CANADA, Inland Waters Directorate
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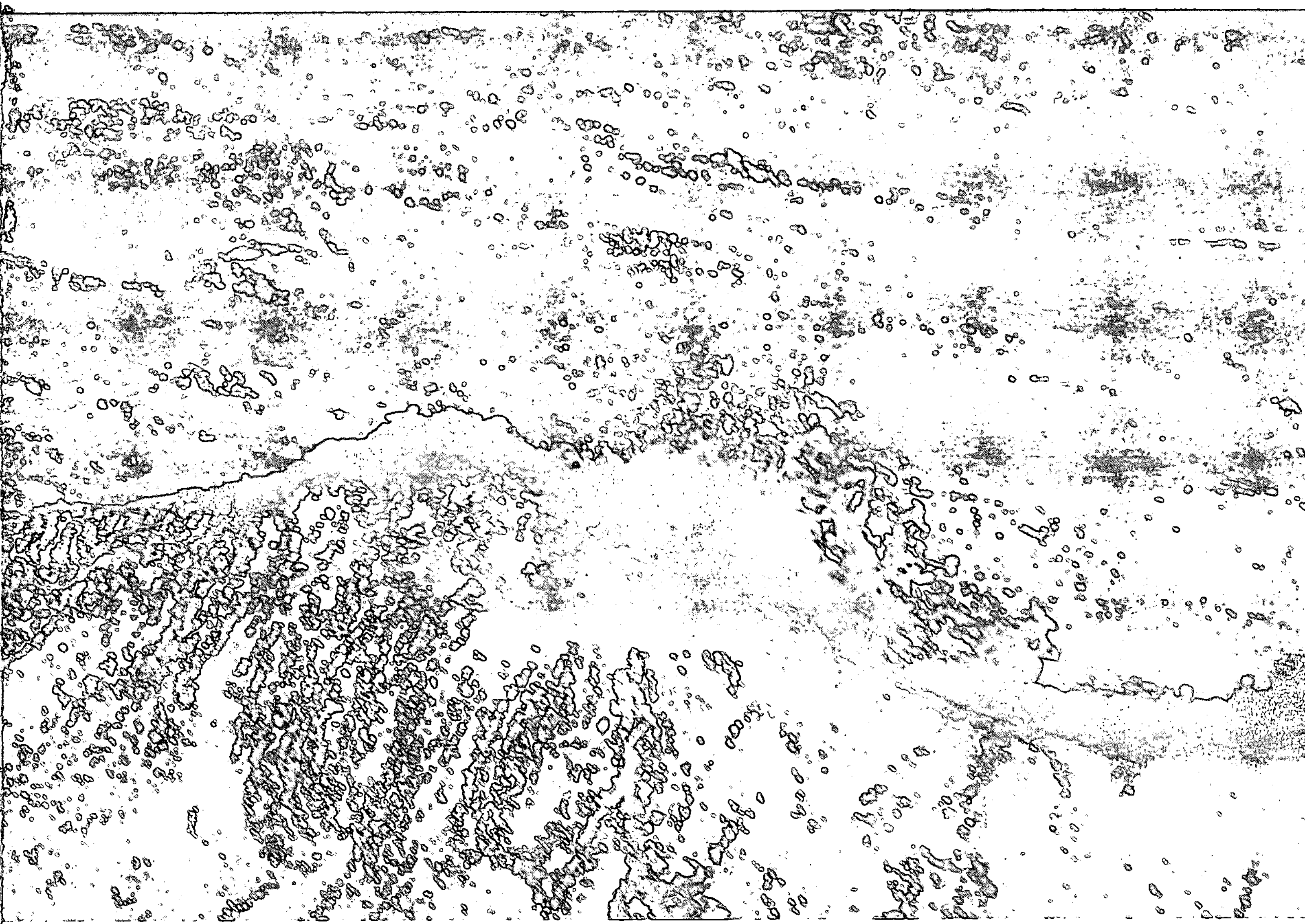
Ice Piling on Lakeshores

With Special Reference to the Occurrences on Lake Simcoe in the Spring of 1973

Gee Tsang



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(Résumé en français)

INLAND WATERS DIRECTORATE,
CANADA CENTRE FOR INLAND WATERS,
BURLINGTON, ONTARIO, 1974

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Abstract

The paper studies the piling of ice on lakeshores. Ice piling is found to be not a static but a dynamic event. Ice piling occurs when ice floes, which gather speed and momentum under the action of wind over an open-water fetch, ram onto the shore or onto shore-fastened ice. The kinetic energy of the ice floes is converted into the potential energy of the ice piles. A strong wind is not necessary for ice piling. Ice piles will occur under a wind of less than 6.75 m/s if other conditions are favourable. Meteorologically, a shift in wind direction from offshore to onshore is a necessary condition for ice piling. The fluctuation of wind strength and direction and the shifting of wind help to loosen the ice floes and promote ice piling. Ice piling occurs in above freezing temperatures only. Equations are derived that give the width of an open-water fetch required for ice piling, the speed of the ice floes for ice piling, the height of an ice pile, and the affecting factors. It is proposed that damage to shoreline properties may be avoided by accelerating the piling of ice at a distance offshore. Field data from the ice pilings on Lake Simcoe in the spring of 1973 are used in the study.

Résumé

Le sujet de ce rapport est l'étude de l'accumulation des glaces sur les rivages des lacs. Cette accumulation se révèle être un phénomène dynamique, et non pas statique. Elle se produit lorsque les glaces flottantes (la banquise) dont la vitesse et l'élan sont augmentés par l'action du vent sur l'eau libre, heurtant le littoral, ou la glace qui s'y est déjà fixée. L'énergie cinétique de la banquise se transforme en énergie potentielle, en s'accumulant. Pour provoquer l'accumulation, le vent ne doit pas forcément être très violent. Elle peut en effet se produire avec un vent de 15 miles/heure (6.75 m/s) si les autres conditions sont favorables. Du point de vue météorologique, il faut un changement de direction du vent, du large vers la côte. La variation de la force et direction du vent et le déplacement du vent aident à desserrer la banquise, facilitant ainsi l'accumulation. Ceci ne se produira qu'avec des températures au dessus de 0°. Certaines équations dérivées nous renseignent sur la largeur requise de l'étendue d'eau libre pour provoquer l'accumulation, sur la vitesse nécessaire de la banquise, sur la hauteur de l'accumulation et sur les facteurs déterminants. Pour éviter la détérioration des propriétés littorales, nous proposons une accélération de l'accumulation, au large des côtes. Les références utilisées proviennent des études faites sur le lac Simcoe au printemps 1973.

Ice Piling on Lakeshores

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INTRODUCTION

In cold regions, ice forms on lakes and reservoirs during winter. Under certain weather conditions, the ice cover breaks up and ice floes are driven by wind and current on shore where they pile up against the shoreline. Ice piling on shores is a spectacular phenomenon, and one that also damages shoreline properties. In the spring of 1973, ice piled up on the shores of Lake Simcoe, Ontario, causing considerable damage and public concern.

This paper will study the structure of an ice pile, the mode of its formation, the meteorological conditions under which piling is likely to happen, the dynamic analysis of the factors that affect piling, and a proposed method of avoiding ice piling damage to shoreline properties. Although Lake Simcoe is used to illustrate the phenomenon, the application of the analysis of this study should be general.

Little is known quantitatively about the piling of ice on lakeshores and seashores. A descriptive paper on ice piling against coastal structures on seashores and lakeshores was written by Bruun and Straumsnes (1970). They reported that a height of 10-15 m may be reached by an ice pile under the action of strong wind and current and that a sloped bottom favours ice piling. Additional work was done by Bruun and Johanneson (1971). They reaffirmed the previous findings of Bruun and Straumsnes and reported an account of the building up of an ice pile in half an hour. In their paper, Bruun and Johanneson considered that the ice first breaks up into ice floes, which, when pushed by wind and current against a shore, accumulate into piles. They drew an analogy between the piling up of rubble in front of a bulldozer and the piling up of ice. Allen (1970) considered the piling of ice to be controlled by the thrust force on the ice floes. From the static equilibrium between the thrust force, the weight of the ice sheets that form the pile, and the friction force, he derived an equation which relates the height of an ice pile to the thrust pressure, the thickness of the ice sheet, the density of the ice pile, and the base angle of the pile. As this report will reveal, ice piling on lakeshores is a more complicated phenomenon than simple static piling. More studies have yet to be carried out to fully understand ice piling problems.

ICE PILING OCCURRENCES ON LAKE SIMCOE IN THE SPRING OF 1973

Geographical Characteristics of Lake Simcoe

Lake Simcoe (Fig. 1), with an area of 743 km^2, is located approximately 50 km north of Toronto, between Georgian Bay of Lake Huron and Lake Ontario. The maximum length of the lake is approximately 47 km. The depth of the lake ranges from generally less than 18 m at the northeastern end to 40 m at Kempenfelt Bay at the

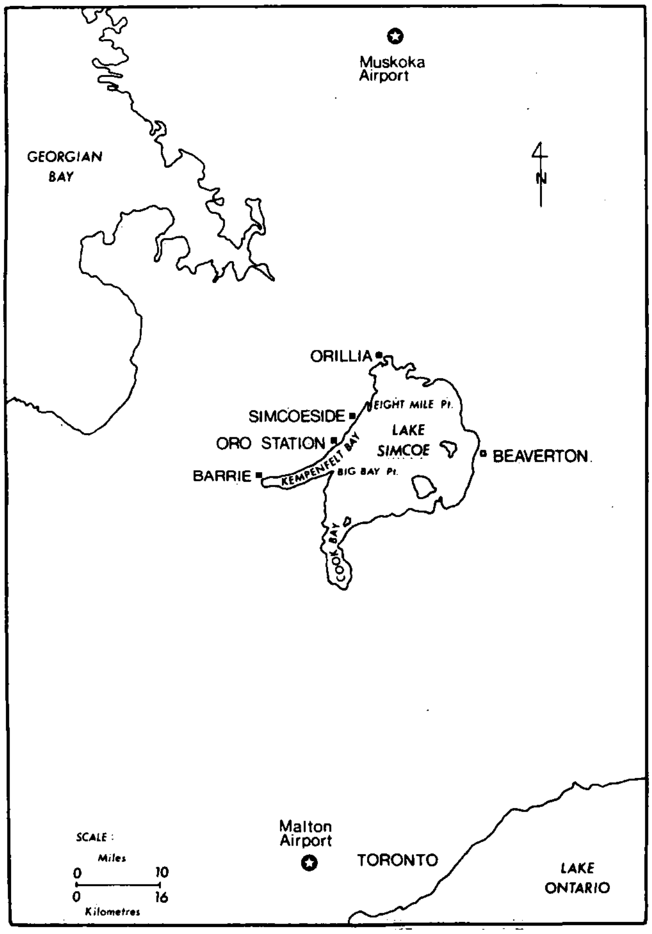


Figure 1. Geographical location of Lake Simcoe.

western end. The depth at the southern Cook Bay is shallow, generally less than 15 m. The depth at the central part of the lake ranges from 18 to 30 m. The lake level is 216.5 m above sea level. The land immediately surrounding Lake Simcoe is less than 305 m above sea level. About 48 km to the north-northeast and about 48 km to the southwest of Lake Simcoe, the lands rise to an elevation of 305-610 m above sea level.

The northeastern shore of Lake Simcoe is mature with many beaches and shoals. The southwestern and the northwestern shores are younger, with beaches and shoals not so evident. Lake Simcoe is a resort area for Toronto residents and numerous cottages are built along the shoreline. Ice forms over Lake Simcoe every winter, reaching a maximum thickness of about 0.5 m by the end of February or early March.

Description of the Ice Piling Occurrences on Lake Simcoe in the Spring of 1973

Based on information obtained from field inspections and interviewing local residents, the ice piling events on Lake Simcoe in the spring of 1973 may be reconstructed.

The first ice piling occurred on March 10, but the exact time could not be established. It was very foggy in the morning of that day. Within the limit of visibility, the nearshore region of the lake was observed to be completely ice covered. A continuous crack approximately 1.6 km from the shore was noted off Eight Mile Point parallel with the shoreline. Under the action of onshore winds, ice piled up against the northwestern shore from Simcoeside to Eight Mile Point, with Simcoeside being the area more affected. The height of the ice piles was not recorded but was believed to be low. Following the ice piling, an open-water gap was noted offshore beyond the shore-fast ice. During the day, the wind also drove the ice sheet from the southeastern shore. At Beaverton, an open-water gap of approximately 150 m was created between the ice sheet and the shore. Ice fishermen were stranded on the ice and had to be rescued.

The next ice piling occurred on March 11 at 1300 hours at the Oro area. At Oro Park, local residents noted ice piles 9 m high. Before the ice piling took place, the nearshore region of the lake was covered by an ice cover about 20 cm thick. The ice piling process was completed in 10 minutes.

The third ice piling was a major one; it began on March 16 at 2300 hours and ended on March 17 at 0130 hours. The area affected was the northwestern shore from Eight Mile Point to Big Bay Point, except for sheltered areas. On March 15 and during the daytime of March 16, the weather was warm and the wind blew from the westerly and southwesterly directions. The lake appeared completely ice covered except for a narrow open-water gap of about 200 m along the lakeshore. The gap was closed as the wind shifted to the opposite direction. Ice piling followed the closing of the gap. Again it was observed to be a fast process, as an ice pile was built in a matter of a few minutes. As a result, ice piles 9 m high were built up at Eight Mile Point and at Big Bay Point. The ice ran up on shoreline properties, overturned boathouses and boat ramps, and damaged some cottages (Fig. 2). During the ice piling, the wind had temporarily shifted to the northerly direction before shifting back northeasterly. This shifting of wind produced two rows of ice piles on the shore of Eight Mile Point (Fig. 3). On March 17, the wind shifted to the northwesterly and westerly directions. The ice sheet was pulled away from the shore, leaving ice piles behind and creating an open-water gap of about 0.75 km wide.

Figure 2. Cottage damaged by ice piling at Lake Simcoe.

Figure 3. Two rows of ice piles off Eight Mile Point.

The fourth ice piling occurred on March 31. On that day local residents at Eight Mile Point noticed a large open-water gap about 750 m wide along the shore. In the afternoon the ice sheet began to move inshore. At that time the wind did not appear to be strong and there was little wave action in the lake. Following the closing up of the water gap, ice began to pile up by rafting (see next section) from 1630 to 1730. The rate of growth of the ice piles was approximately 13 cm/min. When an ice sheet slid over another ice sheet, it moved smoothly without breaking and piling until it hit an obstacle, then it rafted again and started to pile up. Ice rafting and overriding (see next section) were observed during the evening. At one point, at a distance of 30 m from shore, ice was seen to pile to ten layers to approximately 3 m above water, and at a distance of 46 m, it was seen to pile to three layers to approximately 1 m above water. Ice piling continued to take place overnight. Ice piles more than 9 m high were seen the next morning (see Fig. 4).

Figure 4. High pile produced by the fourth ice piling.

PROCESS OF ICE PILING

Based on field inspections and reports from local residents, a typical ice piling on the shore of a lake may be summarized as follows. Ice piling usually follows the shifting of wind from the offshore to the onshore direction. Before ice piling takes place, the nearshore region of the lake is usually covered by a shore-fast ice sheet, the width of which may be from 30 m to 1.5 km. The lake itself is also ice covered. The ice floe on the main part of the lake can be quite large. An open-water gap exists between the shore-fast ice sheet and the main ice body on the lake. The open-water gap is created by the preceding offshore wind. As the wind shifts to the onshore direction, the main ice on the lake drifts towards shore and gradually closes the open-water gap. (On the other side of the lake, the ice sheet is pulled away from the shore, creating an open-water gap.)

As the main ice drifts onshore, the shore-fast ice is also pushed by the wind against the shore. Ice ridges sometimes form as a result of compression. As the wind strengthens and the lake becomes more agitated, longitudinal fractures begin to develop in the direction of the wind. The spacing of the longitudinal fractures is of the order of one hundred metres. The fractures divide the shore-fast ice into ice strips. Under the wind action, the ice strips buckle and crack into ice floes. Fracturing and breaking of the shore-fast ice happen only during the first ice piling on a lake or after a cold spell when the shore-fast ice refreezes into a continuous sheet. When the shore-fast ice strip is narrow, fracturing and cracking of the shore ice do not occur. In addition to the further breaking down of the shore ice and the main ice into smaller floes, when the main ice body from the lake rams onto the shore ice, the ice floes begin to telescope and pile up in the following three ways.

1. Overriding: – the upwind floe rides on the downwind floe (Figs. 5(a) and 6). This mode of telescoping is most common when the wind is strong and the surface of the water agitated. The overriding ice floe slides smoothly over the underridden ice floe until it is stopped by the shore or other obstacles, usually an ice block frozen to the ice sheet or an existing ice pile, and it then begins to pile up. An ice floe that rides up an existing ice pile remains smooth and intact, except for small fractures. When the ice floe overshoots the pile, the overshot part breaks off, disintegrates into small ice cubes, and falls to the onshore side of the pile (Fig. 7). Some of the ice cubes fall to the offshore side of the pile and accumulate at its base. Because these broken ice pieces are repeatedly pushed up the pile by the new ice floes and fall down the slope again, they become much finer than the ice cubes on the onshore side. The number of broken ice pieces on the offshore side of an ice pile is small and the smooth ice floe surface can be easily

exposed by scraping the broken ice pieces from the surface by hand.

2. Undersliding: – The upwind floe slides under the downwind floe (see Fig. 5(b)) until it meets the previous ice floe. The downwind floe then appears to be sitting on a large piece of ice floe. Undersliding of ice floes does not occur as often as does overriding. In the Lake Simcoe example, less than one undersliding floe was observed for every 10 overriding floes. Undersliding results possibly when the downwind floe of two contacting ice floes is momentarily lifted by wave action and the upwind floe slides under it. It is interesting to note that when undersliding occurs the piling seems to stop (Fig. 8).

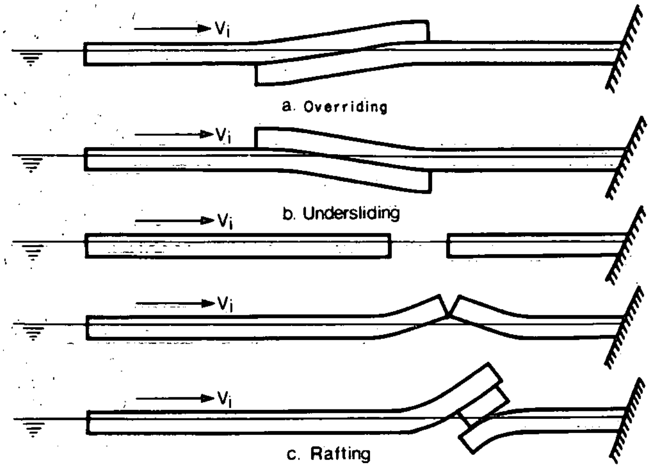


Figure 5. Three modes of telescoping of ice floe.

3. Rafting: – Firstly the drifting ice from the lake presses against the shore-fast ice, then the contacted edges buckle, turn upwards, and break off. The upwind ice floe will then override the downwind floe with the broken ice piece sandwiched between them (Fig. 5(c)). The overriding floe slides easily over the shore-fast ice until it meets an obstacle, when it either rafts again and continues to proceed or begins to pile up. Rafting is the predominant mode of ice floe telescoping when the wind is not strong and little wave action is present in a lake. On Lake Simcoe, rafting was observed on March 31 with the incoming ice floes from the lake moving at a velocity of approximately 0.25 m/s. The broken ice pieces in front of an ice pile formed by rafting were also observed to be fine in size (Fig. 4). This is understandable because the ice pieces broken off by rafting are ground between the two ice floes as the overriding ice floe slides forward.

Figure 6. An overriding ice floe on Lake Simcoe.

Figure 7. Broken ice pieces on the onshore side of an ice pile.

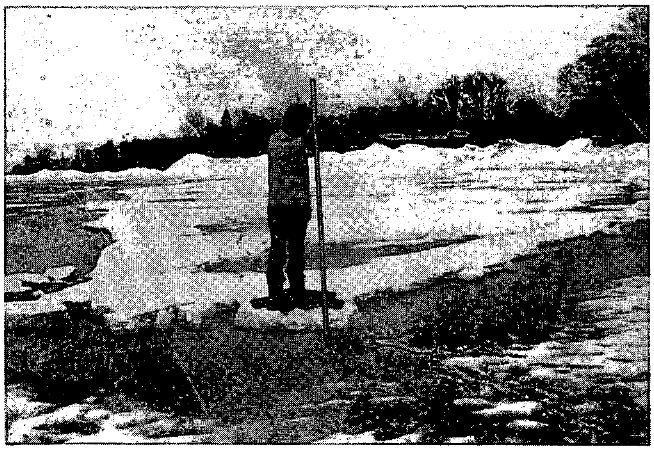
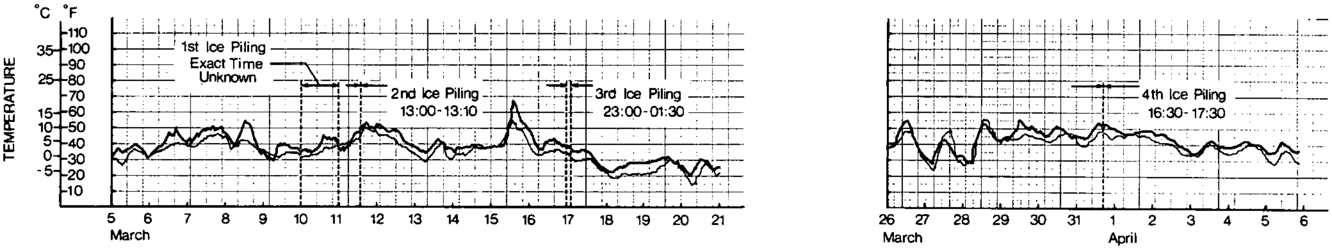


Figure 8. An undersliding ice floe on Lake Simcoe.

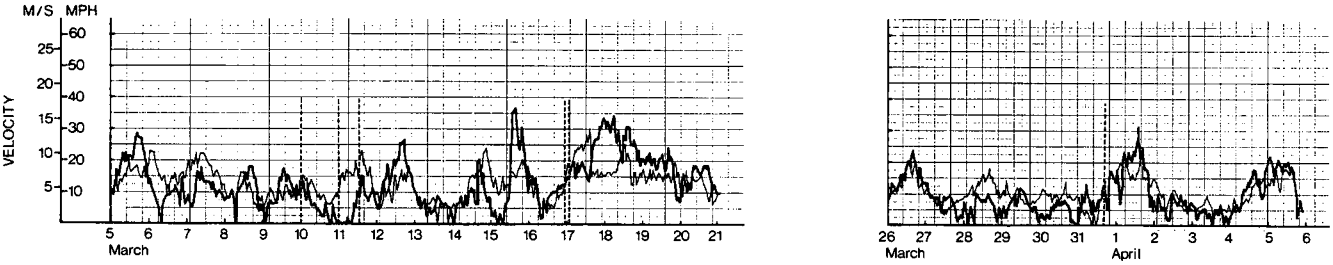
Piling can occur by all the modes above simultaneously, but generally one mode will predominate. An ice pile consists usually of a gentle apron and an abrupt piling. Measurements at Lake Simcoe showed that the gentle apron slope of an ice pile is of the order of 1 in 300. For the ice pile itself, the maximum slopes measured were about 45° and the common slope was about 30°. The onshore and offshore slopes of the ice piles were about the same.

METEOROLOGICAL PARAMETERS THAT AFFECT ICE PILING AND THEIR PHYSICAL INTERPRETATION

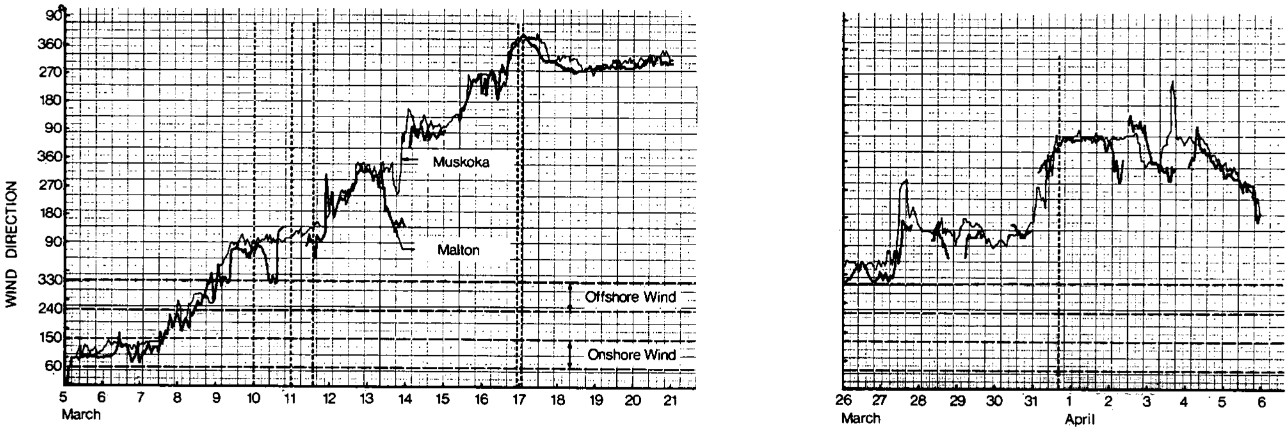
To determine the meteorological parameters that affect ice pilings on Lake Simcoe, wind and temperature data from Malton International Airport weather station and Muskoka Airport weather station were plotted against time as shown on Figure 9. The plotted weather data were



Air Temperature



Wind Velocity



Wind Direction

Figure 9. Weather data from the Malton and Muskoka weather stations (March and April 1973).

hourly averages. The Malton and Muskoka stations are about 160 km apart along an approximately south-north line with Lake Simcoe midway between them (Fig. 1). Figure 9 shows that weather data at the two stations follow a similar trend; consequently the weather data may be applicable to ice piling on Lake Simcoe. Greater fluctuations in velocity are recorded at Malton than at Muskoka, which may be due to the effect of lake breezes from Lake Ontario and the urban environment. To filter the local weather effect for the Malton data, the wind direction and speed were not plotted if the wind was less than 2 m/s and showed great fluctuation. The weather data used covered the periods of March 5-20, 1973, and March 26-April 6, 1973. The first three ice pilings occurred during the first period and the fourth ice piling occurred during the second. The time of the ice piling occurrences is also shown on Figure 9. As the exact time of the first ice piling was not known, two dotted lines are drawn to indicate the period in which the first ice piling took place.

Before an attempt is made to derive a general rule for ice piling, the meteorological conditions associated with each ice piling are noted and their physical interpretations are made as shown below.

First and Second Ice Pilings, March 10 and 11

One can see from Figure 9 that, before the ice piling, the air temperature over Lake Simcoe had been above freezing for several days, except for short intervals. From the beginning of March 7 until noon of March 9, the wind had been continuously shifting from the southeast clockwise back to the southeast. On the day of the ice piling (March 10), immediately prior to the event, the wind over Malton and the wind over Muskoka showed great differences both in speed and direction, which, as shown by a study of the weather maps of the period, was caused by the development of a storm centre over the area. Ice began to pile up when the wind shifted to easterly and began to increase.

From the weather plot, one notes the striking feature that the ice pilings did not occur when the wind was at its strongest nor when it was blowing persistently in the onshore direction. Figure 9 shows that from March 5 to 7, although strong winds had been blowing persistently from the east and southeast, at times reaching a speed of 12.5 m/s, no ice piling was reported. Ice piling is thus obviously not a static consequence of ice sheets being pushed by wind and current against the shore. On the contrary, ice piling is a dynamic event. It occurs when ice sheets, which have attained a sufficiently high velocity under the action of wind and which, because of their great mass, possess great momentum, ram onto the shore-fast ice. Part of the kinetic energy of the incoming ice sheets is used in breaking the shore ice and the ice sheets themselves, part is used to overcome friction, and part is converted into potential energy by piling up on the shore. In fact, only by such a dynamic event can heaps of ice be built up in a matter of a few minutes.

With this understanding in mind, we may now examine the first two ice pilings on Lake Simcoe from a dynamic point of view. Following a persistent southeasterly trend, the wind shifted gradually clockwise through 360° back to the easterly direction. This shifting wind loosened the ice sheets from the shores. From 0000 hour, March 8, over a period of about 20 hours, a northwesterly wind drove ice sheets from the northwestern shore of the lake. As the wind shifted back to the east comparatively quickly, the ice sheets were moved back again towards the northwestern shore. The existence of an open-water fetch or gap caused by the preceding northwesterly wind permitted the ice sheets to gather speed. During the development of the storm centre, although the ice floes did not gain noticeable additional speed, sufficient energy was imparted by the wind to maintain their velocity. The arrival and ramming of the moving ice sheets onto the shore thus caused the first ice piling from Simcoeside to Eight Mile Point. Another patch of ice floes, moving towards Oro Park over a longer fetch of water and under an increasing wind, built up a greater momentum and piled up to a greater height on the Oro shore.

It is evident from Figure 9 that a strong wind is not necessary for ice piling. If the open-water fetch is sufficiently long, the velocity sustained by a wind of 4.5 m/s to 6.75 m/s is sufficient to produce ice piling.

One can easily see that no ice piling will occur if the wind shifts before the ice floes gather speed and hit the shore. It is also clear that, if the open-water fetch is short, a strong wind will be required to impart a sufficiently high velocity to cause ice piling. Similarly, too weak a wind will never produce a velocity great enough for the ice floes to pile up, even if the open-water fetch is long. As velocity is the determining factor, ice piling should also occur when two ice sheets moving at different velocities ram into each other. Further, ice piling will occur on a windless day if the ice floes have acquired sufficient momentum from an earlier wind. In Lake Erie, a huge ice sheet of approximately 1.6 km^2, which was seen from a helicopter to ram onto the shore on a windless day, built ice piles, as high as 4 m in a matter of minutes (Foulds, 1973).

Third Ice Piling, 2300 Hours, March 16 to 0130 Hours, March 17

For this ice piling occurrence, Figure 9 shows that, prior to the ice piling, the wind showed a continuous shifting in direction from easterly clockwise back to northeasterly. From 1300 hours, March 15 to 1300 hours, March 16, the wind blew persistently from the west and northwest. The wind drove the ice away from the northwestern shore and created an open-water fetch. As the wind quickly shifted to northeasterly during the night of March 16, the ice floes were blown towards the shores with a sufficiently high velocity, to produce major ice piles at Eight Mile Point and Big Bay Point. It is noted from Figure 9 that when the ice piling occurred the wind was only slightly more than 6.75 m/s.

Following the ice piling, the wind shifted to the northwesterly direction and blew at great velocities up to 13.5 m/s for several days. However, no ice piling was reported on the southeastern shore. A study of the temperature records shows that during this period the air temperature was below freezing. The ice floes therefore could have been frozen to the shore or to each other. In fact it is evident from Figure 9 that ice piling stopped as soon as the air temperature dropped to the freezing point. This indicates that friction between ice floes increases greatly as soon as the temperature drops to freezing. Thus, ice piling is not likely to take place when the temperature is below freezing.

Fourth Ice Piling, 1630 Hours to 1700 Hours, March 31

The weather plot (Fig. 9) indicates that, before the ice piling, great differences existed in wind strength and direction at the Malton and Muskoka weather stations. A study of the weather maps of the period shows that this difference in wind speed and direction was the consequence of a storm centre staying over the area during the night of March 30 and the morning of March 31, until its departure in the afternoon of March 31. The fluctuation in wind speed and direction associated with the storm centre loosened the ice floes on the lake. Immediately prior to the ice piling, from 1000 hours to 1400 hours, March 31, after having blown for several hours from the northwest, the wind shifted quickly to the east. Ice piling began not long after the wind had shifted to the easterly direction at 1630 hours. Again it is seen from the weather data that the ice piling occurred at a wind velocity of about 6.25 m/s. Although the wind increased to twice this strength later and continued to blow in the same direction, no further ice piling occurred.

Based on the above information, one may draw the following conclusions:

1. Ice piling is not a static but a dynamic process. Under the action of onshore wind, ice floes pick up sufficient momentum over a length of open-water fetch to produce ice piling when they ram onto the shore or the shore-fast ice. During ice piling, the kinetic energy of the ice floes is used to overcome friction and break up ice sheets, and is converted into potential energy of the ice piles.
2. A strong onshore wind is not a requirement for ice piling on lakeshores. Ice piling can occur at wind speeds of less than 6.75 m/s when other conditions are favourable.
3. An open-water fetch is necessary for ice floes to gather sufficient speed for ice piling. A shift of wind from the offshore direction to the onshore direction over a short period allows the ice floes to use the whole open-water fetch to gather speed, and consequently promotes ice piling.
4. Fluctuation of wind strength and direction and shifting of wind help to loosen the ice floes and promote ice piling.
5. Ice piling will not occur at below freezing temperatures.

DYNAMIC ANALYSIS OF ICE PILING

With the physical process of ice piling in mind, we may now proceed to study: (1) the relationship between the speed of the ice floes, size of the ice floes, strength of the wind, and width of the open-water fetch; and (2) the relationship between the height of an ice pile, size of the ice floes, and velocity of the ice floes.

Relationship between the speed of an ice floe, size of the ice floe, strength of the wind, and width of the open-water fetch

From the preceding sections we understand that an open-water fetch is necessary for ice floes to gather sufficient speed to pile up on the shore. Here we shall study the width of the open-water fetch and the time required for an ice floe to obtain the critical velocity for ice piling.

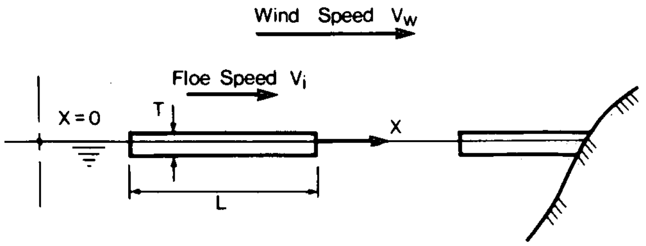


Figure 10. Definition diagram of ice floe moving onshore.

For an ice floe of unit width, length L, thickness T, and moving at a velocity V_i towards the shore under an onshore wind V_w (Fig. 10), the wind drag and the water drag acting on it are, respectively,

$$D_a = C_a \frac{\rho_a}{2} (V_w - V_i)^2 \; L \qquad (1)$$

and

$$D_b = C_b \frac{\rho_b}{2} V_i^2 \, L \qquad (2)$$

where ρ_a is the density of air, ρ_b is the density of water, C_a is the drag coefficient between air and the ice floe, and C_b is the drag coefficient between water and the ice floe. In writing equation (2) it is assumed that there is no current component in the onshore direction. Because the speed of an ice floe is much less than that of the driving wind, V_i may be dropped from equation (1). Using the above drag forces and defining the onshore direction as the positive x direction, the equation of motion for an ice floe may be written as

$$\frac{1}{2} C_a \rho_a V_w^2 - \frac{1}{2} C_b \rho_b \left(\frac{dx}{dt}\right)^2 = \rho_i \, N \, T \frac{d^2x}{dt^2} \qquad (3)$$

where ρ_i is the density of the ice floe and N is the coefficient of virtual mass due to the moving water. The division of the equation above by

$$\frac{1}{2} C_b \rho_b V_w^2$$

gives

$$\frac{\frac{1}{2} C_a \rho_a}{\frac{1}{2} C_b \rho_b} - \left(\frac{\frac{dx}{dt}}{V_w}\right)^2 = \frac{\rho_i \, N \, T}{\frac{1}{2} C_b \rho_b V_w} \frac{d}{dt}\left(\frac{\frac{dx}{dt}}{V_w}\right) \qquad (4)$$

In bringing V_w inside the differentiation sign, it is assumed that the wind speed V_w is constant. The writing of

$$\frac{\frac{1}{2} C_a \rho_a}{\frac{1}{2} C_b \rho_b} = K^2, \qquad \frac{\rho_i \, N \, T}{\frac{1}{2} C_b \rho_b V_w} = A \qquad (5)$$

and

$$\frac{\frac{dx}{dt}}{V_w} = \frac{V_i}{V_w} = V_*$$

changes the equation above to

$$k^2 - V_*^2 = A \frac{dV_*}{dt} \, . \qquad (6)$$

In equation (5), $V_i = dx/dt$ is the velocity of the ice floe. The integration of equation (6) with the boundary condition that $V_* = V_i = 0$ at $t = 0$ gives

$$t = \frac{A}{2K} \ell n \frac{K+V_*}{K-V_*} \qquad (7)$$

which may be further written in a non-dimensional form as

$$t_* = \ell n \frac{1+V_*/K}{1-V_*/K} \qquad (8)$$

where

$$t_* = \frac{t}{\frac{A}{2K}} \qquad (9)$$

is the non-dimensional time.

Equation (8) may also be written as

$$e^{t_*} = \frac{K+V_*}{K-V_*} \qquad (10)$$

from which V_* may be solved as an explicit function of t_*

$$V_* = K \frac{e^{t_*} - 1}{e^{t_*} + 1} \, . \qquad (11)$$

If the position at which the ice floe has zero velocity is chosen as the origin of x, the distance travelled by the ice floe in time t is given by

$$x = \int_0^t V_i \, dt = \frac{AV_w}{2K} \int_0^{t_*} V_* \, dt_* \, . \qquad (12)$$

The substitution of equation (11) into the equation above and the subsequent integration give

$$x_* = \frac{x}{\frac{AV_w}{2K}} = K \, \ell n \frac{(e^{t_*} + 1)^2}{4e^{t_*}} \qquad (13)$$

which relates the time and distance travelled by the ice floe. By substituting equation (10) into equation (13) one obtains the relationship relating x_* and V_*

$$\frac{x_*}{K} = \ell n \frac{1}{1 - (V_*/K)^2} \, . \qquad (14)$$

From equation (8) and (14), curves relating $\frac{V_*}{K}$ and t_* and $\frac{V_*}{K}$ and $\frac{x_*}{K}$ are plotted on Figure 11 as shown by curve

A and curve B, respectively. Figure 11 shows that V_*/K asymptotically approaches unity as t_* increases. This, in fact, is expected as it is evident from equation (6) that, when the acceleration of the ice floe dV_i/dt approaches zero, which results in $dV_*/t = (dV_i/dt)/V_w$ approaching zero, as time progresses, V_* will approach K.

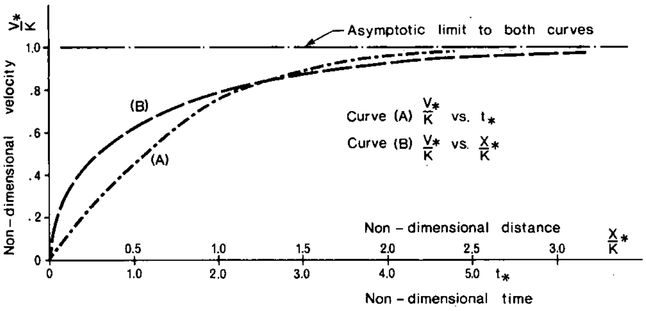


Figure 11. Motion of ice floe under wind.

Because the length of an ice floe is usually much greater than its thickness, the form drag on the ice floe therefore is small compared with the skin drag. For this reason, the drag force in equations (1) and (2) may be approximated by the skin drag alone. For wind and water skin drag on an ice floe, Bruun and Johanneson (1971) proposed the following drag coefficients in combination with the density of air and water:

$$\text{between air and ice} \quad \frac{1}{2} C_a \rho_a = \frac{9.8}{3600} \quad \text{kg/m,}$$

$$\text{between water and ice} \quad \frac{1}{2} C_b \rho_b = 9.8/4 \quad \text{kg/m.}$$

By using the above drag coefficients and a coefficient of virtual mass of 2, for an ice floe of density $\rho_i = 920$ kg/m^3 and thickness $T = 0.3$ m, driven by a wind of $V_w = 6.75$ m/s, the values of K and A are calculated from equation (5) to be 1/30 and 33.4, respectively. With this value of K, one sees that the terminal value of the non-dimensional velocity V_* is 1/30, or that the terminal velocity of the ice floe V_i is 1/30 the driving wind velocity V_w.

We can see from Curve A in Figure 11 that the value of t_* for $V_*/K = 0.9$ is approximately 3. Thus, the time required for the ice floe to obtain 90 per cent of its terminal velocity is

$$t = \frac{A}{2K} t_* = \frac{33.4 \times 3}{2 \times (1/30)} = 1500 \text{ s.} = 25 \text{ min.}$$

From Curve B we can see that the corresponding value of x_*/K at the above value of V_*/K is 1.66. The distance travelled by the ice floe when it attains 90 per cent of its terminal velocity therefore is

$$x = \frac{AV_w}{2K} x_* = \frac{33.4 \times 6.75}{2} \times 1.66 = 190 \text{ m}$$

The time and distance calculated above tend to be underestimated as the time for stopping the offshore motion of the ice floe by the onshore wind, the time for initially loosening the ice floe, and the mutual interference of neighbouring ice floes are not considered. A factor of 2 probably will be sufficient to take these factors into account.

The theoretical calculations above are supported by the Lake Simcoe observations. As mentioned in the earlier parts of the paper, an ice piling occurred almost immediately following the shift in wind direction from offshore to onshore, and the width of the open-water fetch prior to the events was comparable to the calculated width.

The building of an ice pile by moving ice floes

The piling up of ice floes is a process of energy conversion. For a moving ice floe of length L, thickness T, and velocity V_i approaching an ice pile (Fig. 12), the work and energy equation may be written as

$$(N \rho_i T L) V_i^2/2 = W_f + (\rho_i g H^2 T)/(2 \sin\beta) + W_c \quad (15)$$

where the left side of the equation represents the kinetic energy of the ice floe, N again taking into account the virtual mass of the moving water; W_f is the work required to overcome friction; the second term on the right is the potential energy of the part of the floe piling up; and W_c is the work required to fracture the ice floe. The symbol g is the gravitational acceleration and is equal to 9.8 m/s^2. This equation stands whether the oncoming ice floe hits the ice pile directly, or hits some other loosened ice floes in front of the ice pile first, forcing them to pile up. In the latter case, however, telescoping may take place and some of the kinetic energy will be used to overcome the friction between the telescoping ice floes. W_c is difficult to estimate and may be dropped from the equation above by making allowances for the friction term. To determine the work required to overcome friction, from Figure 12, one has

$$W_f = \int_0^{\frac{H}{\sin\beta}} f \rho_i g T S \cos\beta \, dS = \frac{1}{2} f \rho_i g T \frac{\cot\beta}{\sin\beta} H^2$$

where f is the coefficient of friction. Substituting the expression above into equation (15) and noting that the terminal velocity of the ice floe is 1/30 the wind velocity, we find that

$$H = \frac{V_w}{30}\sqrt{\frac{N\ L\ \sin\beta}{g(1+f\ \cot\beta)}}. \qquad (16)$$

The kinetic friction coefficient between two smooth ice slabs sliding at a relative velocity of 4 m/s is 0.02 (Handbook of Chemistry and Physics, Chemical Rubber Publishing Co., 1971). For this study, a friction coefficient of 0.1 may be used to allow for the roughness of the ice floes and the work required to fracture the ice floe. With this value of f and a value of 2 for N, for an ice slab 150 m long driven by a wind of 6.75 m/s, approaching an ice pile of base angle 45°, the height of the ice pile is calculated from equation (16) to be approximately 1 m.

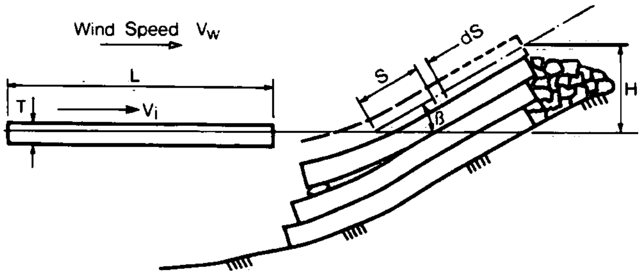


Figure 12. Piling of an ice floe.

The above height is calculated for a two-dimensional example, or for ice piling over the entire frontage of the ice floe. In reality, this is seldom the case. Because of the irregular shape of an ice floe, ice would pile only over part of its frontage while the other part would remain in water. If the portion of the frontage of an ice floe involved in ice piling is 1/10, which is of the order observed in some of the Lake Simcoe ice piles, the calculated height will be 10 m, which is the height attained by some of the Lake Simcoe piles.

Equation (16) indicates that less height will be attained by ice piles of smaller base angle β. This was indeed observed on Lake Simcoe. Field inspections showed that for most higher piles, the base angle was of the order of 45°. Field observations also showed that higher piles were formed by larger floes as shown by equation (16).

DAMAGE ON SHORELINE PROPERTIES BY ICE PILING AND PREVENTIVE MEASURES

By now the physical process of ice piling has become clear. When ice piles, the kinetic energy is converted into potential energy. From a shore erosion point of view, ice piling is good, for otherwise the kinetic energy of the drifting ice floes would be absorbed by the shore itself. For the Lake Simcoe ice pilings, only one lot front about 100 m wide showed signs of beach damage (Fig. 13). At this location, the ice floe broke into small pieces before piling and the broken ice pieces piled up in an irregular manner. Apparently the ice floe had dug into the beach before being crushed and pushed up; this may have occurred because there was no shore-fast ice to shield the beach from the incoming ice floes. For shoreline protection it is therefore desirable to maintain a strip of shore-fast ice.

Figure 13. Shore erosion caused by ice piling.

As mentioned before, Bruun and Straumsnes (1970) found that a sloped shore favours ice piling and recommended vertical walls for coastal structures. From the present study, we see that, although a vertical wall would delay the piling of ice at least until a slope was built by the ice floes, all the kinetic energy of the incoming ice floes has to be absorbed by the walls.

As ice climbs up an ice pile, its kinetic energy is gradually converted into potential energy. If the ice floe possesses greater kinetic energy than the potential energy corresponding to the height of the pile, the ice floe will overshoot the pile. The overshot part then will break and fall to the front of the pile. The broken ice pieces falling to the front possess only the kinetic energy which is associated with their individual mass and are incapable of doing much damage. At Lake Simcoe many trees of 3 or 4 cm diameter were observed to remain intact even when partly buried by the front part of an ice pile. On the other hand, if the overshot part of the ice floe meets an obstacle before breaking off, then the whole ice floe acts as an energy reservoir and the overshooting ice floe is capable of doing great damage (Fig. 2).

It is also noted that, after an ice floe has climbed to its highest point under momentum, it will slide back under gravity, until a static equilibrium is reached; the height of the ice pile is then controlled by static equilibrium. It is because of this that ice piles after an event are not indicators of the greatest height that was attained. If the wind shifts to the offshore direction after an ice piling event, the piling ice may slide back into the lake (see Fig. 14 and compare it with Fig. 2). Sometimes part of the piling ice may break off during backsliding, and remain on shore. The shear surface shows clearly the laminar structure of an ice pile (Fig. 15).

Figure 14. Recession of an ice pile.

Figure 15. Laminar structure of an ice pile.

As damage to shoreline properties is caused by the kinetic energy of the moving ice floes, one could protect the shoreline properties by initiating the conversion of kinetic energy into potential energy well offshore. A possible way of initiating ice piling would be to set up ice wedges in the form of ski jumps at some distance from the shore before the breakup. The ice wedge might then cause an early piling of the incoming floes before they reached the shore. However, systematic investigation of this remedial method has yet to be done.

DISCUSSION

Although some physical insight has been gained from the study and certain conclusions have been reached, much still remains to be done. For instance one could study the effects of the slope of the shore on ice piling, the factors that affect the size of ice floes, and the effect of the geometric shape of an ice floe on ice piling. From the study, ice piling is seen to be a short time event. The gustiness or the instability of the wind could therefore be of some significance for ice piling. The assumption in the study that there was no onshore current should be rectified. If the onshore current should not be neglected, the equation of motion would be more complicated and the coefficient of virtual mass would have a different value.

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