FORMULAE AND TABLES FOR COMPUTING AND PLOTTING DROUGHT FREQUENCY CURVES

TECHNICAL BULLETIN No. 8

W.Q.CHIN

INLAND WATERS BRANCH
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ACKNOWLEDGEMENTS

Tables 1 to 4 have been collated from various published and unpublished sources. Table 2 is from Gumbel (1954). Table 4 is from Coulson (1966). Tables 1 and 3a are contained in any statistical and mathematical text. Table 3b is from Kalinin (1960).

FORMULAE AND TABLES FOR COMPUTING AND PLOTTING DROUGHT FREQUENCY CURVES

INTRODUCTION

Gumbel's limited distribution of the smallest values and the Pearson Type III distribution are two theoretical probability distributions which have been considered as being representative of the probability distribution of low flows or droughts.

The formulae and tables presented herein have been compiled to facilitate the computation and plotting of drought frequency curves using either of these two theoretical probability distributions. For details and the derivations of the various formulae reference may be made to Gumbel (1954) and Matalas (1963).

The formulae which have general applicability are given first, followed by outlines of the procedures for curve fitting; (1) graphically, (2) assuming Gumbel's limited distribution of the smallest values, and (3) assuming Pearson Type III distribution.

DEFINITION OF DROUGHT

The proposed methods for determining the probability distribution of drought flows apply to minimum annual discharges or minimum discharges in a specified season.

The annual (or seasonal) drought, x, can be defined as the minimum instantaneous discharge or the minimum mean discharge for any selected period of time such as one day, five days, a week, or longer. In most practical problems the minimum instantaneous discharge is not too useful. Even the minimum daily discharge is usually not too important since most users of water can withstand a one-day drought. The selection should be dictated by the problem at hand.

MOMENTS OF THE DISTRIBUTION

The moments of the distribution are measures to characterize the distribution. When a random sample is taken from a specified population there are two sets of descriptive measures involved: those assocated with the population and those calculated for the sample.

The following formulae are the best estimators of the population values as derived from the sample (Matalas, 1963).

First Moment about the Origin

This is a measure of location and is defined as the mean value of the variable:

$$\bar{x} = \frac{1}{N} \Sigma x = \text{mean drought} \qquad (1)$$

where x = an annual (or seasonal) drought in a series of observations

and N = sample size or number of years (or seasons) in the series.

Second Moment about the Mean

This is a measure of scale and is defined as the variance of a random sample:

$$S^2 = \frac{1}{N-1} \Sigma (x-\bar{x})^2 = \text{variance} \qquad (2)$$

$$\text{thus } S = \sqrt{\frac{\Sigma (x-\bar{x})^2}{N-1}} = \text{standard deviation} \qquad (3)$$

Third Moment about the Mean

$$m_3 = \frac{1}{N} \Sigma (x-\bar{x})^3 = \text{third moment} \qquad (4)$$

SKEWNESS

Skewness is the degree of asymmetry, or departure from symmetry, of a distribution. There are several formulae for measure of skewness.

The most common measure of skewness uses the third moment about the mean and is expressed in dimensionless form as:

$$g_1 = \frac{N^2}{(N-1)(N-2)} \frac{m_3}{S^3} = \text{coefficient of skew} \qquad (5)$$

GRAPHICAL DISTRIBUTION-FREE METHOD FOR DETERMINING EXPECTED PROBABILITIES

By assigning probability values to each of the ordered observations, a graphical distribution-free method may be used to estimate the frequency curve directly by plotting the data on suitable paper and drawing a best-fit curve by eye through the points. The plotting positions are computed from the formula:

$$P(x) = \frac{M}{N+1} = \text{probability of a drought equal to or more severe than the observation.} \quad (6)$$

where M = the order of the observation when the observations are arranged in increasing value, the smallest drought flow being assigned the value M = 1.

Table 4 lists all values of P(x) in samples ranging in size from N = 10 to N = 60 years. Also listed are values of the corresponding recurrence interval, T, computed from the formula:

$$T = \frac{1}{P(x)} = \text{recurrence interval} \quad (7)$$

GUMBEL'S LIMITED DISTRIBUTION OF THE SMALLEST VALUES

The density function describing the third asymptotic distribution of extreme values is:

$$p(x) = \frac{\alpha}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha-1} \exp\left[-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha}\right] \quad (8)$$

where p(x) = the probability density of drought flows, x;

u = the characteristic drought flow corresponding to a probability of 0.633 or a recurrence interval of 1.58 years;

ε = the lower limit of drought flow;

α = a scale parameter (analagous to the standard deviation of a normal distribution).

Equation 8 may be integrated from ε to x to obtain the probability, P(x), of drought flows being equal to or less than x (see Appendix A). The result of the integration is given in the following equation:

$$P(x) = 1 - e^{-y} \quad (9)$$

where y is an arbitrary transformation introduced to simplify the expression - it is defined as follows:

$$y = \left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha} \quad (10)$$

As a matter of convenience in carrying out the computations described below, Equation 10 may be re-written in the following form:

$$\log (x-\varepsilon) = \log (u-\varepsilon) + \frac{1}{\alpha} \log y. \quad (11)$$

Equation 9 is the equation for a cumulative drought frequency curve, assuming the droughts conform to the third asymptotic distribution of extreme values. The frequency curve may be constructed by assigning a series of arbitrary selected values to P(x) and computing the equivalent values of x, using Equations 9 and 11.

The computations are carried out by first computing a series of values of y (and hence log y) for the selected values of P(x), using Equation 9. The derived values for log y are then substituted in Equation 11, and the equation solved for the equivalent values of x. A preliminary requisite is the computation of values for the parameters u, ε and $1/\alpha$ for use in solving Equation 11. These values are derived from the sample of observed droughts.

The derived x values are plotted as ordinates at the corresponding P(x) values on logarithmic extremal probability paper and a curve drawn through the plotted points to produce the required drought frequency curve. The various steps in the computations are outlined in more detail in the following section.

PROCEDURES FOR CONSTRUCTION OF DROUGHT FREQUENCY CURVE

1. Compute values for $\bar{x}$, S, m_3 and g_1 from the record of observed droughts, using Equations 1, 3, 4 and 5 respectively.

2. Read the value for $1/\alpha$ from Table 2 opposite the appropriate value for g_1. The table is derived from the fact that there is a unique value for $1/\alpha$ for a given value for g_1 (Gumbel 1954).

3. Also from Table 2, read values for two new parameters, A_α and B_α, opposite the appropriate value for g_1. These new parameters are required in subsequent computations.

Like $1/\alpha$, their values depend only on g_1, the coefficient of skew estimated from the observed droughts.

4. Compute values for parameters u and ε from the following equations:

$$u = \bar{x} + A_\alpha S \quad (12)$$

$$\varepsilon = u - B_\alpha S \quad (13)$$

5. For a series of selected values for P(x) compute the equivalent values of y and then log y, using Equation 9. Or, as a matter of convenience, read the required log y values from Table 1.

6. Using Equation 11 and substituting in it the derived values for parameters $1/\alpha$, ε and u, compute the equivalent values for x for the derived log y values. The computations for steps 5 and 6 are usually set up in tabular form, as indicated in the example in Appendix 2.

7. Plot the derived x values as ordinates at the appropriate P(x) values on logarithmic extremal probability paper (log Gumbel paper) and draw a smooth curve, or straight line where possible, through the points to produce the required drought frequency curve.

8. The goodness of fit of the theoretical distribution can be checked by plotting the observed drought values using either Equation 6 or Equation 7 to determine the plotting positions.

Discussion

The theoretical distribution is considered to be applicable if its estimated lower limit ε does not exceed the observed minimum low flow during the period of record. Also it is considered necessary that the estimated lower limit does not take on a large negative value. If ε is negative and small it can safely be assumed to be zero (Gumbel 1954). The theory fails if the lower limit turns out to be larger than the observed smallest drought, providing the observation is reliable.

If the frequency data are plotted on logarithmic extremal probability paper and a straight line can be fitted to the data, the limiting value ε may be assumed to be zero (Gumbel 1954).

PEARSON TYPE III DISTRIBUTION

The Pearson Type III distribution can be expressed in various ways. A representation of this distribution is given by:

$$p(x) = \frac{1}{a\Gamma(b+1)} \left(\frac{x-m}{a}\right)^b \exp\left[-\left(\frac{x-m}{a}\right)\right] \quad (14)$$

where a , m and b are parameters.

With appropriate mathematical treatment and transformation (Matalas 1963), the frequency curve for the Pearson Type III distribution may be computed from the formula:

$$x = \bar{x} + KS \quad (15)$$

where K = the frequency factor for a probability values P(x) and coefficient of skew g_1.

The value of the frequency factor K may be obtained from the following formulae and Table 3-a, which gives the percentile values $\chi^2(P,v)$ for the chi-square distribution with v degrees of freedom:

$$K = \frac{\chi^2(P,v)}{2C} - C \quad (16)$$

where $v = \frac{8}{g_1^2}$ = degrees of freedom (17)

and $C = \frac{2}{g_1}$ = a constant (18)

However, values of the K have been computed and tabulated and one such tabulation is given in Table 3b.

SUMMARY OF PROCEDURE

Compute values of $\bar{x}$, S, m_3, g_1, v and C from formulae (1), (3), (4), (5), (17) and (18), respectively. Then from Table 3-a, the values of $\chi^2(P,v)$ for preassigned values of P(x) are read. The corresponding values of the frequency factor K and the drought x are computed from the formulae (16), and (15) respectively. Alternatively, values of K for the appropriate value of g_1 are read directly from Table 3-b.

EXAMPLES OF THE COMPUTATION AND PLOTTING OF A DROUGHT FREQUENCY CURVE

To illustrate the use of the formulae and tables, the drought frequency curve for the Salmo River near Salmo, B.C., is developed in Appendix 2. To facilitate the plotting of the curve on various types of paper, both the P values and T values are given in all computations. Figure 1 shows a plot of the results on logarithmic extremal probability paper.

TABLE I

REDUCED VARIATE (log y) FOR PROBABILITY P(x)

where $P(x) = 1 - e^{-y}$

Recurrence Interval $T = 1/P(x)$	Probability $P(x)$	Reduced Variate $\log y$
1.01	0.990	0.663
1.05	0.952	0.482
1.10	0.909	0.380
1.20	0.833	0.253
1.30	0.769	0.166
1.40	0.714	0.098
1.50	0.667	0.041
1.58	0.633	0.000
2.00	0.500	-0.159
3.00	0.333	-0.393
4.00	0.250	-0.541
5.00	0.200	-0.652
10.0	0.100	-0.979
15.0	0.067	-1.155
20.0	0.050	-1.292
25.0	0.040	-1.387
30.0	0.033	-1.469
40.0	0.025	-1.602
50.0	0.020	-1.699
75.0	0.013	-1.886
100	0.010	-2.000

TABLE 2

VALUES OF $1/\alpha$, A_α AND B_α

Coefficient of Skew	Scale Parameter	Frequency Factors		Coefficient of Skew	Scale Parameter	Frequency Factors	
g_1	$1/\alpha$	A_α	B_α	g_1	$1/\alpha$	A_α	B_α
				1.054	0.66	0.162	1.649
				1.081	0.67	0.157	1.623
0.007	0.28	0.355	3.573	1.107	0.68	0.152	1.598
0.038	0.29	0.350	3.468	1.134	0.69	0.147	1.573
0.069	0.30	0.346	3.370	1.160	0.70	0.142	1.549
0.099	0.31	0.341	3.277	1.187	0.71	0.136	1.526
0.129	0.32	0.336	3.190	1.214	0.72	0.131	1.503
0.158	0.33	0.331	3.108	1.240	0.73	0.126	1.480
0.188	0.34	0.327	3.030	1.267	0.74	0.121	1.458
0.217	0.35	0.322	2.955	1.294	0.75	0.116	1.436
0.245	0.36	0.317	2.885	1.321	0.76	0.111	1.415
0.274	0.37	0.312	2.818	1.348	0.77	0.106	1.394
0.302	0.38	0.307	2.754	1.375	0.78	0.101	1.374
0.331	0.39	0.302	2.692	1.402	0.79	0.096	1.354
0.359	0.40	0.297	2.634	1.430	0.80	0.092	1.334
0.386	0.41	0.292	2.578	1.457	0.81	0.087	1.314
0.414	0.42	0.287	2.524	1.484	0.82	0.082	1.295
0.442	0.43	0.282	2.472	1.512	0.83	0.077	1.276
0.469	0.44	0.277	2.422	1.540	0.84	0.072	1.258
0.496	0.45	0.271	2.374	1.567	0.85	0.067	1.240
0.523	0.46	0.266	2.328	1.595	0.86	0.063	1.222
0.551	0.47	0.261	2.284	1.623	0.87	0.058	1.204
0.577	0.48	0.256	2.241	1.651	0.88	0.053	1.187
0.604	0.49	0.251	2.199	1.680	0.89	0.049	1.170
0.631	0.50	0.246	2.159	1.708	0.90	0.044	1.154
0.658	0.51	0.240	2.120	1.737	0.91	0.040	1.137
0.684	0.52	0.235	2.082	1.765	0.92	0.035	1.121
0.711	0.53	0.230	2.045	1.794	0.93	0.031	1.105
0.738	0.54	0.225	2.009	1.823	0.94	0.026	1.089
0.764	0.55	0.219	1.975	1.852	0.95	0.022	1.074
0.790	0.56	0.214	1.941	1.881	0.96	0.017	1.059
0.817	0.57	0.209	1.909	1.911	0.97	0.013	1.044
0.843	0.58	0.204	1.877	1.940	0.98	0.009	1.029
0.870	0.59	0.199	1.846	1.970	0.99	0.004	1.014
0.896	0.60	0.193	1.815	2.000	1.00	0.0	1.000
0.922	0.61	0.188	1.786	2.309	1.1	-0.040	0.867
0.949	0.62	0.183	1.757	2.640	1.2	-0.077	0.752
0.975	0.63	0.178	1.729	2.996	1.3	-0.109	0.652
1.002	0.64	0.172	1.702	3.382	1.4	-0.136	0.563
1.028	0.65	0.167	1.675	3.802	1.5	-0.160	0.486

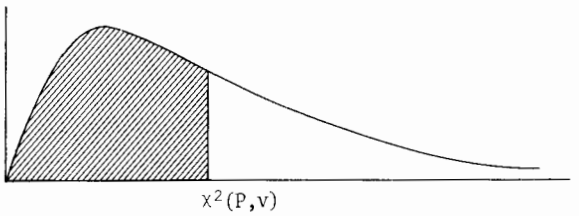
TABLE 2 (Cont.)

VALUES OF $1/\alpha$, A_α and B_α

Coefficient of Skew	Scale Parameter	Frequency Factors		Coefficient of Skew	Scale Parameter	Frequency Factors	
g_1	$1/\alpha$	A_α	B_α	g_1	$1/\alpha$	A_α	B_α
-1.081	0.01	0.448	78.982	-0.398	0.16	0.405	5.763
-1.025	0.02	0.446	39.989	-0.361	0.17	0.401	5.468
-0.971	0.03	0.444	26.986	-0.325	0.18	0.397	5.205
-0.918	0.04	0.442	20.481	-0.289	0.19	0.393	4.969
-0.868	0.05	0.439	16.574	-0.254	0.20	0.389	4.755
-0.819	0.06	0.437	13.967	-0.220	0.21	0.385	4.561
-0.772	0.07	0.434	12.103	-0.186	0.22	0.381	4.383
-0.726	0.08	0.431	10.702	-0.152	0.23	0.377	4.221
-0.681	0.09	0.428	9.611	-0.120	0.24	0.372	4.071
-0.638	0.10	0.425	8.737	-0.087	0.25	0.368	3.933
-0.595	0.11	0.422	8.020	-0.055	0.26	0.364	3.804
-0.554	0.12	0.419	7.421	-0.024	0.27	0.359	3.684
-0.514	0.13	0.415	6.913				
-0.474	0.14	0.412	6.476				
-0.436	0.15	0.408	6.097				

TABLE 3-a

PERCENTILE VALUES [$\chi^2(P,\nu)$]
for
THE CHI-SQUARE DISTRIBUTION
with ν degrees of freedom
(shaded area = P)



ν	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.75}$	$\chi^2_{.50}$	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$
1	7.88	6.63	5.02	3.84	2.71	1.32	.455	.102	.0158	.0039	.0010	.0002	.0000
2	10.6	9.21	7.38	5.99	4.61	2.77	1.39	.575	.211	.103	.0506	.0201	.0100
3	12.8	11.3	9.35	7.81	6.25	4.11	2.37	1.21	.584	.352	.216	.115	.072
4	14.9	13.3	11.1	9.49	7.78	5.39	3.36	1.92	1.06	.711	.484	.297	.207
5	16.7	15.1	12.8	11.1	9.24	6.63	4.35	2.67	1.61	1.15	.831	.554	.412
6	18.5	16.8	14.4	12.6	10.6	7.84	5.35	3.45	2.20	1.64	1.24	.872	.676
7	20.3	18.5	16.0	14.1	12.0	9.04	6.35	4.25	2.83	2.17	1.69	1.24	.989
8	22.0	20.1	17.5	15.5	13.4	10.2	7.34	5.07	3.49	2.73	2.18	1.65	1.34
9	23.6	21.7	19.0	16.9	14.7	11.4	8.34	5.90	4.17	3.33	2.70	2.09	1.73
10	25.2	23.2	20.5	18.3	16.0	12.5	9.34	6.74	4.87	3.94	3.25	2.56	2.16
11	26.8	24.7	21.9	19.7	17.3	13.7	10.3	7.58	5.58	4.57	3.82	3.05	2.60
12	28.3	26.2	23.3	21.0	18.5	14.8	11.3	8.44	6.30	5.23	4.40	3.57	3.07
13	29.8	27.7	24.7	22.4	19.8	16.0	12.3	9.30	7.04	5.89	5.01	4.11	3.57
14	31.3	29.1	26.1	23.7	21.1	17.1	13.3	10.2	7.79	6.57	5.63	4.66	4.07
15	32.8	30.6	27.5	25.0	22.3	18.2	14.3	11.0	8.55	7.26	6.26	5.23	4.60
16	34.3	32.0	28.8	26.3	23.5	19.4	15.3	11.9	9.31	7.96	6.91	5.81	5.14
17	35.7	33.4	30.2	27.6	24.8	20.5	16.3	12.8	10.1	8.67	7.56	6.41	5.70
18	37.2	34.8	31.5	28.9	26.0	21.6	17.3	13.7	10.9	9.39	8.23	7.01	6.26
19	38.6	36.2	32.9	30.1	27.2	22.7	18.3	14.6	11.7	10.1	8.91	7.63	6.84
20	40.0	37.6	34.2	31.4	28.4	23.8	19.3	15.5	12.4	10.9	9.59	8.26	7.43
21	41.4	38.9	35.5	32.7	29.6	24.9	20.3	16.3	13.2	11.6	10.3	8.90	8.03
22	42.8	40.3	36.8	33.9	30.8	26.0	21.3	17.2	14.0	12.3	11.0	9.54	8.64
23	44.2	41.6	38.1	35.2	32.0	27.1	22.3	18.1	14.8	13.1	11.7	10.2	9.26
24	45.6	43.0	39.4	36.4	33.2	28.2	23.3	19.0	15.7	13.8	12.4	10.9	9.89
25	46.9	44.3	40.6	37.7	34.4	29.3	24.3	19.9	16.5	14.6	13.1	11.5	10.5
26	48.3	45.6	41.9	38.9	35.6	30.4	25.3	20.8	17.3	15.4	13.8	12.2	11.2
27	49.6	47.0	43.2	40.1	36.7	31.5	26.3	21.7	18.1	16.2	14.6	12.9	11.8
28	51.0	48.3	44.5	41.3	37.9	32.6	27.3	22.7	18.9	16.9	15.3	13.6	12.5
29	52.3	49.6	45.7	42.6	39.1	33.7	28.3	23.6	19.8	17.7	16.0	14.3	13.1
30	53.7	50.9	47.0	43.8	40.3	34.8	29.3	24.5	20.6	18.5	16.8	15.0	13.8
40	66.8	63.7	59.3	55.8	51.8	45.6	39.3	33.7	29.1	26.5	24.4	22.2	20.7
50	79.5	76.2	71.4	67.5	63.2	56.3	49.3	42.9	37.7	34.8	32.4	29.7	28.0
60	92.0	88.4	83.3	79.1	74.4	67.0	59.3	52.3	46.5	43.2	40.5	37.5	35.5
70	104.2	100.4	95.0	90.5	85.5	77.6	69.3	61.7	55.3	51.7	48.8	45.4	43.3
80	116.3	112.3	106.6	101.9	96.6	88.1	79.3	71.1	64.3	60.4	57.2	53.5	51.2
90	128.3	124.1	118.1	113.1	107.6	98.6	89.3	80.6	73.3	69.1	65.6	61.8	59.2
100	140.2	135.8	129.6	124.3	118.5	109.1	99.3	90.1	82.4	77.9	74.2	70.1	67.3

TABLE 3-b

VALUES OF K AS FUNCTION OF g_1 IN THE PEARSON TYPE III DISTRIBUTION

g_1	Probability																	g_1
	.99	.97	.95	.90	.80	.75	.70	.60	.50	.40	.30	.25	.20	.10	.05	.03	.01	
0.00	2.33	1.88	1.64	1.28	0.84	0.67	0.52	0.25	0.00	-0.25	-0.52	-0.67	-0.84	-1.28	-1.64	-1.88	-2.33	0.00
0.05	2.36	1.90	1.65	1.28	0.84	0.66	0.52	0.24	-0.01	-0.26	-0.52	-0.68	-0.84	-1.28	-1.62	-1.86	-2.29	0.05
0.10	2.40	1.92	1.67	1.29	0.84	0.66	0.51	0.24	-0.02	-0.27	-0.53	-0.68	-0.85	-1.27	-1.61	-1.84	-2.25	0.10
0.15	2.44	1.94	1.68	1.30	0.84	0.66	0.50	0.23	-0.02	-0.28	-0.54	-0.68	-0.85	-1.26	-1.60	-1.82	-2.22	0.15
0.20	2.47	1.96	1.70	1.30	0.83	0.65	0.50	0.22	-0.03	-0.28	-0.55	-0.69	-0.85	-1.26	-1.58	-1.79	-2.18	0.20
0.25	2.50	1.98	1.71	1.30	0.82	0.64	0.49	0.21	-0.04	-0.29	-0.56	-0.70	-0.85	-1.25	-1.56	-1.77	-2.14	0.25
0.30	2.54	2.00	1.72	1.31	0.82	0.64	0.48	0.20	-0.05	-0.30	-0.56	-0.70	-0.85	-1.24	-1.55	-1.75	-2.10	0.30
0.35	2.58	2.02	1.73	1.32	0.82	0.64	0.48	0.20	-0.06	-0.30	-0.56	-0.70	-0.85	-1.24	-1.53	-1.72	-2.06	0.35
0.40	2.61	2.04	1.75	1.32	0.82	0.63	0.47	0.19	-0.07	-0.31	-0.57	-0.71	-0.85	-1.23	-1.52	-1.70	-2.03	0.40
0.45	2.64	2.06	1.76	1.32	0.82	0.62	0.46	0.18	-0.08	-0.32	-0.58	-0.71	-0.85	-1.22	-1.51	-1.68	-2.00	0.45
0.50	2.68	2.08	1.77	1.32	0.81	0.62	0.46	0.17	-0.08	-0.33	-0.58	-0.71	-0.85	-1.22	-1.49	-1.66	-1.96	0.50
0.55	2.72	2.10	1.78	1.32	0.80	0.62	0.45	0.16	-0.09	-0.34	-0.58	-0.72	-0.85	-1.21	-1.47	-1.64	-1.92	0.55
0.60	2.75	2.12	1.80	1.33	0.80	0.61	0.44	0.16	-0.10	-0.34	-0.59	-0.72	-0.85	-1.20	-1.45	-1.61	-1.88	0.60
0.65	2.78	2.14	1.81	1.33	0.80	0.60	0.44	0.15	-0.11	-0.35	-0.60	-0.72	-0.85	-1.19	-1.44	-1.59	-1.84	0.65
0.70	2.82	2.15	1.82	1.33	0.79	0.59	0.43	0.14	-0.12	-0.36	-0.60	-0.72	-0.85	-1.18	-1.42	-1.57	-1.81	0.70
0.75	2.86	2.16	1.83	1.34	0.78	0.58	0.42	0.13	-0.12	-0.36	-0.60	-0.72	-0.86	-1.18	-1.40	-1.54	-1.78	0.75
0.80	2.89	2.18	1.84	1.34	0.78	0.58	0.41	0.12	-0.13	-0.37	-0.60	-0.73	-0.86	-1.17	-1.38	-1.52	-1.74	0.80
0.85	2.92	2.20	1.85	1.34	0.78	0.58	0.40	0.12	-0.14	-0.38	-0.60	-0.73	-0.86	-1.16	-1.36	-1.49	-1.70	0.85
0.90	2.96	2.22	1.86	1.34	0.77	0.57	0.40	0.11	-0.15	-0.38	-0.61	-0.73	-0.85	-1.15	-1.35	-1.47	-1.66	0.90
0.95	2.99	2.24	1.87	1.34	0.76	0.56	0.39	0.10	-0.16	-0.38	-0.62	-0.73	-0.85	-1.14	-1.34	-1.44	-1.62	0.95
1.00	3.02	2.25	1.88	1.34	0.76	0.55	0.38	0.09	-0.16	-0.39	-0.62	-0.73	-0.85	-1.13	-1.32	-1.42	-1.59	1.00
1.05	3.06	2.26	1.88	1.34	0.75	0.54	0.37	0.08	-0.17	-0.40	-0.62	-0.74	-0.85	-1.12	-1.30	-1.40	-1.56	1.05
1.10	3.09	2.28	1.89	1.34	0.74	0.54	0.36	0.07	-0.17	-0.41	-0.62	-0.74	-0.85	-1.10	-1.28	-1.38	-1.52	1.10
1.15	3.12	2.30	1.90	1.34	0.74	0.53	0.36	0.06	-0.18	-0.42	-0.62	-0.74	-0.84	-1.09	-1.26	-1.36	-1.48	1.15
1.20	3.15	2.31	1.91	1.34	0.73	0.52	0.35	0.05	-0.19	-0.42	-0.63	-0.74	-0.84	-1.08	-1.24	-1.33	-1.45	1.20
1.25	3.18	2.32	1.92	1.34	0.72	0.52	0.34	0.04	-0.20	-0.42	-0.63	-0.74	-0.84	-1.07	-1.22	-1.30	-1.42	1.25
1.30	3.21	2.34	1.92	1.34	0.72	0.51	0.33	0.04	-0.21	-0.43	-0.63	-0.74	-0.84	-1.06	-1.20	-1.28	-1.38	1.30
1.35	3.24	2.36	1.93	1.34	0.72	0.50	0.32	0.03	-0.22	-0.44	-0.64	-0.74	-0.84	-1.05	-1.18	-1.26	-1.35	1.35
1.40	3.27	2.37	1.94	1.34	0.71	0.49	0.31	0.02	-0.22	-0.44	-0.64	-0.73	-0.83	-1.04	-1.17	-1.23	-1.32	1.40
1.45	3.30	2.38	1.94	1.34	0.70	0.48	0.30	0.01	-0.23	-0.44	-0.64	-0.73	-0.82	-1.03	-1.15	-1.21	-1.29	1.45
1.50	3.33	2.39	1.95	1.33	0.69	0.47	0.30	0.00	-0.24	-0.45	-0.64	-0.73	-0.82	-1.02	-1.13	-1.19	-1.26	1.50
1.55	3.36	2.40	1.96	1.33	0.69	0.46	0.29	-0.01	-0.24	-0.46	-0.64	-0.73	-0.82	-1.00	-1.12	-1.16	-1.23	1.55
1.60	3.39	2.42	1.96	1.33	0.68	0.46	0.28	-0.02	-0.25	-0.46	-0.64	-0.73	-0.81	-0.99	-1.10	-1.14	-1.20	1.60
1.65	3.42	2.43	1.96	1.32	0.67	0.45	0.27	-0.02	-0.26	-0.46	-0.64	-0.72	-0.81	-0.98	-1.08	-1.12	-1.17	1.65
1.70	3.44	2.44	1.97	1.32	0.66	0.44	0.26	-0.03	-0.27	-0.47	-0.64	-0.72	-0.81	-0.97	-1.06	-1.10	-1.14	1.70
1.75	3.47	2.45	1.98	1.32	0.65	0.43	0.25	-0.04	-0.28	-0.48	-0.64	-0.72	-0.80	-0.96	-1.04	-1.08	-1.12	1.75
1.80	3.50	2.46	1.98	1.32	0.64	0.42	0.24	-0.05	-0.28	-0.48	-0.64	-0.72	-0.80	-0.94	-1.02	-1.06	-1.09	1.80
1.85	3.52	2.48	1.98	1.32	0.64	0.41	0.23	-0.06	-0.28	-0.48	-0.64	-0.72	-0.80	-0.93	-1.00	-1.04	-1.06	1.85
1.90	3.55	2.49	1.99	1.31	0.63	0.40	0.22	-0.07	-0.29	-0.48	-0.64	-0.72	-0.79	-0.92	-0.98	-1.01	-1.04	1.90
1.95	3.58	2.50	2.00	1.30	0.62	0.40	0.21	-0.08	-0.30	-0.48	-0.64	-0.72	-0.78	-0.91	-0.96	-0.99	-1.02	1.95
2.00	3.60	2.51	2.00	1.30	0.61	0.39	0.20	-0.08	-0.31	-0.49	-0.64	-0.71	-0.78	-0.90	-0.95	-0.97	-0.99	2.00
2.05	3.63	2.52	2.00	1.30	0.60	0.39	0.20	-0.09	-0.32	-0.49	-0.64	-0.71	-0.77	-0.89	-0.94	-0.95	-0.96	2.05
2.10	3.65	2.53	2.00	1.29	0.60	0.38	0.19	-0.10	-0.32	-0.49	-0.64	-0.70	-0.76	-0.88	-0.93	-0.93	-0.94	2.10
2.15	3.68	2.54	2.01	1.28	0.59	0.38	0.18	-0.10	-0.32	-0.49	-0.63	-0.70	-0.76	-0.86	-0.92	-0.92	-0.92	2.15
2.20	3.70	2.55	2.01	1.28	0.58	0.37	0.17	-0.11	-0.33	-0.49	-0.63	-0.69	-0.75	-0.85	-0.90	-0.90	-0.90	2.20
2.25	3.72	2.56	2.01	1.27	0.57	0.36	0.16	-0.12	-0.34	-0.49	-0.63	-0.68	-0.74	-0.83	-0.88	-0.88	-0.89	2.25
2.30	3.75	2.56	2.01	1.27	0.56	0.35	0.15	-0.12	-0.34	-0.49	-0.62	-0.68	-0.73	-0.82	-0.86	-0.86	-0.87	2.30
2.35	3.77	2.56	2.01	1.26	0.55	0.34	0.14	-0.13	-0.34	-0.50	-0.62	-0.67	-0.72	-0.81	-0.84	-0.84	-0.85	2.35
2.40	3.79	2.57	2.01	1.25	0.54	0.33	0.13	-0.14	-0.35	-0.50	-0.62	-0.66	-0.71	-0.79	-0.82	-0.82	-0.83	2.40
2.45	3.81	2.58	2.01	1.25	0.54	0.32	0.13	-0.14	-0.35	-0.50	-0.62	-0.66	-0.70	-0.78	-0.80	-0.80	-0.82	2.45
2.50	3.83	2.58	2.01	1.24	0.53	0.32	0.12	-0.15	-0.36	-0.50	-0.61	-0.65	-0.70	-0.77	-0.79	-0.79	-0.80	2.50
2.55	3.85	2.58	2.01	1.23	0.52	0.31	0.11	-0.16	-0.36	-0.50	-0.61	-0.65	-0.69	-0.75	-0.78	-0.78	-0.78	2.55
2.60	3.87	2.59	2.01	1.23	0.51	0.30	0.10	-0.17	-0.37	-0.50	-0.60	-0.64	-0.68	-0.74	-0.76	-0.76	-0.77	2.60
2.65	3.89	2.59	2.01	1.22	0.50	0.29	0.09	-0.18	-0.37	-0.50	-0.60	-0.64	-0.67	-0.73	-0.75	-0.75	-0.75	2.65
2.70	3.91	2.60	2.01	1.21	0.49	0.28	0.08	-0.18	-0.38	-0.50	-0.60	-0.63	-0.67	-0.72	-0.73	-0.73	-0.74	2.70
2.75	3.93	2.61	2.02	1.21	0.48	0.27	0.07	-0.19	-0.38	-0.50	-0.59	-0.63	-0.66	-0.71	-0.72	-0.72	-0.72	2.75
2.80	3.95	2.61	2.02	1.20	0.47	0.27	0.06	-0.20	-0.38	-0.50	-0.59	-0.62	-0.65	-0.70	-0.71	-0.71	-0.71	2.80
2.85	3.97	2.62	2.02	1.20	0.46	0.26	0.05	-0.21	-0.39	-0.50	-0.59	-0.62	-0.64	-0.69	-0.70	-0.70	-0.70	2.85
2.90	3.99	2.62	2.02	1.19	0.45	0.26	0.04	-0.21	-0.39	-0.50	-0.58	-0.61	-0.64	-0.67	-0.68	-0.68	-0.69	2.90
2.95	4.00	2.62	2.02	1.18	0.44	0.25	0.04	-0.22	-0.40	-0.50	-0.58	-0.61	-0.63	-0.66	-0.67	-0.67	-0.68	2.95
3.00	4.02	2.63	2.02	1.18	0.42	0.25	0.03	-0.23	-0.40	-0.50	-0.57	-0.60	-0.62	-0.65	-0.66	-0.66	-0.67	3.00

TABLE 4

RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

M	N = 10		N = 11		N = 12		N = 13		N = 14	
	T	P	T	P	T	P	T	P	T	P
1	11.0	.091	12.0	.083	13.0	.077	14.0	.071	15.0	.067
2	5.5	.182	6.0	.167	6.5	.154	7.0	.143	7.5	.133
3	3.67	.273	4.00	.250	4.33	.231	4.67	.214	5.0	.200
4	2.75	.364	3.00	.333	3.25	.308	3.50	.286	3.75	.267
5	2.20	.455	2.40	.417	2.60	.385	2.80	.357	3.00	.333
6	1.83	.545	2.00	.500	2.17	.462	2.33	.429	2.50	.400
7	1.57	.636	1.71	.583	1.86	.538	2.00	.500	2.14	.467
8	1.38	.727	1.50	.667	1.62	.615	1.75	.571	1.88	.533
9	1.22	.818	1.33	.750	1.44	.692	1.56	.643	1.67	.600
10	1.100	.909	1.20	.833	1.30	.769	1.40	.714	1.50	.667
11	-	-	1.091	.917	1.182	.846	1.27	.786	1.36	.733
12	-	-	-	-	1.083	.923	1.167	.857	1.25	.800
13	-	-	-	-	-	-	1.077	.929	1.154	.867
14	-	-	-	-	-	-	-	-	1.071	.933

M	N = 15		N = 16		N = 17		N = 18		N = 19	
	T	P	T	P	T	P	T	P	T	P
1	16.0	.062	17.0	.059	18.0	.056	19.0	.053	20.0	.050
2	8.0	.125	8.5	.118	9.0	.111	9.5	.105	10.0	.100
3	5.3	.188	5.7	.176	6.0	.167	6.3	.158	6.7	.150
4	4.00	.250	4.25	.235	4.50	.222	4.75	.211	5.0	.200
5	3.20	.312	3.40	.294	3.60	.278	3.80	.263	4.00	.250
6	2.67	.375	2.83	.353	3.00	.333	3.17	.316	3.33	.300
7	2.29	.438	2.43	.412	2.57	.389	2.71	.368	2.86	.350
8	2.00	.500	2.12	.471	2.25	.444	2.38	.421	2.50	.400
9	1.78	.562	1.89	.529	2.00	.500	2.11	.474	2.22	.450
10	1.60	.625	1.70	.588	1.80	.556	1.90	.526	2.00	.500
11	1.45	.688	1.55	.647	1.64	.611	1.73	.579	1.82	.550
12	1.33	.750	1.42	.706	1.50	.667	1.58	.632	1.67	.600
13	1.23	.812	1.31	.765	1.38	.722	1.46	.684	1.54	.650
14	1.143	.875	1.21	.824	1.29	.778	1.36	.737	1.43	.700
15	1.067	.938	1.133	.882	1.20	.833	1.27	.789	1.33	.750
16	-	-	1.062	.941	1.125	.889	1.188	.842	1.25	.800
17	-	-	-	-	1.059	.944	1.118	.895	1.176	.850
18	-	-	-	-	-	-	1.056	.947	1.111	.900
19	-	-	-	-	-	-	-	-	1.053	.950

TABLE 4 - RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

M	N = 20		N = 21		N = 22		N = 23		N = 24	
	T	P	T	P	T	P	T	P	T	P
1	21.0	.048	22.0	.045	23.0	.043	24.0	.042	25.0	.040
2	10.5	.095	11.0	.091	11.5	.087	12.0	.083	12.5	.080
3	7.0	.143	7.3	.136	7.7	.130	8.0	.125	8.3	.120
4	5.2	.190	5.5	.182	5.8	.174	6.0	.167	6.2	.160
5	4.20	.238	4.40	.227	4.60	.217	4.80	.208	5.0	.200
6	3.50	.286	3.67	.273	3.83	.261	4.00	.250	4.17	.240
7	3.00	.333	3.14	.318	3.29	.304	3.43	.292	3.57	.280
8	2.62	.381	2.75	.364	2.88	.348	3.00	.333	3.12	.320
9	2.33	.429	2.44	.409	2.56	.391	2.67	.375	2.78	.360
10	2.10	.476	2.20	.455	2.30	.435	2.40	.417	2.50	.400
11	1.91	.524	2.00	.500	2.09	.478	2.18	.458	2.27	.440
12	1.75	.571	1.83	.545	1.92	.522	2.00	.500	2.08	.480
13	1.62	.619	1.69	.591	1.77	.565	1.85	.542	1.92	.520
14	1.50	.667	1.57	.636	1.64	.609	1.71	.583	1.78	.560
15	1.40	.714	1.47	.682	1.53	.652	1.60	.625	1.67	.600
16	1.31	.762	1.38	.727	1.44	.696	1.50	.667	1.56	.640
17	1.24	.810	1.29	.773	1.35	.739	1.41	.708	1.47	.680
18	1.167	.857	1.22	.818	1.28	.783	1.33	.750	1.39	.720
19	1.105	.905	1.158	.864	1.21	.826	1.26	.792	1.31	.760
20	1.050	.952	1.100	.909	1.150	.870	1.200	.833	1.25	.800
21	-	-	1.048	.955	1.095	.913	1.143	.875	1.190	.840
22	-	-	-	-	1.045	.957	1.091	.917	1.136	.880
23	-	-	-	-	-	-	1.043	.958	1.087	.920
24	-	-	-	-	-	-	-	-	1.042	.960

M	N = 25		N = 26		N = 27		N = 28		N = 29	
	T	P	T	P	T	P	T	P	T	P
1	26.0	.038	27.0	.037	28.0	.036	29.0	.034	30.0	.033
2	13.0	.077	13.5	.074	14.0	.071	14.5	.069	15.0	.067
3	8.7	.115	9.0	.111	9.3	.107	9.7	.103	10.0	.100
4	6.5	.154	6.8	.148	7.0	.143	7.2	.138	7.5	.133
5	5.2	.192	5.4	.185	5.6	.179	5.8	.172	6.0	.167
6	4.33	.231	4.50	.222	4.67	.214	4.83	.207	5.0	.200
7	3.71	.269	3.86	.259	4.00	.250	4.14	.241	4.29	.233
8	3.25	.308	3.38	.296	3.50	.286	3.62	.276	3.75	.267
9	2.89	.346	3.00	.333	3.11	.321	3.22	.310	3.33	.300
10	2.60	.385	2.70	.370	2.80	.357	2.90	.345	3.00	.333
11	2.36	.423	2.45	.407	2.55	.393	2.64	.379	2.73	.367
12	2.17	.462	2.25	.444	2.33	.429	2.42	.414	2.50	.400
13	2.00	.500	2.08	.481	2.15	.464	2.23	.448	2.31	.433
14	1.86	.538	1.93	.519	2.00	.500	2.07	.483	2.14	.467
15	1.73	.577	1.80	.556	1.87	.536	1.93	.517	2.00	.500
16	1.62	.615	1.69	.593	1.75	.571	1.81	.552	1.88	.533
17	1.53	.654	1.59	.630	1.65	.607	1.71	.586	1.76	.567
18	1.44	.692	1.50	.667	1.56	.643	1.61	.621	1.67	.600
19	1.37	.731	1.42	.704	1.47	.679	1.53	.655	1.58	.633
20	1.30	.769	1.35	.741	1.40	.714	1.45	.690	1.50	.667
21	1.24	.808	1.29	.778	1.33	.750	1.38	.724	1.43	.700
22	1.182	.846	1.23	.815	1.27	.786	1.32	.759	1.36	.733
23	1.130	.885	1.174	.852	1.22	.821	1.26	.793	1.30	.767
24	1.083	.923	1.125	.889	1.167	.857	1.21	.828	1.25	.800
25	1.040	.962	1.080	.926	1.120	.893	1.160	.862	1.20	.833
26	-	-	1.038	.963	1.077	.929	1.115	.897	1.154	.867
27	-	-	-	-	1.037	.964	1.074	.931	1.111	.900
28	-	-	-	-	-	-	1.036	.966	1.071	.933
29	-	-	-	-	-	-	-	-	1.034	.967

TABLE 4 - RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

M	N = 30		N = 31		N = 32		N = 33		N = 34	
	T	P	T	P	T	P	T	P	T	P
1	31.0	.032	32.0	.031	33.0	.030	34.0	.029	35.0	.029
2	15.5	.065	16.0	.062	16.5	.061	17.0	.059	17.5	.057
3	10.3	.097	10.7	.094	11.0	.091	11.3	.088	11.7	.086
4	7.8	.129	8.0	.125	8.2	.121	8.5	.118	8.8	.114
5	6.2	.161	6.4	.156	6.6	.152	6.8	.147	7.0	.143
6	5.2	.194	5.3	.187	5.5	.182	5.7	.176	5.8	.171
7	4.43	.226	4.57	.219	4.71	.212	4.86	.206	5.0	.200
8	3.88	.258	4.00	.250	4.12	.242	4.25	.235	4.38	.229
9	3.44	.290	3.56	.282	3.67	.273	3.78	.265	3.89	.257
10	3.10	.323	3.20	.312	3.30	.303	3.40	.294	3.50	.286
11	2.82	.355	2.91	.344	3.00	.333	3.09	.324	3.18	.314
12	2.58	.387	2.67	.375	2.75	.364	2.83	.353	2.92	.343
13	2.38	.419	2.46	.406	2.54	.394	2.62	.382	2.69	.371
14	2.21	.452	2.29	.438	2.36	.424	2.43	.412	2.50	.400
15	2.07	.484	2.13	.469	2.20	.455	2.27	.441	2.33	.429
16	1.94	.516	2.00	.500	2.06	.485	2.12	.471	2.19	.457
17	1.82	.548	1.88	.531	1.94	.515	2.00	.500	2.06	.486
18	1.72	.581	1.78	.562	1.83	.545	1.89	.529	1.94	.514
19	1.63	.613	1.68	.594	1.74	.576	1.79	.559	1.84	.543
20	1.55	.645	1.60	.625	1.65	.606	1.70	.588	1.75	.571
21	1.48	.677	1.52	.656	1.57	.636	1.62	.618	1.67	.600
22	1.41	.710	1.45	.688	1.50	.667	1.55	.647	1.59	.629
23	1.35	.742	1.39	.719	1.43	.697	1.48	.676	1.52	.657
24	1.29	.774	1.33	.750	1.38	.727	1.42	.706	1.46	.686
25	1.24	.806	1.28	.781	1.32	.758	1.36	.735	1.40	.714
26	1.192	.839	1.23	.812	1.27	.788	1.31	.765	1.35	.743
27	1.148	.871	1.185	.844	1.22	.818	1.26	.794	1.30	.771
28	1.107	.903	1.143	.875	1.179	.848	1.21	.824	1.25	.800
29	1.069	.935	1.103	.906	1.138	.879	1.172	.853	1.21	.829
30	1.033	.968	1.067	.938	1.100	.909	1.133	.882	1.167	.857
31	-	-	1.032	.969	1.065	.939	1.097	.912	1.129	.886
32	-	-	-	-	1.031	.970	1.062	.941	1.094	.914
33	-	-	-	-	-	-	1.030	.971	1.061	.943
34	-	-	-	-	-	-	-	-	1.029	.971

TABLE 4 - RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

M	N = 35		N = 36		N = 37		N = 38		N = 39	
	T	P	T	P	T	P	T	P	T	P
1	36.0	.028	37.0	.027	38.0	.026	39.0	.026	40.0	.025
2	18.0	.056	18.5	.054	19.0	.053	19.5	.051	20.0	.050
3	12.0	.083	12.3	.081	12.7	.079	13.0	.077	13.3	.075
4	9.0	.111	9.2	.108	9.5	.105	9.8	.103	10.0	.100
5	7.2	.139	7.4	.135	7.6	.132	7.8	.128	8.0	.125
6	6.0	.167	6.2	.162	6.3	.158	6.5	.154	6.7	.150
7	5.1	.194	5.3	.189	5.4	.184	5.6	.179	5.7	.175
8	4.50	.222	4.62	.216	4.75	.211	4.88	.205	5.0	.200
9	4.00	.250	4.11	.243	4.22	.237	4.33	.231	4.44	.225
10	3.60	.278	3.70	.270	3.80	.263	3.90	.256	4.00	.250
11	3.27	.306	3.36	.297	3.45	.289	3.55	.282	3.64	.275
12	3.00	.333	3.08	.324	3.17	.316	3.25	.308	3.33	.300
13	2.77	.361	2.85	.351	2.92	.342	3.00	.333	3.08	.325
14	2.57	.389	2.64	.378	2.71	.368	2.79	.359	2.86	.350
15	2.40	.417	2.47	.405	2.53	.395	2.60	.385	2.67	.375
16	2.25	.444	2.31	.432	2.38	.421	2.44	.410	2.50	.400
17	2.12	.472	2.18	.459	2.24	.447	2.29	.436	2.35	.425
18	2.00	.500	2.06	.486	2.11	.474	2.17	.462	2.22	.450
19	1.89	.528	1.95	.514	2.00	.500	2.05	.487	2.11	.475
20	1.80	.556	1.85	.541	1.90	.526	1.95	.513	2.00	.500
21	1.71	.583	1.76	.568	1.81	.553	1.86	.538	1.90	.525
22	1.64	.611	1.68	.595	1.73	.579	1.77	.564	1.82	.550
23	1.57	.639	1.61	.622	1.65	.605	1.70	.590	1.74	.575
24	1.50	.667	1.54	.649	1.58	.632	1.62	.615	1.67	.600
25	1.44	.694	1.48	.676	1.52	.658	1.56	.641	1.60	.625
26	1.38	.722	1.42	.703	1.46	.684	1.50	.667	1.54	.650
27	1.33	.750	1.37	.730	1.41	.711	1.44	.692	1.48	.675
28	1.29	.778	1.32	.757	1.36	.737	1.39	.718	1.43	.700
29	1.24	.805	1.28	.784	1.31	.763	1.34	.744	1.38	.725
30	1.200	.833	1.23	.811	1.27	.789	1.30	.769	1.33	.750
31	1.161	.861	1.194	.838	1.23	.816	1.26	.795	1.29	.775
32	1.125	.889	1.156	.865	1.188	.842	1.22	.821	1.25	.800
33	1.091	.917	1.121	.892	1.152	.868	1.182	.846	1.21	.825
34	1.059	.944	1.088	.919	1.118	.895	1.147	.872	1.176	.850
35	1.029	.972	1.057	.946	1.086	.921	1.114	.897	1.143	.875
36	-	-	1.028	.973	1.056	.947	1.083	.923	1.111	.900
37	-	-	-	-	1.027	.974	1.054	.949	1.081	.925
38	-	-	-	-	-	-	1.026	.974	1.053	.950
39	-	-	-	-	-	-	-	-	1.026	.975

TABLE 4 - RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

M	N = 40		N = 41		N = 42		N = 43		N = 44	
	T	P	T	P	T	P	T	P	T	P
1	41.0	.024	42.0	.024	43.0	.023	44.0	.023	45.0	.022
2	20.5	.049	21.0	.048	21.5	.047	22.0	.045	22.5	.044
3	13.7	.073	14.0	.071	14.3	.070	14.7	.068	15.0	.067
4	10.2	.098	10.5	.095	10.8	.093	11.0	.091	11.2	.089
5	8.2	.122	8.4	.119	8.6	.116	8.8	.114	9.0	.111
6	6.8	.146	7.0	.143	7.2	.140	7.3	.136	7.5	.133
7	5.9	.171	6.0	.167	6.1	.163	6.3	.159	6.4	.155
8	5.1	.195	5.2	.190	5.4	.186	5.5	.182	5.6	.178
9	4.56	.220	4.67	.214	4.78	.209	4.89	.205	5.0	.200
10	4.10	.244	4.20	.238	4.30	.233	4.40	.227	4.50	.222
11	3.73	.268	3.82	.262	3.91	.256	4.00	.250	4.09	.244
12	3.42	.293	3.50	.286	3.58	.279	3.67	.273	3.75	.267
13	3.15	.317	3.23	.310	3.31	.302	3.38	.295	3.46	.289
14	2.93	.341	3.00	.333	3.07	.326	3.14	.318	3.21	.311
15	2.73	.366	2.80	.357	2.87	.349	2.93	.341	3.00	.333
16	2.56	.390	2.62	.381	2.69	.372	2.75	.364	2.81	.356
17	2.41	.415	2.47	.405	2.53	.395	2.59	.386	2.65	.378
18	2.28	.439	2.33	.429	2.39	.419	2.44	.409	2.50	.400
19	2.16	.463	2.21	.452	2.26	.442	2.32	.432	2.37	.422
20	2.05	.488	2.10	.476	2.15	.465	2.20	.455	2.25	.444
21	1.95	.512	2.00	.500	2.05	.488	2.10	.477	2.14	.467
22	1.86	.537	1.91	.524	1.95	.512	2.00	.500	2.05	.489
23	1.78	.561	1.83	.548	1.87	.535	1.91	.523	1.96	.511
24	1.71	.585	1.75	.571	1.79	.558	1.83	.545	1.88	.533
25	1.64	.610	1.68	.595	1.72	.581	1.76	.568	1.80	.556
26	1.58	.634	1.62	.619	1.65	.605	1.69	.591	1.73	.578
27	1.52	.659	1.56	.643	1.59	.628	1.63	.614	1.67	.600
28	1.46	.683	1.50	.667	1.54	.674	1.57	.636	1.61	.622
29	1.41	.707	1.45	.690	1.48	.651	1.52	.659	1.55	.644
30	1.37	.732	1.40	.714	1.43	.698	1.47	.682	1.50	.667
31	1.32	.756	1.35	.738	1.39	.721	1.42	.705	1.45	.689
32	1.28	.780	1.31	.762	1.34	.744	1.38	.727	1.41	.711
33	1.24	.805	1.27	.786	1.30	.767	1.33	.750	1.36	.733
34	1.20	.829	1.24	.810	1.26	.791	1.29	.773	1.32	.756
35	1.171	.854	1.20	.833	1.23	.814	1.26	.795	1.29	.777
36	1.139	.878	1.167	.857	1.194	.837	1.22	.818	1.25	.800
37	1.108	.902	1.135	.881	1.162	.860	1.189	.841	1.22	.822
38	1.079	.927	1.105	.905	1.132	.884	1.158	.864	1.184	.844
39	1.051	.951	1.077	.929	1.103	.907	1.128	.886	1.154	.867
40	1.025	.976	1.050	.952	1.075	.930	1.100	.909	1.125	.889
41	-	-	1.024	.976	1.049	.953	1.073	.932	1.098	.911
42	-	-	-	-	1.024	.977	1.048	.955	1.071	.933
43	-	-	-	-	-	-	1.023	.977	1.047	.956
44	-	-	-	-	-	-	-	-	1.023	.978

TABLE 4 - RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

M	N = 45		N = 46		N = 47		N = 48		N = 49	
	T	P	T	P	T	P	T	P	T	P
1	46.0	.022	47.0	.021	48.0	.021	49.0	.020	50.0	.020
2	23.0	.043	23.5	.043	24.0	.042	24.5	.041	25.0	.040
3	15.3	.065	15.7	.064	16.0	.062	16.3	.061	16.7	.060
4	11.5	.087	11.8	.085	12.0	.083	12.2	.082	12.5	.080
5	9.2	.109	9.4	.106	9.6	.104	9.8	.102	10.0	.100
6	7.7	.130	7.8	.128	8.0	.125	8.2	.122	8.3	.120
7	6.6	.152	6.7	.149	6.9	.146	7.0	.143	7.1	.140
8	5.8	.174	5.9	.170	6.0	.167	6.1	.163	6.2	.160
9	5.1	.196	5.2	.191	5.3	.188	5.4	.184	5.6	.180
10	4.60	.217	4.70	.213	4.80	.208	4.90	.204	5.0	.200
11	4.18	.239	4.27	.234	4.36	.229	4.45	.224	4.55	.220
12	3.83	.261	3.92	.255	4.00	.250	4.08	.245	4.17	.240
13	3.54	.283	3.62	.277	3.69	.271	3.77	.265	3.85	.260
14	3.29	.304	3.36	.298	3.43	.292	3.50	.286	3.57	.280
15	3.07	.326	3.13	.319	3.20	.312	3.27	.306	3.33	.300
16	2.88	.348	2.94	.340	3.00	.333	3.06	.327	3.12	.320
17	2.71	.370	2.76	.362	2.82	.354	2.88	.347	2.94	.340
18	2.56	.391	2.61	.383	2.67	.375	2.72	.367	2.78	.360
19	2.42	.413	2.47	.404	2.53	.396	2.58	.388	2.63	.380
20	2.30	.435	2.35	.426	2.40	.417	2.45	.408	2.50	.400
21	2.19	.456	2.24	.447	2.29	.438	2.33	.429	2.38	.420
22	2.09	.478	2.14	.468	2.18	.458	2.23	.449	2.27	.440
23	2.00	.500	2.04	.489	2.09	.479	2.13	.469	2.17	.460
24	1.92	.522	1.96	.511	2.00	.500	2.04	.490	2.08	.480
25	1.84	.544	1.88	.532	1.92	.521	1.96	.510	2.00	.500
26	1.77	.565	1.81	.553	1.85	.542	1.88	.531	1.92	.520
27	1.70	.587	1.74	.574	1.78	.562	1.81	.551	1.85	.540
28	1.64	.609	1.68	.596	1.71	.583	1.75	.571	1.79	.560
29	1.59	.630	1.62	.617	1.66	.604	1.69	.592	1.72	.580
30	1.53	.652	1.57	.638	1.60	.625	1.63	.612	1.67	.600
31	1.48	.674	1.52	.660	1.55	.646	1.58	.633	1.61	.620
32	1.44	.696	1.47	.681	1.50	.667	1.53	.653	1.56	.640
33	1.39	.717	1.42	.702	1.45	.688	1.48	.673	1.51	.660
34	1.35	.739	1.38	.723	1.41	.708	1.44	.694	1.47	.680
35	1.31	.761	1.34	.745	1.37	.729	1.40	.714	1.43	.700
36	1.28	.783	1.31	.766	1.33	.750	1.36	.735	1.39	.720
37	1.24	.804	1.27	.787	1.30	.771	1.32	.755	1.35	.740
38	1.21	.826	1.24	.809	1.26	.792	1.29	.776	1.32	.760
39	1.179	.848	1.21	.830	1.23	.812	1.26	.796	1.28	.780
40	1.150	.870	1.175	.851	1.20	.833	1.22	.816	1.25	.800
41	1.122	.891	1.146	.872	1.171	.854	1.195	.837	1.22	.820
42	1.095	.913	1.119	.894	1.143	.875	1.167	.857	1.190	.840
43	1.070	.935	1.093	.915	1.116	.896	1.140	.878	1.163	.860
44	1.045	.957	1.068	.936	1.091	.917	1.114	.898	1.136	.880
45	1.022	.978	1.044	.957	1.067	.938	1.089	.918	1.111	.900
46	-	-	1.022	.979	1.043	.958	1.065	.939	1.087	.920
47	-	-	-	-	1.021	.979	1.043	.959	1.064	.940
48	-	-	-	-	-	-	1.021	.980	1.042	.960
49	-	-	-	-	-	-	-	-	1.020	.980

TABLE 4 - RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

M	N = 50		N = 51		N = 52		N = 53		N = 54	
	T	P	T	P	T	P	T	P	T	P
1	51.	.020	52.	.019	53.	.019	54.	.019	55.	.018
2	25.5	.039	26.0	.038	26.5	.038	27.0	.037	27.5	.036
3	17.0	.059	17.3	.058	17.7	.057	18.0	.056	18.3	.055
4	12.8	.078	13.0	.077	13.2	.075	13.5	.074	13.8	.073
5	10.2	.098	10.4	.096	10.6	.094	10.8	.093	11.0	.091
6	8.5	.118	8.7	.115	8.8	.113	9.0	.111	9.2	.109
7	7.3	.137	7.4	.135	7.6	.132	7.7	.130	7.9	.127
8	6.4	.157	6.5	.154	6.6	.151	6.8	.148	6.9	.145
9	5.7	.176	5.8	.173	5.9	.170	6.0	.167	6.1	.164
10	5.1	.196	5.2	.192	5.3	.189	5.4	.185	5.5	.182
11	4.64	.216	4.73	.212	4.82	.208	4.91	.204	5.0	.200
12	4.25	.235	4.33	.231	4.42	.226	4.50	.222	4.58	.218
13	3.92	.255	4.00	.250	4.08	.245	4.15	.241	4.23	.236
14	3.64	.274	3.71	.269	3.79	.264	3.86	.259	3.93	.255
15	3.40	.294	3.47	.288	3.53	.283	3.60	.278	3.67	.273
16	3.19	.314	3.25	.308	3.31	.302	3.38	.296	3.44	.291
17	3.00	.333	3.06	.327	3.12	.321	3.18	.315	3.24	.309
18	2.83	.353	2.89	.346	2.94	.340	3.00	.333	3.06	.327
19	2.68	.373	2.74	.365	2.79	.358	2.84	.352	2.89	.345
20	2.55	.392	2.60	.385	2.65	.377	2.70	.370	2.75	.364
21	2.43	.412	2.47	.404	2.52	.396	2.57	.389	2.62	.382
22	2.32	.431	2.36	.423	2.41	.415	2.45	.407	2.50	.400
23	2.22	.451	2.26	.442	2.30	.434	2.35	.426	2.39	.418
24	2.12	.471	2.17	.462	2.21	.453	2.25	.444	2.29	.436
25	2.04	.490	2.08	.481	2.12	.472	2.16	.463	2.20	.455
26	1.96	.510	2.00	.500	2.04	.491	2.08	.481	2.12	.473
27	1.89	.529	1.93	.519	1.96	.509	2.00	.500	2.04	.491
28	1.82	.549	1.86	.538	1.89	.528	1.93	.519	1.96	.509
29	1.76	.569	1.79	.558	1.83	.547	1.86	.537	1.90	.527
30	1.70	.588	1.73	.577	1.77	.566	1.80	.556	1.83	.545
31	1.65	.608	1.68	.596	1.71	.585	1.74	.574	1.77	.564
32	1.59	.627	1.62	.615	1.66	.604	1.69	.593	1.72	.582
33	1.55	.647	1.58	.635	1.61	.623	1.64	.611	1.67	.600
34	1.50	.667	1.53	.654	1.56	.642	1.59	.630	1.62	.618
35	1.46	.686	1.49	.673	1.51	.660	1.54	.648	1.57	.636
36	1.42	.706	1.44	.692	1.47	.679	1.50	.667	1.53	.655
37	1.38	.725	1.41	.712	1.43	.698	1.46	.685	1.49	.673
38	1.34	.745	1.37	.731	1.39	.717	1.42	.704	1.45	.691
39	1.31	.765	1.33	.750	1.36	.736	1.38	.722	1.41	.709
40	1.28	.784	1.30	.769	1.32	.755	1.35	.741	1.38	.727
41	1.24	.804	1.27	.788	1.29	.774	1.32	.759	1.34	.745
42	1.21	.824	1.24	.808	1.26	.792	1.29	.778	1.31	.764
43	1.186	.843	1.21	.827	1.23	.811	1.26	.796	1.28	.782
44	1.159	.863	1.182	.846	1.20	.830	1.23	.815	1.25	.800
45	1.133	.882	1.156	.865	1.178	.849	1.20	.833	1.22	.818
46	1.109	.902	1.130	.885	1.152	.868	1.174	.852	1.196	.836
47	1.085	.922	1.106	.904	1.128	.887	1.149	.870	1.170	.855
48	1.062	.941	1.083	.923	1.104	.906	1.125	.889	1.146	.873
49	1.041	.961	1.061	.942	1.082	.925	1.102	.907	1.122	.891
50	1.020	.980	1.040	.962	1.060	.943	1.080	.926	1.100	.909
51	-	-	1.020	.981	1.039	.962	1.059	.944	1.078	.927
52	-	-	-	-	1.019	.981	1.038	.963	1.058	.945
53	-	-	-	-	-	-	1.019	.981	1.038	.964
54	-	-	-	-	-	-	-	-	1.019	.982

TABLE 4 - RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

M	N = 55		N = 56		N = 57		N = 58		N = 59	
	T	P	T	P	T	P	T	P	T	P
1	56.	.018	57.	.018	58.	.017	59.	.017	60.	.017
2	28.0	.036	28.5	.035	29.0	.034	29.5	.034	30.0	.033
3	18.7	.054	19.0	.053	19.3	.052	19.7	.051	20.0	.050
4	14.0	.071	14.2	.070	14.5	.069	14.8	.068	15.0	.067
5	11.2	.089	11.4	.088	11.6	.086	11.8	.085	12.0	.083
6	9.3	.107	9.5	.105	9.7	.103	9.8	.102	10.0	.100
7	8.0	.125	8.1	.123	8.3	.121	8.4	.119	8.6	.117
8	7.0	.143	7.1	.140	7.2	.138	7.4	.136	7.5	.133
9	6.2	.161	6.3	.158	6.4	.155	6.6	.153	6.7	.150
10	5.6	.179	5.7	.175	5.8	.172	5.9	.169	6.0	.167
11	5.1	.196	5.2	.193	5.3	.190	5.4	.186	5.5	.183
12	4.67	.214	4.75	.211	4.83	.207	4.92	.203	5.0	.200
13	4.31	.232	4.38	.228	4.46	.224	4.54	.220	4.62	.217
14	4.00	.250	4.07	.246	4.14	.241	4.21	.237	4.29	.233
15	3.73	.268	3.80	.263	3.87	.259	3.93	.254	4.00	.250
16	3.50	.286	3.56	.281	3.62	.276	3.69	.271	3.75	.267
17	3.29	.304	3.35	.298	3.41	.293	3.47	.288	3.53	.283
18	3.11	.321	3.17	.316	3.22	.310	3.28	.305	3.33	.300
19	2.95	.339	3.00	.333	3.05	.328	3.11	.322	3.16	.317
20	2.80	.357	2.85	.351	2.90	.345	2.95	.339	3.00	.333
21	2.67	.375	2.71	.368	2.76	.362	2.81	.356	2.86	.350
22	2.55	.393	2.59	.386	2.64	.379	2.68	.373	2.73	.367
23	2.43	.410	2.48	.404	2.52	.397	2.57	.390	2.61	.383
24	2.33	.429	2.38	.421	2.42	.414	2.46	.407	2.50	.400
25	2.24	.446	2.28	.439	2.32	.431	2.36	.424	2.40	.417
26	2.15	.464	2.19	.456	2.23	.448	2.27	.441	2.31	.433
27	2.07	.482	2.11	.474	2.15	.466	2.19	.458	2.22	.450
28	2.00	.500	2.04	.491	2.07	.483	2.11	.475	2.14	.467
29	1.93	.518	1.97	.509	2.00	.500	2.03	.492	2.07	.483
30	1.87	.536	1.90	.526	1.93	.517	1.97	.508	2.00	.500
31	1.81	.554	1.84	.544	1.87	.534	1.90	.525	1.94	.517
32	1.75	.571	1.78	.561	1.81	.552	1.84	.542	1.88	.533
33	1.70	.589	1.73	.579	1.76	.569	1.79	.559	1.82	.550
34	1.65	.607	1.68	.596	1.71	.586	1.74	.576	1.76	.567
35	1.60	.625	1.63	.614	1.66	.603	1.69	.593	1.71	.583
36	1.56	.643	1.58	.632	1.61	.621	1.64	.610	1.67	.600
37	1.51	.661	1.54	.649	1.57	.638	1.59	.627	1.62	.617
38	1.47	.679	1.50	.667	1.53	.655	1.55	.644	1.58	.633
39	1.44	.696	1.46	.684	1.49	.672	1.51	.661	1.54	.650
40	1.40	.714	1.42	.702	1.45	.690	1.48	.678	1.50	.667
41	1.37	.732	1.39	.719	1.41	.707	1.44	.695	1.46	.683
42	1.33	.750	1.36	.737	1.38	.724	1.40	.712	1.43	.700
43	1.30	.768	1.33	.754	1.35	.741	1.37	.729	1.40	.717
44	1.27	.786	1.30	.772	1.32	.759	1.34	.746	1.36	.733
45	1.24	.804	1.27	.789	1.29	.776	1.31	.763	1.33	.750
46	1.22	.821	1.24	.807	1.26	.793	1.28	.780	1.30	.767
47	1.191	.839	1.21	.825	1.23	.810	1.26	.797	1.28	.783
48	1.167	.857	1.188	.842	1.21	.828	1.23	.814	1.25	.800
49	1.143	.875	1.163	.860	1.184	.845	1.20	.830	1.22	.817
50	1.120	.893	1.140	.877	1.160	.862	1.180	.847	1.200	.833
51	1.098	.911	1.118	.895	1.137	.879	1.157	.864	1.176	.850
52	1.077	.929	1.096	.912	1.115	.897	1.135	.881	1.154	.867
53	1.057	.946	1.075	.930	1.094	.914	1.113	.898	1.132	.883
54	1.037	.964	1.056	.947	1.074	.931	1.093	.915	1.111	.900
55	1.018	.982	1.036	.965	1.055	.948	1.073	.932	1.091	.917
56	-	-	1.018	.982	1.036	.966	1.054	.949	1.071	.933
57	-	-	-	-	1.018	.983	1.035	.966	1.053	.950
58	-	-	-	-	-	-	1.017	.983	1.034	.967
59	-	-	-	-	-	-	-	-	1.017	.983

TABLE 4 - RECURRENCE INTERVALS (T) AND PROBABILITIES (P) FOR N = 10 TO N = 60

N = 60						
M	T	P		M	T	P
1	61.	.016		31	1.97	.508
2	30.5	.033		32	1.91	.525
3	20.3	.049		33	1.85	.541
4	15.2	.066		34	1.79	.557
5	12.2	.082		35	1.74	.574
6	10.2	.098		36	1.69	.590
7	8.7	.115		37	1.65	.607
8	7.6	.131		38	1.61	.623
9	6.8	.148		39	1.56	.639
10	6.1	.164		40	1.52	.656
11	5.5	.180		41	1.49	.672
12	5.1	.197		42	1.45	.689
13	4.69	.213		43	1.42	.705
14	4.36	.230		44	1.39	.721
15	4.07	.246		45	1.36	.738
16	3.81	.262		46	1.33	.754
17	3.59	.279		47	1.30	.770
18	3.39	.295		48	1.27	.787
19	3.21	.311		49	1.24	.803
20	3.05	.328		50	1.22	.820
21	2.90	.344		51	1.196	.836
22	2.77	.361		52	1.173	.852
23	2.65	.377		53	1.151	.869
24	2.54	.393		54	1.130	.885
25	2.44	.410		55	1.109	.902
26	2.35	.426		56	1.089	.918
27	2.26	.443		57	1.070	.934
28	2.18	.459		58	1.052	.951
29	2.10	.475		59	1.034	.967
30	2.03	.492		60	1.017	.984

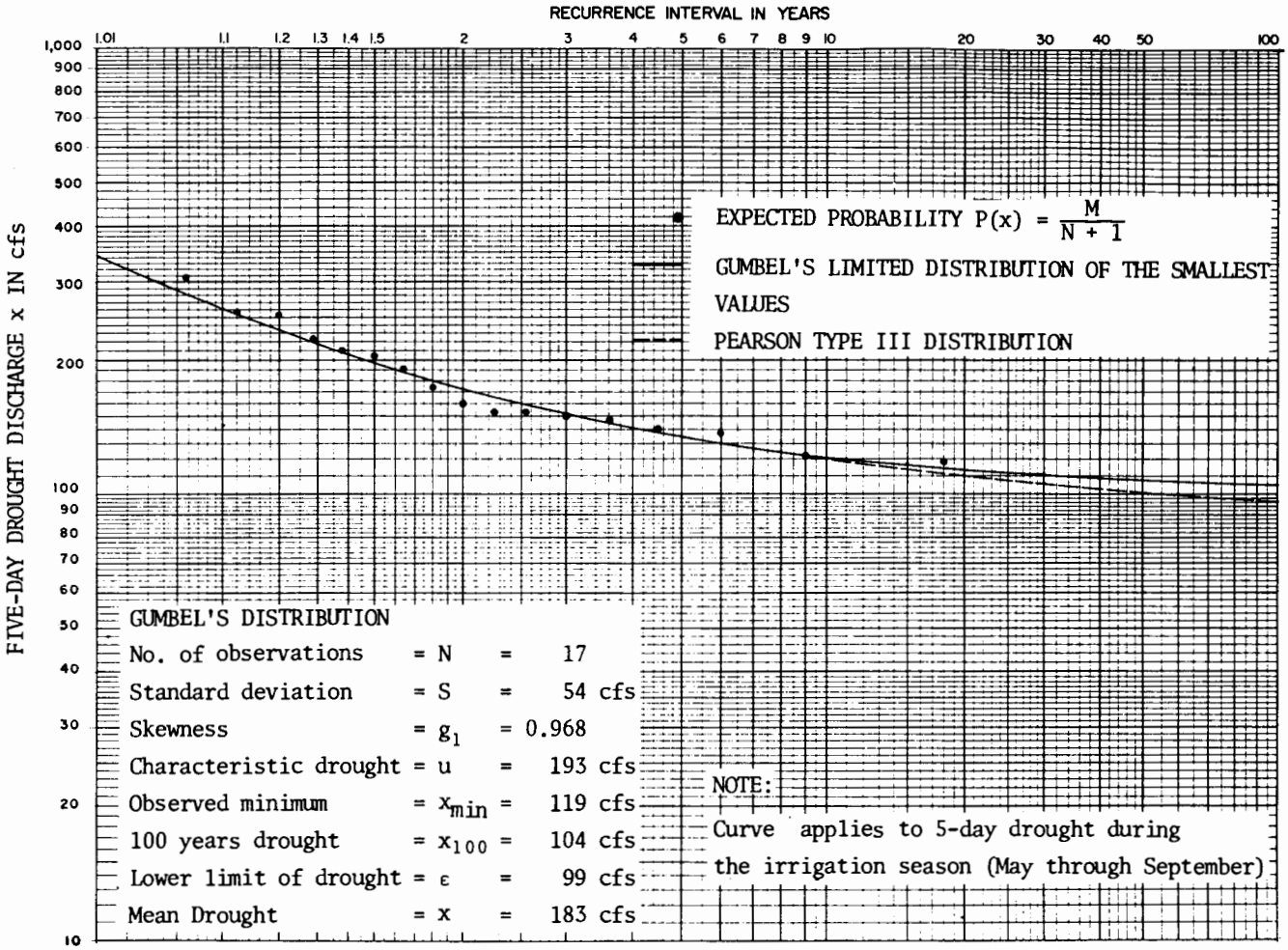


Figure 1. Drought frequency curve for Salmo River near Salmo, B.C.

APPENDIX 1

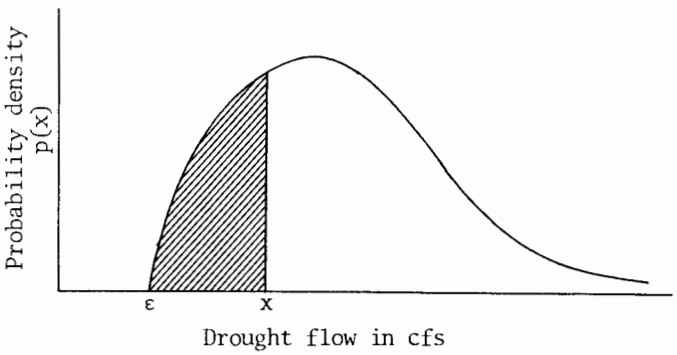
DERIVATION OF EQUATIONS FOR COMPUTING GUMBEL'S CUMULATIVE DISTRIBUTION

Equation 8

Equation 8 in the text gives the probability density function of drought flows as:

$$p(x) = \frac{\alpha}{u - \varepsilon} \left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha-1} e^{-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha}} \qquad (8)$$

This function may be illustrated diagramatically as:



By integrating from ε to some value x, the probability of droughts equal to or less than x would be obtained. The probability of drought flows exceeding x would be obtained by subtracting the result of the integration from 1.

Thus, the probability of droughts equal to or less than x, P(x), is given by:

$$P(x) = \int_{\varepsilon}^{x} \frac{\alpha}{u - \varepsilon} \left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha-1} e^{-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha}} dx \qquad (19)$$

By using the transformation:

$$y = \left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha} \qquad (10)$$

and substituting in Equation 19, Equation 20 is obtained:

$$P(x) = \int_{\varepsilon}^{x} \frac{\alpha}{u - \varepsilon} \left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha-1} e^{-y} dx \qquad (20)$$

This function must be integrated with respect to y between the appropriate limits. Therefore, dy is obtained by differentiating Equation 10 thus:

$$dy = \alpha\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha-1} \frac{1}{u - \varepsilon} dx \qquad (21)$$

and the limits of the integral are found by substituting the limits of x in Equation 10, thus: when $x = \varepsilon$, $y = 0$ and when $x = x$, $y = y$ since for any value of x there is a corresponding value of y as given by Equation 10. By substituting these limits and the equation for dy from Equation 21 into Equation 20, the required integral becomes:

$$P(x) = \int_{0}^{y} e^{-y} dy \qquad (22)$$

Solving the integral in Equation 22:

$$P(x) = \left[- e^{-y}\right]_{0}^{y}$$

$$= - e^{-y} + e^{0}$$

$$= 1 - e^{-y} \qquad (9)$$

which is the probability of drought flows being equal to or less than x.

Equation 11

Equation 11 in the text is obtained as follows:

$$y = \left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{\alpha} \qquad (10)$$

from which

$$\log y = \alpha[\log (x-\varepsilon) - \log (u-\varepsilon)]$$

$$\log (x-\varepsilon) = \log (u-\varepsilon) + \frac{1}{\alpha} \log y \qquad (11)$$

APPENDIX 2

EXAMPLES OF THE COMPUTATION AND PLOTTING OF A DROUGHT FREQUENCY CURVE

EXAMPLE - SALMO RIVER NEAR SALMO, B.C.

Expected Probability Values and Statistical Parameters

Year	Month	5-Day Drought x cfs	Order No. M	Expected Probability T $\frac{N+1}{M}$	Expected Probability P(x) $\frac{M}{N+1}$	$(x-\bar{x})$	$(x-\bar{x})^2$	$(x-\bar{x})^3$
1949	Sept.	119	1	18.00	0.056	- 64	4,096	- 262,144
1950	Sept.	138	3	6.00	0.167	- 45	2,025	- 91,125
1951	Sept.	152	7	2.57	0.389	- 31	961	- 29,791
1952	Sept.	123	2	9.00	0.111	- 60	3,600	- 216,000
1953	Sept.	225	14	1.29	0.778	+ 42	1,764	+ 74,088
1954	Sept.	310	17	1.06	0.944	+127	16,129	+2,048,383
1955	Sept.	253	15	1.20	0.833	+ 70	4,900	+ 343,000
1956	Sept.	193	11	1.64	0.611	+ 10	100	+ 1,000
1957	Sept.	175	10	1.80	0.556	- 8	64	- 512
1958	Sept.	141	4	4.50	0.222	- 42	1,764	- 74,088
1959	Aug.	211	13	1.38	0.722	+ 28	784	+ 21,952
1960	Sept.	150	6	3.00	0.333	- 33	1,089	- 35,937
1961	Sept.	147	5	3.60	0.278	- 36	1,296	- 46,656
1962	Sept.	161	9	2.00	0.500	- 22	484	- 10,648
1963	Sept.	154	8	2.25	0.444	- 29	841	- 24,389
1964	Sept.	259	16	1.13	0.889	+ 76	5,776	+ 438,976
1965	Sept.	208	12	1.50	0.667	+ 25	625	+ 15,625

$\sum x = 3,119$ $\sum (x-\bar{x})^2 = 46,298$ $\sum (x-\bar{x})^3 = 2,151,734$

No. of observations = $N = 17$

Mean drought = $\bar{x} = \frac{\sum x}{N} = 183$ cfs

Standard deviation = $S = \sqrt{\frac{\sum (x-\bar{x})^2}{N-1}} = 54$ cfs

Third Moment = $m_3 = \frac{\sum (x-\bar{x})^3}{N} = 126,573$ cfs

Coefficient of Skew = $g_1 = \frac{N^2}{(N-1)(N-2)} \times \frac{m_3}{S^3} = 0.968$

EXAMPLE - SALMO RIVER NEAR SALMO, B.C.

Gumbel's Limited Distribution of Smallest Values

Statistical Parameters

$\bar{x}$ = 183 cfs $1/\alpha$ = 0.627 (from Table 2)

S = 54 cfs A_α = 0.179 (from Table 2)

g_1 = 0.968 B_α = 1.737 (from Table 2)

$u = \bar{x} + A_\alpha S = 183 + 0.179\ (54) = 193$ cfs

$\varepsilon = u - B_\alpha S = 193 - 1.737\ (54) = 99$ cfs

$\log\ (u-\varepsilon) = \log\ (193-99) = \log\ 94 = 1.973$

Gumbel's Frequency Curve

$$\log\ (x-\varepsilon) = \log\ (u-\varepsilon) + \frac{1}{\alpha} \log\ y$$

Probability		log y	$\frac{1}{\alpha}$ log y	log (x-ε)	x - ε	x
T	P(x)					
1.01	0.990	0.663	.416	2.389	245	344
1.1	0.909	0.380	.238	2.211	163	262✓
1.5	0.667	0.041	.026	1.999	100	199
2.0	0.500	-0.159	- .100	1.873	75	174
3.0	0.333	-0.393	- .246	1.727	53	152
5.0	0.200	-0.652	- .409	1.564	37	136
10.0	0.10	-0.979	- .614	1.359	23	122
20.0	0.05	-1.292	- .810	1.163	15	114
50.0	0.02	-1.699	-1.065	.908	8	107
100.0	0.01	-2.000	-1.254	.719	5	104

EXAMPLE - SALMO RIVER NEAR SALMO, B.C.

Pearson Type III Distribution

Statistical Parameters

$\bar{x}$ = 183 cfs $v = \frac{8}{g_1^2}$ = 8.54

S = 54 cfs $C = \frac{2}{g_1}$ = 2.07

g_1 = 0.968 2C = 4.14

Pearson Type III Frequency Curve

$x = \bar{x} + K S$

where $K = \frac{\chi^2(P,v)}{2C} - C$

Probability		$\chi^2(P,v)$	$\frac{\chi^2(P,v)}{2C}$	K	K S	x
T	P(x)					
1.01	0.990	20.96	5.06	2.99	161	344
1.026	0.975	18.31	4.42	2.35	127	310
1.053	0.95	16.26	3.93	1.86	100	283
1.11	0.90	14.10	3.41	1.34	72	255
1.33	0.75	10.85	2.62	0.55	30	213
2.0	0.50	7.88	1.90	-0.17	- 9	174
4.0	0.25	5.52	1.33	-0.74	- 40	143
10.0	0.10	3.86	0.93	-1.14	- 62	121
20.0	0.05	3.05	0.74	-1.33	- 72	111
40.0	0.025	2.46	0.59	-1.48	- 80	103
100	0.01	1.89	0.46	-1.61	- 87	96

Note: Table 3b gives values of K directly as a function of g_1

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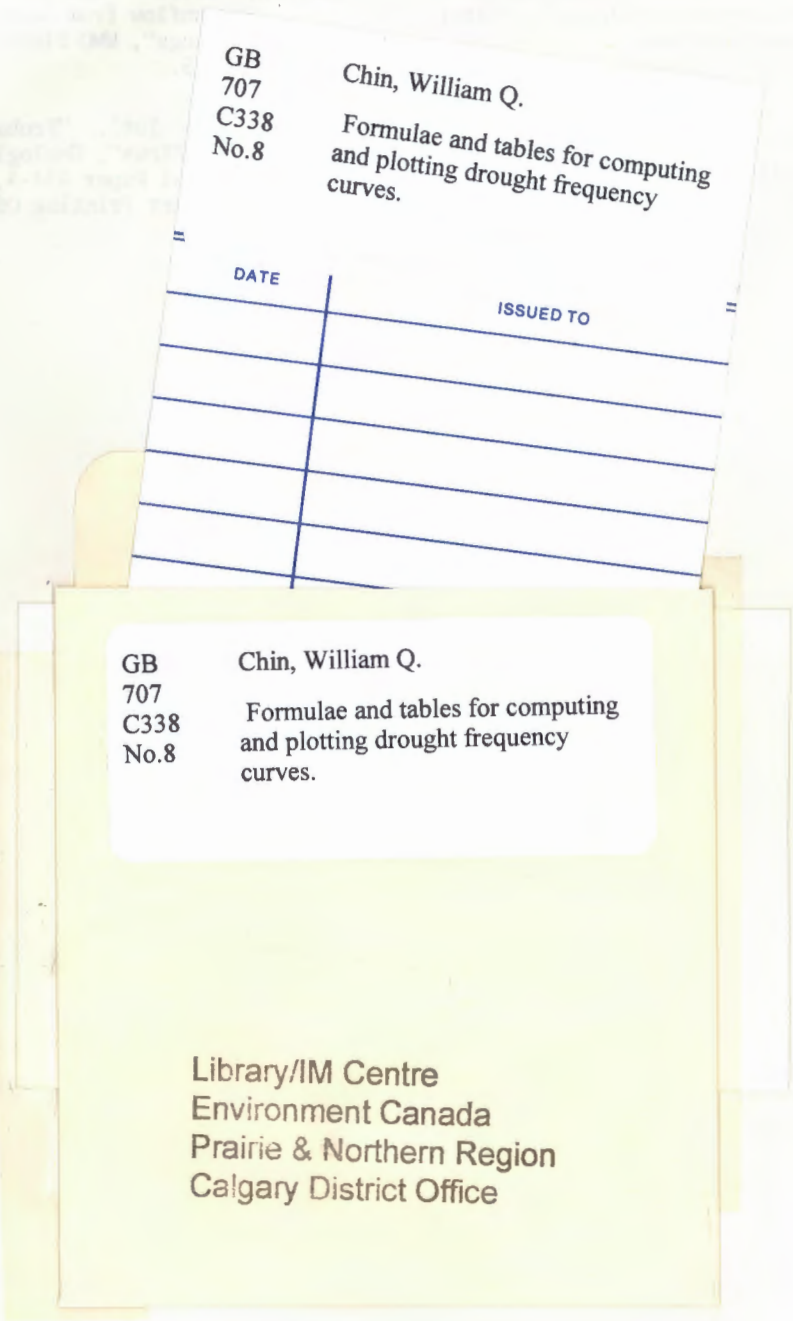

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