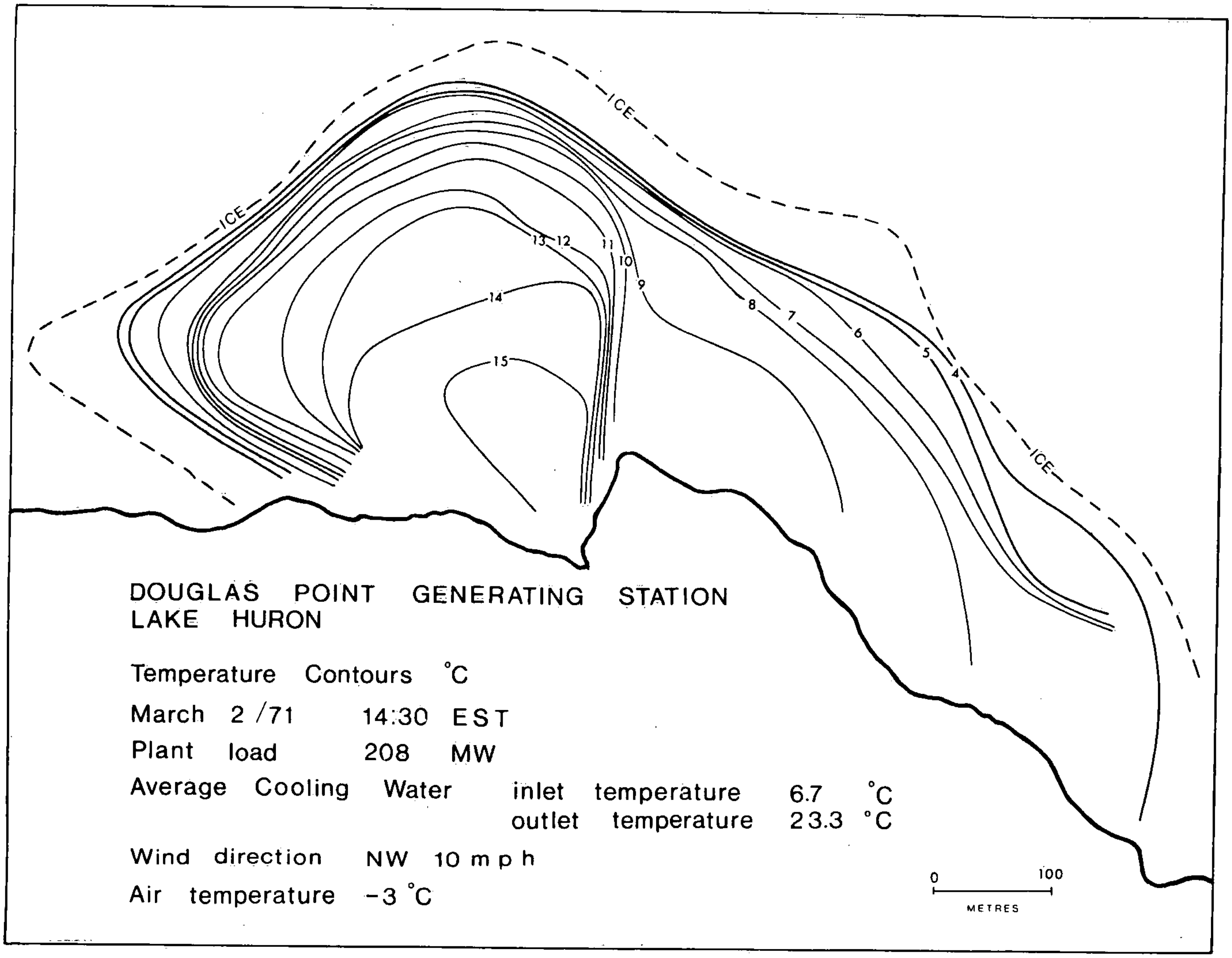


Figure A12. Douglas Point generating station, Lake Huron. Temperature contours, °C – March 2, 1971, 14:30 EST.

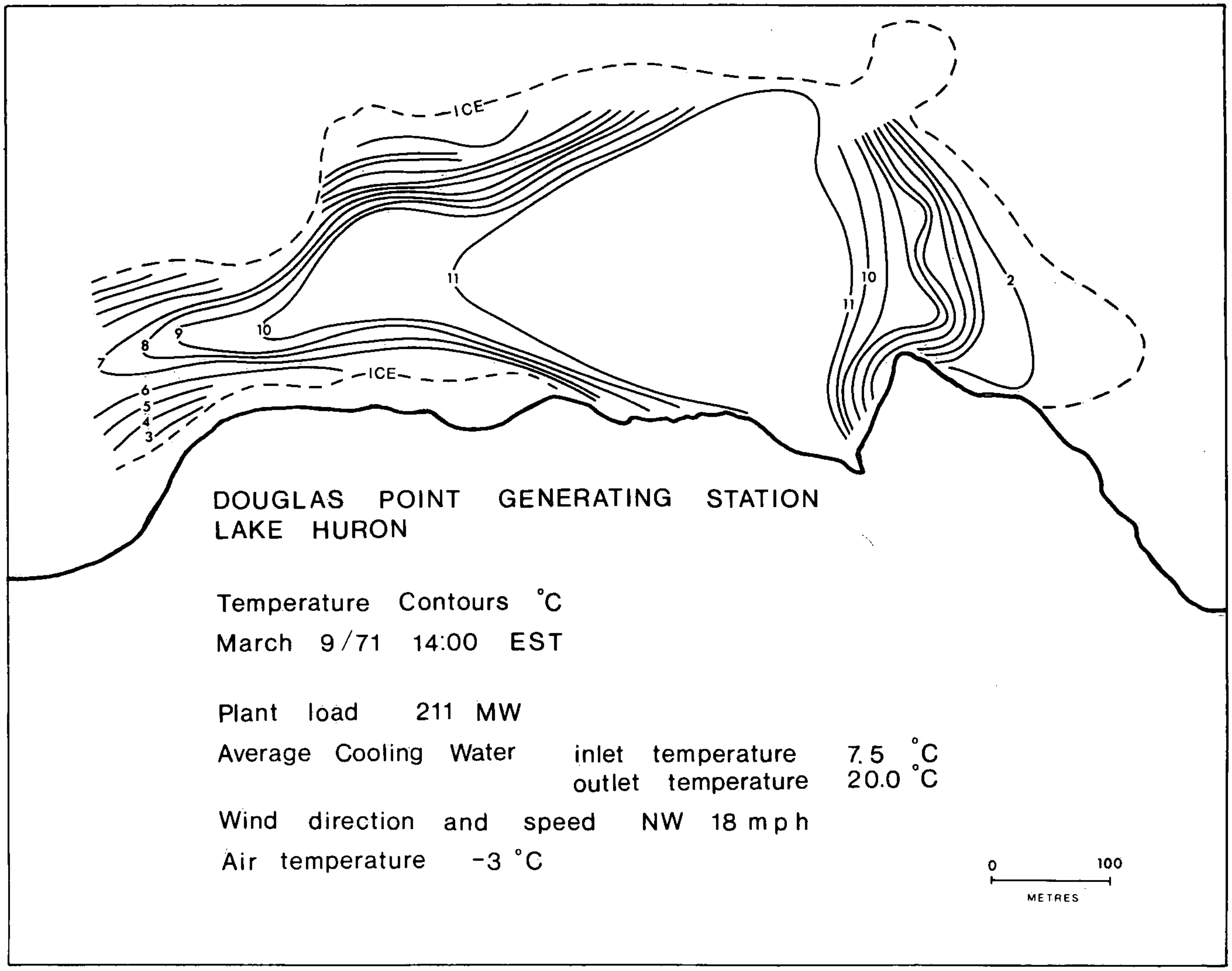


Figure A13. Douglas Point generating station, Lake Huron. Temperature contours, °C – March 9, 1971, 14:00 EST.

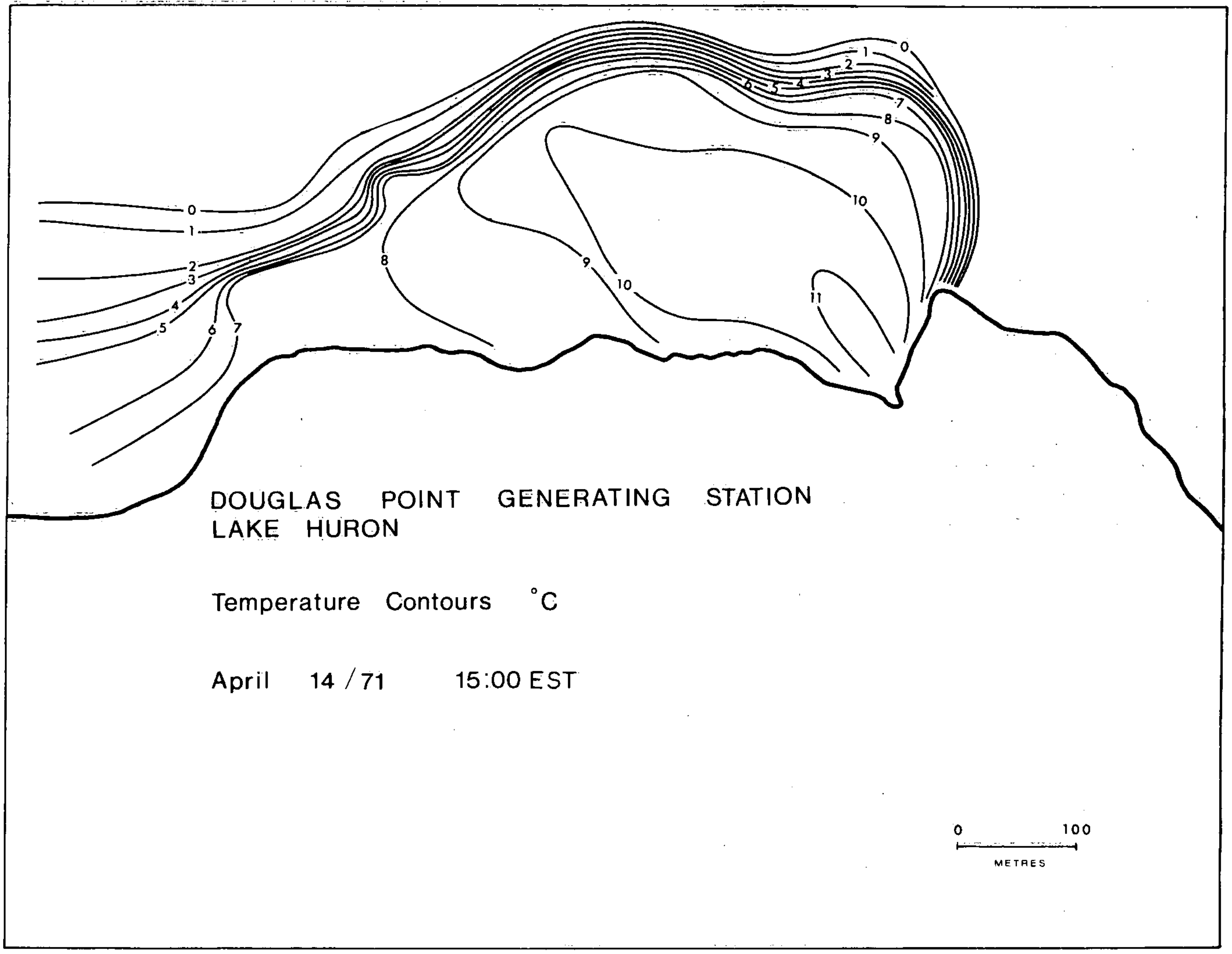
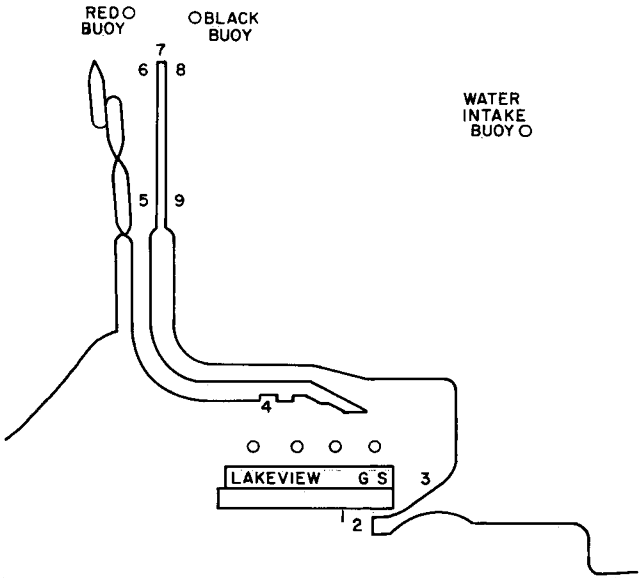
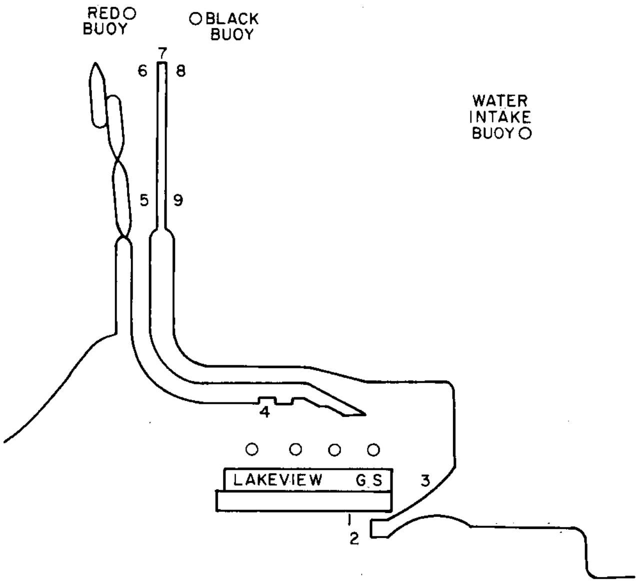


Figure A14. Douglas Point generating station, Lake Huron. Temperature contours, °C – April 14, 1971, 15:00 EST.



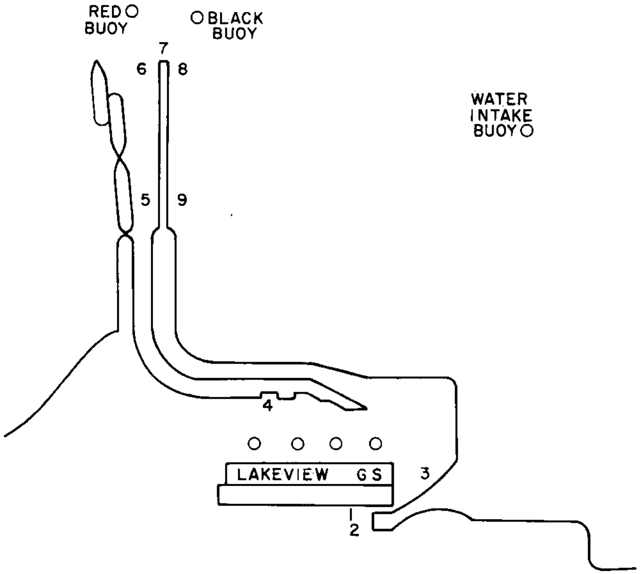
POINT NO.	1	2	3	4	5	6	7	8	9
TEMP. °F	64	62	61.8	46	46	45	45	48	49

Figure A15. Lakeview generating station. Surface water temperature, December 2, 1970, 11:00 – 11:30 EST.



POINT NO.	1	2	3	4	5	6	7	8	9
TEMP. °F	53	53	53.5	—	36.3	36.0	36.3	36.3	37.0

Figure A16. Lakeview generating station. Surface water temperature, March 2, 1971, 12:00 EST.



POINT NO.		1	2	3	4	5	6	7	8	9
TEMP °F	12 30 EST	59	59	58	41.8	41.5	41.5	42	43	44
	13 30 EST	60	60	58	42.2	42.0	42.0	42	44	44.5

Figure A17. Lakeview generating station. Surface water temperature, March 9, 1971, 12:30 EST and 13:30 EST.

APPENDIX B

Heat Transfer Calculations

The heat flux, q/A, from the measurable thermal plume to the atmosphere is the result of evaporation, long-wave back radiation and sensible heat transfer. It may be expressed by

$$q/A = K(T_p - T_L)$$

where q is the heat transfer rate Btu/day
A is the surface area of plume within 1:C isotherm
K is an overall heat transfer coefficient
T_p is the plume surface temperature
T_L is the undisturbed lake surface temperature

Following Edinger and Geyer (1965), the overall heat transfer coefficient in units of Btu/ft^2 day °F is

$$K = (\beta_L + 0.26)\, 11.4\, W + 15.7$$

where β_L is the slope of the saturated vapor pressure curve evaluated at the undisturbed lake surface temperature in (mm Hg/°F).
W is the average wind speed over the plume in mph.

This formula is based on the Lake Hefner equation for evaporative heat transfer and the Bowen ratio for sensible heat transfer. It was assumed in both cases that the presence of the thermal plume did not alter the overlake air temperature or dew point. A first order binomial expansion of the Stefan-Boltsmann equation was used for the long wave back radiation in deriving the above equation.

The plume surface temperature used to calculate the heat flux was an area weighted average

$$T_p = \frac{\Sigma A_i T_i}{\Sigma A_i}$$

where A_i is the area between consecutive isotherms
T_i is the average temperature of the two isotherms.