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# National Hydrology Research Institute

NHRI PAPER NO. 42

IWD TECHNICAL BULLETIN NO. 161

## A Physical Model of Vertical Integration, Drain Discharge and Surface Runoff for Layered Soils

A. Vandenberg

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NATIONAL HYDROLOGY RESEARCH INSTITUTE
SASKATOON, SASKATCHEWAN, 1989

(Disponible en français sur demande)

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The computer model documented in this report was developed as part of the research program on the effects of agricultural land drainage on the basin hydrograph. The model simulates flow through the unsaturated layer to the water table. Subsurface drain discharges can be calculated from the changes in water table elevations that result from this flow. Infiltration from precipitation is the input to the system, and the model accounts for evapotranspiration, soil moisture, and other parameters. The present model, which incorporates a layered soil, represents an extension to a previous model, developed for the special case of a homogeneous soil only.

# Abstract

The elements of the hydrological cycle and their interrelationships which are directly influenced by soil and surface drainage improvements are identified. A model of this partial cycle is constructed, permitting the assessment of the effect of drainage improvement on the total discharge from a drained plot for a given precipitation input. Total discharge is composed of surface runoff and drain discharge.

Particular emphasis is placed on the soil moisture component, infiltration and percolation to the ground-water table. All components except the unsaturated zone are treated as lumped systems, but the unsaturated zone is represented in the model by a stack of up to 50 layers. The moisture transfer between layers is calculated by a forward finite difference type of calculation based on the physical characteristics of the soil layers.

The effect of inhomogeneities in the soil profile on the shape and timing of the discharge is illustrated with a few examples.

The program and related data and documentation files are available on diskette for an IBM-XT and compatible computers.

# Résumé

On veut identifier les éléments du cycle hydrologique et leurs interrelations qui sont directement influencés par des améliorations du drainage en surface et dans le sol. Un modèle de ce cycle partiel est élaboré pour permettre d'évaluer l'effet de l'amélioration du drainage sur le débit total provenant d'une parcelle drainée pour un apport donné en précipitations. Le débit total est composé du ruissellement et du drainage.

Une attention particulière est accordée à la composante d'humidité du sol, à l'infiltration et à la percolation jusqu'à la nappe phréatique. Toutes les composantes à l'exception de la zone non saturée est représentée dans ce modèle par un ensemble superposé de couches dont le nombre peut atteindre 50. Le transfert d'humidité d'une couche à l'autre est calculé par une méthode de différences finies vers l'avant basée sur les caractéristiques physiques des couches du sol.

L'effet d'inhomogénéités dans le profil du sol sur la forme et la chronologie du débit est illustré au moyen de quelques exemples.

Le programme ainsi que les données et les fichiers de documentation connexes sont disponibles sur disquette pour ordinateurs IBM-XT et compatibles.

# List of Symbols

| | | | |
|---|---|---|---|
| $A$ | Drainage intensity = 8 $K_o d/L^2$ | ($day^{-1}$) | Eq. 13 |
| $A_b$ | Coefficient in surface runoff equation | ($cm^{-1}$ $day^{-1}$) | Eq. 18 |
| $D$ | Drain discharge to ditch | (cm/day) | Fig. 2 |
| $d$ | Equivalent depth of aquifer | (cm) | Eq. 13 |
| $E_{max}$ | Maximum evaporation rate | (cm/day) | Precipitation, P |
| $E_{soil}$ | Actual evaporation from upper soil layer | (cm/day) | Precipitation, P |
| $G$ | Ground-water or drain discharge | (cm/day) | Fig. 2 |
| $I$ | Infiltration rate | (cm/day) | Fig. 2 |
| $K, K(\Psi), K(\Theta)$ | Conductivity of soil, function of $\Psi$ and $\Theta$ | (cm/day) | Eq. 1 |
| $K_{b1}$ | Conductivity at a boundary, in the lower element | (cm/day) | Eq. 4 |
| $K_{b2}$ | Conductivity at a boundary, in the upper element | (cm/day) | Eq. 4 |
| $K_i$ | Conductivity of ith element | (cm/day) | Eq. 3 |
| $K_o$ | Saturated conductivity of the soil below the drain | (cm/day) | Eq. 13 |
| $K_s$ | Saturated conductivity of a soil in the profile above the drain | (cm/day) | Eq. 2 |
| $K_{sat}$ | Saturated conductivity of the upper soil element | (cm/day) | Eq. 12 |
| $L$ | Distance between the drains | (cm) | Eq. 13 |
| $m$ | Number of soil elements | | |
| $n_s$(or ns if a subscript) | Number of saturated elements | | Eq. 16 |
| $P$ | Precipitation rate | (cm/day) | Fig. 2 |
| $P_{max}$ | Depth of water on surface below which no runoff occurs | (cm) | Surface runoff, R |
| $Q_i$ | Flow rate from element (i-1) to element i | (cm/day) | Eq. 3 |
| $Q_b$ | Flow between an element node and an adjacent element boundary | (cm/day) | Eq. 4 |
| $q$ | Upward vector of internal moisture flow | (cm/day) | Eq. 1 |
| $R$ | Surface runoff rate | (cm/day) | Fig. 2 |
| $S_1$ S1 | Storage element 1, the atmosphere | (cm) | Fig. 2 |
| $S_2$ | Storage element 2, the surface ponding | (cm) | Fig. 2 |
| $S_3$ | Storage element 3, the soil | (cm) | Fig. 2 |
| $S_{3,i}$ | Storage element $3_i$, the ith soil element | (cm) | |
| $S_4$ | Storage element 4, the drain | (cm) | Fig. 2 |
| $S_5$ | Storage element 5, the ditch | (cm) | Fig. 2 |

| | | | |
|---|---|---|---|
| $t$ | Time | (days) | |
| $Z$ | Elevation above the drain | (cm) | Eq. 1 |
| $Z_i$ | Elevation of centre of ith soil element above the drain | (cm) | Fig. 4 |
| $Z_w$ | Elevation of the water table above the drain | (cm) | Eq. 14 |
| $\alpha$ | Constant in exponential expression for $K(\Psi)$ | ($cm^{-1}$) | Eq. 2 |
| $\Delta t$ | Small time interval between successive inventories | (days) | Eq. 8 |
| $\Delta Z$ | Thickness of a soil element | (cm) | Eq. 3 |
| $\Theta$ | Volumetric moisture content | (cm/cm) | |
| $\Theta_{sat}$ | Moisture content of upper soil element at saturation | (cm/cm) | Eq. 10 |
| $\Theta_m$ | Moisture content of upper soil element | (cm/cm) | Eq. 11 |
| $\Theta_m'$ | Moisture content of upper soil element at end of time step | (cm/cm) | Eq. 11 |
| $\Theta_0$ | Minimum soil moisture content on the $\Psi$-$\Theta$ curve for the upper soil element | (cm/cm) | Precip.,P |
| $\Psi$ | Pressure head | (cm) | Eq. 1 |
| $\Psi_b$ | Pressure head at an element boundary | (cm) | Eq. 4 |
| $\Psi_D$ | Pressure head at depth of drain | (cm) | Eq. 13 |

# A Physical Model of Vertical Integration, Drain Discharge and Surface Runoff for Layered Soils

A. Vandenberg

## INTRODUCTION

In a previous publication (Vandenberg, 1985) a model of infiltration, ground-water discharge and surface runoff for drained fields was described for the special case where the soil could reasonably be assumed to be homogeneous between the surface and the drain. However, practical experience in the field showed that the assumption of homogeneity is seldom warranted, particularly when the uppermost soil layers are intensively cultivated. Thus the existing model was modified to allow simulation of flow in the unsaturated zone for the case of horizontal layering, where each layer has different soil physical properties.

In the case of homogeneous soils, the finite difference expressions describing the moisture accounting process were based on the assumption that, with reasonable accuracy, conductivity can be expressed as an exponential function of pressure. This assumption results in a simplified form of the differential equation describing fluid flow in the unsaturated zone (Wind and Van Doorne,1975). In the case of inhomogeneous soils, this approach seemed to offer little promise, and thus the necessary expressions were derived directly from Darcy's law, on the sole assumption that both the conductivity and the pressure head are, for each layer, fully described as single valued functions of soil moisture. The present program is therefore more flexible, since there are no assumptions about the functional relation between pressure or moisture content and conductivity. This flexibility, however, is bought at the price of the more extensive set of data that must be supplied to the program; instead of the one coefficient $\alpha$ in the exponential expression for unsaturated conductivity, the conductivity function must now be given as a table of conductivity versus pressure for each soil layer.

Thus, where the soil profile can with reasonable accuracy be described as homogeneous, there are now two options: Where the measured unsaturated conductivities can be represented by an exponential expression, the earlier program FLO can be used to advantage. Where the unsaturated conductivities cannot be fitted satisfactorily to an exponential curve, program DRAIN, described in this report, must be used.

The primary objective of this report is documentation of the model. No attempt has as yet been made to verify the model by comparison with actual field data. Although the modelling of soil moisture movement is based on well established physical laws, a number of idealizations and simplifications were necessary to keep the size of the program and the data requirement within limits. The validity of these assumptions can only be established by repeated applications to actual field situations.

Furthermore, the program was developed with one uniformly drained field in mind and does not consider flow routing. Since the discharge of interest will usually be the combination of discharges from many fields, we are still faced with the question of routing the individual discharges to the outlet at which the discharge will be measured. Since timing is an integral factor in the routing process, it cannot be assumed that the total discharge of even a small group of fields can be computed as the sum of the model discharges for the individual fields; i.e., different fields will, in general, have different lag times.

This program has been written primarily for the IBM XT; the FORTRAN code and executable file are available on diskette. Files that give detailed documentation on how to run the program are also provided on the same diskette. The contents of these files are printed in the appendices to this report.

## ELEMENTS OF THE HYDROLOGICAL CYCLE AFFECTED BY DRAINAGE

Any part of the hydrological cycle can be defined by a set of storage reservoirs between which water transfers take place at rates governed by the laws of physics. In general, these physical laws relate the rates of transfer between two adjacent reservoirs to the physical state of the two reservoirs. Once all the transfer functions have been established, the changing state of the system as a whole can be followed through time by a system of double bookkeeping, provided entries to the ledger are made at frequent enough intervals that the state just after the transfers have been entered provides an accurate basis for calculating the transfer rates during the small time until the next entry is made.

The part of the hydrological cycle that is of primary interest in determining the transformation of a precipitation event over a basin into the corresponding discharge event is shown in Figure 1, where the storage reservoirs are represented by rectangular boxes, and the transfer functions by triangles suggesting the prevalent direction of the transfer.

Drainage works, when installed in parts of a basin, will primarily alter transfer functions, such as runoff rate and ground-water discharge rate, but also affect storages, such as surface ponding and soil moisture. In particular, if we wish to study the effect of drainage improvements on the shape of the basin hydrograph, we can limit ourselves to a small subsection of Figure 1, including only those transfer functions and storages most directly altered by drainage improvement. Figure 2 shows such a partial cycle, including the storage reservoirs $S_1$ to $S_5$ and the transfer functions P, I, G, D and R, where:

$S_1$ = Precipitation reservoir
$S_2$ = Surface ponding
$S_3$ = Soil moisture storage
$S_4$ = Storage in the drains
$S_5$ = Storage in the ditch
P = Precipitation
I = Infiltration
G = Ground-water discharge to drain
D = Drain discharge to ditch
R = Runoff over the surface into the ditch

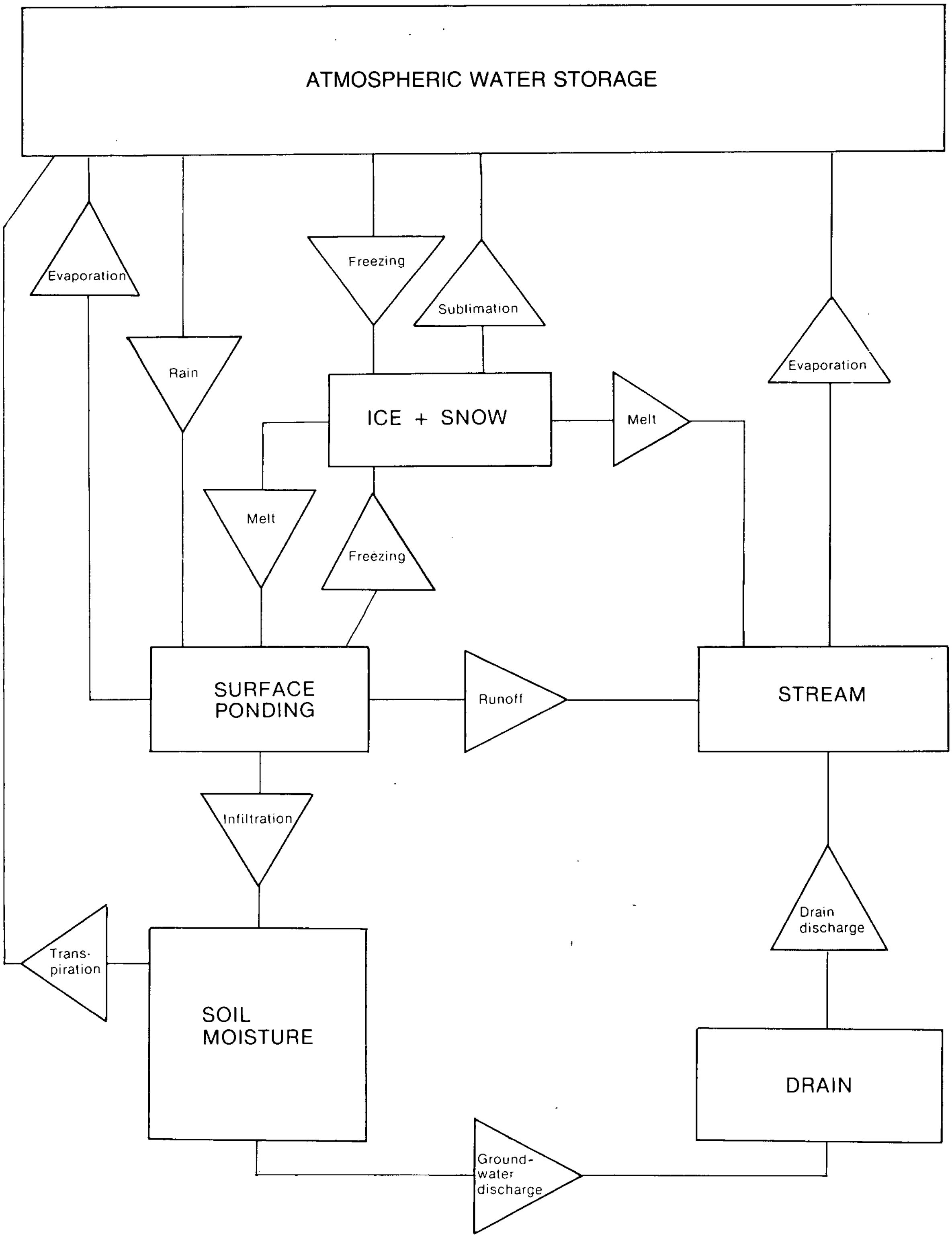


Figure 1. Part of the hydrological cycle affecting streamflow.

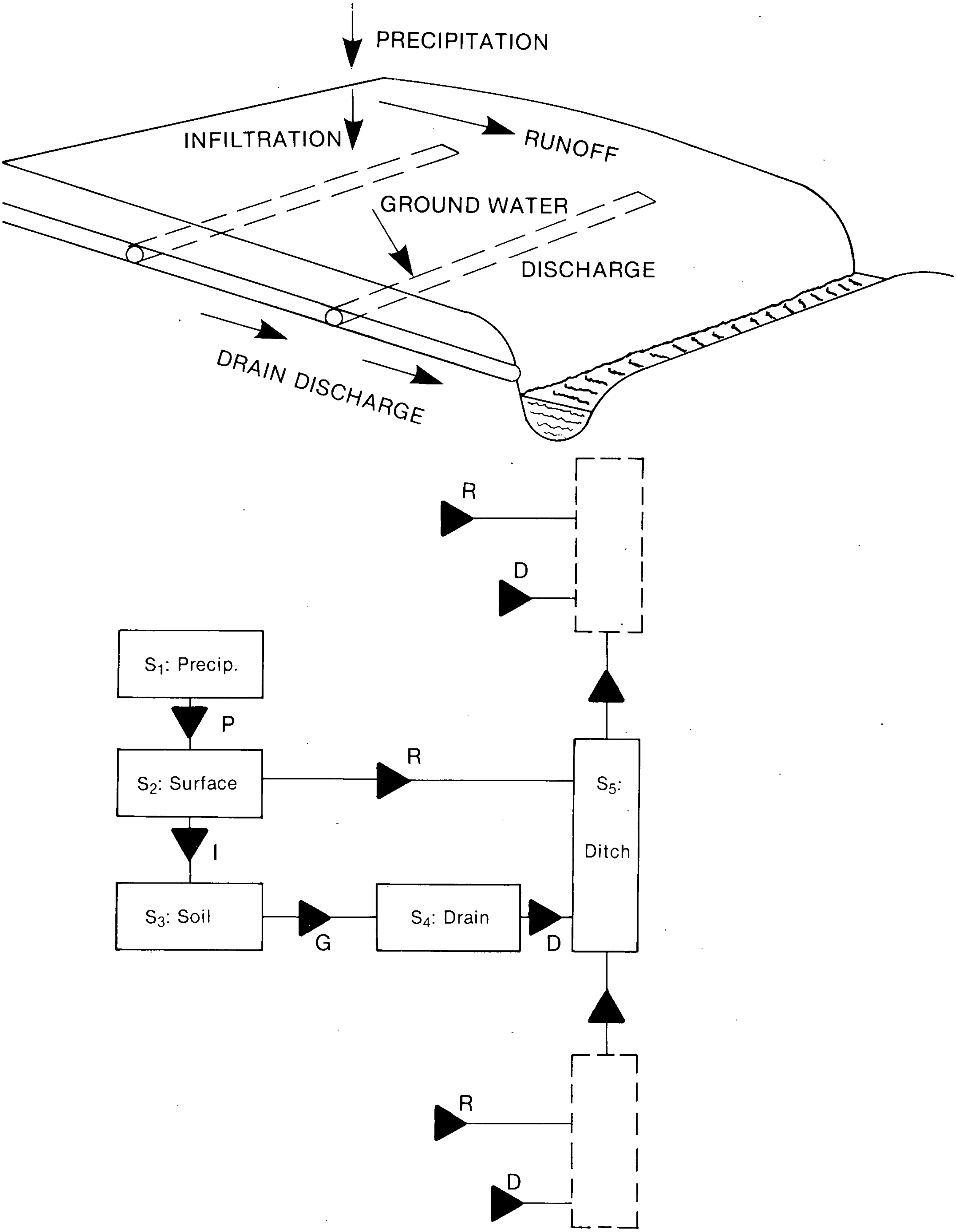


Figure 2. Section of the hydrological cycle affected by drainage improvements.

Figure 2 also indicates how the partial cycle for one field is connected to other fields through the section of ditch, $S_5$; each section of the ditch receives inputs R and D, as well as input from at least one other section. Therefore the state of the section, i.e., its water level, cannot be calculated unless the states of the adjoining sections are also known. This has the further consequence that, if R and D are sensitive to changes in $S_5$, all other storages in the system will also be affected, and a workable model must include all the drained fields which discharge into the same ditch.

Fortunately, in most instances both R and D discharge into the ditch or stream above the water level, and therefore do not depend on the water level in the ditch. Only in cases of extreme flooding, i.e., when the outlet of $S_5$ becomes blocked, will the drain outlet be under water, and then the direction of D may even be reversed. We will not consider such extreme cases here, but limit ourselves to the one field, assuming that all the time R and D are independent of $S_5$, which will be maintained for bookkeeping purposes only. Thus, with the addition of $S_1$ at the input end, we are now dealing with a closed system for which the sum of all the storages must remain constant. $S_1$, like $S_5$, does not influence any transfers, and is only maintained for bookkeeping purposes.

In the next two sections, a detailed analysis is given of the transfer functions: the soil-related functions Q, I and G, and the remaining functions P, D and R. Then the model, basically a moisture-accounting process, is described in its entirety. Some of the model results are shown and the effects of soil heterogeneity on the shape of the discharge hydrograph will be discussed.

## INTERNAL SOIL MOISTURE MOVEMENT, INFILTRATION AND GROUND-WATER DISCHARGE

In the previous section, and in Figures 1 and 2, we have tacitly assumed that for each of the storage elements shown, under isothermal conditions, the state of the element can be equated to the total amount of water in the element, a unique number. Specifically in the case of soil storage, however, soil moisture is typically a function of its position in the soil, notably of its elevation. But the infiltration rate, I, for example, does not depend on the moisture content at some depth, but only on the moisture content near the surface. Similarly, the ground-water discharge, G, does not depend on the moisture content near the surface or at some intermediate depth, but rather on the hydraulic head and therefore on the water content at the depth of the drain. Thus, the lumped system of Figures 1 and 2 can introduce large errors, since soil moisture is distributed unevenly throughout the soil column. Thus, we come to consider the model of Figure 3, with distributed ground-water storage, which is derived from Figure 2 by subdividing $S_3$ into m smaller storage elements $S_{3,i}$ , i = 1 to m, and introducing the (m-1) internal transfer functions $Q_i$, i = 2,m.

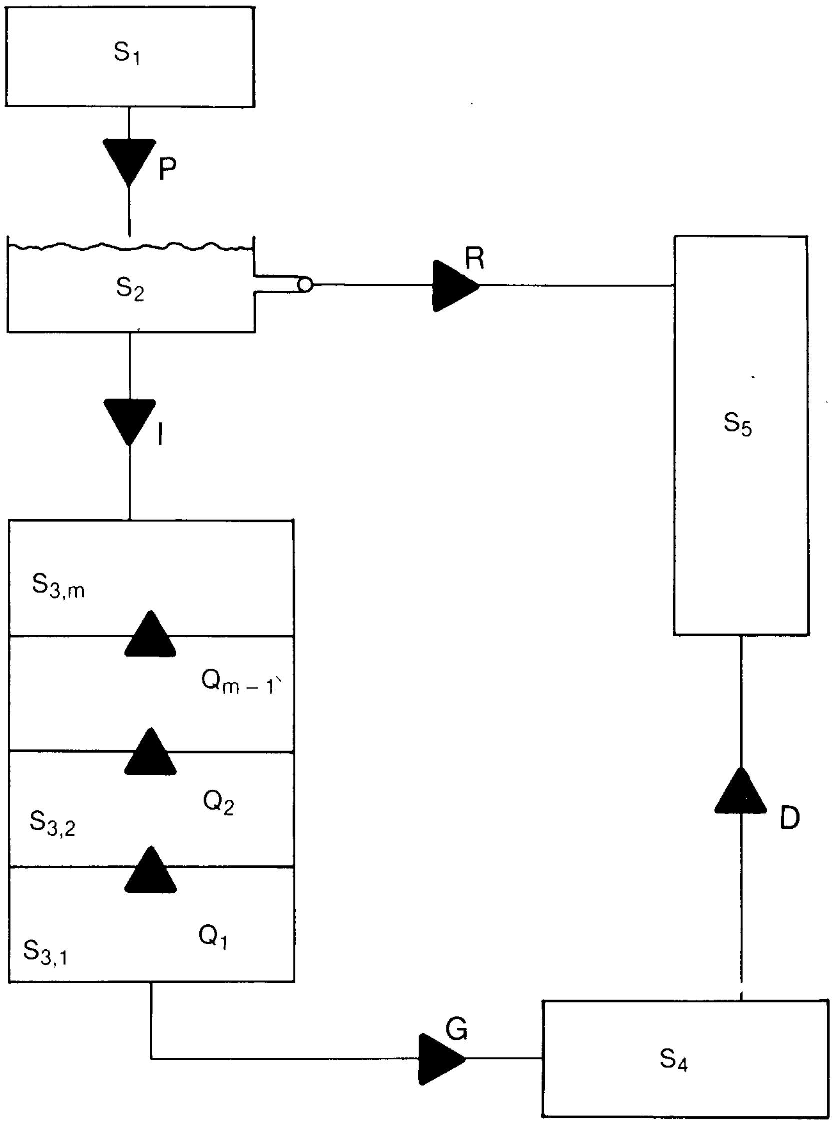


Figure 3. Drainage model with distributed soil moisture storage.

Internal transfer functions, $Q_i$

The internal transfer functions can be derived from Darcy's law, modified for vertical flow in an unsaturated soil:

$$q = -K(\Psi)(d/dZ)(\Psi + Z) = -K(\Psi)(d\Psi/dZ + 1) \quad (1)$$

where

- q = volume of water moving upward through a horizontal plane of unit area per unit of time(dimension L/T)
- K = conductivity, which is a function of pressure head (dimension L/T)
- $\Psi$ = pressure head, negative in unsaturated soil, positive in saturated soil (dimension L)
- Z = vertical coordinate increasing upward (dimension L).

Pressure head is related to moisture content by volume, $\Theta$, but the relation, strictly speaking, is not unique, showing hysteresis; i.e., both the $\Theta(\Psi)$ curve and the $K(\Psi)$ curve for a drying soil are different from the curves for a wetting soil (Hillel, 1980a). However, accounting for hysteresis in a numerical model is difficult, requiring a large amount of extra code and increased run times, and demanding a great deal of extra laboratory data. Therefore, although it is difficult to assess the error introduced in doing so, we will nevertheless base our model on an average $\Psi(\Theta)$ curve which uniquely relates $\Theta$ and $\Psi$ independent of past history. This relation must be determined for each soil in the laboratory or in the field by measuring $\Psi$ at different values of $\Theta$.

Similarly, the relation between K and $\Psi$, or K and $\Theta$ must be established in the laboratory. Except for very dry soils, the $K(\Psi)$ relation can often be expressed by:

$$K = K_s \exp(\alpha\Psi) \qquad (2)$$

where

$K_s$ = conductivity of the saturated soil
$\alpha$ = a constant, characteristic of the particular soil.

For the present model no specific use is made of this equation, although it may be used in data preparation and function evaluation. For the purpose of the model, it is sufficient that both $K(\Theta)$ and $\Psi(\Theta)$ are monotonic, one-valued functions, which must be available to the program in tabular or functional form, such that for a given value of any one of the three variables, $\Theta$, $\Psi$ or K, the other two can be determined.

Equation (1) is used to describe the internal transfer functions $Q_i$. The finite difference form of equation (1) can be written as:

$$Q_i = -\left(\frac{K_{i-1} + K_i}{2}\right)\left(\frac{\Psi_i - \Psi_{i-1}}{\Delta Z} + 1\right) \qquad (3)$$

where

$Q_i$ = the moisture flow rate from the (i-1)th element to the (i)th element

$(K_{i-1} + K_i)/2$ = the average value of K between the (i-1)th and the (i)th element

$\Delta Z$ = the thickness of the soil element.

If the soil is homogeneous, Equation (3) can be used directly to calculate the flow rate between two adjacent elements. However, for the flow between two dissimilar soil elements this procedure is rather inaccurate, since it assumes that at the boundary between the two elements:

- the conductivity is the average of the nodal conductivities, and
- the pressure changes in a nearly linear fashion between the nodes.

With the help of Figure 4 we can see that the latter is certainly not the case, while the first assumption obviously contradicts the assumption of a layered soil with abrupt changes in soil physical properties.

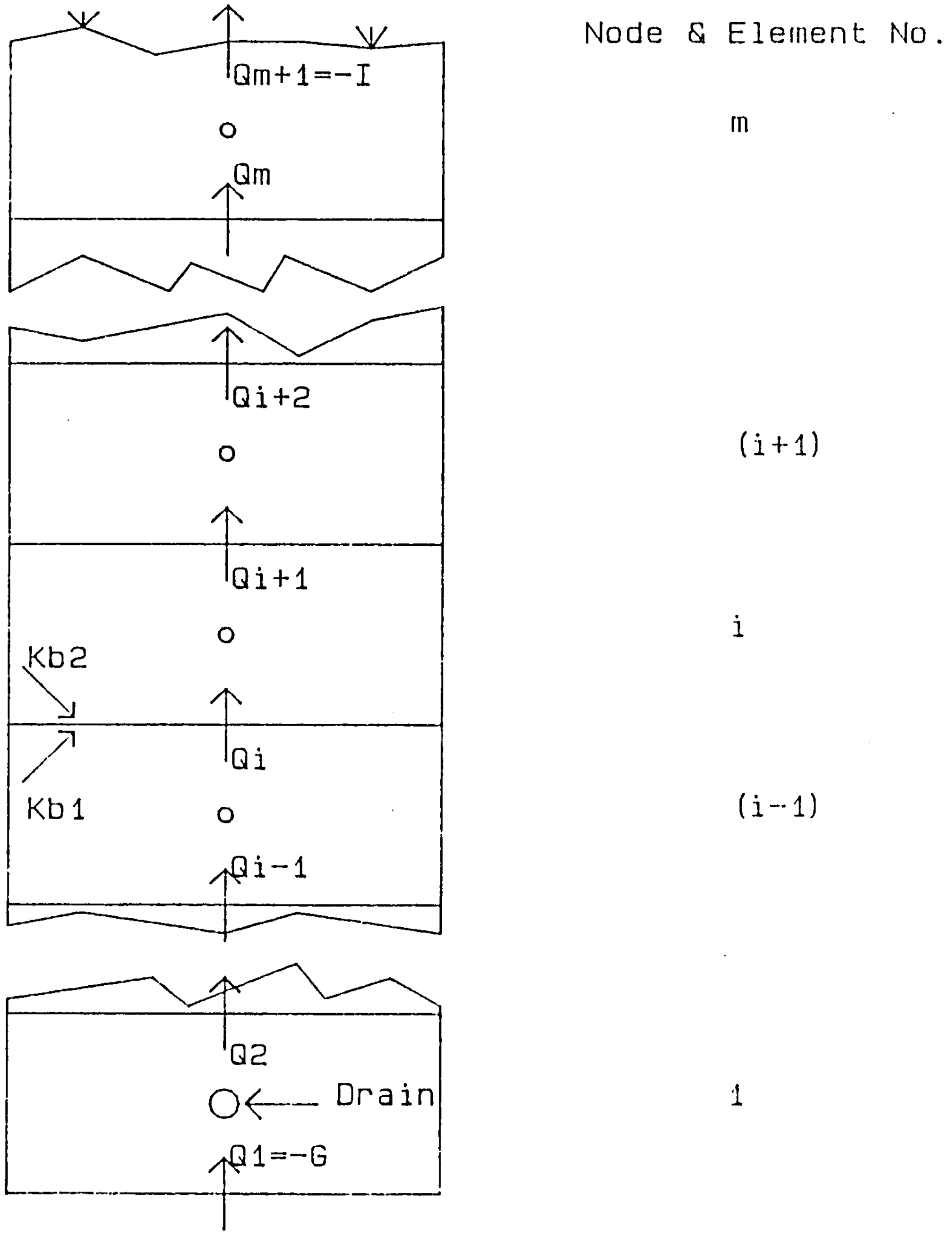


Figure 4. Internal moisture movement and symbol definitions.

For a more precise analysis and calculation of $Q_i$ between dissimilar elements, we consider separately the flow $Q_{b(i-1)}$ between the (i-1)th node and the boundary and the flow $Q_{bi}$ between the boundary and the (i)th node:

$$Q_{(i-1)b} = -\left(\frac{K_{i-1} + K_{b1}}{2}\right)\left(\frac{\Psi_b - \Psi_{i-1}}{(1/2)\Delta Z} + 1\right) \qquad (4)$$

and

$$Q_{bi} = -\left(\frac{K_{b2} + K_i}{2}\right)\left(\frac{\Psi_i - \Psi_b}{(1/2)\Delta Z} + 1\right) \qquad (5)$$

where $K_{b1}$ and $K_{b2}$ are the conductivities at the boundary, in the (i-1)th and in the (i)th element respectively. Since $\Psi_b$, the pressure on the boundary, is the same on both sides, $K_{b1}$ and $K_{b2}$ can be uniquely determined once $\Psi_b$ is known.

Since the flow towards the boundary and the flow away from the boundary must be the same, the right hand sides of Equations (4) and (5) can be equated:

$$(K_{i-1}+K_{b1})(\Psi_b-\Psi_{i-1}+\Delta Z/2) = (K_i+K_{b2})(\Psi_i-\Psi_b+\Delta Z/2)$$

which, written explicitly for $\Psi_b$, becomes:

$$\Psi_b = \frac{(K_i+K_{b2})(\Psi_i+\Delta Z/2) + (K_{i-1}+K_{b1})(\Psi_{i-1}-\Delta Z/2)}{K_i + K_{i-1} + K_{b1} + K_{b2}} \qquad (6)$$

For a first estimate of $\Psi_b$ the assumption is made that

$$K_{b1} = K_{i-1}$$

and

$$K_{b2} = K_i$$

Thus

$\Psi_b$(first estimate)

$$= \frac{K_i(\Psi_i + \Delta Z/2) + K_{i-1}(\Psi_{i-1} - \Delta Z/2)}{K_i + K_{i-1}} \qquad (7)$$

For a better estimate, $K_{b1}$ and $K_{b2}$ can be calculated using the first estimate of $\Psi_b$, followed by application of Equation (6).

The program employs an iterative routine based on Equation (7) to calculate $\Psi_b$ if elements (i-1) and (i) are dissimilar, with a maximum of 4 iterations. The iterations are also terminated whenever the difference between the previously and lastly calculated values of $\Psi_b$ is less than 1 cm. Once $\Psi_b$ has been found with sufficient accuracy, $Q_i$ can be calculated from either Equation (4) or (5).

Now, a rate of flow of moisture $Q_i$ out of the (i-1)th element during a small period of time, $\Delta t$, results in a decrease, $-\Delta(\Theta\Delta Z)$, of the total moisture content, $\Theta\Delta Z$ of the (i-1)th element:

$$\Delta(\Theta_{i-1}\Delta Z) = -Q_i\Delta t$$

Thus

$$\Delta(\Theta_{i-1}\Delta Z) = \Delta t(Q_{i-1} - Q_i)$$

or

$$\Delta\Theta_{i-1} = (Q_{i-1} - Q_i)\Delta t/\Delta Z. \qquad (8)$$

Equation (8) can be used to simulate the future state of all the internal elements with moisture content below saturation. But for the upper element of $S_3$, the flux through the upper surface, that is, infiltration or evaporation, must be calculated by other means, since it is controlled by the state of the storage element $S_2$, the pooled water, as well as by the state of $S_{3,m}$, the uppermost soil layer. And in the case of the lowermost unsaturated element, the flow through its lower surface is determined by saturated flow conditions in the underlying element, and must therefore also be determined separately.

Infiltration, I

Instantaneous infiltration rate into a soil depends primarily on the moisture content of the uppermost soil element, but is limited by the amount of water stored on the surface, $S_2$.

$$I\Delta t \leq S_2 \qquad (9)$$

and by the storage capacity of the upper soil layer

$$I\Delta t \leq \Delta Z(\Theta_{sat} - \Theta'_m) \qquad (10)$$

where

$\Theta_{sat}$ = saturated moisture content of the $m^{th}$ or upper soil storage element, $S_{3,m}$

$\Theta'_m$ = the moisture content of element $S_{3,m}$ taking into account that its water content at the end of the timestep may be taken, thus

$$\Theta'_m = \Theta_m + Q_m \Delta t/\Delta Z. \qquad (11)$$

Furthermore, it must be taken into consideration that, if the level of $S_2$ is above the level of the outflow (Figure 3), surface runoff R will be generated, competing with infiltration for the total available surface storage (ponding).

Within these limits the infiltration rate can be calculated on the assumption that, as long as $S_2 > 0$, the surface of the soil is saturated and has conductivity $K_{sat}$, the saturated conductivity of element $S_{3,m}$. Then, from Equation (1), and with

$$d\Psi/dZ \simeq -\Psi_m/(\Delta Z/2)$$

we obtain

$$I = (K_{sat} + K_m)(\Delta Z - 2\Psi_m)/(2\Delta Z) \quad (12)$$

where I is the moisture movement through the surface in the downward direction.

Ground-water discharge, G

For the simulation of ground-water discharge, G, we use the linear approximation first given by Hooghoudt (1937); and also described in Hillel (1980b):

$$G = A\Psi_D \quad (13)$$

where

$\Psi_D$ = pressure head at the depth of the drains midway between two parallel drains
A = drainage intensity = $8K_od/L^2$ (dimension L/T)
L = distance between drains
d = the "equivalent depth" of the aquifer below the drains (Hillel, 1980a)
$K_o$ = saturated conductivity of soil below the drains.

From Equation (13) we can derive an expression for G in terms of $Z_w$, the height of the water table above the drains midway between the drains, instead of in terms of $\Psi_D$ (Van Wijk, 1980):

$$G = AZ_wK_o/(AZ_w + K_o) \quad (14)$$

and the equivalent equation for $Z_w$:

$$Z_w = GK_o/\{A(K_o - G)\} \quad (15)$$

Equations (14) and (15) contain the two unknowns G and $Z_w$. However, since the saturated zone does not allow for any storage changes, the flow from the lowermost unsaturated element to the water table must be equal to G (Figure 5) and with the use of equation (1) can be expressed as:

$$G = K_{ns+1}\left(\frac{\Psi_{ns+1}}{d - Z_w} + 1\right) \quad (16)$$

where

$K_{ns+1}$ = the conductivity of the lowest unsaturated element
$n_s$ = the number of saturated elements, where a saturated element is an element for which the centre is below the watertable.
$\Psi_{ns+1}$ = the pressure of the lowest unsaturated element
d = $n_s . \Delta Z$ (see Figure 5).

Equating the right hand sides of Equations (14) and (16) leads to a quadratic in $Z_w$:

$$AZ_w^2(K_o/K_{ns+1} - 1) + Z_w\{A(\Psi_{ns+1} + d) - K_o - AdK_o/K_{ns+1}\}$$
$$+ K_o(\Psi_{ns+1} + d) = 0 \quad (17)$$

from which $Z_w$ can be calculated; then G can be calculated from Equation (14).

However, during the operation and testing of the model it appeared that, although Equation (17) works very well whenever the lowermost unsaturated element is the same soil type as the uppermost saturated element, the equation gave problematic results whenever these elements are very dissimilar. In fact, calculated values of $Z_w$ were often less than $(n_s - 1)d$ and thus incompatible with the number of saturated elements. To eliminate this problem, Equation (17) was further simplified, based on the observation that the quadratic term is small in comparison to the other two terms. Thus we assume that

$$A(K_o/K_{ns+1} - 1) = 0$$

which reduces Equation (17) to:

$$Z_w = K_o(d + \Psi_{ns+1})/(K_o - A\Psi_{ns+1}) \quad (17a)$$

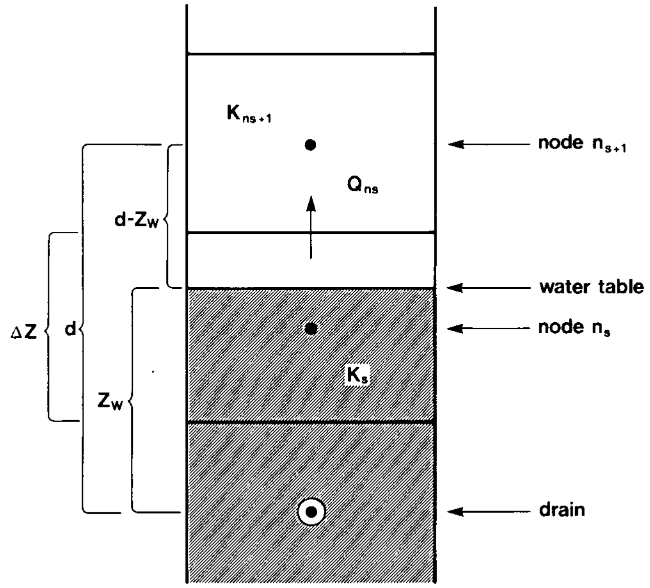


Figure 5. Moisture displacement at the water table.

## OTHER TRANSFER FUNCTIONS

### Precipitation or Evaporation, P

Comparison of Figures 2 and 3 indicates that the transfer function labelled P, the input to the surface reservoir, $S_2$, is in fact composed of a number of transfers, mainly those shown in Figure 2 as rain, melt, evaporation and freezing, the latter two being negative inputs to $S_2$. At present the model does not distinguish between these inputs, but presumes that their algebraic sum is known from other sources and presented at each timestep as input to the model. The construction of the model, however, permits the insertion of additional routines for these functions as they become available.

Whenever there is no precipitation, evaporation may take place from ponded water, or from the upper soil layer if $S_2$ is empty; P may thus be negative. In the model, transpiration, which takes place in the leaves of plants, is not considered, although it transfers moisture from the soil, sometimes at considerable depth, through the root system to the atmosphere. Thus, the model accounts only for evaporation from the ponded water $S_2$ at the given evaporation rate or, if no ponding is present, from the top element of the soil at a reduced rate. At present, the simple formula is used:

$$E_{soil} = E_{max} (\Theta_m - \Theta_o)/(\Theta_{sat} - \Theta_o)$$

where

$E_{soil}$= actual evaporation from the upper soil element.
$E_{max}$ = evaporation from a saturated surface, as given on input
$\Theta_m$ = moisture content of top element
$\Theta_o$ = minimum soil moisture content on the $\Psi - \Theta$ curve
$\Theta_{sat}$ = moisture content at saturation, top element.

Thus actual evaporation will be equal to the given evaporation for a saturated top element only and decreases linearly with decreasing soil moisture content, becoming zero when the soil is at minimum moisture content.

### Drain Discharge, D

In the present version of the model the small storage changes in the drains are not taken into account, and D is assumed to be equal to G at all times. If future applications so warrant, the transfer function $D(S_4)$ can easily be inserted into the model.

### Surface Runoff R

Surface runoff will be generated whenever the depth of water on the surface, $S_2$, exceeds a certain value, designated by the constant $P_{max}$ in the model. $P_{max}$ is usually in the order of a few millimetres.

The runoff velocity $R(S_2)$ is calculated using an equation first proposed by Horton (1938) and described more recently by Huggins and Burney (1982):

$$R = A_b (S_2 - P_{max})^2 \tag{18}$$

where $A_b$ is a constant; R is, however, restricted by the following inequalities:

(1): $R\Delta t < S_2 - P_{max}$, and

(2): $(R + I)\Delta t < S_2$

Condition (2) requires that I and R be determined at the same time. Both I and R are first calculated separately, R being limited by condition (1). Then the sum $(R + I)\Delta t$ is calculated and compared with the available storage $S_2$; if condition (2) is not met, the available storage is prorated over R and I, i.e., both R and I are multiplied by the factor

$$S_2/\{(R + I)\Delta t\}$$

The threshold value $P_{max}$ can have a strong effect on the peak flow. If the threshold value is low, as with good surface drainage, and the soil conductivity is low, the surface storage cannot retain any precipitation and the soil cannot absorb it fast enough; thus most of the precipitation will run off over the surface. Surface runoff is generally fast and thus creates a large peak. If the threshold value is high, as with poor surface drainage and swamp conditions, the excess water cannot run off, and either evaporates or slowly runs off through the soil and the drains.

If, however, not only the surface drainage but subsequently also the subsurface drainage is improved, the peak will become smaller again (Wind and Vandenberg, 1984; Vandenberg, 1985).

## MOISTURE ACCOUNTING

Once the transfer rates between storage elements have been calculated, the newly stored volumes can be calculated by adding the inflows and subtracting the outflows from the previously stored volumes for each element according to the schematic of Figure 3. Since the total volume stored for each soil element is the product of moisture content and thickness, the new moisture content will be:

$$\Theta_i(t + \Delta t) = Q_i(t) + (Q_i - Q_{i+1})\Delta t/\Delta Z \tag{19}$$

for $i = 2, m - 1$

i.e., for all the internal elements (Figure 4). For the upper (the $m^{th}$) element,

$$Q_{m+1} = -I$$

and for the lowermost unsaturated element,

$$Q_i = - G, \text{ the ground water discharge.}$$

From Figure 3 we can read directly:

$$S_1(t + \Delta t) = S_1(t) - P\Delta t \tag{20}$$

$$S_2(t + \Delta t) = S_2(t) + (P - R - I)\Delta t \tag{21}$$

$$S_5(t + \Delta t) = S_5(t) + (R + G)\Delta t \tag{22}$$

Equation 22 occurs in this form, since we have assumed for the moment that

$$D = G$$

In the case of evaporation, i.e., negative P, the value of P in (20) is not necessarily the value given on input, but depends on the presence or absence of pooled water on the surface, and in the absence of surface water, on the moisture content of the upper soil element.

## SIMULATIONS OF A THREE-LAYERED SOIL

In this section, some simulations carried out with the model are described, and the effect of heterogeneity of the soil above the drain on the discharge from the drained field is discussed. These examples are not intended to provide a complete description of how drain discharge is affected by layering of the soil, but rather to show that the model runs satisfactorily even under conditions of extreme heterogeneity.

Basically, the examples deal with a homogeneous sandy soil, into which a low conductivity zone is introduced at different heights above the drain. The only variables between simulations are the type and depth of the low conductivity soil layer and A, Hooghoudt's constant. The other characteristics of the drained field, such as the depth of the drains, the surface runoff coefficient, the maximum pool height, and the conductivity of the subsoil, are all kept constant for all the simulations; a description of the effect of some of these parameters can be found in Vandenberg (1985). A precipitation of 0.6 cm/day for 15 days is the typical input for all simulations. The characteristic curve and the moisture - conductivity relation of the sandy loam and the three low-conductivity soils are given in Table 1. The Ψ-values of the low-conductivity soils are identical; only the conductivities vary. Soil type 3 is 10 times as permeable as soil type 2, while soil type 4 is only half as permeable as soil type 3. The values used for the other parameters are:

$A$ = varying: 0.002, 0.004 or 0.008 $day^{-1}$
$A_b = 0.2\ cm^{-1}\ day^{-1}$
$K_o$ = 2 cm/day
Depth of drain = 100 cm
Maximum pool height = 0.4 cm

Depth of low conductivity zone - varying:
from 0 - 35 cm above the drain,
from 35 - 70 cm above the drain, or
from 65 - 100 cm above the drain.

For all the simulations, the initial state of the system is the state at which there is an equilibrium downward flow of 0.01 cm/day.

At this rate of flow, the initial moisture content available for drainage from the homogeneous sandy soil is approximately 7.4 cm out of a total of 100 x (0.45 - 0.28 cm) = 17 cm for a completely saturated soil, leaving 9.6 cm for storage of precipitation. For a profile with the least permeable soil (type 4) in the lower 35 cm, the total amount of storage initially available is only 6.6 cm. Thus, to accommodate a precipitation event totalling 15 x 0.6 = 9 cm without creating overland flow, the homogeneous sandy soil does not need to generate drain discharge, whereas the soil profile with the low permeability impeding layer has to generate a ground-water discharge of approximately 2.4 cm or a rate of 2.4/15 = 0.16 cm/day.

Table 1. Conductivity and Pressure as Functions of Moisture Content

| | Soil type 1 | | Soil type 2 | | Soil type 3 | | Soil type 4 | |
|---|---|---|---|---|---|---|---|---|
| (%) moisture | Cond. cm/day | Pressure cm | Cond. cm/day | Pressure cm | Cond. cm/day | Pressure cm | Cond. cm/day | Pressure cm |
| 28 | 0.0267 | -217.0 | 0.00008 | -280 | 0.00079 | -280 | 0.00039 | -280 |
| 29 | 0.1577 | -127.0 | 0.00009 | -272 | 0.00087 | -272 | 0.00043 | -272 |
| 30 | 0.2874 | - 97.0 | 0.00010 | -265 | 0.00102 | -265 | 0.00051 | -265 |
| 31 | 0.4203 | - 78.0 | 0.00012 | -255 | 0.00122 | -255 | 0.00061 | -255 |
| 32 | 0.5451 | - 65.0 | 0.00015 | -245 | 0.00149 | -245 | 0.00075 | -245 |
| 33 | 0.6651 | - 55.0 | 0.00019 | -234 | 0.00186 | -234 | 0.00098 | -234 |
| 34 | 0.7217 | - 51.0 | 0.00023 | -223 | 0.00231 | -223 | 0.00115 | -223 |
| 35 | 0.7813 | - 47.0 | 0.00029 | -211 | 0.00294 | -211 | 0.00147 | -211 |
| 36 | 0.8463 | - 43.0 | 0.00037 | -199 | 0.00374 | -199 | 0.00187 | -199 |
| 37 | 0.9077 | - 39.5 | 0.00049 | -186 | 0.00485 | -186 | 0.00242 | -186 |
| 38 | 0.9735 | - 36.0 | 0.00065 | -172 | 0.00641 | -172 | 0.00320 | -172 |
| 39 | 1.0337 | - 33.0 | 0.00087 | -157 | 0.00866 | -157 | 0.00433 | -157 |
| 40 | 1.0976 | - 30.0 | 0.00117 | -142 | 0.01169 | -142 | 0.00584 | -142 |
| 41 | 1.1585 | - 27.3 | 0.0017 | -123 | 0.01709 | -123 | 0.00854 | -123 |
| 42 | 1.3482 | - 19.7 | 0.00265 | -101 | 0.02653 | -101 | 0.01326 | -101 |
| 43 | 1.5359 | - 13.2 | 0.00429 | - 77 | 0.04288 | - 77 | 0.02144 | - 77 |
| 44 | 1.7562 | - 6.5 | 0.00813 | - 52 | 0.08131 | - 52 | 0.04065 | - 52 |
| 45 | 1.8798 | - 3.1 | 0.01189 | - 26 | 0.11890 | - 26 | 0.05945 | - 26 |
| 46 | 2 | 0 | 0.02 | 0 | 0.2 | 0 | 0.1 | 0 |

The results of the 36 simulations, the combined surface runoff and drain discharge, are shown in Figures 6 to 14. They are organized as follows:

| | |
|---|---|
| Figures 6, 7, 8: | Low conductivity layer from 0 - 35 cm above drain |
| Figures 9, 10, 11: | Low conductivity layer from 35 - 70 cm above drain |
| Figures 12, 13, 14: | Low conductivity layer from 65 - 100 cm above drain. |
| Figures 6, 9, 12: | A = 0.008 $day^{-1}$ |
| Figures 7, 10, 13: | A = 0.004 $day^{-1}$ |
| Figures 8, 11, 14: | A = 0.002 $day^{-1}$. |

Each figure represents 4 simulations: No impeding layer, and impeding layers of soil types 2, 3 and 4.

One of the first things to be noted is that, regardless of the other parameters, the total discharge is always primarily surface runoff in the simulations with the least impermeable impeding layer. This is indicated by the sudden rise of the total discharge. For a deeper impeding layer, more precipitation is stored in the sandy soil above the layer, the rise in discharge occurs later in time, and the peak is smaller (Compare, for example, Figures 6, 9 and 12.). The drain discharge for these low permeability layers is never more than 0.03 cm/day.

The simulations with an impeding layer of intermediate permeability (soil type 4) yield surface runoff in all the simulations when the impeding layer is in the middle or the top 35 cm of the profile, but the contribution of drain discharge to total discharge is considerably higher than for soil type 2 (up to 0.12 cm/day). Figure 6 shows a rather curious phenomenon, where an equilibrium flow is established which is slightly higher (0.12 cm/day) than the saturated conductivity of the impeding layer (0.1 cm/day). In the impeding layer, an inverted moisture gradient is established, i.e., the moister soil is near the top of the layer; and, at that particular rate of flow, the flow across the boundary between the two soil layers can also be maintained at the same rate. In other words, Equations 4 and 5 can be satisfied. A few less notorious examples of the same phenomenon are the short "plateaus" on the receding limbs on Figures 9, 10 and 11.

The sudden rises and drops in drain discharge in many of the simulations are caused by a delay in saturation of the impeding layer. The low permeability impedes the flow to deeper layers and causes the sandy soil in the upper layer to become fully saturated; a perched watertable is temporarily formed. However, the drain discharge is determined by the height of the watertable at or below the lower boundary of the impeding layer. Eventually the impeding layer becomes saturated and the watertable suddenly rises dramatically to the height of the perched watertable. The same process happens in reverse during drainage of the profile after the precipitation event ceases.

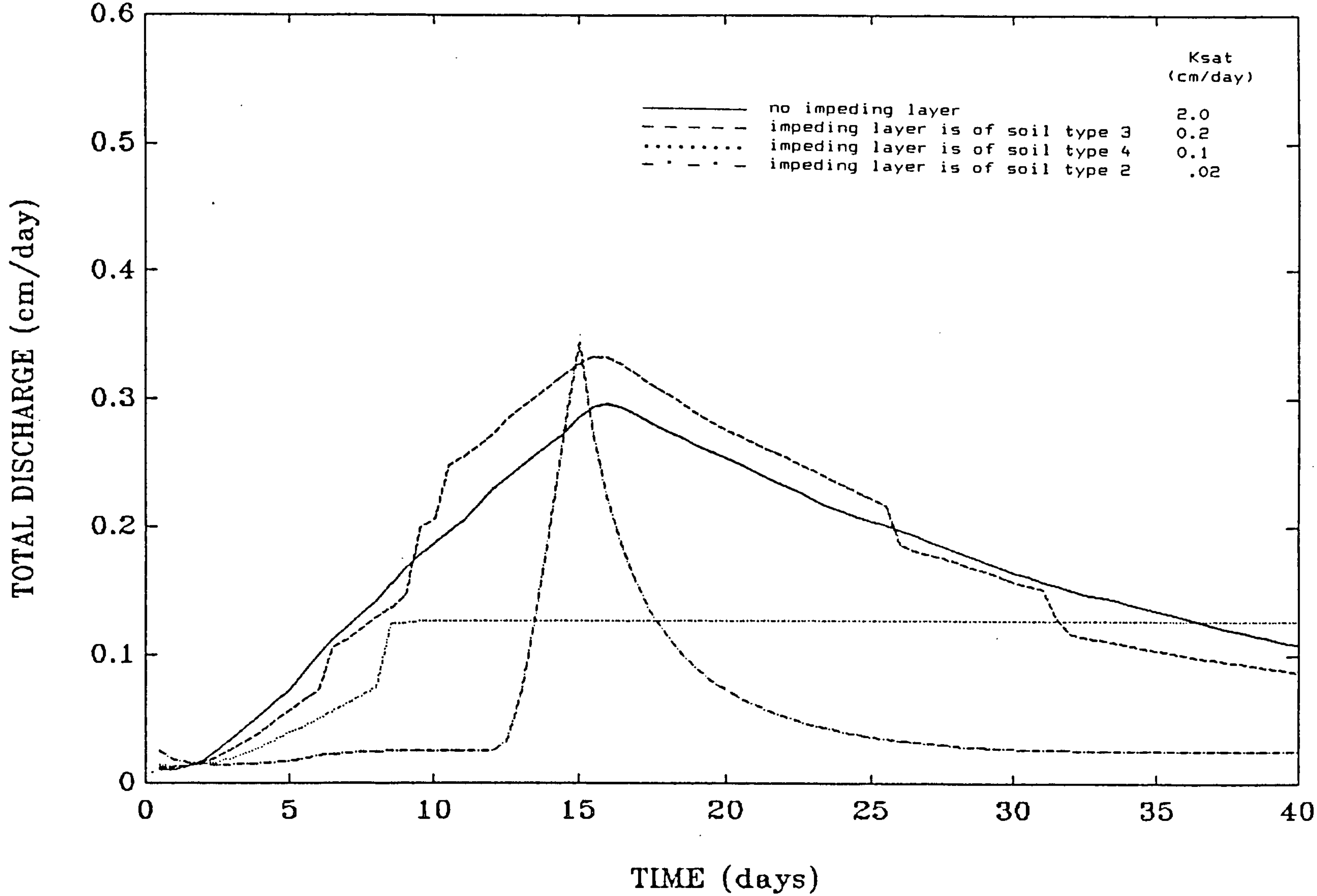


Figure 6. Simulations 1 - 4: Impeding layer from 0 - 35 cm above the drains: A = 0.008/day.

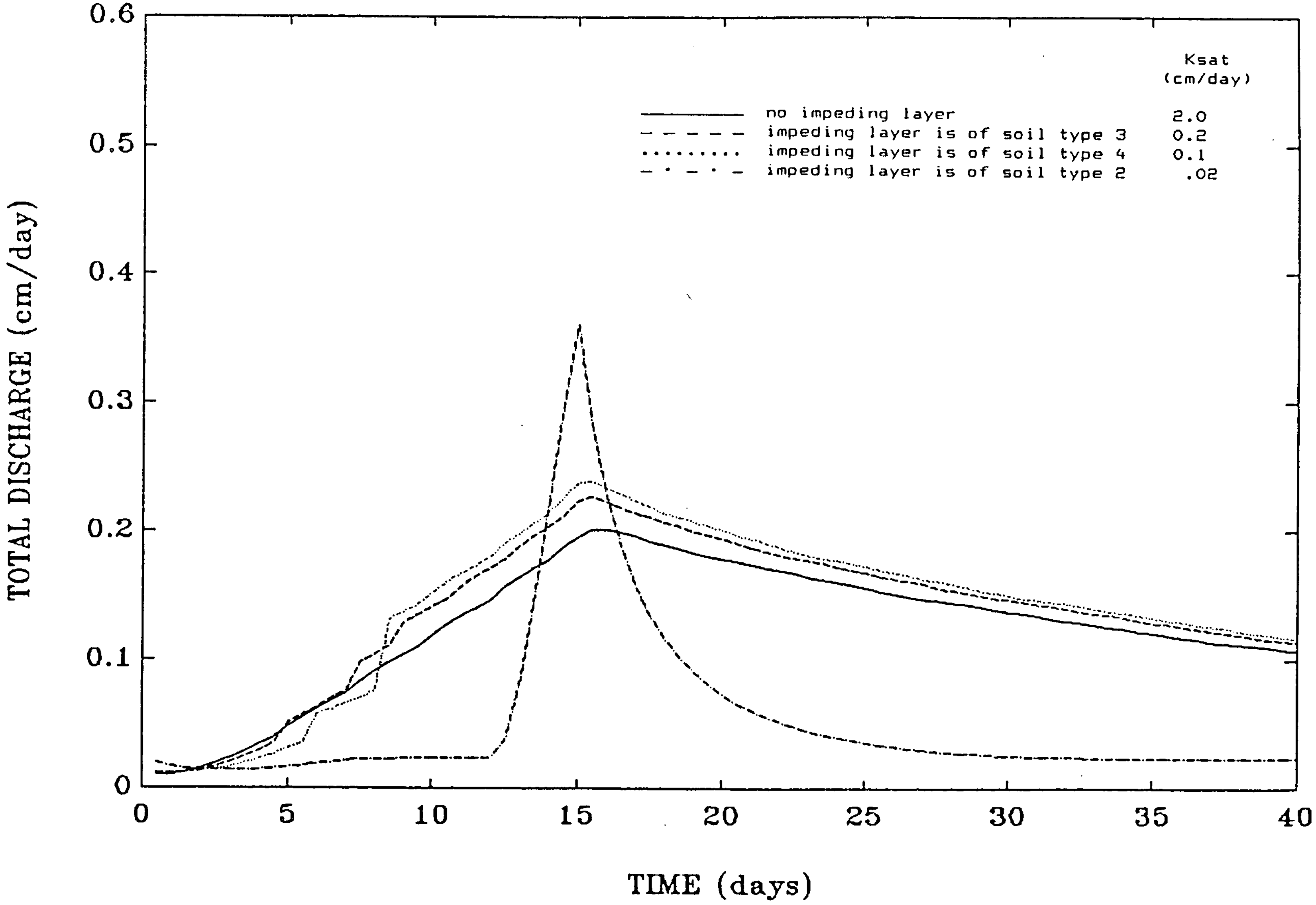


Figure 7. Simulations 5 - 8: Impeding layer from 0 - 35 cm above the drains: A = 0.004/day.

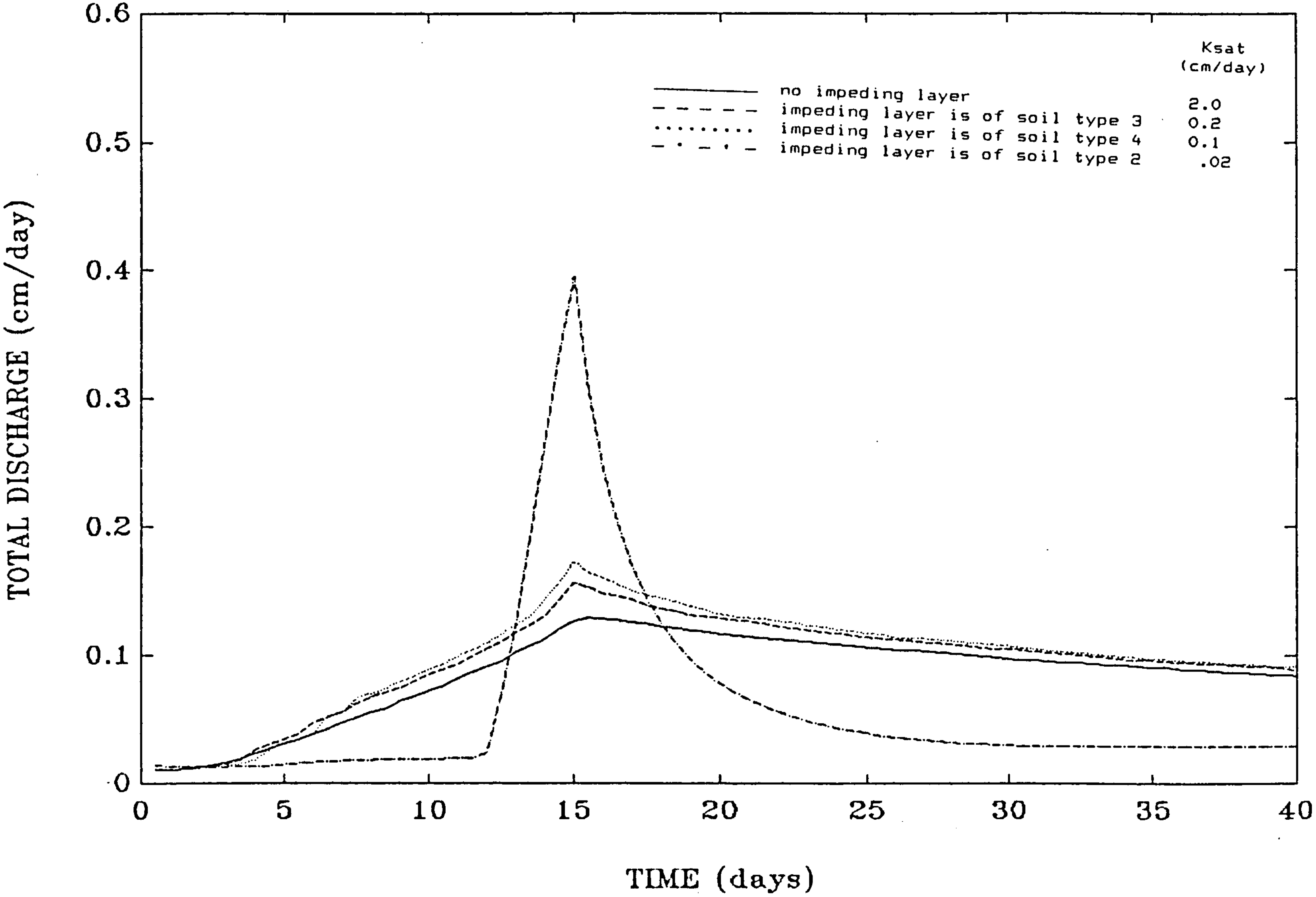


Figure 8. Simulations 9 - 12: Impeding layer from 0 - 35 cm above the drains: A = 0.002/day.

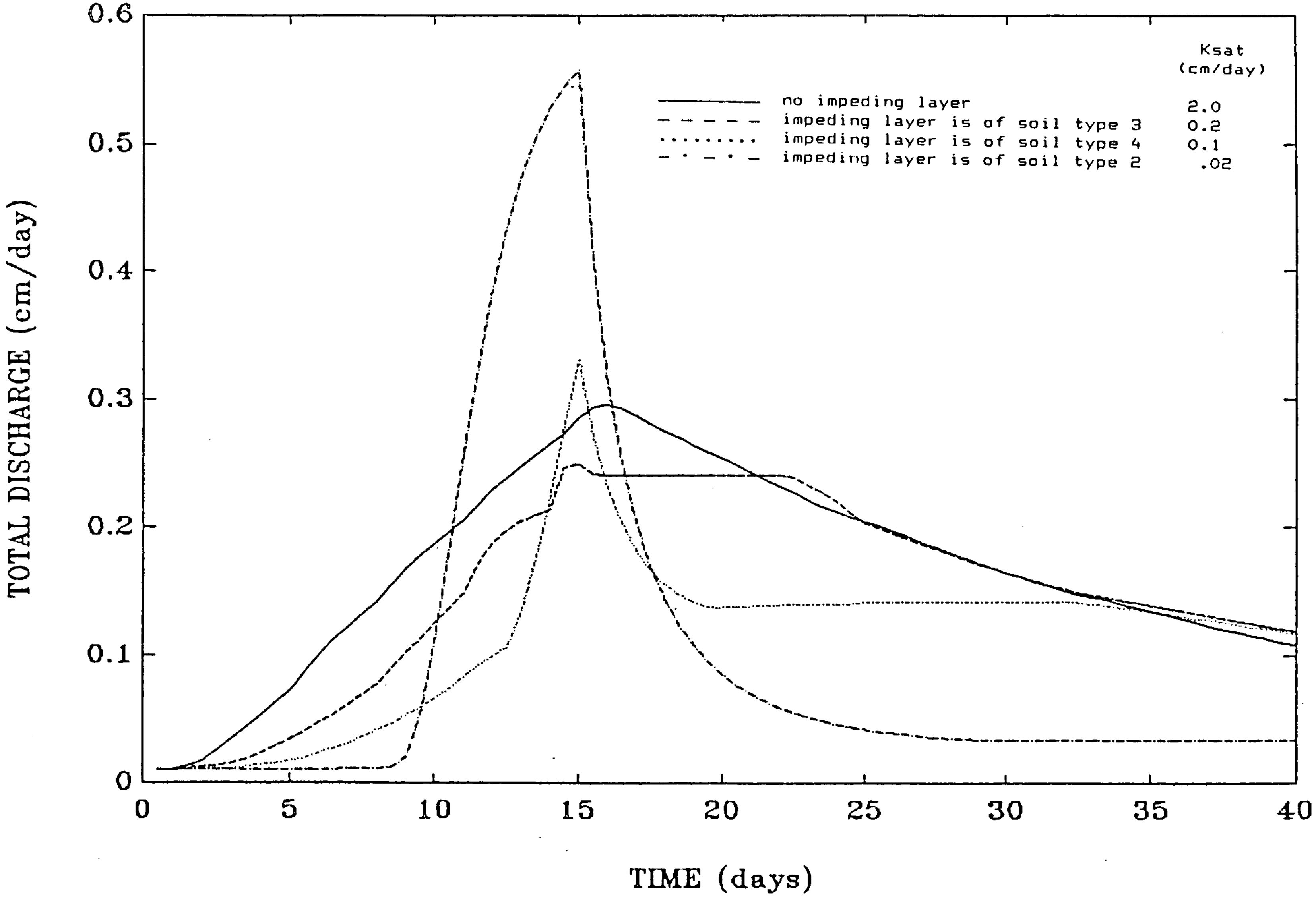


Figure 9. Simulations 13 - 16: Impeding layer from 35 - 65 cm above the drains: A = 0.008/day.

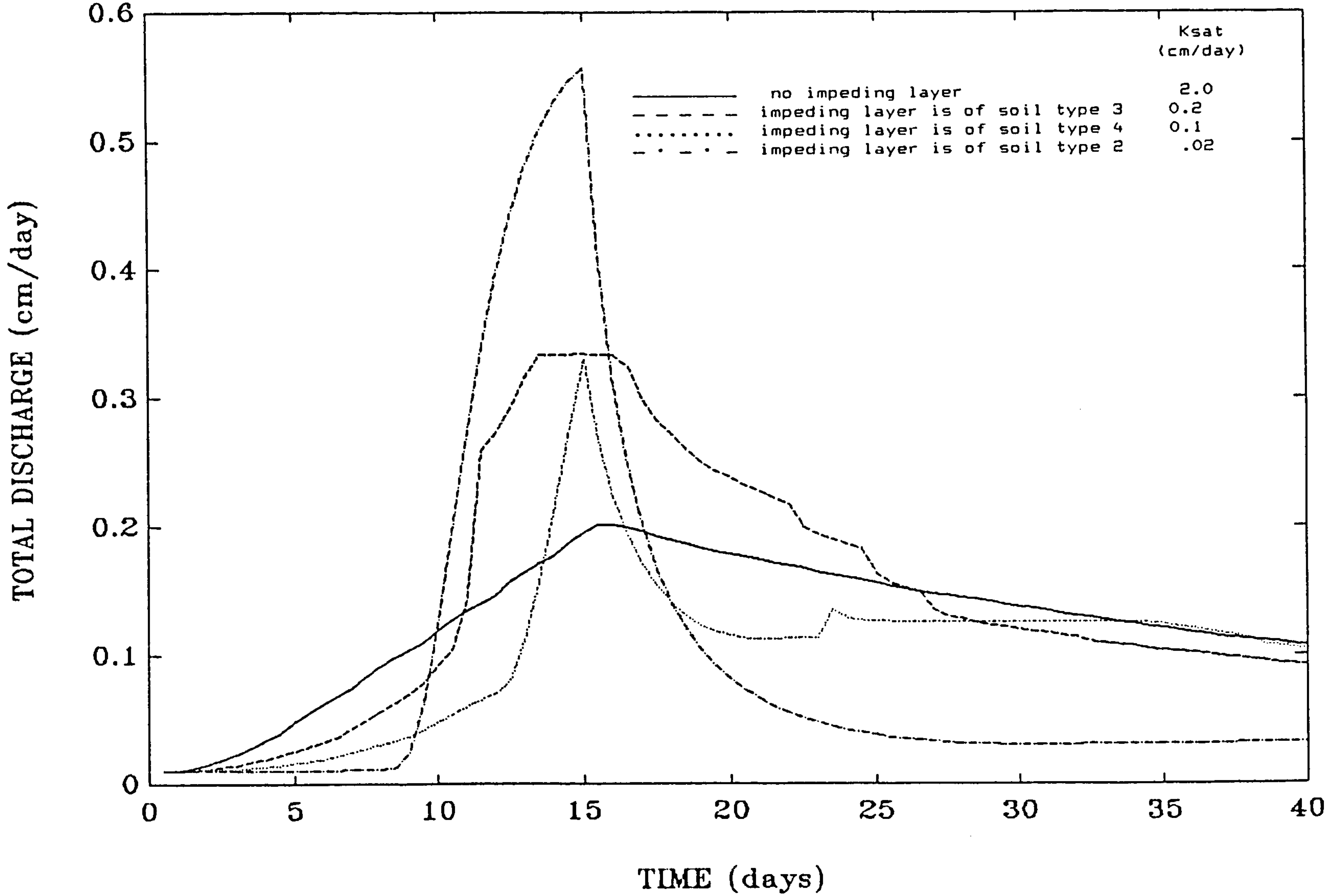


Figure 10. Simulations 17 - 20: Impeding layer from 35 - 65 cm above the drains: A = 0.004/day.

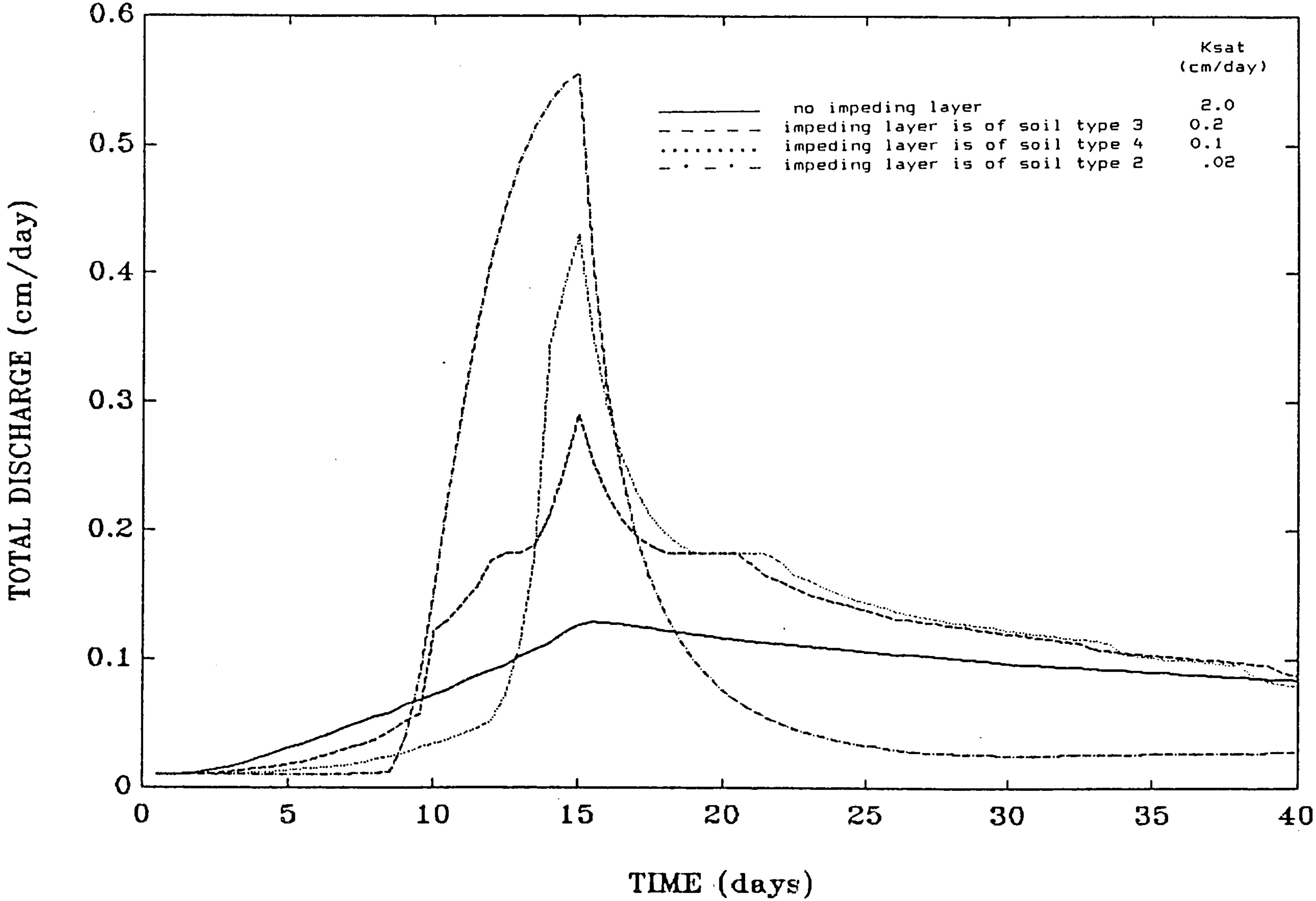


Figure 11. Simulations 21 - 24: Impeding layer from 35 - 65 cm above the drains: A - 0.002/day.

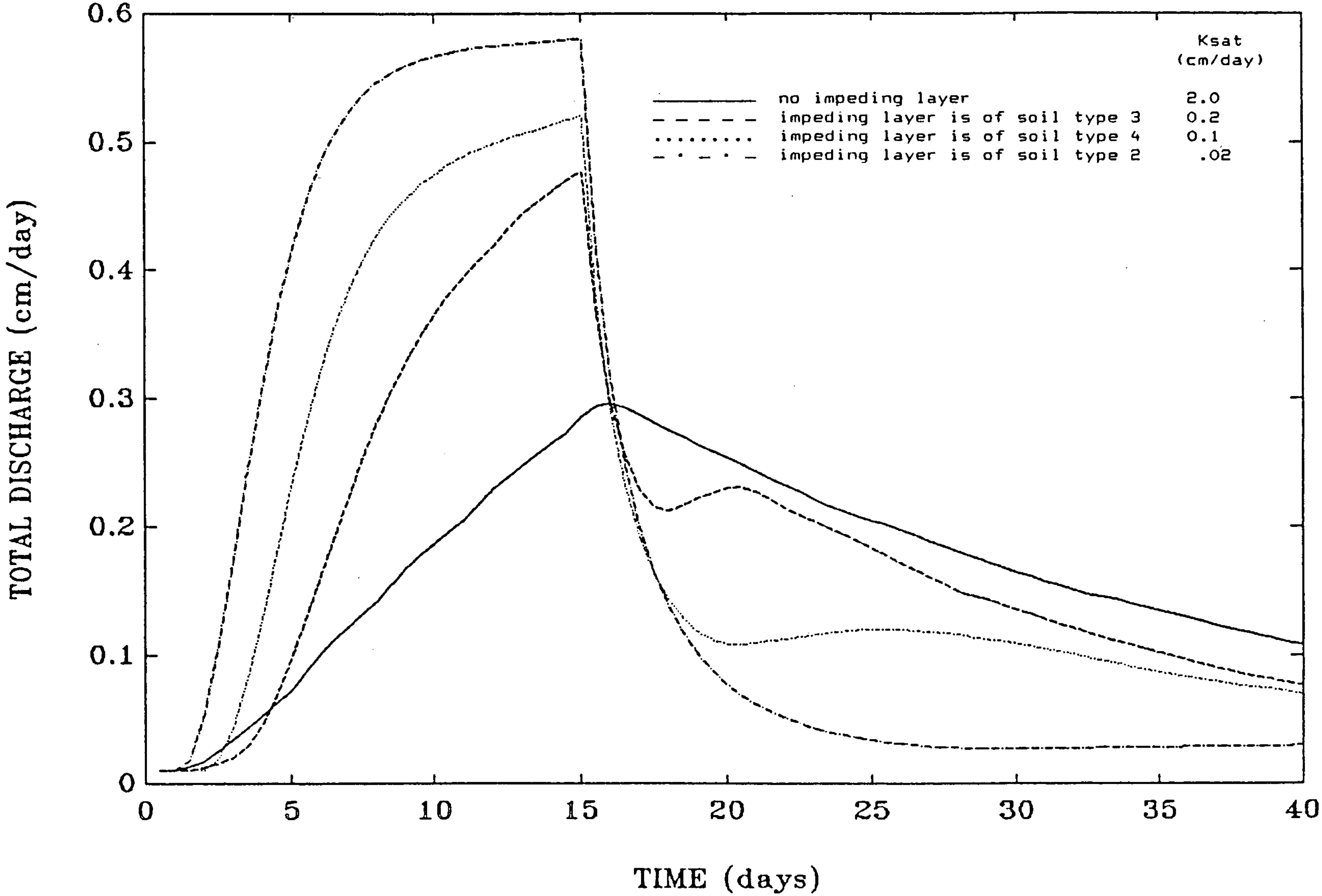


Figure 12. Simulations 25 - 28: Impeding layer from 65 - 100 cm above the drains: A = 0.008/day.

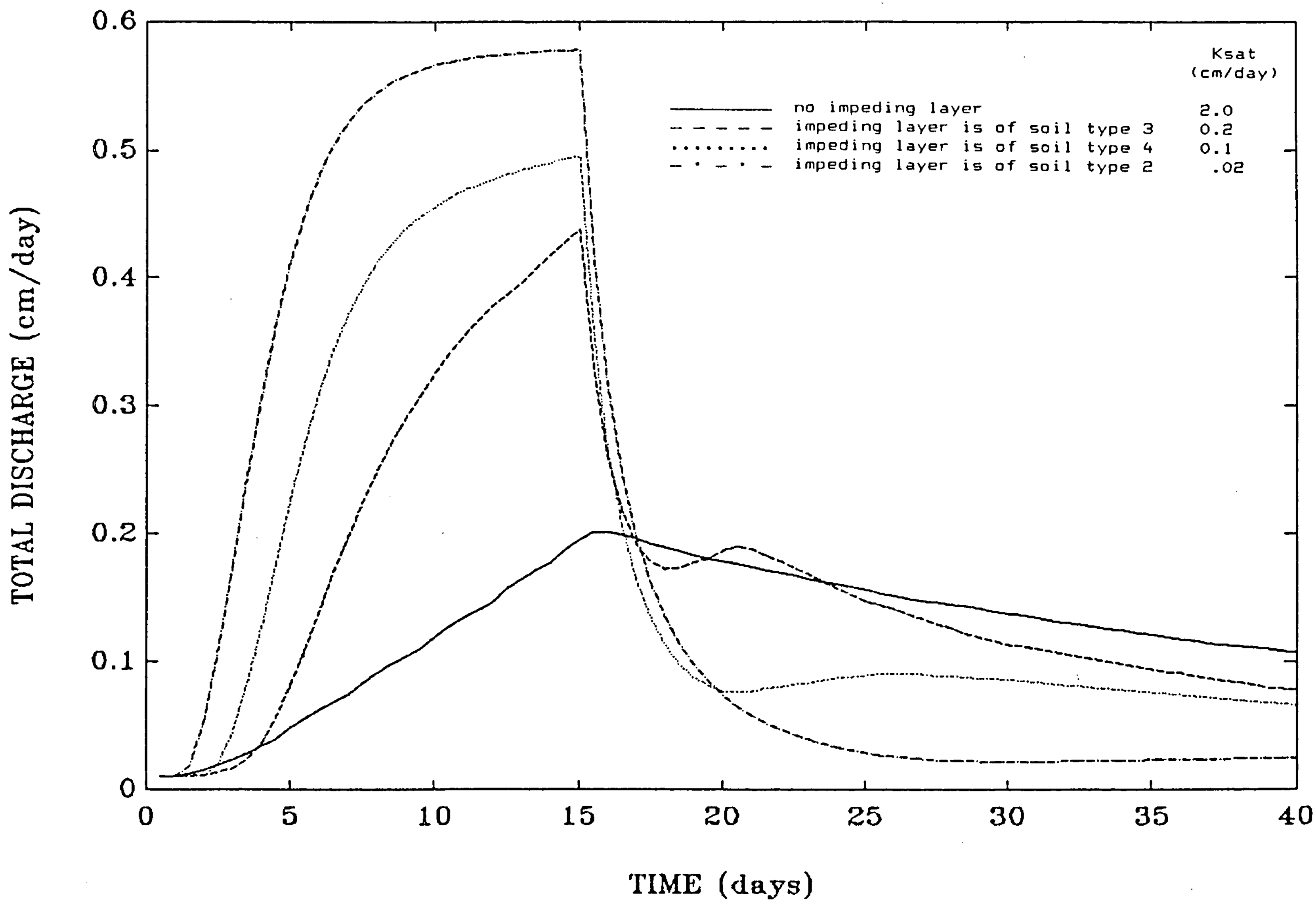


Figure 13. Simulations 29 - 32: Impeding layer from 65 - 100 cm above the drains: A = 0.004/day.

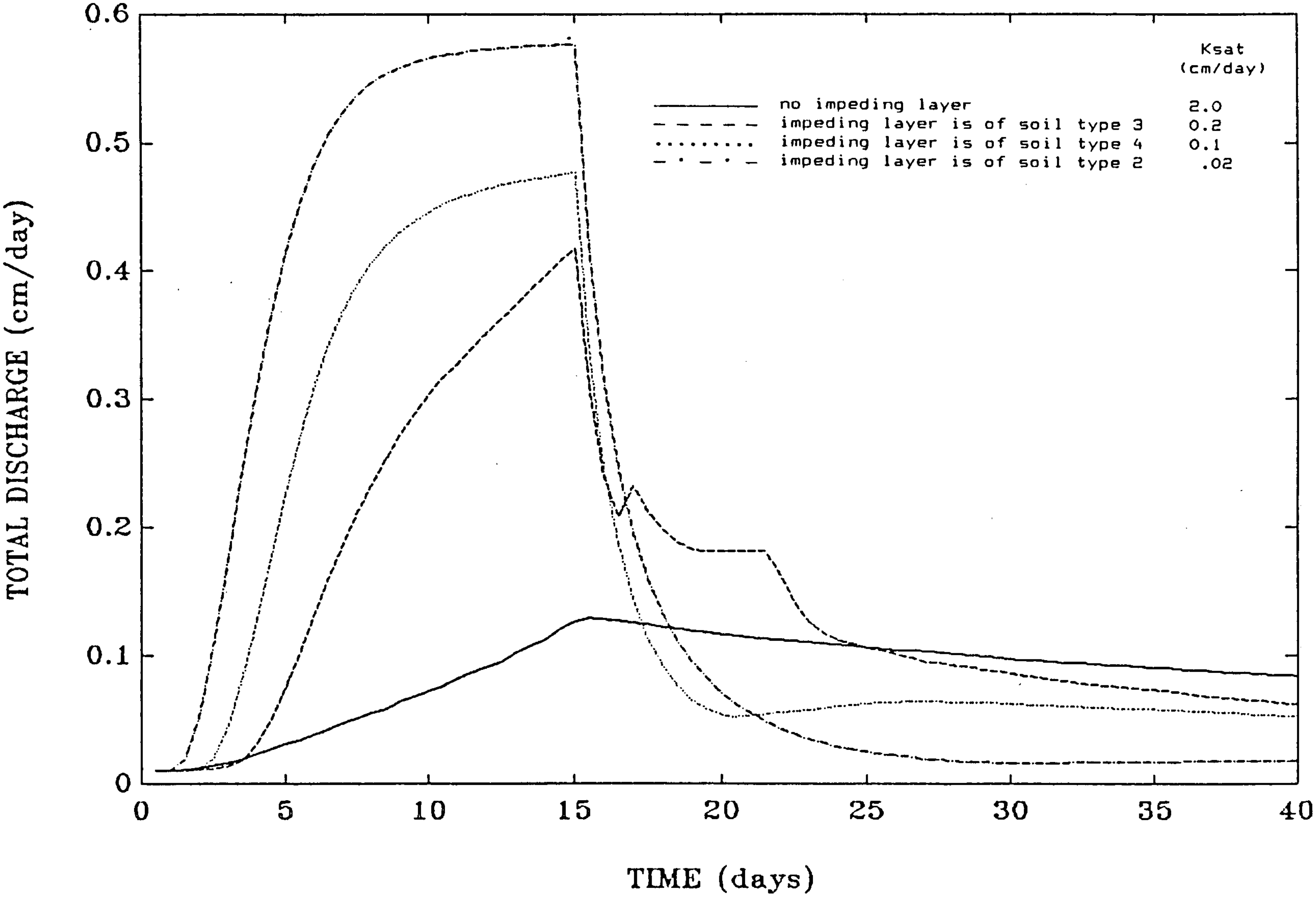


Figure 14. Simulations 33 - 36: Impeding layer from 65 - 100 cm above the drains: A = 0.002/day.

As can be expected, the influence of the impeding layer on the behaviour of the discharge is severest when the layer is situated near the surface (Figures 12, 13 and 14). No storage is available in the soil until the water has passed through the low permeability surface layer and, even thereafter, can only be accessed at the transmission rate imposed by the impeding layer. Thus most of the water will run off the surface before it has a chance to infiltrate. The delay in infiltration is here accompanied by a delay in the arrival of the peak of the drain discharge, which is indicated in Figures 12 to 14 by a secondary peak in the total discharge from 5 to 15 days after the end of the storm and after the peak in surface runoff.

## CONCLUSIONS

The introduction of layered soil in the modelling of vertical unsaturated moisture movement has extended the scope of the model and the drainage situations to which it can be applied. At the same time, the amount of soil physical data that must be gathered before full use of the model can be made has increased drastically. The functions $\Psi(\Theta)$ and $K(\Theta)$ must be determined over the full range of moisture from extreme dryness to complete saturation for each soil type in the profile.

In the examples, it has been demonstrated how a layer of low permeability in the profile impedes the movement of moisture towards the lower unsaturated soil layers, thereby reducing the effective storage. Although this effect is only temporary, the effect on the instantaneous discharge is still large, since rapid access to the storage is required to prevent precipitation running off over the surface. The introduction of an impeding layer in the profile also has some unexpected results on the drain discharge, with respect to the perched watertable that may temporarily develop above the impeding zone. A rapid rise in the drain discharge, when the impeding layer finally becomes saturated and the perched water table suddenly becomes the true watertable, was observed in the model.

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# Appendix A
# Fortran Coding for Program DRAIN

# APPENDIX A

## FORTRAN CODING FOR PROGRAM DRAIN

This appendix is a copy of file DRAIN.FOR on diskette DRAIN-PLOT. DRAIN.FOR is the FORTRAN source code for program DRAIN. For the compilation, the IBM FORTRAN compiler, version 2.00 was used on an IBM PC XT equipped with an 8087 Math Coprocessor.

```
$nofloatcalls
C***********************************************************************
                         PROGRAM DRAIN2
C SIMULATION OF INFILTR.,GROUNDW. DISCHARGE AND SURFACE RUNOFF
C               FROM DRAINED FIELDS:HETEROGENEOUS SOILS.
C
C                               BY
C
C                          A.VANDENBERG
C
C--------------------VERSION FEBRUARY 20/1987---------------------------
C
C
C THIS VERSION INCLUDES RECALCULATION OF THE TIMESTEP AND A FASTER
C ROUTINE FOR INTERPOLATION IN 'TABLE' TO FIND Θ, Ψ AND K, STARTING
C FROM POSITION IN TABLE AT LAST USE, OR ESTIMATE THEREOF. AND IT
C USES A SIMPLIFIED STATEMENT FOR THE CALCULATION OF ZW, ON THE
C ASSUMPTION THAT THE COEFFICIENT OF THE SECOND ORDER TERM IS SMALL
C ENOUGH TO BE NEGLECTED.
C----------------------------------------------------------------------
C  THIS SIMULATION IS A REVISION OF PROGRAM FLO, WHICH WAS PRODUCED
C FOR HOMOGENEOUS SOILS:  PROGRAM DRAIN USES UP TO 5 DIFFERENT SOIL
C TYPES,EACH WITH ITS OWN CHARACTERISTIC CURVE AND ITS OWN
C CONDUCTIVITY-MOISTURE RELATION, INPUT TO THE PROGRAM IN TABULAR
C FORM:IN OTHER RESPECTS INPUTS AND OUTPUTS OF PROGRAM FLO AND
C PROGRAM DRAIN2 ARE VERY SIMILAR.
```

```
C
                    DESCRIPTION OF INPUT VARIABLES
C
C==========================LINES 1 AND 2==================================
C  THE FIRST 2 CARDS OR LINES ARE HEADER LINES TO BE PRINTED ON THE
C OUTPUT FILE AND ONTO TAPE6.
C
C===============================LINE 3====================================
C  NUMSIM      =NUMBER OF SIMULATIONS.ALL OF THE FOLLOWING DATA MUST
C               BE SUPPLIED ONCE FOR EACH OF THE NUMSIM SIMULATIONS.
C===============================LINE 4====================================
C  N           =NUMBER OF SOIL SEGMENTS.
C  DZ          =THICKNESS OF ALL SOIL LAYERS(CM)
C  A           =COEFFICIENT IN HOOGHOUDT)S EQUATION(/DAY)
C  AB          =COEFFICIENT IN SURFACE RUNOFF EQUATION:
C               QS=AB*(POOL-PMAX)**2 (/CM/DAY)
C  PMAX        =POOL DEPTH ABOVE WHICH SURFACE RUNOFF OCCURS(CM)
C  DPRINT      =TIME INTERVAL AT WHICH RESULTS ARE TO BE PRINTED AND
C               WRITTEN TO PLOT TAPE(DAYS).
C  DELT        =LENGTH OF TIME STEP(DAYS)
C  TIME        =TIME AT BEGINNING OF SIMULATION,USUALLY ZERO(DAYS).
C  MT          =NUMBER OF DIFFERENT SOIL TYPES
C  MTE         =NUMBER OF ENTRIES IN THE TABLES OF SOIL PROPERTIES
C  POOL        =INITIAL HEIGHT OF THE WATER POOLED ON THE SURFACE(CM)
C  DITCH       =INITIAL HEIGHT OF WATER IN DITCH(CM)
C  AKO         =CONDUCTIVITY OF THE SUBSOIL(CM/DAY)
```

```
C  DOWNRATE    =RATIO, PUTTING LOWER LIMIT ON INTERVAL IN WHICH DTA
C               IS NOT REPLACED; NORMAL VALUE FROM .5 TO .9
C               A NEGATIVE VALUE CAUSES THE DT-ADJUST ROUTINE TO BE
C               BYPASSED, TAKING THE SUPPLIED VALUE OF DT THROUGHOUT.
C  UPRATE      =RATIO, PUTTING UPPER LIMIT ON INTERVAL IN WHICH DTA
C               IS NOT REPLACED; NORMAL VALUE:> DOWNRATE, < 1.0
C               A VALUE > 1 WILL CAUSE BYPASSING OF THE DT-ROUTINE.
C===============================LINE 5=================================
C  IOPT(1)     =OPTION SELECTOR #1:
C                  =0,THEN N VALUES OF THE INITIAL MOISTURE ARE
C                     EXPECTED IN THE INPUT STREAM.
C                  =1,ONLY A VALUE FOR QS,THE INITIAL STEADY
C                     DISCHARGE IS EXPECTED
C                  =2,N VALUES OF PRESSURE ARE TO BE READ,AND
C                     INITIAL MOISTURE PROFILE IS CALCULATED
C  IOPT(2)       =OPTION SELECTOR #2:
C                  =0,EXTENDED OUTPUT, WITH Θ,K,Ψ PROFILE.
C                  =1,NO MOISTURE PROFILE,BUT INTERNAL FLOWS LISTED
C                  =2,NO INTERNAL FLOW,ONE LINE OF OUTPUT
C  IOPT(3)       =OPTION SELECTOR #3:
C                  =0,NO TAPE6 IS PRODUCED.
C                  =1,TAPE6 IS PRODUCED
C============================LINE 6====================================
C  IND(I),I=1,N =N INTEGERS,RANGING FROM 1 TO MT,INDICATING SOIL
C                 TYPE OF THE I-TH SEGMENT
```

```
C=====================LINE(=FORTRAN LOGICAL RECORD) 7==================
C  TABLE(I,J,K) =THE MATRIX HOLDING THE TABLE OF VALUES OF
C                MOISTURE,CONDUCTIVITY AND PRESSURE,
C                DESCRIBING THE MOISTURE-PRESSURE AND
C                MOISTURE-CONDUCTIVITY RELATIONS FOR EACH
C                SOIL-TYPE.THEY ARE READ IN THIS ORDER:
C                 1)MTE MOISTURE PERCENTAGES,STARTING WITH
C                   THE SMALLEST AND ENDING WITH THE SATU-
C                   RATION MOISTURE CONTENT,THE POROSITY,
C                   SOIL TYPE 1.
C                 2)MTE CONDUCTIVITY VALUES(CM/DAY),SOIL-TYPE 1
C                 3)MTE PRESSURES(CM),SOIL-TYPE 1.
C                THESE ARE FOLLOWED BY THE SAME SEQUENCE FOR
C                SOIL-TYPE 2,3,ETC.
C                NOTE THAT PRESSURE MUST BE NEGATIVE,EXCEPT
C                FOR THE LAST VALUE FOR EACH SOIL-TYPE,WHICH
C                MUST BE THE SATURATION PRESSURE AND BE ZERO.
C===============================LINE 8==================================
C  QS           =ONLY IF IOPT(1)=1:INITIAL STEADY STATE DISCHARGE
C                (CM/DAY)
C==========================LINE 8, ALTERNATE============================
C  WET(I),I=1,N =ONLY IF IOPT(1)=0:N VALUES OF INITIAL = MOISTURE
C==========================LINE 8, ALTERNATE============================
C  FI(I),I=1,N  =ONLY IF IOPT(1)=2:N VALUES OF INITIAL PRESSURE(CM)
C==========================LINE 9, ETC.=================================
C  RAIN         =PRECIPITATION(+) OR EVAPORATION(-)(CM/DAY)
C  TMAX         =TIME(DAYS) UNTIL WHICH FOREGOING RAIN-RATE PREVAILS
```

```
C
C           THE LAST TWO DATA ITEMS MAY BE REPEATED AS MANY TIMES AS
C           NEEDED.AS SOON AS THE NEW VALUE OF TMAX IS LESS THAN THE
C           PREVIOUS ONE,THIS SIMULATION IS STOPPED AND DATA READ-IN
C           FOR THE NEXT SIMULATION STARTS.
C====================END OF DATA
DESCRIPTION======================
      COMMON TABLE(50,3,5),TARG(3),ISTART(50)
      DIMENSION WET(50), PER(50), FI(50)
      DIMENSION IOPT(3), IND(50), Q(50)
      CHARACTER*79 TXT
C
C**************************************************************************
C
C*****************READ-IN AND ECHO OF INPUT DATA**************************
      READ (1,46) TXT
      WRITE(6,47) TXT
      WRITE(*,47) TXT
      READ(1,46) TXT
      WRITE (6,47) TXT
      WRITE(*,47) TXT
      READ (1,*) NUMSIM
      DO 44 KK=1,NUMSIM
      READ (1,*) N,DZ,A,AB,PMAX,DPRINT,DELT,TIME,MT,MTE,POOL,DITCH,
     1AKO,DOWNRATE,UPRATE
      WRITE(*,48)N,DZ,A,AB,PMAX,DPRINT,DELT,TIME,MT,MTE,POOL,DITCH,
```

```
     1AKO,DOWNRATE,UPRATE
      READ (1,*) (IOPT(I),I=1,3)
      READ (1,*) (IND(I),I=1,N)
      READ (1,*) (((TABLE(I,J,K),I=1,MTE),J=1,3),K=1,MT)
      WRITE(*,49)(IOPT(I),I=1,3)
      WRITE(*,49)(IND(I),I=1,N)
      DO 1 K=1,MT
      WRITE(*,50) K
1     WRITE(*,51) ((TABLE(I,J,K),J=1,3),I=1,MTE)
C
C****
C
      DZS=DZ*DZ
      DO 238 I=1,N
238   ISTART(I)=MTE/2
      RNGE=TABLE(MTE,1,IND(N))-TABLE(1,1,IND(N))
      M1=N-1
      DPRINT=DPRINT*.9999
      DEPTH=(N-1.)*DZ
C******
C  SETTING UP OF INITIAL MOISTURE,PRESSURE AND CONDUCT. PROFILES
C******
      IF (IOPT(1).EQ.1) GO TO 5
      IF (IOPT(1).EQ.2) GO TO 3
      READ (1,*) (WET(I),I=1,N)
      DO 2 I=1,N
```

```
      TARG(1)=WET(I)
      CALL LOOKUP (1,IND(I),MTE,I)
      PER(I)=TARG(2)
2     FI(I)=TARG(3)
      GO TO 17
3     READ (1,*) (FI(I),I=1,N)
      DO 4 I=1,N
      TARG(3)=FI(I)
      CALL LOOKUP (3,IND(I),MTE,I)
      PER(I)=TARG(2)
4     WET(I)=TARG(1)
      GO TO 17
5     READ (1,*) QS
C******
C     CALCULATE MAXIMIUM DISCHARGE AND INITIAL WATERTABLE HEIGHT
C******
      QSMAX=A*DEPTH
      QSMAX=QSMAX*AKO/(QSMAX+AKO)
      IF (QS.LE.QSMAX) GO TO 6
      QS=QSMAX
      ZW=DEPTH
      NS=N
      WRITE(*,52) QS
      GO TO 15
6     ZW=QS*AKO/(A*(AKO-QS))
C******
```

```
C       CALCULATE INITIAL PRESSURE PROFILE FROM QS AS GIVEN
C******
      NS=ZW/DZ+1
      DIS=NS*DZ-ZW
      NT=NS+1
      IF (DIS.GT.DZ/2.) GO TO 9
      TARG(2)=TABLE(MTE,2,IND(NT))
      TARG(3)=0.
7     P=TARG(3)
      TARG(3)=DIS*(QS/TARG(2)-1.)
      CALL LOOKUP (3,IND(NT),MTE,NT )
      IF (ABS(P-TARG(3)).GT.0.001) GO TO 7
8     FI(NT)=TARG(3)
      PER(NT)=TARG(2)
      WET(NT)=TARG(1)
      GO TO 11
9     FB=(DIS-.5*DZ)*(QS/TABLE(MTE,2,IND(NS))-1.)
      TARG(3)=FB
      TARG(2)=TABLE(MTE,2,IND(NT))
10    P=TARG(3)
      TARG(3)=DZ/2.*(QS/TARG(2)-1.)+FB
      CALL LOOKUP (3,IND(NT),MTE,NT)
      IF (ABS(P-TARG(3)).GT.0.001) GO TO 10
      GO TO 8
11    N1=NS+2
      DO 14 I=N1,N
      TARG(2)=PER(I-1)
      TARG(3)=0.
```

```
12    P=TARG(3)
      TARG(3)=FI(I-1)+DZ*(QS/(PER(I-1)+TARG(2))-.5)
      CALL LOOKUP (3,IND(I-1),MTE,I-1)
      IF (ABS(P-TARG(3)).GT.0.001) GO TO 12
      FB=TARG(3)
      CALL LOOKUP (3,IND(I),MTE,I)
      AKB=TARG(2)
      TARG(3)=0.
13    P=TARG(3)
      TARG(3)=FB+DZ*(QS/(AKB+TARG(2))-.5)
      CALL LOOKUP (3,IND(I),MTE,I)
      IF (ABS(P-TARG(3)).GT.0.001) GO TO 13
      FI(I)=TARG(3)
      WET(I)=TARG(1)
      PER(I)=TARG(2)
      IF (WET(I).LT.TABLE(MTE,1,IND(I))) GO TO 14
      WRITE(*,53) I
14    CONTINUE
15    DO 16 I=1,NS
      PER(I)=TABLE(MTE,2,IND(I))
      FI(I)=0.
16    WET(I)=TABLE(MTE,1,IND(I))
      GO TO 20
17    NS=0
      DO 18 I=1,N
      IF (WET(I).LT.TABLE(MTE,1,IND(I))) GO TO 19
      NS=NS+1
```

```
18      CONTINUE
19      ZW=(NS-1)*DZ
        NT=NS+1
        N1=NS+2
        ZW=AKO*(DZ*NS+FI(NT))/(AKO-A*FI(NT))
        NS=ZW/DZ+1
        WET(NS)=TABLE(MTE,1,IND(NS))
        PER(NS)=TABLE(MTE,2,IND(NS))
        FI(NS)=0.
        QS=A*ZW*AKO/(AKO+A*ZW)
20      WRITE(*,54) (WET(N+1-I),PER(N+1-I),FI(N+1-I),I=1,N)
        WRITE(*,55) QS,ZW
        DT=DELT
        PRNT=0.
        PRECIP=0.
C******
C        END OF PRELIMINARY CALCULATIONS.
C        START STEPPING THROUGH TIME.
C******
21      READ (1,*) RAIN,TMAX
        TMAX=TMAX*1.0001
        IF (TMAX.LT.TIME) GO TO 44
        WRITE(*,56) TMAX,RAIN
        IPTEST=0
```

```
22    TIME=TIME+DT
      DTA=DT
      IF (TIME.LT.TMAX) GO TO 23
      DTA=TMAX+DTA-TIME
      TIME=TMAX
      IPTEST=1
23    PRNT=PRNT+DTA
      NT=NS+1
      N1=NS+2
C     CALCULATE MAXIMUM ALLOWABLE TIMESTEP
C
C================
C
      IF(DOWNRATE.LT.0.0.OR. UPRATE.GT.1.0)GOTO 72
      IF(IPTEST.EQ.1)GOTO 72
      IF(NT.GT.N)GOTO 72
      DTM=1.E12
      DO 71 I=NT,N
      IF(FI(I).GE.0.)GOTO 71
      TARG(1)=WET(I)-.0001
      CALL LOOKUP(1,IND(I),MTE,I)
      HLO=TARG(3)
      TARG(1)=WET(I)+.0001
      CALL LOOKUP(1,IND(I),MTE,I)
      DENOM=HLO-TARG(3)
```

```
      IF(ABS(DENOM).LT.1.E-10)DENOM=1.E-10
      DTL=ABS(DZS*.0001/(PER(I)*DENOM))
      IF(DTL.LT.DTM)DTM=DTL
71    CONTINUE
      UP=UPRATE*DTM
      DOWN=DOWNRATE*DTM
      IF(DTA.GT.UP.OR.DTA.LT.DOWN)DT=UP
      IF(DTA.LT.1.E-6)STOP 'DTA TOO SMALL'
C******
C       CALCULATE INTERNAL FLOWS
C******
72     Q(NT)=-QS*DTA
       IF (NT.GE.N) GO TO 30
       DO 28 I=N1,N
       IF (IND(I-1).EQ.IND(I)) GO TO 27
       TARG(3)=PER(I)*(FI(I)+DZ/2.)+PER(I-1)*(FI(I-1)-DZ/2.)
       TARG(3)=TARG(3)/(PER(I)+PER(I-1))
       ITCNT=0
24     CALL LOOKUP (3,IND(I),MTE,I)
       ITCNT=ITCNT+1
       AKUP=TARG(2)
       CALL LOOKUP (3,IND(I-1),MTE,I-1)
       AKD=TARG(2)
       FB=(AKUP+PER(I))*(FI(I)+DZ/2.)
       FB=FB+(AKD+PER(I-1))*(FI(I-1)-DZ/2.)
```

```
      FB=FB/(PER(I)+PER(I-1)+AKUP+AKD)
      IF (FB.GT.0.) FB=0.
      IF (ITCNT.LT.4) GO TO 25
      WRITE(*,*) ITCNT,I
      GO TO 26
25    IF (ABS(FB-TARG(3)).LE.1) GO TO 26
      TARG(3)=FB
      GO TO 24
26    FF=DTA
      Q(I)=-(PER(I)+AKUP)*((FI(I)-FB)/DZ+.5)
      GO TO 28
27    Q(I)=DTA*(PER(I)+PER(I-1))/2.
      FF=(FI(I-1)-FI(I))/DZ-1.
28    Q(I)=Q(I)*FF
C******
C       SOIL MOISTURE ACCOUNTING
C******
      DO 29 I=NT,M1
      ADD=(Q(I)-Q(I+1))/DZ
      FULL=TABLE(MTE,1,IND(I))
      IF (ADD+WET(I).LT.FULL-1.E-5) GO TO 29
      ADD=FULL-WET(I)
      Q(I+1)=Q(I)-ADD*DZ
29    WET(I)=WET(I)+ADD
30    WET(N)=WET(N)+Q(N)/DZ
```

```
C******
C       CALCULATE INFILTRATION AND SURFACE DISCHARGE AND THEN
C       CARRY OUT THE NECESSARY BOOKKEEPING TO UPDATE POOL,
C       PRECIP,DITCH AND THE MOISTURE OF THE UPPER SOIL
C       SEGMENT.
C******
      PREVAP=RAIN*DTA
      IF (PREVAP.GT.0.) GO TO 31
      IF (POOL+PREVAP.GE.0) GO TO 31
      PRECIP=PRECIP+POOL
      EVAP=PREVAP+POOL
      EVAP=EVAP*(WET(N)-TABLE(1,1,IND(N)))/RNGE
      POOL=0.
      EMAX=(TABLE(1,1,IND(N))-WET(N))*DZ
      IF (EVAP.LT.EMAX) EVAP=EMAX
      PRECIP=PRECIP-EVAP
      WET(N)=WET(N)+EVAP/DZ
      DISCHA=0.
      RINF=EVAP
      GO TO 34
31    POOL=POOL+PREVAP
      PRECIP=PRECIP-PREVAP
      AMAXI=(TABLE(MTE,1,IND(N))*.9994-WET(N))*DZ
      RINF=(DZ-2.*FI(N))*PER(N)/DZ
      DIF=POOL-PMAX
```

```
      DISCHA=0.
      IF (DIF.LE.0.) GO TO 32
      DISCHA=AB*DIF*DIF*DTA
      IF (DISCHA.GT.DIF) DISCHA=DIF
32    IF (RINF.GT.AMAXI) RINF=AMAXI
      TOT=RINF+DISCHA
      IF (TOT.LE.POOL) GO TO 33
      RINF=RINF*POOL/TOT
      DISCHA=DISCHA*POOL/TOT
33    WET(N)=WET(N)+RINF/DZ
      POOL=POOL-RINF-DISCHA
      DROS=0.
      IF (NT.LT.N) GO TO 34
      IF (WET(N).LT.TABLE(MTE,1,IND(N))) GO TO 34
      DROS=WET(N)-TABLE(MTE,1,IND(N))
34    DITCH=DITCH+DISCHA+QS*DTA+DROS*DZ
      NST=0
      DO 36 I=NT,N
      TARG(1)=WET(I)
      IF (NST.NE.0) GO TO 35
      IF (WET(I).GE.TABLE(MTE,1,IND(I))) GO TO 35
      NST=I
35    CALL LOOKUP (1,IND(I),MTE,I)
      PER(I)=TARG(2)
      WET(I)=TARG(1)
```

```
36    FI(I)=TARG(3)
      NS=NST-1
      NT=NST
      N1=NT+1
      P=PER(NT)
C******
C      CALCULATE WATERTABLE ELEVATION
C******
      ZW=AKO*(FI(NT)+DZ*NS)/(AKO-A*FI(NT))
      NS=ZW/DZ+1
      QS=A*ZW*AKO/(AKO+ZW*A)
      SUM=0.
      DO 37 I=1,N
37    SUM=SUM+WET(I)*DZ
      IF (PRNT.LT.DPRINT) GO TO 42
C******
C      READY DATA FOR OUTPUT
C******
      PRNT=0.
      TOT=QS+DISCHA/DTA
      DO 38 I=NT,M1
38    Q(I)=Q(I)/DTA
      NTT=NT-1
      DO 39 I=1,NTT
```

```
39      Q(I)=-QS
        RINF=RINF/DTA
        DISCHA=DISCHA/DTA
C******
C        PRINT RESULTS
C******
        IF (IOPT(2).LT.2) GO TO 40
        WRITE(*,59) TIME,PRECIP,POOL,DITCH,SUM,RINF,DISCHA,QS,TOT,Q(M1),ZW
        GO TO 41
40      WRITE(*,58) TIME,DTA,QS,ZW,TOT
        WRITE(*,57) PRECIP,POOL,DITCH,SUM,RINF,DISCHA,(Q(I),I=1,M1)
        IF (IOPT(2).EQ.1) GO TO 41
        WRITE(*,54) (WET(N+1-I),PER(N+1-I),FI(N+1-I),I=1,N)
41      IF (IOPT(3).EQ.0) GO TO 42
        WRITE (6,45) TIME,DITCH,POOL,RINF,ZW,QS,DISCHA,TOT
42      IF (IPTEST.EQ.1) GO TO 43
        GO TO 22
43      GO TO 21
44      CONTINUE
        STOP
C
45      FORMAT (F6.1,7E10.3)
46      FORMAT (A79)
47      FORMAT (1H ,A79)
48      FORMAT (1X,I3,7E10.3/2I3,5F10.3)
49      FORMAT (50I2)
```

```
50    FORMAT (1H0,10X,'SOILTYPE ',I1//
     1 5X,'= MOISTURE  CONDUCTIVITY',8X,'PRESSURE'//)
51    FORMAT (1X,F14.4,F14.5,F14.2)
52    FORMAT (1X,' CS TOO LARGE,REPLACED BY MAXIMUM',E12.4)
53    FORMAT (I3,')TH NODE SATURATED: NO EQUILIBRIUM FLOW AT THIS RATE!!
     1')
54    FORMAT (/'      MOISTURE AND CONDUCTIVITY PROFILE'/
     1 13X,'THETA(CM/CM)',8X,'K(CM/DAY)'/(5X,E20.6,5X,F12.6,F15.2))
55    FORMAT ('0  QS=',F12.3,5X,'ZG=',F12.3/)
56    FORMAT ('0FOR THE PERIOD ENDING AT T=',F10.1,
     1 ' DAYS,PRECIP IS',F10.2,' CM/DAY'//)
57    FORMAT (' STORAGES',4F10.5/' INFILT AND RUNOFF',2F10.5/
     1 ' INTERNAL FLOWS'/(9F8.4)/)
58    FORMAT ('0TIME,STEP,GW DISCH.,WATER TABLE,TOT DISCH '/5E15.6/)
59    FORMAT (1X,6F6.1,E10.3,4F8.3)
      END
```

```
      SUBROUTINE LOOKUP (II,JJ,MTE,JS)
C***********************************************************************
C
C  LOOKUP INTERPOLATES BETWEEN ENTRIES OF TABLE TO FIND
C  CORRESPONDING VALUES OF TWO OF THE TABLE ARGUMENTS,
C  TARG,GIVEN THE THIRD ONE;THESE ARGUMENTS,AS WELL AS
C  THE TABULATED VALUES,ARE PASSED THROUGH COMMON.THE
C  SUBROUTINE ARGUMENTS ARE:
C     II=1,2,OR 3:THE INDEX OF TARG(II)!TARG(II) IS THE
C        TABLE ARGUMENT WITH THE GIVEN VALUE.
C     JJ=THE INDEX IND(SEE MAIN PROGRAM) INDICATING THE
C        SOIL TYPE FOR THE INTERPOLATION.
C     MTE=THE NUMBER OF TABLE ENTRIES.
C     JS=THE NODE-INDEX OF THE SOIL ELEMENT FOR WHICH THE INTERPO-
C     LATION IS DONE.NEEDED TO GET THE CORRECT VALUE OF LOCATION
C     IN TABLE.
C***********************************************************************
      COMMON TABLE(50,3,5),TARG(3),ISTART(50)
      P=TARG(II)
      TEST=P-TABLE(ISTART(JS),II,JJ)
      IF(TEST)1,2,3
2     IK=ISTART(JS)
7     DO 4 I=1,3
4     TARG(I)=TABLE(IK,I,JJ)
      RETURN
```

```
1     DO 5 I=2,ISTART(JS)
      JQ=ISTART(JS)+1-I
      IF(P.GT.TABLE(JQ,II,JJ))GOTO 6
5     CONTINUE
      IK=1
      ISTART(JS)=JQ
      GOTO 7
6     FAC=(P-TABLE(JQ,II,JJ))/(TABLE(JQ+1,II,JJ)-TABLE(JQ,II,JJ))
      DO 8 J=1,3
8     TARG(J)=TABLE(JQ,J,JJ)+FAC*(TABLE(JQ+1,J,JJ)-TABLE(JQ,J,JJ))
      ISTART(JS)=JQ
      RETURN
3     DO 9 I=ISTART(JS)+1,MTE
      IF(P.LT.TABLE(I,II,JJ))GOTO 10
9     CONTINUE
      IK=MTE
      ISTART(JS)=MTE
      GOTO 7
10    FAC=(P-TABLE(I-1,II,JJ))/(TABLE(I,II,JJ)-TABLE(I-1,II,JJ))
      DO 11 J=1,3
11    TARG(J)=TABLE(I-1,J,JJ)+FAC*(TABLE(I,J,JJ)-TABLE(I-1,J,JJ))
      ISTART(JS)=I
      RETURN
      END
```

# Appendix B
# Notes for the User of Program DRAIN2

## APPENDIX B

### DRAINOTES, NOTES FOR THE USER OF PROGRAM DRAIN2

This appendix is a copy of the file DRAINOTES on diskette DRAIN-PLOT. In thé text it is assumed that program DRAIN will be run on an IBM-XT or compatible running under DOS.

NOTES ON PROGRAM DRAIN2

DRAIN.EXE is the name of the file which must be loaded into memory for the execution of program DRAIN2. All the data for the run are read from logical unit # 1. Output is on the default output device, normally the screen, but may be rerouted to the printer or any file with the reroute facility of DOS. Also, if IOPT(3) is not zero, output of data suitable for plotting is sent to logical unit #6. Thus, with DRAIN.EXE on the default drive, the input data in a file DRAIN.DAT on a diskette in drive A, output to be routed to the printer, and plot data to be written on a file PLOTDAT.xxx, this would be the command:

```
drain a:drain.dat plotdat.xxx > prn
```

The data set is described on pages 1 and 2 of the FORTRAN source listing, which is in the file DRAIN.FOR. For plotting of data sent to logical device #6, please note the following:

Data are written under format control; the first two records of the file are the two title lines (see page 1 of the source listing), and are expected on the input data for program PLOTGRAF; the next records contain the items: TIME, DITCH, POOL, RINF, ZW, QS, DISCHA, TOT, in that order, where

TIME = time for which data in this record are given (time)
DITCH = accumulated water in the ditch(length)
POOL = water pooled on the surface(length)
RINF = infiltration rate(length/time)
ZW = height of watertable above drain(length)
QS = ground-water discharge(length/time)
DISCHA= rate of surface runoff(length/time)
TOT = sum of QS and DISCHA(length/time)

There is one such record for each time for which output was requested, the same as data output to the default device. The format of the records is (see source listing, page 8, lines, 424 and 431):

(F6.1,7E10.3)

In PLOTGRAF, TIME, which would be the x-coordinate of the plot, is ignored, and data are plotted as if they were equidistant on the time-axis(x-axis); which is true as long as the value of the print-interval(DPRINT) is an integer multiple of the timestep, and is always approximately true.

Thus,to plot, for example, the total discharge, the format given to PLOTGRAF would be:

(66X,E10.3)

skipping the TIME and the first 6 data items, reading only the last item, which is TOT, the total discharge.

When the timestep is recalculated, the number of records is not always entirely predictable. If plotting is considered, the recalculation of the timestep should be omitted, or the records should be counted manually (i.e. without the help of a machine), making sure that, if more than one trace is to be plotted, the number of records is the same for each trace.

# Appendix C

# Notes for the User of Program PLOTGRAF

## APPENDIX C

### PLOTNOTES, NOTES FOR THE USER OF PROGRAM PLOTGRAF

This appendix is a copy of the file PLOTNOTES on the diskette DRAIN-PLOT. In the text it is assumed that the software described is run on an IBM-XT or compatible, with an IBM Proprinter or similar dot-matrix printer to generate the plot.

# PLOTNOTES

PLOT is the name of the file(s) containing program PLOTGRAF for plotting on a dot-matrix printer a string of y- values, function y(x), at equidistant points on the x-axis. The program reads two records with control data from the default device, and y-values are read from unit # 1. Up to five graphs can be shown on the same plot. Scaling is automatic, but the user may specify a minimum value and a maximum value. If all y's are within these limits, scaling will be done using the supplied limits. If any of the y's is outside the limits, the lowest or the highest y-value will replace the limit in question.

==========================================================================

INPUT FROM THE CONSOLE

RECORD # 1 : List directed.

fmt = format specs for reading of the y-values from unit #1; must be in single quotes !!!! Maximum of 30 characters, not counting the quotes.

RECORD # 2 : List directed.

m = number of points to be plotted on each graph; maximum is 200.

n = number of graphs on the plot, maximum = 5.

isize= the number of vertical increments on the plot. Plotting on the printer allows for 130 increments, but 100 is a more suitable number, leaving a sizeable margin. Note that the x-axis is plotted down the page, the y-axis across the page.

sml = forces a lower limit(see above) to be used for scaling.

big = forces an upper limit to be used for scaling.

================================================================================

INPUT FROM UNIT # 1

RECORD # 1: FORMAT(80A1)

79 characters for comments or a title

RECORD # 2: FORMAT(80A1)

79 characters for comments or title

ALL FOLLOWING RECORDS: FORMAT fmt

n x m values of y, m values for one plot followed by m values for the next plot,etc. Read under format control with format fmt, as given in the first record input on the default device.

================================================================================

To run the program using the supplied EXE file and the supplied data file PLOT.DAT, first set the printer for condensed print, and if so wanted, change the LINE SPACING to 1/8 inch spacing (Esc 0) or to 7/72 inch spacing (Esc 1); then enter:

PLOT PLOT.DAT >PRN followed by the input data,for example:

'(66x,E10.3)' 80 5 100 0 .5

File PLOT.DAT contains formatted records containing information prepared by program DRAIN; a record contains several data items recorded at half day intervals;in columns 67-77 the total discharge (in cm) is recorded; there are 400 = 5(n) x 80(m) such records, preceded by the two records containing comments. Thus the program will plot 5 graphs,a symbol for each half day, the size of the y-axis is 100 columns, and since the maximum value of y does not exceed .5, and the minimum is not less than 0., the x-axis will indicate y=0, and the horizontal dashed line at the top of the plot indicates a y-value of 0.5.

Canada