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ABSTRACT

A hydrologic model based on the water and energy balances is proposed for the Lake Ontario local drainage basin (27,100 square miles, comprising the entire local contributing area except for the lake's water surface area of 7,500 square miles). Using a hypothesis which provides estimates of the actual regional evaporation, regional moisture values are obtained which, when routed, simulate the measured monthly outflows from the land area. The model provides a means for studying the hydrology of the basin, simulation of long periods of record and basic relationships which can be used in developing forecasting techniques. Further development of the techniques used is continuing with a view to wider application of the model.

A HYDROLOGIC MODEL OF THE LAKE ONTARIO LOCAL DRAINAGE BASIN

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INTRODUCTION

The use of hydrologic models is a relatively recent development. The purpose of most models to date has been to investigate the response or behaviour of a hydrologic system which can be described mathematically but not necessarily hydrologically, and permit the extension of data beyond available observations.

In its initial stages of development, the model described in this paper was intended for use in a study of the factors related to the persistence of the local supplies to the Great Lakes. Knowledge of persistence would be useful in simulation of supplies to the lakes. In addition, such a model would provide the basic relationships from which methods for forecasting future supplies could be derived.

The present model has been based on a regional concept of actual evaporation which permits the use of the water balance and provides an estimate of the volumetric relationship between precipitation, evaporation and runoff.

Other investigators have used potential evaporation as an estimate of the water loss from evaporation. This approach is satisfactory where conditions are humid and very little restriction is placed on the water available to the soil surface and plants. However, in areas where arid conditions may prevail for periods greater than the unit of the time base of the model, empiricisms must be used to obtain realistic values of the actual water loss due to evaporation.

Although the model developed in this paper is used for the Lake Ontario local drainage basin, the model has much wider application if the concept of regional evaporation is valid. Further development and application of the model to other areas which have widely different hydrologic conditions will permit the verification or indicate the fallacy of the assumptions made.

REVIEW OF THE LITERATURE

In recent work in the hydrologic simulation of runoff data, the water balance has been used as the basic relationship. Dooge (1960) derived a hydrologic model for the computation of low flows from small basins in a humid region where groundwater constitutes a large portion of the total outflow. He used potential evaporation as an estimate of evaporation. By assuming linear groundwater storage, he routed the excess moisture to obtain outflows.

Crawford and Linsley (1966) also used potential evaporation in a small watershed model for use where surface runoff is a large portion of the total runoff. Clark's (1945) routing method was used in modified form to route excess moisture to outflow. Groundwater outflow was simulated by a type of recession curve.

To obtain more reliable estimates of actual evaporation under all conditions of moisture supply, Morton (1965) introduced the concept of evaporation as a regional phenomenon and defined potential evaporation as a characteristic of the air mass passing over a region. The hypothesis proposed, if valid, provides an estimate of the actual evaporation from a region. It permits the development of a model in which moisture excesses and deficiencies have more rational values than those obtained when potential evaporation is used.

The Lake Ontario local drainage basin has been thoroughly described by Morton and Rosenberg (1961). From that description, it can be seen that the Lake Ontario basin has a total area of 34,600 square miles, including 7,500 square miles of lake surface area. The local contributing land area is about 27,100 square miles, and is the area to which the hydrologic model described herein is applied. Of this area, about 12,200 square miles is in Canada and 14,900 square miles, in the United States. Precipitation and runoff throughout the basin are relatively constant with the exception of the eastern end of the area which is influenced by orographic and lake effects, which cause higher precipitation and lower evaporation and result in greater runoff in all seasons of the year from this area.

DATA

The basic data used were of monthly duration since climatological and hydrologic data are most readily available for this time period. Further, it is doubtful whether shorter periods could produce results which would be useful in the regulation of Lake Ontario.

For evaporation computations, the climatological variables of temperature, wind, humidity and sunshine were used. These data were abstracted from the monthly summaries of the Weather Bureau, United States Department of Commerce and the Meteorological Branch, Canada Department of Transport. All available data were used. Snowfall data were also abstracted from the two last-mentioned sources.

Derivation of precipitation data for use in the model is most important since selection and extrapolation of data can lead to errors. In order to obtain precipitation data that would be consistent with runoff data, a double mass plot was made of two sets of precipitation data and runoff. The initial set was derived from reports of the U.S. Weather Bureau and Canadian Meteorological Branch as part of the model study using all stations and weighting with respect to area but not with respect to numbers of stations. The double mass plot of these data versus runoff showed the precipitation data for recent years to be biased. The second set of monthly precipitation data for the Great Lakes basin was available from the United States Corps of Engineers, United States Lake Survey. These data are weighted by stations and basin areas. A double mass plot showed these data to be consistent with the runoff data and they were used in the model.

Since the model required verification, the inflow to the lake from the local contributing area was estimated. Gauged representative basins were used to extrapolate the runoff distribution in ungauged areas. The totals of the gauged runoff and the estimated runoff for the ungauged areas were computed on a monthly basis and used as the values of the local inflow. Runoff data were abstracted from the publications of the United States Geological Survey and the Water Resources Branch.

DEVELOPMENT OF THE MODEL

The objective in developing the model was to adhere as closely as possible to the basic relationships of the water and energy balance of the basin. The elements of the model are shown in the flow chart in Figure 1.

The model is used to obtain an estimate of the actual regional evaporation. Regional evaporation (E_R) is defined in the equation developed by Morton (1965) as

$$E_R = (1-a)\ G - E_P \qquad .....(1)$$

where a is the monthly mean albedo of the surface of the region

G is the total insolation per month in evaporation units

E_P is the potential evaporation of a region

The potential evaporation E_P can be estimated using the method from Penman (1948), the modification to Penman's method proposed by Morton (1965) or using pan data where available.

The values of albedo (a) used in the computation of regional evaporation for the period April through October were determined from data by Morton (1965). These values are shown in Table 1 and are those used in the model. Albedo values for November through March are those obtained initially from values of the albedo of snow published by List, taking into account the accumulation and depletion of the snow cover. These values are modified slightly to reduce to a minimum the error in the 30-year total. The values obtained after modification are not outside the range of values which have been published.

TABLE 1

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Albedo (a)	0.78	0.83	0.61	0.23	0.17	0.13	0.11	0.11	0.14	0.18	0.51	0.66
(1-a)	0.22	0.17	0.39	0.77	0.83	0.87	0.89	0.89	0.86	0.82	0.49	0.34

The water balance of a region may be simplified and written for the period of a month as

$$P - E_R - R = \Delta S \qquad .....(2)$$

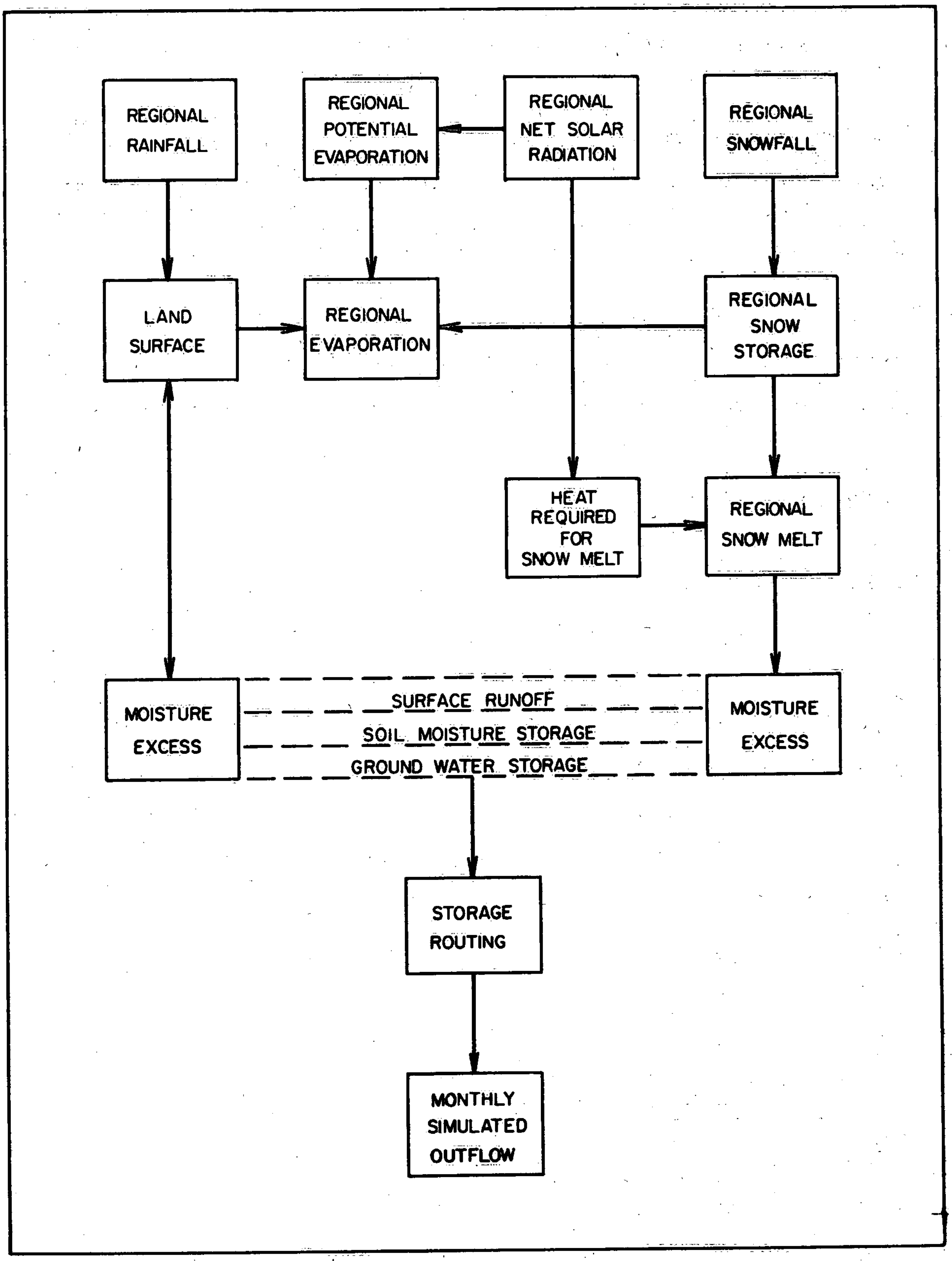

Figure 1. Flow chart of hydrologic model of Lake Ontario basin.

where P is the regional monthly precipitation

E_R is the regional monthly evaporation

R is the regional monthly runoff

ΔS is the change in regional moisture storage

The regional runoff (R) and the change in regional moisture storage (ΔS) are related since ΔS is equal to the change in soil moisture and the change in groundwater storage. These latter quantities are relatively unknown on a regional basis.

Therefore it is assumed that

$$R + \Delta S = M_R \quad .....(3)$$

where M_R is the regional moisture

The use of the concept of regional moisture is based on the reasoning that positive values of regional moisture represent the moisture which is in the snowpack, in groundwater storage and moving through the soil, and which eventually will be measured as runoff from a region. Negative values represent a deficiency in the soil moisture and groundwater which must be recharged before moisture is again available to run off. The concept of regional moisture has the advantage that this value could be used as an evaluation of antecedent conditions which may be useful in short term forecasting.

From (1), (2) and (3) the moisture balance can be written

$$P - [(1-a)\ G - E_P] = M_R \quad .....(4)$$

This is the basic equation of the model as shown in the flow chart in Figure 1.

Equation (4) can be used to estimate the regional moisture when the regional evaporation is equal to or less than the potential evaporation. During the months November through April when evaporation is low and the surface is moist or covered with snow, the regional evaporation equals the potential evaporation ($E_R = E_P$) according to Morton (1965) and equation (1) becomes

$$E_R = \frac{(1-a)\ G}{2} \quad .....(5)$$

and (4) becomes

$$P - \left[\frac{(1-a)\ G}{2}\right] = M_R \quad .....(6)$$

When snow is accumulating on the ground

$$P = P_S + P_R \quad .....(7)$$

where P_S is the precipitation in the form of snow

P_R is the precipitation in the form of rain

Equation (4) can be written for conditions when precipitation accumulates as snow

$$P_S - \left[\frac{(1-a)\ G}{2}\right] = S_A \qquad .....(8)$$

where S_A is the accumulation of snow on the ground

and when precipitation occurs as rain (6) becomes

$$P_R = M_R \qquad .....(9)$$

On most of the basin contributing to Lake Ontario, only limited snowfall accumulates as a snowpack and occasional melting reduces its moisture-holding capacity. The rainfall which occurs during the winter months is assumed to be immediately included in the regional moisture. Only the limited mountainous area of the eastern portion of the basin experiences snow accumulation containing significant moisture storage.

For the months in which snowmelt normally occurs (March through May) the accumulated snow (S_A) is melted during the first month (March) and the heat used in this process is computed using

$$S_M = \frac{S_A \times 80}{596} \qquad ....(10)$$

where S_M is the heat required for snowmelt in evaporation units, and 80 and 596 are the latent heat of fusion of ice and the latent heat of vapourization of water, respectively.

From the heat balance, the heat required for snowmelt should be deducted from that available for evaporation and for March, April and May when significant snowfall is available for snowmelt, equation (8) becomes

$$S_A + P - \left[\frac{(1-a)\ G}{2} - S_M\right] = M_R \qquad ....(11)$$

To sum up the operation of the model during the winter months, it is assumed that evaporation takes place from the snow surface and any rainfall which occurs is immediately included in the regional moisture. The snowfall minus evaporation accumulates until March at which time it normally melts. Any snowfall which occurs after March is assumed to melt within the month in which it occurs and contributes to the regional moisture which is computed using equation (11).

RELATIONSHIP BETWEEN REGIONAL MOISTURE AND OUTFLOW FROM THE CONTRIBUTING BASIN

On a basin the size of the Lake Ontario local contributing area, the volume of the surface runoff contribution to streamflow is small compared to the groundwater flow since surface runoff tends to occur scattered over the basin and seldom would all of a large basin surface area be contributing to streamflow at one time (Betson, 1964). The nearest the basin would come to such an event would probably occur in the spring during the period of snowmelt. Indeed, from Figure 2, it would appear that small departures occur in the winter and spring months indicating errors in this assumption. However,these errors appear as negative in the winter and the resulting error decreases as the season progresses at which time the computed values approach the measured values, and the volumetric relationship remains.

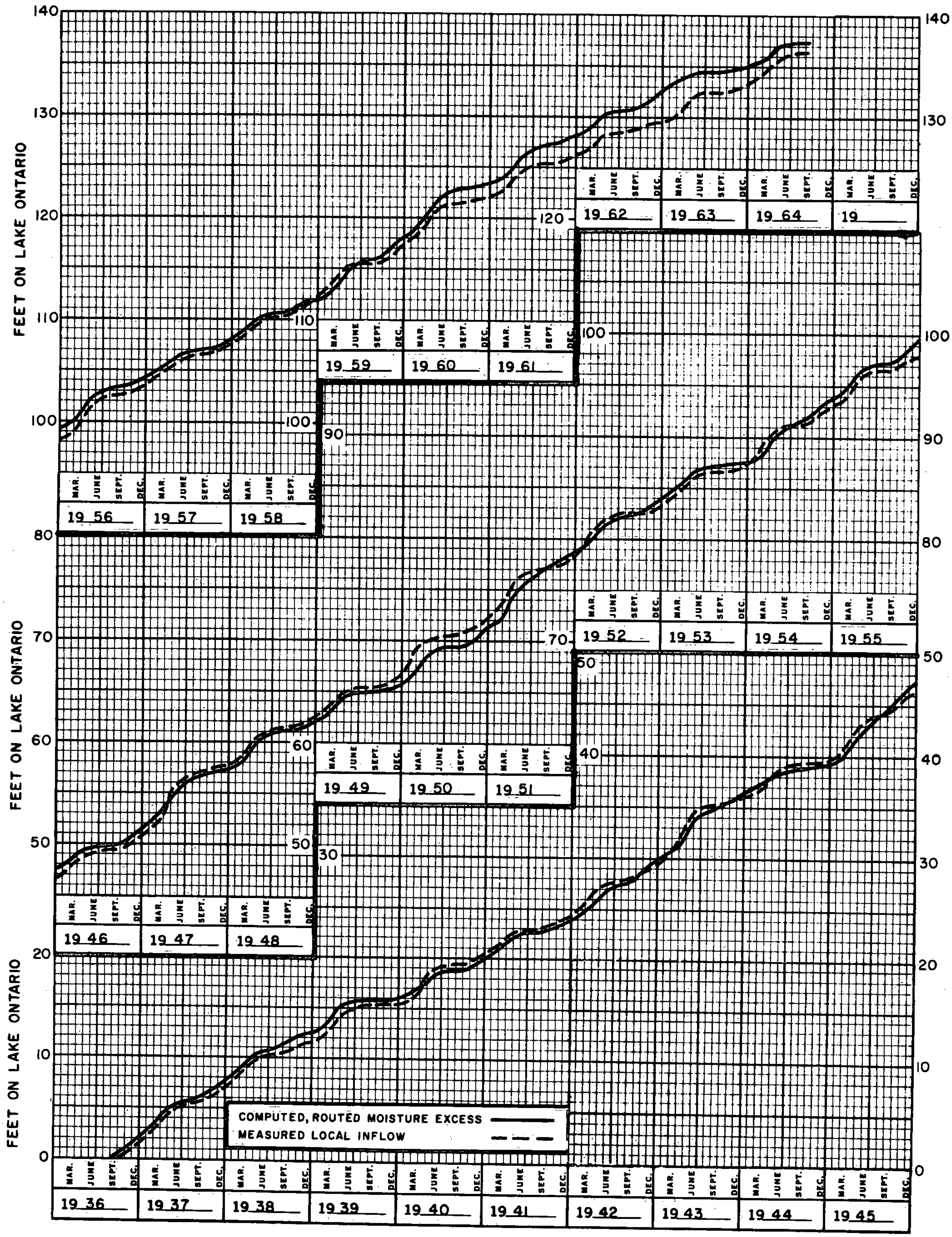

Figure 2. Cumulative local inflow and routed computed moisture excess, 1936-1964.

Therefore, the regional moisture after passing through storage is equivalent in volume to the regional runoff. By a storage routing procedure the regional moisture values become the computed discharges from the contributing basin. Based on the work of Dooge (1960) in which the assumption of linear storage elements permits the use of a linear routing equation, the following equation was used in routing M_R to obtain the computed outflow as a first approximation.

$$Q_n = 0.270\ M_{Rn} + 0.216\ M_{Rn-1} + 0.514\ Q_{n-1} \qquad \ldots(12)$$

where Q is the total outflow from the basin in one month

n refers to the time period of one month

Since linear storages are used, the principle of superposition can be used. Therefore, using trial and error, more storage elements could be used to refine the routing model.

VERIFICATION OF THE MODEL

Two analyses were performed to determine whether the model provided a good representation of the outflow from the local contributing area to the lake. In the first, the cumulative measured outflow was plotted on a time base with the cumulative computed outflow in Figure 2. Good agreement was achieved and no persistent systematic errors resulted. However the computed values show that the estimates of outflow were low during the years 1949, 1950 and 1951. In addition the estimates of outflow were high during the years 1959 through 1964. The errors in the model are probably in the assumptions made, whereas errors in the basic data are probably in the precipitation data.

Secondly, the annual values of measured and computed outflow were plotted on the basis of a September through August water year. This water year was chosen with reference to the measured outflows which consistently showed that the lowest flows occur in August. As a result of a correlation of the annual values, the correlation coefficient of 0.92 with a standard error of ±0.03 was obtained. Figure 3 shows the scatter diagram of these data. Larger errors appear to occur at the extremes as shown by Figure 3.

DISCUSSION

Since verification has demonstrated that the model contains minor inconsistencies, it would be useful to discuss the principal assumptions of the model and the possible errors in the basic data.

The assumptions of the model are:

(1) It is assumed that the albedo of the surface is constant for any given month irrespective of the moisture condition of the surface.

(2) It is assumed that the principal snowmelt period occurs in March and all snowfall in previous months accumulates until then.

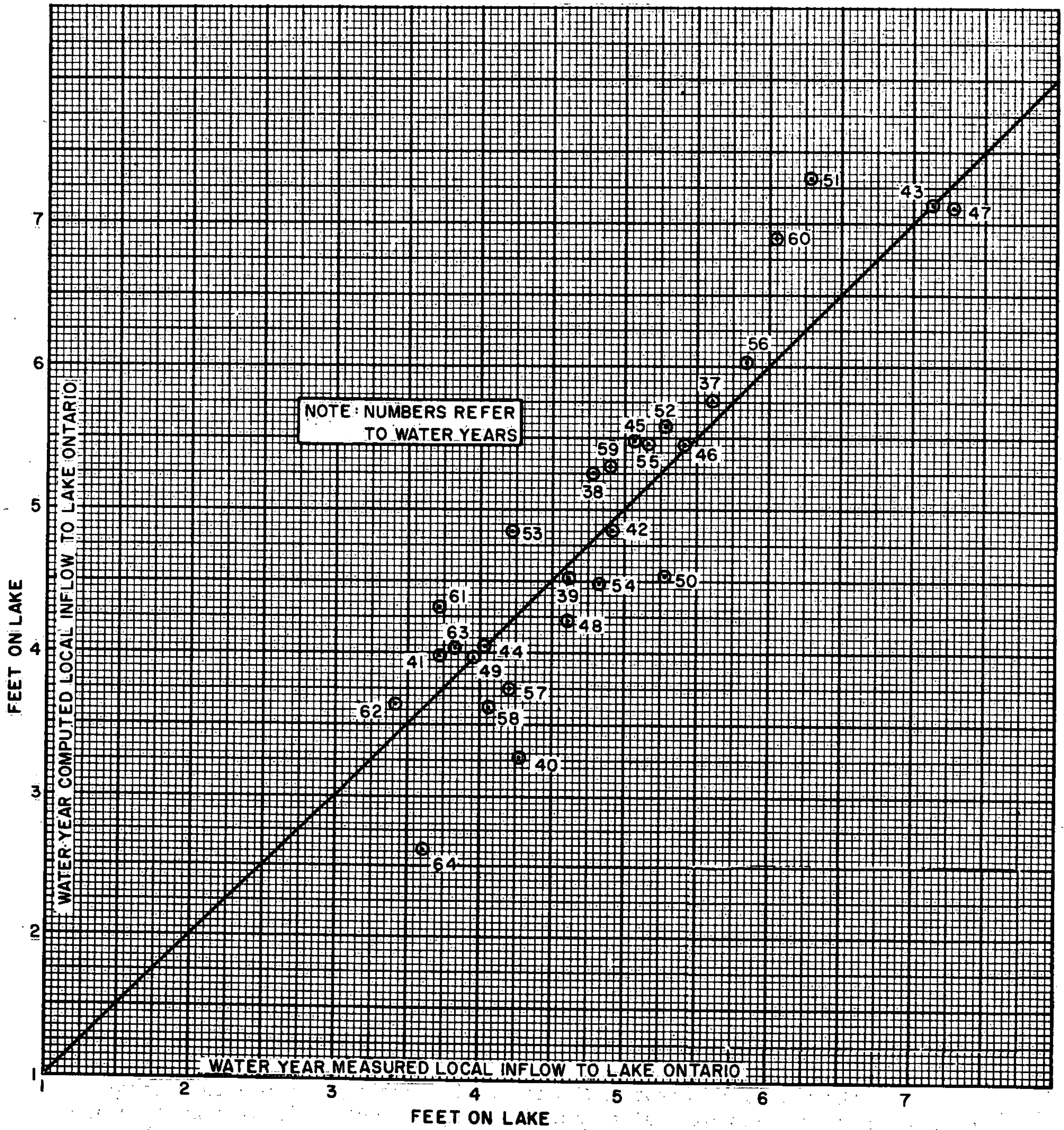

Figure 3. Annual local inflow and annual computed local inflow September-August, 1936-1964. (water year: September-August)

(3) It is assumed that the surface runoff is a relatively small portion of the total runoff.

(4) It is assumed that there are no large deep groundwater aquifers which could carry over large volumes of water for one or more years and the groundwater basin is equivalent to the surface water basin.

If any or all of the basic assumptions were in error, a regular deviation between the computed outflow and the measured outflow would be shown in Figure 2. The errors which are shown are regular and could be attributable to assumption (1) being in error since the deviations occur during the two extremes of the record (see Figure 3). If, as shown by published values, the albedo increased as the surface dried out and decreased as it became wet, this would lead to lower and higher evaporation respectively, [Equation (1)] which could account for the high estimates of outflow in the period 1960-1964 and the low estimates of outflow in the period 1949-1951.

Errors attributable to assumption (2) can be seen by departures of the computed outflow from the measured values during the winter and early spring months of December through May.

Similar errors due to assumption (3) occur in the early spring, and the major portion of the error in estimating the spring peak flow and fall peak flows, where they occur, could be attributed to assumption (3).

From Figure 2 it is doubtful that there are major errors caused by assumption (4) since a regular departure over long periods would be expected as the aquifers were recharged and discharged. If this were happening, the errors shown would be in a direction opposite to those shown in Figure 2, that is, during periods of excess (high moisture), recharge would be taking place with a resultant lower apparent basin outflow and the converse would occur in periods of low moisture with a higher apparent outflow because of groundwater discharge. Further, errors which have been attributed to errors in assumption (1) could also be caused by leakage from groundwater bodies into the basin during periods of high moisture excess, and out of the basin during periods of low moisture since higher water levels in groundwater bodies could cause shifts in the groundwater divide. Further study of the major groundwater bodies in the basin is required to determine whether the errors of the magnitude shown in Figure 1 could be attributable to this assumption.

Excluding measurement errors, the possible errors in the basic data are:

(1) Errors resulting from extrapolation of point precipitation measurements.

(2) Errors resulting from extrapolation of the climatologic observations of temperature, humidity and radiation.

(3) Errors resulting from extrapolation of gauged runoff to ungauged areas.

In the use of observational data it is normally assumed that errors of measurement are randomly distributed. This assumption is also applied here. However, the extrapolation of point measurements requires considerable exercise of judgement. Errors of this type might be suspected from the wider variations of the computed from the measured values at the extremes as shown in Figure 3. If this type of error were present, it would indicate that point precipitation measurements tend to overestimate precipitation during periods of low precipitation and underestimate precipitation during periods of high precipitation. This speculation requires the further assumption that error in the runoff data is minimal. This type of error could be minimized by a thorough study of the climatological and streamflow records so that areas could be classified and more objective extrapolations made which would assist in minimizing this type of error.

Although there are errors which represent deficiencies in the model, this study demonstrates that this type of hydrologic model shows promise for use on large areas but reduction in the area to which it is applied should improve the results. The model verifies relationships which could be useful in forecasting. With further development, the climatological data could be simulated which could provide a useful technique in verifying stochastic models.

This model should be considered as an initial approximation. Further development in the application to smaller areas and verification of the general principles used in other areas will add to its utility.

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