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DEPARTMENT OF THE ENVIRONMENT

A Review of Conceptual Models and Prediction Equations for Reaeration in Open-Channel Flow

Y.L. LAU

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Contents

Foreword

Organic wastes in the waters or changes in flow regime by reservoirs may greatly lower the dissolved oxygen in a river. Where this occurs it may be useful to encourage the supply of oxygen by manipulating the hydraulic parameters rather than by syphons, oxygen bubbles and other mechanical means of artificial aeration.

Reaeration is also a fundamental component of the oxygen balance of a river or stream and consequently knowledge of the phenomenon is vital for correct systems modelling of stream quality.

The relationship between hydraulic parameters and the rate of atmospheric reaeration is not established and this report takes the first step in a programme of the Hydraulics Unit at the Canada Centre for Inland Waters to investigate this mechanism.

Results should be useful for engineers and managers concerned with water quality as giving insight into the factors affecting reaeration and how they can be manipulated to improve dissolved oxygen levels in rivers and canals.

T.M. Dick,
Head, Hydraulics Unit,
Canada Centre for
Inland Waters.

Avant-propos

Les déchets organiques dans les eaux ou les changements qu'apportent au régime d'écoulement les réservoirs, peuvent diminuer grandement l'oxygène dissous d'une rivière. Là où cela se produit, il peut devenir utile de favoriser l'approvisionnement d'oxygène en manipulant les paramètres hydrauliques plutôt que par siphons, bulles d'oxygène ou autre moyen mécanique d'aération artificielle.

La réaération est aussi un composant de base dans le bilan d'oxygène d'une rivière ou d'un cours d'eau et conséquemment la connaissance du phénomène est essentielle à la simulation correcte de modèles des caracté-ristiques d'un cours d'eau,

La relation existant entre les paramètres hydrauliques et le taux de réaération atmosphérique n'est pas établie et cet exposé est un premier pas dans un programme de la Section de l'hydraulique du Centre canadien des eaux intérieures pour étudier ce mécanisme.

Les résultats de cette étude devraient être utiles aux ingénieurs et responsables pour ce qui est de la qualité de l'eau, en ce sens qu'ils leur procureront un aperçu des facteurs affectant la réaération et comment ces mêmes facteurs peuvent être manipulés dans le but d'améliorer le taux d'oxygène dissous dans les rivières et canaux.

T.M. Dick,
Chef, Section de l'Hydraulique,
Centre canadien des eaux
  intérieures.

Abstract

A review of the existing knowledge of the process of atmospheric reaeration is given. The conceptual models which have been proposed to explain the mechanism of reaeration are evaluated. Even though some of these models lead to logical conclusions they cannot as yet be used for the prediction of reaeration rates since they all contain parameters which are very difficult to relate to measurable hydraulic variables. The various empirical and semi-empirical prediction equations are described. However these equations all give very large errors when applied over a wide range of flow conditions. Some areas of research are suggested.

Résumé

Une révision est faite des connaissances acquises dans le processus de la réaération atmosphérique. Les modèles théoriques qui ont été proposés pour expliquer le mécanisme de réaération sont évalués. Même si certains de ces modèles conduisent à des conclusions logiques, on ne peut encore les utiliser pour la prévision de taux de réaération parce qu'ils comportent tous des paramètres qu'il est difficile de concilier avec des variables hydrauliques mesurables. Les équations de prévisions empiriques et semi-empiriques sont décrites; cependant, toutes ces équations produisent des erreurs considérables si on les applique sur un grand écart de conditions d'écoulement. Certains sujets de recherches sont suggérés.

A Review of Conceptual Models and Prediction Equations for Reaeration in Open-Channel Flow

Y.L. LAU

1. INTRODUCTION

There are many factors governing the DO (dissolved oxygen) content of rivers and streams. Besides the consumption of oxygen by the BOD (biochemical oxygen demand) of organic wastes, oxygen can be removed from the water by the respiration of aquatic plants and plankton and by the diffusion of DO into the benthal layer. The replacement of oxygen is largely through atmospheric reaeration and also from photosynthesis of aquatic plants. Evaluation of the DO content of a river depends upon the rates at which the processes mentioned above occur and can be quite complicated because the BOD varies along a river, being governed by the rate of the oxidation process, removal of BOD by sedimentation, addition of BOD along the reach by runoff etc. Therefore, analysis of DO concentration in natural streams has varied from simple one-dimensional solutions to complicated numerical models including temporal as well as spatial variations of the parameters. However, these models can only be as accurate as our understanding of the processes involved since the various rate constants have to be known before any quantitative results can be obtained.

Although atmospheric reaeration is the main source of replenishment of oxygen for a river, the mechanism in which oxygen is absorbed is still not completely understood. Various theories have been proposed to explain this mechanism. Some are more plausible than others but none of these are really suitable for the prediction of a stream's reaeration rate since they all involve parameters such as surface film thickness and surface renewal rate which are very difficult to measure or to relate to common hydraulic variables. For this reason many empirical and semi-empirical equations have been put forth for the prediction of the reaeration rate. These equations have been derived mainly through regression analysis of some experimental data. However these equations have all been shown to give very large errors when applied to conditions outside of the experimental range from which they were derived. Therefore more research is required before we can further our understanding of the process of reaeration. This in turn would allow more realistic modelling of a stream's DO balance. Because the DO content is one of the principal indicators of a river's quality this knowledge is required before administrative decisions such as the required degree of waste treatment, necessity for artificial aeration etc. can be judiciously made. This report will give a review of the process of reaeration and point out some areas for future research. An extensive survey of the literature including methods of determining DO and the reaeration coefficient, analysis of errors and a compilation of available experimental data has been presented by Bennett and Rathbun (1971). This review will not be as extensive but will attempt to give more critical evaluation of some of the more significant works on reaeration.

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2. MECHANISM OF OXYGEN ABSORPTION

2.1. Absorption Into a Stagnant Layer of Liquid

A brief outline of the process of gas absorption into a stagnant liquid layer is germane at this point since many of the theories for reaeration evolved from this mechanism of absorption.

It can be shown that when a gas is in contact with a liquid surface, the number of gas molecules striking the liquid surface is given by the relationship.

$$N = \frac{P}{\sqrt{2\pi \, mk \, T}} \qquad \qquad \dots\dots(1)$$

where N = number of gas molecules striking the surface per unit time.
P = partial pressure of the gas.
m = mass of the gas molecule.
k = Boltzmann constant.
T = absolute temperature.

Some of the gas molecules striking the surface will be absorbed and some will rebound back into the gas phase. The ratio of the number of absorbed to the number of rebounded molecules decreases as the surface becomes saturated. By making some simple assumptions about this ratio, it can be shown that the time required to saturate a new surface is only of the order of microseconds and for all intents and purposes, the surface in contact with the atmosphere can be considered to be saturated with oxygen at all times. The O_2 saturation concentration of water at $20°C$ is approximately 9 mg/litre.

In the absence of instabilities, oxygen entering the surface has to find its way into the body of liquid by means of molecular diffusion. This can be described by the usual Fickian model of diffusion which states that the rate of transfer of the diffusing substance through unit area of a section is proportional to the concentration gradient normal to the section, i.e.

$$F = -D_m \, \frac{\partial C}{\partial y} \qquad \qquad \dots\dots(2)$$

where F = rate of mass transfer per unit area.
D_m = molecular diffusion coefficient.
C = concentration, mass of diffusing substance per unit volume.
y = space co-ordinate in the direction of diffusion.

From equation (2) and considering mass balance, the equation for the diffusion of O_2 into the stagnant layer is

$$\frac{\partial C}{\partial t} = D_m \, \frac{\partial^2 C}{\partial y^2} \qquad \qquad \dots\dots(3)$$

where t = time co-ordinate.

If we now consider diffusion into a semi-infinite fluid, with concentration C_O initially, the boundary conditions are

$$C = C_O \qquad t < o$$
$$C = C_s \qquad y = o, \; t > o \qquad\qquad C_s = \text{saturation concentration}$$
$$C \rightarrow C_O \qquad y \rightarrow \infty$$

The solution to the problem is

$$C_s - C = (C_s - C_O) \; \text{erf} \; \frac{y}{2\sqrt{D_m t}} \qquad\qquad \dots\dots(4)$$

From equation (4), the rate of absorption at the surface is

$$F = -D_m \left(\frac{\partial C}{\partial y} \right)_{y=o} = \sqrt{\frac{D_m}{\pi t}} \; (C_s - C_O) \qquad\qquad \dots\dots(5)$$

Hence the mass transfer rate is proportional to $(C_s - C_O)$ and one can write

$$F = K_L \; (C_s - C_O) \qquad\qquad \dots\dots(6)$$

and in this case

$$K_L = \sqrt{\frac{D_m}{\pi t}} \qquad\qquad \dots\dots(7)$$

K_L is generally referred to as the liquid film coefficient.

It can be seen that the mass transfer rate by molecular diffusion decreases with time. Calculations show that after one second or so this mass transfer rate will be orders of magnitude less than that given by the dynamic capture of oxygen molecules described earlier.

Experimental measurement of absorption in a stagnant liquid layer can result in a much higher absorption rate than that given by equation (5). This is due to the fact that the absorbed gas induces a density and/or viscosity gradient which may cause instability at the interface. Convective cells can develop and a much higher mass transfer rate is obtained as a result. Ruckenstein (1971) showed that a roll cell model can explain the large increase in the rate of absorption observed experimentally and that for large times equation (3) can be used with a constant diffusion coefficient much larger than the molecular one to describe the situation. The molecular diffusion coefficient D_m for oxygen in water is approximately equal to 2×10^{-5} cm^2/sec at 20°C.

2.2. Absorption Into Bodies of Fluid in Motion

This section will describe the various models which had been developed to explain the absorption of a gas into a body of fluid which is not stagnant. It will become obvious that the models all contain some intangible parameter which cannot be directly related to commonly measured hydraulic variables. Hence while these models can be used to explain the process, they are not too useful for the prediction of the reaeration rate for streams.

2.2.1. Two Film Model. Lewis and Whitman (1924).

This model assumes that the interface between the gas and the liquid consists of a laminar gas film and a laminar liquid film. The gas passes through the liquid film by molecular diffusion and is distributed into the body of the liquid by turbulent diffusion. Since the turbulent diffusivity is very much larger than the molecular diffusivity, the concentration gradient in the main body of the fluid must be very small and can be assumed to be constant. The resistance of the gas phase is also very small as compared to the resistance in the liquid phase and so it is assumed that the concentration on top of the liquid film is equal to the saturation concentration C_S. Since this liquid film is very thin the concentration is assumed to vary linearly across it.

Hence

$$F = -D_m \left(\frac{\partial C}{\partial y} \right)_{y=0} = D_m \frac{C_S - C_O}{\delta} \qquad \ldots\ldots(8)$$

where C_O = concentration in the main body of the fluid.
δ = thickness of the laminar film.

$$K_L = \frac{D_m}{\delta} \qquad \ldots\ldots(9)$$

This model postulates the existence of a stable laminar film at the interface. Except for flows with very small Reynolds number, the existence of such a film is highly suspect and this model has generally been discarded when considering absorption in natural streams.

A similar liquid film coefficient was deduced by Levich (1962) although the concept of a film was not employed explicitly. Kishinevsky (1955) has criticized these film models as being unrealistic and asserted that an effective diffusion coefficient which embodies the effect of eddy diffusion should be used in the diffusion equation.

2.2.2. Penetration Theory. Higbie (1935).

Higbie proposed a model in which the fluid at the interface was continuously replaced by fluid elements from below so that unsteady state molecular diffusion was occurring all the time at the interface. He argued that before the dissolved gas had time to penetrate the liquid film and set up the linear concentration gradient as proposed in the film theory, the surface fluid element would have already been replaced with fluid from below. Supposing that the main body of the fluid is initially at ten percent saturation $(0.1C_S)$ and that the surface film is 0.5 mm thick, the time required for the gas to diffuse through a liquid film can be estimated from equation (4), the molecular diffusion solution, by calculating the time for the concentration at $y = 0.05$ cm to increase to say $0.15 \ C_S$. The result is 16.7 seconds. It is certainly reasonable to assume that the surface element will be replaced well before this stage has been reached.

Higbie's model assumed that the transient molecular diffusion took place up to a time t_e, at which time the surface element was completely mixed with the main body of the fluid below and a fresh new surface was

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exposed and the diffusion process repeated itself. Therefore an average absorption rate can be calculated from that given by equation (5).

$$F_{av} = \frac{1}{t_e} \int_0^{t_e} \sqrt{\frac{D_m}{\pi t}} \; (C_s - C_o) \; dt = 2 \sqrt{\frac{D_m}{\pi t_e}} \; (C_s - C_o) \qquad \ldots\ldots(10)$$

$$\therefore \; K_L = 2 \sqrt{\frac{D_m}{\pi t_e}} \qquad\qquad\qquad\qquad\qquad \ldots\ldots(11)$$

In this model, the surface elements are all exposed for an average time t_e before being mixed and this is the major weakness of the penetration theory. With the randomness inherent in turbulent flows the idea that all the surface elements will be replaced at the same time is not a very acceptable one. For applications to reaeration prediction the average exposure time t_e still has to be found.

2.2.3. Surface Renewal Theory. Danckwerts (1951).

Danckwerts also disputed the existence of a stable liquid film at the surface. He postulated that fluid at the surface consisted of elements which had been exposed for varying periods of time and that these elements were continually being swept into the bulk of the fluid by the eddies. He assumed that the rate of production of new surface was constant for a particular system and that the chance of any surface element being replaced was independent of its "age" (time of exposure). The fraction of the area of a surface which had age between θ and $\theta+d\theta$ was equal to the fractional area which previously had age between θ and $\theta-d\theta$ minus that portion which had been replaced during the interval $d\theta$. The fractional area was given a distribution $\phi(\theta)$ and the surface renewal rate was termed r.

$$\therefore \; \phi(\theta) = \phi(\theta-d\theta) \; (1 - rd\theta)$$

$$= (\phi[\theta] - \frac{d\phi}{d\theta} d\theta) \; (1 - rd\theta) \qquad\qquad \ldots\ldots(12)$$

From equation (12) and the condition that

$$\int_0^{\infty} \phi(\theta) \; d\theta = 1 \qquad\qquad\qquad\qquad \ldots\ldots(13)$$

Danckwerts obtained the distribution of surface age as

$$\phi(\theta) = re^{-r\theta} \qquad\qquad\qquad\qquad\qquad \ldots\ldots(14)$$

According to the surface renewal theory, the surface elements absorbed gas by molecular diffusion up to the moment they were replaced. Therefore, using the result of equation (5), the average absorption rate was

$$F = \int_0^{\infty} (C_s - C_o) \sqrt{\frac{D_m}{\pi\theta}} \; re^{-r\theta} \; d\theta$$

$$= (C_s - C_o) \; \sqrt{D_m r} \qquad\qquad\qquad\qquad \ldots\ldots(15)$$

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Hence

$$K_L = \sqrt{D_m r} \qquad \qquad \ldots\ldots(16)$$

The difficulty encountered in trying to use this theory is in obtaining the surface renewal rate r. Most measurements were made in stirring jars using the assumption that the rate was proportional to the rate of rotation of the stirrer but these values of r could not be applied to rivers and streams.

It can be noted that both the penetration model and the surface renewal model predicted liquid film coefficients varying with $D_m^{\frac{1}{2}}$. In fact, the theoretical development for the two were similar except that in the penetration model the surface elements all reached the same age before being replaced and the distribution function was

$$\phi(\theta) = \frac{1}{t_e} \quad o \leq \theta \leq t_e \qquad \qquad \ldots\ldots(17)$$

$$\phi(\theta) = o \quad t_e < \theta$$

Although both Higbie (1935) and Danckwert (1951) reasoned that molecular diffusion took place only close to the surface, they both used the absorption rate from equation (5) which meant that their models, contrary to their thinking, essentially incorporated the idea of an infinitely deep molecular diffusion layer which was disturbed at various times.

2.2.4. Film-Penetration Model. Dobbins (1956).

In the penetration and surface renewal models, some surface age distribution function was applied to the absorption rate obtained from diffusion into a semi-infinite fluid. Dobbins (1956) reasoned that since the concentration gradient in the bulk of the fluid was so small when compared with that in the surface layer, the bulk concentration could be regarded as constant and hence the diffusion equation should be applied to the surface layer only. He envisaged the existence of a thin film of thickness δ whose elements were continually being replaced and which absorbed gas by molecular diffusion before being replaced. The equation solved was still equation (3) but with different boundary conditions.

$$\frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} \qquad (3)$$

with

$$
\begin{array}{lll}
C = C_o & t = o & o < y < \delta \\
C = C_s & t > o & y = o \\
C = C_o & t > o & y = \delta
\end{array}
$$

the solution is

$$C - C_o = (C_s - C_o) \sum_{n=o}^{\infty} \left[\operatorname{erf} \frac{(2n + 2)\,\delta - y}{2\sqrt{D_m t}} - \operatorname{erf} \frac{2n\delta + y}{2\sqrt{D_m t}} \right] \;\ldots(18)$$

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Dobbins used equation (18) and the surface age distribution $\phi(\theta)$ obtained by Danckwerts (1951) [equation (14)] to derive the surface absorption rate.

$$F = \int_0^\infty -D_m \left(\frac{\partial C}{\partial y}\right)_{y=o} \phi(\theta) d\theta$$

$$= (C_s - C_o) \sqrt{D_m r} \coth \sqrt{\frac{r\delta^2}{D_m}} \qquad \ldots\ldots(19)$$

$$\therefore K_L = \sqrt{D_m r} \coth \sqrt{\frac{r\delta^2}{D_m}} \qquad \ldots\ldots(20)$$

At large renewal rates, the coth term ≈ 1 and equation (20) gives the same result as the surface renewal model. At small renewal rates, since $\coth(x) \to \frac{1}{x}$ for small x, equation (20) gives $K_L = \frac{D_m}{\delta}$ which is the result of the two-film theory of Lewis and Whitman (1924). The film-penetration model is more correct in concept but one can see that there are now two parameters, r and δ awaiting to be related to the common hydraulic variables.

O'Connor and Dobbins (1956) reasoned that the coth term in equation (20) was close to unity for most practical cases and therefore the liquid film coefficient was proportional to the square root of the renewal rate. They performed reaeration measurements in a cylinder in which turbulence was generated by an oscillating lattice of aluminum screenings. It was argued that the rate of surface renewal was proportional to the vertical velocity fluctuations which would be proportional to the speed of oscillation of the lattice. They plotted K_L versus (speed of oscillation)$^{\frac{1}{2}}$ and because the plot was roughly linear, claimed to have verified that $K_L \sim \sqrt{r}$. However if their data is plotted in terms of K_L versus speed of oscillation on a ln-ln basis, it can be seen that the slope actually varies from $\frac{1}{2}$ to 2. This may indicate that K_L does not always vary with $\sqrt{r}$ or the relationship between r and the speed of oscillation is not as straightforward as it was assumed.

2.2.5. Surface Renewal-Damped Eddy Viscosity Model. King (1966).

King (1966) attempted to develop a generalized model for the mass transfer process. The reasoning behind this model is that near the surface there is a zone of primary resistance to mass transfer. Eddies larger than this zone contribute to the surface renewal while small eddies add to the gradient diffusion within this zone in the form of eddy diffusivity. In this manner the concept of a film with the accompanying discontinuity in mass transport is avoided. The governing differential equation then becomes

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial y} \left[(D_m + \epsilon_y) \frac{\partial C}{\partial y} \right] \qquad \ldots\ldots(21)$$

King (1966) assumed that in the very thin surface layer the eddy diffusivity ϵ_y can be put equal to ay^n where a and n are constants. Hence the equation to be solved is

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial y} \left[(D_m + ay^n) \frac{\partial C}{\partial y} \right] \qquad \ldots\ldots(22)$$

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with boundary conditions

$$C = C_o \quad t = 0, \quad y > 0$$
$$C = C_s \quad t > 0, \quad y = 0$$
$$C = C_o \quad t > 0, \quad y \to \infty$$

Solutions to equation (22) with the above boundary conditions were obtained for some special cases. For instance when t, the surface age, is large and n is sufficiently large, steady state transfer results and

$$-D_m \left(\frac{\partial C}{\partial y}\right)_{y=0} = K_L (C_s - C_o) = -(D_m + ay^n) \frac{\partial C}{\partial y} \qquad \ldots \ldots (23)$$

Integration gives

$$K_L = D_m^{1-\frac{1}{n}} \frac{a^{\left(1+\frac{1}{n}\right)}}{\pi} \sin \left(\frac{\pi}{n}\right) \qquad \ldots \ldots (24)$$

Equation (24) gives asymtotic values of K_L at large t for various values of n.

For small t and a, the solution reduces to the classical expression

$$K_L = \sqrt{\frac{D_m}{\pi t}} \qquad (7)$$

King (1966) showed that the surface renewal and film penetration models were special cases of this generalized model in which either surface age was assumed small or the exponent n taken to be infinite. The predictions given by the damped eddy diffusivity model are only for a given element with surface age t while in an area of surface there is a distribution of surface ages. If all the elements have large surface age, equation (24) shows that the age does not matter but that the exponent n governs the film coefficient. If all the surface ages are low enough, an average age $\bar{t}$ can be defined for use in equation (7). For the surface renewal model of Danckwerts (1951), $\bar{t} = \frac{1}{\pi r}$ while for the penetration model of Higbie (1935), $\bar{t} = \frac{t_e}{4}$.

Even though this model has achieved an improvement in concept by assuming a continuous eddy diffusivity profile, thus eliminating the surface film, it has not achieved any improvement for use in the prediction of reaeration since there are now three parameters - a, n and t_e which have to be evaluated.

2.2.6. Large Eddy Model. Fortescue and Pearson (1967).

This model attempts to incorporate the effect of the convective transport by the eddies. The surface layer is assumed to compose of a

single row of counter-rotating, square-celled eddies. The velocity distribution within each cell is assumed to be

$$u = A \sin\left[\frac{\pi x}{L}\right] \cos\left[\frac{\pi y}{L}\right]$$

$$v = -A \cos\left[\frac{\pi x}{L}\right] \sin\left[\frac{\pi y}{L}\right] \qquad \dots\dots(25)$$

$$w = 0$$

where   L is the dimension of the cell.
        A is the maximum velocity within the cell.

The velocity distribution given by equation (25) is used in the mass transfer equation

$$u(x,y) \frac{\partial C}{\partial x} + v(x,y) \frac{\partial C}{\partial y} = D_m \left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right] \qquad \dots\dots(26)$$

with boundary conditions

$$C = C_s, \quad y = 0$$

$$\frac{\partial C}{\partial x} = 0, \quad x = NL, \quad N = \text{an integer}$$

$$C = C_o, \quad y = L$$

Fortescue and Pearson (1967) assumed that L was equal to the integral length scale of the turbulent flow field and A was equal to twice the root-mean-square turbulence intensity. Knowing these two parameters, equation (26) can be solved numerically and the average film coefficient over one roll cell can be computed as

$$K_{rc} = -\frac{D_m}{(C_s - C_o)} \frac{1}{L} \int_o^L \left(\frac{\partial C}{\partial y}\right)_{y=o} dx \qquad \dots\dots(27)$$

A known turbulent velocity field was created by Fortescue and Pearson (1967) by inserting grids of cylindrical rods at the beginning of a 2-foot long rectangular channel. Equation (26) was solved numerically and it was found that over their experimental range, the film coefficient for a roll cell could be approximated by

$$K_{rc} = 1.46 \left[\frac{D_m \sqrt{\overline{u^2}}}{L}\right]^{\frac{1}{2}} \qquad \dots\dots(28)$$

Since the turbulence was decaying, equation (28) had to be integrated and averaged over the test length to obtain an average coefficient for the test section. The calculated coefficients agreed reasonably well with the experimental results. Variation of the film coefficient with Reynolds number was shown although K must also be functions of other dimensionless parameters. Also, since the turbulent velocity field was used, equation (26) should have included the turbulent diffusion coefficient as well as the molecular one.

The concentration contours from the numerical solution displayed a high concentration gradient at the top of the eddy with a layer of fluid which swept downwards near the edge. This illustrated how a series of eddies was able to convect absorbed gas into the bulk of the fluid and to increase the mass transfer rate. However, a row of squared celled eddies is certainly not a true picture of the gas-liquid interface. The problem of defining the circulation pattern, length and velocity scale and the necessity of numerical solution even for a simplified model limits the large eddy model to an explanatory role.

2.2.7. Stochastic Model of Alternating Film and Eddying States. Rudis and Machek (1971).

This model is almost entirely statistical in its derivation. The main assumption is that any part of the surface of a liquid in open-channel flow alternates between periods with undisturbed surface and periods in which eddying occurs and surface renewal is evident. Hence at any time t, a surface element may be in a "film" state or an "eddying" state. If it is in a film state then the time which the surface has been exposed to the gas depends on how long ago the film state began. If the surface happens to be in an eddying state at time t, then its exposure period is equal to the time elapsed since the last renewal took place.

Hence

$$k(\theta) = P \cdot g(\theta) + (1-P) \, f(u) \, h\left(\theta/u\right) \qquad \ldots\ldots(29)$$

where
- $k(\theta)$ = probability that the exposure period is equal to (θ).
- P = probability that at time t the surface is in film state.
- $1-P$ = probability that at time t the surface is in eddying state.
- $g(\theta)$ = probability density of the time $t-\theta$ between t and the beginning of that film state.
- $f(u)$ = probability density of the time interval $t-u$ between t and the start of the eddying state.
- $h\left(\theta/u\right)$ = probability of the exposure period being θ given that the eddying state started at time $t-u$.

Rudis and Machek (1971) assumed exponential distributions for the probability densities $g(\theta)$ and $f(\theta)$, i.e.

$$g(\theta) = \beta e^{-\beta\theta} \qquad \ldots\ldots(30)$$

and

$$f(\theta) = \alpha e^{-\alpha\theta} \qquad \ldots\ldots(31)$$

$\frac{1}{\beta}$ and $\frac{1}{\alpha}$ are the mean duration of the eddying and film states respectively. It was also assumed that the number of renewals during an eddying state of duration u has a Poisson distribution, i.e.

$$P_\nu(u) = \frac{e^{-\lambda u}(\lambda u)^\nu}{\nu!} \qquad \nu = 0,1,2,3\ldots\ldots \qquad \ldots\ldots(32)$$

λ is the mean number of renewals in a unit time and is equivalent to the

renewal rate r introduced by Danckwerts. From these assumptions and equation (29), Rudis and Machek (1971) derived the expression for k (θ).

$$k(\theta) = \frac{\beta}{\alpha+\beta} \left[\alpha e^{-\beta\theta} + (\alpha+\lambda) \, e^{-(\alpha+\lambda)\theta} \right] \qquad \ldots\ldots(33)$$

The absorption rate from equation (5) is again used to find a mean absorption rate.

$$F = \int_{0}^{\infty} (C_S - C_O) \sqrt{\frac{D_m}{\pi\theta}} \, k(\theta) \, d\theta$$

$$= (C_S - C_O) \frac{\sqrt{D_m}}{1+\rho} \left[\sqrt{\beta} + \rho \sqrt{\frac{\beta}{\rho} + \lambda} \right] \qquad \ldots\ldots(34)$$

where $\quad \rho = \dfrac{\beta}{\alpha} = \dfrac{\text{expected duration of eddying state}}{\text{expected duration of film state}}$

$$\therefore \; K_L = \frac{\sqrt{D_m}}{1+\rho} \left[\sqrt{\beta} + \rho \sqrt{\frac{\beta}{\rho} + \lambda} \right] \qquad \ldots\ldots(35)$$

The unknown parameters in this model are the constants α, β, and λ. These depend upon the characteristics of the open-channel flow and must be determined experimentally. Rudis and Machek (1971) estimated α and β by photographing the surface of water flowing in a tilting flume. They correlated the values of α and β with Reynolds number and the bottom roughness. For the surface renewal rate λ, it was assumed that the equation of Danckwerts (1951) was correct for conditions in a mixing vessel. Four measurements of K_L were made in a mixing vessel and λ was calculated from

$$K_L = \sqrt{D_m\lambda} \qquad (16)$$

λ was also assumed to depend on Reynolds number only. Reaeration experiments were performed in a tilting flume and the measured K_L values were compared in a K_L versus Reynolds number plot with the calculated K_L using equation (35).

However, the theoretical values of K_L were not calculated at the same Reynolds number as the experimental points and it can only be said that they seem to follow the same trend.

The expression for K_L as developed by Rudis and Machek (1971) approaches the limit $K_L = \sqrt{D_m\lambda}$ as the eddy duration increases and film state duration tends to zero. This is in accord with the Danckwerts surface renewal theory. However, at the other extreme when the duration of the film state approaches infinity, equation (35) shows that K_L tends to zero. This is certainly not correct and is a result of using the absorption rate $\sqrt{D_m/\pi\theta}$ from equation (5). It can be shown that if the absorption rate is calculated from the solution for C given by Dobbins (1956) in equation (18) and this is used in conjunction with k (θ) from equation (33), the expression for K_L will be

$$K_L = \frac{\alpha}{\alpha+\beta} \sqrt{D_m\beta} \, \coth\left(\delta\sqrt{\frac{\beta}{D_m}} \right) + \frac{\beta}{\alpha+\beta} \sqrt{D_m(\alpha+\lambda)} \, \coth\left(\delta\sqrt{\frac{\alpha+\lambda}{D_m}} \right) \qquad \ldots\ldots(36)$$

This expression for K_L will approach the limits $K_L = \dfrac{D_m}{\delta}$ as given by the two film theory and $K_L = \sqrt{D_m \lambda}$ as given by the surface renewal theory for the two extremes of all film state or all eddying state.

From the experimental data of Rudis and Machek (1971), the values of α, β, and λ for a Reynolds number of 16,000 were 0.61, 0.182 and 20 respectively. This gave $\dfrac{\alpha}{\alpha+\beta} = 0.77$ and $\dfrac{\beta}{\alpha+\beta} = 0.23$ which meant that for 77% of the time the surface was in film state.

The contribution to K_L from the film state came to approximately 24%. These values are rather higher than what one would expect for a turbulent open-channel flow. Another perplexing thing is that from the published data, the fraction of the time in which the surface is in eddying state decreased with increasing bottom roughness. This seems unreasonable and leads one to suspect the experimental method of obtaining α and β.

The stochastic model separates a film state and an eddying state whereas the surface renewal and film-penetration models assume eddying state at all times. Hence although the upper limit for K_L are the same, the stochastic model predicts lower values of K_L for most of the experimental range.

2.2.8. General Remarks

From the various theories on the absorption mechanism it can be concluded that the main resistance to gas absorption is in a small region near the surface where molecular diffusion plays the dominant role. The surface in contact with the gas can be assumed to be saturated at all times and because the dissolved gas can be mixed within the main part of the fluid very much faster than it can diffuse through the surface layer, the concentration within the bulk of the fluid can be considered to be uniform. Hence the molecular diffusion layer acts effectively as a "bottleneck" in the transport of the dissolved gas into the fluid.

The theories presented all began with a diffusion equation and imposed boundary conditions appropriate to either a semi-infinite layer or a film of thickness δ. The instantaneous absorption rate $-D_m \left(\dfrac{\partial C}{\partial y} \right)_{y=0}$ was then modified by making assumptions about the time which an element was in contact with the gas and the overall liquid film coefficient was found from the definition of the mass flux F.

$$F = K_L(C_s - C_o) \qquad (6)$$

In the derivations of the absorption rate, the concentration in the bulk of the fluid was always assumed to be constant and not changing with time. It was argued that the molecular diffusion process occurred over only a short period before surface renewal would mix the surface elements into the bulk of the liquid and therefore the bulk concentration could be considered constant during the time. This assumption, or rather the boundary conditions employed, has left out the mechanism through which the gas diffusing through the film can be transferred to the main body of the fluid.

Only the Dobbins film-penetration model (1956) and the stochastic model of Rudis and Machek (1971) [with the corrections given here] can be reduced to the limits given by the surface renewal and film theories, and on this basis these two models can be considered to have wider applicability than the others. King's model of damped eddy diffusivity and the large eddy model are both suitable only for explanation of the mechanism.

Even though some refinements may be made on these conceptual models one is still left with the problem of determining the parameters such as film thickness δ, renewal rate r, and eddying and film state durations etc. It is questionable whether renewal rates obtained in a stirred cell can be applied to an open channel. Obviously, work has to be done to relate these parameters to measurable hydraulic variables before the conceptual models can be successfully applied to predict reaeration rates in natural channels.

3. EMPIRICAL AND SEMI-EMPIRICAL EQUATIONS FOR PREDICTION OF THE REAERATION RATE

In the previous chapter the various conceptual models on the mechanism of oxygen absorption have been shown to be unsuitable for the prediction of reaeration rates in rivers and streams. Because of the difficulty in applying the results of these theories a number of empirical and semi-empirical equations have been developed with the object of relating the rate of reaeration to the commonly measured stream variables such as depth, discharge, slope etc. These equations are not concerned with the mechanism behind the absorption process but are meant mainly as tools for design and management. The derivation of these equations is mainly through regression-correlation analysis of experimental data to establish relationships between the reaeration rate and the hydraulic parameters which are considered to be pertinent. These equations would obviously give the best fit to the data from which they were derived. Unfortunately most of them do not work very well when applied to data from a different range of flow conditions. Bennett and Rathbun (1971) have compiled the available flume and field data and performed error estimates on the various prediction equations. It was shown that over the entire range of data, most of the equations gave a standard error of estimate greater than fifty percent.

3.1. The Reaeration Constant and the DO Balance Equation.

Before proceeding to describe the various prediction equations it is worthwhile to see how experimental measurements can be related to the liquid film coefficient since the mass transfer rate is generally not directly measured and conditions vary with different experimental configurations.

Consider oxygen being absorbed into a stirred tank. The DO concentration can be described by equation (21)

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial y}\left[(D_m + \varepsilon_y)\frac{\partial C}{\partial y}\right] \qquad (21)$$

However the concentration in the stirred tank is usually assumed to be uniform owing to the high degree of turbulent mixing. Hence

equation (21) can be integrated over the whole volume giving an equation for the mean bulk concentration.

$$\frac{\partial \bar{C}}{\partial t} = \frac{A_s}{V} \left[-(D_m + \epsilon_y) \left(\frac{\partial C}{\partial y}\right)_{y=0} \right] = \frac{A_s}{V} K_L (C_s - \bar{C}) \qquad \ldots\ldots(37)$$

where $\bar{C}$ = average concentration of the whole volume of the tank.
  A_s = area of the gas-liquid interface.
  V = volume of liquid in the tank.

Experimental measurement of the mean concentration in the tank will therefore allow the parameter $\frac{A_s K_L}{V}$ to be evaluated. This parameter is termed K_2, the reaeration constant. Hence the equation for the oxygen deficit ($C_s - \bar{C}$) in a stirred tank is generally written as

$$\frac{\partial D}{\partial t} = -K_2 D \qquad \ldots\ldots(38)$$

where D is the average O_2 deficit.

Strictly speaking, K_2 and K_L only describe the condition at the interface and are not part of the governing equation (21). However in equation (38), the change in oxygen deficit is considered as a first order reaction for the whole body of liquid instead of being an absorption process at the surface.

It should be noted that K_2 is the parameter which can be determined experimentally. Because of the difficulty in estimating the interfacial area under highly agitated conditions, accurate determination of the liquid film coefficient K_L cannot be obtained.

For the diffusion of oxygen in a 2-dimensional, uniform, open-channel flow the equation for the conservation of mass can be written as

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left[(D_m + \epsilon_x)\frac{\partial C}{\partial x}\right] + \frac{\partial}{\partial y}\left[(D_m + \epsilon_y)\frac{\partial C}{\partial y}\right] + S \qquad \ldots\ldots(39)$$

where ϵ_x and ϵ_y are the longitudinal and vertical eddy diffusivities respectively. The term S is a source or sink term representing the effect of BOD and plants etc.

Theoretically, one could use equation (39) to solve for the concentration of DO in the flow without introducing K_L or K_2 provided one knew ϵ_x and ϵ_y completely. However solution of equation (39) can be rather difficult to achieve in closed form and since the bulk concentration is known to be rather uniform, equation (39) can be averaged over the depth of flow, resulting in the equation

$$\frac{\partial \bar{C}}{\partial t} + \bar{U} \frac{\partial \bar{C}}{\partial x} = \overline{(D_m + \epsilon_x)} \frac{\partial^2 \bar{C}}{\partial x^2} + \frac{1}{h}\left[-(D_m + \epsilon_y)\left(\frac{\partial C}{\partial y}\right)_{y=0}\right] + \bar{S}$$

$$= \overline{(D_m + \epsilon_x)} \frac{\partial^2 \bar{C}}{\partial x^2} + \frac{K_L}{h}(C_s - \bar{C}) + \bar{S} \qquad \ldots\ldots(40)$$

Terms with an overbar indicate average over the depth.

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The longitudinal diffusion term is often neglected since it is usually small compared with the convection term $\bar{U} \frac{\partial \bar{C}}{\partial x}$ and the equation for the DO concentration can be written as

$$\frac{\partial \bar{C}}{\partial t} + \bar{U} \frac{\partial \bar{C}}{\partial x} = K_2(C_S - \bar{C}) + \bar{S} \qquad \ldots\ldots(41)$$

or

$$\frac{\partial \bar{D}}{\partial t} + \bar{U} \frac{\partial \bar{D}}{\partial x} = -K_2\bar{D} - \bar{S} \qquad \ldots\ldots(42)$$

K_2, the reaeration constant, is actually equal to $\frac{K_L}{h}$. It is usually obtained from a solution of the concentration equation and experimental measurements of DO concentration. The observed K_2 values are then subjected to multiple regression analysis, assuming some functional dependence on the hydraulic variables, to obtain a prediction equation.

For studies in natural streams, steady state and uniform flow conditions are generally assumed. The source-sink term is usually made up of two components, K_1L and D_B. K_1 is the BOD reaction rate constant, L is the amount of BOD and D_B is a constant representing the net effect of plant respiration and photosynthesis. The term L has to be obtained from a solution of the BOD balance equation and is a function of x. Solutions for the DO and BOD balance equations have been presented by Dobbins (1964) and Camp (1963). In order to obtain K_2 from field experiments, all the other rate constants have to be known. Methods for determining these constants have been given by Dobbins (1964) and Bennett and Rathbun (1971).

When conditions in a river are such that the entire cross-section is well mixed and a cross-sectional average can be used to represent the O_2 deficit without being very much in error, a longitudinal dispersion term is generally included in the mass transport equation, i.e.

$$\frac{\partial \bar{\bar{D}}}{\partial t} + \bar{\bar{U}} \frac{\partial \bar{\bar{D}}}{\partial x} = D_L \frac{\partial^2 \bar{\bar{D}}}{\partial x^2} - K_2\bar{\bar{D}} \qquad \ldots\ldots(43)$$

where D_L is the longitudinal dispersion coefficient and the double overbar indicates cross-sectional average.

Dobbins (1964) has shown that the effect of D_L is negligible for most natural streams. However when D_L is very large, such as in tidal estuaries, the dispersion term should be taken into account.

It should be noted that in most of the engineering literature, the reaeration coefficient k_2 is used where

$$k_2 = \frac{K_2}{2 \cdot 31} = \frac{K_L}{2 \cdot 31 \ h} \qquad \ldots\ldots(44)$$

This k_2 is used to facilitate the use of logarithm to the base 10 instead of base e.

In recirculating flumes in the laboratory, conditions can be controlled so that atmospheric reaeration is the only factor affecting the DO content. However, because the water is reaerating only in the flume but

not in the return passage, the concentration in the flume will be varying both with distance and time. The O_2 deficit can be described by equation (42) with $\bar{S}=0$. The solution for the oxygen deficit $\bar{D}$, when the appropriate boundary conditions are applied, is

$$\bar{D} = \bar{D}_o \; e^{-K_2 \beta t} \; e^{-K_2 (1-\beta) \frac{x}{U}} \qquad \qquad \dots\dots(45)$$

where

$\bar{D}_o$ = initial O_2 deficit

and

$$\beta = \frac{\text{volume of liquid in the flume}}{\text{total volume in the system}}$$

Hence if measurement of the deficit at one point in the flume is made and $\ln D$ versus t is plotted, the slope is equal to $K_2 \beta$ which is less than K_2. To find K_2, measurements have to be made at two points in the flume, with a time lag equal to the travel time between the two points, i.e., one has to follow a parcel of fluid down the flume.

In a straight-through flume where the water is not recirculated, conditions are steady in time and the variation of concentration with distance would yield the reaeration rate. However, these flumes must necessarily be quite small, being limited in size by the availability of deoxygenated water.

In a stirring tank or a circular flume such as the one used by Isaacs and Gaugy (1968), the conditions change only with time and measurements need to be made at only one location.

3.2. Prediction Equations

3.2.1. O'Connor-Dobbins Equation (1956)

This equation was the first attempt to relate the film coefficient from the film-penetration model of Dobbins (1956) to ordinary hydraulic variables. The film coefficient given by that model is

$$K_L = \sqrt{D_m r} \; \coth \sqrt{\frac{r \delta^2}{D_m}} \qquad (20)$$

O'Connor and Dobbins (1956) reasoned that for most practical cases the coth term was close to unity and so only the surface renewal rate r had to be evaluated. They considered a time scale for mixing to be equal to $\frac{\ell}{v'}$, where ℓ is the mixing length and v' is the rms vertical velocity fluctuation. The surface renewal rate was assumed to be equal to the inverse of the time scale. Hence

$$r = \frac{v'}{\ell}$$

For non-isotropic turbulence, Prandtl's definition of mixing length was used to give

$$r = \frac{v'}{\ell} = \left(\frac{\partial U}{\partial y}\right)_{y=0} \qquad \qquad \cdots\cdots(46)$$

From the logarithmic velocity distribution, they obtained

$$r = \frac{\sqrt{hgS}}{\kappa h} \qquad \qquad \cdots\cdots(47)$$

where h = mean depth of flow.
       g = gravitational constant.
       S = slope of the channel.
       κ = 0.4, the Von-Karman universal constant.

Substituting equation (47) into the expression $K_L = \sqrt{D_m r}$ gives

$$K_L = \frac{D_m^{\frac{1}{2}} S^{\frac{1}{4}} g^{\frac{1}{4}}}{\kappa^{\frac{1}{2}} h^{\frac{1}{4}}} \qquad \qquad \cdots\cdots(48)$$

and

$$k_2 = \frac{D_m^{\frac{1}{2}} S^{\frac{1}{4}} g^{\frac{1}{4}}}{2\cdot31 \; \kappa^{\frac{1}{2}} h^{\frac{5}{4}}} \qquad \qquad \cdots\cdots(49)$$

In case of relatively deep channels where velocity gradients may be very small, O'Connor and Dobbins (1956) assumed that isotropic turbulence may be approached and they used some measurements by Kalinske (1941) of the mixing length and velocity fluctuations which gave approximately $v' = 0\cdot1\,U$ and $\ell = 0\cdot1\,h$. These values were assumed to be constant throughout the depth.

Therefore

$$K_L = \frac{D_m U}{h} \qquad \qquad \cdots\cdots(50)$$

and

$$k_2 = \frac{D_m^{\frac{1}{2}} U^{\frac{1}{2}}}{2\cdot31 \; h^{\frac{3}{2}}} \qquad \qquad \cdots\cdots(51)$$

O'Connor and Dobbins (1956) arbitrarily assumed that if the Chezy coefficient for a river is less than 17, the non-isotropic turbulence formula for k_2, equation (49), could be applied and equation (51) for isotropic turbulence could be applied if the Chezy coefficient is greater than 17. This method of separating the flow by its Chezy coefficient has been widely criticized as being arbitrary and unsound.

There is also no reason why the renewal rate should be exactly equal to the inverse of the time scale and even if this happens to be true the value of r obtained from equation (47) is still subject to question. Although a logarithmic velocity distribution can satisfactorily describe the velocity profile throughout most of the depth, it may not be accurate right up to the surface due to the effect of secondary currents and especially if there is

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wind blowing on the water surface. Therefore the actual velocity gradient at the surface may be very different from its theoretical value.

It has been pointed out by Watt (1972) that an equation similar to (47) can be derived without knowing the velocity distribution. Writing $r = \frac{v'}{L_L}$ where L_L is the integral length scale of turbulence, and assuming that $v' \sim U_*$ the shear velocity and $L_L \sim h$ one gets

$$r \sim \frac{U_*}{h} \sim \frac{\sqrt{ghS}}{h}$$

This result does not lend any more support to equation (48) for K_L but simply indicates that such a relationship for the renewal rate can be arrived at without having to differentiate the velocity profile.

From the data of Kalinske (1941) it can be seen that the values of $\frac{v'}{U}$ actually varied with depth. For some streams with depth around ten feet, $\frac{v'}{U}$ at the surface is approximately equal to 0.1 while in other cases this value may be close to 0.2. Therefore, even though equations (49) and (51) have been shown to correlate fairly well with some field observations of k_2, they cannot be applied over a wide range of flow conditions.

3.2.2. Churchill-Elmore-Buckingham Equation (1962).

Perhaps the most reliable set of field data on the reaeration rate was obtained by Churchill *et al.* (1962). Their experiments were carried out in stretches of rivers below dams and impoundments where the discharges were practically devoid of BOD and the DO concentration were low because of prolonged storage under stratified conditions. The DO concentration was sampled from boats and 16 different reaches on 5 rivers were tested. The flow depths varied from about 2 to 11 feet and the velocities varied from 1.8 to 5 feet per second.

A large number of regression equations were presented using various parameters but none of these could correlate the data significantly better than the others. The equation recommended by Churchill *et al.* (1962) was

$$k_2 = 5.026 \ \frac{U^{0.969}}{h^{1.673}} \ 1.0241^{(T-20)} \qquad \ldots\ldots(52)$$

where k_2 is in days^{-1},
 U in feet per second,
 h in feet, and
 T in °C.

This equation had a multiple correlation coefficient of 0.822.

Churchill *et al.* (1962) reported that there were no measurable gradients of DO in the vertical direction. However the lateral cross-stream DO concentrations exhibited definite U-shaped patterns, with higher concentrations at the banks and lower concentrations in the middle. Variations were often as high as 15%. This was attributed to a variety of factors, including incomplete mixing of tributary inflows, unequal effects of plant

respiration and photosynthesis and increased aeration near the banks because of shallower depths. However there was no evidence whether any or all of these factors were significant.

3.2.3. Krenkel and Orlob Equation (1963).

The rate of reaeration in a two-dimensional open-channel flow was measured by Krenkel and Orlob (1963) in a 1-foot wide by 60-foot long tilting flume. The water was first deoxygenated with sodium sulphite and the subsequent rise in DO concentration was measured at two points along the flume with a time lag equal to the travel time between the two points. This procedure enabled the calculation of k_2. The longitudinal dispersion coefficient D_L was measured by the injection of dye into the flume and measuring the change of dye concentration with time at a point downstream of the injection. The measured values of k_2 were correlated with D_L and the depth of flow h and the equation given by Krenkel and Orlob (1963) for the reaeration coefficient was

$$k_{2(20°)} = 1 \cdot 138 \times 10^{-5} \frac{D_L^{1.321}}{h^{2.32}} \qquad \ldots\ldots(53)$$

where k_2 is in minutes^{-1},
D_L in ft^2/min., and
h in feet.

However, it is very doubtful whether the flume was long enough for the tracer to have been well mixed over the cross section. Calculations according to the criterion given by Fischer (1967) shows that the distance downstream from the source to where the one-dimensional diffusion equation would start to apply has to be about 90 feet for the kind of flow conditions used by Krenkel and Orlob.. Hence the values of D_L measured by Krenkel and Orlob were probably not correct and equation (53) cannot be expected to apply to other data.

Krenkel and Orlob (1963) also reasoned that the reaeration rate was related to E, the energy expenditure per unit mass of fluid. E = USg where U is the mean velocity and S the slope. A regression analysis was performed and gave the equation

$$k_{2(20°)} = 1 \cdot 141 \times 10^{-4} \frac{E^{0.408}}{h^{0.66}} \qquad \ldots\ldots(54)$$

where k_2 is in minutes^{-1},
E in ft^2/min.^{3}, and
h in feet.

It should be noted that neither equation 53 or 54 is dimensionally homogeneous.

3.2.4. Dobbins Equation (1964).

In another attempt to relate the parameters in the film-penetration theory to ordinary hydraulic variables Dobbins (1964) argued that the thickness of the liquid film must be proportional to the size of the smallest energy dissipating eddies and that the rate of energy expenditure to overcome

surface tension and replace the surface liquid must be proportional to the turbulent energy dissipated per unit mass of liquid near the surface. From these arguments he obtained

$$\delta = C_3 \left[\frac{\nu^3}{E_s}\right]^{\frac{1}{4}} = C_4 \left[\frac{\nu^3}{E}\right]^{\frac{1}{4}} \qquad \ldots\ldots(55)$$

where ν is the kinematic viscosity of the liquid,
E_s and E are the energy dissipated per unit mass of liquid at the surface and by the entire flow respectively, and
C_3 and C_4 are constants.

The renewal rate r was given as

$$r = \frac{C_5}{C_4^3} \frac{\rho \nu^{\frac{3}{4}} E^{\frac{3}{4}}}{\sigma} \qquad \ldots\ldots(56)$$

where C_5 is a constant depending on the properties of the fluid and,
σ is the surface tension.

C_4 is the constant which varies with the dynamics of the flow. From experiments with helium and nitrogen in a cylinder with an oscillating grid to generate turbulence Dobbins obtained the value of 14.3 for C_5. From reported values of k_2 in natural streams and flume measurements of Krenkel and Orlob (1963), the value of C_4 was evaluated and plotted against $\left[\frac{\nu^3}{E}\right]^{\frac{1}{4}} h^{-1}$. Dobbins (1964) gave the equation for C_4 as

$$C_4 = 0\cdot65 + 15000 \left(\frac{\left[\frac{\nu^3}{E}\right]^{\frac{1}{4}}}{h}\right)^2 \qquad \ldots\ldots(57)$$

Hence from the Dobbins equation for K_L equation (20), the equation for k_2 is

$$k_2 = \frac{1}{2\cdot31h} \left(\frac{C_5 \; D_m \; \rho \; \nu^{\frac{3}{4}} \; E^{\frac{3}{4}}}{C_4^3 \; \sigma}\right)^{\frac{1}{2}} \coth \sqrt{\frac{C_5 \; \rho \; \nu^{\frac{9}{4}} \; E^{\frac{1}{4}}}{C_4 \; D_m \; \sigma}} \qquad \ldots\ldots(58)$$

with $C_5 = 14.3$ and C_4 given by equation (57). This equation is obviously very cumbersome to use.

The data from which the equation for C_4 was derived actually contained a lot of scatter. Values of C_4 for the rivers varied from 0.3 to 1.2 but was assumed to be constant at 0.65. Since both r and δ depend upon C_4 this must raise some doubts as to the accuracy of the predictions. However Dobbins (1964) showed that there was fair agreement between some stream measurements of K_L and his predictions.

3.2.5. Isaacs and Gaudy Equation (1968).

Isaacs and Gaudy (1968) measured reaeration in a circular flume in which the inner and outer walls could be driven at different speeds to

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simulate streamflow conditions. They did some dimensional analysis but in the end expressed k_2 as

$$k_2 = C\ U^m\ h^n \qquad \qquad \dots(59)$$

Multiple regression analysis on their data yielded values of 1.0027 and -1.4859 for m and n respectively. Since these were close to 1 and -3/2, another analysis was performed keeping the exponents fixed at 1 and -3/2. The resulting equation was

$$k_2 = 3\cdot053\ \frac{U}{h^{\frac{3}{2}}}\ (1\cdot0241)^{(T-20)} \qquad \qquad \dots(60)$$

where k_2 is in day^{-1},
U   in feet per second,
h   in feet, and
T   in °C.

Isaacs and Gaudy (1968) applied the same regression analysis to other published data including those of Churchill *et al.* (1962) and Krenkel and Orlob (1963). The constants came out to be 3.739 and 2.44 respectively, indicating among other things that U and h alone were insufficient to describe the reaeration process.

Although the apparatus used by Isaacs and Gaudy (1968) has the advantage that conditions do not vary in the flow direction, the flow pattern in the circular flume has been criticized as not being similar to natural flows because of the different directions in which shear is being exerted on the sides and at the bottom and because of the resulting high velocity gradients near the side walls. The width to depth ratios used varied from 2 to 2/3 which is also not characteristic of natural streams.

3.2.6. Thackston and Krenkel Equation (1969).

It was assumed by Thackston and Krenkel (1969) that k_2 was proportional to the renewal rate which they argued must depend on the eddy diffusion coefficient of mass at the surface ε_{yo} and the depth h. Therefore they proposed that $k_2 \sim \dfrac{\varepsilon_{yo}}{h^2}$. Asuming Reynolds analogy, so that $\varepsilon_{yo} \sim k_y \sim U_* h$ where k_y was the eddy diffusivity of momentum, they concluded that $k_2 \sim \dfrac{U_*}{h}$.

Reaeration experiments were performed in a 2-foot wide, 60-foot long tilting flume. Tracer experiments were also performed to measure the longitudinal dispersion coefficient D_L. The experimental values of k_2 were correlated with $\dfrac{D_L}{h^2}$ and with $\dfrac{U_*}{h}$. The equations given by Thackston and Krenkel (1969) were

$$k_2 = 1\cdot5\ \times\ 10^{-5}\ \frac{D_L}{h^2} \qquad \qquad \dots(61)$$

and

$$k_2 = 2\cdot15\ \times\ 10^{-4}\ \frac{U_*}{h} \qquad \qquad \dots(62)$$

To take into account that the interfacial area may be larger than the projected surface area, the values of $\frac{k_2 h}{U_*}$ were plotted against Froude number $\frac{U}{\sqrt{gh}}$. There was quite a lot of scatter but Thackston and Krenkel (1969) gave the formula

$$k_2 = 1 \cdot 25 \times 10^{-4} \ (1 + F_r^{\frac{1}{2}}) \ \frac{U_*}{h} \qquad\qquad \dots\dots(63)$$

as the prediction equation for k_2 where F_r is the Froude number.

Thackston and Krenkel compared their data with the ones obtained by Krenkel and Orlob (1963) in a 1-foot wide flume. Multiple regression analysis were applied to both sets of data but although the exponents obtained for the parameters were very much the same the constant C for the data of Krenkel and Orlob was found to be always greater by a factor of 2.2 to 3.5. Regression analysis applied to the natural stream data of Churchill (1963) yielded an equation very much the same as equation (62). On this basis Thackston and Krenkel (1969) claimed that the data from their 2-foot wide flume were representative of natural stream data while those from the 1-foot wide flume of Krenkel and Orlob (1963) were not. Since the width was the only significant difference between the two flumes they argued that some unknown factor related to the width made the one-foot flume unrepresentative of open-channel flow. It is more likely that the prediction equations used are not general enough and that other parameters which affect the reaeration rate are missing from the regression analysis.

3.3. Error Estimates for the Prediction Equations

In addition to the studies described in the previous pages reaeration measurements were also made by Owens *et al.* (1964) in several small streams and by Negulescu and Rojanski (1969) in a recirculating flume. They also presented prediction equations for k_2 using velocity and depth. These and other equations described previously are summarized below.

O'Connor-Dobbins

$$k_2 = \frac{D_m^{\frac{1}{2}} \ S^{\frac{1}{4}} \ g^{\frac{1}{4}}}{2 \cdot 31 \ h^{\frac{5}{4}}} \qquad \text{(non-isotropic turbulence)} \qquad (49)$$

O'Connor-Dobbins

$$k_2 = \frac{D_m^{\frac{1}{2}} \ U^{\frac{1}{2}}}{2 \cdot 31 \ h^{\frac{3}{2}}} \qquad \text{(isotropic turbulence)} \qquad (51)$$

Churchill *et al.*

$$k_2 = 5 \cdot 026 \ \frac{U^{0.969}}{h^{1.673}} \ 1 \cdot 0241^{(T-20)} \qquad (52)$$

Krenkel-Orlob

$$k_{2 \ (20^\circ C)} = 3 \cdot 659 \ \frac{D_L^{1.321}}{h^{2.32}} \qquad (53)$$

Krenkel-Orlob

$$k_{2 (20°C)} = 24.66 \frac{E^{0.408}}{h^{0.66}} \qquad (54)$$

Dobbins

$$k_2 = \frac{1}{2.31 h} \sqrt{\frac{C_5 D_m \rho v^{\frac{3}{4}} E^{\frac{3}{4}}}{C_4^3 \sigma}} \; coth \sqrt{\frac{C_5 \rho v^{\frac{9}{4}} E^{\frac{1}{4}}}{C_4 \sigma D_m}} \qquad (58)$$

Isaacs-Gaudy

$$k_2 = 3.053 \frac{U}{h^{\frac{3}{2}}} 1.0241^{(T-20)} \qquad (60)$$

Thackston-Krenkel

$$k_2 = 1.5 \times 10^{-5} \frac{D_L}{h^2} \qquad (61)$$

Thackston-Krenkel

$$k_2 = 2.15 \times 10^{-4} \frac{U_*}{h} \qquad (62)$$

Thackston-Krenkel

$$k_2 = 1.25 \times 10^{-4} (1 + F_r^{\frac{1}{2}}) \frac{U_*}{h} \qquad (63)$$

Owens $et\ al.$

$$k_2 = 9.41 \frac{U^{0.67}}{h^{1.85}} 1.0241^{(T-20)} \qquad \dots\dots(64)$$

Negulescu-Rojanski

$$k_2 = 4.74 \left(\frac{U}{h}\right)^{0.85} \qquad \dots\dots(65)$$

Note that for equations (52), (53), (54), (59), (64) and (65) which are not dimensionally homogeneous, the constants have been adjusted so that k_2 is given in days^{-1} with U in feet per second, h in feet, D_L in feet square per second and E in feet^2/second^3.

Bennett and Rathbun (1971) compiled the available field and laboratory data and evaluated the performance of the various prediction equations by comparing their standard error of estimate over the whole range of data.

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They defined standard errors of estimate E_S and E_{SL} as follows:

$$E_S = \sqrt{\dfrac{\sum\limits_{i=1}^{n} [(k_2)_e - (k_2)_c]_i^2}{n}} \qquad \qquad \dots\dots(66)$$

$$E_{SL} = \sqrt{\dfrac{\sum\limits_{i=1}^{n} [\log_{10}(k_2)_e - \log_{10}(k_2)_c]_i^2}{n}} \qquad \dots\dots(67)$$

where $(k_2)_e$ = estimated k_2 from prediction equation,
$(k_2)_c$ = calculated k_2 from experiment, and
n = number of k_2 values used.

A percent error E_p was defined as

$$E_p = 100 \, [1 - 10^{-E_{SL}}] \qquad \qquad \dots\dots(68)$$

This value of the percent error is a conservative estimate of the error of the prediction equation.

For each of the prediction equations, Bennett and Rathbun (1971) calculated E_S and E_p first for the original data sets from which the equations were derived and then for all of the data sets. Their results were tabulated and are reproduced here in Table 1. It can be seen that when applied to the whole range of data none of the equations is very satisfactory. The value of E_p varies from 86 percent to 43 percent and some of the values of E_S are greater than the maximum k_2 in the data. Bennett and Rathbun (1971) performed multiple regression analysis on the field and laboratory flume data separately, using width, depth, slope, velocity and hydraulic radius as parameters and concluded that all the k_2 prediction equations obtained from flume data were significantly different statistically from those obtained from field data. Whether this occurs because there are differences in the basic physical process or not is not known. However there are definite deficiencies in the field data since on the one hand almost none of them have accounted for plant effects, sedimentation and benthal demand adequately but on the other hand, flume data might have been affected by secondary currents or by corrosive oxidation in the flume. The equation which gave the best fit to all available field data was given by Bennett and Rathbun (1971) as

$$k_2 = 4 \cdot 605 \, \dfrac{U^{0.413} \, S^{0.273}}{h^{1.408}} \qquad \qquad \dots\dots(69)$$

where k_2 is in days^{-1}.

Equation (69) has standard error of estimate E_p equal to 31.5 percent. For routine use as a prediction equation, it was suggested that the equation

$$k_2 = 8 \cdot 76 \, \dfrac{U^{0.607}}{h^{1.689}} \qquad \qquad \dots\dots(70)$$

serves just as well since it has a value of E_p equal to 37.5 percent and does not require knowledge of the slope.

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TABLE 1

Error Analysis of the Various Prediction Equations as Published by Bennett and Rathbun (1971)

Reference	Predicted k_2 days^{-1}	Number data points used in original publication	k_2 days^{-1} Max.	Min.	U feet per second Max.	Min.	H feet Max.	Min.	S feet per foot Max.	Min.	Standard error of estimate E_S days^{-1}	Standard error of estimate E_p percent	Number data points which can be used	k_2 days^{-1} Max.	Min.	U feet per second Max.	Min.	H feet Max.	Min.	S feet per foot Max.	Min.	Standard error of estimate E_S days^{-1}	Standard error of estimate E_p percent
O'Connor and Dobbins (1958)	$\frac{480\, D_m^{1/2}\, S^{1/4}}{H^{5/4}}$ (C < 17)	13	2.60	0.14	0.73	0.19	8.60	1.90	1.40×10^{-3}	9.50×10^{-5}	0.34	45	38	82.92	0.14	1.37	0.13	33.69	0.06	2.01×10^{-2}	9.50×10^{-5}	33.7	46
O'Connor and Dobbins (1958)	$\frac{127\, (D_m U)^{1/2}}{H^{3/2}}$ (C > 17)	22	4.8	0.14	4.20	0.53	24.20	0.90	3.60×10^{-3}	2.70×10^{-5}	0.42	44	169	115.3	0.14	5.00	0.24	24.20	0.04	2.40×10^{-2}	2.70×10^{-5}	115.0	71
Dobbins (1964b)	$\frac{0.12\, B_E\, A\, E^{3/8}\, \coth\left(\frac{B\, E^{1/8}}{C_4^{1/2}}\right)}{H\, C_4^{3/2}}$	123	115.3	0.14	5.00	0.13	24.20	0.08	2.40×10^{-2}	2.70×10^{-5}	7.74	33	207	115.3	0.14	5.00	0.24	24.20	0.04	2.40×10^{-2}	2.70×10^{-5}	21.0	43
Fortescue and Pearson (1967)	$429 \left(\frac{D_m U}{H^3}\right)^{1/2}$	38	4.8	0.02	4.20	0.19	37.00	0.90	---------	---------	2.87	70	239	115.3	0.02	5.00	0.13	37.00	0.04	---------	---------	395.2	86
Krenkel (1960)	$24.55\, E^{0.408}\, H^{-0.66}$	58	115.3	10.63	2.14	0.13	0.20	0.08	2.40×10^{-2}	7.50×10^{-4}	7.53	15	207	115.3	0.14	5.00	0.24	24.20	0.04	2.40×10^{-2}	2.70×10^{-5}	16.4	51
Thackston (1966)	$18.58\, \frac{U_*}{H}$	40	82.93	7.66	2.32	3.65	0.23	0.04	2.04×10^{-2}	6.50×10^{-4}	9.96	25	207	115.3	0.14	5.00	0.24	24.20	0.04	2.40×10^{-2}	2.70×10^{-5}	20.6	45
Thackston (1966)	$10.80\, (1 + F^{0.5})\, \frac{U_*}{H}$	105	82.93	0.14	5.00	0.19	24.20	0.04	2.04×10^{-2}	2.70×10^{-5}	5.90	37	207	115.3	0.14	5.00	0.24	24.20	0.04	2.40×10^{-2}	2.70×10^{-5}	20.0	45
Churchill and others (1962)	$5.026\, U^{0.969}\, H^{-1.673}$	30	5.58	0.22	5.00	1.85	11.41	2.12	2.35×10^{-3}	1.26×10^{-4}	0.52	28	239	115.3	0.02	5.00	0.13	37.00	0.04	---------	---------	140.3	73
Owens and others (1964)	$10.90\, U^{0.73}\, H^{-1.75}$	32	49.17	0.31	1.83	0.13	2.44	0.39	---------	---------	5.46	35	239	115.3	0.02	5.00	0.13	37.00	0.04	---------	---------	419.8	84
Owens and others (1964)	$9.41\, U^{0.67}\, H^{-1.85}$	68	57.7	0.22	5.00	0.13	11.41	0.34	---------	---------	5.00	32	239	115.3	0.02	5.00	0.13	37.00	0.04	---------	---------	474.9	85
Isaacs and Gaudy (1968)	$3.739\, \frac{U}{H^{3/2}}$	30	5.56	0.22	5.00	1.85	11.41	2.12	---------	---------	0.55	30	239	115.3	0.02	5.00	0.13	37.00	0.04	---------	---------	55.3	63
Isaacs and Gaudy (1968)	$2.440\, \frac{U}{H^{3/2}}$	58	115.3	0.11	2.14	0.13	0.20	0.08	---------	---------	10.44	29	239	115.3	0.02	5.00	0.13	37.00	0.04	---------	---------	30.8	65
Negulescu and Rojanski (1969)	$4.74 \left(\frac{U}{H}\right)^{0.85}$	23	18.70	0.65	1.90	0.29	3.11	0.16	---------	---------	1.63	29	239	115.3	0.02	5.00	0.13	37.00	0.04	---------	---------	21.2	61

Columns 3–12 fall under "Data from source publication" (Derivation data range, with Standard error of estimate E_S days^{-1} and Standard error of estimate E_p percent); columns 13–24 fall under "Total applicable data" (Number data points which can be used, Data range used, Standard error of estimate E_S days^{-1} and Standard error of estimate E_p percent).

4. CONCLUSIONS

From this review it is clear that a completely satisfactory method of predicting atmospheric reaeration is still not available. The conceptual models of Dobbins (1956) and Rudis and Machek (1971) seem to follow the most logical course but like all the other conceptual models, they suffer from the drawback of having to employ parameters which are difficult, and sometimes impossible to measure. On the other hand the prediction equations which are generally very simple to apply, have been shown to give large errors when applied over a wide range of flow conditions. The most likely reason for the lack of success of these prediction equations is that not all the parameters which affect reaeration have been taken into account. Velocity and depth, the two commonly used parameters, are not sufficient for the complete description of flow properties in an open channel.

Refinements can probably be made on some of the conceptual models. Improvement in boundary conditions could be made to take into account the transfer of oxygen into the bulk of the fluid. A continuous eddy diffusivity profile similar to that of King (1966) but with a liquid of finite depth is also a possibility. However, research still has to be done to relate the different parameters to hydraulic variables such as velocity, slope, depth, turbulence intensity etc. Parameters such as the surface renewal rate should be measured in open channel flow if possible, instead of in stirring tanks.

It is likely that the prediction equations have not taken into account all the parameters which are of significance and hence they all have limited applicability. Although dimensional analysis has been employed, it has always been used only as a means to justify a certain form of the equation for regression analysis. The data have not been subjected to rigorous dimensional analysis and experimentation by varying only one dimensionless parameter at a time to investigate the effect of each parameter on the reaeration rate. The data compiled by Bennett and Rathbun (1971) may be utilized for such an analysis.

The discovery of Churchill *et al.* (1963) that there is considerable variation in the cross-stream reaeration rate should be investigated. If this is a physical process common to all natural streams it has implications as to where sewage and BOD wastes can be strategically released. It follows that the effect of channel shapes on reaeration may be investigated.

There are other factors affecting the DO balance of a stream which have not been looked at in this review. The effect of wind blowing over the water surface, effect of surfactants, temperature, temporal variation of plant effects, etc. all contribute to the reaeration rate. It is obvious that more research has to be done before we can successfully model the DO balance of a natural stream.

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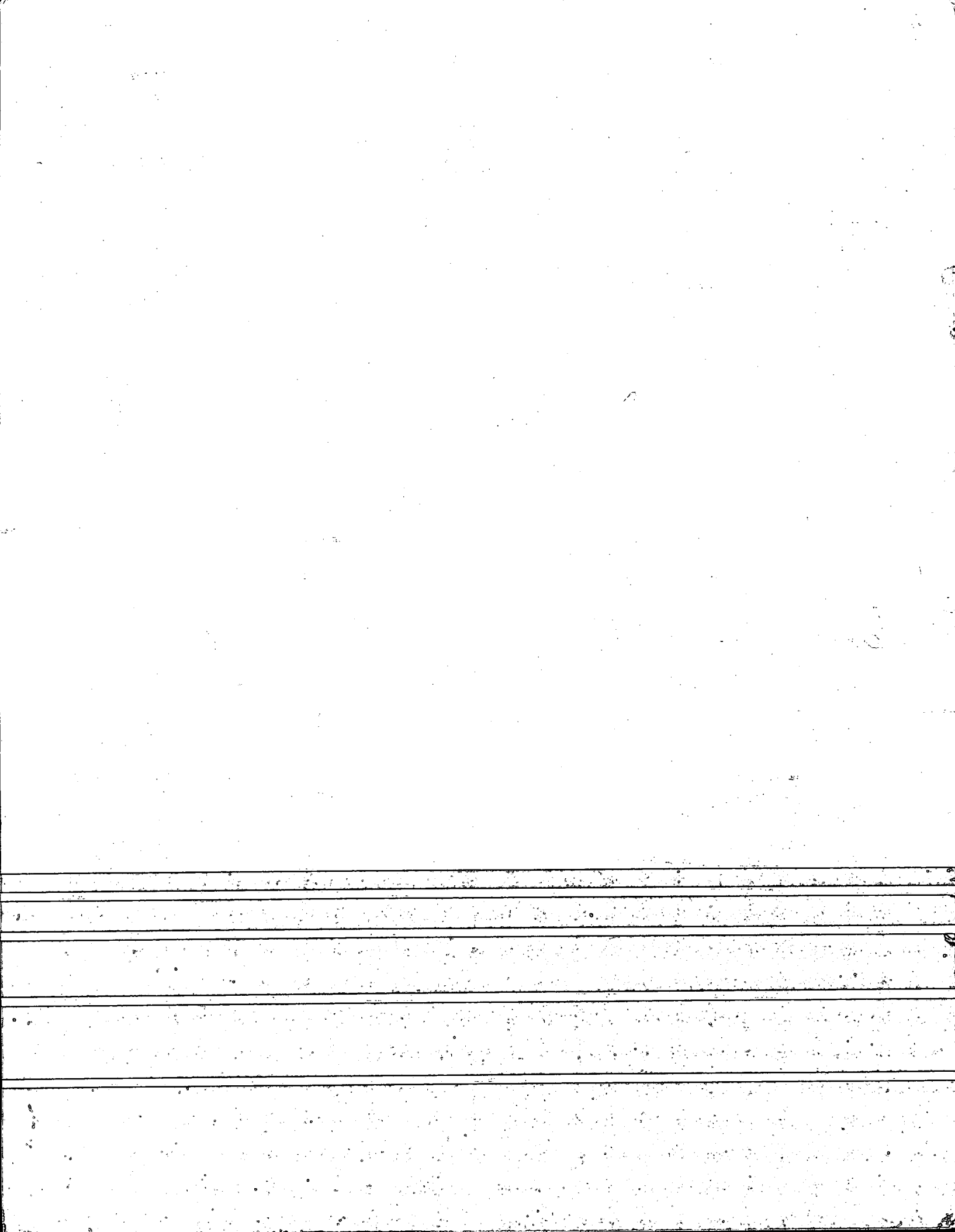
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