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NATURAL FLOW OF NORTH SASKATCHEWAN RIVER AT ALBERTA - SASKATCHEWAN BOUNDARY BY THE RIM-STATION METHOD

TECHNICAL BULLETIN No. 1

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WATER RESOURCES BRANCH
DEPARTMENT OF NORTHERN AFFAIRS AND NATIONAL RESOURCES
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1. Introduction

Estimates of the natural flow in the North and South Saskatchewan Rivers at the Alberta-Saskatchewan Boundary may be required for use in the division of the waters between the provinces. Adjustment of measured flows at the boundary for the effect of upstream uses is a difficult problem and therefore the reliability of the resulting estimates of natural flow is questionable in basins such as that of the South Saskatchewan River, where there is a complex pattern of water use developments. An alternative, sometimes called the "rim-station" method, has been suggested. Measurements of natural flow at "rim-stations" or key stations in the basin above the regions of development, are correlated against computed or recorded natural flows at the boundary. The correlations would be used to compute natural flow data at the boundary in future years, when developments between the rim-stations and the boundary increase in complexity.

Prior to the completion of the Brazeau reservoir in October 1961, there was no significant interference with the natural flow of the North Saskatchewan River in Alberta. Thus the records in this basin prior to October 1961 provide an opportunity to develop correlations by the rim-station method. This report describes the results of a preliminary investigation into regression equations.

The station on the North Saskatchewan River at Rocky Mountain House was chosen as the rim-station. Simple regression and multiple regression techniques involving precipitation were developed. Standard errors in the simple regressions are relatively large and the improvement obtained by multiple regressions involving precipitation on the basin below the rim-station proved to be less significant than expected. However, the equations presented in this report permit estimation of flow at the boundary with a standard error of 8% to 12% for the months of June to October, 20% for the months of November to March, 32% for April and 24% for May.

2. Types of Investigations

The investigations carried out fall into the following general categories:

(a) Simple Regressions

Linear regressions of flows at the boundary on flows at the rim-station, North Saskatchewan River at Rocky Mountain House.

(b) Stepwise Multiple Regressions

Regressions of flows at the boundary on flows at the rim-station and on other variables associated with precipitation on the basin between the rim-station and the boundary.

3. Simple Regression Equations

Results of the simple regression analysis are shown in Table 1. The equations may be used to obtain monthly natural flow at the Alberta-Saskatchewan Boundary from the measured flow at Rocky Mountain House. Standard errors of estimate are shown in Table 3.

The equations involve deviations from the long-term mean of the logs of the monthly flows. The figures for long-term means were derived from the period 1944 to 1961, or some lesser period for those months where records for the full period were not available. The long-term means are given in Table 4.

4. Multiple Regression Equations

The multiple regression equations shown in Table 2 permit somewhat more accurate estimates of monthly natural flows at the Alberta-Saskatchewan Boundary. They contain variables associated with precipitation on that part of the basin lying between Rocky Mountain House and the Boundary. Standard errors of estimate for the multiple regressions are shown in Table 3 and the improvement over the simple regressions is indicated.

5. The Precipitation Variables for the Multiple Regressions

The following two precipitation variables are used in the multiple regressions:

X_2 - Excess precipitation

X_3 - Total precipitation

(a) Excess Precipitation - X_2

All precipitation data are obtained from the eleven Department of Transport meteorological stations shown in the headings in Table 5. Total rainfall in any 24-hour period at any of these reporting stations is reduced by the appropriate allowance for infiltration shown in Table 6. The remainder, if any, is considered to be excess precipitation. It is multiplied by the appropriate Thiessen weight shown in Table 7 to produce the excess for the zone in question. The values of the excess for the zones are divided into portions applicable to the current and following months respectively, using the appropriate ratios from Table 5. The sum of the portions applicable to the current month plus the portions brought forward from the preceding month is the required value for X_2 for the multiple regression equations. An example of the calculation of X_2 for the month of August 1956 is shown in Table 8.

(b) Total Precipitation - X_3

The total reported precipitation at each of the eleven meteorological stations in the two months immediately preceding the month in question is adjusted by multiplying by the appropriate Thiessen weight. The sum of the adjusted values from the eleven stations is the required value of X_3 for the multiple regression equations.

TABLE 1

Simple Regression Equations

Month	Equation	Note
November to March incl.	$Y = 0.915\ X_1$	Y = Deviation of the log of a single month's flow in North Saskatchewan at Alta.-Sask. Boundary from the long-term mean of the logs of flows for that month, as given in Table 4.
April	$Y = 1.025\ X_1$	
May	$Y = 1.194\ X_1$	
June	$Y = 1.295\ X_1$	
July	$Y = 1.077\ X_1$	X_1 = Deviation of the log of the same month's flow in North Saskatchewan River at Rocky Mountain House from the long-term mean of the logs of flows for that month, as given in Table 4.
August	$Y = 1.299\ X_1$	
September	$Y = 1.284\ X_1$	
October	$Y = 0.988\ X_1$	

TABLE 2

Multiple Regression Equations

Month	Equation
April	$Y = 0.9963\ X_1 + 0.1478\ X_3 - 0.2271$
May	$Y = 0.9166\ X_1 + 0.1164\ X_3 - 0.2080$
June	$Y = 1.020\ X_1 + 0.2105\ X_2 + 0.01071\ X_3 - 0.07623$
July	$Y = 1.001\ X_1 + 0.1349\ X_2 + 0.008323\ X_3 - 0.07243$
August	$Y = 1.269\ X_1 + 0.05481\ X_2 + 0.0001333\ X_3 - 0.02042$
September	$Y = 1.135\ X_1 + 0.07885\ X_2 + 0.02091\ X_3 - 0.1410$
October	$Y = 0.9713\ X_1 + 0.01439\ X_3 - 0.06515$

Y = Deviation of the log of a single month's flow at Alta.-Sask. Boundary from long-term mean of logs of flows for that month, as given in Table 4.

X_1 = Deviation of the log of the same month's flow at Rocky Mountain House from the long-term mean of flows for that month, as given in Table 4.

X_2 = Excess precipitation factor for the month, as determined by method described in this report.

X_3 = Total precipitation on the basin between Rocky Mountain House and the Alta.-Sask. Boundary for the two preceding months, as determined by method described in this report.

TABLE 3

Standard Errors of Estimate for the Regression Equations

Month	Simple Regressions		Multiple Regressions	
	Log Units	Per cent	Log Units	Per cent
Nov. to March incl.	0.088	20	*	*
April	0.148	35	0.136	32
May	0.136	32	0.103	24
June	0.073	17	0.039	9
July	0.045	10	0.036	8
August	0.049	12	0.050	12
September	0.051	12	0.044	10
October	0.053	12	0.049	11

* No multiple regression equations developed for winter months.

TABLE 4

Long-term Means of Logs of Discharges for Calendar Months

Month	Long-term Mean of Logs of Discharge	
	Rocky Mountain House	Alta.-Sask. Boundary
January	2.955	3.198
February	2.942	3.106
March	2.942	3.139
April	3.298	3.747
May	3.766	4.016
June	4.102	4.300
July	4.103	4.277
August	3.975	4.147
September	3.736	3.943
October	3.443	3.684
November	3.194	3.418
December	3.051	3.161

Note: Data for Alta.-Sask. Boundary taken from gauging station at Frenchman Butte up to September 1958 and from station at Lea Park from October 1958.

TABLE 5

Per Cent of Excess Precipitation Occurring as Runoff at Boundary in Current Month

Date of Rain		Meteorological Station			
May July Aug. Oct.	Apr. June Sept.	Lloydminster Ranfurly Vegreville	Edmonton Wetaskiwin Athabasca	Calmar Sion	Rocky Mtn. House Edson Entrance
31	30	6%	0%	0%	0%
30	29	16	6	0	0
29	28	25	16	6	0
28	27	33	25	16	6
27	26	40	33	25	16
26	25	46	40	33	25
25	24	52	46	40	33
24	23	57	52	46	40
23	22	62	57	52	46
22	21	66	62	57	52
21	20	70	66	62	57
20	19	73	70	66	62
19	18	76	73	70	66
18	17	79	76	73	70
17	16	82	79	76	73
16	15	84	82	79	76
15	14	86	84	82	79
14	13	88	86	84	82
13	12	89	88	86	84
12	11	91	89	88	86
11	10	92	91	89	88
10	9	93	92	91	89
9	8	94	93	92	91
8	7	95	94	93	92
7	6	96	95	94	93
6	5	97	96	95	94
5	4	98	97	96	95
4	3	98	98	97	96
3	2	99	98	98	97
2	1	99	99	98	98
1		100	99	99	98

TABLE 6

Assumed Infiltration Rates

Meteorological Station	Infiltration ins. per 24 hours	Note
Rocky Mountain House	0.6 ins.	Reported 24-hour rainfalls at the meteorological stations are reduced by the quantities shown above. The remainders, if any, are adjusted by the appropriate Thiessen weights from Table 7 and the adjusted values then divided between the current and following months by application of ratios from Table 5. Sum of current month values plus total carryover from previous month gives value for X_2 for the multiple regression equations.
Edson	0.6	
Entrance	0.6	
Calmar	0.8	
Sion	0.8	
Edmonton	1.0	
Wetaskiwin	1.0	
Athabasca	1.0	
Lloydminster	1.2	
Ranfurly	1.2	
Vegreville	1.2	

TABLE 7

Thiessen Weights

Meteorological Station	Thiessen Weight
Calmar	0.113
Edmonton (A)	0.126
Lloydminster	0.077
Ranfurly	0.148
Rocky Mountain House	0.134
Sion	0.079
Vegreville	0.170
Wetaskiwin	0.014
Athabasca	0.042
Edson	0.043
Entrance	0.054
	1.000

TABLE 8

Sample Computation of X_2

August 1956

Date of rain	Meteorological Station	Excess Precip. ins.	Adj. Precip. ins.	Allocation to months: Current Month ins.	Next Month ins.
(1)	(2)	(3)	(4)	(5)	(6)
August 2	Rocky Mtn. House	0.04	0.0054	0.0053	0.0001
15	Edson	0.04	0.0017	0.0013	0.0004
22	Edson	0.12	0.0052	0.0027	0.0025
16	Entrance	0.80	0.0432	0.0328	0.0104
23	Entrance	0.40	0.0216	0.0099	0.0117
4	Calmar	0.12	0.0136	0.0132	0.0004
4	Sion	1.02	0.0806	0.0782	0.0024
3	Edmonton	0.03	0.0038	0.0037	0.0001
4	Edmonton	0.01	0.0013	0.0013	0.0000
16	Edmonton	0.64	0.0807	0.0666	0.0141
3	Ranfurly	0.15	0.0222	0.0220	0.0002
	From August Precip.			0.2370	
	Brought forward from July			0.0347	
	Value of X_2 for August			0.2717	
	Carried forward to September				0.0423

Note 1 - Value shown in column (3) is reported precip. reduced by appropriate allowance for infiltration from Table 6.

Note 2 - Value shown in column (4) is value from column (3) multiplied by appropriate Thiessen weight from Table 7.

Note 3 - Value in column (5) is value from column (4) multiplied by appropriate ratio from Table 5.

Note 4 - Value in column (6) are differences in values from columns (4) and (5).

6. Comments on the Simple Regression Method

The station North Saskatchewan River at Rocky Mountain House was selected as the rim-station; the monthly flow at that station is the independent variable in all the regressions. The dependent variable, monthly flow at the Alberta-Saskatchewan Boundary, was obtained from the station at Frenchman Butte prior to October 1958 and at Lea Park thereafter.

The method used for correlation is that described by Langbein (1960). Monthly discharges at each station are converted to logarithms. Long-term means of these logarithms are computed for each calendar month at each station and also the deviation of the log of each month's discharge from the appropriate long-term mean. A linear regression is computed for the deviations of the dependent station on the deviations of the independent station. A FORTRAN program has been written for the correlation method. It is presented in a separate report.

Correlations were carried out for various groupings of the available monthly data from April 1944 to September 1961. The following groups were tested:

(a) All months in a single group.

(b) All months in the winter period November to March.

(c) All Aprils and Mays, the break-up period.

(d) All months in the open-water period June to October.

(e) Individual months from March to October inclusive.

The correlations of the individual months demonstrated that there is an appreciable difference in the variance of the logs of flows from month to month and also a seasonal change in the slope of the regression line. The correlations based on groups of months were therefore discarded, except in the case of the winter period November to March.

7. Comments on the Multiple Regressions

It was reasoned that multiple regressions involving data for precipitation on that part of the basin lying between the streamflow stations might improve the errors of estimate in the simple regressions. It was decided to investigate by stepwise multiple regression techniques the effects of winter snowfall and summer precipitation on the subsequent flows at the Boundary.

Many different combinations of the monthly precipitation data were investigated and two alternative methods of dealing with heavy summer precipitation were tested. The method of treatment used in the multiple regression equations in Table 2 evolved from these investigations. Lower standard errors of estimate for the study period April 1944 to September 1961 were produced by this treatment than by any of the other alternatives studied.

No attempt was made to develop multiple regression equations for the winter months November to March inclusive.

It may be noted in Table 3 that the multiple regressions permit more accurate estimates of Boundary flows for the April to October period than can be obtained from the simple regressions, most significant improvement being for the high-flow month of June.

8. Factors Affecting the Errors of Estimate

The inherent errors in the streamflow data constitute a limiting factor on the errors of estimate for the regressions. The magnitude of the measurement errors is not known but they will be greatest in the ice forming and break-up periods, somewhat lower in mid-winter and lower still in the summer months. It is possible that the large standard errors of estimate for the month of April and the lower but still large errors of estimate for the winter months and the month of May may be partially attributable to measurement errors in the streamflow data.

REFERENCE

Langbein, W.B., 1960. Hydrologic data networks and methods of extrapolating or extending available hydrologic data. Hydrologic Networks and Methods. W.M.O. Flood Control Series No. 15.

The short period of overlapping record, while satisfactory for the simple regressions, is not very satisfactory for stepwise multiple regressions involving numerous independent variables because of the loss of a degree of freedom as each new variable is added.

Errors in the estimates of annual or seasonal flow at the Boundary obtained by summing the monthly estimates will be appreciably lower than the errors for the individual months indicated in Table 3. An investigation of the standard errors in annual estimates has not been made for this report but it is estimated that it will be in the neighborhood of 5%. It is important to note that the lowest errors of estimate are those for the months of June and July, which are the months of highest flow.

SUPPLEMENT TO
TECHNICAL BULLETIN NO. 1

Water Resources Branch
Department of Northern Affairs and National Resources

Estimates of annual flow of North Saskatchewan River at the Alberta-Saskatchewan Boundary for water years 1954 to 1964 inclusive have now been computed from the multiple regression equations for individual months presented in Technical Bulletin No. 1. Each month's flow was estimated from the appropriate equation and the estimated annual flows obtained by summing the monthly estimates. The simple regression equations were used for the months of November to March, no multiple regressions having been developed for these months. The period used for the tests contains three years, 1962 to 1964, which were not used in deriving the regression equations.

A comparison of the estimated and actual annual flows at the Boundary is given in Table 9. Actual flows for the years 1962 to 1964 were obtained by adjusting the measured flows for storage in Brazeau Reservoir, which came into operation in water year 1962.

The 11 year sample indicates that the monthly regression equations can be used to predict the total annual flow with an average error of -4%. Indicated standard deviation in the prediction errors is 5% of the total flow. Thus about two-thirds of the predictions of annual flow at the Alberta-Saskatchewan Boundary should lie within the range of error of +1% to -9%.

TABLE 9

Errors of Estimate of Annual Flow
at Alberta-Saskatchewan Boundary
(Obtained by Monthly Multiple Regression Equations)

Water Year	Measured Flow at Boundary acre-ft.	Estimated Flow at Boundary acre-ft.	Error in Estimate acre-ft.	Error in Estimate %
1954	9,195,000	8,239,700	-955,300	-10.39
1955	6,374,000	5,985,900	-388,100	- 6.09
1956	5,377,000	4,980,700	-396,300	- 7.37
1957	4,601,000	4,198,600	-402,400	- 8.75
1958	6,338,000	6,108,100	-229,900	- 3.63
1959	4,888,000	4,901,000	+ 13,000	+ 0.27
1960	4,780,000	4,714,500	- 65,500	- 1.37
1961	4,574,000	4,910,700	+336,700	+ 7.36
1962	5,152,800*	4,796,400	-356,400	- 6.91
1963	6,134,100*	6,194,600	+ 60,500	+ 0.99
1964	5,943,200*	5,425,600	-517,600	- 8.71

* Adjusted for Brazeau Reservoir Storage.

Average error of estimate	-4.0%
Standard deviation in error of estimate	5.4%