INLAND WATERS BRANCH

DEPARTMENT OF THE ENVIRONMENT

Temperature Distribution Due to the Release of Heated Effluents into Channel Flow

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Foreword

Ever increasing demands for energy are resulting in some situations in which water systems receive large quantities of waste heat in restricted localities. Waste heat is often dissipated in rivers or lakes, from which it eventually reaches the atmosphere. The elevated temperatures in the receiving waters are cause for concern and in recognition of this, the Canada Centre for Inland Waters has initiated a research program to assess the effect of the waste heat on the environment.

This report examines the mixing of warm water outfalls with the receiving river and proposes a method for computing the temperature distribution downstream. The relative importance of the direct transfer of heat to the atmosphere and the effect of mixing on the river temperature is also examined.

Other studies at the Canada Centre for Inland Waters related to waste heat include the study of thermal plumes in lakes, the use of infrared remote sensing for detecting movement of heated effluents, a contracted study of the present and future waste heat load in the Great Lakes up to the year 2000 and an ongoing follow-up study of waste heat discharges for the rest of Canada and of potential beneficial ways of using some of the waste heat.

These studies will contribute to the knowledge required to assess the effects of dispersing of waste heat into the environment, and serve to help establish guidelines to minimize any adverse effects of such discharges.

J.P. Bruce,
Director, C.C.I.W

Summary

Existing temperature prediction equations for heated discharges into rivers are shown to be inadequate. An improved method of estimating river temperature is presented in which the initial jet mixing is accounted for by assuming a virtual source to be located some distance downstream of the outfall. The temperature and width of the virtual source and its distance from the outfall are estimated from published data on heated jets. A solution for the heat diffusion equation is given, using the conditions at the virtual source as boundary conditions. This solution can be used to predict the distribution of temperature in the river from the virtual source downstream.

Résumé

Les équations existantes de la température pour les débits thermiques dans les rivières se révèlent insuffisantes. On présente une méthode améliorée pour estimer la température d'une rivière dans laquelle on tient compte du mélange initial du jet en supposant une source virtuelle située à une certaine distance en aval de l'embouchure. On estime la température et la largeur de la source virtuelle et sa distance de l'embouchure selon des données publiées sur les jets thermiques. On fournit une solution pour l'équation de la diffusion thermique, en utilisant les conditions qui se trouvent à la source virtuelle comme conditions aux limites. On peut utiliser cette solution pour prédire la distribution de la température dans la rivière de la source virtuelle en aval.

Introduction

The discharge of heated effluents and its effects on the receiving waters has become a problem of major concern and in recent years a great deal of research has been carried out to investigate the extent of these effects. One of the main goals is to be able to predict changes in the temperature distribution due to such thermal discharges.

In practice, receiving water bodies can be classified into three main types, each with its own but by no means completely distinct characteristics. These are (a) reservoirs and lakes, (b) rivers and streams and (c) cooling ponds. Edinger and Geyer [1965] has discussed these different types of receiving waters and presented simple analysis of the temperature distribution for each type. An extensive survey of the many aspects of thermal pollution, including lists of bibliography, was made by Parker and Krenkel [1969]. The present report deals more specifically with thermal discharges into rivers, which are generally more tractable because the gravitational force dominates and essentially establishes the flow pattern and turbulence characteristics. Although a number of equations have been proposed, there is still a need for a rapid but reasonably accurate method of predicting temperature distribution in a river and the aim of this report is to provide such a method as well as to point out some of the areas where knowledge is required before the problem can be better understood and improvements of the temperature prediction can be made.

Parker and Krenkel [1969] in their extensive review, has already summarized various equations used for predicting stream temperatures. Most of these either used the energy budget approach or some form of an equation balancing convection and surface heat transfer which required an empirical heat exchange coefficient.

The energy budget approach was used by Raphael [1962], Messinger [1963], Garrison and Elder [1965] among others. Essentially this method finds the change in heat content in a section of river by summing the inflow and outflow of heat and the various surface heat transfer terms. This requires measurement of all the surface heat transfer terms including incoming and reflected solar radiation, evaporation and convection. To use the energy budget approach requires a good deal of precise and reliable instrumentation which makes this method infeasible in many cases.

In the second method of temperature prediction, the net surface heat transfer is assumed to be directly proportional to the difference between the surface temperature and a fictitious equilibrium temperature. The proportionality constant K can be evaluated if an evaporation formula for the river is known. This simplification enables the heat transfer equation to be solved quite readily and was used by Duttweiler [1963], Edinger and Geyer [1965], Velz and Gannon [1960] and others. These models invariably result in an exponential decay of the temperature distribution.

All the models mentioned above are one-dimensional, assuming temperature distributions which are uniform vertically and laterally. This kind of model can be applied only after the effluent has been mixed throughout the depth and has spread across the whole width of the river. Even then, the variation of velocity in a river cross section would give rise to an apparent diffusion in the direction of flow. This convective transport, termed longitudinal dispersion, is usually quite significant but none of the above models took it into account. Except in very small streams where the thermal discharge represents all or most of the flow, the one-dimensional model is unlikely to be very useful. For many cases, the heated effluent discharged at one bank would have been diluted to an insignificant excess temperature even before the plume has reached the opposite bank and one has to investigate the plume while it is spreading laterally across the river. Obviously, none of the above one-dimensional models can be used for this investigation. One must resort to a two or three-dimensional model in order to estimate the temperature distribution in the river with any degree of accuracy.

Edinger and Polk [1969] did consider lateral mixing of heated water discharged along a bank into a river of uniform velocity. They presented solutions for the 2-dimensional case with and without surface heat transfer and also a solution for the 3-dimensional case. Constant lateral and vertical turbulent mixing coefficients were used. The solutions given were derived for delta function type sources at the origin where the temperature becomes infinite. Infinite temperatures at the source may be acceptable when dealing with the decay of temperature very far downstream but not if interest is in the plume which is still growing across the river. For example, the solution given by Edinger and Polk for the 2-dimensional case with no surface heat transfer is

$$\frac{T(x,y)}{T_P} = \frac{W}{\sqrt{\pi}}\frac{Q_P}{Q_R} \exp\left(-\frac{y^2}{\frac{4xD_y}{U}}\right)\sqrt{\frac{U}{xD_y}}$$

where T is the temperature,

T_P is the plant discharge temperature,

$\frac{Q_P}{Q_R}$ is the plant to river discharge ratio,

W is the width of river,

y is the cross stream co-ordinate,

x is the downstream co-ordinate,

U is the velocity of the river and,

D_y is the lateral turbulent mixing coefficient.

Taking typical values used by Edinger & Polk for a river:

$$W = 500 \text{ ft.}, \frac{Q_P}{Q_R} = 0.3, D_y = 10^5 \text{ft}^2/\text{day}, U = 1 \text{ ft/sec.}$$

the temperature along the bank, $T(x,o)$, is greater than the plant temperature until a distance x exceeding 6,000 feet is reached. At this point the plume

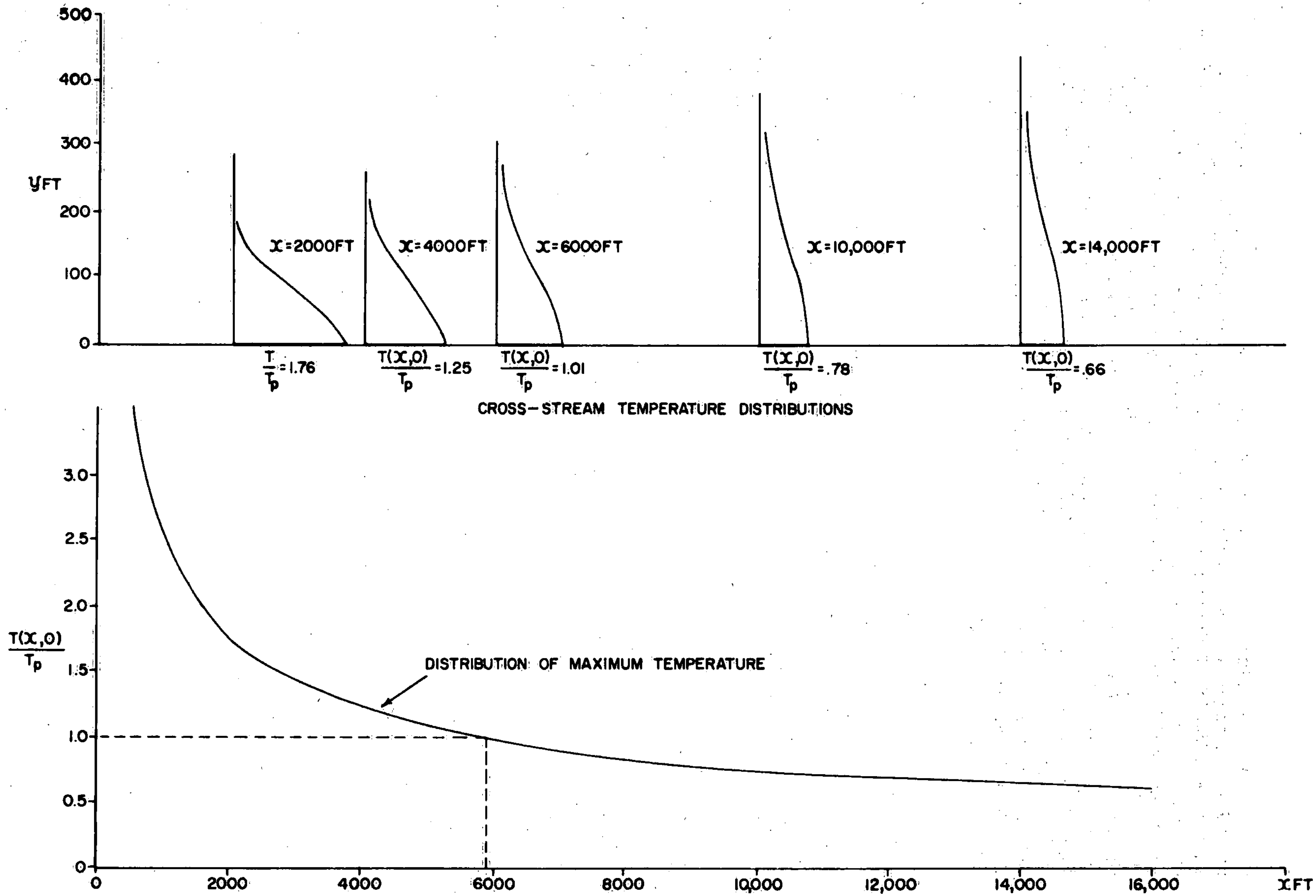

FIGURE I. TEMPERATURE DISTRIBUTION AS GIVEN BY THE SOLUTION OF EDINGER AND POLK FOR A TYPICAL CASE W=500FT; $\frac{Q_p}{Q_R}=0.3$; $D_y=10^5\ FT^2/DAY$; U= IFPS.

as given by the solution has spread 250 feet across the river, as shown in Figure 1. Edinger and Polk tried to circumvent the problem by not looking at the temperature distribution but instead at the area enclosed by some particular temperature contour. They considered the point where $\bar{T}(x,o) = T_p$ to be the source location and integrated along a temperature contour from that point on downstream. It has been indicated earlier that at x = 6,000, the plume has already spread 250 feet laterally. Using this 250 feet wide temperature distribution to model the spread of a discharge which may be only 20 or 30 feet wide and of uniform temperature can hardly be considered satisfactory. In order to achieve a reasonable prediction of river temperature, more realistic boundary conditions will have to be imposed on the theoretical model.

CHAPTER 2

Analytical Solution

Consider hot water being discharged at a river bank either parallel or at an angle to the flowing stream. At the initial stages the warm jet will spread, entraining cooler water from the sides and the bottom. The mechanisms of jet growth and temperature distribution at this stage are largely governed by the jet-induced turbulence. The spreading of warm jets has been studied by various investigators including Tami *et al.* [1969], Fan [1967], Zeller [1967] and Jen *et al* [1964]. However, knowledge concerning the rate of mixing and entrainment of the jet near its source is far from being complete. Most of the published data are from field studies or for jets discharged into still surroundings in the laboratory. As will be shown later these data are not generally applicable to discharge conditions in a river and there is a great need for experimental data which could be used for engineering design.

The initial spreading of jets is governed by the jet-induced turbulence and cannot be described by the heat diffusion equation. However, further downstream when the jet momentum has been largely dissipated, the ambient river turbulence governs the diffusion process and from there on the spreading of the heated plume is represented by the diffusion equation. Therefore, to investigate this downstream spreading of the plume using the diffusion equation, one needs to account for the temperature decay due to jet mixing. One way of doing this is to estimate the location where the diffusion equation would start to apply and, using available information on jet spreading, estimate what the temperature there would be. The temperature distribution at that location is then approximated by a constant width virtual source and the heat diffusion equation can be solved using conditions at the virtual source as boundary conditions. Figure 2 gives an illustration of this situation.

Ellison and Turner [1959], have shown that when a surface jet is emitted over a denser fluid, vertical entrainment of the denser fluid into the jet ceased when the densimetric Froude number became less than 1·2. The densimetric Froude number F_r is defined as

$$F_r = \frac{U}{\sqrt{(\Delta\rho/\rho)gd}}$$

where U is the velocity of the plume relative to the underlying fluid,

$\Delta\rho/\rho$ is the ratio of the density difference between jet and ambient to the density of the ambient fluid and,

d is the depth of the plume.

If the initial value of F_r at the outfall is less than 1·2, no vertical mixing will take place and the jet would spread laterally, remaining essentially constant in depth. On the other hand, if F_r at the outfall

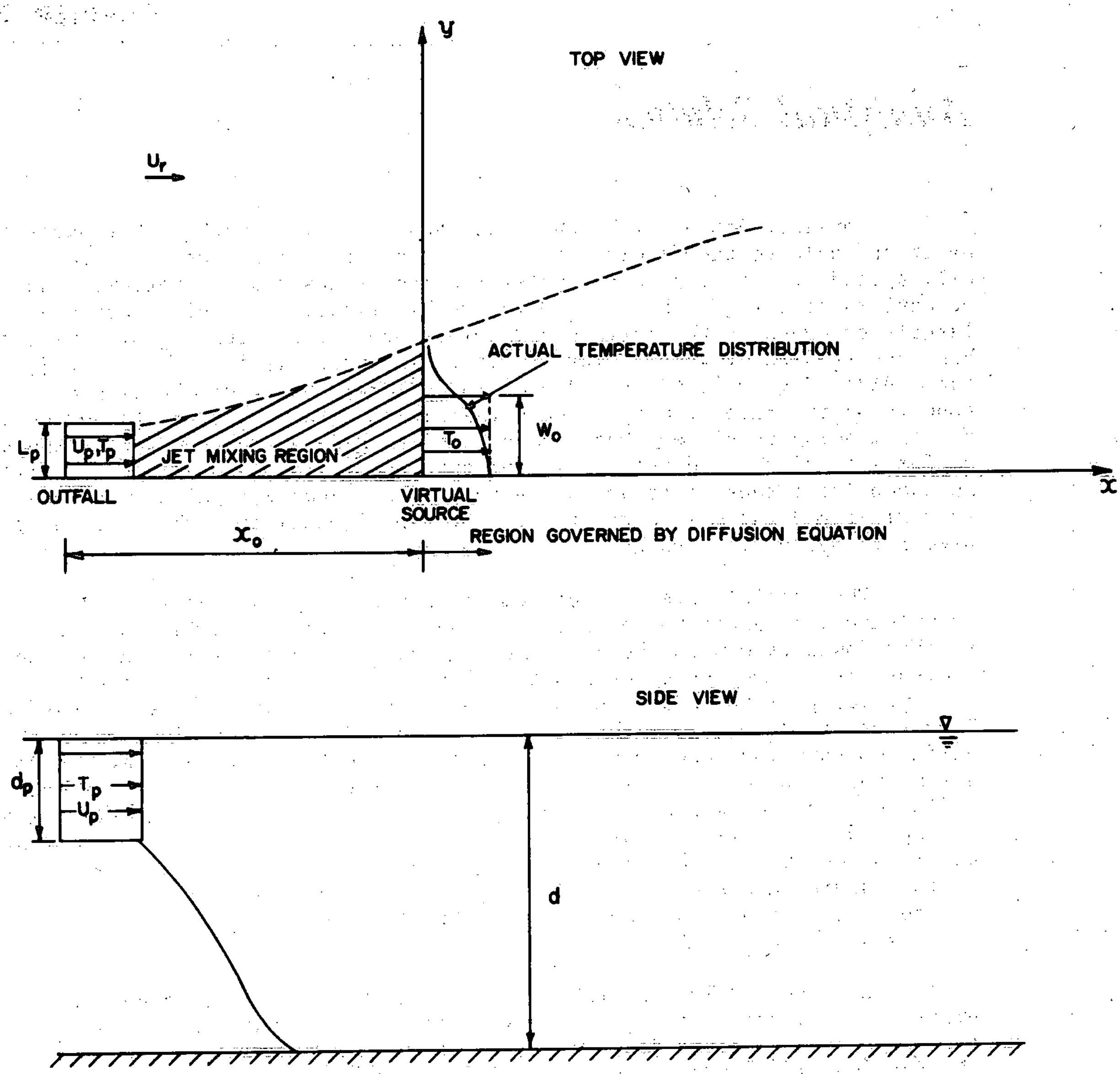

FIGURE 2. DEFINITION SKETCH

is greater than 1·2, vertical entrainment will increase the depth of the jet as it moves downstream. Because of vertical and lateral entrainment $\Delta\rho/\rho$ is decreasing with distance from the outfall. At the same time because the jet is losing its momentum, U is also decreasing with distance from the outfall. The combination of these two effects may result in the value of F_r decreasing with distance from the outfall. This often happens in lakes where the longshore current velocities are usually low enough that at some distance from the outfall F_r becomes less than 1·2 and a constant depth plume results.

For discharge into rivers, with jet velocities typically in the range of five feet per second, excess temperature at the outfall of 10°F to 20°F and thickness of discharge around five feet, the value of F_r at the outlet is usually around 5 or 6. Hence the jet will always grow in depth initially and because of the small depths of most rivers, the jet would almost always grow to the full depth of the river. Measurements of buoyant effluent by Honstead [1957] showed that prompt mixing existed in the vertical direction. Measurements of dissolved oxygen in rivers by Churchill *et al* [1962] also indicated no vertical concentration gradient. Hence it will be assumed in the present analysis that temperature is well mixed vertically beginning at the virtual source.

With the assumptions that the river has a constant mean velocity and depth, that the flow is well mixed vertically, that the lateral turbulent mixing coefficient is constant and horizontal diffusion of heat can be neglected as compared to horizontal convection, the equation for the steady state diffusion of heat can be written as

$$U \frac{\partial T}{\partial x} = D_y \frac{\partial^2 T}{\partial y^2} - \frac{K}{\rho g C_p d} T \qquad (1)$$

The boundary conditions for a finite width, constant temperature source, with no temperature gradient at the banks are

$T = T_o \qquad x = 0 \qquad 0<y<W_o$

$\frac{\partial T}{\partial y} = 0 \qquad y = 0 \text{ and } y = W$

where T = temperature excess, over that of the ambient river.

x = downstream co-ordinate measured from the virtual source.

y = lateral co-ordinate measured from the bank.

U = velocity of the river.

D_y = lateral turbulent mixing coefficient.

K = surface heat transfer coefficient.

ρ = density.

g = acceleration due to gravity.

C_p = specific heat.

d = depth of river.

T_o = temperature excess at the virtual source.

W_o = width of the virtual source.

W = width of the river.

Figure 2 is a definition sketch of the co-ordinate system.

Putting $T' = \frac{T}{T_o}$, $x' = \frac{x}{W_o}$ $y' = \frac{y}{W_o}$ equation (1) becomes

$$\frac{\partial T'}{\partial x'} = \frac{1}{P_e}\frac{\partial^2 T'}{\partial y'^2} - P_1 T'$$

where $P_e = \frac{UW_o}{D_y}$ (a Peclet number) and $P_1 = \frac{KW_o}{\rho g C_p U d}$

The surface heat transfer term can be eliminated by making the substitution

$T' = \theta \exp(-P_1 x')$

resulting in the equation

$$\frac{\partial \theta}{\partial x'} = \frac{1}{P_e}\frac{\partial^2 \theta}{\partial y'^2} \qquad (2)$$

with the boundary conditions

$\theta = 1$ $\quad x = 0 \quad$ $0<y'<1$

$\frac{\partial \theta}{\partial y'} = 0$ $\quad y' = 0$ and $y' = \frac{W}{W_o}$

Solution to equation (2) can be obtained by use of the Laplace transform.

$$\theta = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left\{\operatorname{erf}\frac{\frac{2nW}{W_o}+1+y'}{2\sqrt{x'/P_e}} + \operatorname{erf}\frac{\frac{2nW}{W_o}+1-y'}{2\sqrt{x'P_e}}\right\} \qquad (3)$$

$$\therefore T' = \frac{T(x,y)}{T_o} = \frac{e^{-P_1 x'}}{2}\sum_{n=-\infty}^{\infty}\left\{\operatorname{erf}\frac{\frac{2nW}{W_o}+1+y'}{2\sqrt{x'/P_e}} + \operatorname{erf}\frac{\frac{2nW}{W_o}+1-y'}{2\sqrt{x'/P_e}}\right\} \qquad (4)$$

Equation (4) can also be written as

$$\frac{T(x,y)}{T_o} = \frac{1}{2}\left\{\operatorname{erf}\frac{W_o-y}{2\sqrt{xD_y/U}} + \operatorname{erf}\frac{W_o+y}{2\sqrt{xD_y/U}} + \sum_{n=1}^{\infty}\left[\operatorname{erf}\frac{2nW+W_o+y}{2\sqrt{xD_y/U}} - \operatorname{erf}\frac{2nW-W_o-y}{2\sqrt{xD_y/U}}\right.\right.$$

$$\left.\left. + \operatorname{erf}\frac{2nW+W_o-y}{2\sqrt{xD_y/U}} - \operatorname{erf}\frac{2nW-W_o+y}{2\sqrt{xD_y/U}}\right]\right\} e^{-\frac{K}{\rho g C_p U d}x} \qquad (5)$$

A closer inspection of equation (5) reveals that for most practical cases, many of the terms can be left out and the solution simplified. Take for instance a typical case of a river with U = 1 foot per second,

$$d = 15 \text{ ft.}, \quad K = 100 \frac{\text{BTU}}{\text{ft}^2 \text{day}^\circ\text{F}}, \quad W = 300 \text{ ft.}, \quad W_o = 30 \text{ ft., and}$$

$$D_y = 10^5 \text{ft.}^2/\text{day},$$

one gets

$$\frac{K}{\rho g C_p U d} = 1.236 \times 10^{-6} \ \frac{1}{\text{ft.}}$$

For distances downstream of the virtual source of 1 mile and 3 miles, the exponential term $e^{\frac{-K}{\rho g C_p U d} x}$ is equal to 0.993 and 0.980 respectively. This means that the temperature distribution at 3 miles is 0.98 times that obtained by not considering any surface heat transfer. If velocities are very low, say around 0.1 foot per second and depth of plume is only a few feet, a situation often encountered in lakes, this exponential term can be quite important as will be shown in section 4. However, such low velocities and small depths are seldom found in rivers and the values quoted above are much more typical. One can conclude then that the effect of surface heat transfer on the temperature distribution in a river is quite minimal. This fact can be further illustrated by calculating the amount of heat lost to the atmosphere up to any distance x. Denoting this heat loss as Q,

$$Q = \int_0^x \int_0^W KT dy dx \quad \text{with T given by equation (5).}$$

This gives

$$Q = \rho g C_p U d W_o T_o \ (1-e^{-P_1 x'})$$

$$\therefore \ \frac{\text{Heat lost to atmosphere}}{\text{total heat input}} = 1-e^{-P_1 x'} = 1-e^{-\frac{K}{\rho g C_p U d} x}$$

One can see that this ratio is very small even for relatively large x.

The terms behind the summation sign in equation (5) can be looked at next. The value of the error function increases towards one as the argument increases, and is already equal to 0.995 when the argument reaches a value of 2. The terms being summed alternate in sign and the term with the smallest argument is $\operatorname{erf} \frac{2nW-W_o-y}{2\sqrt{xD_y/U}}$. Hence one can establish the criterion that if $\frac{2nW-W_o-y}{2\sqrt{xD_y/U}} > 2$ one can safely neglect the terms being summed which come about due to the finite width of the river. If interest is in the maximum temperature which occurs at y = 0, this criterion shows that the summation terms can be disregarded up to a downstream distance of 3.3 miles. Even if the temperature at y = 150 feet is considered, these terms are negligible up to 2 miles downstream. Therefore only in very exceptional cases do we have to take those terms into account.

With the reasoning from the above paragraphs the equation for the temperature downstream of the virtual source can be reduced to

$$\frac{T(x,y)}{T_o} = \frac{1}{2}\left\{\text{erf}\,\frac{W_o-y}{2\sqrt{xD_y/U}} + \text{erf}\,\frac{W_o+y}{2\sqrt{xD_y/U}}\right\} \tag{6}$$

which is the solution for the case of an infinitely wide stream with no surface heat transfer, as was obtained by Csanady [1970]. Equation (6) can be written in terms of the non-dimensionalized variables as

$$T' = \frac{1}{2}\left\{\text{erf}\,\frac{1-y'}{2\sqrt{x'/P_e}} + \text{erf}\,\frac{1+y'}{2\sqrt{x'/P_e}}\right\} \tag{7}$$

This equation is plotted in Figure 3 for various values of $\frac{x'}{P_e}$.

In Figure 4 the variation of temperature at $y' = 0$ with downstream distance is plotted for various values of P_e, giving an indication of the distribution of maximum temperature with distance from the virtual source which is in general the most vital information. This plot may also be used in conjunction with a temperature survey to give an estimate of D_y for the given stretch of river.

Using the typical values of flow rate, source width, etc. as given previously, the cross-stream distribution of temperature at a few locations downstream as well as the downstream distribution of the maximum temperature are calculated from equation (6) and are plotted in Figures 5 and 6 respectively. It can be seen from the actual values of x and y involved that the terms due to surface heat transfer and finite width of river can be left out without any loss of accuracy. One can also see that at a downstream distance of 5,000 feet, the maximum excess temperature is only about 0.2 times the virtual source temperature but the plume still has not yet spread completely across the river. This means that one has to go further downstream before the one-dimensional model is applicable. Since the virtual source temperature is usually some fraction of the actual discharge temperature, this illustrates that there is very little useful information which one can obtain from the one-dimensional model.

The solution given in this section enables one to calculate the temperature distribution due to a heated discharge when the velocity and lateral turbulent mixing coefficient for the river are known together with the width and temperature of the virtual source and its distance from the outfall. In the next section a method for estimating the characteristics of the virtual source will be established.

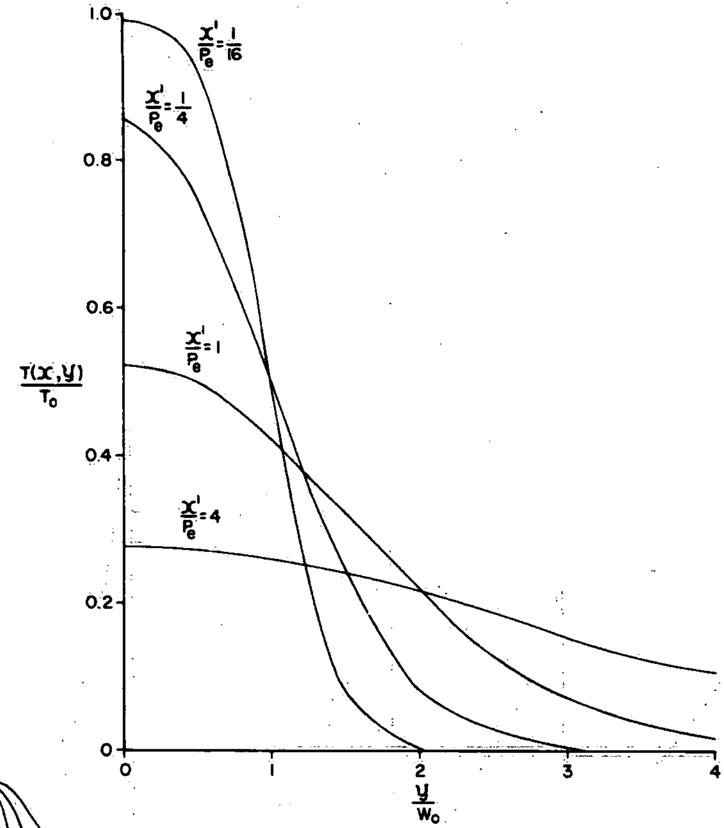

FIGURE 3. NON-DIMENSIONALIZED TEMPERATURE DISTRIBUTION

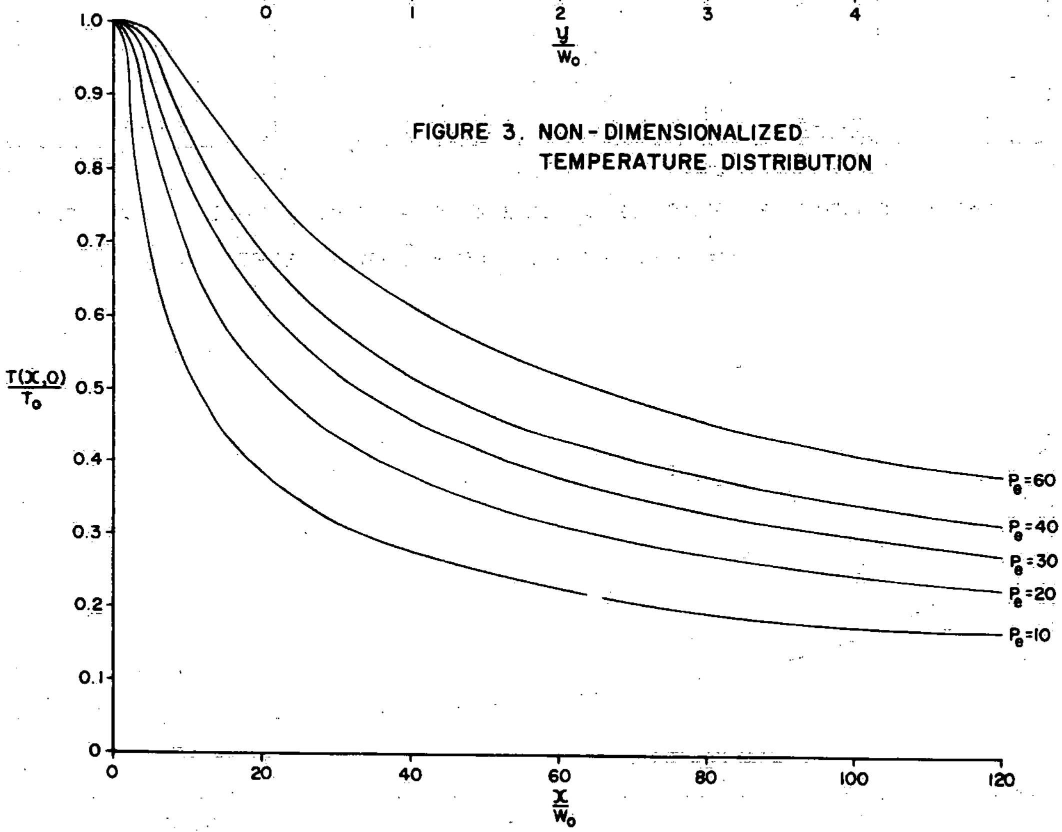

FIGURE 4. MAXIMUM TEMPERATURE DISTRIBUTION DOWNSTREAM FROM THE VIRTUAL SOURCE FOR DIFFERENT VALUES OF P_e

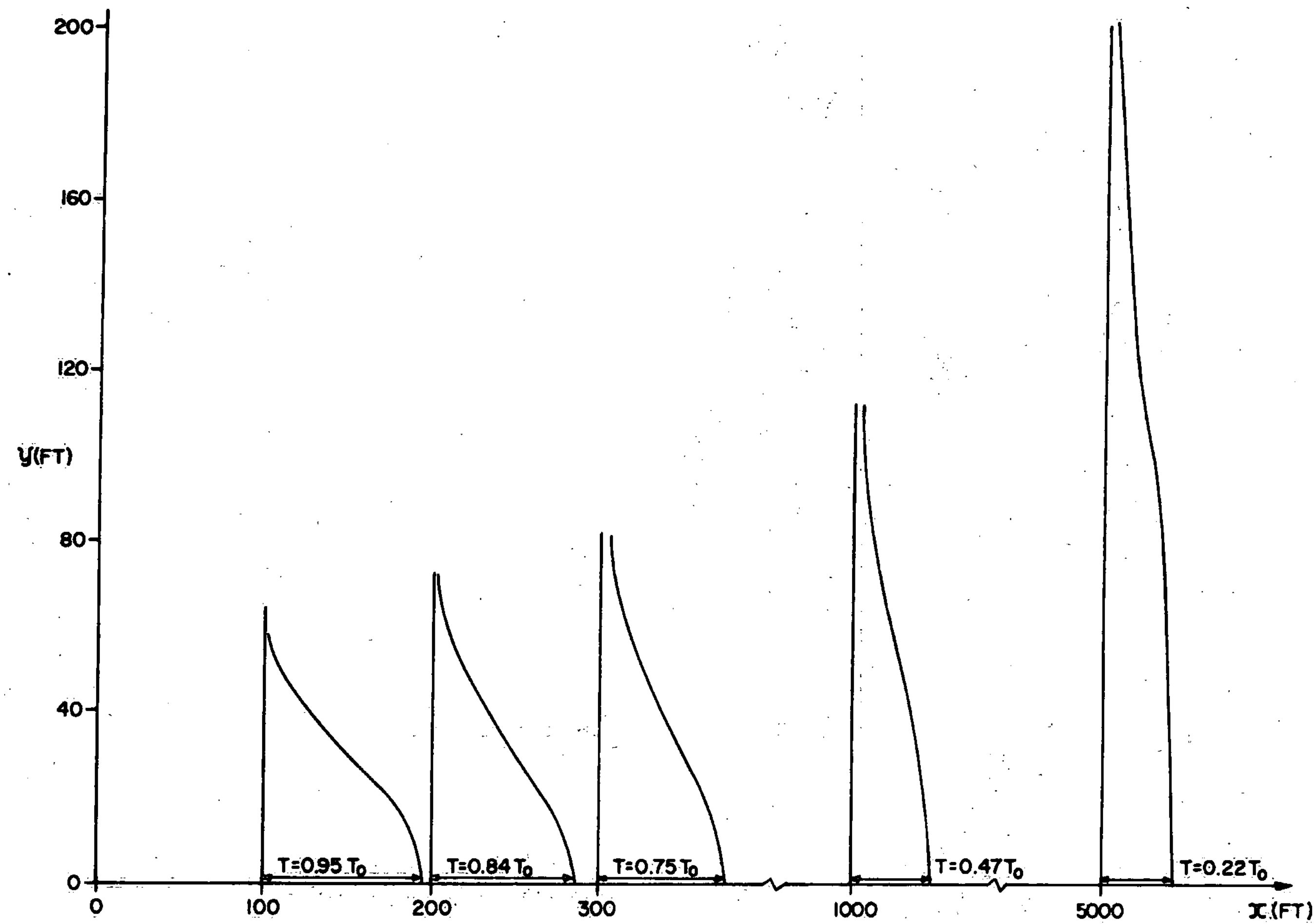

FIGURE 5. TEMPERATURE DISTRIBUTION AT VARIOUS DOWNSTREAM DISTANCES FOR A TYPICAL CASE

W = 300FT, U = 1 FPS, $D_y = 10^6$ FT^2/DAY, W_0 = 30 FT.

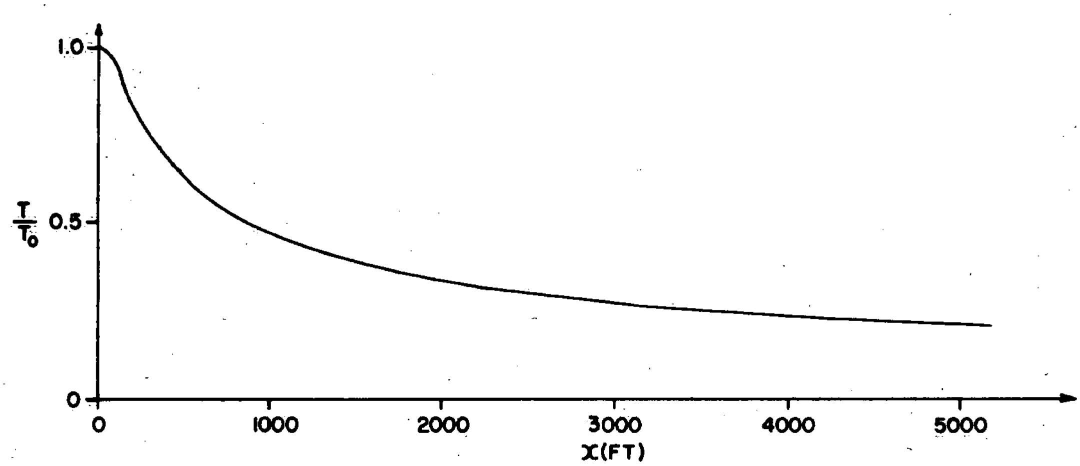

FIGURE 6. VARIATION OF MAXIMUM TEMPERATURE WITH DISTANCE FROM THE VIRTUAL SOURCE FOR A TYPICAL CASE

Estimation of the Virtual Source Conditions

The solution given in the previous section is derived from the diffusion equation with boundary condition such that there is a finite width source of uniform temperature throughout the whole depth. Although this is still an approximation, it would give more realistic answers than the one-dimensional models cited and the solution of Edinger and Polk which imposed an infinite line source at the origin. The problem discussed in this section is the estimation of the excess temperature T_o and width W_o of the virtual source and its distance from the actual outfall. Real discharge configurations vary a great deal but in general they fall into one of the following categories: (1) surface discharge parallel to stream, (2) cross-stream surface discharge and, (3) submerged discharge. Discussion will follow the categories listed above. Figure 7 gives an illustration of these discharge configurations.

Surface Discharge Parallel to Stream

Suppose the heated effluent is discharged from an outfall of width L_p and depth d_p with excess temperature of the plant equal to T_p and at a velocity U_p. The conclusions of Ellison and Turner [1959] regarding vertical entrainment should still apply as long as the discharge velocity is greater than the ambient river velocity and the densimetric Froude number at the outlet F_p is defined using the velocity relative to that of the ambient fluid. Thus

$$F_p = \frac{U_p - U_r}{\sqrt{(\Delta\rho/\rho) g d_p}}$$

where U_r is the velocity of the river.

Experiments of surface discharge of round heated jets into quiescent cooler water by Jen *et al* [1966] showed that the bottom of the jet grew linearly with distance from the orifice with a slope from 0.18 to 0.125 and that the jet centreline temperature decreased inversely as distance. These tests were conducted for a range of F_p between 18 and 180. Later tests by Tamai *et al* [1969] showed that for values of F_p less than about 10 of the jets did not continue to grow far downstream.

However, the observed initial jet entrainment for these experiments cited above was very rapid. Stolzenbach and Harleman [1971] also experimented with surface discharges of heated water and found somewhat slower vertical entrainment. Nevertheless, because the ratio of river depth to depth of discharge is usually not large, the heated effluent should be able to penetrate to the bottom except for values of F_p so small that no initial entrainment can take place at all. The data of Stolzenbach and Harleman [1971] showed that vertical growth of the jets were severely inhibited for values of F_p less than 2.0. This value of F_p will be taken as the limiting value for which no increase in the depth of the jet occurs.

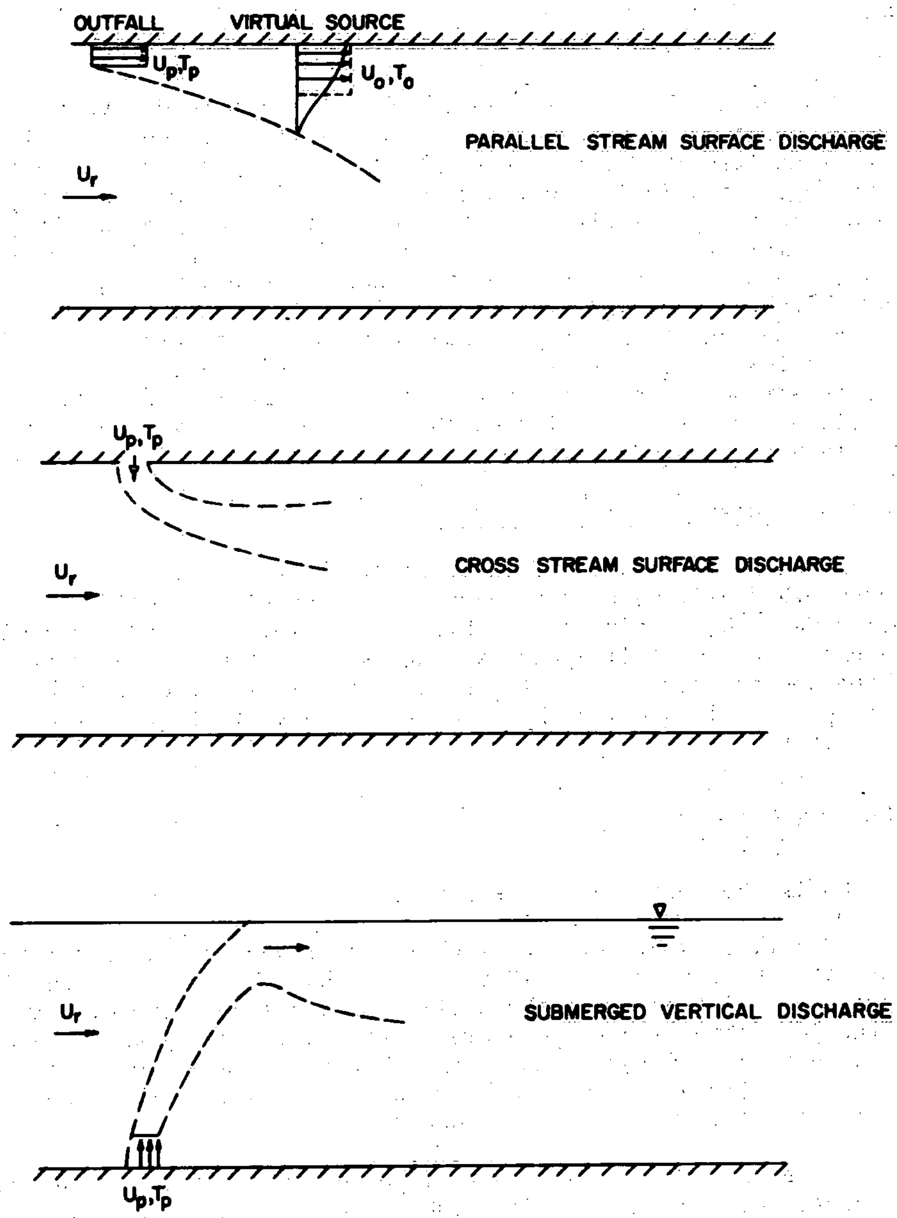

FIGURE 7. SKETCHES OF POSSIBLE DISCHARGE CONFIGURATIONS

The problem of selecting the location of the virtual source is a difficult one. The distance from the outfall, where ambient turbulence begins to dominate, would vary with each particular outlet design. Small jets issuing from multiport diffusers will obviously lose their momentum faster than one large jet. The values of eddy diffusivity obtained from heated discharge measurements are not too consistent. Therefore one can only attempt to make the best possible approximation using available data.

Experiments on finite width slot jets by Yevdjevich [1966] indicated that the centreline velocity decayed initially with $x^{-1/2}$ as in a two-dimensional jet but further downstream the velocity decreased as x^{-1} as in a round jet. In this latter range the velocity decay along the centreline can be described by the relationship

$$\frac{U_c}{U_o} = 7.5 \left(\frac{L_o}{d_o}\right)^{.435} \frac{d_o}{x}$$

where U_c is the centreline velocity,

U_o is the jet efflux velocity,

L_o is the longer side of the jet, and,

d_o is the shorter side.

For the present purpose of defining a virtual source, the distance x_o is assumed to be that distance at which the centreline velocity of a jet with width L_p depth d_p and efflux velocity U_p has decayed to a value equal to the ambient river velocity U_r. Therefore x_o can be calculated from the equation

$$x_o = 7.5 \left(\frac{L_p}{d_p}\right)^{.435} d_p \frac{U_p}{U_r} \qquad (8)$$

If it so happens that the depth of discharge at the outfall is greater than the width, then the roles of d_p and L_p in equation (8) will have to be interchanged. However, this is seldom the case.

Jen *et al* [1966] experimented with heated surface jets with values of F_p ranging between 18 and 180 and presented the equation

$$T_m/T_p = 7.0\ D_o/x$$

where T_m was the centreline excess temperature, and

D_o was either the diameter of the discharge or 4 times its hydraulic radius.

However, the above equation fitted mostly data with values of x/D_o greater than 50. One would expect the virtual source to be closer to the outfall than 50 D_o, where the decay of the centreline temperature may not be as rapid. Stolzenbach and Harleman [1971] presented numerical solutions and experimental data with varying F_p and aspect ratio of the discharge canal. Taking those points which have values of x/D_o between 40 and 10 and replotting them, it can be seen that they follow more or less a minus half instead of a minus one slope, Figure 8. The intercept varies between runs

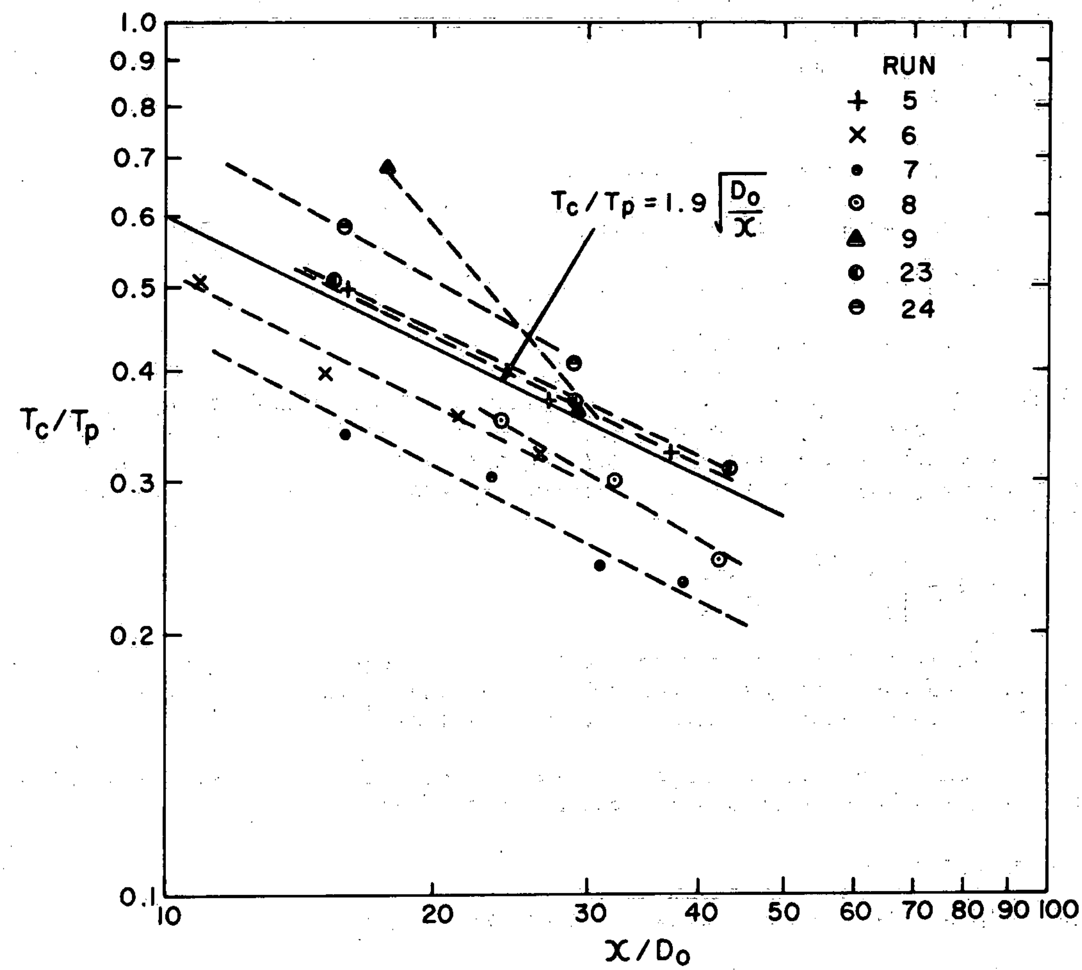

FIGURE 8. DECAY OF CENTRELINE TEMPERATURE - DATA OF STOLZENBACH AND HARLEMAN (1971)

but an average value is about 1.9. The values of F_p for these data points are between 2 and 10 which are more typical of discharge into rivers. Hence if x_o given by equation (8) is such that x_o/D_o <40, the virtual source temperature will be estimated from the equation

$$T_o = 1.9 \sqrt{D_o/x_o}\ T_p \qquad (9)$$

If x_o/D_o >40 the equation given by Jen can be used.

The width of the virtual source W_o can now be estimated by equating the heat content at the outfall with that at the virtual source.

$$W_o = \frac{U_p L_p d_p T_p}{U_r T_o d} \qquad (10)$$

As an example, take the case of a river with velocity U_r = 1 foot per sec., d = 15 feet and a discharge with U_p = 5 feet per sec., T_p = 20°F, L_p = 20 feet, and d_p = 3 feet, with ambient temperature of 70°F, F_p = 7.9, and from equations (8), (9) and (10), x_o = 256 feet, T_o = 7.2°F and W_o = 55.6 feet. Therefore in this particular case, the solution of the diffusion equation would be applied starting from the virtual source which is 256 feet downstream of the outfall and with W_o and T_o having values of 58.6 feet and 7.2°F respectively. One can also see that there is a significant difference between T_o and T_p which cannot be ignored as was done by Edinger and Polk [1969].

In cases where F_p <2.0, vertical entrainment will be suppressed and the heated effluent will flow on top with a constant depth equal to the depth of discharge. Since

$$\frac{U_p - U_r}{\sqrt{(\Delta\rho/\rho) g d_p}} < 2.0,$$

the relative velocity $U_p - U_r < 2.0 \sqrt{(\Delta\rho/\rho) g d_p}$. Assuming a maximum temperature difference of 20°F, one can see that the relative velocity will not be much greater than 1 foot per second. With such small relative velocity, it should be possible to neglect the jet diffusion phase when considering the far field temperature distribution and assume that the heat diffusion equation can be applied starting from the outfall. Hence, if F_p <2.0, the solution given in equation (7) can be applied starting from the outfall, with the river depth d replaced by the depth of discharge d_p and the values of T_o and W_o are those of T_p and L_p respectively.

Cross-Stream Discharge

In many cases the heated effluent is discharged from the bank perpendicular or at any angle to the stream. In order to properly define a virtual source, the lateral and vertical spreading as well as the trajectory of heated surface jets in a cross flow have to be known. Unfortunately a satisfactory model of this type of flow does not exist at present. Carter [1969] made laboratory studies of the trajectory of a heated discharge at right angles to a flowing stream. In his experiments the heated discharge occupied the entire depth of the channel and the trajectory of such a jet cannot be expected to apply to other cases. He made no measurements of the lateral spreading and of course none for vertical mixing.

Zeller [1967] made measurements of the plume from a power plant discharge into a lake. Because lake current was driven by wind, the rate of plume spread was found to vary with wind speed. Zeller's measurements were mostly made at a distance of 400 feet or more from the outfall, where the jet had probably lost much of its momentum. Stolzenbach and Harleman [1971] presented numerical solutions and a few test results for surface discharge with a weak cross flow. The distribution of maximum temperature did not differ significantly from the cases with no cross flow. There were no measurements of vertical spreading or width of the jet.

There is very little knowledge concerning the vertical entrainment and turbulence characteristics of a horizontal buoyant jet in a cross flow; the effect of the exit Froude number; the jet and river velocity ratios, etc. Therefore, only very crude estimates of the virtual source conditions can be made at present. The data of Carter [1969] showed that for jet to river velocity ratio as high as 5 the jet was bent around within a downstream distance of ten times the width of discharge. Therefore it can be assumed that at the virtual source, the plume is essentially moving parallel to the ambient river. Entrainment is also known to increase when a jet is discharged into a cross flow, as was found by Keffer and Baines [1963]. It follows that the virtual source ought to be closer to the actual outfall than in the case of the parallel stream discharge. However, until more information is available one may do just as well by making a rough estimate of the virtual source conditions by using the same method as for discharge parallel to the river flow as shown in the previous section.

Submerged Discharge

Consider a heated jet discharging vertically from the bottom of a river. the situation is much the same as in the previously discussed cross-stream case, except that the buoyancy force is now acting in the direction of the jet discharge. Another significant difference is that jet travel is limited by the free surface of the river and the jet is very quickly forced to turn into a horizontal stream. Theoretical and experimental work on jet in a cross stream such as those of Abramovich [1963], Bosanquet *et al* [1950], Keffer and Baines [1963], and Pratte and Baines [1967] considered jets discharged into infinitely deep cross flows. These studies are applicable to stack effluents emitted into the atmosphere but not to the present configuration of vertical discharge in a river. In fact there are no available data on the velocity and temperature and thickness of the jet as it reaches the free surface.

Fan [1967] measured the trajectories of jets of salt water injected vertically downward into horizontal flows of fresh water. It can be seen from his data that for the values of Froude number and velocity ratio being considered here, the jet discharge would reach the surface almost immediately, or within a downstream distance of ten or twenty feet. Again for lack of better information one can estimate the virtual source by considering the effluent as being discharged at the surface in the direction of the river flow.

Conceivably, a submerged discharge may take place at a considerable distance from the river bank. Under these circumstances, the x axis in equation 7 can be considered as the plume centreline and the temperature distribution in one half of the plume can be calculated using equation 7. The value of W_o to be used is one half that calculated from equation 10.

CHAPTER 4

The Heat Exchange Coefficient and the Lateral Turbulent Diffusivity

The heat diffusion equation used in section 2 involved two physical parameters besides the depth and mean velocity of the river. These are the surface heat exchange coefficient K and the lateral turbulent diffusion coefficient D_y. It would not be possible to obtain numerical values for the temperature distribution without knowledge of these coefficients. They will be discussed briefly in this section.

Surface heat transfer involves the net absorbed radiation, back radiation, evaporation and convection. These mechanisms of heat transfer have been discussed by various authors including Edinger and Geyer [1965] and Sundaram *et al* [1969] and formulae for the derivation of K have been given. The coefficient K is defined as the net surface heat transfer divided by the difference between the surface temperature and the equilibrium temperature. If terms of second order in temperature difference are neglected K can be evaluated if the rate of evaporation is known. Studies in lakes by Anderson *et al* [1952] and Harbeck [1962] etc. have shown that evaporation can be estimated knowing only water surface temperature, wind speed, air temperature and humidity. Evaporation rates have not been measured in rivers. It has been shown in section 2 that for the calculation of river temperature, the exponential term due to surface heat transfer has typical values of .993 and .980 respectively at 1 mile and 3 miles downstream and therefore its effects are negligible. However this does not imply that surface heat transfer is never important. Current velocity in lakes can often be as low as 0.1 foot per second and with a plume depth of 5 feet the corresponding values of the exponential term at 1 mile and 3 miles are .850 and .614 respectively. Therefore there are instances when the surface heat transfer should be taken in account. In general, it can be neglected for the calculation of river temperatures. A representative value for K is around 150 $Btu/ft^2/°F/day$ and this can be used for evaluating the exponential term if desired.

The lateral turbulent diffusion coefficient in open channels have been measured by Elder [1959], Glover [1964], Fischer [1967], Sayre and Chang [1968] and Prych [1970]. Although individual data varied, the value of D_y was found in all cases to be proportional to U_*d where U_* is the shear velocity and d is the depth. The constant of proportionality for straight channels varied between 0.1 and 0.25. Values up to 0.7 has been reported for curved channels. Prych [1970] experimented with the lateral spreading of buoyant and non-buoyant streams and showed that in the case of a buoyant discharge, there is an increase in the initial rate of spreading due to secondary currents induced by the lateral density gradient. However, the actual magnitude of this effect on D_y has not been determined. A value of 0.25 U_*d should be a reasonable estimate of D_y for any stretch of river that is relatively straight. For a river with a slope of 0.0005 and depth of 15 feet, this gives a value of 1.8 $ft^2/sec.$ for D_y.

More accurate determination of D_y for a particular river can be accomplished by experiments with tracers *in situ*. The coefficient is usually calculated from the formula

$$D_y = \frac{U}{2} \frac{d\sigma^2}{dx} \tag{11}$$

where σ^2 is the variance of the lateral distribution of the depth averaged concentration.

Discussions regarding measurements of D_y can be found in the articles quoted above.

In oceans and other large bodies of water, the eddy diffusivity has been shown to increase with the width of the diffusing plume. Al Saffar [1966] compiled measurements in the ocean and showed that the data could be well represented by Richardson's 4/3 power law. However, for all but the largest of rivers, the plume width is the same order of magnitude as the width of the river and a constant D_y can accurately describe the diffusion process.

It can be seen from equation (6) that the term $xD_y/_U$ determines the temperature distribution. Therefore if a river has the same velocity but twice the value of D_y, the distance x required to achieve a particular temperature drop will be halved. This effect is particularly significant for the first few thousand feet from the outfall where the temperature decay is more rapid.

Conclusions

It was pointed out in the introduction that solutions of the one-dimensional heat equation should not be applied to the prediction of temperature in rivers downstream of heated discharges because in most cases the effluent would have been diluted to insignificant levels before the one-dimensional equation becomes applicable. The two-dimensional solution of Edinger and Polk [1969] was also shown to be unrealistic because of the infinite line source boundary conditions and because the initial jet mixing was not taken into consideration. This was confirmed by the calculations of sections 2 and 3 in which the lateral spreading of heated effluent from a virtual source of finite width was considered. The virtual source is an approximation of the temperature decay due to the initial jet mixing. It is believed that this method should give more accurate estimates of the temperature distribution in a river. The solution given can also be used to give a rough estimate of temperature in a lake due to a heated discharge when there is a steady longshore current. However, both velocity and the turbulent diffusivity in a lake varies with the wind and predictions of temperature are therefore much more difficult.

To estimate the conditions at the virtual source requires knowledge of the initial spreading of heated jets under different discharge configurations. It is seen that much of this knowledge is still not yet available. The spreading and temperature decay of a heated surface jet in a parallel stream still needs evaluation. Also unknown are the vertical and lateral entrainment rates of heated surface jets in cross flows and their variations with densimetric Froude number, velocity ratio, etc. Submerged discharge in a flowing medium of limited depth should also be investigated. Such information would allow better estimates to be made of the virtual source conditions and hence better evaluation of the downstream decay of temperature. However, this information will become vital if water temperature standards for rivers are going to be set which will limit the maximum temperature within a prescribed mixing zone because then the initial jet mixing for different discharge configurations will have to be known quantitatively.

Until more data are available, the temperature distribution in a river can be calculated by first estimating the temperature and width of the virtual source and its distance from the outfall and then using equation (7) or Figure 3 to evaluate the distribution from the virtual source downstream.

If the discharge represents most or all of the total flow of the river, there is no need to consider lateral spreading. Under these circumstances the one-dimensional equation can be used but the effect of longitudinal dispersion should be included.

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