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DEPARTMENT OF THE ENVIRONMENT

# *An Engineering Study of Crustal Movement around Lake Superior*

*G.W. KITE*

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INLAND WATERS BRANCH
DEPARTMENT OF THE ENVIRONMENT
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# Abstract

Crustal movement has been measured in the Great Lakes area since the middle of the nineteenth century using long term lake level gauges. A possible cause of this movement is isostatic adjustment of the earth's crust following the last glacial retreat.

One method of computing relative crustal movement is to take differences in lakes levels between pairs of gauges and compute first order linear trends. This study analyses the validity of this method and also examines methods of converting the resulting relative movements into absolute movements for the Lake Superior region.

It was found that time series created by taking differences in mean monthly elevations at two lake level gauges are made up of three main components; a first order linear trend, periodicity in the mean (chiefly the annual cycle) and a large random component.

The results of the analyses show that points around the northeastern shoreline of Lake Superior are rising relative to a geologic datum. For example, Port Arthur (Thunder Bay) is found to be rising at a rate of 1.14 feet per 100 years.

Included as related topics are brief discussions of 1) previous investigations, 2) isostasy, 3) other geophysical measurements within the Great Lakes area having possible relevance to vertical crustal movement.

This, the first of two reports on this subject, develops the method and uses Lake Superior as an example. The second report presents results for all of the Great Lakes. The study was conducted by the Central Region, Engineering Division, as part of an ongoing investigation into the hydrology and hydraulics of the Great Lakes.

# Acknowledgments

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# INTRODUCTION

It is well known that the earth's crust is not stable; earthquakes and fault movements are reported almost daily. Available evidence indicates that movements of the crust have been taking place ever since it was formed. The geological record shows an alternation of cycles of emergence followed by erosion and of inundation followed by sedimentation. Mountain ranges have been uplifted and eroded, the eroded material being deposited possibly in areas that were subsiding. At three different times high mountain ranges have been uplifted in the northern Great Lakes region (Moore, 1948).

Less well known but nonetheless important is a slow long-term vertical movement of the crust. Even at the present time movement of the earth's crust is being observed at many places. In North America, the Atlantic coast is subsiding between Saint John, New Brunswick, and Key West, Florida. The northern shore of the Gulf of Mexico is subsiding; the western shore is rising. In general, the Pacific coast is subsiding except for the area north of Seattle, which is rising slightly. In South America, the Pacific coast between Peru and Cape Horn is rising while to the east, Brazil is subsiding. In Europe, the Baltic area is rising while France and the Mediterranean area are subsiding; Britain is tilting, with the north rising and the south subsiding. Japan and the Philippine Islands are rising; the west coast of India is subsiding. These facts are deduced from long-term observations of tide gauges at points along the coasts of the regions mentioned (Gutenberg, 1941).

Inland, away from the sea level datum, accurate levelling at different periods of time can be used to measure vertical movements between points not too far apart. When measuring differences in elevation by first order levelling the accuracy of the vertical measurement is given as $\pm 0.015\sqrt{M}$ feet where M is the distance, in miles, over which the measurement is made. The distance between Thunder Bay (Port Arthur) and Marquette, both on Lake Superior, is about 420 miles measured around the lake as a levelling survey would go. It is known to a good degree of accuracy that the area around Thunder Bay is rising relative to the area around Marquette at a rate of 0.57 foot per 100 years. From the equation given above it would not be possible for a surveyor to detect a movement of less than ±0.30 foot between these two points and so, to be certain that movement had occurred, two sets of levels would have to be run with a period of not less than 50 years between the surveys.

Fortunately an alternate method of detecting crustal movement exists in areas where there are large lakes. By taking differences between levels of the same lake as measured at two different points over a long period of time a measurement of the change in relative vertical position of the two lake gauges is obtained. Thus, crustal movement around the Great Lakes is measured not because it is greater there than elsewhere but because of the large number of lake level gauges available with long periods of record. Other means are also available for estimating vertical crustal movements over geologic time periods.

In the Great Lakes region, investigation of post-glacial crustal movement has been continuing since the late nineteenth century. Geologists have approached the subject by studying contemporaneous shore features such as raised beaches, wave-cut cliffs, deltas, etc. formed by the glacial lakes. Engineers, on the other hand, have usually been more interested in finding out why the records from their lake elevation gauges tend to change in time with respect to other gauges. From the engineer's viewpoint, crustal movement is only one of a series of possible factors such as local land movement, gauge settling, wind set-up, seiches, tides, instrument and observer errors, etc. which could affect the readings of his gauges.

Naturally enough, these two approaches do not always produce harmonious results. The geologist works with a time-scale of thousands of years; the engineer with tens of years.

One point where opinions differ is based on the geologic evidence provided is based on the geologic evidence provided by the Nipissing zero isobase. The Nipissing Shoreline, formed between 6,000 and 4,000 years ago, is the most recent prominent shoreline of the ancestral Great Lakes (Farrand, 1960); it follows fairly closely the shoreline of the present Lakes Superior and Michigan-Huron. An isobase is a form of contour-line joining points of equal elevation along the geologic shoreline and the zero isobase is a contour-line joining points along the shoreline at which the gradient of the shoreline changes. South of the zero isobase, the beaches are horizontal; north of the zero isobase, the beaches are warped upward. Geologists interpret this to mean that there has been no recent (post-glacial) differential crustal movement south of this line but engineers note that lake level gauges are showing long-term trends both north and south of this zero isobase. The Nipissing zero isobase has been traced from the Bayfield peninsula on Lake Superior in an east-south-east direction crossing Lake Michigan near Escanaba and Lake Huron near Alpena and Port Elgin; its projection eastward would pass just north of Toronto.

A second point of differing opinions lies in the interpretation of the trends computed in time series made up of differences in lake levels. It has been suggested (Maclean, 1961) that these trends are due not to crustal movement, but to the effect of wind set-up. On an hourly basis this is probably true, but in order to produce consistent linear trends measured over periods of, in some cases, more than 100 years, a consistently increasing or consistently decreasing set-up would be required, that is, there would need to be a significant long-term change in wind set-up, a hypothesis which has not been proved.

Another problem facing researchers in the Great Lakes region is the difficulty, at present, of correlating lines of equal crustal movement of one lake with those of a second lake, because the magnitude of error of land levelling between the lakes is greater than the movement being measured.

The most likely cause of the vertical movement measured in the Great Lakes area is isostatic recovery following the retreat of the last continental glaciation. It may be of use to engineers therefore to present here a very brief outline of the mechanism by which this uplift is probably occuring.

From the modern study of earthquakes, a great deal of information about the structure of the earth has been gained. The seismic evidence, in agreement with geologic indications, shows that the earth has a crust composed of several distinct layers. The outermost layer is of granitic composition and is covered locally by sedimentary rocks of various thicknesses. It rests on a layer whose composition approximates vitreous basalt, and this in turn rests on an ultrabasic rock, peridotite at a depth some 30 km. below the surface. The average thicknesses of the granitic and basaltic layers are about 10 and 20 km., respectively. The sedimentary layer is discontinuous, and the granitic layer is not everywhere present below the oceans. From a consideration of the density of the various layers, more information can be obtained about the earth's core. The density of the granitic layer is about 2.67 and the density of the basalt and peridotite not much greater. The average density of the entire earth, however, is 5.5 and therefore, the core must be composed of heavier material than the crustal material in order to compensate for the lighter layers;

calculations give a density of approximately 8 for the core material. This figure is slightly greater than the density of iron and slightly less than the density of nickel, so the core probably consists of a mixture of these two elements and, for this reason, is called nifel, from the chemical symbols.

Similarly, the granitic and sedimentary layers are called sial from the initial letters of the preponderant oxides, silica and alumina; the basaltic layer is called sima from silica and magnesia. The sial is crystalline and rigid but the sima lacks rigidity and will yield slowly to long-continued stresses. The marked break at the base of the crust, between the sima and the mantle, is known as the Mohorovicic discontinuity.

Geologists, having the above concept of the earth's structure, visualize continents as large sial "rafts" floating on a "sea" of sima. Since the density of sima (nearly 3.0) is not much greater than that of sial (about 2.7), the "rafts" are largely submerged.

Large topographical features such as mountains, which rise about the general level of the continent, have a correspondingly greater depth of sial beneath them; while shallow features such as continental shelves penetrate only a small distance into the sima. The state of balance which is maintained between adjacent columns of matter in the crust and supported by the basaltic substratum, is called isostasy. Details of the process can be found in any elementary geology textbook, for example Blyth (1960). It appears probable that very large topographical features on the earth's surface are bounded by faults and supported by the upward pressure of the substratum, i.e., they are isostatically compensated on a regional scale. The Alps and the Rockies, for example, are essentially balanced in this way. Smaller local features, however, may be supported not by isostasy but by the rigidity of the crust.

Considering an individual column of the earth's crust, an analogy can be made to a weighted rod floating vertically in water. The ratio of the total length of the rod to its height above the water surface is a constant, depending on the densities of the materials involved. If the rod is pushed deeper into the water, a volume of water will be displaced and a force will be felt attempting to push it back to its original position. It is postulated that this occurs to the earth's crust. As a glacier advances over a continent it pushes the sial "raft" down into the sima, displacing the latter and creating a balancing up-thrust force. Later, as the glacier retreats, the forces will again be out of equilibrium and the up-thrust force will push the sial "raft" back to its original position. Due to the fluid nature of the sima and the rigidity of the sial this upward movement is likely to be a series of short fast rises superimposed on a long steady rise.

It is probable that in most cases the degree of restoration will not be complete due to the weight of surface water and glacially deposited drift. In the case of the Great Lakes region, the drift is commonly believed to be several hundred feet thick in some places. The opposite situation will, of course, occur in the Canadian Shield, the source region for the drift deposited to the south. There, since material was removed during glaciation, uplift probably proceeded to a greater degree although not sufficient to return the area to its original elevation.

In the Great Lakes region, retreat of the Labrador sector of the Laurentide Ice Sheet from its point of farthest advance, about 150 miles south of Lake Michigan, began some 17,000 years ago. At first the glacial lakes, formed of meltwater between the basin rims and the ice front, discharged southwards into the Mississippi River. Subsequent advances and retreats of the ice front together with associated isostatic activity produced a complex system of lakes with discharges varying in direction with time. Lakes approximately occupying the present Lake Huron basin, for example, discharged at different times through the Mattawa-Ottawa Rivers, through the Kirkfield-Fenelon Falls and Trent Valley river systems, and various other outlet channels as well as the present St. Clair-Detroit Rivers.

Many descriptions of the glacial and post-glacial history of the Great Lakes have been written. In 1958 Hough published a revised lake history including material still considered geologically controversial. Prest (1970) employed all known Canadian data in producing his description of the lake history. This very comprehensive report includes major changes in the previously accepted sequence of events, as well as changes in the Canadian and U.S. shorelines of the glacial lakes. Prest's history ties the Great Lakes glacial events to those occurring in the rest of Canada, and of particular interest to this study, to the present Lake Winnipeg and James Bay areas.

# A BRIEF REVIEW OF PREVIOUS RESEARCH INTO CRUSTAL MOVEMENT IN THE GREAT LAKES REGION

The credit for first calling attention to evidences of modern crustal movement in the Great Lakes region goes to G.R. Stuntz, a land surveyor of Wisconsin, who, in 1853, observed that the river beds at the western end of Lake Superior were growing deeper, indicating a tilting of the area with the western end of the lake subsiding. At that time the only estimates available of the rate of the crustal movement were from geological evidence such as raised beaches. This evidence, however, enabled investigators to perform rough calculations regarding the effects of the movement. Thus, Dr. J.W. Spencer in 1894 was able to tell the American Association for the Advancement of Science that: "the end of the [Niagara] falls seems destined, if we read the future by the past, to be effected, not by the erosion expending itself on the rocks, but by terrestrial deformation turning the drainage of all the upper lakes into the Mississippi, by way of Chicago, just as the Huron waters were lately turned from the Ottawa into the Niagara drainage; and at the recent rate it would seem that about 5,000 or 6,000 years at the most will be needed".

The first investigation of crustal movement by measuring progressive trends in lake levels was made by Dr. Grove Karl Gilbert in 1896. Dr. Gilbert drew up plans for a measuring system to cover the whole of the Great Lakes basin and derived operating methods designed to eliminate the various sources of error such as solar and lunar tides, wind effects, pressure differential effects and errors in the measuring equipment. Through lack of equipment, Gilbert worked with only four pairs of gauging stations, which he placed so that the lines joining the pairs were in the direction of the maximum movement since the Nipissing epoch of the upper lakes.

In his 1926 report "Regulation of the Great Lakes", J.R. Freeman stated that he was led to study "this matter of tilting of the earth in relation to lake levels" while seeking an explanation for the apparent change in the relative elevations of Lake Huron and Lake Erie. Freeman chose twenty pairs of gauges with long accurate records, each pair of gauges measuring water level on either side of one of the Great Lakes. Using monthly mean and yearly mean elevations, as compared to Gilbert's daily figures, Freeman derived rates of crustal movement in feet per hundred miles per hundred years and wrote, "Whatever the cause, continuous progressive tilting upward toward the north at the rate of about half a foot per one hundred miles per century in the southern part of the Great Lakes region, with indications of double this rate over some parts of the lake system, is proved beyond all doubt". With only twenty pairs of gauges available, Freeman had to follow Gilbert's assumption that each lake basin was tipping in its entirety at the same rate without consideration of local topography and variation in gravity.

The next major investigation was carried out by Sherman Moore of the U.S. Lake Survey in 1922. In a paper published in the May and June, 1922, issue of the "Military Engineer", Moore wrote that his investigations were prompted by the discovery in the winter of 1919-20 that the levels of Lake Erie at Port Colborne were between 0.10 and 0.20 foot lower than the corresponding levels at Cleveland, although the two gauges had been carefully set to the same datum by water levels in 1875. Further investigation showed that the gauge at Harbor Beach was reading lower levels than that at Milwaukee and that the Marquette gauge was showing lower levels than the gauge at Duluth. Moore then checked other gauges and found that in every case gauges to the north and east were giving lower readings. "The only logical conclusion", Moore wrote, "appeared to be that there was in progress a movement of the earth's crust over the entire region, a tilting movement that was causing a relative rise in the land to the north and east". Having chosen reliable gauges at widely spaced points with records covering long periods of time, Moore calculated rates of vertical movement between the gauges. Using the five summer months, June to October, to reduce interference from wind, he plotted the annual differences between each pair of gauges and drew in best-fitting straight lines. This gave the rates of crustal movement between the pairs of gauges. On each lake, Moore separated these rates into a north-south component of movement and an east-west component. Using a least squares method, Moore obtained one representative north-south (vertical) movement and one representative east-west (vertical) movement for each lake; resolving these two gave one resultant rate of movement for the shoreline of each lake together with the direction of movement for each lake. For example, Moore's results gave an average rate of movement for Lake Erie of 0.46 foot per 100 miles per 100 years in a direction 31° north of west. Unlike Gilbert and Freeman, Moore recognized that the rate of movement was not uniform over the whole Great Lakes Basin.

In 1948, in a paper published by the Geological Society of America, Moore gave the results of further investigations that he had carried out on ninety-one long-term gauges on the Great Lakes. Moore plotted the annual differences in five monthly mean summer elevations between each pair of long-term gauges on each lake and drew best-fitting straight lines through the points. The rates of movement indicated by these lines were then adjusted for harmony using a least squares technique. The average correction required was 0.03 foot per 100 years and the maximum correction to any rate was 0.06 foot per 100 years. With the rates at the principal gauges on each lake established, Moore determined the movement at other points around the lakes by comparison with at least two of the principal points. Then, Moore went further and attempted to determine the rate of movement between each of the lakes by comparing the results of level lines run at widely separated times. Compared to levelling by water, land levels are relatively inaccurate but this method may give an indication of the between-lake movement. Even with rates of movement determined over the whole of the Great Lakes Basin, the measurements are only relative and so Moore tied them in to present day sea level (that is, 1948 sea level) using level lines run from Oswego to New York. This enabled Moore to convert all his rates of movement to absolute units in terms of sea level. Moore's report on the crustal movement of Lake Superior showed that the land on the U.S. side of Lake Superior was subsiding with respect to sea level at rates per 100 years amounting to 0.09 foot at Point Iroquois, 0.62 foot at Marquette and 1.03 foot at Duluth. Partly because of this investigation by Moore, the P-5 rule curve for regulating Lake Superior was replaced by the so-called 1949 rule, which was designed to reduce the frequency of high levels on the lake and minimize the damages experienced at U.S. harbors due to high lake levels. Moore's lines of zero crustal movement passed through Newfoundland; St. John, New Brunswick; Kingston, Ontario, and across the top of the great Lakes. Areas north of this line are rising relative to sea level, those south of the line are subsiding. Regarding the line, Moore stated, "If this line of zero movement is considered as the arc of a circle, which it approximates, the focus is about 200 miles east of James Bay. If the circle is continued it will pass close to Churchill, where the records of the tide gauge, when properly interpreted, show zero movement".

In 1954 the Canadian Hydrographic Service published a report by C. Price on the assessment of crustal movement on Lake Ontario. Price took five gauging stations and, using the mean elevation during the four summer months June to September to eliminate possible discrepancies due to ice, floods, spring and autumn gale conditions etc., plotted the annual differences in elevation in the following various ways:

a) straight annual difference in elevation versus time for each combination of gauges.

b) five-year moving mean gauge difference versus time for each combination of gauges.

c) difference between elevation of each gauge and the five gauge mean versus time.

d) difference between five-year moving mean elevation and the five gauge five-year moving mean elevation for each location versus time.

e) difference between ten-year moving mean elevation at each location and the five gauge ten-year moving mean elevation versus time.

Price used a method of adjusting the resulting rates of crustal movement based on the assumption that at all points equi-distant from some straight "hinge-line" the rates of movement must be equal.

In May 1957, the Vertical Control Subcommittee of the Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data published its first interim report on crustal movement in the Great Lakes region. The report presented the findings of a comparison carried out between the methodology and results of Moore (1948), and Price (1954). Several differences in technique and data were found between these two studies, and the Subcommittee related these differences to the differences in the calculated rates of crustal movement determined by each author. The Subcommittee were of the opinion that the differences between the two investigations were too large to reconcile and decided to carry out an independent study utilising a method which combined the best features of both reports. The method finally adopted was as follows:

a) Four-month means (June to September) were calculated for each long-term gauge for the full period of record.

b) The differences in mean water surface elevations for each pair of gauges on each lake were determined and plotted.

c) A least squares straight line was fitted to the data on each graph.

d) A least squares adjustment was made to show a closure of rates of movement within any triangle.

e) The triangulation was extended from the main gauging stations on each lake to include smaller stations with shorter periods of record.

Two reports were issued for each of the Great Lakes. The first of the two reports for each lake gave the calculated rates of crustal movement between principal harbours around the lake, and the second report gave the calculated movement at the smaller stations.

This investigation is certainly the most thorough study of crustal movement in the Great Lakes region undertaken to date. However, of the 118 gauges used by the Subcommittee only 31 had periods of record exceeding 20 years, which must be considered the very minimum period of significance. In fact, as discussed in the section on current methodology in this report, statistical analyses of the autocorrelation of many of the gauge records indicate that the effective lengths of the series are often very much less than the periods of record. In 11 of the sets of gauges for which the subcommittee computed regression lines there were only 2 years of record common to both gauges. These data are of little value for computing long-term trends from time series for which it is known that there are highly significant short term components due to meteorological effects, wind set-up, instrument and operator errors, gauge location peculiarities, etc.

A doctoral dissertation by W.F. Maclean of the University of Michigan, published in 1961 by the Great Lakes Research Division of the Institute of Science and Technology, was mainly a critique of existing data and techniques. Maclean contended that the previously measured rates of crustal movement merely represented the change in the net difference of the accumulated effects and errors, because Great Lakes water-level gauge records are not corrected for meteorological effects, wind set-up, or instrument and operator errors. Maclean noted that geologists have found that the "hinge line" of the ancient Nipissing beach lies in the northern sector of the Great Lakes, and concluded, "it is reasonable to assume that the value of the rates of modern crustal movement, even the existence of modern crustal movement around the Great Lakes, is in doubt".

The Vertical Control Subcommittee, together with the Lake Levels Subcommittee, published a review of Maclean's report in 1964. Their main criticism of Maclean's report was his interpretation of the effect of meteorological conditions on long-term gauge records. Quoting from the subcommittees' review, "Without doubt metecrological effects are the most significant errors in water level transfers. They do not, however, have any effect on the determination of crustal movement rates over long periods .... There are many plots of gauge differences covering periods of 50 to 100 years that show a pronounced slope in the best fitting line drawn through them. This slope can only be explained by relative movement of the land at the gauge sites. The normal line in such a plot demonstrating only average net wind set-up between the two points would be a horizontal line at a fixed distance above or below the zero axis equal to the amount of the average net set-up. To obtain a sloping line the wind would have to increase or decrease progressively over a long period, and this is not physically true". The report concluded that crustal movement was present in the Great Lakes region and that the only satisfactory method of measuring it was by water level differences.

A report on the use of high accuracy levelling to measure rates of movement over short distances was written in the Canadian Surveyor by Frost and Lilly (1966). By re-levelling several level lines covering a triangular area Quebec - Lac St. Jean - La Malbaie, the authors determined that there had been changes in elevation at points within this area at rates of up to 1.96 feet per century. The movement detected by Frost and Lilly appears to be of two types. There is a circular area in the centre of Laurentide Park which is sinking relative to Quebec, while in the Lac St. Jean area the land is rising relative to Quebec. While two measurements up to 50 years apart cannot be considered as reliable as a continuous record, this study shows a way of extending the study of crustal movement away from the lake and sea coasts.

The Engineering Division, Inland Waters Branch, began a systematic study of crustal movement around the Great Lakes in 1967. The first step was to update the figures derived by the Vertical Control Subcommittee in 1956 using the same method of averaging the elevations over the four summer months, and using all the same stations, but using a digital computer to calculate least squares linear regression lines on each set of elevation differences.

A second program was written to work from basic data and compute rates of crustal movement for all main gauging stations. The output from this program was in the form of plots of gauge differences as time - series, as well as printed statistical output. This second program was used to analyse data from only those pairs of stations having continuous common periods of record greater than 20 years.

From the printed results of the two methods, two further types of plot were obtained. For the first type of plot, lines joining a base station to each of the other stations on the lake were drawn on a map and divided according to the rates of crustal movement between the stations. From these, lines of equal rates of movement were drawn on the map after the style of contour lines.

The second method developed in 1967 was to plot rates of crustal movement at points around a lake relative to one reference station on the lake in feet per 100 miles per 100 years versus the whole circle bearings of the gauging stations relative to the reference station. This method was used to produce graphs for all the Great Lakes and it was found in each case that some form of circular curve was present, implying that the measured crustal movement was basin-wide at least. This work is reported in Kite (1967).

Gale (1970) published a further account of the detection of vertical crustal movement in the Quebec - Lac St. Jean - La Malbaie area, with a spur westwards to Senneterre, by comparison of geodetic surveys run at different times. Gale's results are in general agreement with those of Frost and Lilly (1966), although a slightly different computation method was used and an additional survey, run in 1966, was available. Gale found that the maximum rate of vertical movement (which occurred at Senneterre) was 54.6 cms/50 years (3.58 feet/100 years).

It is also reported in this study that re-levelling of the Trans-Canada line (begun in 1966) shows no evidence of significant movement between Vancouver and Calgary, Toronto and Montreal or in the New Brunswick area, although all the data had not then been analysed.

Walcott (1971) reviews recent published data on vertical crustal movement in North America from water level observations in the Great Lakes; tidal records along the Atlantic Coast and at Churchill, on Hudson's Bay; geodetic re-levelling; and gravity measurements. Walcott compares these results with radio-carbon dating of material found in positions somehow related to past lake or sea levels, such as sea-shells, wood, and other organic materials found on old strand-lines or delta foreset beds. Walcott concludes that the available evidence for recent vertical movements is consistent with the broad uplift of the land that has occurred over at least the last 7,000 years and that this movement is postglacial rebound.

Of particular interest in this study is Walcott's research into the elastic yield of the earth caused by fluctuating water levels in the Great Lakes. Thus the measured value of water level on a lake will comprise both the actual water level and a deformation of the ground caused by that level. It has been found that the maximum contribution of elastic yield to the measured rates of crustal movement is around 10%.

## PROCEDURE

The previous studies using lake levels to investigate the Great Lakes have three major weaknesses:

(a) The assumption has been made, without any real justification, that time series made up of differences in lake elevations as recorded at different points around the lake's shoreline can be adequately represented by first order linear trends.

(b) Trends in differences in levels can only indicate relative movement between gauges. To convert these rates of movement to absolute rates of movement a stable datum must be used. Moore (1948) is the only investigator to have used a datum. He used mean sea level as a datum and sets of long distance levels to transfer the datum to the Great Lakes. As discussed previously, however, levelling over large distances cannot detect vertical crustal movement unless the sets of levels are run at large time intervals.

(c) Related to (b) above, no previous investigation has accurately related vertical movement on one lake to movement on another lake by deliberately using a common datum.

The purpose of this study is to attempt to solve these problems in the following ways:

(a) A complete statistical analysis of the various types of time-series present will be made to identify the important components and test the hypothesis that a first order linear trend can adequately represent time series of differences in lake levels.

(b) A common datum should be found for measurements of relative crustal movement or all the Great Lakes and possibly the Nipissing zero isobase provides a starting point for this.

(c) For practical use, it is of great importance to relate the absolute movement of land around a lake to the movement of the lake surface. In this way the effects of vertical crustal movement on power, navigation and shore property can be evaluated. To do this it will be necessary to determine rates of movement of each of the lake surfaces.

(d) It is difficult at present to correlate lines of equal crustal movement of one lake with those of a second lake, because the magnitude of error of land levelling between the lakes is greater than the movement being measured. However, information from other geophysical phenomena may result in a distribution pattern independent of land/water boundaries so that, by transfer between the sets of distributions, a complete map of crustal movement can be built up and perhaps referred to sea level or to some absolute value. There is also the chance that if these various types of measurement are really measuring the various effects of the same phenomenon, then by comparing the distributions of these effects, researchers may be able to better determine the nature of the basic phenomenon.

## A) Determination of relative rates of crustal movement

## I) Analysis of a sample set of data

### (a) Introduction

In order to test the validity of previous investigators use of first order linear trends as the dominant component of time series of differences in gauge elevations it was decided to subject a sample set of data to a detailed time-series analysis.

Gauging stations around Lake Superior were selected on the basis of length of continuous record; stations with periods of less than 20 years were not used. Previous studies used only four or five summer months of record each year in order to avoid periods of excessive wind set-up and ice conditions. This is somewhat artificial, however, since these natural adversities can only provide increased stochastic and periodic components to the time series, and any trend present will still show up if all twelve months of the year are used. For this reason, and because many of the analysis techniques to be used require a continuous record, the entire available data were used with no pre-selection of months.

The use of monthly mean elevations in the analysis is really a compromise between the additional information content of daily data with its correspondingly high random component and vastly increased computation time and the other extreme, annual or 5-year mean elevations with their reduced information content and smoothing bias. Mean monthly data introduce their own bias, however, since all months do not have equal numbers of days and this is reflected in the averaging process.

The gauges used, and their periods of record are listed in Table 1. The pair of gauging stations having the longest common period of record, Marquette and Duluth, 111 years, were used as base stations in the study. In the absence of other criteria it was assumed that having the longest common period meant that the computed rate of movement between the pair of gauges would have greater precision than the rates of movement computed between pairs of gauges with shorter common periods of record.

Mean monthly differences in elevation were computed between the two reference gauges and between each of the two reference gauges and all other gauges on the lake. The next step was to determine the statistical make-up of these time-series in order to compute representative rates of relative movement. The most simple procedure, which has been used in all previous studies, would be to fit a first-order linear trend only, but it is possible that periodic or stochastic components could be more important in some of these time series than trend components. In order to evaluate the relative importance of each of these three components, the following procedure was followed to analyse a sample set of data.

In applying relatively more sophisticated statistical techniques than previous studies care must be taken that the data is used in the most efficient manner i.e. a more detailed analysis will always yield more information but the information obtained may not be worth the effort put into obtaining it. For this reason the detailed analysis was confined to a sample data set and not applied wholesale to the available data.

1) Various techniques were used to fit a polynomial to the time series. The best fitting polynomial was then subtracted from the time series.

2) The residual from step (1) was analysed for periodicity in the mean and variance. Any periodicities found to be significant were removed.

3) The residual from step (2) was checked for distribution.

## Table 1

## LIST OF GAUGING STATIONS AROUND LAKE SUPERIOR AND THE PERIODS OF RECORD USED

| Station | From* | To* | No. of Months |
|---|---|---|---|
| Port Arthur** | 1860 | 1970 | 1331 |
| Michipicoten | 1915 | 1970 | 653 |
| Sault Ste Marie*** | 1908 | 1970 | 747 |
| Point Iroquois | 1930 | 1970 | 408 |
| Marquette | 1860 | 1970 | 1332 |
| Keweenaw L.E. | 1890 | 1961 | 829 |
| Houghton | 1892 | 1963 | 295 |
| Two Harbors | 1941 | 1970 | 350 |
| Duluth | 1860 | 1970 | 1211 |

* Inclusive (note that because of gaps in the data the number of months of record does not always correspond to the number of years).

** Thunder Bay

*** Later dropped because of drawdown effects.

Using this procedure a reasonable breakdown of the time series as to trend, periodic component and stochastic component was obtained. The steps are described in more detail using, as an example, the mean monthly differences in elevation between Marquette and Duluth for the period 1860 to 1970 inclusive. In each of the three steps, autocorrelation and spectral analysis techniques were used to help determine the magnitude of the relevant component of the time series. Since these two techniques require continuous data, i.e., no missing values are allowed, the period of record used for these purposes was 1888 to 1970 inclusive. The statistical techniques used are well known and are included in most text books on statistics, see for example, Yevjevich (1972, a and b). For this reason, the techniques themselves are not described in detail, only some particular aspects of the techniques are discussed.

The sample used for this investigation, mean monthly differences between Lake Superior elevations measured at Marquette and at Duluth, are listed in Table 2 and plotted in Figure 1. Note that in Figure 1 a constant of 0.30 foot has been subtracted from each measurement in order to fit the data to the plot without changing the ordinate scale.

(b) Statistical techniques used
Autocorrelation

In detecting pattern of movement, it is logical to question whether or not successive values of a time series are interdependent. A measure of this dependence is given by the autocorrelation coefficient. For a discrete time series this is computed as

$$r_k = \frac{\sum_{i=1}^{N-k} (x_i - \bar{x}_i)(x_{i+k} - \bar{x}_{i+k})}{(N - k)\, s_i\, s_{i+k}} \qquad (1)$$

where $$\bar{x}_i = \frac{1}{N-k} \sum_{i=1}^{N-k} x_i \qquad (2)$$

## CRUSTAL MOVEMENT - GREAT LAKES BASIN
## DIFFERENCES IN MEAN MONTHLY ELEVATIONS, MARQUETTE - DULUTH (1860-1970)

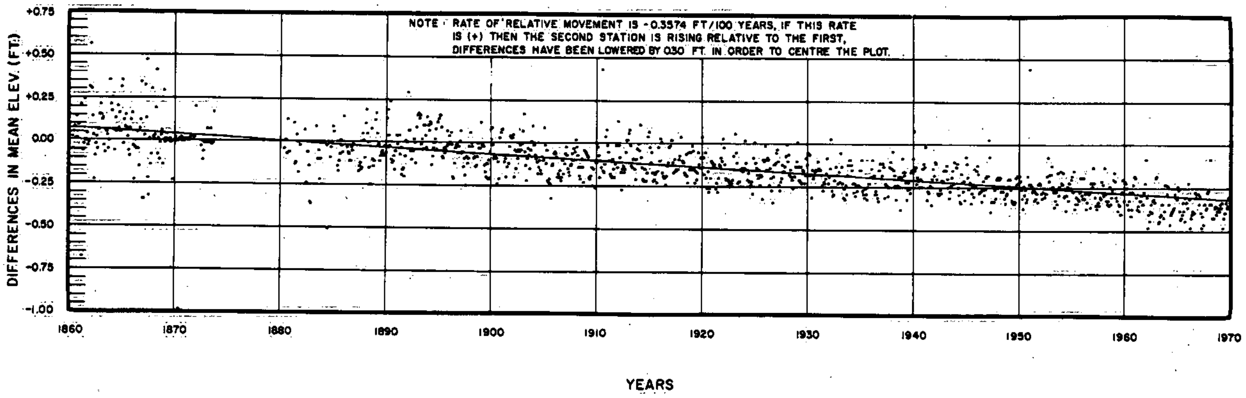


FIGURE I. ORIGINAL TIME SERIES.

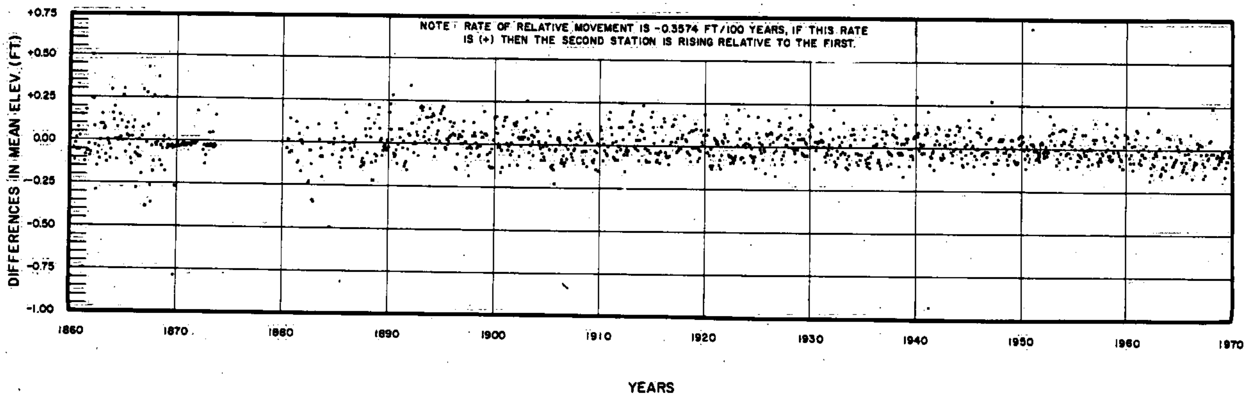


FIGURE 2. ORIGINAL TIME SERIES, LESS LINEAR TRENDS.

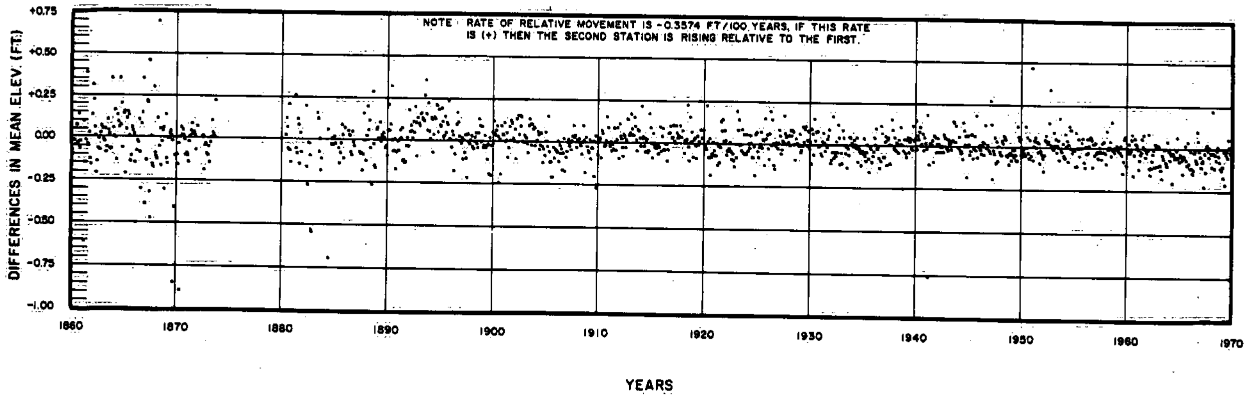


FIGURE 3. ORIGINAL TIME SERIES, LESS LINEAR TRENDS, LESS PERIODICITIES.

## Table 2

## DIFFERENCES IN MEAN MONTHLY ELEVATIONS
## MARQUETTE MINUS DULUTH

( 100.0 SIGNIFIES ORIGINAL DATA IS MISSING AND NO DIFFERENCE IS POSSIBLE )

| YEAR | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEP | OCT | NOV | DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1860 | 100.00 | 100.00 | 100.00 | .32 | .25 | .35 | .38 | .31 | .44 | .31 | .39 | 100.00 |
| 1861 | 100.00 | .51 | -0.38 | .24 | .28 | .54 | .30 | .30 | .34 | .27 | .29 | .61 |
| 1862 | .86 | .60 | .44 | .33 | .07 | .21 | .28 | .32 | .25 | .36 | .45 | .51 |
| 1863 | .45 | .42 | .43 | .27 | .23 | .36 | .35 | .33 | .25 | .30 | .65 | .51 |
| 1864 | .47 | .55 | .36 | .24 | .43 | .35 | .36 | .50 | .37 | .46 | .35 | .61 |
| 1865 | .65 | .28 | .42 | .38 | .45 | .37 | .32 | .24 | .21 | .37 | .42 | .46 |
| 1866 | .27 | .07 | .26 | .30 | .25 | .35 | .25 | .15 | .37 | .42 | .64 | -0.04 |
| 1867 | -0.04 | .33 | .62 | .43 | .77 | .08 | -0.02 | .31 | .16 | .22 | .60 | .32 |
| 1868 | .31 | .33 | .49 | .71 | .22 | .31 | .19 | .29 | .33 | .36 | .16 | .59 |
| 1869 | .29 | .31 | .34 | .30 | .30 | .30 | .31 | .29 | -0.45 | .07 | .30 | .30 |
| 1870 | .32 | .30 | .30 | .30 | -0.69 | .31 | .31 | .32 | .30 | .31 | .30 | .31 |
| 1871 | .31 | .31 | .31 | .30 | .31 | .32 | .31 | .31 | .31 | .32 | .32 | .50 |
| 1872 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | .27 | .30 | .19 | .24 | .29 | .30 | .29 |
| 1873 | .37 | .30 | .30 | .29 | .37 | .28 | .30 | .29 | .47 | 100.00 | 100.00 | 100.00 |
| 1874 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1875 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1876 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1877 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1878 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1879 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1880 | 100.00 | 100.00 | 100.00 | 100.00 | .26 | .31 | .31 | .23 | .43 | .24 | .24 | 100.00 |
| 1881 | 100.00 | 100.00 | 100.00 | .41 | .17 | .18 | .34 | .30 | .12 | .30 | .28 | 100.00 |
| 1882 | 100.00 | 100.00 | 100.00 | .37 | .05 | .06 | .24 | .24 | .16 | -0.05 | -0.06 | 100.00 |
| 1883 | 100.00 | 100.00 | 100.00 | 100.00 | .39 | .30 | .20 | .25 | .08 | .20 | 100.00 | 100.00 |
| 1884 | 100.00 | 100.00 | 100.00 | 100.00 | .28 | -0.21 | 100.00 | 100.00 | .21 | .27 | .30 | .39 |
| 1885 | .36 | 100.00 | 100.00 | 100.00 | .16 | .13 | .19 | .23 | .31 | .28 | .24 | .45 |
| 1886 | 100.00 | 100.00 | 100.00 | .27 | .24 | .23 | .19 | .19 | .27 | .25 | .29 | 100.00 |
| 1887 | 100.00 | 100.00 | 100.00 | 100.00 | .13 | .16 | .18 | .14 | .15 | .35 | .31 | .21 |
| 1888 | .41 | .49 | .35 | .30 | .15 | .05 | .05 | .38 | .20 | .16 | .19 | .43 |
| 1889 | .47 | .35 | .28 | .29 | .19 | .22 | .24 | .25 | .23 | .22 | .28 | .24 |
| 1890 | .44 | .49 | .34 | .28 | .54 | .20 | .18 | .19 | .22 | .14 | .25 | .34 |
| 1891 | .29 | .34 | .34 | .23 | .27 | .11 | .14 | .15 | .10 | .22 | .31 | .30 |
| 1892 | .59 | .37 | .37 | .25 | .20 | .26 | .25 | .32 | .30 | .18 | .39 | [illegible] |
| 1893 | .47 | .44 | .46 | .29 | .32 | .26 | .40 | .40 | .34 | .27 | .45 | .40 |
| 1894 | .37 | .42 | .41 | .17 | .22 | .27 | .30 | .32 | .38 | .26 | .42 | .44 |
| 1895 | .46 | .46 | .39 | .22 | .21 | .27 | .24 | .26 | .28 | .37 | .29 | .34 |
| 1896 | .30 | .36 | .29 | .10 | .09 | .18 | .27 | .29 | .29 | .24 | .28 | .31 |
| 1897 | .37 | .24 | .25 | .16 | .21 | .11 | .05 | .22 | .18 | .21 | .28 | .36 |
| 1898 | .30 | .28 | .24 | .21 | .22 | .16 | .17 | .22 | .30 | .27 | .27 | .42 |
| 1899 | .29 | .29 | .28 | .17 | .13 | .10 | .18 | .09 | .20 | .09 | .25 | .38 |
| 1900 | .32 | .39 | .29 | .21 | .23 | .22 | .21 | .06 | .19 | .09 | .33 | .36 |
| 1901 | .38 | .38 | .23 | .19 | .21 | .10 | .16 | .23 | .18 | .28 | .34 | .33 |
| 1902 | .33 | .35 | .10 | .26 | .15 | .25 | .21 | .26 | .25 | .29 | .26 | .30 |
| 1903 | .37 | .47 | .25 | .17 | .08 | .21 | .27 | .23 | .15 | .26 | .31 | .40 |
| 1904 | .29 | .29 | .23 | .21 | .19 | .12 | .16 | .20 | .14 | .12 | .22 | .20 |
| 1905 | .39 | .29 | .18 | .26 | .08 | .07 | .09 | .06 | -0.02 | .17 | .18 | .24 |
| 1906 | .25 | .24 | .22 | .10 | .12 | .10 | .14 | .08 | .10 | .20 | .12 | .18 |
| 1907 | .24 | .28 | .23 | .15 | .13 | .11 | .17 | .18 | .24 | .20 | .27 | .25 |
| 1908 | .32 | .28 | .23 | .16 | .11 | .09 | .16 | .21 | .16 | .07 | .26 | .25 |
| 1909 | .24 | .23 | .22 | .10 | .09 | .11 | .16 | .05 | .15 | .20 | .04 | .19 |

Table 2 (Cont'd.)

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1910 | .30 | .28 | .23 | .10 | .20 | .16 | .73 | .14 | .13 | .18 | .34 | .38 |
| 1911 | .29 | .25 | .20 | .12 | .11 | .10 | .21 | .15 | .10 | .18 | .29 | .16 |
| 1912 | .43 | .29 | .22 | .13 | .03 | .15 | .17 | .13 | .15 | .25 | .32 | .32 |
| 1913 | .36 | .38 | .29 | .17 | .16 | .21 | .21 | .15 | .09 | .21 | .27 | .30 |
| 1914 | .26 | .41 | .22 | .16 | .09 | .08 | .15 | .15 | .08 | .13 | .34 | .37 |
| 1915 | .23 | .19 | .25 | .13 | .08 | .09 | .12 | .13 | .07 | .14 | .28 | .20 |
| 1916 | .33 | .31 | .13 | .11 | .10 | .10 | .06 | .11 | .17 | .19 | .28 | .29 |
| 1917 | .32 | .32 | .19 | .13 | .21 | .14 | .15 | .15 | .15 | .21 | .23 | .34 |
| 1918 | 1.15 | .20 | .19 | .13 | .12 | .24 | .11 | .07 | .25 | .15 | .25 | .15 |
| 1919 | .29 | .26 | .15 | .07 | .14 | .09 | .13 | .25 | .18 | .16 | .22 | .34 |
| 1920 | .22 | .22 | .12 | .25 | .10 | .04 | .14 | .02 | .04 | .10 | .20 | .19 |
| 1921 | .17 | .18 | .16 | .04 | .07 | .02 | .10 | .18 | .17 | .22 | .20 | .33 |
| 1922 | .28 | .18 | .10 | .09 | .05 | .09 | .08 | .06 | .05 | .07 | .09 | .23 |
| 1923 | .15 | .36 | .25 | .19 | .07 | .08 | .08 | .15 | .05 | .16 | .14 | .16 |
| 1924 | .32 | .19 | .15 | .09 | .13 | .08 | .05 | .06 | .08 | .01 | .22 | .31 |
| 1925 | .19 | .16 | .12 | .10 | .15 | .06 | .12 | .11 | -0.02 | .21 | .20 | .29 |
| 1926 | .21 | .15 | .22 | .21 | .06 | .16 | .10 | .03 | -0.04 | .11 | .18 | .19 |
| 1927 | .25 | .27 | .20 | .06 | .03 | .08 | .10 | .19 | .14 | .14 | .17 | .23 |
| 1928 | .30 | .20 | .24 | .12 | .15 | .11 | .09 | .04 | .11 | .11 | .17 | .20 |
| 1929 | .31 | .17 | .09 | .08 | .17 | .07 | .15 | .14 | .07 | .12 | .22 | .18 |
| 1930 | .25 | .12 | .24 | .12 | -0.01 | .15 | .10 | .12 | .14 | .15 | .15 | .20 |
| 1931 | .16 | .15 | .15 | .12 | .06 | .03 | .08 | .07 | .10 | .13 | .22 | .07 |
| 1932 | .11 | .16 | .33 | .03 | 0 | .01 | .09 | .05 | .17 | .12 | .07 | .14 |
| 1933 | .08 | .25 | .17 | .07 | -0.01 | .04 | -0.02 | .06 | .01 | .12 | .18 | .21 |
| 1934 | .21 | .18 | .15 | .07 | .02 | .07 | 0 | .06 | .03 | .06 | .06 | .15 |
| 1935 | .15 | .21 | .03 | .01 | -0.01 | .04 | .01 | -0.02 | .05 | .07 | .07 | .11 |
| 1936 | .16 | .21 | .11 | .12 | .01 | .03 | .01 | -0.01 | -0.03 | .15 | .19 | .10 |
| 1937 | .12 | .19 | .17 | -0.01 | .07 | .06 | .06 | 0 | .05 | .17 | .20 | .16 |
| 1938 | .26 | .15 | .12 | .06 | -0.02 | .05 | .06 | .11 | .05 | -0.08 | .11 | .22 |
| 1939 | .14 | .12 | .14 | .08 | -0.02 | .04 | .08 | .04 | .02 | .14 | .15 | .22 |
| 1940 | .38 | .15 | .17 | .07 | .11 | .09 | .05 | .05 | .10 | .05 | .16 | .19 |
| 1941 | .17 | .29 | -0.85 | -0.02 | .03 | .02 | .05 | .06 | -0.05 | .10 | .16 | .12 |
| 1942 | .20 | .19 | .10 | .02 | .01 | .01 | .04 | .02 | .09 | .09 | .15 | .15 |
| 1943 | .15 | .18 | .10 | .05 | -0.01 | .03 | .06 | .11 | .15 | .07 | .20 | 1.26 |
| 1944 | .17 | .19 | .05 | .01 | -0.03 | -0.04 | .13 | .08 | .03 | .09 | -0.02 | .23 |
| 1945 | .19 | .11 | .04 | .07 | .08 | -0.04 | .02 | .06 | 0 | .16 | .06 | .15 |
| 1946 | .18 | .13 | .02 | .03 | .03 | -0.03 | .04 | .09 | .01 | .04 | .11 | .12 |
| 1947 | .12 | .33 | .23 | -0.09 | -0.03 | -0.03 | .06 | -0.03 | .04 | -0.01 | .10 | .11 |
| 1948 | .11 | .10 | .11 | .03 | .06 | .03 | .05 | -0.01 | -0.04 | .01 | .02 | .10 |
| 1949 | .06 | .10 | .04 | -0.02 | -0.03 | -0.04 | -0.04 | -0.03 | .11 | .07 | .12 | .08 |
| 1950 | .12 | .09 | .04 | -0.01 | -0.07 | .06 | .01 | .04 | -0.03 | -0.01 | .12 | .07 |
| 1951 | .74 | .07 | .06 | .03 | .04 | -0.02 | .03 | -0.01 | .02 | -0.03 | .07 | .08 |
| 1952 | .07 | .02 | -0.01 | .03 | .05 | 0 | .02 | .02 | .12 | .23 | .09 | .13 |
| 1953 | .13 | .18 | .06 | .03 | -0.06 | -0.02 | .03 | -0.04 | .14 | .01 | .06 | .15 |
| 1954 | .08 | .07 | .14 | 0 | .01 | -0.08 | .01 | .06 | -0.02 | .08 | .05 | .16 |
| 1955 | .12 | .03 | .15 | -0.07 | -0.04 | 0 | -0.04 | -0.01 | -0.02 | .03 | .11 | .01 |
| 1956 | -0.01 | .11 | -0.01 | .02 | -0.07 | -0.03 | .03 | .02 | .02 | -0.06 | .08 | .06 |
| 1957 | .17 | .08 | 0 | -0.05 | -0.07 | -0.04 | -0.03 | -0.05 | 0 | -0.01 | .09 | .07 |
| 1958 | .01 | .17 | .02 | -0.02 | -0.04 | -0.02 | -0.05 | .06 | -0.04 | 0 | .04 | .10 |
| 1959 | .14 | .09 | 0 | -0.02 | -0.08 | -0.04 | -0.05 | -0.10 | -0.10 | .03 | .10 | -0.02 |
| 1960 | .09 | .08 | .05 | -0.04 | -0.07 | -0.02 | .03 | -0.11 | -0.07 | .03 | .07 | .12 |
| 1961 | .11 | 0 | -0.07 | -0.05 | -0.08 | -0.02 | -0.02 | .01 | -0.04 | -0.04 | .03 | .08 |
| 1962 | .22 | 0 | -0.18 | -0.10 | -0.14 | -0.08 | -0.04 | -0.09 | -0.02 | -0.01 | -0.03 | .11 |
| 1963 | .12 | .01 | -0.15 | -0.14 | -0.11 | -0.10 | -0.04 | -0.04 | -0.12 | -0.08 | .07 | .07 |
| 1964 | .02 | .06 | -0.04 | -0.16 | -0.10 | -0.09 | -0.08 | -0.07 | -0.06 | .02 | 0 | -0.01 |
| 1965 | .06 | .04 | -0.08 | -0.13 | -0.12 | -0.10 | -0.03 | -0.08 | -0.14 | 0 | -0.03 | -0.06 |
| 1966 | .08 | -0.01 | -0.18 | -0.12 | -0.10 | -0.08 | -0.09 | -0.09 | .03 | 0 | .02 | .02 |
| 1967 | -0.04 | .01 | -0.09 | -0.18 | -0.11 | -0.16 | -0.04 | -0.04 | -0.07 | -0.06 | .03 | .04 |
| 1968 | -0.04 | .22 | -0.04 | -0.12 | -0.12 | -0.12 | -0.06 | -0.05 | -0.10 | -0.08 | -0.06 | -0.08 |
| 1969 | -0.04 | -0.06 | -0.03 | -0.18 | -0.12 | -0.09 | -0.09 | -0.03 | -0.06 | -0.05 | 0 | -0.03 |
| 1970 | .04 | .04 | -0.08 | -0.20 | -0.17 | -0.13 | -0.07 | -0.01 | -0.05 | -0.12 | -0.02 | -0.05 |

## Table 3

# DIFFERENCE IN ELEVATIONS LESS LINEAR TRENDS

## MARQUETTE MINUS DULUTH

( 100.0 SIGNIFIES ORIGINAL DATA IS MISSING AND NO DIFFERENCE IS POSSIBLE )

| YEAR | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEP | OCT | NOV | DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1860 | 100.00 | 100.00 | 100.00 | -0.05 | -0.12 | -0.02 | .01 | -0.06 | .07 | -0.06 | .02 | 100.00 |
| 1861 | 100.00 | .14 | -0.75 | -0.13 | -0.09 | .18 | -0.06 | -0.06 | -0.02 | -0.09 | -0.07 | .25 |
| 1862 | .50 | .24 | .08 | -0.03 | -0.29 | -0.15 | -0.08 | -0.04 | -0.11 | -0.00 | .09 | .15 |
| 1863 | .09 | .06 | .07 | -0.09 | -0.13 | .00 | -0.01 | -0.03 | -0.11 | -0.06 | .29 | .15 |
| 1864 | .11 | .19 | .00 | -0.11 | .08 | -0.00 | .01 | .15 | .02 | .11 | -0.00 | .26 |
| 1865 | .30 | -0.07 | .07 | .03 | .10 | .02 | -0.03 | -0.11 | -0.14 | .02 | .07 | .11 |
| 1866 | -0.08 | -0.28 | -0.09 | -0.05 | -0.10 | .00 | -0.10 | -0.20 | .02 | .07 | .29 | -0.39 |
| 1867 | -0.38 | -0.01 | .28 | .09 | .43 | -0.26 | -0.36 | -0.03 | -0.18 | -0.12 | .26 | -0.02 |
| 1868 | -0.03 | -0.01 | .15 | .37 | -0.12 | -0.03 | -0.15 | -0.05 | -0.01 | .02 | -0.18 | .25 |
| 1869 | -0.05 | -0.03 | .00 | -0.04 | -0.04 | -0.04 | -0.03 | -0.05 | -0.79 | -0.27 | -0.03 | -0.03 |
| 1870 | -0.01 | -0.03 | -0.03 | -0.03 | -1.02 | -0.02 | -0.02 | -0.01 | -0.03 | -0.02 | -0.03 | -0.02 |
| 1871 | -0.02 | -0.02 | -0.02 | -0.03 | -0.02 | -0.01 | -0.02 | -0.02 | -0.02 | -0.01 | -0.01 | .17 |
| 1872 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | -0.06 | -0.03 | -0.14 | -0.08 | -0.03 | -0.02 | -0.03 |
| 1873 | .05 | -0.02 | -0.02 | -0.03 | .05 | -0.04 | -0.02 | -0.03 | .15 | 100.00 | 100.00 | 100.00 |
| 1874 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1875 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1876 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1877 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1878 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1879 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1880 | 100.00 | 100.00 | 100.00 | 100.00 | -0.04 | .01 | .01 | -0.07 | .13 | -0.06 | -0.06 | 100.00 |
| 1881 | 100.00 | 100.00 | 100.00 | .12 | -0.12 | -0.11 | .05 | .01 | -0.17 | .01 | -0.01 | 100.00 |
| 1882 | 100.00 | 100.00 | 100.00 | .08 | -0.24 | -0.23 | -0.05 | -0.05 | -0.13 | -0.34 | -0.35 | 100.00 |
| 1883 | 100.00 | 100.00 | 100.00 | 100.00 | .10 | .01 | -0.09 | -0.04 | -0.21 | -0.09 | 100.00 | 100.00 |
| 1884 | 100.00 | 100.00 | 100.00 | 100.00 | -0.00 | -0.49 | 100.00 | 100.00 | -0.07 | -0.01 | .02 | .11 |
| 1885 | .08 | 100.00 | 100.00 | 100.00 | -0.12 | -0.15 | -0.09 | -0.05 | .03 | .00 | -0.04 | .17 |
| 1886 | 100.00 | 100.00 | 100.00 | -0.01 | -0.04 | -0.05 | -0.09 | -0.09 | -0.00 | -0.02 | .02 | 100.00 |
| 1887 | 100.00 | 100.00 | 100.00 | 100.00 | -0.14 | -0.11 | -0.09 | -0.13 | -0.12 | .08 | .04 | -0.06 |
| 1888 | .14 | .22 | .08 | .03 | -0.12 | -0.22 | -0.22 | .11 | -0.07 | -0.11 | -0.08 | .16 |
| 1889 | .20 | .08 | .01 | .02 | -0.08 | -0.04 | -0.02 | -0.01 | -0.03 | -0.04 | .02 | -0.02 |
| 1890 | .18 | .23 | .08 | .02 | .28 | -0.06 | -0.08 | -0.07 | -0.04 | -0.12 | -0.01 | .08 |
| 1891 | .03 | .08 | .08 | -0.03 | .01 | -0.15 | -0.12 | -0.11 | -0.16 | -0.04 | .05 | .04 |
| 1892 | .33 | .11 | .11 | -0.00 | -0.05 | .01 | -0.00 | .07 | .05 | -0.07 | .14 | .21 |
| 1893 | .22 | .19 | .21 | .04 | .07 | .01 | .15 | .15 | .09 | .02 | .20 | .15 |
| 1894 | .12 | .17 | .16 | -0.08 | -0.03 | .02 | .05 | .07 | .13 | .01 | .17 | .19 |
| 1895 | .22 | .22 | .15 | -0.02 | -0.03 | .03 | -0.00 | .02 | .04 | .13 | .05 | .10 |
| 1896 | .06 | .12 | .05 | -0.14 | -0.15 | -0.06 | .03 | .05 | .05 | .00 | .04 | .07 |
| 1897 | .13 | .00 | .01 | -0.08 | -0.03 | -0.13 | -0.19 | -0.02 | -0.06 | -0.03 | .05 | .13 |
| 1898 | .07 | .05 | .01 | -0.02 | -0.01 | -0.07 | -0.06 | -0.01 | .07 | .04 | .04 | .19 |
| 1899 | .06 | .06 | .05 | -0.06 | -0.10 | -0.13 | -0.05 | -0.14 | -0.03 | -0.14 | .02 | .15 |
| 1900 | .09 | .16 | .06 | -0.02 | .00 | -0.01 | -0.02 | -0.16 | -0.03 | -0.13 | .11 | .14 |
| 1901 | .16 | .16 | .01 | -0.03 | -0.01 | -0.12 | -0.06 | .01 | -0.04 | .06 | .12 | .11 |
| 1902 | .11 | .13 | -0.12 | .04 | -0.07 | .03 | -0.01 | .04 | .03 | .07 | .04 | .08 |
| 1903 | .15 | .25 | .03 | -0.05 | -0.14 | -0.00 | .06 | .02 | -0.06 | .05 | .10 | .19 |
| 1904 | .08 | .08 | .02 | -0.00 | -0.02 | -0.09 | -0.05 | -0.01 | -0.07 | -0.09 | .01 | -0.01 |
| 1905 | .18 | .08 | -0.03 | .05 | -0.13 | -0.14 | -0.12 | -0.15 | -0.23 | -0.04 | -0.03 | .03 |
| 1906 | .04 | .03 | .01 | -0.10 | -0.08 | -0.10 | -0.06 | -0.12 | -0.10 | -0.00 | -0.08 | -0.02 |
| 1907 | .04 | .08 | .03 | -0.05 | -0.07 | -0.09 | -0.03 | -0.02 | .04 | .00 | .07 | .05 |
| 1908 | .12 | .08 | .03 | -0.04 | -0.09 | -0.11 | -0.04 | .01 | -0.04 | -0.13 | .06 | .05 |
| 1909 | .05 | .04 | .03 | -0.09 | -0.10 | -0.08 | -0.03 | -0.14 | -0.04 | .01 | -0.15 | -0.00 |

## Table 4

### DIFFERENCE IN ELEVATIONS LESS LINEAR TRENDS, LESS PERIODIC COMPONENTS
### MARQUETTE MINUS DULUTH

( 100.0 SIGNIFIES ORIGINAL DATA IS MISSING AND NO DIFFERENCE IS POSSIBLE )

| YEAR | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEP | OCT | NOV | DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1860 | 100.00 | 100.00 | 100.00 | -0.02 | -0.05 | .07 | .07 | -0.04 | .14 | -0.07 | -0.03 | 100.00 |
| 1861 | 100.00 | .09 | -0.61 | -0.15 | -0.02 | .38 | -0.04 | -0.05 | .03 | -0.13 | -0.16 | .13 |
| 1862 | .31 | .22 | .06 | .01 | -0.21 | -0.14 | -0.07 | -0.01 | -0.07 | .03 | .07 | .05 |
| 1863 | -0.01 | -0.02 | .05 | -0.09 | -0.06 | .10 | .04 | .02 | -0.07 | -0.07 | .35 | .06 |
| 1864 | .01 | .16 | -0.00 | -0.13 | .13 | .09 | .06 | .35 | .07 | .20 | -0.06 | .14 |
| 1865 | .15 | -0.21 | .05 | .11 | .15 | .13 | .01 | -0.14 | -0.11 | .06 | .04 | .02 |
| 1866 | -0.15 | -0.50 | -0.08 | -0.02 | -0.03 | .11 | -0.09 | -0.30 | .08 | .15 | .35 | -0.39 |
| 1867 | -0.39 | -0.13 | .22 | .21 | .46 | -0.32 | -0.47 | .01 | -0.16 | -0.17 | .30 | -0.09 |
| 1868 | -0.11 | -0.13 | .11 | .69 | -0.05 | .05 | -0.17 | -0.02 | .04 | .06 | -0.31 | .14 |
| 1869 | -0.12 | -0.15 | -0.00 | -0.00 | .03 | .04 | .01 | -0.02 | -0.85 | -0.41 | -0.11 | -0.10 |
| 1870 | -0.10 | -0.16 | -0.03 | .01 | -0.89 | .06 | .02 | .05 | .02 | -0.01 | -0.10 | -0.09 |
| 1871 | -0.10 | -0.14 | -0.02 | .01 | .04 | .09 | .02 | .04 | .03 | .01 | -0.07 | .07 |
| 1872 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | .01 | .01 | -0.19 | -0.04 | -0.03 | -0.09 | -0.10 |
| 1873 | -0.05 | -0.14 | -0.02 | .01 | .11 | .03 | .02 | .01 | .22 | 100.00 | 100.00 | 100.00 |
| 1874 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1875 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1876 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1877 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1878 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1879 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1880 | 100.00 | 100.00 | 100.00 | 100.00 | .03 | .12 | .07 | -0.06 | .21 | -0.07 | -0.14 | 100.00 |
| 1881 | 100.00 | 100.00 | 100.00 | .26 | -0.05 | -0.08 | .12 | .09 | -0.14 | .04 | -0.07 | 100.00 |
| 1882 | 100.00 | 100.00 | 100.00 | .20 | -0.16 | -0.27 | -0.02 | -0.02 | -0.09 | -0.53 | -0.54 | 100.00 |
| 1883 | 100.00 | 100.00 | 100.00 | 100.00 | .16 | .12 | -0.07 | .00 | -0.18 | -0.11 | 100.00 | 100.00 |
| 1884 | 100.00 | 100.00 | 100.00 | 100.00 | .06 | -0.69 | 100.00 | 100.00 | -0.03 | .01 | -0.03 | .02 |
| 1885 | -0.02 | 100.00 | 100.00 | 100.00 | -0.05 | -0.14 | -0.08 | -0.02 | .09 | .03 | -0.11 | .07 |
| 1886 | 100.00 | 100.00 | 100.00 | .05 | .03 | .03 | -0.07 | -0.09 | .05 | -0.01 | -0.04 | 100.00 |
| 1887 | 100.00 | 100.00 | 100.00 | 100.00 | -0.07 | -0.08 | -0.08 | -0.18 | -0.09 | .16 | -0.00 | -0.12 |
| 1888 | .03 | .20 | .06 | .12 | -0.05 | -0.25 | -0.26 | .29 | -0.02 | -0.15 | -0.16 | .06 |
| 1889 | .08 | .01 | .01 | .10 | -0.01 | .03 | .02 | .04 | .01 | -0.05 | -0.03 | -0.09 |
| 1890 | .06 | .21 | .06 | .09 | .32 | .00 | -0.07 | -0.06 | .01 | -0.17 | -0.07 | -0.01 |
| 1891 | -0.06 | .00 | .06 | .01 | .07 | -0.14 | -0.12 | -0.13 | -0.13 | -0.03 | .02 | -0.04 |
| 1892 | .18 | .05 | .09 | .05 | .01 | .11 | .05 | .20 | .11 | -0.09 | .13 | .10 |
| 1893 | .09 | .15 | .16 | .13 | .13 | .12 | .27 | .36 | .16 | .06 | .22 | .05 |
| 1894 | .01 | .13 | .13 | -0.07 | .04 | .14 | .13 | .21 | .21 | .05 | .18 | .09 |
| 1895 | .09 | .19 | .11 | .02 | .03 | .14 | .05 | .10 | .10 | .24 | .01 | .01 |
| 1896 | -0.04 | .06 | .03 | -0.18 | -0.08 | .00 | .10 | .17 | .11 | .03 | .00 | -0.01 |
| 1897 | .02 | -0.11 | .00 | -0.07 | .04 | -0.10 | -0.22 | .04 | -0.01 | -0.01 | .00 | .03 |
| 1898 | -0.03 | -0.05 | -0.00 | .02 | .05 | -0.02 | -0.04 | .05 | .13 | .09 | -0.00 | .08 |
| 1899 | -0.04 | -0.03 | .03 | -0.04 | -0.03 | -0.11 | -0.02 | -0.19 | .02 | -0.20 | -0.03 | .05 |
| 1900 | -0.01 | .12 | .05 | .04 | .06 | .09 | .03 | -0.24 | .01 | -0.19 | .09 | .04 |
| 1901 | .04 | .11 | -0.00 | .01 | .05 | -0.10 | -0.04 | .09 | .01 | .12 | .11 | .02 |
| 1902 | .00 | .07 | -0.10 | .13 | -0.00 | .15 | .04 | .15 | .09 | .15 | .00 | -0.00 |
| 1903 | .04 | .25 | .02 | -0.01 | -0.06 | .09 | .13 | .10 | -0.02 | .10 | .08 | .08 |
| 1904 | -0.02 | -0.00 | .01 | .06 | .04 | -0.05 | -0.02 | .05 | -0.03 | -0.12 | -0.04 | -0.08 |
| 1905 | .06 | .00 | -0.03 | .15 | -0.06 | -0.12 | -0.12 | -0.21 | -0.21 | -0.03 | -0.09 | -0.04 |
| 1906 | -0.05 | -0.06 | .01 | -0.12 | -0.02 | -0.07 | -0.04 | -0.16 | -0.07 | .02 | -0.17 | -0.09 |
| 1907 | -0.06 | -0.00 | .02 | -0.02 | -0.00 | -0.05 | .01 | .03 | .10 | .03 | .04 | -0.03 |
| 1908 | .01 | .00 | .02 | -0.00 | -0.02 | -0.07 | -0.00 | .10 | .01 | -0.18 | .03 | -0.03 |
| 1909 | -0.05 | -0.06 | .01 | -0.10 | -0.04 | -0.03 | .00 | -0.20 | .00 | .04 | -0.27 | -0.07 |

## Table 3 (Cont'd.)

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1910 | .11 | .09 | .04 | -0.09 | .01 | -0.03 | .54 | -0.05 | -0.06 | -0.01 | .15 | .19 |
| 1911 | .10 | .06 | .01 | -0.07 | -0.08 | -0.09 | .02 | -0.04 | -0.09 | -0.00 | .11 | -0.02 |
| 1912 | .25 | .11 | .04 | -0.05 | -0.15 | -0.03 | -0.01 | -0.05 | -0.03 | .07 | .14 | .14 |
| 1913 | .18 | .20 | .11 | -0.01 | -0.02 | .03 | .03 | -0.03 | -0.09 | .03 | .09 | .12 |
| 1914 | .08 | .23 | .04 | -0.02 | -0.09 | -0.10 | -0.02 | -0.02 | -0.09 | -0.04 | .17 | .20 |
| 1915 | .11 | .02 | .08 | -0.04 | -0.09 | -0.08 | -0.05 | -0.04 | -0.10 | -0.03 | .11 | .03 |
| 1916 | .16 | .14 | -0.04 | -0.06 | -0.07 | -0.07 | -0.11 | -0.06 | .00 | .02 | .11 | .12 |
| 1917 | .15 | .15 | .02 | -0.04 | .05 | -0.02 | -0.01 | -0.01 | -0.01 | .05 | .07 | .18 |
| 1918 | .99 | .04 | .03 | -0.03 | -0.04 | .08 | -0.05 | -0.09 | .09 | -0.01 | .09 | -0.01 |
| 1919 | .13 | .10 | -0.01 | -0.09 | -0.02 | -0.07 | -0.03 | .09 | .02 | .00 | .06 | .18 |
| 1920 | .06 | .07 | -0.03 | .10 | -0.05 | -0.11 | -0.01 | -0.13 | -0.11 | -0.05 | .05 | .04 |
| 1921 | .02 | .03 | .01 | -0.11 | -0.08 | -0.13 | -0.05 | .03 | .02 | .07 | .05 | .18 |
| 1922 | .13 | .03 | -0.05 | -0.06 | -0.10 | -0.06 | -0.07 | -0.09 | -0.10 | -0.08 | -0.05 | .09 |
| 1923 | .01 | .22 | .11 | .05 | -0.07 | -0.06 | -0.06 | .01 | -0.09 | .02 | -0.00 | .02 |
| 1924 | .18 | .05 | .01 | -0.05 | -0.01 | -0.06 | -0.09 | -0.03 | -0.06 | -0.13 | .08 | .17 |
| 1925 | .05 | .02 | -0.02 | -0.04 | .01 | -0.08 | -0.02 | -0.03 | -0.15 | .08 | .07 | .16 |
| 1926 | .08 | .02 | .09 | .08 | -0.07 | .03 | -0.03 | -0.10 | -0.17 | -0.02 | .05 | .06 |
| 1927 | .12 | .14 | .07 | -0.07 | -0.10 | -0.05 | -0.03 | .05 | .01 | .01 | .04 | .10 |
| 1928 | .17 | .07 | .11 | -0.01 | .02 | -0.01 | -0.03 | -0.08 | -0.01 | -0.01 | .05 | .08 |
| 1929 | .19 | .05 | -0.03 | -0.04 | .05 | -0.05 | .03 | .02 | -0.05 | -0.00 | .10 | .06 |
| 1930 | .13 | .00 | .12 | .00 | -0.13 | .03 | -0.02 | .00 | .02 | .03 | .03 | .08 |
| 1931 | .04 | .03 | .04 | .01 | -0.05 | -0.08 | -0.03 | -0.04 | -0.01 | .02 | .11 | -0.04 |
| 1932 | -0.00 | .05 | .22 | -0.08 | -0.11 | -0.10 | -0.02 | -0.06 | .06 | .01 | -0.04 | .03 |
| 1933 | -0.03 | .14 | .06 | -0.04 | -0.12 | -0.07 | -0.13 | -0.05 | -0.10 | .01 | .07 | .10 |
| 1934 | .11 | .08 | .05 | -0.03 | -0.08 | -0.03 | -0.10 | -0.04 | -0.07 | -0.04 | -0.04 | .05 |
| 1935 | .05 | .11 | -0.07 | -0.09 | -0.11 | -0.06 | -0.09 | -0.12 | -0.05 | -0.03 | -0.03 | .01 |
| 1936 | .06 | .11 | .01 | .02 | -0.09 | -0.07 | -0.09 | -0.11 | -0.13 | .06 | .10 | .01 |
| 1937 | .03 | .10 | .08 | -0.10 | -0.02 | -0.03 | -0.03 | -0.09 | -0.04 | .08 | .11 | .07 |
| 1938 | .17 | .06 | .03 | -0.03 | -0.11 | -0.04 | -0.03 | .02 | -0.04 | -0.17 | .02 | .13 |
| 1939 | .05 | .03 | .05 | -0.01 | -0.11 | -0.05 | -0.01 | -0.04 | -0.06 | .06 | .07 | .14 |
| 1940 | .30 | .07 | .09 | -0.01 | .03 | .01 | -0.03 | -0.03 | .02 | -0.03 | .08 | .11 |
| 1941 | .09 | .21 | -0.93 | -0.10 | -0.05 | -0.06 | -0.03 | -0.02 | -0.13 | .02 | .08 | .04 |
| 1942 | .12 | .11 | .02 | -0.06 | -0.06 | -0.06 | -0.03 | -0.05 | .02 | .02 | .08 | .08 |
| 1943 | .08 | .11 | .03 | -0.02 | -0.08 | -0.04 | -0.01 | .04 | .08 | .00 | .13 | 1.19 |
| 1944 | .10 | .12 | -0.02 | -0.06 | -0.10 | -0.11 | .06 | .01 | -0.04 | .02 | -0.09 | .16 |
| 1945 | .12 | .05 | -0.02 | .01 | .02 | -0.10 | -0.04 | -0.00 | -0.06 | .10 | -0.00 | .09 |
| 1946 | .12 | .07 | -0.04 | -0.03 | -0.03 | -0.09 | -0.02 | .03 | -0.05 | -0.02 | .05 | .06 |
| 1947 | .06 | .27 | .17 | -0.15 | -0.09 | -0.09 | .00 | -0.09 | -0.02 | -0.07 | .04 | .06 |
| 1948 | .06 | .05 | .06 | -0.02 | .01 | -0.02 | -0.00 | -0.06 | -0.09 | -0.04 | -0.03 | .05 |
| 1949 | .01 | .05 | -0.01 | -0.07 | -0.08 | -0.09 | -0.09 | -0.08 | .06 | .02 | .07 | .03 |
| 1950 | .07 | .04 | -0.01 | -0.06 | -0.12 | .01 | -0.04 | -0.01 | -0.07 | -0.05 | .08 | .03 |
| 1951 | .70 | .03 | .02 | -0.01 | -0.00 | -0.06 | -0.01 | -0.05 | -0.02 | -0.07 | .03 | .04 |
| 1952 | .03 | -0.02 | -0.05 | -0.01 | .01 | -0.04 | -0.02 | -0.02 | .08 | .19 | .05 | .09 |
| 1953 | .09 | .14 | .02 | -0.01 | -0.10 | -0.05 | -0.00 | -0.07 | .11 | -0.02 | .03 | .12 |
| 1954 | .05 | .04 | .11 | -0.03 | -0.02 | -0.11 | -0.02 | .03 | -0.05 | .05 | .02 | .07 |
| 1955 | .09 | .00 | .12 | -0.10 | -0.07 | -0.03 | -0.07 | -0.04 | -0.05 | .00 | .08 | -0.02 |
| 1956 | -0.04 | .08 | -0.04 | -0.00 | -0.09 | -0.05 | .01 | -0.00 | -0.00 | -0.08 | .06 | .04 |
| 1957 | .15 | .06 | -0.02 | -0.07 | -0.09 | -0.06 | -0.05 | -0.07 | -0.02 | -0.03 | .07 | .05 |
| 1958 | -0.01 | .15 | .00 | -0.04 | -0.06 | -0.04 | -0.07 | .04 | -0.06 | -0.02 | .02 | .08 |
| 1959 | .13 | .08 | -0.01 | -0.03 | -0.09 | -0.05 | -0.06 | -0.11 | -0.11 | .02 | .09 | -0.03 |
| 1960 | .08 | .07 | .04 | -0.05 | -0.08 | -0.03 | .02 | -0.12 | -0.08 | .02 | .06 | .11 |
| 1961 | .10 | -0.01 | -0.08 | -0.06 | -0.09 | -0.03 | -0.03 | .00 | -0.05 | -0.04 | .03 | .08 |
| 1962 | .22 | -0.00 | -0.18 | -0.10 | -0.14 | -0.08 | -0.04 | -0.09 | -0.02 | -0.01 | -0.03 | .11 |
| 1963 | .12 | .01 | -0.15 | -0.14 | -0.11 | -0.10 | -0.04 | -0.04 | -0.12 | -0.08 | .07 | .07 |
| 1964 | .02 | .06 | -0.04 | -0.16 | -0.10 | -0.09 | -0.08 | -0.06 | -0.05 | .03 | .01 | -0.00 |
| 1965 | .07 | .05 | -0.07 | -0.12 | -0.11 | -0.09 | -0.02 | -0.07 | -0.13 | .01 | -0.02 | -0.05 |
| 1966 | .09 | .00 | -0.17 | -0.11 | -0.09 | -0.07 | -0.08 | -0.08 | .04 | .01 | .03 | .03 |
| 1967 | -0.03 | .02 | -0.08 | -0.17 | -0.09 | -0.14 | -0.02 | -0.02 | -0.05 | -0.04 | .05 | .06 |
| 1968 | -0.02 | .24 | -0.02 | -0.10 | -0.10 | -0.10 | -0.04 | -0.03 | -0.08 | -0.06 | -0.04 | -0.06 |
| 1969 | -0.02 | -0.04 | -0.01 | -0.16 | -0.10 | -0.07 | -0.07 | -0.01 | -0.04 | -0.03 | .02 | -0.01 |
| 1970 | .06 | .06 | -0.05 | -0.17 | -0.14 | -0.10 | -0.04 | .02 | -0.02 | -0.09 | .01 | -0.02 |

Table 4 (Cont'd.)

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1910 | .00 | .01 | .03 | -0.09 | .07 | .05 | .84 | -0.02 | -0.02 | .01 | .15 | .09 |
| 1911 | -0.00 | -0.02 | .00 | -0.05 | -0.01 | -0.04 | .09 | .00 | -0.05 | .02 | .09 | -0.09 |
| 1912 | .11 | .04 | .02 | -0.03 | -0.08 | .05 | .03 | -0.03 | .02 | .14 | .14 | .04 |
| 1913 | .06 | .17 | .08 | .05 | .04 | .15 | .10 | .02 | -0.05 | .08 | .07 | .03 |
| 1914 | -0.02 | .22 | .03 | .04 | -0.02 | -0.05 | .02 | .02 | -0.06 | -0.05 | .17 | .09 |
| 1915 | -0.00 | -0.09 | .06 | -0.01 | -0.02 | -0.03 | -0.02 | -0.01 | -0.06 | -0.02 | .09 | -0.05 |
| 1916 | .04 | .09 | -0.04 | -0.03 | -0.00 | -0.01 | -0.11 | -0.04 | .06 | .06 | .10 | .03 |
| 1917 | .04 | .11 | .01 | .00 | .10 | .06 | .03 | .04 | .04 | .10 | .04 | .08 |
| 1918 | .70 | -0.06 | .02 | .01 | .02 | .23 | -0.02 | -0.10 | .16 | .01 | .07 | -0.08 |
| 1919 | .02 | .03 | -0.01 | -0.08 | .05 | -0.01 | .01 | .25 | .08 | .03 | .03 | .08 |
| 1920 | -0.03 | -0.02 | -0.03 | .22 | .01 | -0.08 | .03 | -0.18 | -0.08 | -0.06 | .01 | -0.04 |
| 1921 | -0.07 | -0.07 | .00 | -0.12 | -0.01 | -0.11 | -0.02 | .13 | .08 | .14 | .01 | .08 |
| 1922 | .02 | -0.07 | -0.04 | -0.03 | -0.03 | .01 | -0.04 | -0.09 | -0.06 | -0.10 | -0.13 | -0.00 |
| 1923 | -0.08 | .19 | .08 | .14 | -0.01 | -0.00 | -0.04 | .08 | -0.05 | .06 | -0.06 | -0.06 |
| 1924 | .06 | -0.04 | .00 | -0.02 | .05 | .01 | -0.08 | -0.08 | -0.01 | -0.18 | .06 | .07 |
| 1925 | -0.04 | -0.08 | -0.02 | .00 | .07 | -0.02 | .03 | .02 | -0.13 | .15 | .03 | .06 |
| 1926 | -0.02 | -0.09 | .06 | .19 | -0.01 | .15 | .01 | -0.12 | -0.14 | -0.01 | .01 | -0.02 |
| 1927 | .01 | .09 | .05 | -0.05 | -0.03 | .02 | .01 | .17 | .07 | .05 | .00 | .01 |
| 1928 | .05 | -0.01 | .09 | .05 | .08 | .08 | .00 | -0.09 | .04 | .00 | .01 | -0.01 |
| 1929 | .06 | -0.04 | -0.03 | -0.01 | .11 | .02 | .09 | .11 | -0.01 | .03 | .08 | -0.02 |
| 1930 | .02 | -0.11 | .09 | .07 | -0.06 | .15 | .03 | .08 | .08 | .08 | -0.01 | -0.00 |
| 1931 | -0.05 | -0.06 | .02 | .07 | .01 | -0.03 | .00 | -0.01 | .04 | .05 | .09 | -0.11 |
| 1932 | -0.09 | -0.04 | .17 | -0.07 | -0.04 | -0.06 | .02 | -0.04 | .12 | .04 | -0.11 | -0.05 |
| 1933 | -0.11 | .09 | .04 | .00 | -0.05 | -0.01 | -0.13 | -0.02 | -0.06 | .05 | .05 | .02 |
| 1934 | -0.00 | -0.00 | .03 | .01 | -0.02 | .05 | -0.10 | -0.01 | -0.03 | -0.04 | -0.12 | -0.03 |
| 1935 | -0.05 | .04 | -0.06 | -0.09 | -0.04 | .00 | -0.08 | -0.16 | -0.00 | -0.02 | -0.10 | -0.06 |
| 1936 | -0.04 | .05 | .00 | .10 | -0.02 | -0.01 | -0.07 | -0.13 | -0.09 | .12 | .07 | -0.07 |
| 1937 | -0.07 | .02 | .06 | -0.11 | .04 | .05 | .00 | -0.11 | .00 | .16 | .09 | -0.01 |
| 1938 | .05 | -0.03 | .02 | .01 | -0.04 | .04 | .01 | .11 | .01 | -0.25 | -0.03 | .04 |
| 1939 | -0.04 | -0.06 | .04 | .05 | -0.04 | .03 | .04 | -0.02 | -0.02 | .12 | .03 | .04 |
| 1940 | .15 | -0.02 | .07 | .04 | .09 | .11 | .01 | .01 | .07 | -0.02 | .05 | .02 |
| 1941 | -0.01 | .19 | -0.76 | -0.10 | .02 | .01 | .01 | .04 | -0.09 | .06 | .06 | -0.04 |
| 1942 | .01 | .05 | .01 | -0.03 | .00 | -0.00 | .00 | -0.03 | .07 | .05 | .05 | -0.01 |
| 1943 | -0.02 | .04 | .02 | .03 | -0.01 | .03 | .04 | .15 | .14 | .03 | .12 | .92 |
| 1944 | -0.01 | .06 | -0.02 | -0.03 | -0.03 | -0.07 | .14 | .09 | .01 | .07 | -0.18 | .06 |
| 1945 | .01 | -0.05 | -0.03 | .07 | .08 | -0.07 | -0.01 | .06 | -0.02 | .19 | -0.06 | .00 |
| 1946 | .01 | -0.01 | -0.04 | .01 | .03 | -0.04 | .02 | .13 | -0.00 | -0.00 | .01 | -0.02 |
| 1947 | -0.04 | .27 | .13 | -0.18 | -0.02 | -0.04 | .06 | -0.09 | .03 | -0.08 | .00 | -0.03 |
| 1948 | -0.04 | -0.05 | .04 | .02 | .07 | .06 | .05 | -0.05 | -0.05 | -0.04 | -0.10 | -0.03 |
| 1949 | -0.08 | -0.04 | -0.01 | -0.05 | -0.01 | -0.04 | -0.08 | -0.08 | .12 | .06 | .04 | -0.04 |
| 1950 | -0.03 | -0.05 | -0.01 | -0.03 | -0.05 | .12 | -0.00 | .06 | -0.03 | -0.06 | .05 | -0.05 |
| 1951 | .47 | -0.07 | .01 | .04 | .06 | .00 | .03 | -0.03 | .03 | -0.09 | -0.02 | -0.04 |
| 1952 | -0.06 | -0.14 | -0.05 | .05 | .07 | .04 | .02 | .04 | .15 | .34 | .02 | .01 |
| 1953 | -0.01 | .09 | .01 | .05 | -0.03 | .01 | .04 | -0.07 | .17 | -0.01 | -0.02 | .03 |
| 1954 | -0.05 | -0.06 | .08 | .01 | .04 | -0.08 | .02 | .13 | -0.01 | .11 | -0.03 | -0.01 |
| 1955 | -0.01 | -0.11 | .09 | -0.10 | -0.00 | .06 | -0.05 | -0.00 | -0.00 | .03 | .06 | -0.08 |
| 1956 | -0.11 | .01 | -0.03 | .06 | -0.03 | .01 | .06 | .06 | .05 | -0.11 | .02 | -0.04 |
| 1957 | .03 | -0.03 | -0.02 | -0.06 | -0.02 | .00 | -0.02 | -0.06 | .03 | -0.02 | .04 | -0.03 |
| 1958 | -0.09 | .10 | -0.00 | .00 | .01 | .04 | -0.05 | .15 | -0.01 | .00 | -0.02 | -0.00 |
| 1959 | .01 | -0.00 | -0.02 | .01 | -0.03 | .01 | -0.04 | -0.15 | -0.08 | .06 | .06 | -0.10 |
| 1960 | -0.02 | -0.01 | .03 | -0.02 | -0.01 | .05 | .08 | -0.16 | -0.04 | .06 | .03 | .02 |
| 1961 | -0.00 | -0.12 | -0.07 | -0.03 | -0.02 | .06 | .01 | .08 | .00 | -0.05 | -0.02 | -0.01 |
| 1962 | .09 | -0.12 | -0.15 | -0.11 | -0.07 | -0.03 | -0.01 | -0.11 | .03 | .01 | -0.10 | .02 |
| 1963 | .01 | -0.10 | -0.13 | -0.17 | -0.04 | -0.06 | -0.00 | -0.00 | -0.08 | -0.10 | .04 | -0.01 |
| 1964 | -0.07 | -0.02 | -0.04 | -0.20 | -0.03 | -0.04 | -0.06 | -0.05 | -0.01 | .07 | -0.05 | -0.07 |
| 1965 | -0.03 | -0.04 | -0.06 | -0.14 | -0.04 | -0.05 | .02 | -0.07 | -0.10 | .04 | -0.09 | -0.11 |
| 1966 | -0.01 | -0.11 | -0.14 | -0.12 | -0.02 | -0.01 | -0.06 | -0.08 | .10 | .05 | -0.01 | -0.04 |
| 1967 | -0.11 | -0.08 | -0.07 | -0.21 | -0.03 | -0.13 | .02 | .02 | -0.01 | -0.04 | .01 | -0.02 |
| 1968 | -0.10 | .22 | -0.02 | -0.11 | -0.03 | -0.06 | -0.01 | .01 | -0.04 | -0.07 | -0.11 | -0.12 |
| 1969 | -0.10 | -0.17 | -0.01 | -0.20 | -0.03 | -0.01 | -0.05 | .06 | .01 | -0.02 | -0.02 | -0.08 |
| 1970 | -0.03 | -0.02 | -0.05 | -0.23 | -0.07 | -0.07 | -0.01 | .10 | .03 | -0.12 | -0.05 | -0.09 |

$$\bar{X}_{i+k} = \frac{1}{N-k} \sum_{i=1}^{N-k} X_{i+k} \qquad (3)$$

$$S_i^2 = \frac{1}{N-k} \sum_{i=1}^{N-k} (X_i - \bar{X}_i)^2 \qquad (4)$$

$$S_{i+k}^2 = \frac{1}{N-k} \sum_{i=1}^{N-k} (X_{i+k} - \bar{X}_{i+k})^2 \qquad (5)$$

Note that using the above equations instead of the more common

$$r_k = \frac{\sum_{i=1}^{N-k} (X_i - \bar{X})\ (X_{i+k} - \bar{X})}{(N-k)\ S^2} \qquad (6)$$

takes into consideration the fact that most hydrologic time series include some degree of non-stationarity. This means that for small values of k the results will be less biased. For large values of k, however, some information is lost due to the term N-k, and so a secondary bias is introduced. This secondary bias can be minimised by keeping the ratio k/N less than 0.1.

Plotting $r_k$ versus k produces a correlogram which theoretically can be used to determine the make-up of the original time series. In practice, sampling fluctuations and superposition of harmonics tend to complicate the issue. Confidence limits for the correlogram were computed using the method by Anderson (1941):

$$CL_\alpha = \frac{-1 \pm n_\alpha \sqrt{N-k-2}}{N-k-1} \qquad (7)$$

where N is the number of observed values in the time series,
L is the lag, and
$n_\alpha$ is the standard normal deviate for a two tail test at a significance level $\alpha$.

Spectral analysis

Spectral analysis is a technique used to determine the distribution of the total variance of a time series with frequency. The traditional method is to derive spectral estimates from the autocovariance, since it has been determined, e.g. Wold (1954), that for a continuous function the spectral density is the Fourier transform of the autocovariance function, c(k).

$$V(f) = \int_{-\infty}^{\infty} e^{-2\pi ifk} c(k)dk \qquad (8)$$

or, since c(k) is an even function

$$V(f) = 2 \int_{o}^{\infty} c(k)\cos 2\pi fkdk \qquad (9)$$

A second technique known as the Fast Fourier Transform has been developed to compute the spectral density directly (Cooley and Tukey, 1965).

In practice one estimate of the spectrum is derived from a discrete series of autocovariances as:

$$V_k = \frac{n}{m} \left[ C_o + C_m \cos k\pi + 2 \sum_{j=1}^{m-1} C_j \cos \frac{kj\pi}{m} \right] \qquad (10)$$

at $k = o, m$ ; $n = 0.5$, and

for $o < k < m$ ; $n = 1.0$.

The spectral estimate must be refined by applying a filter or kernal function. This can be done either by multiplying the covariances by the kernal function before the Fourier transformation or by performing the transformation and then forming linear combinations of $V_k$ using the transform of the kernal function. In this study a "Hamming" smoothing function was used with no pre-whitening.

As with the correlogram, a plot of spectral density versus frequency can be used to detect trend and periodicity in a time series. Confidence limits for the plot of the spectral estimates were computed using the method described by Jenkins (1961):

$$CL_{\alpha}\ (N,k) = \frac{\chi^2_{100-\alpha}\ (\gamma)}{\gamma} \qquad (11)$$

$$CL'_{\alpha}\ (N,k) = \frac{\chi^2_{\alpha}\ (\gamma)}{\gamma} \qquad (12)$$

where $CL_{\alpha}$ and $CL'_{\alpha}$ are factors by which the mean estimated spectrum is to be multiplied,
$\gamma$ is defined as the equivalent degrees of freedom,
$\gamma = 2N/k$
N and k are the number of observed values and the required lag as before, and
$\alpha$ is the required confidence level, and
$\chi^2 \alpha\ (\gamma)$ is the $\alpha$% value of the Chi-square distribution with $\gamma$ degrees of freedom.

The spectral estimates are normally plotted on a logarithmic scale for two reasons:-

(a) It has been found that for most natural time series low frequencies are of more importance than high frequencies and a logarithmic scale brings out this difference.

(b) Confidence limits can be plotted more easily on a logarithmic scale since this involves only adding a constant factor to the mean spectral estimate.

## Variate difference

The basic principle behind the variate difference method of trend removal is that taking successive differences between values of a variable will eventually eliminate any trend provided that there is no significant periodic component present.

If the original time series consists of a stochastic and a time-dependent component then tne series can be written as:

$$\zeta_t = a + bt + \varepsilon_t \tag{13}$$

and consists of $\zeta_1, \zeta_2, \zeta_3, \zeta_4, \dots\dots\dots\dots\zeta_n$.

If the difference between successive values of the series $\Delta^1_{\zeta_1} = \zeta_2 - \zeta_1$, $\Delta^1_{\zeta_2} = \zeta_3 - \zeta_2$ are taken, then a new random series with a variance $\sigma^2_{\Delta^1_\zeta}$ will be created.

Similarly, further differences can be taken $\Delta^2_{\zeta_1} = \Delta^1_{\zeta_2} - \Delta^1_{\zeta_1} = \zeta_1 + \zeta_3 - 2\zeta_2$, etc., until, in general, the rth difference and the r + 1th series is created:

$$\Delta^r_{\zeta_i} = \zeta_{i+r} - \binom{r}{1}\zeta_{i+r-1} + \binom{r}{2}\zeta_{i+r-2} + \dots.(-1)^r\zeta_i \tag{14}$$

and the variance of $\Delta^r_{\zeta_i}$ becomes $E\{\Delta^r_{\zeta_i}\}^2$ (15)

where $E\{\Delta^r_{\zeta_i}\}^2 = \sigma_\varepsilon^{\,2}\left[1 + \binom{r}{1}^2 + \binom{r}{2}^2 + \dots + \binom{r}{r-1}^2 + 1\right]$ (16)

where $\varepsilon$ is the stochastic component of the series.

$$\text{var}\ \Delta^r_{\zeta_i} = \sigma_\varepsilon^{\ 2} \binom{2r}{r} = \sigma_\varepsilon^{\ 2} \left[\frac{2r(2r-1\ \ldots.\ (2r-r+1)}{1.2.3.\ldots\ldots.r}\right] \qquad (17)$$

$$\text{so that}\quad \sigma_\varepsilon^{\ 2} = \frac{\text{var}\ \Delta^r_{\zeta_i}}{\binom{2r}{r}} = \frac{\text{var}\ \Delta^r_{\zeta_i}\ (r!)}{(2r)!} \qquad (18)$$

Thus, the method of variate differences is to plot

$$\frac{\text{var}\ \Delta^r_{\zeta_i}}{\binom{2r}{r}} \quad \text{v.} \quad r.$$

After an initial peak the series should oscillate around a fixed line. This will be the value of $\sigma_\varepsilon^{\ 2}$, the variance of the stochastic component. The number of points which are significantly different from this line will give the degree of the polynomial which makes up the trend.

## Polynomial regression

A polynomial regression program was used to generate powers of the variable and calculate polynomials of successively increasing degree such as:

$$\begin{aligned} y &= a + bx \\ y &= a + bx + cx^2 \end{aligned} \qquad (19)$$

up to the fourth power. In the equations above, y is the monthly difference in elevation and x is a decimal indication of the corresponding year and month. If there is no reduction in the residual sum of squares between two successive degrees of polynomials, the program terminates the problem before completing the analysis for the highest degree polynomial specified.

The output of the polynomial regression program includes the following information:

1. Regression coefficients for successive degree polynomials.

2. Analysis-of-variance table for each successive degree polynomial.

3. Table of residuals for the final degree polynomial.

Stepwise regression

A stepwise regression program was also run using up to fourth-order powers of time. This program computes a sequence of multiple linear regression equations in a stepwise manner. At each step one variable is added to the regression equation. The added variable is the one which makes the greatest reduction in the error sum of squares. Equivalently, it is the variable which has the highest partial correlation with the dependent variable partialed on the variables which have already been added; and, equivalently, it is the variable which when added will have the highest F value. The F-levels for inclusion and deletion can be specified by the user or, if not specified, values of 0.01 and 0.005 will be used. The F-levels used should be computed using standard tables of the F distribution to find F $(1, \gamma, \alpha)$ where $\gamma$ is calculated as a weighted average number of degrees of freedom during the stepwise computations i.e. N - V where N is the number of cases and V is the number of independent variables expected in the final step of the regression.

Output from the stepwise regression program includes

(a) optional output prior to performing regression
    (1) means and standard deviations
    (2) covariance matrix
    (3) correlation matrix

(b) at each step in the regression
    (1) multiple correlation coefficient
    (2) standard error of estimate
    (3) analysis of variance table
    (4) for variables in the equation
        (a) regression coefficient
        (b) standard error
        (c) F to remove
    (5) for variables not in the equation
        (a) tolerance
        (b) partial correlation coefficient
        (c) F to enter

(c) optional output after performing regression
    (1) summary table
    (2) list of residuals
    (3) plots of residuals versus input variables

## Periodic Analysis

The theoretical treatment of the periodogram is discussed in many statistical text books and, except for a section on the method used to determine the distribution of the variance, is given only a very brief treatment in this report.

A time series $x_t$, $t = 1,2....n$ is considered to be composed only of periodic sine and cosine functions of various amplitudes and phases arranged so that:

$$x_t = \bar{x} + \sum_{j=1}^{k} (A_j \cos\frac{2\pi jt}{n} + B_j \sin\frac{2\pi jt}{n}) \qquad (20)$$

where $x_t$ is the observed value of the time series at time t
$k$ is the number of harmonics corresponding to a base period $\omega$
$\bar{x}$ is the mean value of the time series, and

$$A_j = \frac{1}{n} \sum_{t=1}^{n} x_t \cos\frac{2\pi jt}{k} \qquad (21)$$

$$B_j = \frac{1}{n} \sum_{t=1}^{n} x_t \sin\frac{2\pi jt}{k} \qquad (22)$$

The period, $\omega$, is usually a period of 24 hours, 12 months or 365 days in hydrology and is broken down into a number of harmonics. The maximum number of harmonics in any period is $\omega/2$, so that 365 days consists of 182 harmonics, and 12 months consists of the 6 harmonics, 12, 6, 4, 3, 2.4 and 2 months. Usually only five or six harmonics are calculated for any period since they commonly explain up to 95 per cent of the variance due to periodicity.

A large proportion of hydro-statistics is devoted to analysing data with a maximum period of 12 months, and for this period (or any other period for which the long-term means of sub-periods are known) simplified equations are available to find $A_j$ and $B_j$ in equation 20.

$$A_j = \frac{1}{m} \sum_{i=1}^{m} (x_i - \bar{x}) \cos \frac{2\pi ij}{m} \qquad (23)$$

$$B_j = \frac{1}{m} \sum_{i=1}^{m} (x_i - \bar{x}) \sin \frac{2\pi ij}{m} \qquad (24)$$

where $\bar{x}$ is the overall mean of the series

$x_i$ is the long-term mean for the sub-period i

e.g., for a 12 month period, $\bar{x}$ is the long-term annual mean of x, and $x_i$ is the long-term mean of the ith month, January, February, etc.

Once A and B have been calculated for each required harmonic, the amplitude and phase of that harmonic can be found;

$$C_i = \sqrt{A_i^2 + B_i^2} \qquad (25)$$

$$\theta = \sin^{-1} (A_i/C_i) \qquad (26)$$

and, with this information, the periodogram, which is a graph of $C_i^2$ versus frequency, can be plotted.

The total variance of the series can now be explained a little more. If the amplitude squared of the ith harmonic is given by $C_i^2$

then the variance of the ith harmonic is:

$$\text{var } h_i = C_i^2/2 \qquad (27)$$

The variance of the means is given by:

$$\text{var } \bar{x} = \frac{1}{\omega} \sum_{j=1}^{\omega} (\bar{x}_j - \bar{\bar{x}})^2 \tag{28}$$

where $\bar{\bar{x}}$ is, for example, the long-term annual mean of a series and
$\bar{x}_j$, where $j=1, 2 \ldots \omega$, is a long-term monthly mean of the series.

$$P_i = \frac{\text{var } h_i \text{ x } 100}{\text{var } \bar{x}} \tag{29}$$

is the percentage of the periodic variance which is explained by the ith harmonic and

$$P_p = \frac{\text{var } \bar{x} \text{ x } 100}{\text{var } x} \tag{30}$$

is the percentage of the total variance of the series which is described by the variance of the periodic movement in the mean.

If var $x_s$ is the variance of the periodic movement of the standard deviation of the series, then var x - var $\bar{x}$ - var $x_s$ will be the proportion of the total series variance which is not due to periodicity in the mean and standard deviation, but is due to the stochastic component of the series.

This last part of the total variance can be reached from a different direction, let:

$$\varepsilon_i = \frac{x_i - \bar{x}}{(S_i - S)} \tag{31}$$

where $\bar{x}$ is the mean of the series $x_i$
$S$ is the mean of the standard deviations $S_i$,

Then $$P_r = \frac{\text{var } \varepsilon_i \text{ x } 100}{\text{var } x} \tag{32}$$

will be the percentage of the total variance which is explained by the random component.

### (c) Results of Analysis

As a first look at the time series the autocorrelation and spectral analysis programs were run on the raw differences in mean monthly elevation, Marquette minus Duluth (Table 5). The correlogram, Figure 4, shows a highly significant annual cycle with some downward trend apparent. This is confirmed by the spectral density plot, Figure 7, which also shows a highly significant trend component present. On the basis of these two sets of information it was obvious that an important component of the time series is a trend. In order to determine the order of trend present the variate difference technique previously described was used. The results of this program are given in Figure 10 and Table 8. While the results are not conclusive (probably due to the presence of a periodic component in the time series) they do indicate that the most important trends are first and second order.

In order to remove the trend, two programs, a polynomial regression and a stepwise regression, were used.

The results of the polynomial regression program are given in Table 10. The standard error of estimate decreases from 0.1194 for a first order linear trend to 0.1186 for a fourth order equation, while the multiple correlation coefficient increases from 0.6792 to 0.6857 over the same three steps. These are not significant changes and do not justify the increased difficulties which would be encountered in fitting a trend of greater than first order. The multiple correlation coefficient of around 0.68 does not seem large, indicating that only 46 per cent of the variance in the time series is explained by a linear first-order trend and, therefore, tests were performed on the correlation coefficient. The actual length of data is, in this case, 1211 months, but after correction for serial correlation within the data the effective data length is only 94 months. The effective data length is used in the tests instead of the actual data length and is computed as:

$$N' = \frac{N}{1 + 2r_1r_1' + 2r_2r_2' + \ldots\ldots\ldots} \qquad (33)$$

where N is the actual record length

$r_1$ is the first serial correlation coefficient for the

## Table 5

### AUTOCOVARIANCE AND SPECTRAL DENSITY ESTIMATES FOR ORIGINAL DIFFERENCES IN ELEVATION

### MARQUETTE MINUS DULUTH

| LAG | AUTO COVARIANCE | RAW POWER SPECTRUM | SMOOTHED POWER SPECTRUM | PERIOD (MONTHS) | FREQUENCY LIMITS ( CYCLES/MONTH ) | | FREQUENCY ( CYCLES/ MONTH ) |
|---|---|---|---|---|---|---|---|
| | | | | | LOWER | UPPER | |
| 0 | 2.114E-02 | 4.410E-01 | 2.484E-01 | 9.960E 02 | -6.944E-03 | 6.944E-03 | 0 |
| 1 | 1.388E-02 | 2.235E-02 | 1.141E-01 | 1.440E 02 | 0 | 1.389E-02 | 6.944E-03 |
| 2 | 1.183E-02 | 2.376E-03 | 7.862E-03 | 7.200E 01 | 6.944E-03 | 2.083E-02 | 1.389E-02 |
| 3 | 1.006E-02 | 6.251E-03 | 4.137E-03 | 4.800E 01 | 1.389E-02 | 2.778E-02 | 2.083E-02 |
| 4 | 8.806E-03 | 9.326E-04 | 2.382E-03 | 3.600E 01 | 2.083E-02 | 3.472E-02 | 2.778E-02 |
| 5 | 8.192E-03 | 1.917E-03 | 2.201E-03 | 2.880E 01 | 2.778E-02 | 4.167E-02 | 3.472E-02 |
| 6 | 7.877E-03 | 4.134E-03 | 3.451E-03 | 2.400E 01 | 3.472E-02 | 4.861E-02 | 4.167E-02 |
| 7 | 7.959E-03 | 3.381E-03 | 3.400E-03 | 2.057E 01 | 4.167E-02 | 5.556E-02 | 4.861E-02 |
| 8 | 8.728E-03 | 2.710E-03 | 2.546E-03 | 1.800E 01 | 4.861E-02 | 6.250E-02 | 5.556E-02 |
| 9 | 9.598E-03 | 1.326E-03 | 1.850E-03 | 1.600E 01 | 5.556E-02 | 6.944E-02 | 6.250E-02 |
| 10 | 1.173E-02 | 2.219E-03 | 1.833E-03 | 1.440E 01 | 6.250E-02 | 7.639E-02 | 6.944E-02 |
| 11 | 1.319E-02 | 1.433E-03 | 1.841E-02 | 1.309E 01 | 6.944E-02 | 8.333E-02 | 7.639E-02 |
| 12 | 1.412E-02 | 7.445E-02 | 4.163E-02 | 1.200E 01 | 7.639E-02 | 9.028E-02 | 8.333E-02 |
| 13 | 1.340E-02 | 4.762E-03 | 1.964E-02 | 1.108E 01 | 8.333E-02 | 9.722E-02 | 9.028E-02 |
| 14 | 1.124E-02 | -2.494E-04 | 1.601E-03 | 1.029E 01 | 9.028E-02 | 1.042E-01 | 9.722E-02 |
| 15 | 9.523E-03 | 2.783E-03 | 2.204E-03 | 9.600E 00 | 9.722E-02 | 1.111E-01 | 1.042E-01 |
| 16 | 8.421E-03 | 3.297E-03 | 2.566E-03 | 9.000E 00 | 1.042E-01 | 1.181E-01 | 1.111E-01 |
| 17 | 7.646E-03 | 6.332E-04 | 1.489E-03 | 8.471E 00 | 1.111E-01 | 1.250E-01 | 1.181E-01 |
| 18 | 7.585E-03 | 1.691E-03 | 1.858E-03 | 8.000E 00 | 1.181E-01 | 1.319E-01 | 1.250E-01 |
| 19 | 7.640E-03 | 3.473E-03 | 2.686E-03 | 7.579E 00 | 1.250E-01 | 1.389E-01 | 1.319E-01 |
| 20 | 8.255E-03 | 1.830E-03 | 2.272E-03 | 7.200E 00 | 1.319E-01 | 1.458E-01 | 1.389E-01 |
| 21 | 9.633E-03 | 2.108E-03 | 1.863E-03 | 6.857E 00 | 1.389E-01 | 1.528E-01 | 1.458E-01 |
| 22 | 1.152E-02 | 1.322E-03 | 1.858E-03 | 6.545E 00 | 1.458E-01 | 1.597E-01 | 1.528E-01 |
| 23 | 1.313E-02 | 2.866E-03 | 5.467E-03 | 6.261E 00 | 1.528E-01 | 1.667E-01 | 1.597E-01 |
| 24 | 1.370E-02 | 1.572E-02 | 9.747E-03 | 6.000E 00 | 1.597E-01 | 1.736E-01 | 1.667E-01 |
| 25 | 1.299E-02 | 2.612E-03 | 5.375E-03 | 5.760E 00 | 1.667E-01 | 1.806E-01 | 1.736E-01 |
| 26 | 1.108E-02 | 1.522E-03 | 2.096E-03 | 5.538E 00 | 1.736E-01 | 1.875E-01 | 1.806E-01 |
| 27 | 9.105E-03 | 2.927E-03 | 2.444E-03 | 5.333E 00 | 1.806E-01 | 1.944E-01 | 1.875E-01 |
| 28 | 7.706E-03 | 2.234E-03 | 2.306E-03 | 5.143E 00 | 1.875E-01 | 2.014E-01 | 1.944E-01 |
| 29 | 7.075E-03 | 1.854E-03 | 1.991E-03 | 4.966E 00 | 1.944E-01 | 2.083E-01 | 2.014E-01 |
| 30 | 7.018E-03 | 2.069E-03 | 2.007E-03 | 4.800E 00 | 2.014E-01 | 2.153E-01 | 2.083E-01 |
| 31 | 7.214E-03 | 2.013E-03 | 1.806E-03 | 4.645E 00 | 2.083E-01 | 2.222E-01 | 2.153E-01 |
| 32 | 7.819E-03 | 1.056E-03 | 1.845E-03 | 4.500E 00 | 2.153E-01 | 2.292E-01 | 2.222E-01 |
| 33 | 7.949E-03 | 3.530E-03 | 3.039E-03 | 4.364E 00 | 2.222E-01 | 2.361E-01 | 2.292E-01 |
| 34 | 1.132E-02 | 3.869E-03 | 3.137E-03 | 4.235E 00 | 2.292E-01 | 2.431E-01 | 2.361E-01 |
| 35 | 1.259E-02 | 1.024E-03 | 1.884E-03 | 4.114E 00 | 2.361E-01 | 2.500E-01 | 2.431E-01 |
| 36 | 1.323E-02 | 1.919E-03 | 2.172E-03 | 4.000E 00 | 2.431E-01 | 2.569E-01 | 2.500E-01 |
| 37 | 1.250E-02 | 3.916E-03 | 3.142E-03 | 3.892E 00 | 2.500E-01 | 2.639E-01 | 2.569E-01 |
| 38 | 1.109E-02 | 2.548E-03 | 2.603E-03 | 3.789E 00 | 2.569E-01 | 2.708E-01 | 2.639E-01 |
| 39 | 8.933E-03 | 1.422E-03 | 1.530E-03 | 3.692E 00 | 2.639E-01 | 2.778E-01 | 2.708E-01 |
| 40 | 7.176E-03 | 9.826E-04 | 1.659E-03 | 3.600E 00 | 2.708E-01 | 2.847E-01 | 2.778E-01 |

Table 5 (Cont'd.)

| LAG | AUTO COVARIANCE | RAW POWER SPECTRUM | SMOOTHED POWER SPECTRUM | PERIOD (MONTHS) | FREQUENCY LIMITS ( CYCLES/MONTH ) | | FREQUENCY ( CYCLES/ MONTH ) |
|---|---|---|---|---|---|---|---|
| | | | | | LOWER | UPPER | |
| 41 | 6.541E-03 | 3.484E-03 | 2.481E-03 | 3.512E 00 | 2.778E-01 | 2.917E-01 | 2.847E-01 |
| 42 | 6.965E-03 | 1.624E-03 | 2.268E-03 | 3.429E 00 | 2.847E-01 | 2.986E-01 | 2.917E-01 |
| 43 | 7.259E-03 | 2.565E-03 | 1.884E-03 | 3.349E 00 | 2.917E-01 | 3.056E-01 | 2.986E-01 |
| 44 | 7.593E-03 | 5.447E-04 | 1.553E-03 | 3.273E 00 | 2.986E-01 | 3.125E-01 | 3.056E-01 |
| 45 | 9.173E-03 | 2.910E-03 | 2.524E-03 | 3.200E 00 | 3.056E-01 | 3.194E-01 | 3.125E-01 |
| 46 | 1.078E-02 | 3.598E-03 | 2.913E-03 | 3.130E 00 | 3.125E-01 | 3.264E-01 | 3.194E-01 |
| 47 | 1.303E-02 | 1.308E-03 | 1.977E-03 | 3.064E 00 | 3.194E-01 | 3.333E-01 | 3.264E-01 |
| 48 | 1.313E-02 | 1.928E-03 | 1.628E-03 | 3.000E 00 | 3.264E-01 | 3.403E-01 | 3.333E-01 |
| 49 | 1.186E-02 | 1.242E-03 | 1.610E-03 | 2.939E 00 | 3.333E-01 | 3.472E-01 | 3.403E-01 |
| 50 | 1.029E-02 | 2.158E-03 | 1.899E-03 | 2.880E 00 | 3.403E-01 | 3.542E-01 | 3.472E-01 |
| 51 | 8.480E-03 | 1.946E-03 | 2.040E-03 | 2.824E 00 | 3.472E-01 | 3.611E-01 | 3.542E-01 |
| 52 | 7.049E-03 | 2.141E-03 | 1.669E-03 | 2.769E 00 | 3.542E-01 | 3.681E-01 | 3.611E-01 |
| 53 | 6.854E-03 | 2.817E-04 | 1.462E-03 | 2.717E 00 | 3.611E-01 | 3.750E-01 | 3.681E-01 |
| 54 | 6.687E-03 | 3.555E-03 | 2.726E-03 | 2.667E 00 | 3.681E-01 | 3.819E-01 | 3.750E-01 |
| 55 | 6.776E-03 | 3.226E-03 | 2.976E-03 | 2.618E 00 | 3.750E-01 | 3.889E-01 | 3.819E-01 |
| 56 | 7.559E-03 | 1.812E-03 | 2.196E-03 | 2.571E 00 | 3.819E-01 | 3.958E-01 | 3.889E-01 |
| 57 | 8.596E-03 | 2.069E-03 | 2.089E-03 | 2.526E 00 | 3.889E-01 | 4.028E-01 | 3.958E-01 |
| 58 | 1.006E-02 | 2.415E-03 | 2.464E-03 | 2.483E 00 | 3.958E-01 | 4.097E-01 | 4.028E-01 |
| 59 | 1.180E-02 | 2.975E-03 | 2.548E-03 | 2.441E 00 | 4.028E-01 | 4.167E-01 | 4.097E-01 |
| 60 | 1.237E-02 | 1.678E-03 | 1.891E-03 | 2.400E 00 | 4.097E-01 | 4.236E-01 | 4.167E-01 |
| 61 | 1.173E-02 | 1.306E-03 | 1.379E-03 | 2.361E 00 | 4.167E-01 | 4.306E-01 | 4.236E-01 |
| 62 | 9.470E-03 | 1.251E-03 | 1.440E-03 | 2.323E 00 | 4.236E-01 | 4.375E-01 | 4.306E-01 |
| 63 | 8.337E-03 | 2.019E-03 | 2.104E-03 | 2.286E 00 | 4.306E-01 | 4.444E-01 | 4.375E-01 |
| 64 | 6.812E-03 | 3.157E-03 | 2.301E-03 | 2.250E 00 | 4.375E-01 | 4.514E-01 | 4.444E-01 |
| 65 | 5.898E-03 | 5.746E-04 | 1.247E-03 | 2.215E 00 | 4.444E-01 | 4.583E-01 | 4.514E-01 |
| 66 | 6.113E-03 | 9.175E-04 | 1.144E-03 | 2.182E 00 | 4.514E-01 | 4.653E-01 | 4.583E-01 |
| 67 | 6.365E-03 | 2.247E-03 | 2.150E-03 | 2.149E 00 | 4.583E-01 | 4.722E-01 | 4.653E-01 |
| 68 | 6.648E-03 | 3.154E-03 | 2.542E-03 | 2.118E 00 | 4.653E-01 | 4.792E-01 | 4.722E-01 |
| 69 | 8.199E-03 | 1.401E-03 | 1.740E-03 | 2.087E 00 | 4.722E-01 | 4.861E-01 | 4.792E-01 |
| 70 | 1.002E-02 | 1.123E-03 | 1.621E-03 | 2.057E 00 | 4.792E-01 | 4.931E-01 | 4.861E-01 |
| 71 | 1.131E-02 | 3.009E-03 | 2.458E-03 | 2.028E 00 | 4.861E-01 | 5.000E-01 | 4.931E-01 |
| 72 | 1.197E-02 | 2.500E-03 | 2.734E-03 | 2.000E 00 | 4.931E-01 | 5.069E-01 | 5.000E-01 |

## Table 6

### AUTOCOVARIANCE AND SPECTRAL DENSITY ESTIMATES FOR DIFFERENCES IN ELEVATION LESS LINEAR TRENDS

### MARQUETTE MINUS DULUTH

| LAG | AUTO COVARIANCE | RAW POWER SPECTRUM | SMOOTHED POWER SPECTRUM | PERIOD (MONTHS) | FREQUENCY LIMITS ( CYCLES/MONTH ) LOWER | UPPER | FREQUENCY ( CYCLES/ MONTH ) |
|---|---|---|---|---|---|---|---|
| 0 | 1.102E-02 | 8.384E-03 | 8.278E-03 | 9.960E 02 | -6.944E-03 | 6.944E-03 | 0 |
| 1 | 3.820E-03 | 8.153E-03 | 7.459E-03 | 1.440E 02 | 0 | 1.389E-02 | 6.944E-03 |
| 2 | 1.831E-03 | 4.907E-03 | 5.218E-03 | 7.200E 01 | 6.944E-03 | 2.083E-02 | 1.389E-02 |
| 3 | 1.054E-04 | 3.016E-03 | 3.453E-03 | 4.800E 01 | 1.389E-02 | 2.778E-02 | 2.083E-02 |
| 4 | -1.131E-03 | 3.026E-03 | 2.663E-03 | 3.600E 01 | 2.083E-02 | 3.472E-02 | 2.778E-02 |
| 5 | -1.769E-03 | 1.456E-03 | 2.498E-03 | 2.880E 01 | 2.778E-02 | 4.167E-02 | 3.472E-02 |
| 6 | -2.129E-03 | 4.419E-03 | 3.358E-03 | 2.400E 01 | 3.472E-02 | 4.861E-02 | 4.167E-02 |
| 7 | -2.088E-03 | 2.768E-03 | 3.229E-03 | 2.057E 01 | 4.167E-02 | 5.556E-02 | 4.861E-02 |
| 8 | -1.270E-03 | 3.120E-03 | 2.652E-03 | 1.800E 01 | 4.861E-02 | 6.250E-02 | 5.556E-02 |
| 9 | -3.947E-04 | 1.438E-03 | 2.066E-03 | 1.600E 01 | 5.556E-02 | 6.944E-02 | 6.250E-02 |
| 10 | 1.721E-03 | 2.486E-03 | 2.063E-03 | 1.440E 01 | 6.250E-02 | 7.639E-02 | 6.944E-02 |
| 11 | 3.165E-03 | 1.696E-03 | 1.850E-02 | 1.309E 01 | 6.944E-02 | 8.333E-02 | 7.639E-02 |
| 12 | 4.149E-03 | 7.399E-02 | 4.127E-02 | 1.200E 01 | 7.639E-02 | 9.028E-02 | 8.333E-02 |
| 13 | 3.491E-03 | 4.033E-03 | 1.921E-02 | 1.108E 01 | 8.333E-02 | 9.722E-02 | 9.028E-02 |
| 14 | 1.363E-03 | 4.609E-05 | 1.523E-03 | 1.029E 01 | 9.028E-02 | 1.042E-01 | 9.722E-02 |
| 15 | -3.384E-04 | 2.482E-03 | 2.065E-03 | 9.600E 00 | 9.722E-02 | 1.111E-01 | 1.042E-01 |
| 16 | -1.420E-03 | 3.105E-03 | 2.339E-03 | 9.000E 00 | 1.042E-01 | 1.181E-01 | 1.111E-01 |
| 17 | -2.205E-03 | 3.995E-04 | 1.310E-03 | 8.471E 00 | 1.111E-01 | 1.250E-01 | 1.181E-01 |
| 18 | -2.264E-03 | 1.653E-03 | 1.737E-03 | 8.000E 00 | 1.181E-01 | 1.319E-01 | 1.250E-01 |
| 19 | -2.200E-03 | 3.271E-03 | 2.577E-03 | 7.579E 00 | 1.250E-01 | 1.389E-01 | 1.319E-01 |
| 20 | -1.571E-03 | 1.871E-03 | 2.230E-03 | 7.200E 00 | 1.319E-01 | 1.458E-01 | 1.389E-01 |
| 21 | -1.797E-04 | 2.030E-03 | 1.793E-03 | 6.857E 00 | 1.389E-01 | 1.528E-01 | 1.458E-01 |
| 22 | 1.710E-03 | 1.156E-03 | 1.738E-03 | 6.545E 00 | 1.458E-01 | 1.597E-01 | 1.528E-01 |
| 23 | 3.335E-03 | 2.813E-03 | 5.335E-03 | 6.261E 00 | 1.528E-01 | 1.667E-01 | 1.597E-01 |
| 24 | 3.910E-03 | 1.544E-02 | 9.542E-03 | 6.000E 00 | 1.597E-01 | 1.736E-01 | 1.667E-01 |
| 25 | 3.265E-03 | 2.433E-03 | 5.200E-03 | 5.760E 00 | 1.667E-01 | 1.806E-01 | 1.736E-01 |
| 26 | 1.427E-03 | 1.499E-03 | 1.986E-03 | 5.538E 00 | 1.736E-01 | 1.875E-01 | 1.806E-01 |
| 27 | -5.078E-04 | 2.777E-03 | 2.355E-03 | 5.333E 00 | 1.806E-01 | 1.944E-01 | 1.875E-01 |
| 28 | -1.884E-03 | 2.262E-03 | 2.283E-03 | 5.143E 00 | 1.875E-01 | 2.014E-01 | 1.944E-01 |
| 29 | -2.426E-03 | 1.837E-03 | 1.993E-03 | 4.966E 00 | 1.944E-01 | 2.083E-01 | 2.014E-01 |
| 30 | -2.485E-03 | 2.088E-03 | 2.010E-03 | 4.800E 00 | 2.014E-01 | 2.153E-01 | 2.083E-01 |
| 31 | -2.293E-03 | 1.998E-03 | 1.821E-03 | 4.645E 00 | 2.083E-01 | 2.222E-01 | 2.153E-01 |
| 32 | -1.688E-03 | 1.137E-03 | 1.892E-03 | 4.500E 00 | 2.153E-01 | 2.292E-01 | 2.222E-01 |
| 33 | -1.550E-03 | 3.555E-03 | 3.101E-03 | 4.364E 00 | 2.222E-01 | 2.361E-01 | 2.292E-01 |
| 34 | 1.799E-03 | 4.000E-03 | 3.215E-03 | 4.235E 00 | 2.292E-01 | 2.431E-01 | 2.361E-01 |
| 35 | 3.061E-03 | 1.033E-03 | 1.922E-03 | 4.114E 00 | 2.361E-01 | 2.500E-01 | 2.431E-01 |
| 36 | 3.734E-03 | 1.933E-03 | 2.150E-03 | 4.000E 00 | 2.431E-01 | 2.569E-01 | 2.500E-01 |
| 37 | 3.022E-03 | 3.775E-03 | 3.075E-03 | 3.892E 00 | 2.500E-01 | 2.639E-01 | 2.569E-01 |
| 38 | 1.643E-03 | 2.574E-03 | 2.572E-03 | 3.789E 00 | 2.569E-01 | 2.708E-01 | 2.639E-01 |
| 39 | -4.771E-04 | 1.366E-03 | 1.556E-03 | 3.692E 00 | 2.639E-01 | 2.778E-01 | 2.708E-01 |
| 40 | -2.226E-03 | 9.848E-04 | 1.617E-03 | 3.600E 00 | 2.708E-01 | 2.847E-01 | 2.778E-01 |

Table 6 (Cont'd.)

| LAG | AUTO COVARIANCE | RAW POWER SPECTRUM | SMOOTHED POWER SPECTRUM | PERIOD (MONTHS) | FREQUENCY LIMITS ( CYCLES/MONTH ) LOWER | UPPER | FREQUENCY ( CYCLES/ MONTH ) |
|---|---|---|---|---|---|---|---|
| 41 | -2.844E-03 | 3.354E-03 | 2.386E-03 | 3.512E 00 | 2.778E-01 | 2.917E-01 | 2.847E-01 |
| 42 | -2.445E-03 | 1.515E-03 | 2.175E-03 | 3.429E 00 | 2.847E-01 | 2.986E-01 | 2.917E-01 |
| 43 | -2.162E-03 | 2.545E-03 | 1.864E-03 | 3.349E 00 | 2.917E-01 | 3.056E-01 | 2.986E-01 |
| 44 | -1.839E-03 | 6.112E-04 | 1.581E-03 | 3.273E 00 | 2.986E-01 | 3.125E-01 | 3.056E-01 |
| 45 | -2.781E-04 | 2.894E-03 | 2.528E-03 | 3.200E 00 | 3.056E-01 | 3.194E-01 | 3.125E-01 |
| 46 | 1.338E-03 | 3.585E-03 | 2.885E-03 | 3.130E 00 | 3.125E-01 | 3.264E-01 | 3.194E-01 |
| 47 | 3.611E-03 | 1.233E-03 | 1.934E-03 | 3.064E 00 | 3.194E-01 | 3.333E-01 | 3.264E-01 |
| 48 | 3.737E-03 | 1.929E-03 | 1.613E-03 | 3.000E 00 | 3.264E-01 | 3.403E-01 | 3.333E-01 |
| 49 | 2.569E-03 | 1.250E-03 | 1.620E-03 | 2.939E 00 | 3.333E-01 | 3.472E-01 | 3.403E-01 |
| 50 | 1.040E-03 | 2.181E-03 | 1.904E-03 | 2.880E 00 | 3.403E-01 | 3.542E-01 | 3.472E-01 |
| 51 | -7.249E-04 | 1.909E-03 | 2.020E-03 | 2.824E 00 | 3.472E-01 | 3.611E-01 | 3.542E-01 |
| 52 | -2.147E-03 | 2.119E-03 | 1.654E-03 | 2.769E 00 | 3.542E-01 | 3.681E-01 | 3.611E-01 |
| 53 | -2.339E-03 | 3.076E-04 | 1.466E-03 | 2.717E 00 | 3.611E-01 | 3.750E-01 | 3.681E-01 |
| 54 | -2.487E-03 | 3.533E-03 | 2.727E-03 | 2.667E 00 | 3.681E-01 | 3.819E-01 | 3.750E-01 |
| 55 | -2.382E-03 | 3.251E-03 | 3.002E-03 | 2.618E 00 | 3.750E-01 | 3.889E-01 | 3.819E-01 |
| 56 | -1.560E-03 | 1.884E-03 | 2.226E-03 | 2.571E 00 | 3.819E-01 | 3.958E-01 | 3.889E-01 |
| 57 | -4.889E-04 | 2.003E-03 | 2.062E-03 | 2.526E 00 | 3.889E-01 | 4.028E-01 | 3.958E-01 |
| 58 | 9.761E-04 | 2.377E-03 | 2.432E-03 | 2.483E 00 | 3.958E-01 | 4.097E-01 | 4.028E-01 |
| 59 | 2.773E-03 | 2.988E-03 | 2.542E-03 | 2.441E 00 | 4.028E-01 | 4.167E-01 | 4.097E-01 |
| 60 | 3.410E-03 | 1.659E-03 | 1.875E-03 | 2.400E 00 | 4.097E-01 | 4.236E-01 | 4.167E-01 |
| 61 | 2.846E-03 | 1.269E-03 | 1.349E-03 | 2.361E 00 | 4.167E-01 | 4.306E-01 | 4.236E-01 |
| 62 | 6.544E-04 | 1.229E-03 | 1.409E-03 | 2.323E 00 | 4.236E-01 | 4.375E-01 | 4.306E-01 |
| 63 | -4.061E-04 | 1.974E-03 | 2.082E-03 | 2.286E 00 | 4.306E-01 | 4.444E-01 | 4.375E-01 |
| 64 | -1.896E-03 | 3.188E-03 | 2.303E-03 | 2.250E 00 | 4.375E-01 | 4.514E-01 | 4.444E-01 |
| 65 | -2.773E-03 | 5.556E-04 | 1.234E-03 | 2.215E 00 | 4.444E-01 | 4.583E-01 | 4.514E-01 |
| 66 | -2.539E-03 | 8.728E-04 | 1.109E-03 | 2.182E 00 | 4.514E-01 | 4.653E-01 | 4.583E-01 |
| 67 | -2.222E-03 | 2.216E-03 | 2.124E-03 | 2.149E 00 | 4.583E-01 | 4.722E-01 | 4.653E-01 |
| 68 | -1.874E-03 | 3.158E-03 | 2.542E-03 | 2.118E 00 | 4.653E-01 | 4.792E-01 | 4.722E-01 |
| 69 | -2.738E-04 | 1.421E-03 | 1.761E-03 | 2.087E 00 | 4.722E-01 | 4.861E-01 | 4.792E-01 |
| 70 | 1.576E-03 | 1.162E-03 | 1.659E-03 | 2.057E 00 | 4.792E-01 | 4.931E-01 | 4.861E-01 |
| 71 | 2.933E-03 | 3.061E-03 | 2.480E-03 | 2.028E 00 | 4.861E-01 | 5.000E-01 | 4.931E-01 |
| 72 | 3.643E-03 | 2.432E-03 | 2.722E-03 | 2.000E 00 | 4.931E-01 | 5.069E-01 | 5.000E-01 |

## Table 7

## AUTOCOVARIANCE AND SPECTRAL DENSITY ESTIMATES FOR DIFFERENCES IN ELEVATION LESS TRENDS AND PERIODICITIES

### MARQUETTE MINUS DULUTH

| LAG | AUTO COVARIANCE | RAW POWER SPECTRUM | SMOOTHED POWER SPECTRUM | PERIOD (MONTHS) | FREQUENCY LIMITS ( CYCLES/MONTH ) |  | FREQUENCY ( CYCLES/ MONTH ) |
|---|---|---|---|---|---|---|---|
|  |  |  |  |  | LOWER | UPPER |  |
| 0 | 1.001E 00 | 1.318E 00 | 1.403E 00 | 9.960E 02 | -6.944E-03 | 6.944E-03 | 0 |
| 1 | 1.810E-01 | 1.502E 00 | 1.282E 00 | 1.440E 02 | 0 | 1.389E-02 | 6.944E-03 |
| 2 | 1.175E-01 | 7.265E-01 | 8.252E-01 | 7.200E 01 | 6.944E-03 | 2.083E-02 | 1.389E-02 |
| 3 | 1.308E-01 | 3.799E-01 | 4.954E-01 | 4.800E 01 | 1.389E-02 | 2.778E-02 | 2.083E-02 |
| 4 | 1.158E-01 | 5.356E-01 | 4.451E-01 | 3.600E 01 | 2.083E-02 | 3.472E-02 | 2.778E-02 |
| 5 | 9.765E-02 | 2.977E-01 | 3.843E-01 | 2.880E 01 | 2.778E-02 | 4.167E-02 | 3.472E-02 |
| 6 | 5.018E-02 | 4.365E-01 | 3.991E-01 | 2.400E 01 | 3.472E-02 | 4.861E-02 | 4.167E-02 |
| 7 | 4.067E-02 | 4.127E-01 | 4.235E-01 | 2.057E 01 | 4.167E-02 | 5.556E-02 | 4.861E-02 |
| 8 | 9.485E-02 | 4.359E-01 | 4.025E-01 | 1.800E 01 | 4.861E-02 | 6.250E-02 | 5.556E-02 |
| 9 | 3.436E-02 | 3.139E-01 | 3.151E-01 | 1.600E 01 | 5.556E-02 | 6.944E-02 | 6.250E-02 |
| 10 | 8.030E-02 | 1.971E-01 | 3.632E-01 | 1.440E 01 | 6.250E-02 | 7.639E-02 | 6.944E-02 |
| 11 | 6.291E-02 | 8.024E-01 | 5.870E-01 | 1.309E 01 | 6.944E-02 | 8.333E-02 | 7.639E-02 |
| 12 | 8.595E-02 | 4.713E-01 | 5.141E-01 | 1.200E 01 | 7.639E-02 | 9.028E-02 | 8.333E-02 |
| 13 | 1.021E-01 | 3.264E-01 | 3.231E-01 | 1.108E 01 | 8.333E-02 | 9.722E-02 | 9.028E-02 |
| 14 | 5.955E-02 | 1.669E-01 | 2.071E-01 | 1.029E 01 | 9.028E-02 | 1.042E-01 | 9.722E-02 |
| 15 | 6.202E-02 | 1.820E-01 | 2.336E-01 | 9.600E 00 | 9.722E-02 | 1.111E-01 | 1.042E-01 |
| 16 | 7.960E-02 | 4.212E-01 | 3.053E-01 | 9.000E 00 | 1.042E-01 | 1.181E-01 | 1.111E-01 |
| 17 | 5.163E-02 | 1.563E-01 | 2.465E-01 | 8.471E 00 | 1.111E-01 | 1.250E-01 | 1.181E-01 |
| 18 | 2.705E-02 | 2.835E-01 | 2.678E-01 | 8.000E 00 | 1.181E-01 | 1.319E-01 | 1.250E-01 |
| 19 | 1.971E-02 | 3.426E-01 | 3.137E-01 | 7.579E 00 | 1.250E-01 | 1.389E-01 | 1.319E-01 |
| 20 | -7.700E-04 | 2.759E-01 | 2.829E-01 | 7.200E 00 | 1.319E-01 | 1.458E-01 | 1.389E-01 |
| 21 | 4.940E-02 | 2.397E-01 | 2.310E-01 | 6.857E 00 | 1.389E-01 | 1.528E-01 | 1.458E-01 |
| 22 | 5.698E-02 | 1.658E-01 | 2.210E-01 | 6.545E 00 | 1.458E-01 | 1.597E-01 | 1.528E-01 |
| 23 | 4.808E-02 | 3.320E-01 | 2.733E-01 | 6.261E 00 | 1.528E-01 | 1.667E-01 | 1.597E-01 |
| 24 | 4.538E-02 | 2.428E-01 | 2.752E-01 | 6.000E 00 | 1.597E-01 | 1.736E-01 | 1.667E-01 |
| 25 | 7.904E-02 | 2.945E-01 | 2.700E-01 | 5.760E 00 | 1.667E-01 | 1.806E-01 | 1.736E-01 |
| 26 | 3.904E-02 | 2.395E-01 | 2.758E-01 | 5.538E 00 | 1.736E-01 | 1.875E-01 | 1.806E-01 |
| 27 | 5.652E-02 | 3.422E-01 | 3.188E-01 | 5.333E 00 | 1.806E-01 | 1.944E-01 | 1.875E-01 |
| 28 | 1.643E-02 | 3.432E-01 | 2.983E-01 | 5.143E 00 | 1.875E-01 | 2.014E-01 | 1.944E-01 |
| 29 | -7.281E-03 | 1.488E-01 | 2.230E-01 | 4.966E 00 | 1.944E-01 | 2.083E-01 | 2.014E-01 |
| 30 | 8.506E-04 | 2.768E-01 | 2.465E-01 | 4.800E 00 | 2.014E-01 | 2.153E-01 | 2.083E-01 |
| 31 | 1.114E-02 | 2.731E-01 | 2.534E-01 | 4.645E 00 | 2.083E-01 | 2.222E-01 | 2.153E-01 |
| 32 | -1.190E-02 | 1.838E-01 | 2.657E-01 | 4.500E 00 | 2.153E-01 | 2.292E-01 | 2.222E-01 |
| 33 | -6.336E-02 | 4.505E-01 | 4.023E-01 | 4.364E 00 | 2.222E-01 | 2.361E-01 | 2.292E-01 |
| 34 | 3.449E-02 | 5.076E-01 | 4.167E-01 | 4.235E 00 | 2.292E-01 | 2.431E-01 | 2.361E-01 |
| 35 | 2.205E-02 | 1.693E-01 | 2.741E-01 | 4.114E 00 | 2.361E-01 | 2.500E-01 | 2.431E-01 |
| 36 | 3.622E-02 | 2.867E-01 | 2.916E-01 | 4.000E 00 | 2.431E-01 | 2.569E-01 | 2.500E-01 |
| 37 | 2.478E-02 | 4.251E-01 | 3.694E-01 | 3.892E 00 | 2.500E-01 | 2.639E-01 | 2.569E-01 |
| 38 | 7.226E-02 | 3.212E-01 | 3.200E-01 | 3.789E 00 | 2.569E-01 | 2.708E-01 | 2.639E-01 |
| 39 | 3.627E-02 | 2.119E-01 | 2.256E-01 | 3.692E 00 | 2.639E-01 | 2.778E-01 | 2.708E-01 |
| 40 | -5.121E-03 | 1.622E-01 | 2.446E-01 | 3.600E 00 | 2.708E-01 | 2.847E-01 | 2.778E-01 |

## Table 7 (Cont'd.)

| LAG | AUTO COVARIANCE | RAW POWER SPECTRUM | SMOOTHED POWER SPECTRUM | PERIOD (MONTHS) | FREQUENCY LIMITS ( CYCLES/MONTH ) |  | FREQUENCY ( CYCLES/ MONTH ) |
|---|---|---|---|---|---|---|---|
|  |  |  |  |  | LOWER | UPPER |  |
| 41 | -5.234E-02 | 4.707E-01 | 3.283E-01 | 3.512E 00 | 2.778E-01 | 2.917E-01 | 2.847E-01 |
| 42 | 2.856E-02 | 1.599E-01 | 2.708E-01 | 3.429E 00 | 2.847E-01 | 2.986E-01 | 2.917E-01 |
| 43 | 9.146E-03 | 3.311E-01 | 2.389E-01 | 3.349E 00 | 2.917E-01 | 3.056E-01 | 2.986E-01 |
| 44 | -5.215E-02 | 1.015E-01 | 2.082E-01 | 3.273E 00 | 2.986E-01 | 3.125E-01 | 3.056E-01 |
| 45 | -2.232E-03 | 3.356E-01 | 2.782E-01 | 3.200E 00 | 3.056E-01 | 3.194E-01 | 3.125E-01 |
| 46 | -4.680E-02 | 3.201E-01 | 2.996E-01 | 3.130E 00 | 3.125E-01 | 3.264E-01 | 3.194E-01 |
| 47 | 2.468E-02 | 2.155E-01 | 2.602E-01 | 3.064E 00 | 3.194E-01 | 3.333E-01 | 3.264E-01 |
| 48 | -1.214E-02 | 3.054E-01 | 2.401E-01 | 3.000E 00 | 3.264E-01 | 3.403E-01 | 3.333E-01 |
| 49 | -5.635E-02 | 1.113E-01 | 1.710E-01 | 2.939E 00 | 3.333E-01 | 3.472E-01 | 3.403E-01 |
| 50 | -7.706E-03 | 1.767E-01 | 1.748E-01 | 2.880E 00 | 3.403E-01 | 3.542E-01 | 3.472E-01 |
| 51 | 5.292E-03 | 2.341E-01 | 2.589E-01 | 2.824E 00 | 3.472E-01 | 3.611E-01 | 3.542E-01 |
| 52 | -4.452E-03 | 3.992E-01 | 2.894E-01 | 2.769E 00 | 3.542E-01 | 3.681E-01 | 3.611E-01 |
| 53 | 1.656E-02 | 8.713E-02 | 2.502E-01 | 2.717E 00 | 3.611E-01 | 3.750E-01 | 3.681E-01 |
| 54 | -1.641E-02 | 4.841E-01 | 3.614E-01 | 2.667E 00 | 3.681E-01 | 3.819E-01 | 3.750E-01 |
| 55 | -2.820E-02 | 3.476E-01 | 3.446E-01 | 2.618E 00 | 3.750E-01 | 3.889E-01 | 3.819E-01 |
| 56 | 3.647E-03 | 1.982E-01 | 2.668E-01 | 2.571E 00 | 3.819E-01 | 3.958E-01 | 3.889E-01 |
| 57 | -2.763E-02 | 3.470E-01 | 2.975E-01 | 2.526E 00 | 3.889E-01 | 4.028E-01 | 3.958E-01 |
| 58 | -7.809E-02 | 2.806E-01 | 3.017E-01 | 2.483E 00 | 3.958E-01 | 4.097E-01 | 4.028E-01 |
| 59 | -5.239E-02 | 3.061E-01 | 2.688E-01 | 2.441E 00 | 4.028E-01 | 4.167E-01 | 4.097E-01 |
| 60 | -6.900E-02 | 1.693E-01 | 2.244E-01 | 2.400E 00 | 4.097E-01 | 4.236E-01 | 4.167E-01 |
| 61 | 1.375E-02 | 2.721E-01 | 2.223E-01 | 2.361E 00 | 4.167E-01 | 4.306E-01 | 4.236E-01 |
| 62 | -7.626E-02 | 1.583E-01 | 1.791E-01 | 2.323E 00 | 4.236E-01 | 4.375E-01 | 4.306E-01 |
| 63 | 6.179E-02 | 1.348E-01 | 2.016E-01 | 2.286E 00 | 4.306E-01 | 4.444E-01 | 4.375E-01 |
| 64 | 1.532E-02 | 4.017E-01 | 2.611E-01 | 2.250E 00 | 4.375E-01 | 4.514E-01 | 4.444E-01 |
| 65 | -3.096E-02 | 5.733E-02 | 1.596E-01 | 2.215E 00 | 4.444E-01 | 4.583E-01 | 4.514E-01 |
| 66 | 1.416E-03 | 1.576E-01 | 1.368E-01 | 2.182E 00 | 4.514E-01 | 4.653E-01 | 4.583E-01 |
| 67 | 2.733E-02 | 1.675E-01 | 2.131E-01 | 2.149E 00 | 4.583E-01 | 4.722E-01 | 4.653E-01 |
| 68 | -3.053E-02 | 3.757E-01 | 3.074E-01 | 2.118E 00 | 4.653E-01 | 4.792E-01 | 4.722E-01 |
| 69 | -2.095E-03 | 2.871E-01 | 2.759E-01 | 2.087E 00 | 4.722E-01 | 4.861E-01 | 4.792E-01 |
| 70 | 4.546E-03 | 1.500E-01 | 2.357E-01 | 2.057E 00 | 4.792E-01 | 4.931E-01 | 4.861E-01 |
| 71 | -2.068E-02 | 3.856E-01 | 2.750E-01 | 2.028E 00 | 4.861E-01 | 5.000E-01 | 4.931E-01 |
| 72 | -1.948E-02 | 1.402E-01 | 2.531E-01 | 2.000E 00 | 4.931E-01 | 5.069E-01 | 5.000E-01 |

Table 8

## RESULTS OF VARIATE DIFFERENCE TECHNIQUE ON ORIGINAL DIFFERENCES IN ELEVATION

MARQUETTE MINUS DULUTH

| DIFFERENCE NUMBER R | MEAN OF DIFFERENCES | VARIANCE OF DIFFERENCES | VARIANCE DIVIDED BY $\binom{2R}{R}$ |
|---|---|---|---|
| 1 | -.29215-03 | 19819-01 | .99097-02 |
| 2 | .33417-04 | .54241-01 | .90402-02 |
| 3 | -.20903-03 | .17521-00 | .87603-02 |
| 4 | .10042-03 | .60356-00 | .86224-02 |
| 5 | -.27638-03 | .21508+01 | .85350-02 |
| 6 | -.92205-04 | .78273+01 | .84711-02 |
| 7 | .13674-02 | .28898+02 | .84203-02 |
| 8 | -.22166-02 | .10782+03 | .83776-02 |
| 9 | .11059-01 | .40553+03 | .83408-02 |
| 10 | -.12380-01 | .15353+04 | .83098-02 |

VARIANCE OF ORIGINAL SERIES IS .26451-01

Table 9

RESULTS OF SIMPLE LINEAR REGRESSION ON ORIGINAL DIFFERENCES IN ELEVATION
MARQUETTE MINUS DULUTH

| | |
|---|---|
| LENGTH OF CONTINUOUS DIFFERENCES IS - | 111 YEARS |
| FIRST YEAR OF JOINT PERIOD OF RECORD IS - | 1860 |
| LONG-TERM MEAN DIFFERENCE IN LEVELS IS - | .16 FEET |
| SLOPE OF FIRST ORDER LINEAR TREND IS - | -0.0036 FEET PER YEAR |
| INTERCEPT OF TREND IS - | .3748 FEET |
| CORRELATION COEFFICIENT RELATING MONTHLY ELEVATION DIFFERENCES TO TIME IS - | -0.69 |
| RATE OF RELATIVE MOVEMENT IS - | -0.3597 FEET PER 100 YEARS |

Table 10

## RESULTS OF POLYNOMIAL REGRESSION ON ORIGINAL DIFFERENCES IN ELEVATION MARQUETTE MINUS DULUTH

```
NUMBER OF OBSERVATIONS  1211

                                   POLYNOMIAL REGRESSION OF NUMBER  1

INTERCEPT                                  7.061132

REGRESSION COEFFICIENT  1                 -.003597

MULTIPLE CORRELATION COEFFICIENT           .685638

STANDARD ERROR OF ESTIMATE                 .119165

                              ANALYSIS OF VARIANCE FOR   1  DEGREE POLYNOMIAL

  SOURCE OF VARIATION        DEGREE OF      SUM OF       MEAN          F          IMPROVEMENT IN TERMS
                              FREEDOM       SQUARES      SQUARE        VALUE       OF SUM OF SQUARES

DUE TO REGRESSION                1         15.23079     15.23079    1072.56212          15.23079
DEVIATION ABOUT REGRESSION    1209         17.16826       .01420
TOTAL                         1210         32.39904



                                   POLYNOMIAL REGRESSION OF NUMBER  2

INTERCEPT                                -51.870746

REGRESSION COEFFICIENT  1                   .057919

REGRESSION COEFFICIENT  2                 -.000016

MULTIPLE CORRELATION COEFFICIENT            .691840

STANDARD ERROR OF ESTIMATE                  .118250

                              ANALYSIS OF VARIANCE FOR   2  DEGREE POLYNOMIAL

  SOURCE OF VARIATION        DEGREE OF      SUM OF       MEAN          F          IMPROVEMENT IN TERMS
                              FREEDOM       SQUARES      SQUARE        VALUE       OF SUM OF SQUARES

DUE TO REGRESSION                2         15.50758      7.75379     554.51542            .27679
DEVIATION ABOUT REGRESSION    1208         16.89146       .01398
TOTAL                         1210         32.39904
```

Table 10 (Cont'd.)

## POLYNOMIAL REGRESSION OF NUMBER 3

| | |
|---|---|
| INTERCEPT | 1228.332992 |
| REGRESSION COEFFICIENT 1 | -1.947720 |
| REGRESSION COEFFICIENT 2 | .001031 |
| REGRESSION COEFFICIENT 3 | -.000000 |
| MULTIPLE CORRELATION COEFFICIENT | .692450 |
| STANDARD ERROR OF ESTIMATE | .118203 |

### ANALYSIS OF VARIANCE FOR 3 DEGREE POLYNOMIAL

| SOURCE OF VARIATION | DEGREE OF FREEDOM | SUM OF SQUARES | MEAN SQUARE | F VALUE | IMPROVEMENT IN TERMS OF SUM OF SQUARES |
|---|---|---|---|---|---|
| DUE TO REGRESSION | 3 | 15.53494 | 5.17831 | 370.62284 | .02736 |
| DEVIATION ABOUT REGRESSION | 1207 | 16.86411 | .01397 | | |
| TOTAL | 1210 | 32.39904 | | | |

## POLYNOMIAL REGRESSION OF NUMBER 4

| | |
|---|---|
| INTERCEPT | 27498.257156 |
| REGRESSION COEFFICIENT 1 | -56.801593 |
| REGRESSION COEFFICIENT 2 | .043978 |
| REGRESSION COEFFICIENT 3 | -.000015 |
| REGRESSION COEFFICIENT 4 | .000000 |
| MULTIPLE CORRELATION COEFFICIENT | .692510 |
| STANDARD ERROR OF ESTIMATE | .118242 |

### ANALYSIS OF VARIANCE FOR 4 DEGREE POLYNOMIAL

| SOURCE OF VARIATION | DEGREE OF FREEDOM | SUM OF SQUARES | MEAN SQUARE | F VALUE | IMPROVEMENT IN TERMS OF SUM OF SQUARES |
|---|---|---|---|---|---|
| DUE TO REGRESSION | 4 | 15.53762 | 3.88441 | 277.82913 | .00269 |
| DEVIATION ABOUT REGRESSION | 1206 | 16.86142 | .01398 | | |
| TOTAL | 1210 | 32.39904 | | | |

## PLOTS OF AUTOCORRELATION COFFICIENTS UP TO LAG 72 FOR DIFFERENCES IN MEAN MONTHLY ELEVATIONS, MARQUETTE MINUS DULUTH, SHOWING 95% CONFIDENCE LIMITS

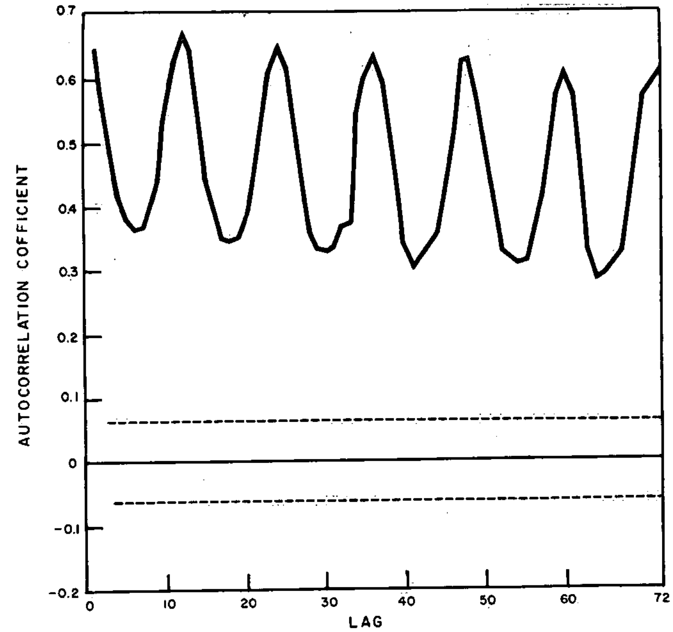


FIGURE 4. CORRELOGRAM OF ORIGINAL TIME SERIES

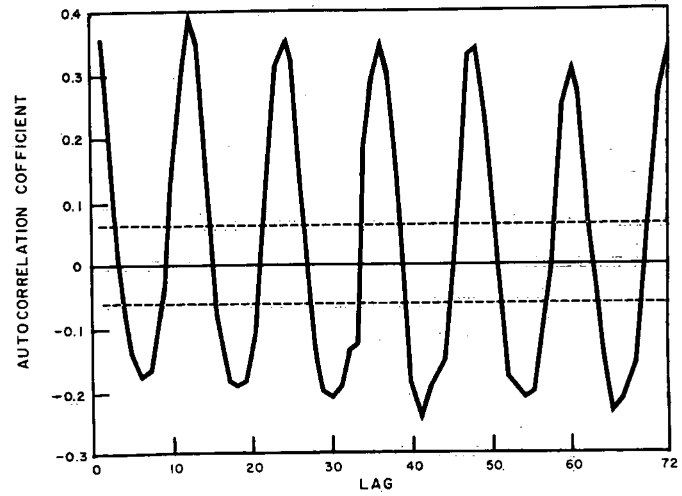


FIGURE 5. CORRELOGRAM OF TIME SERIES WITH LINEAR TRENDS REMOVED

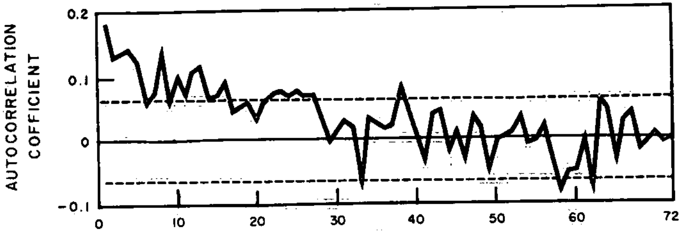


FIGURE 6. CORRELOGRAM OF TIME SERIES LESS LINEAR TRENDS LESS PERIODIC COMPONENTS

# SPECTRAL ANALYSIS OF DIFFERENCES IN MEAN MONTHLY ELEVATION, MARQUETTE MINUS DULUTH, SHOWING UPPER 95 % CONFIDENCE LIMIT

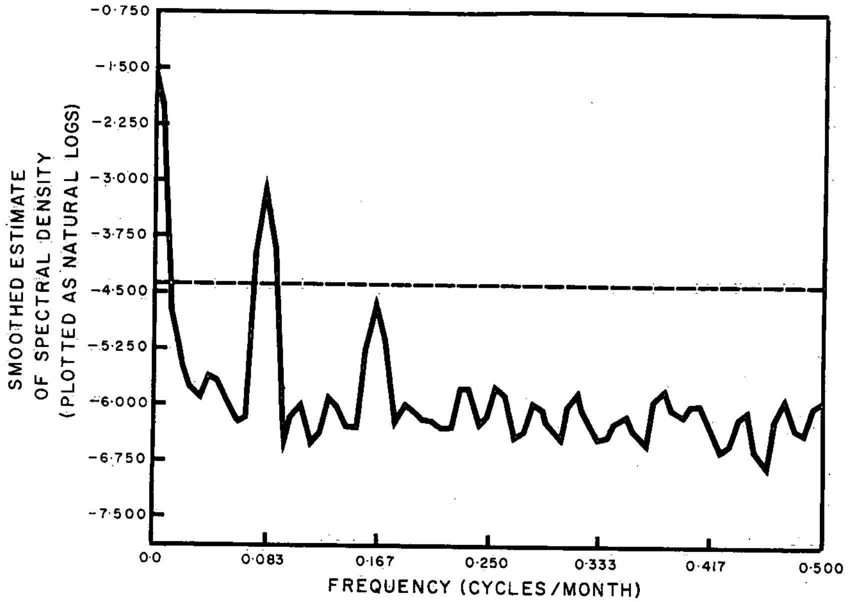


FIGURE 7. SPECTRAL ESTIMATE OF ORIGINAL TIME SERIES.

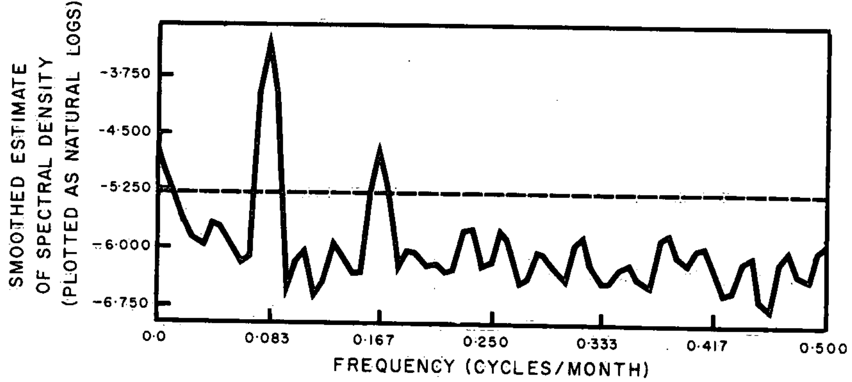


FIGURE 8. SPECTRAL ESTIMATE OF TIME SERIES WITH LINEAR TRENDS REMOVED.

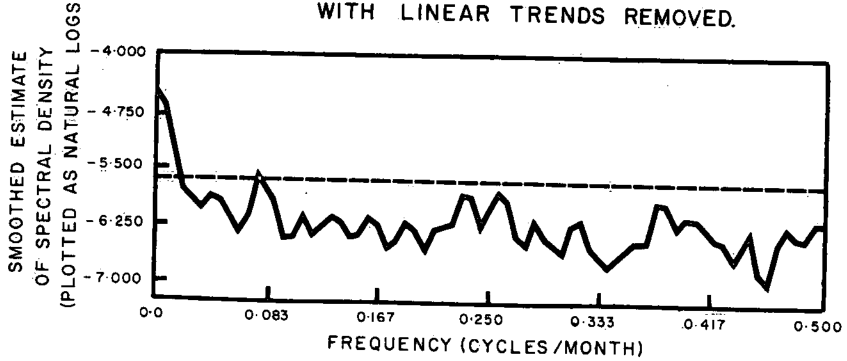
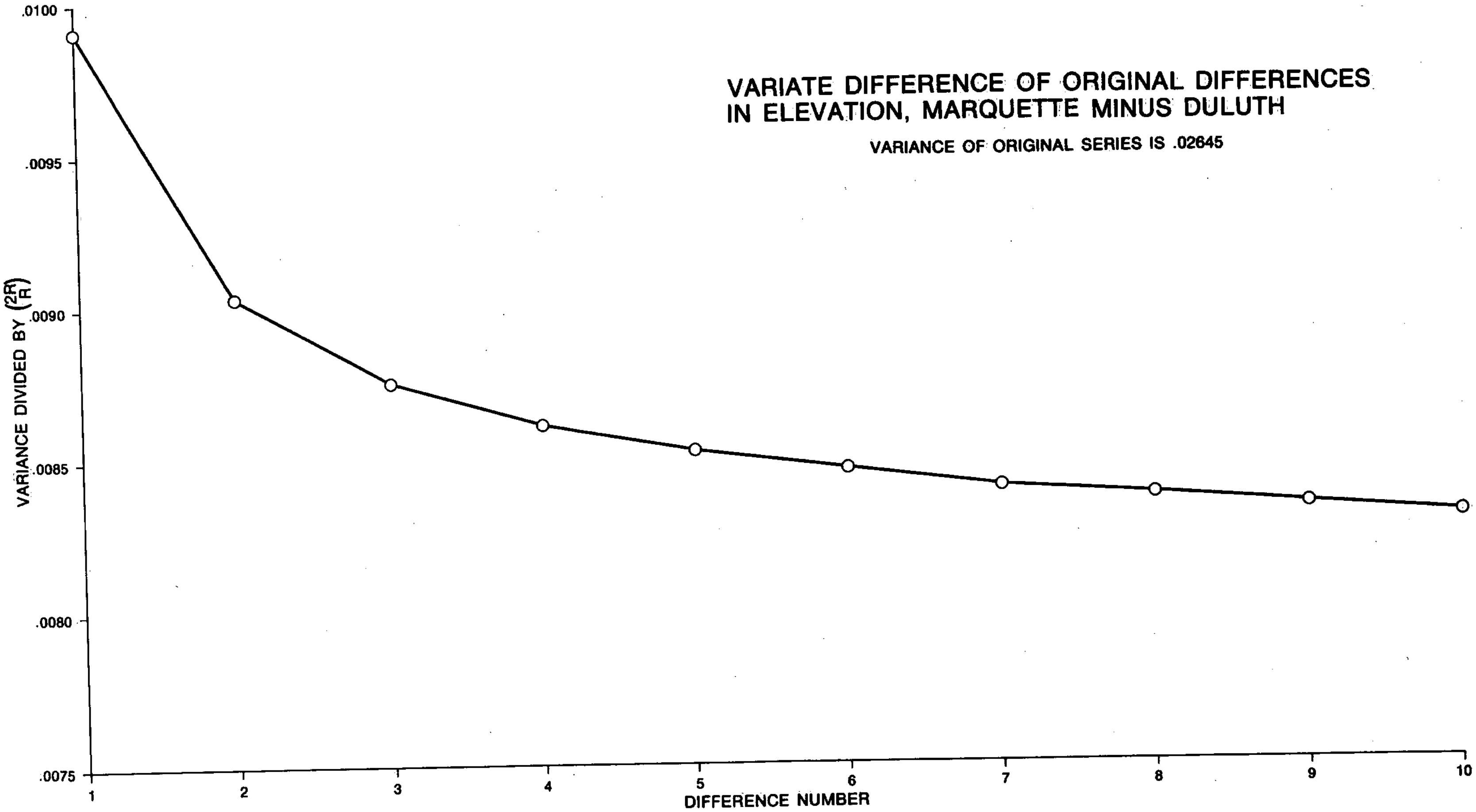


FIGURE 9. SPECTRAL ESTIMATE OF TIME SERIES LESS LINEAR TRENDS LESS PERIODIC COMPONENT. (ADJUSTED TO ORIGINAL VARIANCE)



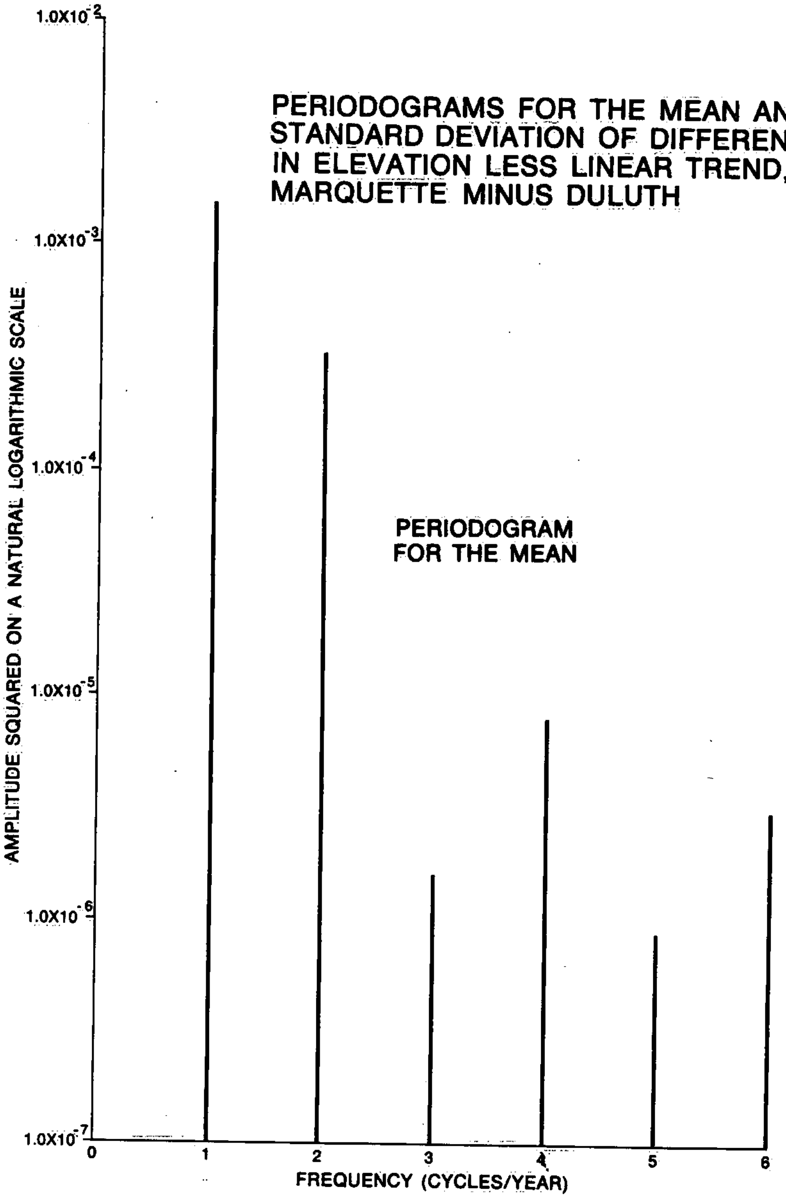
VARIATE DIFFERENCE OF ORIGINAL DIFFERENCES IN ELEVATION, MARQUETTE MINUS DULUTH

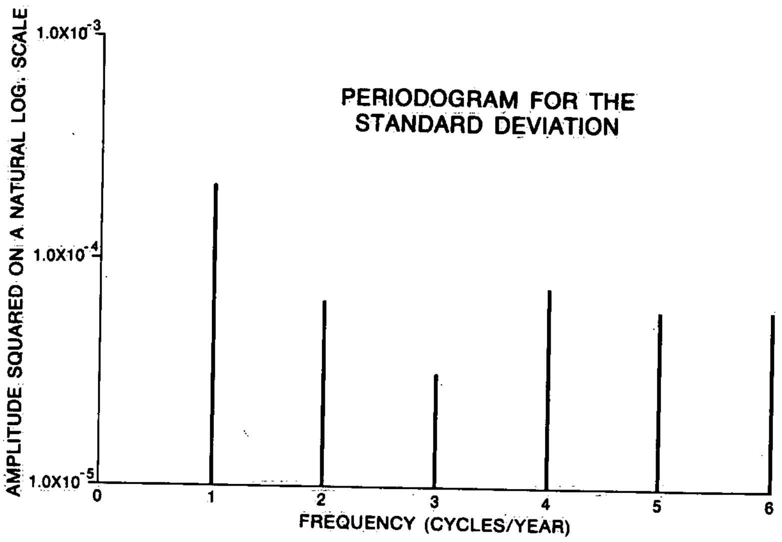
VARIANCE OF ORIGINAL SERIES IS .02645

FIGURE 10

FIGURE 11

PERIODOGRAMS FOR THE MEAN AND STANDARD DEVIATION OF DIFFERENCES IN ELEVATION LESS LINEAR TREND, MARQUETTE MINUS DULUTH





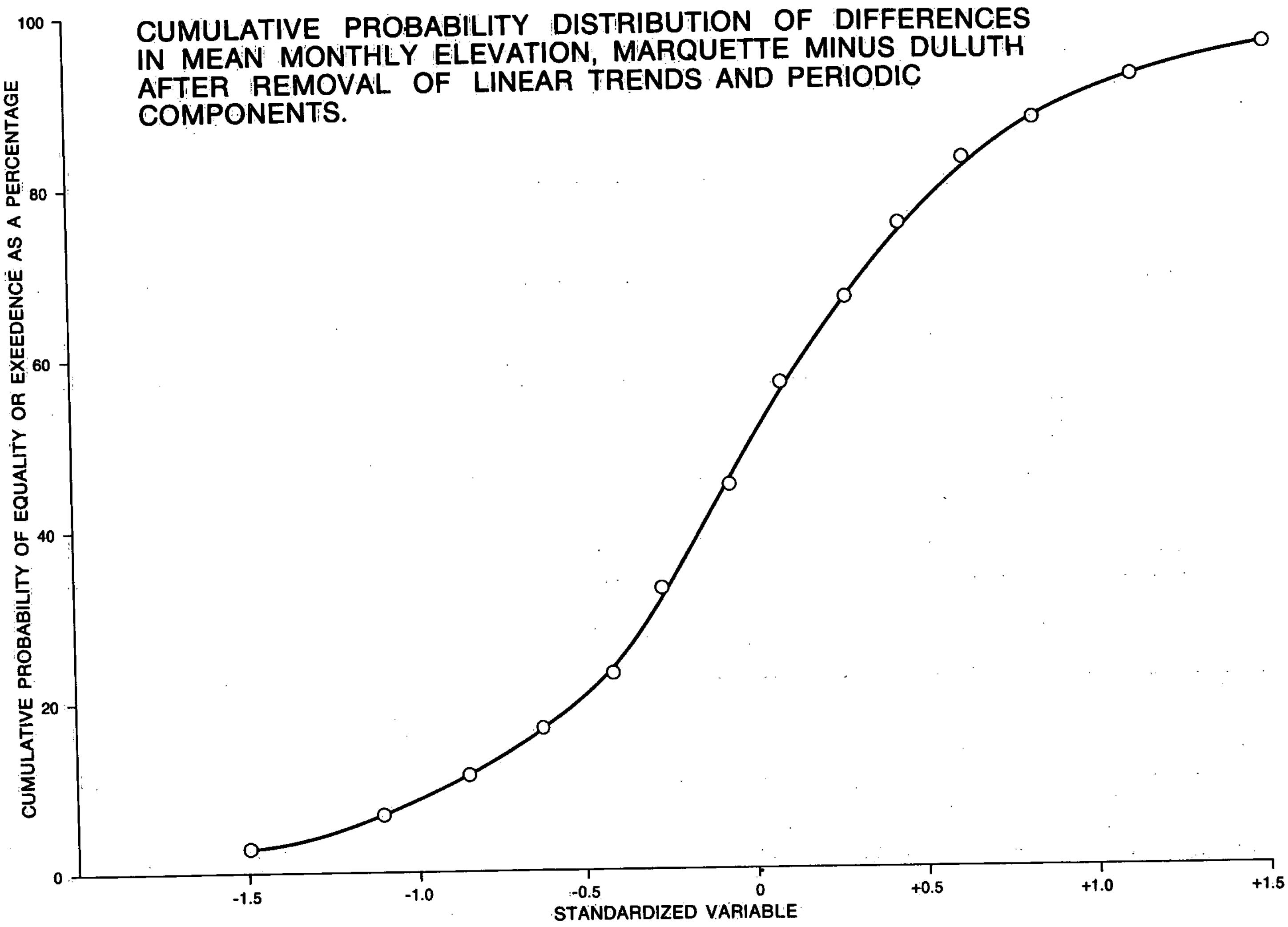

CUMULATIVE PROBABILITY DISTRIBUTION OF DIFFERENCES IN MEAN MONTHLY ELEVATION, MARQUETTE MINUS DULUTH AFTER REMOVAL OF LINEAR TRENDS AND PERIODIC COMPONENTS.

FIGURE 12

first regression variable, and
$r_1'$ is the first serial correlation coefficient for the second regression variable, etc.

Since the gauge differences used in the regression were computed at equal time intervals of one month, the values of $r_1$, $r_2$, $r_3$ ...... $r_n$ for the time variable are all unity, which explains the drastic difference between raw data record length and effective record length. Because the yearly cycle obviously dominates the correlogram of raw data, Figure 4, the value of n, used to determine the effective data length, was taken as 12. If n had been taken as 100, the number of lags used to compute serial correlations, then the effective length would be reduced to 11 units. This could obviously be continued until the effective data length is reduced to zero; therefore, for this purpose, one annual cycle of serial correlations is considered to be representative. The results of the three tests on the correlation coefficient; approximate normal function test, Student t test and Fisher Z test; are given in Table 12.

Table 11 includes the analysis of variance results for each step of the computations. It can be seen that the first-order linear regression is highly significant since the computed F value is 1072 compared to a tabulated F (1, 1198, 0.95) of 3.85.

Due to the 100 per cent intercorrelation between the powers of time and to the slightly increasing correlation between the differences in elevation and $(\text{time})^n$ as n increases (as shown in the polynomial regression results), the stepwise program selected $(\text{time})^4$ as the most significant variable and discarded lower powers as contributing nothing additional to the explained variance. While this is correct, the additional explained variance due to using higher powers of time is offset by the extra complications involved in both computation and interpretation of results. The output from the stepwise regression program is given in Table 11. The original data with the computed linear trends removed are listed in Table 3 and plotted on Figure 2 while the corresponding autocovariance and spectral analyses are given in Table 6 and Figures 5 and 8.

With the linear trends removed from the time series the next step was to determine whether or not a significant proportion of the variance could be explained by periodicity. Looking at the long-term monthly means of the differences in gauge elevations, Marquette minus Duluth, after the linear trends have been removed (Table

13) there is a very apparent annual cycle. In the winter months of November, December, January, February and March Marquette is higher than Duluth while in the summer months this is reversed, Duluth being higher than Marquette. This could be due to the pattern of ice formation in Lake Superior, Duluth being well within the firm pack ice while Marquette is on the fringe of the permanent open water. This may be an indication of seasonal wind patterns. It is also significant that the long-term monthly standard deviations are higher in the winter months than in the summer months, indicating a lower accuracy of winter measurements or increased random component.

To examine this apparent periodicity more closely a form of harmonic analysis of the mean and variance of the time series was used. This shows (Table 14) that 63 percent of the variance of the time series can be explained by harmonics in the mean and standard deviation. Of the variance in the mean 83 percent is explained by the annual cycle. Note that on Figure 11 the squared amplitudes are plotted on a logarithmic scale. The periodicities in the mean and standard deviation were then removed by standardizing the data on a monthly basis. This process succesfully removed all of the annual cycle and its harmonics. Table 4 and Figure 3 show the data for Marquette minus Duluth after removal of linear trends and periodic components.

Figures 6 and 9 and Table 7 show the correlogram and spectral analyses of the differences in elevation after removal of all linear trends and all periodicities of 12 months or less. It is apparent from these figures that some trend still remains, probably non-linear. It is also interesting to note that all three spectrum plots, Figures 7,8 and 9, indicate the possibility of a 2 year cycle at a low level of significance. Note that on Figure 9 the spectral densities have been adjusted so that the total variance of the time series remains as it was before standardization. This is to ensure a continuity of scale on Figures 7,8 and 9.

The set of residuals was then checked against the normal and log-normal distributions using Chi-square and Kolmogorov-Smirnov tests (Tables 15, 16 and 17) and then tested for independence (Table 20). The results indicated that the residuals are neither normally distributed nor log-normally distributed. Further tests (Tables 18,19) indicated that no simple linear Markov models were present. Figure 12 shows the cumulative probability distribution of this residual component.

In summary the variance of the original time series made up of differences in mean monthly elevation, Marquette minus Duluth, was found to be made up shown in Table 21.

In view of this analysis, and assuming that other time series of differences in mean monthly elevations around Lake Superior will have a similar make-up, it was decided that, for the purpose of this report, the first-order linear trend is acceptable as being the most important component of the time series. For the other time series, then, a regression was run on the original time series to determine the first order linear trends. In only three cases, Keweenaw L.E. with respect to Marquette, Houghton with respect to Marquette and Two Harbours with respect to Duluth, was it determined that the first order linear trend was not significant. The first two cases were expected and the importance of the third case will be discussed later.

At this stage it would be statistically very easy to compute confidence limits on the simple linear regression coefficients (rates of crustal movement) from the available least squares analysis. This is not done because, as discussed below, the computed rates of movement are to be adjusted and the adjustment will depend not only upon the confidence of the correlation but upon the length of record used.

## II) Adjustment of Relative Rates of Movement

### (a) Triangulation

Following the brief statistical analysis each pair of gauging stations had a computed rate of relative movement between them. Due to the differences in the periods of record common to each pair of gauges, these rates of movement are not in harmony with each other.

The relative rates of movement were therefore adjusted using a least squares triangulation technique in which each rate of movement between two points was assigned a "weight" or measure of relative reliability. These weights were computed for each rate of movement as $R^2N$ where R is the simple linear correlation coefficient relating differences in mean monthly elevations between two gauges to time and N is the number of months of record common to both stations. In this way the applied weight increases as the proportion of the variance of the time series is explained by the linear trend (as measured by the coefficient of determination, $R^2$) and increases as

## Table 11

## RESULTS OF STEP-WISE POLYNOMIAL REGRESSION ON ORIGINAL DIFFERENCES IN ELEVATION

## MARQUETTE MINUS DULUTH

---

```
SUB PROBLEM     1
 DEPENDENT VARIABLE           5
 MAXIMUM NUMBER OF STEPS     10
 F-LEVEL FOR INCLUSION    0.010000
 F-LEVEL FOR DELETION     0.005000
 TOLERANCE LEVEL          0.001000


  STEP NUMBER     1
  VARIABLE ENTERED     4

  MULTIPLE R              0.6807
  STD. ERROR OF EST.      0.1192

  ANALYSIS OF VARIANCE

                         DF    SUM OF SQUARES    MEAN SQUARE     F RATIO
          REGRESSION      1        14.690           14.690      1033.322
          RESIDUAL     1197        17.017            0.014

                    VARIABLES IN EQUATION

   VARIABLE       COEFFICIENT   STD. ERROR    F TO REMOVE

   (CONSTANT        1.88478)
   YR**4   4      -0.00000        0.00000       1033.3218 (9)

                    VARIABLES NOT IN EQUATION

   VARIABLE       PARTIAL CORR.     TOLERANCE      F TO ENTER

   YEAR    1         0.10464          0.0005        13.2409 (9)
   YR**2   2         0.10303          0.0002        12.8310 (9)
   YR**3   3         0.10524          0.0001        13.3949 (9)

 F-LEVEL OR TOLERANCE INSUFFICIENT FOR FURTHER COMPUTATION
```

---

## Table 12

## RESULTS OF TESTS ON THE CORRELATION COEFFICIENT BETWEEN DIFFERENCES IN ELEVATION AND TIME

### MARQUETTE MINUS DULUTH

| | |
|---|---|
| COMPUTED VALUE OF CORRELATION COEFFICIENT "r" | 0.6856 |
| NUMBER OF DATA POINTS | 1211 |
| EFFECTIVE NUMBER OF DATA POINTS | 94 |

| TEST | RESULTS |
|---|---|
| APPROXIMATE NORMAL | $r>0.0$ |
| STUDENT t, P = 99% | $r>0.0$ |
| FISHER'S Z TRANSFORM, P = 99% | $0.5<\rho<0.8$ |

## Table 13

## MONTHLY MEANS AND STANDARD DEVIATIONS FOR DIFFERENCES IN ELEVATION LESS LINEAR TRENDS

### MARQUETTE MINUS DULUTH

| Month | Long Term Monthly Mean | Long Term Monthly Standard Deviation |
|---|---|---|
| January | 0.1129 | 0.1346 |
| February | 0.0849 | 0.0691 |
| March | 0.0078 | 0.1251 |
| April | -0.0494 | 0.0491 |
| May | -0.0636 | 0.0612 |
| June | -0.0636 | 0.0483 |
| July | -0.0326 | 0.0806 |
| August | -0.0358 | 0.0633 |
| September | -0.0393 | 0.0686 |
| October | -0.0132 | 0.0609 |
| November | 0.0451 | 0.0601 |
| December | 0.0862 | 0.1365 |

## Table 14

## RESULTS OF PERIODIC ANALYSIS OF DIFFERENCES IN ELEVATION LESS LINEAR TRENDS

### MARQUETTE MINUS DULUTH

| | |
|---|---|
| TOTAL VARIANCE OF THE PERIODIC SERIES | .1088-01 |
| VARIANCE OF THE MEANS | .3662-02 |
| VARIANCE OF THE STANDARD DEVIATIONS | .9841-03 |
| PERCENT OF THE TOTAL VARIANCE WHICH IS EXPLAINED BY THE VARIANCE OF THE MEANS | .3363+02 |
| PERCENT OF THE TOTAL VARIANCE WHICH IS EXPLAINED BY THE VARIANCE OF THE STANDARD DEVIATIONS | .8987+01 |
| PERCENT OF THE TOTAL VARIANCE WHICH IS NOT EXPLAINED BY THE VARIANCE IN THE MEAN AND STANDARD DEVIATION | .5739+02 |

### PERIODOGRAM COMPONENTS FOR THE MEAN

| HARMONIC | A | B | C SQUARED | PERIOD | PHASE |
|---|---|---|---|---|---|
| 1 | .3737-01 | .1156-01 | .1530-02 | .1200+02 | .1271+01 |
| 2 | .7646-02 | .1519-01 | .2892-03 | .6000+01 | .4663+00 |
| 3 | -.6067-03 | -.8541-03 | .1098-05 | .4000+01 | -.6177+00 |
| 4 | -.2757-02 | -.9416-03 | .8490-05 | .3000+01 | -.1242+01 |
| 5 | .6739-03 | -.6359-03 | .8586-06 | .2400+01 | .8144+00 |
| 6 | -.1765-02 | -.6549-08 | .3115-05 | .2000+01 | -.1571+01 |

### HARMONIC VARIANCES IN THE MEAN

| HARMONIC | VARIANCE | PERCENT OF TOTAL PERIODIC VARIANCE |
|---|---|---|
| 1 | .7650-03 | .8348+02 |
| 2 | .1446-03 | .1578+02 |
| 3 | .5488-06 | .5989-01 |
| 4 | .4245-05 | .4633+00 |
| 5 | .4293-06 | .4685-01 |
| 6 | .1558-05 | .1700+00 |

Table 14 (Cont'd.)

## PERIODOGRAM COMPONENTS FOR THE STANDARD DEVIATION

| HARMONIC | A | B | C SQUARED | PERIOD | PHASE |
|---|---|---|---|---|---|
| 1 | .1191-01 | .6575-02 | .1850-03 | .1200+02 | .1066+01 |
| 2 | .3179-02 | .8382-02 | .8036-04 | .6000+01 | .3625+00 |
| 3 | .5872-02 | -.1183-03 | .3450-04 | .4000+01 | .1551+01 |
| 4 | .7414-02 | .5155-02 | .8154-04 | .3000+01 | .9632+00 |
| 5 | .4264-02 | .7451-02 | .7370-04 | .2400+01 | .5198+00 |
| 6 | -.8594-02 | -.4033-08 | .7387-04 | .2000+01 | -.1571+01 |

## HARMONIC VARIANCES IN THE STANDARD DEVIATIONS

| HARMONIC | VARIANCE | PERCENT OF TOTAL PERIODIC VARIANCE |
|---|---|---|
| 1 | .9251-04 | .3498+02 |
| 2 | .4018-04 | .1519+02 |
| 3 | .1725-04 | .6522+01 |
| 4 | .4077-04 | .1541+02 |
| 5 | .3685-04 | .1393+02 |
| 6 | .3693-04 | .1396+02 |

## Table 15

## CUMULATIVE PROBABILITY DISTRIBUTION OF THE RESIDUALS OF THE DIFFERENCES IN ELEVATIONS

## MARQUETTE MINUS DULUTH

| Z | PERCENT PROBABILITY OF THE TIME SERIES BEING LESS THAN OR EQUAL TO Z |
|---|---|
| -.1502+01 | .3920+01 |
| -.1111+01 | .8442+01 |
| -.8414+00 | .1518+02 |
| -.6225+00 | .2111+02 |
| -.4301+00 | .2955+02 |
| -.2527+00 | .3789+02 |
| -.8310-01 | .4794+02 |
| .8387-01 | .5839+02 |
| .2534+00 | .6603+02 |
| .4309+00 | .7317+02 |
| .6232+00 | .8050+02 |
| .8422+00 | .8583+02 |
| .1112+01 | .9075+02 |
| .1502+01 | .9417+02 |

## Table 16

## PROBABILITY DENSITY DISTRIBUTION OF THE RESIDUALS OF THE DIFFERENCES IN ELEVATION

## MARQUETTE MINUS DULUTH

| LOW POINT | HIGH POINT | MID POINT | PROBABILITY DENSITY |
|---|---|---|---|
| | -.1502+01 | | .3920+01 |
| -.1502+01 | -.1111+01 | -.1306+01 | .4523+01 |
| -.1111+01 | -.8414+00 | -.9762+00 | .6734+01 |
| -.8414+00 | -.6225+00 | -.7319+00 | .5930+01 |
| -.6225+00 | -.4301+00 | -.5263+00 | .8442+01 |
| -.4301+00 | -.2527+00 | -.3414+00 | .8342+01 |
| -.2527+00 | -.8310-01 | -.1679+00 | .1005+02 |
| -.8310-01 | .8387-01 | .3862-03 | .1045+02 |
| .8387-01 | .2534+00 | .1686+00 | .7638+01 |
| .2534+00 | .4309+00 | .3421+00 | .7136+01 |
| .4309+00 | .6232+00 | .5271+00 | .7337+01 |
| .6232+00 | .8422+00 | .7327+00 | .5327+01 |
| .8422+00 | .1112+01 | .9769+00 | .4925+01 |
| .1112+01 | .1502+01 | .1307+01 | .3417+01 |
| .1502+01 | | | .5829+01 |

## Table 17

## RESULTS OF CHI-SQUARE AND KOLMOGOROV-SMIRNOV TESTS ON THE DISTRIBUTION OF THE RESIDUALS OF THE DIFFERENCES IN ELEVATION

### MARQUETTE MINUS DULUTH

| | |
|---|---|
| COMPUTED VALUE OF CHI-SQUARE BASED ON THE NORMAL DISTRIBUTION IS - | .9273+02 |
| NUMBER OF DEGREES OF FREEDOM IS - | 12 |
| COMPUTED VALUE OF KOLMOGOROV-SMIRNOV PARAMETER BASED ON THE NORMAL DISTRIBUTION IS - | .7169-01 |
| SAMPLE SIZE IS - | 995 |
| EXPECTED VALUE OF THE SAMPLE IS - | .3862-03 |
| STANDARD DEVIATION OF THE SAMPLE IS - | .1000+01 |

TESTS USING A LOG-NORMAL DISTRIBUTION CANNOT BE MADE BECAUSE OF NEGATIVE VARIATES WITHIN THE SAMPLE DISTRIBUTION

## Table 18

## SERIAL CORRELATION COEFFICIENTS OF THE RESIDUALS, MARQUETTE MINUS DULUTH AND THEORETICAL SERIAL CORRELATION COEFFICIENTS OF FIRST AND SECOND ORDER LINEAR MARKOV MODELS

COLUMN (1) - ORDER
(2) - SAMPLE SERIAL CORRELATION COEFFICIENT
(3) - THEORETICAL FIRST ORDER LINEAR MARKOV SERIAL CORRELATION COEFFICIENTS
(4) - THEORETICAL SECOND ORDER LINEAR MARKOV SERIAL CORRELATION COEFFICIENTS

| (1) | (2) | (3) | (4) |
|---|---|---|---|
| 1 | .181 | .181 | .181 |
| 2 | .118 | .033 | .118 |
| 3 | .131 | .006 | .035 |
| 4 | .116 | .001 | .016 |
| 5 | .098 | .000 | .006 |
| 6 | .050 | .000 | .002 |
| 7 | .041 | .000 | .001 |
| 8 | .096 | .000 | .000 |
| 9 | .035 | .000 | .000 |
| 10 | .081 | .000 | .000 |

## Table 19

### RESULTS OF CHI-SQUARE TESTS OF THE RESIDUALS, MARQUETTE MINUS DULUTH, AGAINST FIRST AND SECOND ORDER LINEAR MARKOV MODELS

---

(A) FIRST ORDER LINEAR MARKOV MODEL COMPUTED VALUE OF CHI-SQUARE IS - 40.33484

DEGREES OF FREEDOM ARE - 9

TABULATED VALUES OF CHI-SQUARE ARE -

| 99 PCT | 95 PCT | 5 PCT | 1 PCT |
|---|---|---|---|
| 2.088 | 3.325 | 16.919 | 21.666 |

(B) SECOND ORDER LINEAR MARKOV MODEL COMPUTED VALUE OF CHI-SQUARE IS - 27.19726

DEGREES OF FREEDOM ARE - 8

TABULATED VALUES OF CHI-SQUARE ARE -

| 99 PCT | 95 PCT | 5 PCT | 1 PCT |
|---|---|---|---|
| 1.646 | 2.733 | 15.507 | 20.090 |

## Table 20

### RESULTS OF RANDOMNESS TESTS ON THE STOCHASTIC COMPONENT OF THE DIFFERENCES IN ELEVATION

### MARQUETTE MINUS DULUTH

| TEST NO. | STANDARD NORMAL VARIABLE | R. VALUE | R. MEAN | R. STANDARD DEVIATION |
|---|---|---|---|---|
| 1 | -5.02 | 678.00 | 746.96 | 13.64 |
| 2 | -5.64 | 587.50 | 663.00 | 13.29 |
| 3 | .93 | 488.00 | 497.00 | 9.11 |

THE STD. NORMAL VARIABLE (K) INDICATES THE NUMBER OF STD. DEVIATIONS THAT THE TEST VALUE IS BELOW THE EXPECTED (MEAN) VALUE IF THE OBSERVATIONS ARE INDEPENDENT. VALUES OF K LESS THAN -2.33 OCCUR WITH A PROBABILITY OF LESS THAN 0.01 FOR A SEQUENCE OF INDEPENDENT OBSERVATIONS (USE K GREATER THAN 2.58 FOR TEST 3 - 2-TAIL PROBABILITY).

SPECIFICALLY, SUCH VALUES OF K INDICATE -

TEST 1 - A TENDENCY FOR OBSERVATIONS TO CLUSTER INTO THE RESPECTIVE CATEGORIES.

TEST 2 - A TENDENCY FOR DIRECTIONS OF MOVEMENT TO PERSIST (IE. - CLUSTERING NOT EXCLUSIVELY DUE TO A FEW LARGE CHANGES.)

TEST 3 - AN UPWARD OR DOWNWARD TREND DUE TO A RELATIVE SHORTAGE OF - OR + SIGNS RESPECTIVELY (TREND DUE TO LARGER MOVEMENTS IN ONE DIRECTION IS NOT DETECTED BY THIS TEST).

# Table 21

## PERCENT OF VARIANCE OF ORIGINAL TIME SERIES EXPLAINED BY TIME SERIES COMPONENTS

### MARQUETTE MINUS DULUTH

| COMPONENTS | PERCENT |
|---|---|
| (a) Linear trends | |
| (1) first order | 47.01 |
| (2) second order | 0.85 |
| (3) third order | 0.09 |
| (4) fourth order | 0.01 |
| | 47.96 |
| (b) Periodicity in the mean | |
| (1) first harmonic | 11.59 |
| (2) second harmonic | 2.19 |
| (3) third harmonic | 0.01 |
| (4) fourth harmonic | 0.06 |
| (5) fifth harmonic | - |
| (6) sixth harmonic | 0.02 |
| | 13.87 |
| (c) Periodicity in the standard deviation | |
| (1) first harmonic | 1.30 |
| (2) second harmonic | 0.56 |
| (3) third harmonic | 0.24 |
| (4) fourth harmonic | 0.57 |
| (5) fifth harmonic | 0.52 |
| (6) sixth harmonic | 0.52 |
| | 3.71 |
| (d) Residual | 34.46 |
| | 100.00 |

the period of common record increases. The larger the explained variance and the longer the period of common record the greater is the reliability that can be attached to the corresponding rate of relative movement and so the higher is the weighting factor used in the adjustment. This method assumes that the accuracy of the method of determining a rate of movement is independent of the distance between gauges.

For Lake Superior the triangle of gauging stations having the highest set of weights attached to them is Marquette - Duluth - Port Arthur. These three stations were therefore used in the initial step of the triangular adjustment.

The residual or observation errors can be written as:

$$\begin{aligned} &a + 0.3597 \\ &b - 0.7088 \\ &(b-a) - 1.1639 \end{aligned} \qquad (34)$$

where a is the probable rate of movement between Marquette and Duluth, and

b is the probable rate of movement between Marquette and Port Arthur.

Using the weights previously derived (based on $R^2N$) the sum of the squares of the observation errors is:-

$$E = \frac{564}{2096}(a + 0.3597)^2 + \frac{662}{2096}(b - 0.7088)^2 + \frac{870}{2096}((b-a) - 1.1639)^2 \qquad (35)$$

Simplifying, differentiating with respect to each of the unknowns and equating to zero yields two normal equations:

$$\begin{aligned} 1.3684a - 0.8302b + 1.1599 &= 0 \\ 1.4618b - 0.8302b - 1.4139 &= 0 \end{aligned} \qquad (36)$$

Solving these two equations simultaneously the probable rates of movement a, b and b-a can be determined:

$$\begin{aligned} a &= -0.3979 \\ b &= +0.7413 \\ b-a &= +1.1392 \end{aligned} \tag{37}$$

Using these adjusted rates of movement the rates between other gauging stations are corrected so as to be in agreement. The adjusted and unadjusted rates of movment derived are listed in Table 22.

In order to provide a comparison with the results of previous investigations Tables 23 and 24 were drawn up showing rate of crustal movement relative to Marquette and to Port Arthur respectively.

(b) Circular distribution

As explained in the review of the 1967 Engineering Division study, if crustal movement were acting on a lake basin as an entire unit and if the movement were in only one direction, say the northern side of the basin was rising relative to the southern side, then from a position A in the centre of the region, the relative movement at points on the circumference of a circle with centre A would, when plotted against angular position, form a sine curve. In a practical application this form of plot needs some modifications because gauging stations are not available at the lake centres, and the gauges which are available are not distributed around the circumference of a circle. The method developed in 1967 was to plot rates of crustal movement at points around a lake relative to one reference station on the lake in feet per 100 miles per 100 years versus the whole circle bearings of the gauging stations relative to the reference station. This method was used to produce graphs for all the Great Lakes and in each case it was found that some form of circular curve was present, implying that the measured crustal movement was basin-wide at least. The diagram produced for Lake Superior using this second method of presentation is shown in Figure 14. On Lake Superior the best gauge to use for this presentation would be Houghton, with its fairly central position, however the period of record at Houghton is too small to permit its use as a main station. The two gauges sharing next best position are Keweenaw Lower Entrance and Port Arthur (Thunder Bay) and, of these, Port Arthur was used because of its longer period of record.

## Table 22

Computed and computed-adjusted rates of relative crustal movement around Lake Superior

| GAUGE B | GAUGE A | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Marquette | | | | Duluth | | | |
| | Rates of movement A-B | | | | | | | |
| | Correlation coefficient | Unadjusted rate, feet per 100 yrs. | Adjusted rate, feet per 100 yrs. | No. of months of common record | Correlation Coefficient | Unadjusted rate, feet per 100 yrs. | Adjusted rate feet per 100 yrs. | No. of months of common record |
| Port Arthur | 0.7396 | 0.7088 | 0.7413 | 1210 | 0.8940 | 1.1639 | 1.1392 | 1089 |
| Michipicoten | 0.8306 | 0.9902 | 1.0134 | 653 | 0.8827 | 1,4345 | 1.4113 | 653 |
| Sault Ste Marie | 0.5755 | 0.5525 | - | 742 | 0.7727 | 0.9913 | - | 742 |
| Point Iroquois | 0.5261 | 0.3314 | 0.3659 | 408 | 0.6871 | 0.7983 | 0.7638 | 408 |
| Marquette | | | | | 0.6856 | 0.3597 | 0.3979 | 1211 |
| Keweenaw L.E. | 0.0897* | 0.0393 | 0.0393 | 829 | 0.7233 | 0.4371 | 0.4371 | 829 |
| Houghton | 0.1607* | -0.0826 | -0.0111 | 295 | 0.5856 | 0.4583 | 0.3868 | 295 |
| Duluth | 0.6856 | -0.3597 | -0.3979 | 1211 | | | | |
| Two Harbors | 0.3978 | -0.5394 | -0.4134 | 350 | 0.1441* | 0.1106 | 0.0154 | 350 |

* Statistically non-significant regressions.

## Table 23

Crustal movement around Lake Superior in feet per 100 years relative to Marquette

| Gauge | Author | | | | | | | | Length of common Period of record as of Jan. 1971 (months) |
|---|---|---|---|---|---|---|---|---|---|
| | Gutenberg, 1933 | Gutenberg, 1941 | Moore, 1948 | V.C.S., 1957 (2) unadjusted | V.C.S., 1957 (2) adjusted | Kite, 1967 | Kite, 1971 unadjusted | Kite, 1971 adjusted | |
| Port Arthur | +0.97 | +0.65 | +0.68 | +0.60 | +0.65 | +0.59✓ | +0.71 | +0.74 | 1210 |
| Michipicoten Harbor | +0.70 | +0.49 | +0.55 | +1.01 | +1.04 | +1.01✓ | +0.99 | +1.01 | 653 |
| Point Iroquois | | | +0.53 | +0.39 | +0.27 | +0.33 | +0.33 | +0.36 | 408 |
| Whitefish Point | | | | -0.43 | -0.42 | +0.16 | | | |
| Grand Marais | | | | +0.20 | +0.13 | 0.00 | | | |
| Munising | | | -0.46 | +0.46 | +0.42 | +0.51 | | | |
| Presque Isle | | | | +0.04 | -0.01 | -0.14 | | | |
| Keweenaw L.E. | | | | -0.06 | -0.05 | -0.08✓ | +0.04 | +0.04 | 829 |
| Houghton | | | +0.05 | -0.08 | +0.03 | -0.02 | -0.08 | +0.01 | 295 |
| Keweenaw U.E. | | | | +0.14 | +0.09 | +0.27 | | | |
| Grand Traverse Bay | | | | +0.28 | +0.30 | -0.30 | | | |
| Copper Harbor | | | +0.36 | +0.33 | +0.31 | +0.31 | | | |
| Ontonagon | | | -0.08 | -0.04 | -0.16 | -0.20 | | | |
| Eagle Harbor | | | | +0.06 | +0.14 | +0.19 | | | |
| Black River | | | | -0.89 | -0.87 | -0.41 | | | |
| Ashland | | | -0.41 | -0.63 | -0.62 | -0.55 | | | |
| Cornucopia | | | | -0.40 | -0.44 | -0.60 | | | |
| Port Wing | | | -0.24 | -0.26 | -0.32 | -0.29 | | | |

Table 23 (Cont'd.)

| Gauge | Author | | | | | | | | Length of common Period of record as of Jan. 1971 (months) |
|---|---|---|---|---|---|---|---|---|---|
| | Gutenberg, 1933 | Gutenberg, 1941 | Moore, 1948 | (2) V.C.S., 1957 unadjusted | (2) V.C.S., 1957 adjusted | Kite, 1967 | Kite, 1971 unadjusted | Kite, 1971 adjusted | |
| Duluth | -0.59 | -0.39 | -0.40 | -0.34 | -0.31 | -0.36 | -0.36 | -0.40 | 1211 |
| Two Harbors | | | -0.21 | -0.14 | -0.28 | -0.47✓ | -0.54 | -0.41 | 350 |
| Beaver Bay | | | | +0.06 | +0.08 | -0.37 | | | |
| Lutsen | | | | -0.67 | -0.61 | -0.63 | | | |
| Grand Marais | | | +0.25 | +0.20 | +0.09 | -0.08 | | | |
| Rock Harbor | | | | +0.51 | +0.47 | | | | |
| Washington Harbor | | | | +0.33 | +0.33 | | | | |
| Sault Ste. Marie | +0.39 | | | | | +0.61✓ | +0.55 | (4) | 742 |
| Isle Royal | | | +0.39 | | | | | | |

Notes: (1) A + ve rate of movement indicates a rise of the land adjacent to the gauging station with respect to Marquette.

(2) V.C.S. is an abbreviation for Vertical Control Subcommittee, Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data.

(3) Check (✓) marks signify those stations which, in 1967, had over 20 years of record. Other figures for 1967 are given merely to update V.C.S. results, they are not considered reliable.

(4) Figures for Sault Ste. Marie are not considered reliable because of drawdown.

## Table 25

## ABSOLUTE RATES OF CRUSTAL MOVEMENT REFERENCED TO MEAN SEA LEVEL AT NEW YORK

| STATION | RATE OF MOVEMENT FT./100 YEARS |
|---|---|
| Port Arthur | +0.28 |
| Michipicoten | +0.55 |
| Sault Ste. Marie | +0.11 |
| Point Iroquois | -0.09 |
| Marquette | -0.46 |
| Keweenaw L.E. | -0.42 |
| Houghton | -0.47 |
| Duluth | -0.86 |
| Two Harbors | -0.87 |
| Mean lake level as presently computed | -0.12 |

## Table 26

## RESULTS OF TESTS ON THE CORRELATION COEFFICENT BETWEEN DIFFERENCES IN ELEVATION AND TIME

DULUTH MINUS TWO HARBORS

| | |
|---|---|
| COMPUTED VALUE OF CORRELATION COEFFICIENT "r" | 0.1441 |
| NUMBER OF DATA POINTS | 350 |
| EFFECTIVE NUMBER OF DATA POINTS | 67 |

| TEST | RESULT |
|---|---|
| APPROXIMATE NORMAL | $r \ngtr 0.0$ |
| STUDENT t, P = 95% | $r \ngtr 0.0$ |
| FISHER Z, P = 95% | $-0.15 \leq r \leq 0.35$ |

zero. This would seem to tie in very well with the geological evidence regarding the Nipissing zero isobase mentioned in the introduction.

From all three viewpoints, the present uncertainty of levelling to mean sea level, the insignificant rate of movement between Duluth and Two Harbors, and the geological evidence, the best solution to the problem of computing absolute rates of crustal movement is to use the Nipissing zero isobase as it is currently (Hough, 1958; MacLean, 1961) envisaged for Lake Superior. It must be emphasized that this is only a guess; the period of record at Two Harbors is around 30 years which is not really long enough to permit accurate determinations of rates of movement.

It is also perfectly possible that although no differential movement can be proved, both points or even all the points on the lake are also subject to a uniform rate of movement which cannot be measured.

Rates of movement relative to the zero isobase will be the same as those computed relative to Duluth since Duluth is presumed to be beyond the influence of isostatic adjustment following the last glacial retreat. These rates of movement are shown on Figure 13 and listed in Table 27.

## C) Determination of Movement of the Lake Surface

### I) Computation of mean lake elevation

Gauging stations have been measuring local water surface elevations on Lake Superior since about 1860. Only since 1900, however, has an attempt been made to combine gauge records and produce a daily mean lake elevation.

For the period January 1900 to date the daily mean lake levels have been computed using the stations shown in Table 28.

End-of-month mean lake elevations have then been computed as in Table 29 with the following definitions:

a - 10 day mean utilizing the daily mean lake elevations for the last five days of the month referenced and the first five daily means of the following month.

## Table 24

Crustal Movement around Lake Superior
in feet per 100 years relative to Port Arthur

| Gauge | Author | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Gutenberg, 1933 | Gutenberg, 1941 | Moore, 1948 (2) | V.C.S., 1957 (3) unadjusted | V.C.S., 1957 (3) adjusted | Kite, 1967 | Kite, 1971 unadjusted | Kite, 1971 adjusted | Length of common period of record as of January, 1971 (months) |
| Michipicoten Harbor | -0.25 | -0.06 | -0.13 | +0.28 | +0.39 | +0.42 ✓ | +0.45 | +0.27 | 653 |
| Point Iroquois | | | -0.15 | -0.36 | -0.38 | -0.26 | -0.14 | -0.37 | 408 |
| Whitefish Point | | | | -1.06 | -1.07 | -0.43 | | | |
| Grand Marais | | | | -0.59 | -0.52 | -0.59 ✓ | | | |
| Munising | | | -1.14 | -0.26 | -0.23 | -0.08 | | | |
| Marquette | -0.97 | | -0.68 | -0.60 | -0.65 | -0.59 ✓ | -0.71 | -0.74 | 1210 |
| Presque Isle | | | | -0.72 | -0.66 | -0.73 | | | |
| Keweenaw L.E. | | | | -0.70 | -0.70 | -0.67 ✓ | -0.45 | -0.70 | 708 |
| Houghton | | | -0.63 | -0.51 | -0.62 | -0.61 ✓ | -0.38 | -0.75 | 174 |
| Keweenaw U.E. | | | | -0.60 | -0.56 | -0.32 | | | |
| Grand Traverse Bay | | | | -0.33 | -0.35 | -0.89 | | | |
| Copper Harbor | | | -0.32 | -0.35 | -0.34 | -0.28 | | | |
| Ontonagon | | | -0.76 | -0.93 | -0.81 | -0.75 | | | |
| Eagle Harbor | | | | -0.43 | -0.51 | -0.40 | | | |
| Black River | | | | -1.50 | -1.52 | -1.00 | | | |
| Ashland | | | -1.09 | -1.27 | -1.27 | -1.14 | | | |

Table 24 (Cont'd.)

| Gauge | Author | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Gutenberg, 1933 | Gutenberg, 1941 | Moore, (2) 1948 | V.C.S., 1957 (3) unadjusted | V.C.S., 1957 (3) adjusted | Kite, 1967 | Kite, 1971 unadjusted | Kite, 1971 adjusted | Length of common period of record as of January, 1971 (months) |
| Cornucopia | | | | -1.12 | -1.09 | -1.19 | | | |
| Port Wing | | | -0.92 | -1.03 | -0.97 | -0.88 | | | |
| Duluth | -1.49 | -1.10 | -1.08 | -0.92 | -0.96 | -0.95 ✓ | -1.16 | -1.14 | 1089 |
| Two Harbors | | | -0.89 | -1.07 | -0.93 | -1.06 ✓ | -0.78 | -1.15 | 350 |
| Beaver Bay | | | | -0.56 | -0.57 | -0.96 | | | |
| Lutsen | | | | -1.19 | -1.26 | -1.22 | | | |
| Grand Marais | | | -0.43 | -0.67 | -0.56 | -0.67 | | | |
| Rock Harbor | | | | -0.21 | -0.18 | | | | |
| Washington Harbor | | | | | | | | | |
| Sault Ste. Marie | | | | | | +0.02 ✓ | +0.02 | (5) | 720 |
| Isle Royal | | | -0.29 | | | | | | |

Notes:

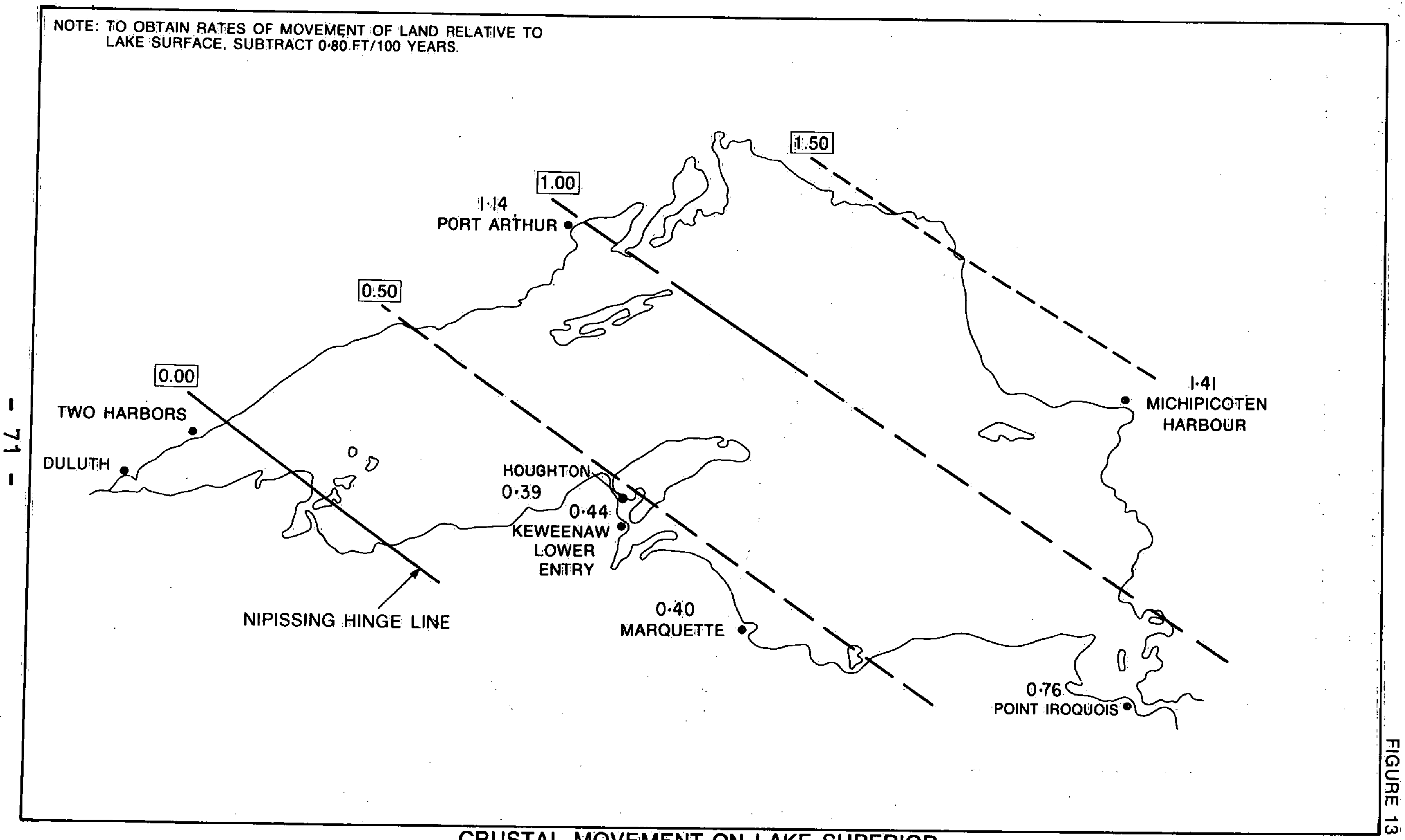
(1) A + ve rate of movement indicates a rise of the land adjacent to the gauging station with respect to Port Arthur.

(2) These figures were obtained by the Vertical Control Subcommittee by triangulation from Moore's 1948 results.

(3) V.C.S. is an abbreviation for Vertical Control Subcommittee, Coodinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data.

(4) Check (✓) marks signify those stations which, in 1967, had over 20 years of record. Other figures for 1967 are given merely to update V.C.S. results, they are not considered reliable.

(5) Figures for Sault Ste. Marie are not considered reliable because of drawdown.

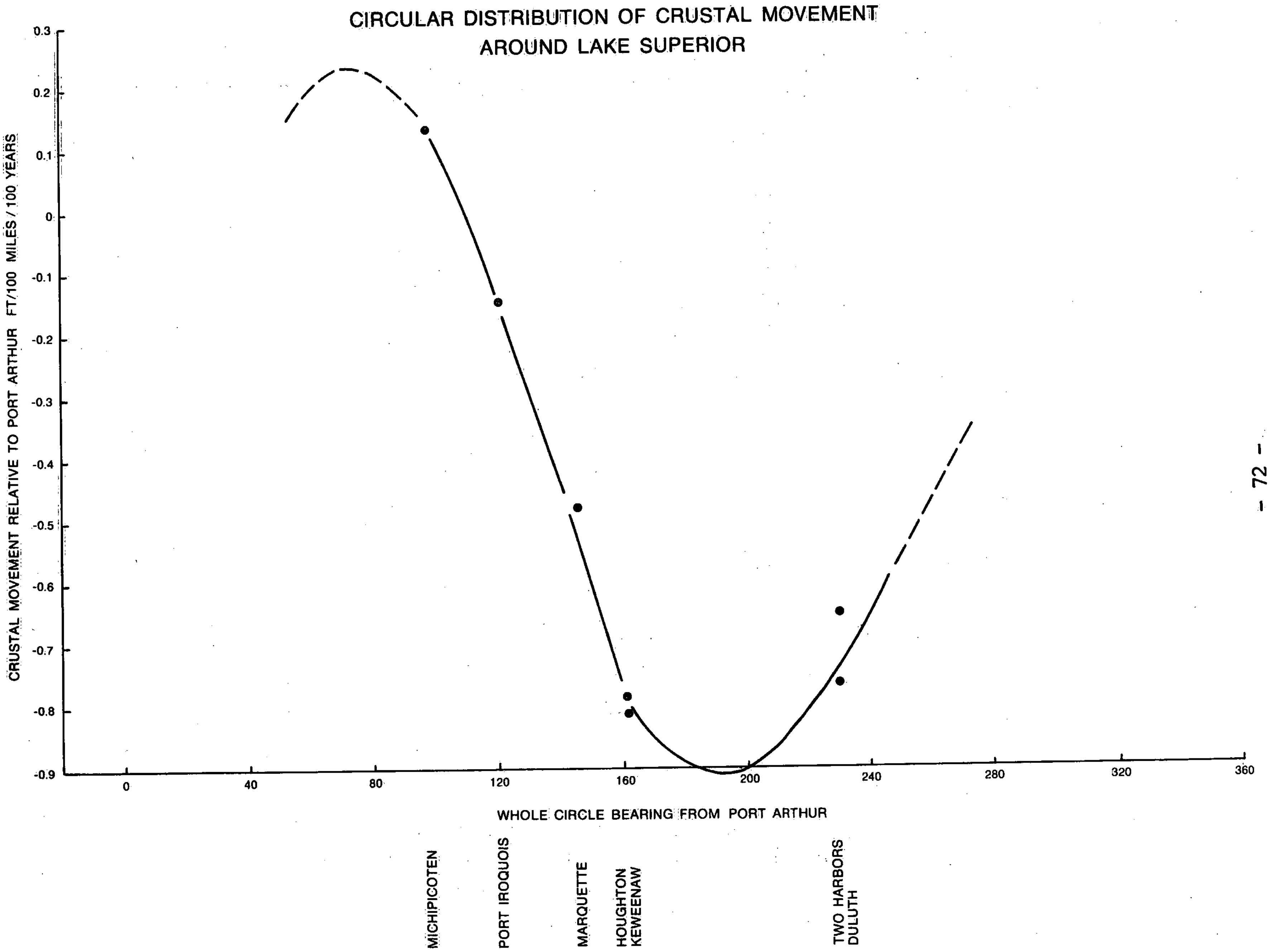


NOTE: TO OBTAIN RATES OF MOVEMENT OF LAND RELATIVE TO LAKE SURFACE, SUBTRACT 0·80 FT/100 YEARS.

FIGURE 13

CRUSTAL MOVEMENT ON LAKE SUPERIOR
LINES OF EQUAL RATE OF ABSOLUTE MOVEMENT, FT/100 YEARS

FIGURE 14



CIRCULAR DISTRIBUTION OF CRUSTAL MOVEMENT
AROUND LAKE SUPERIOR

## B) Determination of Absolute Rates of Movement

### I) By land levelling

In order to obtain absolute rates of crustal movement the relative rates as determined in this and other studies must be referred to some constant datum. The most obvious datum to use is mean sea level. Changes in sea level can, however, be introduced by factors such as uplift of areas now under the sea, sedimentation, changes in volume of glaciers and polar ice caps, spreading of the continents, long-term variations in temperature or pressure etc. Over geologic time mean sea level has undoubtedly changed many times, notably as the volume of water locked up in the glaciers has changed. It seems improbable, however, that any great change in the volume of water in the oceans has taken place in the last hundred years or so. Tidal gauges at various locations do show a change with time in the relationship between a gauge datum on land and the mean sea level (Gutenberg, 1941). If the mean sea level is not changing significantly then the land around the gauges must be moving.

From mean sea level, land levelling can be used to obtain the relative elevation of a point in the area suspected of movement. Comparing this figure with the figure obtained from a similar set of land levels obtained at some other time, an estimate of the rate of crustal movement can be obtained. As usual the process is complicated in practice because of the numerous possible errors. Among other factors, the possible error in the rate of movement determined in this manner is proportional to the number of years between levelling surveys; the greater the time difference, the less will be the uncertainty in the result. However, using sets of levels taken many years ago introduces the problem of unequal errors; level lines run today using modern equipment and techniques are far more accurate than level lines run in the nineteenth century.

The only study on record presenting computed rates of crustal movement in the Great Lakes region relative to mean sea level is that by Moore in 1948. The levels used by Moore were taken at different times within the period 1857 to 1941. Moore used levels from New York to Rensselaer to Oswego, Cleveland, Harbor Beach and Sault Ste. Marie. The most uncertain section of the levelling is that between Lakes Erie and Huron although other sections have been subject to criticism (MacLean, 1961, page 111).

Using Moore's figures, which are at present the best available, the rate of movement of Point Iroquois, on Lake Superior, with respect to mean sea level at New York is -0.09 foot/100 years. Table 25, rates of absolute movement for other points on Lake Superior, is based on Moore's result for Point Iroquois.

It is interesting to note that in only a few years time (1972-73) it will be possible to check these figures using modern first order levelling. The I.G.L.D. levelling from Father Point, Quebec, to the Great Lakes is now being re-run and will be finished in a year or so. A comparison of the two sets of levels run 15 years apart should give a reasonable measure of absolute rates of crustal movement bearing in mind the relatively short period between the levels and the limit of accuracy of the levelling. In the meantime, however, Moore's figures are the only ones available. Some of these figures for water level transfers are not in agreement with recent studies but there is little point in correcting these while others remain which cannot be checked.

## II) Nipissing zero isobase

A second procedure was followed in another attempt to derive absolute rates of movement. If, as seems likely, the movement being measured is the result of isostatic adjustment following the retreat of the last glaciers, then the maximum uplift on Lake Superior would be in the northeast (assuming that the maximum thickness of the last glacial advance was over eastern Hudson's Bay). There would be no current subsidence in the region if we follow this hypothesis and, somewhere along the southern boundary of the region, there would be a line of zero movement establishing the present southern limit of the presumably diminishing circle of influence of the uplift. If this line of zero movement could be established then all rates of crustal movement presently determined could be referred to this line.

In an earlier attempt to find a zero isobase or a line with no crustal movement, Gutenberg in 1941 examined his computed values of relative movement and, where two stations exhibited zero relative movement, assumed they were both to the south of the zero isobase.

On Lake Superior the present results indicate that the true long-term rate of movement between Duluth and Two Harbors is zero. Table 26 shows that the correlation coefficient between differences in mean monthly elevations and time is not significantly different from

## Table 27

### ABSOLUTE RATES OF CRUSTAL MOVEMENT AROUND LAKE SUPERIOR REFERRED TO THE NIPISSING ZERO ISOBASE

| STATION | RATE OF CRUSTAL MOVEMENT FT./100 YEARS |
|---|---|
| Port Arthur | +1.14 |
| Michipicoten | +1.41 |
| Sault Ste. Marie | +0.97 |
| Point Iroquois | +0.76 |
| Marquette | +0.40 |
| Keweenaw L.E. | +0.44 |
| Houghton | +0.39 |
| Duluth | 0.00 |
| Two Harbors | 0.00 |
| Mean lake level as presently computed | +0.74 |

## Table 28

### GAUGING STATIONS USED TO COMPUTE MEAN ELEVATION OF LAKE SUPERIOR FOR DIFFERENT PERIODS OF TIME

| PERIOD | FROM | TO | GAUGING STATION |
|---|---|---|---|
| I | Dec. 1899 - | Apr. 1914 | 1, 2 |
| II | May 1914 - | Nov. 1914 | 1, 2, 5 |
| III | Dec. 1914 - | Nov. 1917 | 2, 5 |
| IV | Dec. 1917 - | Dec. 1950 | 1, 2, 4, 5 |
| V | Jan. 1951 - | Dec. 1964 | 1, 2, 3, 4, 5 |
| VI | Dec. 1964 - | | 1, 2, 3, 4, 5 |

where station
1 is Duluth
2 is Marquette
3 is Point Iroquois
4 is Michipicoten
5 is Port Arthur

## Table 29

## LIST OF TYPES OF MEAN LAKE SUPERIOR ELEVATION COMPUTED AT DIFFERENT TIMES

| PERIOD | E-O-M MEAN |
|---|---|
| I | a |
| II | a |
| III | a |
| IV | b |
| V | b |
| VI | a, b* |

* For this period a coordinated mean is arrived at based on both the 10 and 4 day end-of-month means.

## Table 30

## GAUGE CORRECTIONS FOR I.G.L.D.

| GAUGE | CORRECTION, FEET |
|---|---|
| Duluth | IGLD = USLS - 1.76 |
| Marquette | IGLD = USLS - 1.71 |
| Point Iroquois | IGLD = USLS - 1.63 |
| Michipicoten | IGLD = USLS - 1.56 |
| Port Arthur | IGLD = USLS - 1.66 |

b = 4-day mean averaging the daily mean lake elevations for the last two days of the month referenced and the first two daily means of the following month.

Monthly mean lake elevations have been computed for each period using the same stations as in Table 28 and averaging the month's daily values.

A further complication is introduced by the fact that the vertical control data on which the various gauges have been based have changed over the years. Some of the more common or widespread data have been the Canadian Geodetic Datum, U.S. Coast and Geodetic Survey Datum, U.S. Lake Survey 1903 Datum, U.S. Lake Survey 1935 Datum and the Georgian Bay Ship Canal Datum. The most recent change in datum occurred when the International Great Lakes Datum was introduced, using 1955 as a reference year. One of the reasons for the latest change is to compensate for the cumulative crustal movement prior to the date of the new datum. Because the crust of the earth in the Great Lakes region is moving with respect to sea level, and because the rate of movement is not uniform throughout the area, the elevations of gauging stations and bench marks are changing with respect to each other. A new datum will re-set the relative crustal movement between the gauges to zero at that particular instant at which the new datum is put into effect. In fact, because of the relatively low velocity of crustal movement a period of a few years could be allowed to effect the new datum.

The corrections made to convert lake gauging stations from U.S.L.S. 1935 datum to I.G.L.D. 1955 datum on Lake Superior are given in Table 30.

These corrections derived in Table 30 lead to the fact that a different correction must be applied to the mean lake levels for each period, as shown in Table 31.

For the purposes of this study, however, the only effect of the latest change in datum is to force all relative crustal movement between gauges to be near zero in 1955. Provided that the datum correction has been carried out uniformly for each station over the entire periods of record previous to 1955 then the datum change will not affect the measured rates of relative crustal movement.

Using the rates of relative crustal movement determined earlier in the study, a rate of movement of

## Table 31

## CORRECTIONS FOR I.G.L.D. FOR MEAN LAKE SUPERIOR ELEVATION FOR DIFFERENT PERIODS

| PERIOD | CORRECTION, FEET |
|---|---|
| I | -1.74 |
| II | -1.71 |
| III | -1.68 |
| IV | -1.67 |
| V | -1.66 |

## Table 32

## RATES OF MOVEMENT OF MEAN LAKE SUPERIOR ELEVATIONS RELATIVE TO MARQUETTE

| PERIOD | RATE OF MOVEMENT FT./100 YEARS |
|---|---|
| I | -0.20 |
| II | +0.11 |
| III | +0.37 |
| IV | +0.34 |
| V, VI | +0.34 |

the mean lake elevation can be calculated for each period of record.

Table 32 shows that, relative to Marquette the rate of movement of the mean lake level has been changing over the last 70 years due to the fact that different gauges have been used to compute the mean. It follows that any studies using this series of mean elevations have used a series containing an acceleration or second order trend of the order of 1 foot/100 years/100 years.

A more important set of statistics is the change in the rate of movement of mean lake level relative to the outlet of the lake. There are two gauging stations near the Lake Superior outlet, Point Iroquois and Sault Ste. Marie. About 14 miles separates these two gauges and yet relative to Marquette there is a difference of over 0.2 foot per 100 years in the computed rates of crustal movement. This is presumably due to the drawdown of the lake surface between the two gauges as the lake enters the St. Mary's River. As far as record length is considered it would be preferable to use the Sault Ste. Marie gauge as representing conditions at the lake outlet since it started operating in 1860 while the Point Iroquois gauge is a relative newcomer, with continuous records starting in 1950. The effect of the drawdown makes it impossible to use this gauge, however, and so Table 33 gives rates of movement relative to Point Iroquois.

This shows that, since 1900, the computed mean monthly elevation of Lake Superior has been changing relative to the lake outlet at different rates. Because of the different combinations of gauges used to compute the lake mean there has been an effective acceleration of movement of around 0.75 foot/100 years/100 years. This means that the outlet of Lake Superior is now falling relative to computed mean lake level at a slower rate than it was 50 years ago.

## II) Effects of regulation

The control structure across the outlet of Lake Superior was completed in August 1922 under the orders of approval issued by the I.J.C. in 1914. No formal regulation plan was followed for the first 6 years, all major interests agreed on each operation undertaken. On July 31, 1928 the Lake Superior Board of Control adopted an operating rule curve known as "Tentative Rule Curve D". Due to the Long Lake diversion into Lake Superior a new rule curve "P5" was adopted by the Board in 1941. A further diversion, Ogoki, and the study of crustal

## Table 33

## RATES OF MOVEMENT OF MEAN LAKE SUPERIOR ELEVATIONS RELATIVE TO POINT IROQUOIS

| PERIOD | RATE OF MOVEMENT FT./100 YEARS |
|---|---|
| I | -0.57 |
| II | -0.25 |
| III | -0.00 |
| IV | -0.03 |
| V, VI | -0.02 |

## Table 34

## DETERMINATION OF FIRST ORDER LINEAR TREND IN GAUGE RECORDS USING DIFFERENT PERIODS OF TIME

(Upper figure of each pair below is the linear trend in feet per 100 years, lower figure of each pair is the corresponding correlation coefficient).

| STATION | PERIOD USED (MONTHS) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1211 | 1210 | 829 | 742 | 653 | 408 | 350 | 295 |
| Marquette | 0.70<br>0.34 | 0.69<br>0.36 | 0.38<br>0.12 | 0.82<br>0.23 | 0.63<br>0.16 | -0.55<br>0.13 | -2.07<br>0.31 | -0.15<br>0.03 |
| Duluth | 1.06<br>0.47 | | | | | | | |
| Port Arthur | | -0.01<br>0.00 | | | | | | |
| Keweenaw L.E.* | | | 0.34<br>0.11 | | | | | |
| Sault Ste. Marie | | | | 0.26<br>0.08 | | | | |
| Michipicoten | | | | | -0.35<br>0.09 | | | |
| Point Iroquois | | | | | | -0.88<br>0.21 | | |
| Two Harbors | | | | | | | -1.53<br>0.23 | |
| Houghton* | | | | | | | | -0.06<br>0.01 |

* Note: These periods of record do not extend to 1970.

movement around Lake Superior by Sherman Moore (Moore, 1948) led to a further curve "1949 Rule" being initiated in May, 1951. At the time that the "1949 Rule" was introduced it was decided that the gauges at Marquette, Duluth, Port Arthur and Michipicoten would be corrected for crustal movement. Marquette, Duluth, Port Arthur and Michipicoten were to be corrected by -0.08, -0.14, -0.06 and -0.10 foot respectively from 1950 on. It appears that advance daily elevations for the gauges at Duluth and Marquette are corrected for crustal movement while published mean daily elevations for these two gauges are not corrected. The present (summer, 1971) corrections are -0.12 foot for Duluth and -0.08 foot for Marquette. The Canadian gauges at Port Arthur (now Thunder Bay) and Michipicoten Harbour are not corrected for crustal movement.

Negative corrections to the gauges at Marquette and Duluth can only aggravate the problem of flooding in the southwest corner of the lake since this will lower the computed mean lake level. The rule curve will then specify a discharge lower than it should be which will result in a higher lake level and more flooding. The effect on lake regulation will be to increase the percent of time at which lake elevations are near the maximum. Even given the correct algebraic sign the U.S. corrections do not effectively correct for rates of crustal movement, only for differences in mean elevation. In explanation, if the equation relating gauge difference, y, to time, t, is expressed by a general polynomial of the form:

$$y = a + bt + ct^2 + \ldots\ldots \qquad (38)$$

consisting of at least a constant, a velocity term and an acceleration term, then the corrections applied to the U.S. gauges adjust the constant term only. Every few years this constant would need adjusting to account for the higher order terms.

Presently the rule curve followed is the "1955 Modified Rule of 1949" which provides improved benefits to power and navigation. Currently studies are continuing on the development of a more sophisticated plan to regulate Lake Superior in conjunction with other lakes in the Great Lakes chain.

## III) Calculation of trend in mean lake surface elevation

It should be noted that the last figure given in Table 27, mean lake level as presently computed, is based on the use of five lake gauges as explained in an earlier section of this report. The technique of obtaining a trend in differences in gauge elevations cannot take into account any overall change in lake level. The rates of movement computed assume a level lake and therefore if any rate of movement of lake level occurred during the period of interest this would have to be subtracted from the rates of movement computed for each gauging station.

If any overall movement of the lake surface has occurred (and we would expect that there would be a rise in lake level as the land around the lake rises) then this would show up as a trend in each gauge record, that is, if a gauge shows a trend of y ft/100 years and has an absolute rate of movement, as previously computed, of x ft/100 years then it can be said that lake level is moving at a rate of x-y ft/100 years. Table 34 shows the results of a simple linear regression program applied to each of the gauging stations around Lake Superior for the period of record of each gauge. The rate of movement of lake level at Marquette, the station with the longest continuous record, was also computed at each period of record.

The recorded correlation coefficients indicate that these results are not very accurate. This is to be expected since trends in the original gauge records are very much more susceptible to change with time than are the differences between gauge elevations used to compute relative rates of movement. This is shown very clearly in the results for Marquette where the rate of movement calculated over a period of 350 months back from December 1970 is -2.07 feet/100 years. Over increasingly longer periods of time the linear trend changes through -0.55, +0.63, +0.82, +0.69, +0.70. The results corresponding to gauging stations at Houghton and Keweenaw Lower Entrance are neglected here because the records at these gauges do not extend to the present time. On the basis of Table 34 a reasonable rate of movement of the mean lake surface elevation might be around 0.7 foot per 100 years.

A second estimate of the rate of movement of the lake surface level can be obtained by considering the inflow - outflow - storage relationships for the lake. Since no natural stage-discharge relationship exists for Lake Superior it is impossible to determine whether or not any trend exists in the time series of lake outflow. However, since regulated outflow records have been kept, these records can be substituted for natural outflows and analysed accordingly, provided that no deliberate attempt has been made to introduce a trend in the regulation. No linear first-order trend showed up in the analysis. Barring a major change in climate in the last 100 years, which cannot at present be substantiated, the lake inflows should not contain any trend. The time series of lake inflows will, however, contain two step functions or jumps in 1937 and 1943, due to the Long Lake and Ogoki diversions. The result of there being no trend in lake input and no change in lake output must be that there has been no change in storage.

If there has been no change in storage then the result of tipping the lake by crustal movement can be determined. Taking the longest possible cross-section of Lake Superior at right angles to the iso-lines of rate of movement it is clear that an upward movement of the land by x feet over a given period at one end of the section relative to the land at the other end of the section must result in a rise in mean lake level of x/2 feet over the same period of time. This assumes that the lake basin moves as a complete unit. Taking a cross-section of Lake Superior between Duluth and Michipicoten, which is approximately at right angles to the expected iso-lines of crustal movement produces the result that the mean lake surface must be rising at a rate of around 0.8 foot per 100 years.

If, on the other hand, the lake were in a natural condition the following analysis would hold: The lake outlet (assume that the gauge at Point Iroquois represents lake outlet conditions) is rising due to crustal movement relative to the southwest end of the lake. Assuming that a simple weir relationship controls the outflow of Lake Superior then the increase in outlet elevation will decrease the lake outflows. The outflow will remain decreased until the level of Lake Superior has risen sufficiently to restore outlet conditions. Thus, to flow the same long-term mean outflow, which is required by continuity considerations, the mean lake level must be continuously rising to counteract the effects of crustal movement. In practice the crustal movement effect will act continuously and the mean lake level will respond continuously without any significant change in outflow being noticed. This means that the mean level of Lake Superior must be increasing at a rate equal in magnitude and direction to the rate at which the land around the lake outlet is rising relative to some absolute datum. On Lake Superior, Point Iroquois is rising relative to Duluth - Two Harbors at a rate of around 0.80 foot per 100 years and so this must be the rate of increase of the mean lake level.

Concluding this section, the first method; finding first order linear trends in gauge records showed a rise in mean lake level of around 0.7 foot per 100 years.

Depending on the degree of regulation involved, either of the second or third methods could be used. In this case both methods give the same result, around 0.8 foot per 100 years. An independent investigation by Rowe (1969) using 5-year mean elevations at Marquette agrees quite well with these results.

### D) Determination of rate of movement of the land relative to the mean lake surface.

Combining the absolute rates of movement derived in Section B with the rate of movement of the mean lake level derived in Section C resulted in a set of rates of movement of points around the lake relative to the mean lake level, as shown in Table 35.

### E) Comparison with other evidence

#### Introduction

The chief purpose of the report has been to compute rates of crustal movement for points around Lake Superior. This was achieved by computing relative rates of movement around the lake, referring these to an absolute level and accounting for the change in lake level.

As mentioned in the introduction it is very difficult to interpolate the absolute movements computed for two lakes to obtain movements between the lakes. This is because river level gauges cannot be used and land levelling is not of sufficient accuracy. Information from other geophysical phenomena may be of use in bridging the gaps between lake gauges.

The few following paragraphs summarise the information used in this comparison. It should be emphasised that this comparison was made to fill in gaps in engineering knowledge and therefore is, from any other than an engineering viewpoint, probably inadequate.

#### Raised Shorelines

The most striking comparison to be made with the present results is the one with the raised shorelines observed in the Lake Superior basin. The Nipissing zero isobase has been used in this study to convert relative rates of vertical crustal movement to rates of absolute movement. The positions of other Nipissing isobases have been computed (Farrand, 1960) as shown on Figure 15. The

## Table 35

## RATES OF MOVEMENT OF LAND RELATIVE TO THE MEAN LAKE SURFACE

| STATION | RATE OF MOVEMENT FT./100 YEARS |
|---|---|
| Port Arthur | +0.34 |
| Michipicoten | +0.61 |
| Point Iroquois | -0.04 |
| Marquette | -0.40 |
| Keweenaw L.E. | -0.36 |
| Houghton | -0.41 |
| Duluth | -0.80 |
| Two Harbors | -0.80 |

## Table 36

## COMPARISON OF CRUSTAL MOVEMENT RATES COMPUTED FROM LAKE LEVEL GAUGES WITH RATES COMPUTED FROM RAISED BEACHES

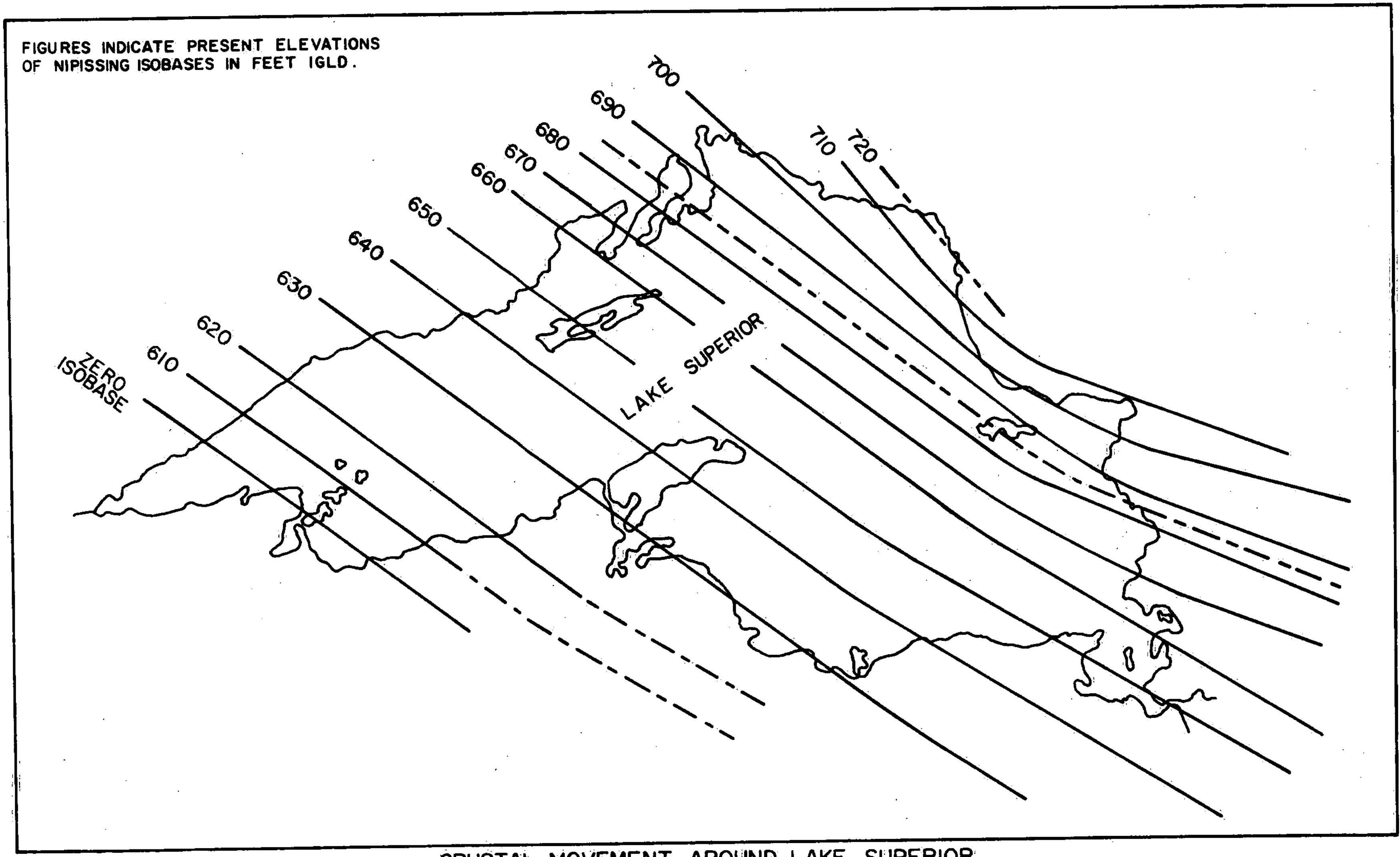
| STATION | RATE FROM LAKE LEVEL GAUGES, FT./100 YEARS* | RATE FROM RAISED BEACHES, FT./100 YEARS** |
|---|---|---|
| Port Arthur | 1.14 | 0.90 |
| Michipicoten | 1.41 | 1.83 |
| Sault Ste. Marie | 0.97 | 0.72 |
| Point Iroquois | 0.76 | 0.70 |
| Marquette | 0.40 | 0.42 |
| Keweenaw L.E. | 0.44 | 0.38 |
| Houghton | 0.39 | 0.45 |
| Duluth*** | 0.00 | 0.00 |
| Two Harbors*** | 0.00 | 0.00 |

* From table 27, the rate at Sault Ste. Marie is not considered reliable

** From isobases derived by Farrand (1960) and a date for the Nipissing beaches derived by Prest (1970).

*** South of the Nipissing zero isobase and therefore not in the region of post-Nipissing isostatic uplift.

FIGURE 15



FIGURES INDICATE PRESENT ELEVATIONS OF NIPISSING ISOBASES IN FEET IGLD.

CRUSTAL MOVEMENT AROUND LAKE SUPERIOR
LOCATION OF NIPISSING ISOBASES ( After Farrand, 1960 )

similarity of the pattern shown on Figure 15, reproduced from Farrand's work, and Figure 13 of this study is immediately apparent.

Since the directions of vertical crustal movement indicated from raised shorelines and from lake level gauges are similar, then probably the same tilting process has been operating throughout post-Nipissing time. Knowing the total uplift in the Nipissing shoreline at a given location and the approximate length of time during which this uplift has occurred, approximately 6,000 years (Prest, 1970) for the main Lake Nipissing, and assuming a linear process, an average rate of uplift at that location can be computed. Table 36 provides a comparison between rates of vertical movement computed from Farrand's 1960 shoreline data and rates of movement computed from lake level gauges in the present study. Considering that one set of figures results from 6,000 years of record and the other from less than 100 years of record, the agreement is quite satisfactory and suggests that uplift has proceeded uniformly in the Lake Superior basin at these approximate rates for several thousand years.

## Geology of faults

A map showing the generalized geological compilation of the Lake Superior region is shown in Figure 16. This figure is based on Weber and Goodacre (1966) and Innes (1960). The major faults are the Douglas and Lake Owen trending SW-NE although other faults are shown. None of the hydrometric stations used are within these two faults.

## Gravity anomalies

A gravity anomaly is the difference between the acceleration of gravity measured at some point on the surface of the earth ($g_m$) and theoretical gravity computed for the same point ($g_t$). Gravity anomalies are usually measured in milligals, where 1 milligal is .001 cms/sec$^2$, so that g is about 980,000 milligals. Modern gravimeters can measure differences in the order of one tenth of a milligal.

Researchers commonly use three types of gravity anomaly depending on the assumptions used in computing theoretical gravity. Before any of these assumptions are used normal gravity at sea level corresponding to the latitude of the station in question must be computed,

$$\gamma_0 = 978.049\ (1 + 0.0052884 \sin^2\phi - 0.0000059 \sin^2 2\phi)\ \text{cm/sec}^2. \qquad (39)$$

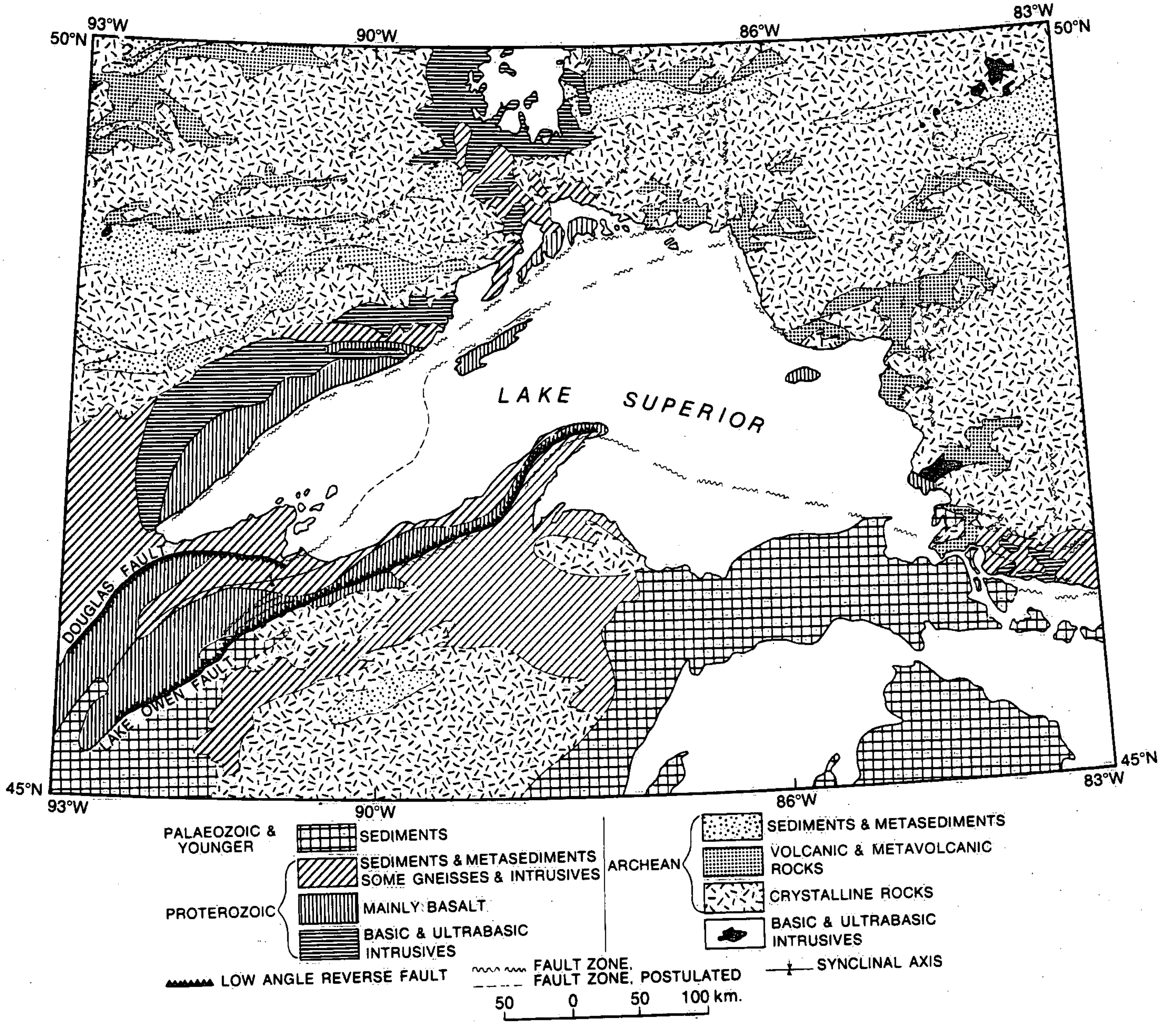


FIGURE 16

GENERALIZED GEOLOGICAL COMPILATION OF LAKE SUPERIOR REGION where $\phi$ is the latitude of the station.

The simplest form of anomaly, the free air method of reduction, modifies $\gamma_o$ to account for the elevation of the station above sea level. Theoretical gravity as computed by this method is then given by:

$$g_t = \gamma_o - (0.09409 + 0.00007 \cos 2\phi) \times 10^{-3}h \qquad (40)$$

where h is the elevation of the station above sea level, in feet.

A second method of obtaining $g_t$, the Bouguer method of reduction, takes into account not only the elevation of the station but the attraction of the material lying between the plane of the station and sea level. This is assumed to be represented by the attraction of an infinite sheet with a thickness equal to the elevation of the station and with an average density of 2.67. Neglecting the curvature of the earth and the effect of local topography the theoretical gravity computed by this method could be obtained by subtracting the term $0.03406 \times 10^{-3}h$ from $g_t$ as computed in the free air method.

The third and most complicated method of computing theoretical gravity involves, in addition to the two corrections previously described, a correction for isostasy. To determine theoretical gravity by the isostatic method of reduction it is necessary to compute the attraction of the topography and compensation for the whole earth. Theoretical gravity will then be given by:

$g_t = \gamma_o$ - free air effect + attraction of the topography and compensation for the whole earth.

Using any of these reduction methods the gravity anomaly is then obtained as $g_m - g_t$.

The free air anomaly is of little use in this study because it takes no account of the attraction of the ground material. The Bouguer and isostatic methods of reduction are suitable for indicating deep-seated effects such as those due to varying crustal thicknesses, incomplete isostatic recovery of the crust following glaciation etc. Crustal thickness is in general related to both surface elevation and Bouguer gravity anomaly values. Where the thickness of the crust is found to be greater than normal the regional free-air and isostatic anomalies are usually positive, and where the thickness of the crust is found to be less than normal the regional free-air and isostatic anomaly values are usually negative. These observations, however, do not correspond to the results which would be expected on the assumption that the isostatic anomalies represent a departure of a homogeneous crust and mantle from isostasy. On this assumption a positive anomaly would indicate undercompensation due to a deficiency in compensating mass (too thin a crust).

An examination of maps of isostatic anomaly distribution computed for the Lake Superior region revealed no pattern which could be correlated with the results of this study. Recent work by Walcott (1971), amongst others, does indicate that the region of maximum uplift is associated with an extensive free air anomaly.

### Earthquake activity

Maps showing lines of equal acceleration as percent of g, the gravitational attraction, with 100 year average return period have been prepared for various regions of North America. For the Lake Superior region the maps by Milne and Davenport (1969) are good examples. The relationship used between A, the acceleration amplitude, and P, the probability that A will not be exceeded in any given year is a double exponential extreme value distribution of the form:

$$\log_e^A = V - \frac{1}{a} (\log_e (-\log_e P)) \tag{42}$$

where V and 1/a are constants which depend on the location.

There is insufficient data available on earthquake activity in the Lake Superior region to make any comparison with other phenomena although any correspondence seems unlikely.

## CONCLUSIONS

This report presents a methodology developed to compute the magnitude of crustal movement around the Great Lakes. The basic data used are long-term records of lake elevations and geologic indications of the southern limits of post-glacial uplift. The development of the methodology for gauges around Lake Superior leads to the following conclusions:

(a) Time series created by taking differences in mean monthly lake elevations at two gauges are generally made up of the following three components:

   (1) A dominant first order linear trend with less significant higher order linear trends.

   (2) Periodicity in the mean and standard deviation, chiefly the annual and six-monthly cycles.

   (3) A residual.

(b) In the test case analysed in this study, Marquette minus Duluth, the variance of the time series was found to be divided as follows:

   (1) Trends - 48%
   (2) Periodic components - 18%
   (3) Residual - 33%

(c) From the results of the test analysis it was concluded that the time series made up of differences in mean monthly lake elevations can be adequately represented by first order linear trends for all practical purposes.

(d) Rates of relative movement computed by previous investigators are generally correct for those pairs of gauges having periods of common record of around 20 years or more. Gauges with records of less than this can give very inaccurate results.

(e) For Lake Superior the Nipissing zero isobase can be used as a datum to convert the relative movements between gauging stations to absolute movements (see Figure 13).

(f) By computing the movement of the mean lake surface the change in position of land around the lake relative to the mean lake surface can be computed.

(g) Very good agreement was found between the rates of absolute movement computed in this study and rates of movement computed from the present elevations of Nipissing shorelines. This indicates that the post-Nipissing uplift has proceeded uniformly in the Lake Superior basin, at approximately the rates measured today, for several thousand years. No similarities were found in this study between the rates of absolute crustal movement as determined in this study and the computed values of other phenomena such as gravity anomalies, earthquake frequency or positioning of geologic structures within the Great Lakes area.

(h) The results presented in this study reflect current knowledge and current data. In two or three years time it will be possible to verify the absolute rates of crustal movement used in this study. A second set of levels from Father Point to the Great Lakes will have been run by that time and, within the limits set by the accuracy of levelling and the small time period between sets of levels (15 years) it will be possible to compute absolute changes in elevation of key points around the lake.

(i) The current practice of correcting recorded elevations at Marquette and Duluth for purposes of regulating Lake Superior lowers the computed mean lake level. This results in lower regulated outflows and correspondingly increases the risk of flooding in the southwestern end of Lake Superior.

(j) The effects of crustal movement on Lake Superior power, navigation and shore property interests in the future, assuming a continuation of present conditions would be as follows:

| | | |
|---|---|---|
| power | - | insignificant effect, since the mean lake level is rising approximately at the same rate as the land near the outlet and there would therefore be no change in available head. |
| navigation | - | the benefits to navigation would be mixed, Duluth-Superior Harbor and Marquette would |

have increased harbor depths, while Thunder Bay and Michipicoten would have decreased depths. Channel depth in the St. Mary's River will remain relatively unchanged.

Canadian Shoreline

The Canadian shoreline is generally rising relative to mean lake level and the following effects might be expected: -

| | | |
|---|---|---|
| erosion | - | decrease in damages through lower water levels. |
| inundation | - | decrease in damages through higher water levels occurring less frequently. |
| recreation beaches | -<br>- | benefit since more beach area will be exposed. |
| recreation boating | - | increase in costs, more dredging will be needed to counteract the effect of shallower water. |
| water supply | - | increase in costs since water water need to be pumped through a greater head. |
| sewer outfalls | - | decrease in costs because of the greater head available; modified, to some extent, by the need to provide longer sewer lines. |

United States Shoreline

The U.S. shoreline is generally falling relative to mean lake level and in the above examples, the consequences of crustal movement would be reversed.

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