Subsurface Disposal of Waste in Canada—II

Disposal-Formation and Injection-Well Hydraulics

R. O. van Everdingen

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LIST OF SYMBOLS USED

C = bulk compressibility of disposal formation, in vol/vol/atmosphere
h = thickness of the disposal formation (effective thickness), in cm (ft)
h_c = thickness of confining beds, in cm (ft)
I = natural hydraulic gradient, dimensionless
P_f = natural fluid pressure in disposal formation, at well bore, in psi (atm)
P_{fr} = pressure loss due to friction of moving fluid inside well tubing, in psi (atm)
P_o = pressure exerted by weight of saturated overburden material, in psi (atm)
P_w = pressure exerted by weight of waste column in injection well, in psi (atm)
P_i = well-head pressure during injection of waste fluid, in psi (atm)
ΔP = increase in pressure in disposal formation, at the well bore, as a result of waste injection, in psi (atm)
q_t = injection rate, in "dimensionless units"
q_T = injection rate, in cm^3/second (US gal/minute)
Q_t = injected volume, in "dimensionless units"
Q_T = injected volume, in cm^3 (US gallons)
r = distance from injection well, in multiples of well radius r_w
r_1 = distance from injection well, in cm (feet)
r_e = radius of cylinder of injected waste in subsurface, in km (miles)
r_t = radius of injection tubing (or casing), in cm (inch)
r_w = well radius, in cm (inch)
t = time, in dimensionless units
T = time in seconds
T_e = useful life of waste-injection installation, years
T_t = time needed for waste to become "harmless" (if at all), years
V = velocity of fluid movement, in cm/year (ft/year)
Z = depth of top of disposal interval below surface, in meters (feet)
Z_f = height of static formation-water column above top of disposal interval, in meters (feet)
Z_w = height of waste water column above top of disposal interval, in meters (feet)
ϕ = porosity of the disposal formation, expressed as a fraction
ϕ_c = porosity of confining beds, expressed as a fraction
ϕ_o = average porosity of the overburden, expressed as a fraction
κ = permeability of disposal formation, in darcys
κ_c = permeability of confining beds, in darcys
μ = viscosity of waste fluid, in centipoises
ρ_f = density of natural fluid in disposal formation, in g/cm^3
ρ_{fo} = density of natural fluids in overburden (average), in g/cm^3
ρ_{mo} = density of mineral grains in overburden (average), in g/cm^3
ρ_o = average density of overburden, in g/cm^3
ρ_w = density of waste fluid, in g/cm^3

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INTRODUCTION

An adequate understanding of injection-well and disposal-formation hydraulics is a prerequisite for the formulation of criteria for the selection of sites and formations for subsurface disposal of liquid wastes. It is also needed for any evaluation of the subsurface waste-disposal potential of a larger region. Correct interpretation of injection-well performance is impossible without such knowledge (van Everdingen and Freeze, 1971).

Pressure build-up in the disposal formation, as a result of waste injection, may cause reactivation of abandoned and inadequately plugged oil, gas or water wells, even if these do not penetrate the confining beds. Pressure build-up will also lead to an increased discharge rate from the disposal formation, whether from outcrops or through leaky confining beds. The salinity of the discharge may increase gradually, and it can be expected that eventually waste material will appear in the discharge area. The latter will take place even if injection is discontinued. It can further be expected that waste material (or waste-derived material) will be discharged over a period of time that is appreciably longer than the period of operation of the injection well.

Most of the equations used at present to describe pressure build-up in a disposal formation during waste injection, or the movement of waste through such a formation, imply a number of simplifying assumptions. These generally include: a homogeneous disposal formation having isotropic permeability and infinite areal extent, impermeable confining beds, and permeability and porosity invariant with time. They further assume constant density and viscosity for the injected liquid, and disregard possible density differences between native and injected fluids, as well as effects of potential chemical reactions between injected liquid and formation fluids and rocks. The effect of natural hydraulic, thermal, or osmotic gradients is generally not taken into account.

Attempts to include many of the relevant variables in mathematical models have been described by Kumar and Kimbler (1970; dispersion and gravitational segregation accompanying injection of freshwater into a saline aquifer), Bredehoeft and Pinder (1971; physical-chemical description of moving groundwater in a porous medium by simultaneous solution of differential equations describing the transport of mass, energy and momentum), Henry and Kohout (1971; streamline, velocity, temperature and salinity distribution before and during waste injection into a deep, saline, geothermally-heated aquifer), and by Whitherspoon and Neuman (1971; pressure build-up during waste injection into a multilayered aquifer/aquitard system).

Each of the above models requires the use of a high-speed digital computer. In many cases accurate values for a number of the input parameters are not available initially. They are determined through a trial-and-error process by comparison of the model results with observed distributions of pressure (salinity, temperature etc.).

To facilitate initial evaluations of waste-disposal potential, the following discussions make use of the relatively well-known non-equilibrium approach, in which the disposal formation is assumed to be homogeneous, isotropic and of infinite areal extent, with non-leaky confining beds (van Everdingen, 1968). The fluids referred to are assumed to be of uniform density, with their viscosity being a function of temperature and, to a lesser degree, of pressure. It is further assumed in the calculations that the injected liquids will not react with the formation rock or water. If such reactions do take place, both permeability and porosity will be affected.

Where necessary, an attempt is made to point out where differences between behaviour under actual and "ideal" conditions may be significant.

PRESSURE BUILD-UP vs. TIME, AT INJECTION WELL

The quantity of fluid that can be stored per unit volume of the formation is determined by the porosity of the formation; the quantity that can be stored per unit area of the formation is determined by the porosity and the thickness of the formation. The differential pressure required to inject fluid at unit rate into an initially fluid-saturated formation, is determined by formation thickness and permeability, and by fluid viscosity. The pressure-time-distance relationship is further dependent on the areal extent of the formation, the radius of the well, and the compressibilities of the fluids and the formation.

The movement of the slightly compressible fluids dealt with in subsurface disposal obeys a differential equation which states that the difference in volume of fluid flowing in and out of an annular space between two concentric rings around a well bore is equal to the volumetric expansion of the fluids in the annulus. The fundamental equation is linear; this allows flexibility in the use of a solution; it may be multiplied or divided by a constant, or shifted in time to fit a multitude of conditions; solutions may be superposed.

Table I. Values of parameters used in examples (maximum and minimum values for some parameters).

Parameter	Units	Example	Maximum	Minimum
C	vol/vol/atm	0.45×10^{-4}	–	–
	vol/vol/psi	3×10^{-6}	–	–
ρ_f	g/cm^3	1.1	1.2	1.0
ρ_{fo}	g/cm^3	1.1	1.2	1.0
ρ_{mo}	g/cm^3	2.68	2.87	2.65
ρ_o	g/cm^3	2.44	2.87	2.16
ρ_w	g/cm^3	1.2	1.5	0.8
h	cm	2000	–	–
	feet	65.57	–	–
I (horizontal)	–	0.01	10^{-1}	10^{-3}
I^1 (vertical)	–	1.0	10	10^{-2}
κ	darcy	0.1	5.0	0.01
P_f	psi	1250	–	–
q_T	cm^3/s	6000	60,000	375
	USgal/min	95.1	951.0	5.94
r_w	cm	7.62	–	–
	inch	3	–	–
r_t	cm	3.81	20.32	1.905
	inch	1 1/2	8	3/4
Z	m	1000	–	–
	feet	3280.8	–	–
μ	centipoise	0.5	1.0	0.2
ϕ	fraction	0.15	0.05	0.30
ϕ_o	fraction	0.1	0.05	0.35

In order to simplify the use of the solutions of the differential equation for the wide variety of conditions found in the natural subsurface system and for variations in the injection rate, the time t, the injection rate q_t, and the injected volume Q_t are expressed in "dimensionless units". Numerical values for t, q_t and Q_t can be derived from actual time T, actual rate q_T and actual volume Q_T through the following conversions (van Everdingen, 1968):

$$t = \kappa T / (\mu \phi C r_w^2) \quad (1)$$

$$q_t = q_T \mu / (2 \pi \kappa h) \quad (2)$$

$$\text{and } Q_t = Q_T / (2 \pi \phi C h r_w^2) \quad (3)$$

In the following paragraphs examples of the use of these conversions and of the related pressure and volume functions will be given. A single set of values will be used in these examples (Table I); for some parameters maximum and minimum values (also listed in Table I) are selected, to demonstrate the range in magnitude of the effects of waste injection. Figure 1 enables calculation of waste density and formation water density under subsurface conditions of temperature and pressure; Figure 2 provides information on fluid viscosity at various temperatures and salt concentrations; Figure 3 presents information on the porosity, temperature, and pressure dependence of compressibility.

For the values assigned to the various parameters the following calculations can be made:

$$t = 0.1\ T / (0.15 \times 0.5 \times 0.45 \times 10^{-4} \times 58.06) = 510.327758\ T$$

$$\text{and } q_t = 0.5 \times 6000/(2\pi \times 0.1 \times 2000) = 2.387324$$

In Table II the calculation of t is carried out for times ranging from 1 day to 100 years; column 2 gives T in seconds, and column 3 gives the corresponding values of t. The values for P_t (cumulative pressure change, in atmospheres, at the well bore, when *unit* rate of injection has been maintained since time T = zero, into a formation of unit thickness) are calculated from the P_t function (van Everdingen, 1968):

$$P_t = 1/2 \ln t + 0.4045 \text{ (for times with } t > 100) \quad (4)$$

and given in column 4. Multiplication by 14.696 gives P_t in psi (column 5); multiplication by q_t produces the pressure

Table II. Pressure increase ΔP at various times, under constant injection rate. $q_T = 6000\ cm^3/s$; $\kappa = 0.1$ darcy; $\phi = 15\%$; $\mu = 0.5$ cp; h = 2000 cm; $r_w = 3$ in(7.62 cm); $q_t = 2.3873$.

Time	T (seconds)	t	P_t, atm	P_t, psi	ΔP, psi
1 day	86,400	44.09×10^6	9.2053	135.28	322.96
Years					
1	31,560,000	161.06×10^8	12.1557	178.64	426.47
2	63,120,000	322.12×10^8	12.5023	183.73	438.63
3	94,680,000	483.18×10^8	12.705	186.71	445.74
4	126,240,000	64.42×10^9	12.849	188.83	450.79
5	157,800,000	80.53×10^9	12.960	190.47	454.71
10	315,600,000	161.06×10^9	13.307	195.56	466.87
20	631,200,000	322.12×10^9	13.654	200.65	479.02
50	1,578,000,000	80.53×10^{10}	14.112	207.39	495.10
100	3,156,000,000	161.06×10^{10}	14.458	212.47	507.26

increase ΔP in the formation at the well bore (column 6). The results are plotted in Figure 4 (line marked "ΔP at well"). If the formation is of limited areal extent, the pressure *vs.* time curve will rise above that represented by equation 4, as soon as the pressure front reaches the aquifer boundaries; a constant injection rate will, in that case, result in a constant rate of pressure increase with time. A pressure fall-off test should produce a straight-line plot for observed pressure *vs.* the logarithm of shut-in time T_1 divided by total time T. Extension of this line to the point where $T_1/T = 1$ should give a pressure equal to the original fluid pressure in the formation. If the reservoir has a limited extent, the pressure for $T_1/T = 1$ will be larger than the original formation pressure. The reservoir size can then be computed from the excess pressure and the total quantity of fluid injected. The results will have a significant effect on the decision on further use of the injection operation.

A sudden rapid increase in injection pressure at constant injection rate, or a decrease in injection rate at constant injection pressure, is usually indicative of plugging of the formation by suspended solids contained in the waste stream, by mobilized clay particles, or by the products of chemical or biological reactions. A gradual decrease in injection pressure or a gradual increase in the intake rate may be indicative of an increase in formation permeability through solution. A sudden decrease in injection pressure (or increase in the intake rate), on the other hand, may indicate tubing or casing failure, or the occurrence of hydraulic fracturing. The latter is dealt with in some detail later in this report. Continuous recording of both injection (tubing) pressure, and pressure in the fluid-filled annulus between injection tubing and well casing will enable early detection of pressure changes which may be indicative of trouble.

Table III. Pressure increase ΔP, in psi, at various times, at various distances from the injection well. $\kappa = 0.1$ darcy; $\phi = 15\%$; $\mu = 0.5$ cp; h = 2000 cm; r_w = 3 in (7.62 cm); $q_t = 2.3873$; q_T = 6000 cm^3/s.

Time (years)	Distance			
	at well	305m 1000ft	3,050m 10,000ft	30,500m 100,000ft
1	426.5	135.5	55.14	0.448
10	466.9	175.9	95.14	18.42
100	507.3	216.3	135.5	55.14

It may be useful also to demonstrate the influence of the injection rate q_T on the pressure increase at the well. All other conditions remaining the same, the pressure increase at the well bore will be directly proportional to q_T. This is shown in Figure 5, for q_T ranging from about 3 to 1000 U.S. gpm, with other conditions as given in Table I.

PRESSURE BUILD-UP vs. DISTANCE AND TIME

The increase in pressure in the disposal formation as a result of waste injection, at a distance r from the well, at time t, is given (van Everdingen, 1968) by:

$$P_t = - 1/2\, Ei\,(- r^2/4t) \qquad (5)$$

where Ei(x) is the exponential integral $\int_x^\infty e^{-y}/y\, dy$,

r is *expressed in multiples of the well radius,* and
t is again dimensionless time.

Corresponding values for ΔP are found after multiplication by 14.696 q_t. Results for distances of 1000, 10,000, and 100,000 ft, for 1, 10 and 100 years after the start of injection, are given in Table III and plotted in Figure 4. Values for $-Ei\,(-r^2/4t)$ for various values of $r^2/4t$ are listed in Table IV.

If the confining beds overlying the disposal formation possess a measurable permeability, the influence of leakage through the confining beds on pressure build-up in the disposal formation can be taken into account through the use of the modified exponential integral: $Eim\,(x,y) = \int_x^\infty \frac{1}{y} \exp\,(- z - \frac{y^2}{4z})\, dz$. For this purpose the parameter r_1/B is introduced, which takes into account the relation between permeability and thickness of both the disposal formation and the confining beds:

$$r_1/B = r_1/\sqrt{(\kappa h/\kappa_c h_c^{-1})} \qquad (6)$$

Methods of determining the confining bed (or caprock) permeability can be found in Witherspoon *et al.* (1967). The pressure build-up is now given by

$$P_t = - 1/2\, Eim\,(- r^2/4t, r_1/B) \qquad (7)$$

Values for the modified function for a range of combinations of $r^2/4t$ and r_1/B are presented in Table V (Hantush, 1956). In addition to the r_1/B factor, the model developed by Witherspoon and Neuman (1971) uses a β factor which accounts for contrasts between aquifer and confining beds that result from the combined effects of permeabilities and storage coefficients.

CUMULATIVE VOLUME INJECTABLE UNDER CONSTANT PRESSURE

To determine the cumulative volume Q_T that can be injected in time T under a constant pressure increase ΔP, the time T is again converted to dimensionless units using equation 1; the value of the dimensionless volume Q_t is determined using the Q_t function, which gives the cumulative volume of fluid produced from or injected into a formation of unit thickness, if from time zero onward the pressure at the well bore was decreased (or increased) by 1 atmosphere (van Everdingen, 1968). Values for the Q_t function are given in Table VI and plotted in Figure 6. Then using equation 3, Q_t is converted to Q_T in cm^3 and m^3. The results of such a calculation, using values for the

TABLE IV. Values of exponential integral – Ei $(-r^2/4t)$ for values of $N = r^2/4t$ between 1.0×10^{-15} and 9.9

N	$N \times 10^{-15}$	$N \times 10^{-14}$	$N \times 10^{-13}$	$N \times 10^{-12}$	$N \times 10^{-11}$	$N \times 10^{-10}$	$N \times 10^{-9}$	$N \times 10^{-8}$	$N \times 10^{-7}$	$N \times 10^{-6}$	$N \times 10^{-5}$	$N \times 10^{-4}$	$N \times 10^{-3}$	$N \times 10^{-2}$	$N \times 10^{-1}$	N
1.0	33.9616	31.6590	29.3564	27.0538	24.7512	22.4486	20.1460	17.8435	15.5409	13.2383	10.9357	8.6332	6.3315	4.0379	1.8229	0.2194
1.1	33.8662	31.5637	29.2611	26.9585	24.6559	22.3533	20.0507	17.7482	15.4456	13.1430	10.8404	8.5379	6.2363	3.9436	1.7371	.1860
1.2	33.7792	31.4767	29.1741	26.8715	24.5689	22.2663	19.9637	17.6611	15.3586	13.0560	10.7534	8.4509	6.1494	3.8576	1.6595	.1584
1.3	33.6992	31.3966	29.0940	26.7914	24.4889	22.1863	19.8837	17.5811	15.2785	12.9759	10.6734	8.3709	6.0695	3.7785	1.5889	.1355
1.4	33.6251	31.3225	29.0199	26.7173	24.4147	22.1122	19.8096	17.5070	15.2044	12.9018	10.5993	8.2968	5.9955	3.7054	1.5241	.1162
1.5	33.5561	31.2535	28.9509	26.6483	24.3458	22.0432	19.7406	17.4380	15.1354	12.8328	10.5303	8.2278	5.9266	3.6374	1.4645	.1000
1.6	33.4916	31.1890	28.8864	26.5838	24.2812	21.9786	19.6760	17.3735	15.0709	12.7683	10.4657	8.1634	5.8621	3.5739	1.4092	.08631
1.7	33.4309	31.1283	28.8258	26.5232	24.2206	21.9180	19.6154	17.3128	15.0103	12.7077	10.4051	8.1027	5.8016	3.5143	1.3578	.07465
1.8	33.3738	31.0712	28.7686	26.4660	24.1634	21.8608	19.5583	17.2557	14.9531	12.6505	10.3479	8.0455	5.7446	3.4581	1.3098	.06471
1.9	33.3197	31.0171	28.7145	26.4119	24.1094	21.8068	19.5042	17.2016	14.8990	12.5964	10.2939	7.9915	5.6906	3.4050	1.2649	.05620
2.0	33.2684	30.9658	28.6632	26.3607	24.0581	21.7555	19.4529	17.1503	14.8477	12.5451	10.2426	7.9402	5.6394	3.3547	1.2227	.04890
2.1	33.2196	30.9170	28.6145	26.3119	24.0093	21.7067	19.4041	17.1015	14.7989	12.4964	10.1938	7.8914	5.5907	3.3069	1.1829	.04261
2.2	33.1731	30.8705	28.5679	26.2653	23.9628	21.6602	19.3576	17.0550	14.7524	12.4498	10.1473	7.8449	5.5443	3.2614	1.1454	.03719
2.3	33.1286	30.8261	28.5235	26.2209	23.9183	21.6157	19.3131	17.0106	14.7080	12.4054	10.1028	7.8004	5.4999	3.2179	1.1099	.03250
2.4	33.0861	30.7835	28.4809	26.1783	23.8758	21.5732	19.2706	16.9680	14.6654	12.3628	10.0603	7.7579	5.4575	3.1763	1.0762	.02844
2.5	33.0453	30.7427	28.4401	26.1375	23.8349	21.5323	19.2298	16.9272	14.6246	12.3220	10.0194	7.7172	5.4167	3.1365	1.0443	.02491
2.6	33.0060	30.7035	28.4009	26.0983	23.7957	21.4931	19.1905	16.8880	14.5854	12.2828	9.9802	7.6779	5.3776	3.0983	1.0139	.02185
2.7	32.9683	30.6657	28.3631	26.0606	23.7580	21.4554	19.1528	16.8502	14.5476	12.2450	9.9425	7.6401	5.3400	3.0615	.9849	.01918
2.8	32.9319	30.6294	28.3268	26.0242	23.7216	21.4190	19.1164	16.8138	14.5113	12.2087	9.9061	7.6038	5.3037	3.0261	.9573	.01686
2.9	32.8968	30.5943	28.2917	25.9891	23.6865	21.3839	19.0813	16.7788	14.4762	12.1736	9.8710	7.5687	5.2687	2.9920	.9309	.01482
3.0	32.8629	30.5604	28.2578	25.9552	23.6526	21.3500	19.0474	16.7449	14.4423	12.1397	9.8371	7.5348	5.2349	2.9591	.9057	.01305
3.1	32.8302	30.5276	28.2250	25.9224	23.6198	21.3172	19.0146	16.7121	14.4095	12.1069	9.8043	7.5020	5.2022	2.9273	.8815	.01149
3.2	32.7984	30.4958	28.1932	25.8907	23.5881	21.2855	18.9829	16.6803	14.3777	12.0751	9.7726	7.4703	5.1706	2.8965	.8583	.01013
3.3	32.7676	30.4651	28.1625	25.8599	23.5573	21.2547	18.9521	16.6495	14.3470	12.0444	9.7418	7.4395	5.1399	2.8668	.8361	.008939
3.4	32.7378	30.4352	28.1326	25.8300	23.5274	21.2249	18.9223	16.6197	14.3171	12.0145	9.7120	7.4097	5.1102	2.8379	.8147	.007891
3.5	32.7088	30.4062	28.1036	25.8010	23.4985	21.1959	18.8933	16.5907	14.2881	11.9855	9.6830	7.3807	5.0813	2.8099	.7942	.006970
3.6	32.6806	30.3780	28.0755	25.7729	23.4703	21.1677	18.8651	16.5625	14.2599	11.9574	9.6548	7.3526	5.0532	2.7827	.7745	.006160
3.7	32.6532	30.3506	28.0481	25.7455	23.4429	21.1403	18.8377	16.5351	14.2325	11.9300	9.6274	7.3252	5.0259	2.7563	.7554	.005448
3.8	32.6266	30.3240	28.0214	25.7188	23.4162	21.1136	18.8110	16.5085	14.2059	11.9033	9.6007	7.2985	4.9993	2.7306	.7371	.004820
3.9	32.6006	30.2980	27.9954	25.6928	23.3902	21.0877	18.7851	16.4825	14.1799	11.8773	9.5748	7.2725	4.9735	2.7056	.7194	.004267
4.0	32.5753	30.2727	27.9701	25.6675	23.3649	21.0623	18.7598	16.4572	14.1546	11.8520	9.5495	7.2472	4.9482	2.6813	.7024	.003779
4.1	32.5506	30.2480	27.9454	25.6428	23.3402	21.0376	18.7351	16.4325	14.1299	11.8273	9.5248	7.2225	4.9236	2.6576	.6859	.003349
4.2	32.5265	30.2239	27.9213	25.6187	23.3161	21.0136	18.7110	16.4084	14.1058	11.8032	9.5007	7.1985	4.8997	2.6344	.6700	.002969
4.3	32.5029	30.2004	27.8978	25.5952	23.2926	20.9900	18.6874	16.3848	14.0823	11.7797	9.4771	7.1749	4.8762	2.6119	.6546	.002633
4.4	32.4800	30.1774	27.8748	25.5722	23.2696	20.9670	18.6644	16.3619	14.0593	11.7567	9.4541	7.1520	4.8533	2.5899	.6397	.002336
4.5	32.4575	30.1549	27.8523	25.5497	23.2471	20.9446	18.6420	16.3394	14.0368	11.7342	9.4317	7.1295	4.8310	2.5684	.6253	.002073
4.6	32.4355	30.1329	27.8303	25.5277	23.2252	20.9226	18.6200	16.3174	14.0148	11.7122	9.4097	7.1075	4.8091	2.5474	.6114	.001841
4.7	32.4140	30.1114	27.8088	25.5062	23.2037	20.9011	18.5985	16.2959	13.9933	11.6907	9.3882	7.0860	4.7877	2.5268	.5979	.001635
4.8	32.3929	30.0904	27.7878	25.4852	23.1826	20.8800	18.5774	16.2748	13.9723	11.6697	9.3671	7.0650	4.7667	2.5068	.5848	.001453
4.9	32.3723	30.0697	27.7672	25.4646	23.1620	20.8594	18.5568	16.2542	13.9516	11.6491	9.3465	7.0444	4.7462	2.4871	.5721	.001291
5.0	32.3521	30.0495	27.7470	25.4444	23.1418	20.8392	18.5366	16.2340	13.9314	11.6289	9.3263	7.0242	4.7261	2.4679	.5598	.001148
5.1	32.3323	30.0297	27.7271	25.4246	23.1220	20.8194	18.5168	16.2142	13.9116	11.6091	9.3065	7.0044	4.7064	2.4491	.5478	.001021
5.2	32.3129	30.0103	27.7077	25.4051	23.1026	20.8000	18.4974	16.1948	13.8922	11.5896	9.2871	6.9850	4.6871	2.4306	.5362	.0009086
5.3	32.2939	29.9913	27.6887	25.3861	23.0835	20.7809	18.4783	16.1758	13.8732	11.5706	9.2681	6.9659	4.6681	2.4126	.5250	.0008086
5.4	32.2752	29.9726	27.6700	25.3674	23.0648	20.7622	18.4596	16.1571	13.8545	11.5519	9.2494	6.9473	4.6495	2.3948	.5140	.0007198
5.5	32.2568	29.9542	27.6516	25.3491	23.0465	20.7439	18.4413	16.1387	13.8361	11.5336	9.2310	6.9289	4.6313	2.3775	.5034	.0006409
5.6	32.2388	29.9362	27.6336	25.3310	23.0285	20.7259	18.4233	16.1207	13.8181	11.5155	9.2130	6.9109	4.6134	2.3604	.4930	.0005708
5.7	32.2211	29.9185	27.6159	25.3133	23.0103	20.7082	18.4056	16.1030	13.8004	11.4978	9.1953	6.8932	4.5958	2.3437	.4830	.0005085
5.8	32.2037	29.9011	27.5985	25.2959	22.9934	20.6908	18.3882	16.0856	13.7830	11.4804	9.1779	6.8758	4.5785	2.3273	.4732	.0004532
5.9	32.1866	29.8840	27.5814	25.2789	22.9763	20.6737	18.3711	16.0685	13.7659	11.4633	9.1608	6.8588	4.5615	2.3111	.4637	.0004039
6.0	32.1698	29.8672	27.5646	25.2620	22.9595	20.6569	18.3543	16.0517	13.7491	11.4465	9.1440	6.8420	4.5448	2.2953	.4544	.0003601
6.1	32.1533	29.8507	27.5481	25.2455	22.9429	20.6403	18.3378	16.0352	13.7326	11.4300	9.1275	6.8254	4.5283	2.2797	.4454	.0003211
6.2	32.1370	29.8344	27.5318	25.2293	22.9267	20.6241	18.3215	16.0189	13.7163	11.4138	9.1112	6.8092	4.5122	2.2645	.4366	.0002864
6.3	32.1210	29.8184	27.5158	25.2133	22.9107	20.6081	18.3055	16.0029	13.7003	11.3978	9.0952	6.7932	4.4963	2.2494	.4280	.0002555
6.4	32.1053	29.8027	27.5001	25.1975	22.8949	20.5923	18.2898	15.9872	13.6846	11.3820	9.0795	6.7775	4.4806	2.2346	.4197	.0002279
6.5	32.0898	29.7872	27.4846	25.1820	22.8794	20.5768	18.2742	15.9717	13.6691	11.3665	9.0640	6.7620	4.4652	2.2201	.4115	.0002034

TABLE IV (cont'd). Values of exponential integral $-\mathrm{Ei}\,(-r^2/4t)$ for values of $N = r^2/4t$ between 1.0×10^{-15} and 9.9

N	$N\times10^{-15}$	$N\times10^{-14}$	$N\times10^{-13}$	$N\times10^{12}$	$N\times10^{-11}$	$N\times10^{-10}$	$N\times10^{-9}$	$N\times10^{-8}$	$N\times10^{-7}$	$N\times10^{-6}$	$N\times-10^{-5}$	$N\times10^{-4}$	$N\times10^{-3}$	$N\times10^{2}$	$N\times10^{-1}$	N
6.6	32.0745	29.7719	27.4693	25.1667	22.8641	20.5616	18.2590	15.9564	13.6538	11.3512	9.0487	6.7467	4.4501	2.2058	.4036	.0001816
6.7	32.0595	29.7569	27.4543	25.1517	22.8491	20.5465	18.2439	15.9414	13.6388	11.3362	9.0337	6.7317	4.4351	2.1917	.3959	.0001621
6.8	32.0446	29.7421	27.4395	25.1369	22.8343	20.5317	18.2291	15.9265	13.6240	11.3214	9.0189	6.7169	4.4204	2.1779	.3883	.0001448
6.9	32.0300	29.7275	27.4249	25.1223	22.8197	20.5171	18.2145	15.9119	13.6094	11.3068	9.0043	6.7023	4.4059	2.1643	.3810	.0001293
7.0	32.0156	29.7131	27.4105	25.1079	22.8053	20.5027	18.2001	15.8976	13.5950	11.2924	8.9899	6.6879	4.3916	2.1508	.3738	.0001155
7.1	32.0015	29.6989	27.3963	25.0937	22.7911	20.4885	18.1860	15.8834	13.5808	11.2782	8.9757	6.6737	4.3775	2.1376	.3668	.0001032
7.2	31.9875	29.6849	27.3823	25.0797	22.7771	20.4746	18.1720	15.8694	13.5668	11.2642	8.9617	6.6598	4.3636	2.1246	.3599	.00009219
7.3	31.9737	29.6711	27.3685	25.0659	22.7633	20.4608	18.1582	15.8556	13.5530	11.2504	8.9479	6.6460	4.3500	2.1118	.3532	.00008239
7.4	31.9601	29.6575	27.3549	25.0523	22.7497	20.4472	18.1446	15.8420	13.5394	11.2368	8.9343	6.6324	4.3364	2.0991	.3467	.00007364
7.5	31.9467	29.6441	27.3415	25.0389	22.7363	20.4337	18.1311	15.8286	13.5260	11.2234	8.9209	6.6190	4.3231	2.0867	.3403	.00006583
7.6	31.9334	29.6308	27.3282	25.0257	22.7231	20.4205	18.1179	15.8153	13.5127	11.2102	8.9076	6.6057	4.3100	2.0744	.3341	.00005886
7.7	31.9203	29.6178	27.3152	25.0126	22.7100	20.4074	18.1048	15.8022	13.4997	11.1971	8.8946	6.5927	4.2970	2.0623	.3280	.00005263
7.8	31.9074	29.6048	27.3023	24.9997	22.6971	20.3945	18.0919	15.7893	13.4868	11.1842	8.8817	6.5798	4.2842	2.0503	.3221	.00004707
7.9	31.8947	29.5921	27.2895	24.9869	22.6844	20.3818	18.0792	15.7766	13.4740	11.1714	8.8689	6.5671	4.2716	2.0386	.3163	.00004210
8.0	31.8821	29.5795	27.2769	24.9744	22.6718	20.3692	18.0666	15.7640	13.4614	11.1589	8.8563	6.5545	4.2591	2.0269	.3106	.00003767
8.1	31.8697	29.5671	27.2645	24.9619	22.6594	20.3568	18.0542	15.7516	13.4490	11.1464	8.8439	6.5421	4.2468	2.0155	.3050	.00003370
8.2	31.8574	29.5548	27.2523	24.9497	22.6471	20.3445	18.0419	15.7393	13.4367	11.1342	8.8317	6.5298	4.2346	2.0042	.2996	.00003015
8.3	31.8453	29.5427	27.2401	24.9375	22.6350	20.3324	18.0298	15.7272	13.4246	11.1220	8.8195	6.5177	4.2226	1.9930	.2943	.00002699
8.4	31.8333	29.5307	27.2282	24.9256	22.6230	20.3204	18.0178	15.7152	13.4126	11.1101	8.8076	6.5057	4.2107	1.9820	.2891	.00002415
8.5	31.8215	29.5189	27.2163	24.9137	22.6112	20.3086	18.0060	15.7034	13.4008	11.0982	8.7957	6.4939	4.1990	1.9711	.2840	.00002162
8.6	31.8098	29.5072	27.2046	24.9020	22.5995	20.2969	17.9943	15.6917	13.3891	11.0865	8.7840	6.4822	4.1874	1.9604	.2790	.00001936
8.7	31.7982	29.4957	27.1931	24.8905	22.5879	20.2853	17.9827	15.6801	13.3776	11.0750	8.7725	6.4707	4.1759	1.9498	.2742	.00001733
8.8	31.7868	29.4842	27.1816	24.8790	22.5765	20.2739	17.9713	15.6687	13.3661	11.0635	8.7610	6.4592	4.1646	1.9393	.2694	.00001552
8.9	31.7755	29.4729	27.1703	24.8678	22.5652	20.2626	17.9600	15.6574	13.3548	11.0523	8.7497	6.4480	4.1534	1.9290	.2647	.00001390
9.0	31.7643	29.4618	27.1592	24.8566	22.5540	20.2514	17.9488	15.6462	13.3437	11.0411	8.7386	6.4368	4.1423	1.9187	.2602	.00001245
9.1	31.7533	29.4507	27.1481	24.8455	22.5429	20.2404	17.9378	15.6352	13.3326	11.0300	8.7275	6.4258	4.1313	1.9087	.2557	.00001115
9.2	31.7424	29.4398	27.1372	24.8346	22.5320	20.2294	17.9268	15.6213	13.3217	11.0191	8.7166	6.4148	4.1205	1.8987	.2513	.000009988
9.3	31.7315	29.4290	27.1264	24.8238	22.5212	20.2186	17.9160	15.6135	13.3109	11.0083	8.7058	6.4040	4.1098	1.8888	.2470	.000008948
9.4	31.7208	29.4183	27.1157	24.8131	22.5105	20.2079	17.9053	15.6028	13.3002	10.9976	8.6951	6.3934	4.0992	1.8791	.2429	.000008018
9.5	31.7103	29.4077	27.1051	24.8025	22.4999	20.1973	17.8948	15.5922	13.2896	10.9870	8.6845	6.3828	4.0887	1.8695	.2387	.000007185
9.6	31.6998	29.3972	27.0946	24.7920	22.4895	20.1869	17.8843	15.5817	13.2791	10.9765	8.6740	6.3723	4.0784	1.8599	.2347	.000006439
9.7	31.6894	29.3868	27.0843	24.7817	22.4791	20.1765	17.8739	15.5713	13.2688	10.9662	8.6637	6.3620	4.0681	1.8505	.2308	.000005771
9.8	31.6792	29.3766	27.0740	24.7714	22.4688	20.1663	17.8637	15.5611	13.2585	10.9559	8.6534	6.3517	4.0579	1.8412	.2269	.000005173
9.9	31.6690	29.3664	27.0639	24.7613	22.4587	20.1561	17.8535	15.5509	13.2483	10.9458	8.6433	6.3416	4.0479	1.8320	.2231	.000004637

(From U. S. Geological Survey Water-Supply Paper 887)

TABLE V. Values of modified exponential integral for various values of $r^2/4t$ and r_1/B, for the leaky case.

$r^2/4t$ \ r_1/B	0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01
0	∞	14.0474	12.6611	11.8502	11.2748	10.8286	10.4640	10.1557	9.8887	9.6532	9.4425
.000001	13.2383	13.0031	12.4417	11.8153	11.2711	10.8283	10.4640	10.1557	9.8887		
.000002	12.5451	12.4240	12.1013	11.6716	11.2259	10.8174	10.4619	10.1554	9.8886	9.6532	
.000003	12.1397	12.0581	11.8322	11.5098	11.1462	10.7849	10.4509	10.1523	9.8879	9.6530	9.4425
.000004	11.8520	11.7905	11.6168	11.3597	11.0555	10.7374	10.4291	10.1436	9.8849	9.6521	9.4422
.000005	11.6289	11.5795	11.4384	11.2248	10.9642	10.6822	10.3993	10.1290	9.8786	9.6496	9.4413
.000006	11.4465	11.4053	11.2866	11.1040	10.8764	10.6240	10.3640	10.1094	9.8686	9.6450	9.4394
.000007	11.2924	11.2570	11.1545	10.9951	10.7933	10.5652	10.3255	10.0862	9.8555	9.6382	9.4361
.000008	11.1589	11.1279	11.0377	10.8962	10.7151	10.5072	10.2854	10.0602	9.8398	9.6292	9.4313
.000009	11.0411	11.0135	10.9330	10.8059	10.6416	10.4508	10.2446	10.0324	9.8219	9.6182	9.4251
.00001	10.9357	10.9109	10.8382	10.7228	10.5725	10.3963	10.2038	10.0034	9.8024	9.6059	9.4176
.00002	10.2426	10.2301	10.1932	10.1332	10.0522	9.9530	9.8386	9.7126	9.5781	9.4383	9.2961
.00003	9.8371	9.8288	9.8041	9.7635	9.7081	9.6392	9.5583	9.4671	9.3674	9.2611	9.1499
.00004	9.5495	9.5432	9.5246	9.4940	9.4520	9.3992	9.3366	9.2653	9.1863	9.1009	9.0102
.00005	9.3263	9.3213	9.3064	9.2818	9.2480	9.2052	9.1542	9.0957	9.0304	8.9591	8.8827
.00006	9.1440	9.1398	9.1274	9.1069	9.0785	9.0426	8.9996	8.9500	8.8943	8.8332	8.7673
.00007	8.9899	8.9863	8.9756	8.9580	8.9336	8.9027	8.8654	8.8224	8.7739	8.7204	8.6625
.00008	8.8563	8.8532	8.8439	8.8284	8.8070	8.7798	8.7470	8.7090	8.6661	8.6186	8.5669
.00009	8.7386	8.7358	8.7275	8.7138	8.6947	8.6703	8.6411	8.6071	8.5686	8.5258	8.4792
.0001	8.6332	8.6308	8.6233	8.6109	8.5937	8.5717	8.5453	8.5145	8.4796	8.4407	8.3983
.0002	7.9402	7.9390	7.9352	7.9290	7.9203	7.9092	7.8958	7.8800	7.8619	7.8416	7.8192
.0003	7.5348	7.5340	7.5315	7.5274	7.5216	7.5141	7.5051	7.4945	7.4823	7.4686	7.4534
.0004	7.2472	7.2466	7.2447	7.2416	7.2373	7.2317	7.2249	7.2169	7.2078	7.1974	7.1859
.0005	7.0242	7.0237	7.0222	7.0197	7.0163	7.0118	7.0063	6.9999	6.9926	6.9843	6.9750
.0006	6.8420	6.8416	6.8403	6.8383	6.8353	6.8316	6.8271	6.8218	6.8156	6.8086	6.8009
.0007	6.6879	6.6876	6.6865	6.6848	6.6823	6.6790	6.6752	6.6706	6.6653	6.6594	6.6527
.0008	6.5545	6.5542	6.5532	6.5517	6.5495	6.5467	6.5433	6.5393	6.5347	6.5295	6.5237
.0009	6.4368	6.4365	6.4357	6.4344	6.4324	6.4299	6.4269	6.4233	6.4192	6.4146	6.4094
.001	6.3315	6.3313	6.3305	6.3293	6.3276	6.3253	6.3226	6.3194	6.3157	6.3115	6.3069
.002	5.6394	5.6393	5.6389	5.6383	5.6374	5.6363	5.6350	5.6334	5.6315	5.6294	5.6271
.003	5.2349	5.2348	5.2346	5.2342	5.2336	5.2329	5.2320	5.2310	5.2297	5.2283	5.2267
.004	4.9482	4.9482	4.9480	4.9477	4.9472	4.9467	4.9460	4.9453	4.9443	4.9433	4.9421
.005	4.7261	4.7260	4.7259	4.7256	4.7253	4.7249	4.7244	4.7237	4.7230	4.7222	4.7212
.006	4.5448	4.5448	4.5447	4.5444	4.5441	4.5438	4.5433	4.5428	4.5422	4.5415	4.5407
.007	4.3916	4.3916	4.3915	4.3913	4.3910	4.3908	4.3904	4.3899	4.3894	4.3888	4.3882
.008	4.2591	4.2590	4.2590	4.2588	4.2586	4.2583	4.2580	4.2576	4.2572	4.2567	4.2561
.009	4.1423	4.1423	4.1422	4.1420	4.1418	4.1416	4.1413	4.1410	4.1406	4.1401	4.1396
.01	4.0379	4.0379	4.0378	4.0377	4.0375	4.0373	4.0371	4.0368	4.0364	4.0360	4.0356
.02	3.3547	3.3547	3.3547	3.3546	3.3545	3.3544	3.3543	3.3542	3.3540	3.3538	3.3536
.03	2.9591	2.9591	2.9591	2.9590	2.9590	2.9589	2.9589	2.9588	2.9587	2.9585	2.9584
.04	2.6813	2.6812	2.6812	2.6812	2.6812	2.6811	2.6810	2.6810	2.6809	2.6808	2.6807
.05	2.4679	2.4679	2.4679	2.4679	2.4678	2.4678	2.4678	2.4677	2.4676	2.4676	2.4675
.06	2.2953	2.2953	2.2953	2.2953	2.2952	2.2952	2.2952	2.2952	2.2951	2.2950	2.2950
.07	2.1508	2.1508	2.1508	2.1508	2.1508	2.1508	2.1507	2.1507	2.1507	2.1506	2.1506
.08	2.0269	2.0269	2.0269	2.0269	2.0269	2.0269	2.0269	2.0268	2.0268	2.0268	2.0267
.09	1.9187	1.9187	1.9187	1.9187	1.9187	1.9187	1.9187	1.9186	1.9186	1.9186	1.9185
.1	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229	1.8228	1.8228	1.8228	1.8227
.2	1.2227	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226
.3	0.9057	0.9057	0.9057	0.9057	0.9057	0.9057	0.9057	0.9057	0.9056	0.9056	0.9056
.4	7024	7024	7024	7024	7024	7024	7024	7024	7024	7024	7024
.5	5598	5598	5598	5598	5598	5598	5598	5598	5598	5598	5598
.6	4544	4544	4544	4544	4544	4544	4544	4544	4544	4544	4544
.7	3738	3738	3738	3738	3738	3738	3738	3738	3738	3738	3738
.8	3106	3106	3106	3106	3106	3106	3106	3106	3106	3106	3106
.9	2602	2602	2602	2602	2602	2602	2602	2602	2602	2602	2602
1.0	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194
2.0	489	489	489	489	489	489	489	489	489	489	489
3.0	130	130	130	130	130	130	130	130	130	130	130
4.0	38	38	38	38	38	38	38	38	38	38	38
5.0	11	11	11	11	11	11	11	11	11	11	11
6.0	4	4	4	4	4	4	4	4	4	4	4
7.0	1	1	1	1	1	1	1	1	1	1	1
8.0	0	0	0	0	0	0	0	0	0	0	0

TABLE V (cont'd). Values of modified exponential integral for various values of $r^2/4t$ and r_1/B, for the leaky case

$r^2/4t$ \ r_1/B	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	0.05
0	9.4425	8.6319	8.0569	7.6111	7.2471	6.9394	6.6731	6.4383	6.2285
.000001									
.000002									
.000003	9.4425								
.000004	9.4422								
.000005	9.4413								
.000006	9.4394								
.000007	9.4361	8.6319							
.000008	9.4313	8.6318							
.000009	9.4251	8.6316							
.00001	9.4176	8.6313	8.0569						
.00002	9.2961	8.6152	8.0558	7.6111	7.2471				
.00003	9.1499	8.5737	8.0483	7.6101	7.2470				
.00004	9.0102	8.5168	8.0320	7.6069	7.2465	6.9394	6.6731		
.00005	8.8827	8.4533	8.0080	7.6000	7.2450	6.9391	6.6730		
.00006	8.7673	8.3880	7.9786	7.5894	7.2419	6.9384	6.6729	6.4383	
.00007	8.6625	8.3233	7.9456	7.5754	7.2371	6.9370	6.6726	6.4382	6.2285
.00008	8.5669	8.2603	7.9105	7.5589	7.2305	6.9347	6.6719	6.4381	6.2284
.00009	8.4792	8.1996	7.8743	7.5402	7.2222	6.9316	6.6709	6.4378	6.2283
.0001	8.3983	8.1414	7.8375	7.5199	7.2122	6.9273	6.6693	6.4372	6.2282
.0002	7.8192	7.6780	7.4972	7.2898	7.0685	6.8439	6.6242	6.4143	6.2173
.0003	7.4534	7.3562	7.2281	7.0759	6.9068	6.7276	6.5444	6.3623	6.1848
.0004	7.1859	7.1119	7.0128	6.8929	6.7567	6.6088	6.4538	6.2955	6.1373
.0005	6.9750	6.9152	6.8346	6.7357	6.6219	6.4964	6.3626	6.2236	6.0821
.0006	6.8009	6.7508	6.6828	6.5988	6.5011	6.3923	6.2748	6.1512	6.0239
.0007	6.6527	6.6096	6.5508	6.4777	6.3923	6.2962	6.1917	6.0807	5.9652
.0008	6.5237	6.4858	6.4340	6.3695	6.2935	6.2076	6.1136	6.0129	5.9073
.0009	6.4094	6.3757	6.3294	6.2716	6.2032	6.1256	6.0401	5.9481	5.8509
.001	6.3069	6.2765	6.2347	6.1823	6.1202	6.0494	5.9711	5.8864	5.7965
.002	5.6271	5.6118	5.5907	5.5638	5.5314	5.4939	5.4516	5.4047	5.3538
.003	5.2267	5.2166	5.2025	5.1845	5.1627	5.1373	5.1084	5.0762	5.0408
.004	4.9421	4.9345	4.9240	4.9105	4.8941	4.8749	4.8530	4.8286	4.8016
.005	4.7212	4.7152	4.7068	4.6960	4.6829	4.6675	4.6499	4.6302	4.6084
.006	4.5407	4.5357	4.5287	4.5197	4.5088	4.4960	4.4814	4.4649	4.4467
.007	4.3882	4.3839	4.3779	4.3702	4.3609	4.3500	4.3374	4.3233	4.3077
.008	4.2561	4.2524	4.2471	4.2404	4.2323	4.2228	4.2118	4.1994	4.1857
.009	4.1396	4.1363	4.1317	4.1258	4.1186	4.1101	4.1004	4.0894	4.0772
.01	4.0356	4.0326	4.0285	4.0231	4.0167	4.0091	4.0003	3.9905	3.9795
.02	3.3536	3.3521	3.3502	3.3476	3.3444	3.3408	3.3365	3.3317	3.3264
.03	2.9584	2.9575	2.9562	2.9545	2.9523	2.9501	2.9474	2.9444	2.9409
.04	2.6807	2.6800	2.6791	2.6779	2.6765	2.6747	2.6727	2.6705	2.6680
.05	2.4675	2.4670	2.4662	2.4653	2.4642	2.4628	2.4613	2.4595	2.4576
.06	2.2950	2.2945	2.2940	2.2932	2.2923	2.2912	2.2900	2.2885	2.2870
.07	2.1506	2.1502	2.1497	2.1491	2.1483	2.1474	2.1464	2.1452	2.1439
.08	2.0267	2.0264	2.0260	2.0255	2.0248	2.0240	2.0231	2.0221	2.0210
.09	1.9185	1.9183	1.9179	1.9174	1.9169	1.9162	1.9154	1.9146	1.9136
.1	1.8227	1.8225	1.8222	1.8218	1.8213	1.8207	1.8200	1.8193	1.8184
.2	1.2226	1.2225	1.2224	1.2222	1.2220	1.2218	1.2215	1.2212	1.2209
.3	0.9056	0.9056	0.9055	0.9054	0.9053	0.9052	0.9050	0.9049	0.9047
.4	7024	7023	7023	7022	7022	7021	7020	7019	7018
.5	5598	5597	5597	5597	5596	5596	5595	5594	5594
.6	4544	4544	4543	4543	4543	4542	4542	4542	4541
.7	3738	3738	3737	3737	3737	3737	3736	3736	3735
.8	3106	3106	3106	3106	3105	3105	3105	3105	3104
.9	2602	2602	2602	2602	2601	2601	2601	2601	2601
1.0	0.2194	0.2194	0.2194	0.2194	0.2193	0.2193	0.2193	0.2193	0.2193
2.0	489	489	489	489	489	489	489	489	489
3.0	130	130	130	130	130	130	130	130	130
4.0	38	38	38	38	38	38	38	38	38
5.0	11	11	11	11	11	11	11	11	11
6.0	4	4	4	4	4	4	4	4	4
7.0	1	1	1	1	1	1	1	1	1
8.0	0	0	0	0	0	0	0	0	0

$r^2/4t$ \ r_1/B	0.055	0.06	0.065	0.07	0.075	0.08	0.085	0.09	0.095	0.10
0	6.0388	5.8658	5.7067	5.5596	5.4228	5.2950	5.1750	5.0620	4.9553	4.8541
.000001										
.000002										
.000003										
.000004										
.000005										
.000006										
.000007										
.000008										
.000009										
.00001										
.00002										
.00003										
.00004										
.00005										
.00006										
.00007										
.00008										
.00009										
.0001	6.0388	5.8658	5.7067	5.5596	5.4228	5.2950				
.0002	6.0338	5.8637	5.7059	5.5593	5.4227	5.2949	5.1750	5.0620	4.9553	
.0003	6.0145	5.8527	5.6999	5.5562	5.4212	5.2942	5.1747	5.0619	4.9552	4.8541
.0004	5.9818	5.8309	5.6860	5.5476	5.4160	5.2912	5.1730	5.0610	4.9547	4.8539
.0005	5.9406	5.8011	5.6648	5.5330	5.4062	5.2848	5.1689	5.0585	4.9532	4.8530
.0006	5.8948	5.7658	5.6383	5.5134	5.3921	5.2749	5.1621	5.0539	4.9502	4.8510
.0007	5.8468	5.7274	5.6081	5.4902	5.3745	5.2618	5.1526	5.0471	4.9454	4.8478
.0008	5.7982	5.6873	5.5755	5.4642	5.3542	5.2461	5.1406	5.0381	4.9388	4.8430
.0009	5.7500	5.6465	5.5416	5.4364	5.3317	5.2282	5.1266	5.0272	4.9306	4.8368
.001	5.7026	5.6058	5.5071	5.4075	5.3078	5.2087	5.1109	5.0133	4.9208	4.8292
.002	5.2991	5.2411	5.1803	5.1170	5.0517	4.9848	4.9166	4.8475	4.7778	4.7079
.003	5.0025	4.9615	4.9180	4.8722	4.8243	4.7746	4.7234	4.6707	4.6169	4.5622
.004	4.7722	4.7406	4.7068	4.6710	4.6335	4.5942	4.5533	4.5111	4.4676	4.4230
.005	4.5846	4.5590	4.5314	4.5022	4.4713	4.4389	4.4050	4.3699	4.3335	4.2960
.006	4.4267	4.4051	4.3819	4.3573	4.3311	4.3036	4.2747	4.2446	4.2134	4.1812
.007	4.2905	4.2719	4.2518	4.2305	4.2078	4.1839	4.1588	4.1326	4.1053	4.0771
.008	4.1707	4.1544	4.1368	4.1180	4.0980	4.0769	4.0547	4.0315	4.0073	3.9822
.009	4.0638	4.0493	4.0336	4.0169	3.9991	3.9802	3.9603	3.9395	3.9178	3.8952
.01	3.9675	3.9544	3.9403	3.9252	3.9091	3.8920	3.8741	3.8552	3.8356	3.8150
.02	3.3205	3.3141	3.3071	3.2997	3.2917	3.2832	3.2742	3.2647	3.2547	3.2442
.03	2.9370	2.9329	2.9284	2.9235	2.9183	2.9127	2.9069	2.9007	2.8941	2.8873
.04	2.6652	2.6622	2.6589	2.6553	2.6515	2.6475	2.6432	2.6386	2.6338	2.6288
.05	2.4554	2.4531	2.4505	2.4478	2.4448	2.4416	2.4383	2.4347	2.4310	2.4271
.06	2.2852	2.2833	2.2812	2.2790	2.2766	2.2740	2.2713	2.2684	2.2654	2.2622
.07	2.1424	2.1408	2.1391	2.1372	2.1352	2.1331	2.1308	2.1284	2.1258	2.1232
.08	2.0198	2.0184	2.0169	2.0153	2.0136	2.0118	2.0099	2.0078	2.0056	2.0034
.09	1.9125	1.9114	1.9101	1.9087	1.9072	1.9056	1.9040	1.9022	1.9003	1.8983
.1	1.8175	1.8164	1.8153	1.8141	1.8128	1.8114	1.8099	1.8084	1.8067	1.8050
.2	1.2205	1.2201	1.2196	1.2192	1.2186	1.2181	1.2175	1.2168	1.2162	1.2155
.3	0.9045	0.9043	0.9040	0.9038	0.9035	0.9032	0.9029	0.9025	0.9022	0.9018
.4	7016	7015	7014	7012	7010	7008	7006	7004	7002	7000
.5	5593	5592	5591	5590	5588	5587	5586	5584	5583	5581
.6	4540	4540	4539	4538	4537	4536	4535	4534	4533	4532
.7	3735	3734	3734	3733	3733	3732	3732	3731	3730	3729
.8	3104	3104	3103	3103	3102	3102	3101	3101	3100	3100
.9	2600	2600	2600	2599	2599	2599	2598	2598	2597	2597
1.0	0.2193	0.2192	0.2192	0.2192	0.2191	0.2191	0.2191	0.2191	0.2190	0.2190
2.0	489	489	489	489	489	489	489	489	488	488
3.0	130	130	130	130	130	130	130	130	130	130
4.0	38	38	38	38	38	38	38	38	38	38
5.0	11	11	11	11	11	11	11	11	11	11
6.0	4	4	4	4	4	4	4	4	4	4
7.0	1	1	1	1	1	1	1	1	1	1
8.0	0	0	0	0	0	0	0	0	0	0

TABLE V (cont'd). Values of modified exponential integral for various values of $r^2/4t$ and r_1/B, for the leaky case

$r^2/4t$ \ r_1/B	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0	4.8541	4.0601	3.5054	3.0830	2.7449	2.4654	2.2291	2.0258	1.8488	1.6981	1.5550	1.4317	1.3210	1.2212	1.1307	1.0485	0.9735	0.9049
.0001																		
.0002																		
.0003	4.8541																	
.0004	4.8539																	
.0005	4.8530																	
.0006	4.8510	4.0601																
.0007	4.8478	4.0600																
.0008	4.8430	4.0599																
.0009	4.8368	4.0598																
.001	4.8292	4.0595	3.5054															
.002	4.7079	4.0435	3.5043	3.0830	2.7449													
.003	4.5622	4.0092	3.4969	3.0821	2.7448													
.004	4.4230	3.9551	3.4806	3.0788	2.7444	2.4654	2.2291											
.005	4.2960	3.8821	3.4567	3.0719	2.7428	2.4651	2.2290											
.006	4.1812	3.6384	3.4274	3.0614	2.7398	2.4644	2.2289	2.0258										
.007	4.0771	3.7529	3.3947	3.0476	2.7350	2.4630	2.2286	2.0257										
.008	3.9822	3.6903	3.3598	3.0311	2.7284	2.4608	2.2279	2.0256	1.8488									
.009	3.8952	3.6302	3.3239	3.0126	2.7202	2.4576	2.2269	2.0253	1.8487									
.01	3.8150	3.5725	3.2875	2.9925	2.7104	2.4534	2.2253	2.0248	1.8486	1.6981	1.5550	1.4317	1.3210	1.2212	1.1307	1.0485		
.02	3.2442	3.1158	2.9521	2.7658	2.5688	2.3713	2.1809	2.0023	1.8379	1.6883	1.5530	1.4309	1.3207	1.2210	1.1306	1.0484	0.9735	0.9049
.03	2.8873	2.8017	2.6896	2.5571	2.4110	2.2578	2.1031	1.9515	1.8062	1.6695	1.5423	1.4251	1.3177	1.2195	1.1299	1.0481	9733	9048
.04	2.6288	2.5655	2.4816	2.3802	2.2661	2.1431	2.0155	1.8869	1.7603	1.6379	1.5213	1.4117	1.3094	1.2146	1.1270	1.0465	9724	9044
.05	2.4271	2.3776	2.3110	2.2299	2.1371	2.0356	1.9283	1.8181	1.7075	1.5985	1.4927	1.3914	1.2955	1.2052	1.1210	1.0426	9700	9029
.06	2.2672	2.2218	2.1673	2.1002	2.0227	1.9369	1.8452	1.7497	1.6524	1.5551	1.4593	1.3663	1.2770	1.1919	1.1116	1.0362	9657	9001
.07	2.1232	2.0894	2.0435	1.9867	1.9206	1.8469	1.7673	1.6835	1.5973	1.5101	1.4232	1.3380	1.2551	1.1754	1.0993	1.0272	9593	8956
.08	2.0034	1.9745	1.9351	1.8861	1.8290	1.7646	1.6947	1.6206	1.5436	1.4650	1.3860	1.3078	1.2310	1.1564	1.0847	1.0161	9510	8895
.09	1.8983	1.8732	1.8389	1.7961	1.7460	1.6892	1.6272	1.5609	1.4918	1.4206	1.3486	1.2766	1.2054	1.1358	1.0682	1.0032	9411	8819
.1	1.8050	1.7829	1.7527	1.7149	1.6704	1.6198	1.5644	1.5048	1.4422	1.3774	1.3115	1.2451	1.1791	1.1140	1.0505	0.9890	0.9297	0.8730
.2	1.2155	1.2066	1.1944	1.1789	1.1602	1.1387	1.1145	1.0879	1.0592	1.0286	0.9964	0.9629	0.9284	0.8932	0.8575	8216	7857	7501
.3	0.9018	0.8969	0.8902	0.8817	0.8713	0.8593	0.8457	0.8306	0.8142	0.7964	7775	7577	7369	7154	6932	6706	6476	6244
.4	7000	6969	6927	6874	6809	6733	6647	6551	6446	6332	6209	6080	5943	5801	5653	5501	5345	5186
.5	5581	5561	5532	5496	5453	5402	5344	5278	5206	5128	5044	4955	4860	4761	4658	4550	4440	4326
.6	4532	4518	4498	4472	4441	4405	4364	4317	4266	4210	4150	4086	4018	3946	3871	3793	3712	3629
.7	3729	3719	3704	3685	3663	3636	3606	3572	3534	3493	3449	3401	3351	3297	3242	3183	3123	3060
.8	3100	3092	3081	3067	3050	3030	3008	2982	2953	2922	2889	2853	2815	2774	2732	2687	2641	2592
.9	2597	2591	2583	2572	2559	2544	2527	2507	2485	2461	2436	2408	2378	2347	2314	2280	2244	2207
1.0	0.2190	0.2186	0.2179	0.2171	0.2161	0.2149	0.2135	0.2120	0.2103	0.2085	0.2065	0.2043	0.2020	0.1995	0.1970	0.1943	0.1914	0.1885
2.0	488	488	487	486	485	484	482	480	477	475	473	470	467	463	460	456	452	448
3.0	130	130	130	130	130	130	129	129	128	128	127	127	126	125	125	124	123	123
4.0	38	38	38	38	38	38	38	37	37	37	37	37	37	37	37	36	36	36
5.0	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
6.0	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
7.0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE V (cont'd). Values of modified exponential integral for various values of $r^2/4t$ and r_1/B, for the leaky case

$r^2/4t$ \ r_1/B	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	6.0	7.0	8.0	9.0
0	0.8420	0.4276	0.2278	0.1247	0.0695	0.0392	0.0223	0.0128	0.0074	0.0025	0.0008	0.0003	0.0001
.01													
.02													
.03	0.8420												
.04	8418												
.05	8409												
.06	8391												
.07	8360	0.4276											
.08	8316	4275											
.09	8259	4274											
.1	0.8190	0.4271	0.2278										
.2	7148	4135	2268	0.1247	0.0695								
.3	6010	3812	2211	1240	694								
.4	5024	3411	2096	1217	691	0.0392							
.5	4210	3007	1944	1174	681	390	0.0223						
.6	3543	2630	1774	1112	664	386	222	0.0128					
.7	2996	2292	1602	1040	639	379	221	127					
.8	2543	1994	1436	961	607	368	218	127	0.0074				
.9	2168	1734	1281	881	572	354	213	125	73				
1.0	0.1855	0.1509	0.1139	0.0803	0.0534	0.0338	0.0207	0.0123	0.0073	0.0025			
2.0	444	394	335	271	210	156	112	77	51	21	0.0008	0.0003	
3.0	122	112	100	86	71	57	45	34	25	12	6	2	
4.0	36	34	31	27	24	20	16	13	10	6	3	2	0.0001
5.0	11	10	10	9	8	7	6	5	4	2	1	1	0
6.0	4	3	3	3	3	2	2	2	2	1	1	0	
7.0	1	1	1	1	1	1	1	1	1	0	0		
8.0	0	0	0	0	0	0	0	0	0				

Table VI. Cumulative volume Q_t injected per atmosphere pressure increase into a formation of unit thickness, as a function of dimensionless time t.

t	Q_t	t	Q_t
10^{-2}	0.117	10^6	1.4662×10^5
10^{-1}	0.404	10^7	1.2568×10^6
1	1.568	10^8	1.1021×10^7
10^1	7.402	10^9	0.9797×10^8
10^2	4.3025×10^1	10^{10}	0.8798×10^9
10^3	2.9262×10^2	10^{11}	0.8014×10^{10}
10^4	2.1989×10^3	10^{12}	0.7407×10^{11}
10^5	1.7594×10^4	10^{13}	0.6933×10^{12}

various parameters similar to those in earlier examples, are presented in Table VII. The numerical factor for conversion from Q_t to Q_T, for $\Delta P = 200$ psi, is

$$Q_T = \frac{200}{14.696} \times 2\pi \times 0.15 \times 0.45 \times 10^{-4} \times 2000 \times 58.06\, Q_t$$

$$= 67.02264\, Q_t$$

The last two columns in Table VII give the additional quantity injected during consecutive years (averages for years between 5 and 10, 10 and 20, 20 and 50, and 50 and 100 years). The slight irregularity in the decrease in annually-injected volumes is due to the averaging procedure in addition to the inherent inaccuracy in the determination of Q_t from Figure 6.

RELATIVE INFLUENCE OF κ, ϕ AND μ ON PRESSURE BUILD-UP

Values of the physical parameters involved in pressure build-up may vary over a wide range. Viscosity of the formation fluids can be about 1 centipoise (cp) (mNs/m^2) in a shallow reservoir at low temperature and as low as 0.2 cp in a reservoir at a depth of 10,000 ft (3048 m) with a temperature of 90°C (203°F); densities may range from about 1.0 to about 1.25 g/cm^3. Formation permeability can be as high as several darcys or as low as a few millidarcys (permeability of confining beds may range down to less than 10^{-6} darcy); porosity values range from a few per cent to over 30%; formation thickness may vary between a few meters and a few hundred meters.

The relative influence of the parameters κ, ϕ and μ on the pressure increase ΔP, through their influence on the dimensionless t and q_t, can be demonstrated by comparison of pressure increases after 1 day (86,400 s) of fluid injection at a rate of 6000 cm^3/s (95.1 U.S. gpm) into a disposal formation with a thickness h = 2000 cm (65.57 ft) (Table VIII). The ranges for the three parameters are as indicated in Table I. Dimensionless time t increases with increasing permeability κ, and with decreasing porosity ϕ and fluid viscosity μ. Dimensionless injection rate q_t decreases with increasing permeability κ and with decreasing viscosity μ.

In the last two lines of Table VIII the maximum and minimum values for t and q_t are given; these are obtained by using the extreme values for κ, ϕ and μ listed in Table I. First of all it should be noted, that the minimum value for q_t corresponds to the maximum value for t, and *vice versa.* Secondly, the large range in values of t (12.758 T to 38,275 T, or a factor of 3000) has a much smaller influence on the pressure increase ΔP than the smaller range in values of q_t (0.09549 to 47,746, or a factor of 500). This is because the *logarithm* of t is used in equation 4 to calculate P_t (pressure increase per unit rate of injection and unit thickness), which is then multiplied by q_t to obtain ΔP.

Finally, it should be pointed out that the extremes for t, q_t and ΔP calculated here are somewhat unrealistic, because in practice the porosity ϕ and permeability κ often increase or decrease together. The variation in t and q_t is therefore largely a function of the potential variation in κ.

HYDRAULIC FRACTURING AND SAFE INJECTION PRESSURE

When a constant injection rate is maintained in an injection well, the fluid pressure in the formation increases

Table VII. Cumulative volume Q_T injected under pressure $\Delta P = 200$ psi, after various times T. $\kappa = 0.1$ darcy; $\phi = 15\%$; $\mu = 0.5$ cp; h = 2000 cm; r_w = 3 in (7.62 cm).

Time years	t	Q_t	Q_T, cm^3	Q_T, m^3	Injection, m^3/year	Injection USgal/year
1	161.06×10^8	1.40×10^9	93.8×10^9	93.8×10^3	93.8×10^3	24,777,915
2	322.12×10^8	2.72×10^9	182.3×10^9	182.3×10^3	88.5×10^3	23,379,461
3	483.18×10^8	4.01×10^9	268.7×10^9	268.8×10^3	86.5×10^3	22,840,344
4	644.2×10^8	5.27×10^9	353.2×10^9	353.2×10^3	84.4×10^3	22,298,406
5	805.3×10^8	6.50×10^9	435.6×10^9	435.6×10^3	82.4×10^3	21,780,063
10	161.06×10^9	1.26×10^{10}	844.5×10^9	844.5×10^3	av. 81.8×10^3	21,603,043
20	322.12×10^9	2.46×10^{10}	1648.7×10^9	1649.0×10^3	av. 80.4×10^3	21,246,055
50	805.3×10^9	5.90×10^{10}	3954.3×10^9	3954.0×10^3	av. 76.8×10^3	20,300,017
100	161.06×10^{10}	1.16×10^{11}	7774.6×10^9	7775.0×10^3	av. 76.4×10^3	20,185,896

Table VIII. Influence of variation in values of κ, ϕ and μ, on values of t and q_t, and ΔP, for T = 1 day.

κ millidarcy	ϕ fraction	μ centipoise	t	1/2 ln t + 0.4045		q_t	ΔP, psi
				atmosphere	psi		
100	15	0.5	510.3 T	9.21	135.3	2.3873	322.96
1000	15	0.5	5103 T	10.36	152.2	0.23873	36.34
10	15	0.5	51.03 T	8.05	118.4	23.873	2825.7
100	30	0.5	255.1 T	8.86	130.2	2.3873	310.8
100	5	0.5	1531 T	9.75	143.4	2.3873	342.2
100	15	1.0	255.1 T	8.86	130.2	4.7746	621.6
100	15	0.2	1275.8 T	9.66	142.0	0.95492	135.6
1000 (max)	5 (min)	0.2 (min)	38,275 T (max)	11.36	167.0	0.09549 (min)	15.95 (min)
10 (min)	30 (max)	1.0 (max)	12.758 T (min)	7.36	108.2	47.746 (max)	5165.0 (max)

with time. Ultimately, it may reach a critical value at which "hydraulic fracturing" of the formation takes place.

Fractures induced by fluid pressure in an injection well tend to follow planes parallel to the largest and intermediate principal compressive stresses in the subsurface, and perpendicular to the direction of the least principal stress. The orientation of hydraulically-induced fractures is thus strongly influenced by the prevailing state of stress in the surrounding rocks (Hubbert, 1971).

At shallow depth in regions under lateral compression the least principal stress is nearly vertical, which leads to horizontal orientation of hydraulically-induced fractures. At greater depths, in tectonically relaxed regions, often characterized by some normal faulting, the greatest principal stress is usually nearly vertical, the intermediate and least principal stresses nearly horizontal. The orientation of hydraulic fractures in such a region can be expected to be vertical, perpendicular to the least principal stress, and parallel to the strike of any existing normal faults in the area. The effectiveness of confining strata will undoubtedly be adversely affected by the occurrence of this type of hydraulic fracture.

Another consequence of fluid injection under high pressure may be differential movement of parts of the rock mass along joints, fractures or bedding planes that deviate somewhat from a direction perpendicular to the least principal stress. Shear stress across such a discontinuity will have some finite value. If residual or active tectonic stresses are present in the rock, slippage and possibly minor earthquakes may result from increases in fluid pressure (Hubbert, 1971).

During injection tests in which the injection pressure is increased gradually to test the intake capacity of the disposal formation, the occurrence of hydraulic fracturing causes a sudden increase in injection rate or a sudden drop in injection pressure. This is the result of a sudden increase in the effective permeability of the formation caused by: (1) parting of the rock along bedding planes; (2) opening of joints and fractures. However, a sudden increase in injection rate may also reflect sudden compaction of adjacent or included shale beds, or the yielding of improperly set packers in the borehole, or casing failure. If the formation is re-tested at a lower pressure after hydraulic fracturing has taken place, the original relationship of pressure *vs.* input rate should theoretically be re-established.

Hydraulic fracturing is used to advantage in the petroleum industry, to increase the permeability of oil-and-gas producing formations; the permeability is increased permanently by adding sand or another "propping" agent to the injection fluid, to maintain the fractures in an open position. A re-test in this case should reflect the increased permeability of the formation around the well bore. Hydraulic fracturing is also used in the solution mining of evaporite beds, to create horizontal communication between brine wells.

The direction in which hydraulic fractures are *initiated* can be controlled by proper "notching" of the formation (Aughenbaugh and Pullen, 1970). The direction of *propagation* of such fractures, however, is essentially a function of the existing stress field in the rocks, and the physical characteristics of the rocks involved; it may be predictable, but it cannot really be controlled. It should further be realized that horizontal hydraulic fracturing, followed by propping (either by sand or grout, or by continued hydraulic pressure) reduces the effective horizontal compressive stresses to a certain extent, while it increases the vertical compressive stress near the borehole (McClain, 1970). This may eventually result in one of the horizontal stress axes becoming the direction of minimum compressive stress, leading to increased likelihood of future vertical fracturing.

The use of the hydraulic fracturing technique to increase the permeability of a subsurface disposal formation should therefore be discouraged until assurance can be given, that confining beds will not be damaged in the process. It is therefore necessary to limit injection pressures to values well below the critical pressure, to prevent accidental occurrence of hydraulic fracturing.

In general it is assumed that hydraulic fracturing will occur when the fluid pressure in the formation exceeds the pressure P_o that results from the weight of the saturated rock column overlying the formation (minus any vertical strain resulting from possible horizontal compressive stresses affecting the overburden column). The value of P_o thus determines the maximum limit of the total pressure allowable in the formation during injection. This total pressure equals the sum of the injection pressure P_i, maintained at the wellhead, and the pressure P_w of the column of waste fluid in the hole, minus the pressure loss P_{fr}, which results from friction between the moving fluid and the stationary wall of tubing or well casing:

$$P_o = P_i\,(\max) + P_w - P_{fr} \quad (8)$$

or

$$P_i\,(\max) = P_o - P_w + P_{fr} \quad (9)$$

In most cases it will be impossible to give more than a first approximation of the overburden pressure. Assuming an average mineral density ρ_{mo}, an average fluid density ρ_{fo}, and an average porosity ϕ_o for the overburden material, the average overburden density ρ_o can be calculated from the relation:

$$\rho_o = \rho_{mo} + (\rho_{fo} - \rho_{mo})\,\phi_o \quad (10)$$

The pressure exerted by a unit thickness of this average overburden material is:

$$\delta P_o = \rho_o \times 0.434 \text{ psi/ft or } \rho_o \times 1.424 \text{ psi/m} \quad (11)$$

and the pressure exerted by the full overburden column of thickness Z is:

$$P_o = Z \times \delta P_o \text{ (psi)} \quad (12)$$

Figure 7 presents in graphical form the values for both ρ_o and δP_o that result from mineral densities ranging from 2.65 to 2.87, for porosities between 0 and 35% with fluid densities ranging from 1.0 to 1.2.

During injection the weight of each foot (meter) of the column of waste fluid in the disposal well will exert a pressure given by:

$$\delta P_w = \rho_w \times 0.434 \text{ psi/ft or } \rho_w \times 1.424 \text{ psi/m} \quad (13)$$

Values for δP_w, for ρ_w between 0.8 and 1.5, are plotted in Figure 8. The pressure P_w for a waste column of height Z_w is:

$$P_w = Z_w \times \delta P_w \text{ (psi)} \quad (14)$$

If a positive injection pressure P_i is maintained at the wellhead during injection, then $Z_w = Z$ (depth to top of disposal formation) and

$$P_w = Z \times \delta P_w \text{ (psi)}\ . \quad (15)$$

Figure 9 enables the determination of the loss of pressure per foot (meter) of smooth pipe, δP_{fr}, for standard diameters ranging from 1 1/2 to 8 inches and for flow rates between 1.6 and 1600 gallons per minute. The total pressure loss for a particular combination of pipe diameter, flow rate and well depth is:

$$P_{fr} = Z \times \delta P_{fr} \text{ (psi)} \quad (16)$$

Substituting equations 12, 15 and 16 into equation 9 gives:

$$(P_i) \max = Z(\delta P_o - \delta P_w + \delta P_{fr}) \quad (17)$$

At this point in the calculations it appears prudent to apply a safety factor through multiplication of δP_o by 0.75 to obtain the hopefully safe maximum allowable well-head pressure:

$$(P_i) \text{ safe} = Z\,(0.75\,\delta P_o - \delta P_w + \delta P_{fr}) \quad (18)$$

Combining equation 18 with equations 10, 11 and 13, and using the example values from Table I, gives:

$$(P_i) \text{ safe} = 3280.8\ [\![\, 0.75 \times 0.434\,[2.68 + (1.1 - 2.68) \times 0.15] - 0.434 \times 1.2 + 0.0107]\!] = 935.3 \text{ psi}$$

for Z in feet, and gradients and losses in psi/ft;

$$(P_i) \text{ safe} = 1000\ [\![\, 0.75 \times 1.424\,[2.68 + (1.1 - 2.68) \times 0.15] - 1.424 \times 1.2 + 0.0351]\!] = 935.4 \text{ psi}$$

for Z in meters, and gradients and losses in psi/m.

The injection-pressure monitor on waste-injection wells should preferably be equipped with an automatic shut-off valve, activated when P_i reaches the (P_i) safe value.

WELL AND FORMATION STIMULATION

Although hydraulic fracturing is not an acceptable formation stimulation technique for a subsurface waste-disposal operation, there are other methods that can be used to improve the intake capacity of a disposal formation: injection of acid, jetting, nitro-shooting, back-washing. A stimulation method is acceptable only as long as it improves the formation permeability, without affecting that of the confining beds. The effect of stimulation will be an increase in the permeability and porosity of the formation for some distance surrounding the well bore. In the computation of the effect it is generally assumed that, within a certain radius around the well, permeability is increased enough to be considered infinite in comparison with the original permeability of the formation, while porosity is unaffected.

The pressure increase per unit rate of injection as a function of time, after stimulation, is given by P_t^1. To use the following method, it is necessary to determine the new effective radius of the well r_{ws}, from pressure observations during injection tests at constant rate, both before and after stimulation. Assuming that r_{ws} equals 25 ft (or 100 r_w), then the value of dimensionless time t will decrease by a factor $(r_{ws}/r_w)^2 = 10{,}000$ (equation 1). For values of T between 1 day and 5 years, Table IX gives a comparison of P_t^1 and ΔP^1 with the values of P_t and ΔP that would have

Table IX. Reduction of injection pressure by stimulation. Stimulated radius r_{ws}= 25 ft (762.5 cm); q_T = 6000 cm^3/s; κ = 0.10 darcy; μ = 0.5 cp; ϕ = 15%; h = 2000 cm.

Time	After stimulation			Before stimulation		Improvement
	t^1	P^1_t atm	ΔP^1 psi	P_t atm	ΔP psi	$\Delta P - \Delta P^1$ psi
1 day	44.09 x 10^2	4.60	161.4	9.21	323.0	161.6
Years						
1	161.06 x 10^4	7.55	264.9	12.16	426.5	161.6
2	322.12 x 10^4	7.90	277.1	12.50	438.6	161.5
3	483.18 x 10^4	8.10	284.2	12.71	445.7	161.5
4	644.2 x 10^4	8.24	289.2	12.85	450.8	161.6
5	805.3 x 10^4	8.36	293.1	12.96	454.7	161.6

Table X. Radius r_e of waste cylinder around injection well after various injection periods T; distance r_n traversed by waste under natural gradient I; and total distance moved in downstream direction. q_T = 6000 cm^3/s; ϕ = 15%; I = 0.01; μ = 0.5 cp; κ = 0.1 darcy; h = 2000 cm.

T (years)	r_e		r_n		Total	
	miles	km	miles	km	miles	km
1	0.088	0.142	0.00252	0.00405	0.0905	0.146
2	0.125	0.200	0.00503	0.00810	0.130	0.208
3	0.153	0.245	0.00755	0.0121	0.160	0.257
4	0.176	0.283	0.0101	0.0162	0.186	0.299
5	0.197	0.317	0.0126	0.0202	0.210	0.337
10	0.278	0.448	0.0252	0.0405	0.303	0.488
20	0.394	0.634	0.0503	0.0810	0.444	0.715
50	0.623	1.002	0.126	0.2025	0.749	1.205
100	0.881	1.417	0.252	0.405	1.133	1.822

prevailed without stimulation; the values of other parameters are the same as in earlier examples. The last column of the table indicates a constant reduction in pressure has been obtained, equal to 161.6 psi for each 6000 cm^3/s injected. This result can also be obtained from equation 4:

$$P_t - P^1_t = 1/2 \, 1n\,(t/t^1) = 1/2 \, 1n\, 10{,}000 = 4.6 \text{ atm} = 67.6 \text{ psi}.$$

$$\text{Thus } \Delta P - \Delta P^1 = 2.387 \times 67.6 = 161.5 \text{ psi}.$$

WASTE MOVEMENT

The next point of interest is the question of how far injected waste can be expected to move in the disposal formation. Warner (1965) and McLean (1968) both presented equations to calculate the radius of the cylinder of waste around an injection well. The two equations give identical results; the one given by McLean is used here, because its application is the easiest. It applies to the injection of a fluid into a homogeneous, isotropic disposal formation; injected fluid and original formation fluid are assumed to be incompressible, non-mixing, of identical density and viscosity, and not subject to a natural hydraulic gradient. The radius of the waste cylinder in miles, after time T, is:

$$(r_e) \text{ miles} = 0.0236\,[q^1_T \; T/H\phi]^{1/2} \quad (19)$$

where q^1_T is injection rate in thousands of gallons per day, T is time in years, H is formation thickness in feet and ϕ is porosity as a fraction. If q_T is expressed in cm^3/s, and thickness h in cm, then the radius in km is given by:

$$(r_e) \text{ km} = 0.03168\,[q_T \; T/h\phi]^{1/2} \quad (20)$$

Table X gives the radius of the waste cylinder around an injection well, both in miles and km, after various periods of injection at a rate of 6000 cm^3/s, into a formation 2000 cm thick, with a porosity of 15%. The results are also presented in Figure 10.

If the formation is not homogeneous and isotropic, waste "plumes" will move farther in some directions than the distances given here for the various times. If differences exist between the densities of the waste fluid and the formation fluid in a particular case, the waste may move much farther than indicated by the results of Table X, in either the upper or the lower portion of the disposal formation. The answers obtained by the use of equations 19 and 20 can thus be regarded as indicating minimum distances; in a number of cases the waste can be expected to move farther from the injection well in some directions.

If the bulk compressibility of the saturated formation were taken into account, waste-movement distances would become slightly smaller:

$$(r_e)\ \text{km} = 0.3168\,[q_T\ T/h\phi(1+C\Delta P)]^{1/2} \qquad (21)$$

Use of equation 21 produces values that normally differ by less than 2% from those obtained from equation 20.

A major portion of the relevant current literature on subsurface disposal of waste treats the effects of injection only in terms of gradients imposed by the fluid injection; the presence of natural hydrodynamic (and/or osmotic or thermal) gradients is usually disregarded. In a number of cases it may be found that natural flow rates are of the same order of magnitude as those caused by continuing waste injection. Using Darcy's law, the bulk flow velocity in a subsurface formation is given by:

$$\bar{V} = \kappa I/\mu \qquad (22)$$

In a porous medium with porosity ϕ the average velocity of flow through interconnected pores is larger by a factor approaching $1/\phi$:

$$V = \kappa I/\mu\phi \qquad (23)$$

To express the average velocity in cm/year, equation 23 is multiplied by 83.21 (the equivalent of 1 darcy in cm/day for pure water at 20° C or 68° F) and by 365:

$$V = 30371.65\kappa I/\mu\phi \qquad (24)$$

For $I = 0.01$, $\mu = 0.5$ cp, $\phi = 15\%$ and $\kappa = 0.1$ darcy, $V =$ 404.96 cm/year (or 13.29 ft/year). Values for the distance r_n traversed by the waste under these conditions in the downstream direction are listed in Table X and plotted in Figure 10. It can be seen, that r_n gains in importance as time goes by. It is assumed in this calculation, that no dispersion takes place, and that the formation is homogeneous and isotropic.

Figure 11 presents in graphical form the average flow velocities of injected waste under a natural hydrodynamic gradient in a homogeneous isotropic aquifer, for various values of the gradient I, the porosity ϕ and the permeability κ. The lower horizontal axis of this graph indicates how far downstream the waste will move in a period of 1000 years under the natural gradient; this is independent of whether *injection is continued* or not. It presents a fair approximation of where the waste will be after this period of time. In any particular case the calculation will require detailed knowledge of the natural gradients in the disposal formation, as well as of the permeability and porosity distribution.

Kostin (1972) has presented a similar reasoning to determine the radius of the first of three "sanitary protective zones" to be established around each waste-injection site. Within these zones development of water supplies would be subject to varying degrees of restriction. In all areas where groundwater is a valuable resource, however, it appears to make more sense to turn Kostin's argument around, and to prevent waste-injection operations near locations where groundwater resources are being exploited.

The radius of the first zone is calculated from (20) and (24) for time T_e, the expected lifetime of the prospective injection operation. It represents the distance that will be travelled by the waste during this time under natural and injection-induced gradients:

$$R_1 = 0.03168\,[q_T\ T_e/h\phi]^{1/2} + 0.30372\,\kappa I\ T_e/\mu\phi \qquad (25)$$

The distance between a prospective waste-injection operation and any existing or anticipated groundwater developments should not be less than that indicated by equation 25.

The outer radius of the second zone represents R_1 plus the distance travelled by the waste under the natural hydrodynamic gradient between the time of termination of waste injection T_e and the time T_t at which the waste can be expected to have become "harmless":

$$R_2 = R_1 + 0.30372\,\kappa I\ (T_t - T_e)/\mu\phi =$$
$$0.03168\,[q_T\ T_e/h\phi]^{1/2} + 0.30372\kappa I T_t/\mu\phi \qquad (26)$$

The time T_t at which the waste has become harmless (by neutralization, adsorption, oxidation, reduction, dilution, radioactive decay, biodegradation, etc.) has to be determined experimentally in most cases. For many present-day waste components, no data are available on which to base calculations of T_t. Kostin (1972) states that T_t is generally much larger than T_e. Some waste components may not lose their harmful qualities at all. Equation 26 represents the minimum distance to be maintained between a prospective waste-injection site and any groundwater developments that make use of the first aquifers overlying or underlying the disposal formation.

The outer radius of the third zone represents R_2 plus the distance the injected waste would travel under the influence of development of the disposal formation by pumping at the original injection rate, in a location on the outer edge of this belt, during the whole period $T_e + T_t$:

$$R_3 = R_2 + 0.03168\,[q_T(T_e + T_t)/h\phi]^{1/2} =$$
$$0.03168\,[\sqrt{T_e + T_t} + \sqrt{T_e}]\,[q_T/h\phi]^{1/2} +$$
$$0.30372\kappa I T_t/\mu\phi \qquad (27)$$

Equation 27 gives the minimum distance that would have to be maintained between a prospective waste injection site and existing or anticipated groundwater developments producing from the disposal formation itself. Using values of 20 and 50 years for T_e and T_t, respectively, and the example values for the other parameters listed in Table I, R_1 = 0.715 km (0.444 mile), R_2 = 0.836 km (0.520 mile), and R_3 = 2.022 km (1.256 mile). It should be stressed here that the two factors with major influence on the above answers are κ and I; values of both may range over several orders of magnitude. In a cavernous limestone, for instance, using a value of κ = 5 darcy, and the same values for other parameters as before, the above distances would increase to R_1 = 4.68 km (2.91 mile), R_2 = 10.76 km (6.69 mile), and R_3 = 70.03 km (43.52 mile).

In addition to movement of the waste within the disposal formation, the waste may tend to move across confining beds, if an appreciable gradient exists in that direction (either natural or imposed). Some evidence exists for the occurrence of osmotic effects (osmotic flow, salt filtering) across shale layers under the influence of either dissolved-solids-concentration gradients or pressure gradients. It has also been found that the permeability of rocks that contain an appreciable fraction of clay minerals increases with increasing pressure gradient, with increasing salt concentration, and with decreasing clay content; and that it depends strongly on the types of clay minerals in the rock, and the type of dissolved solids in the fluid (van Everdingen, 1968). However, for the moment the movement across the confining beds will only be treated in terms of true Darcy-type flow under a normal hydraulic gradient; the permeability is assumed to be constant, and independent of the pressure, the dissolved-solids concentration and the ion type.

Figure 12 enables determination of average flow velocity V_c for injected waste under a natural hydraulic gradient across a confining bed, for a range of values of the gradient I', the permeability κ_c and the porosity ϕ_c. The lower horizontal axis of the graph indicates the thickness h_c of the confining bed required to "confine" the waste for a period of 1000 years. With the present state of the art, these figures can be regarded as no more than an approximation; if non-Darcy behaviour is a significant factor, the actual figures may turn out to be either larger or smaller.

It was stated in the introduction, that waste or waste-derived material would eventually appear in the natural discharge areas of a disposal formation. In the absence of adsorption and other modifying reactions, such "waste" discharge will continue, even when waste injection is discontinued, until most of the waste material has passed through the formation. Leakage of waste through confining beds under the influence of injection-pressure build up can be expected to decrease or even stop, fairly soon after injection is discontinued, and as soon as excess pressure has been dissipated. If a natural gradient through the confining beds exists, however, leakage of waste will continue, although at a reduced rate.

Pre-existing or specially installed wells in the disposal formation, the confining beds, and/or in overlying formations could be employed to monitor both pressure build-up and waste movement. The natural discharge areas should be monitored for the appearance of waste or waste-derived materials in shallow groundwater, springs and surface waters (van Everdingen and Freeze, 1971).

CONCLUSIONS

1. The magnitude and extent of the pressure increase in a disposal formation during continuous waste injection depends on: formation permeability, porosity, compressibility and thickness, on fluid viscosity and compressibility, on the radius of the well, and on the areal extent of the formation. They may further be affected by reactions between fluids and rocks in the formation.
2. Undesirable changes in the performance of a disposal system, whether by plugging of the well or the formation, by hydraulic fracturing, or by packer, tubing or casing failure, usually give rise to relatively rapid changes in injection pressure and/or rate.
3. Wellhead injection pressure and flow rate, waste density and temperature, and the pressure in the fluid-filled annulus between injection tubing and well casing, should be measured at the wellhead and checked regularly, to ensure early dectection of problems. They should preferably be recorded continuously to enable a full *post-mortem* analysis of sudden failures. Pressure recording should continue for some time after use of the injection well is stopped.
4. Automatic warning signals and/or shut-off valves should be activated by the wellhead monitor when the injection pressure reaches a previously established "maximum safe" injection pressure; potential waste-density fluctuations, affecting the weight of the waste column in the well, and thus the pressure in the formation, should be taken into account in setting the triggering pressure for the monitor. A sudden decrease in the injection pressure/injection rate ratio, indicating possibly unexpected hydraulic fracturing, or tubing, packer or casing failure, should also cause the trigging of the warning signal and/or automatic shut-off.
5. The technique of hydraulic fracturing should not be used to increase the receptive capacity of a waste disposal formation, because of the inherent risk of causing damage to overlying or underlying confining beds. Other well- and formation-stimulation techniques may be used only if they do not affect the integrity of the confining beds.
6. The radius of the waste "cylinder" in the disposal formation around an injection well as calculated from equation 19 is a minimum value. As a result of irregularities normally existing in a disposal formation, as well as possible differences in density between waste and formation fluids, the waste may move much farther and faster in some portions of the formation. The use of lost-circulation zones for waste injection, for example, will lead to "unexpected", rapid waste movement, often in unpredictable directions. Such zones in relatively

shallow formations should therefore not be used, even though they facilitate relatively high injection rates at low pressure.

7. Injected waste will move through the disposal formation under the influence of existing natural gradients, even after injection has been discontinued. Although the rate of movement may be slow, the waste may cross formation boundaries and eventually it may appear in the natural discharge areas of the flow system. Monitoring of observations wells, and of surface waters in discharge areas, for the appearance of waste components or degradation products, should therefore be continued for a considerable time after disposal has been stopped. This applies particularly in the case of toxic wastes.
8. The disadvantages of selection of the deepest suitable formation at any particular site for waste disposal are largely financial: higher drilling costs, higher costs for more and larger casing, tubing, and cementing, etc., and higher cost of materials of higher pressure rating. They are, however, outweighed by the gain in safety margins: more aggregate thickness of confining beds, less chance of accidental hydraulic fracturing, decreased rate of waste movement under natural gradients, and often a larger distance to any natural discharge areas. In some cases, however, the paucity of subsurface data for greater depths may necessitate the drilling of a number of test holes to define subsurface conditions more adequately.

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Figures 1 to 12

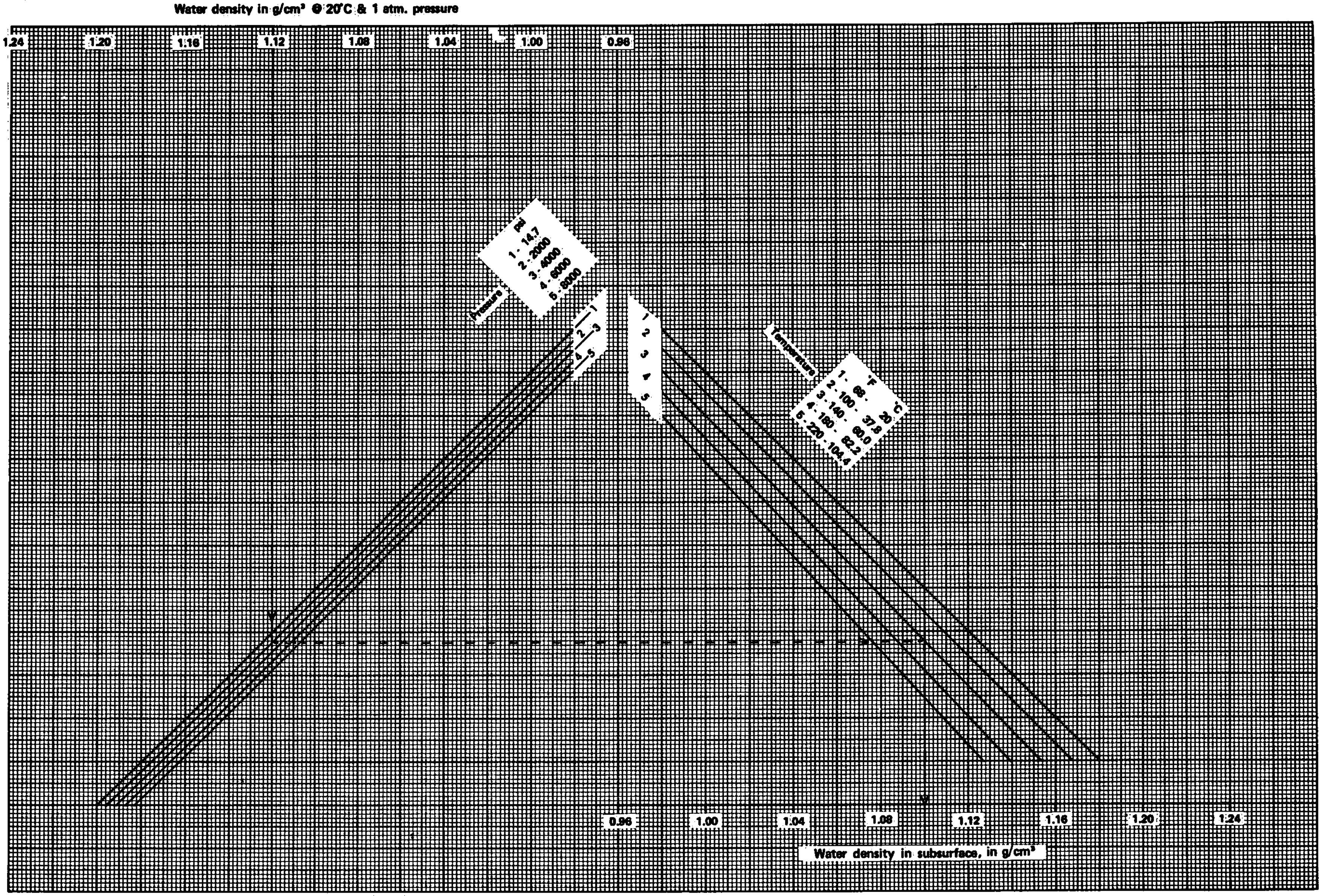

Figure 1. Determination of ρ_f and ρ_w (g/cm^3) under subsurface conditions, using density at surface, reservoir pressure and reservoir temperature.

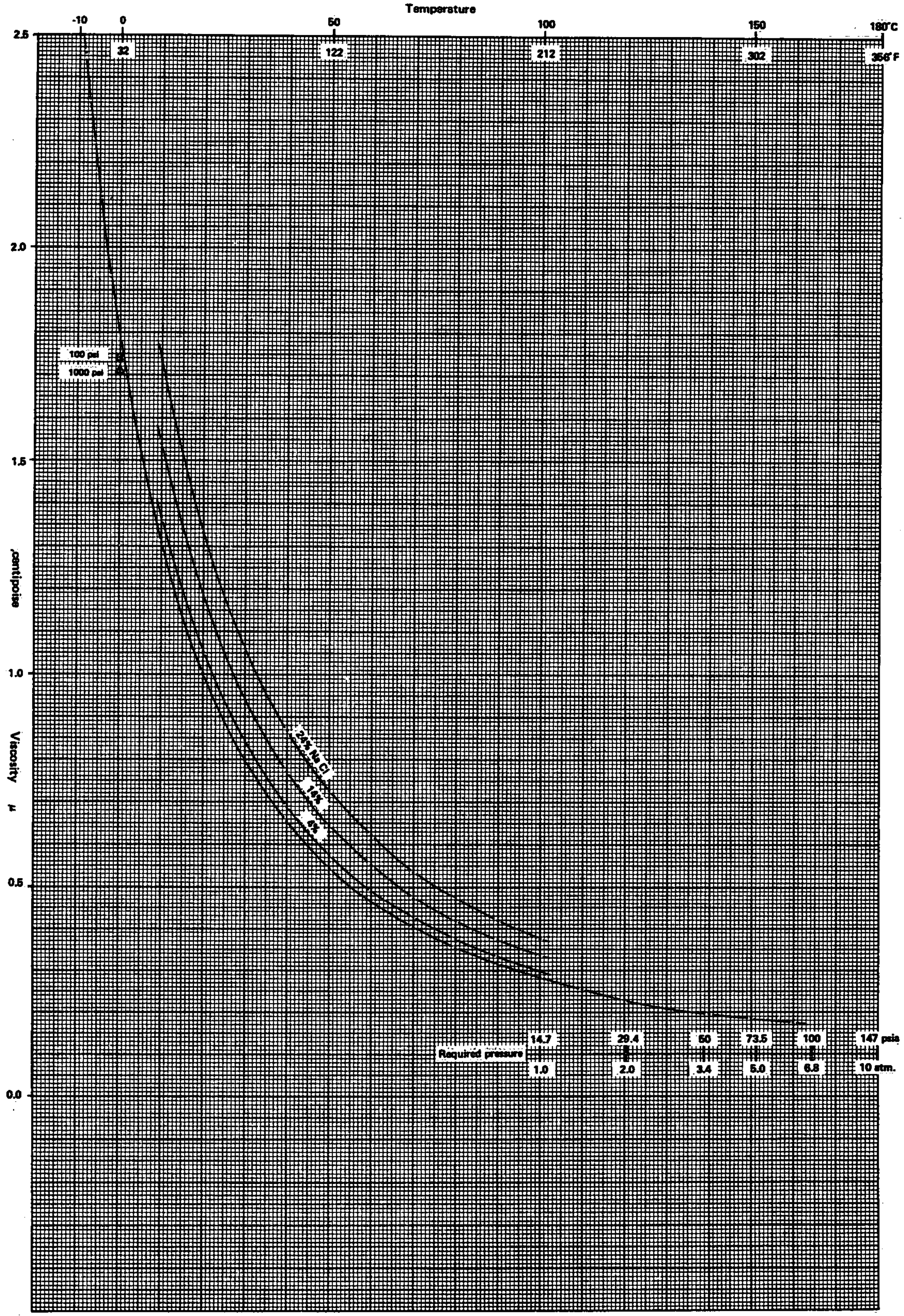

Figure 2. Viscosity *vs.* temperature, for pure water and aqueous NaCl solutions. (Data from: Handbook of Chemistry and Physics, 50th Edition, The Chemical Rubber Company, 1970).

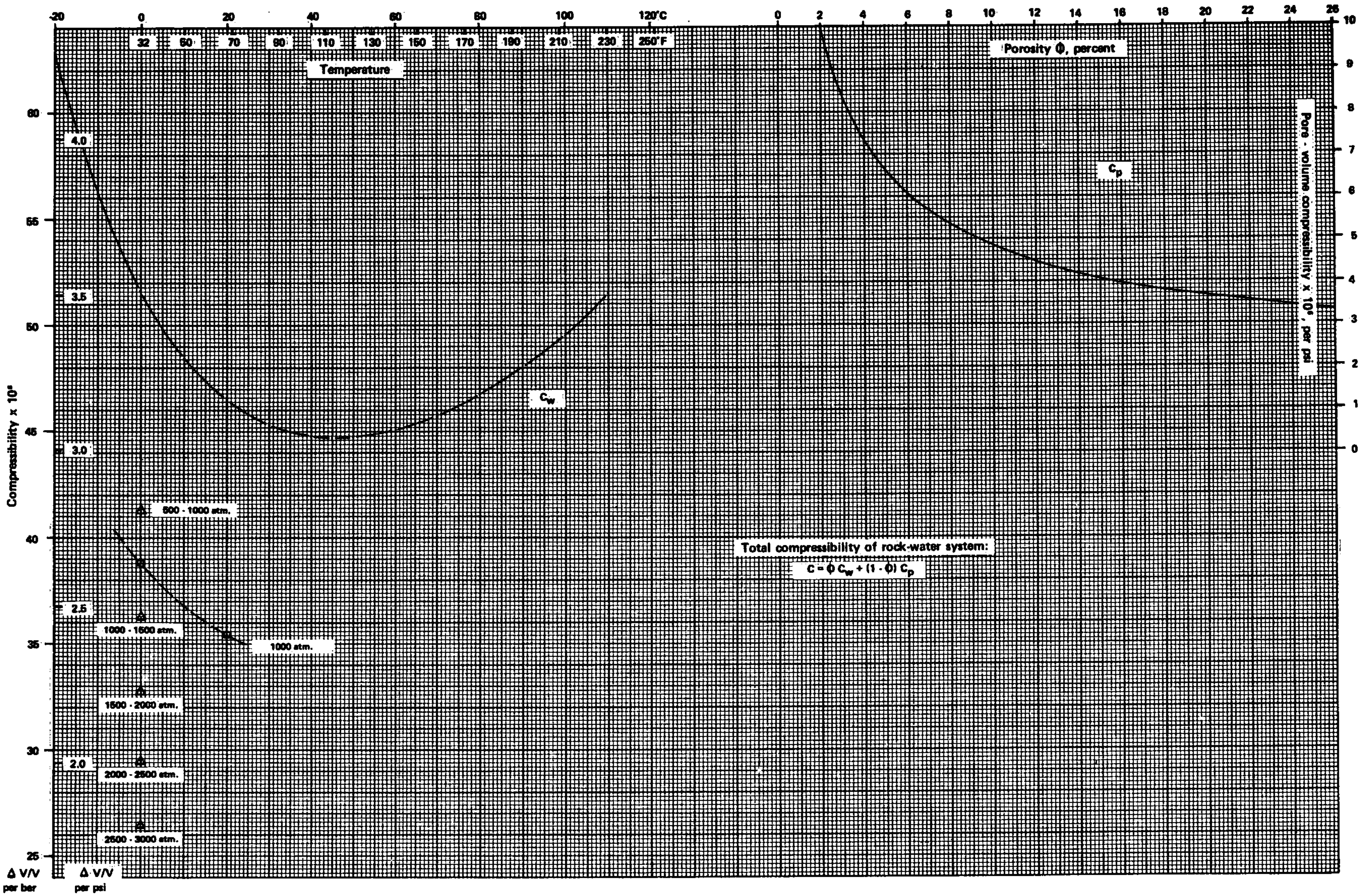

Figure 3. Compressibility of water *vs.* temperature (and pressure), and pore-volume compressibility *vs.* porosity. (C_W data from: Handbook of Chemistry and Physics, 50th Edition; Cp after Hall, M. N., 1953: Compressibility of reservoir rocks, Trans AIME 198, 309).

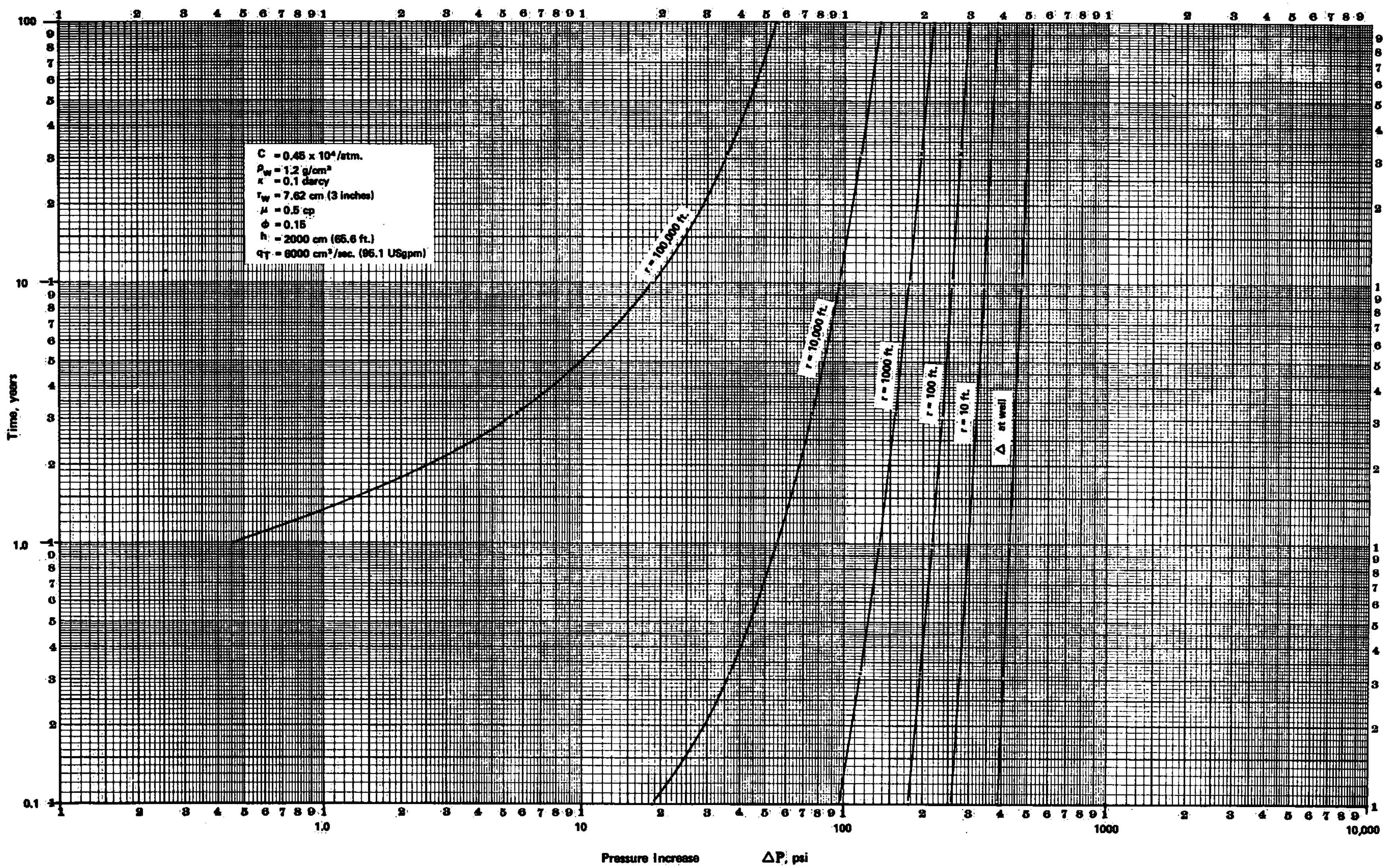

Figure 4. Pressure increase ΔP, in disposal formation, *vs.* time T, at various distances r from the centre of the disposal well.

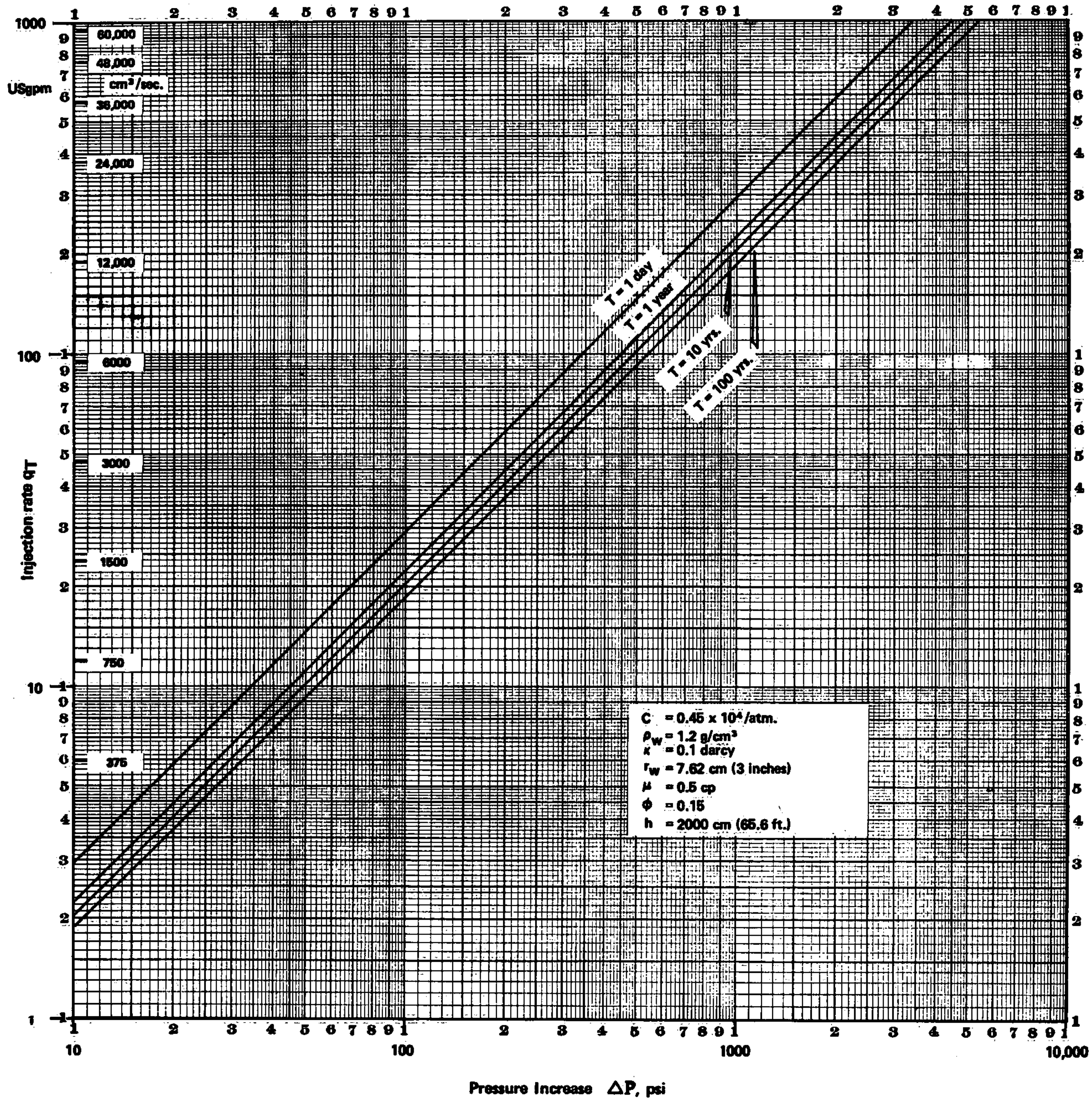

Figure 5. Pressure increase ΔP, in disposal formation at well bore, *vs.* injection rate q_T, after various times T.

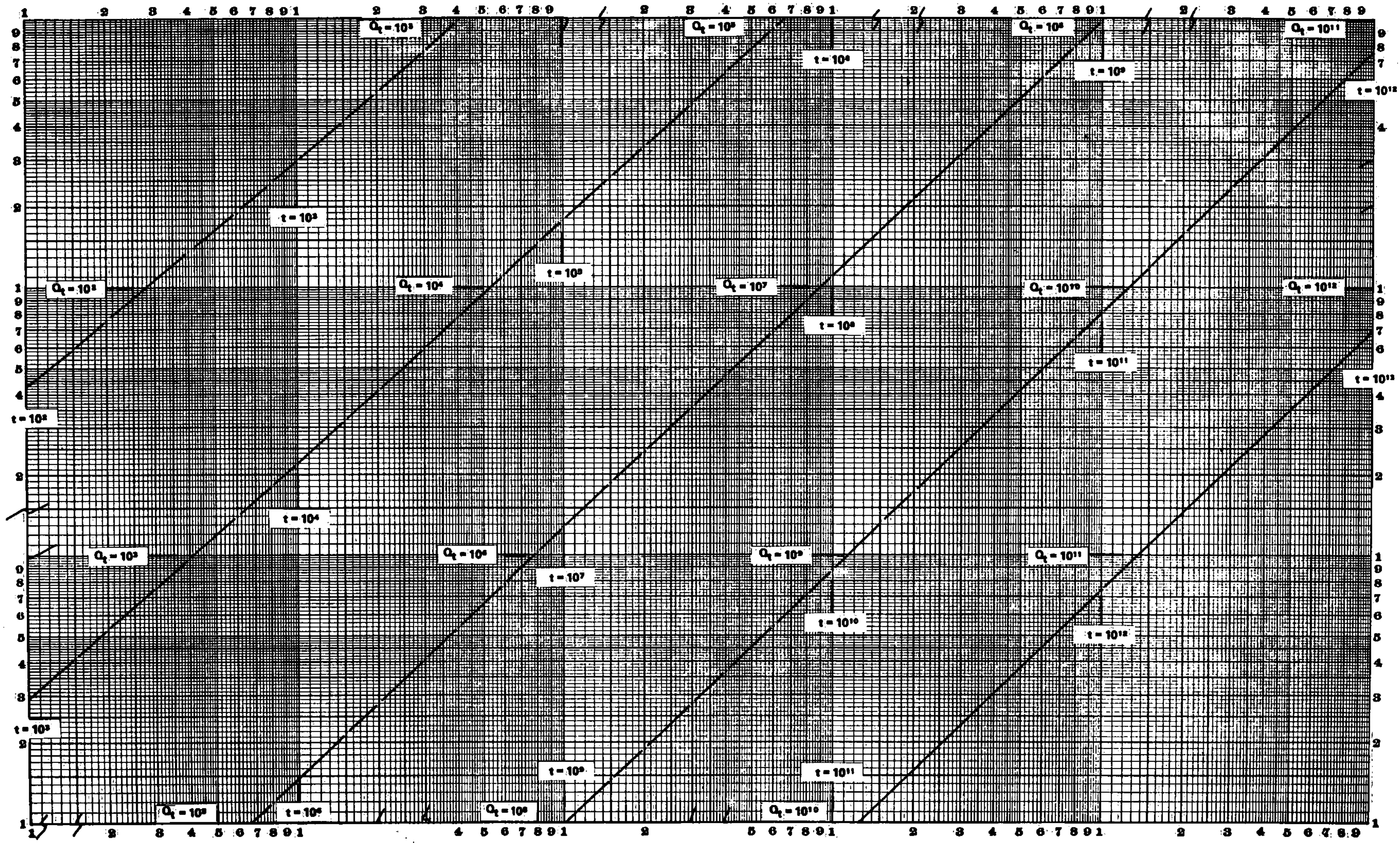

Figure 6. Dimensionless cumulative volume Q_t as a function of dimensionless time t (after van Everdingen, A. F., 1968).

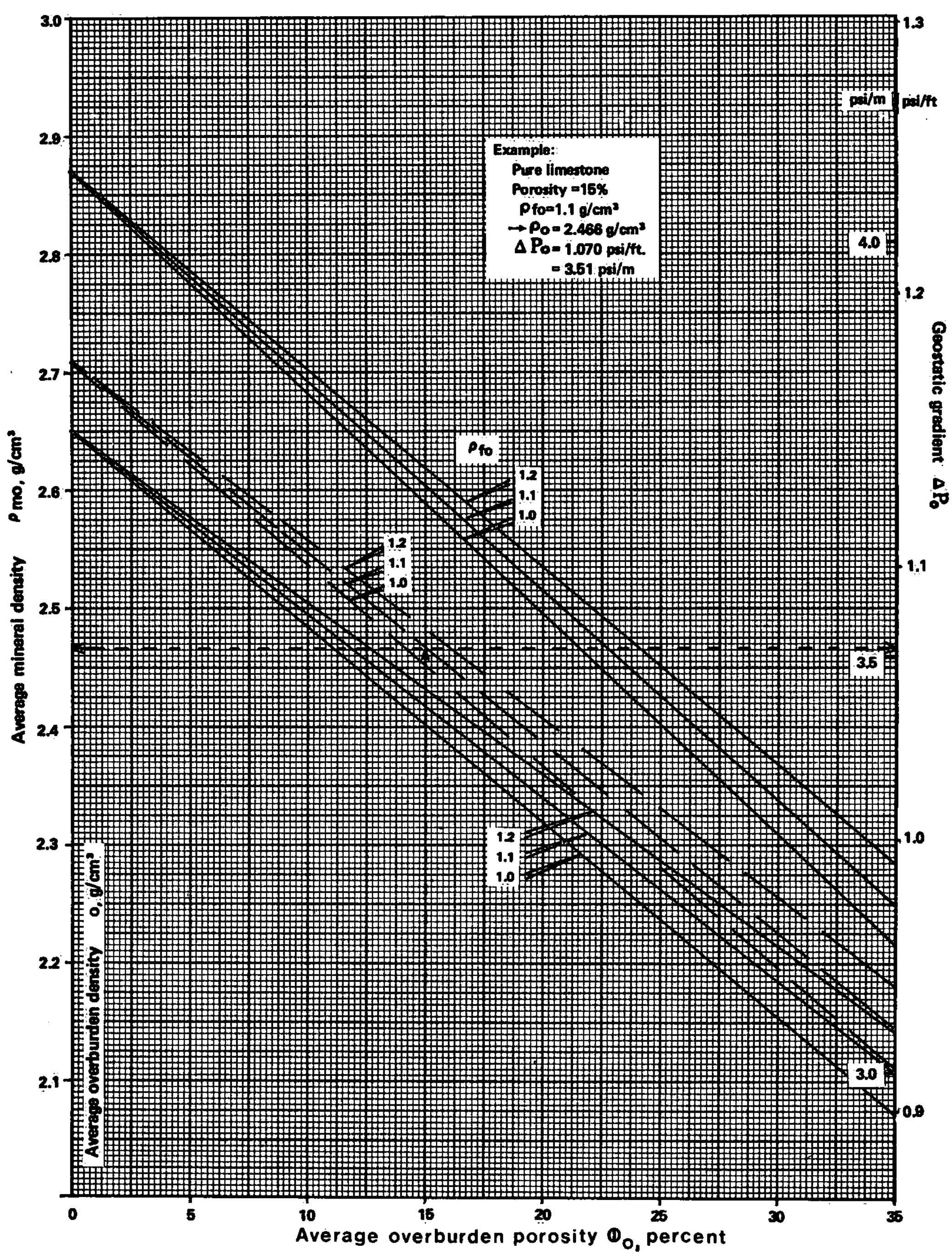

Figure 7. Determination of average overburden density ρ_o and geostatic gradient δP_o.

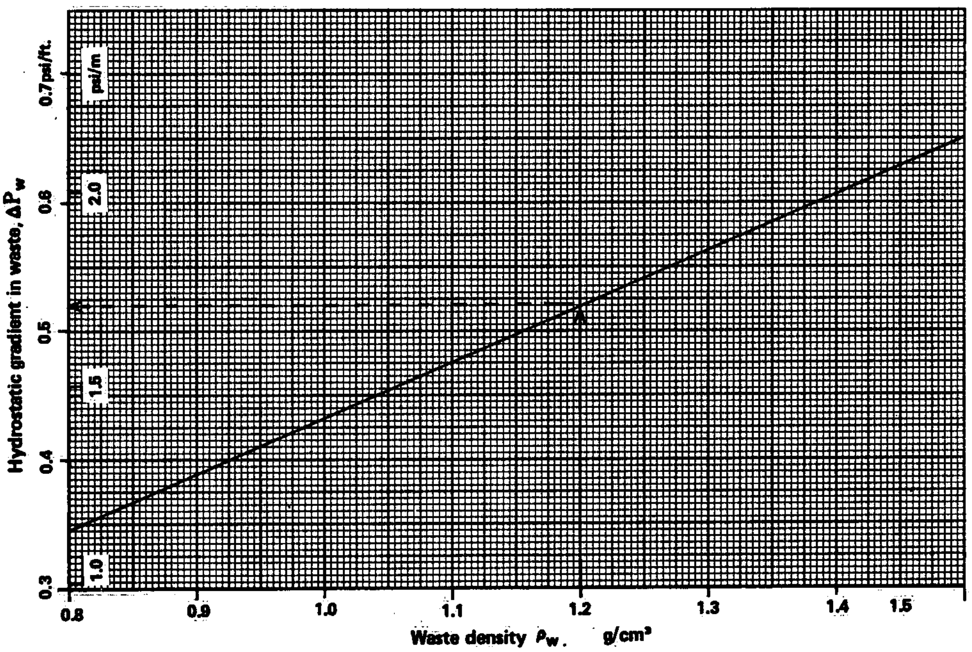

Figure 8. Determination of hydrostatic gradient δP_W in waste column in an injection well.

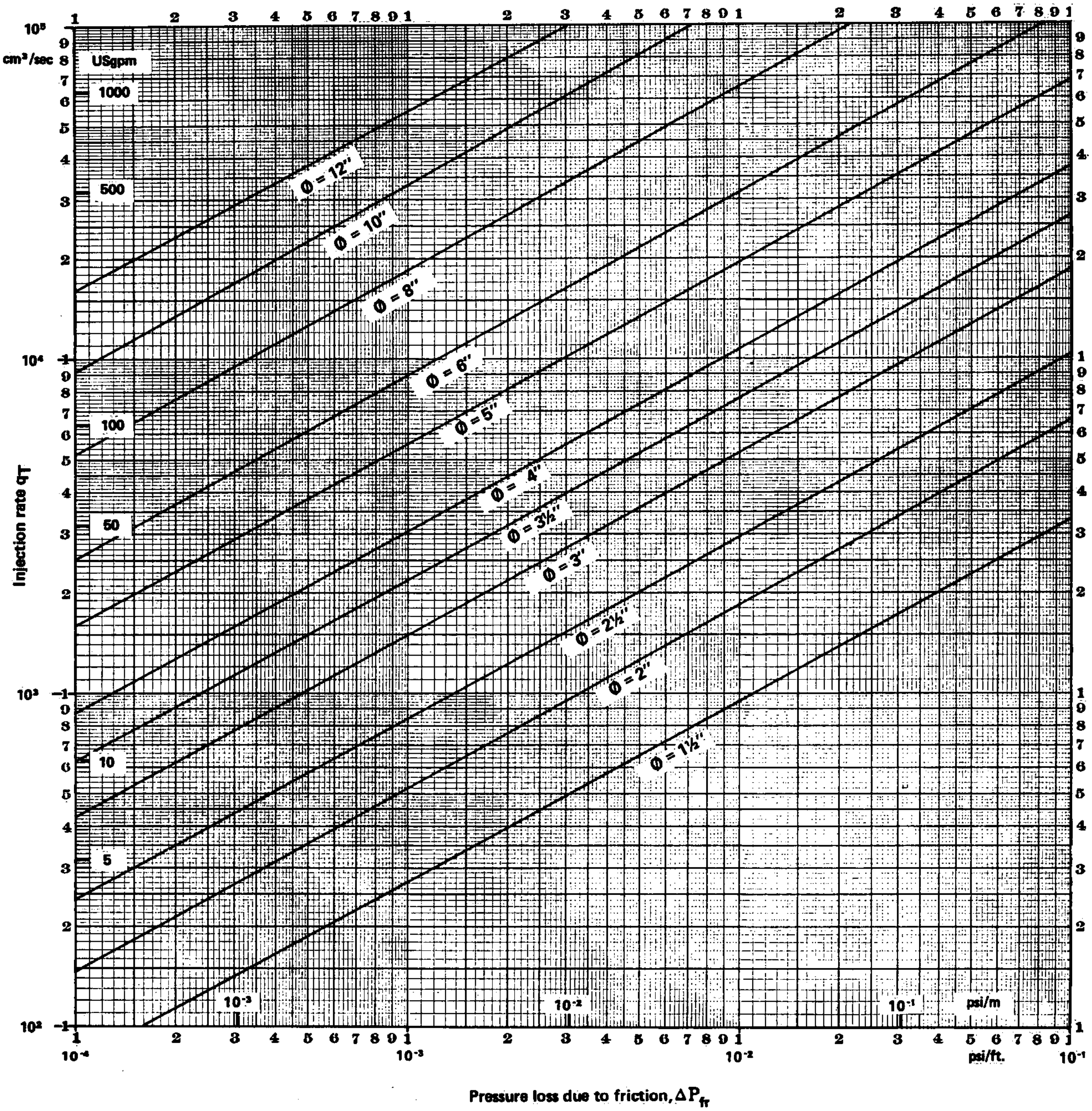

Figure 9. Pressure loss δP_{fr}, in psi/ft and psi/m, due to friction of fluid flowing at rate q_T in smooth pipe of various diameters. (Data from: "Water Well Handbook", 3rd Edition, Missouri Water Well Drillers Association, 1965, p. 34).

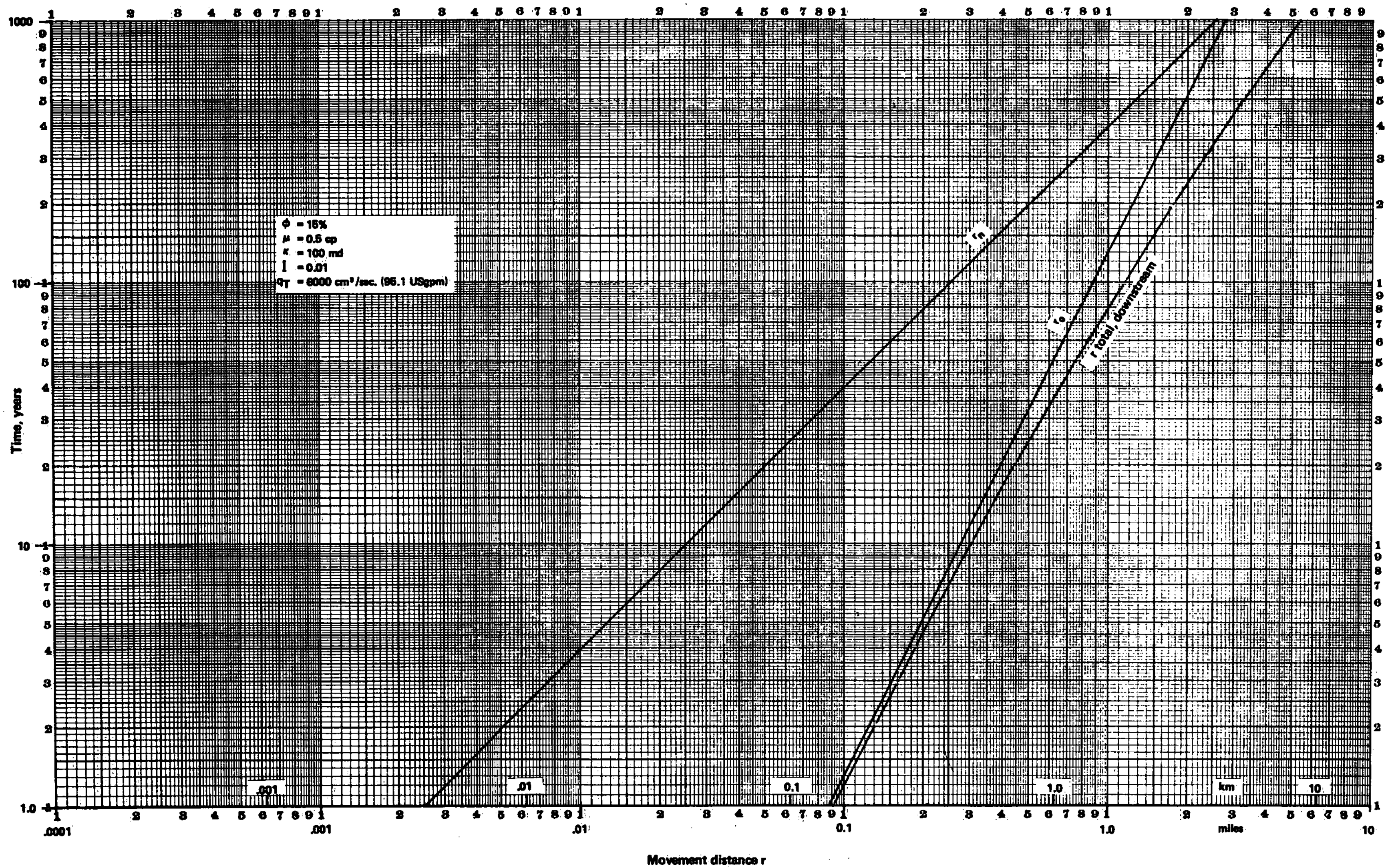

Figure 10. Waste movement distances *vs.* time; r_e as a result of injection; r_n as a result of existing natural hydraulic gradient; $r_{total} = r_e + r_n$.

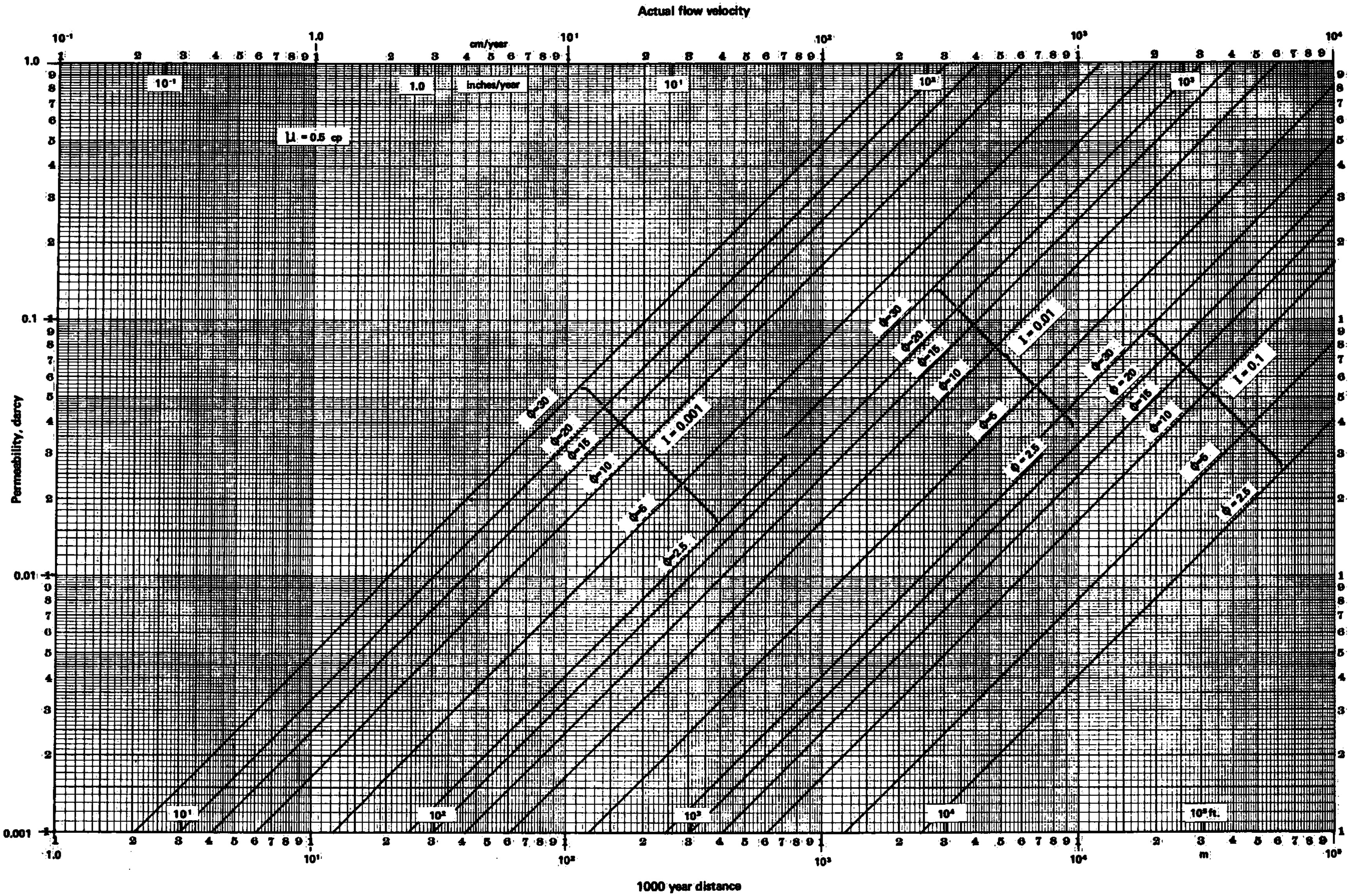

Figure 11. Flow velocity V in disposal formation *vs.* permeability κ, for various values of gradient I and porosity ϕ (viscosity $\mu = 0.5$ centipoise).

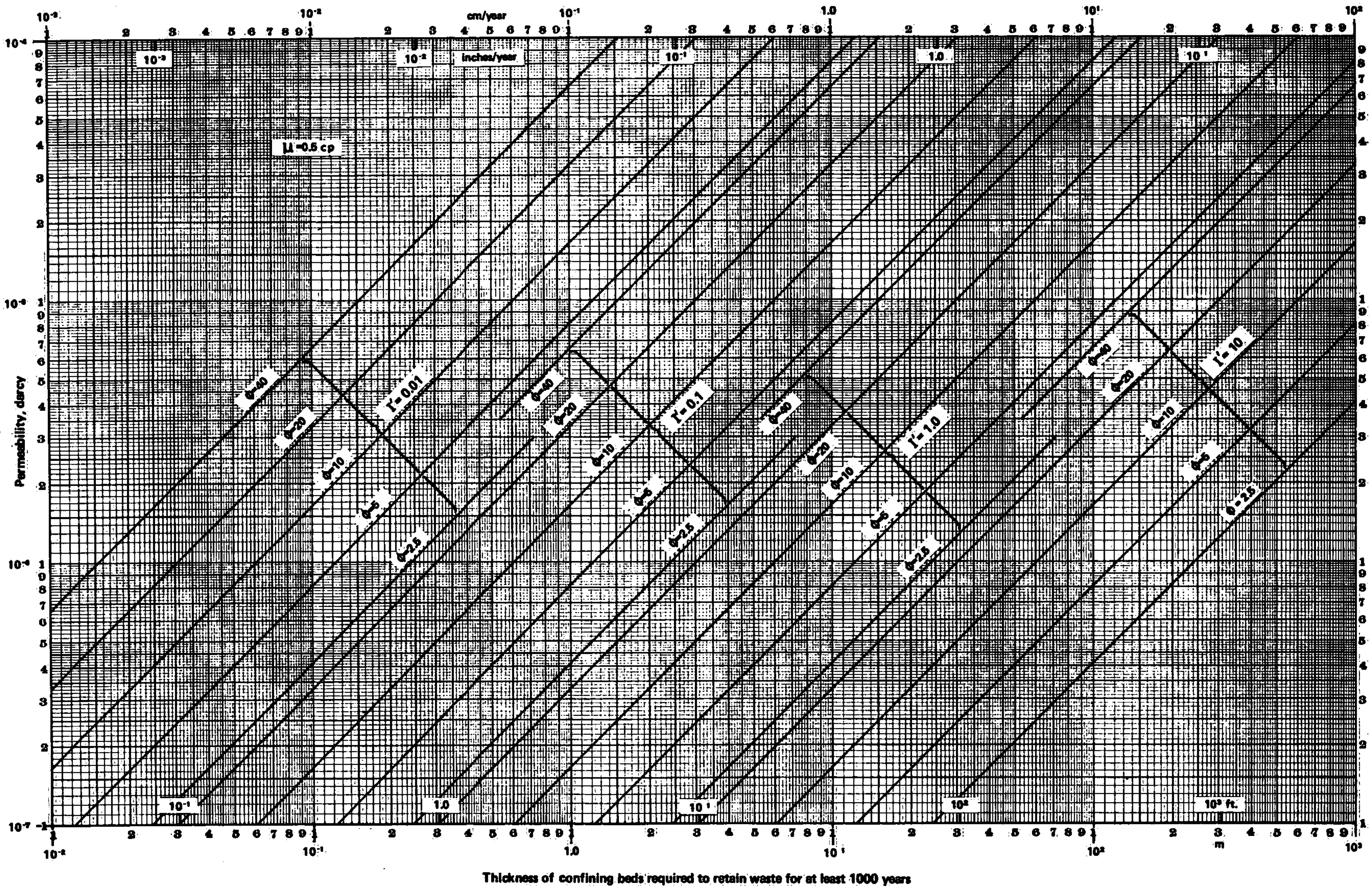

Figure 12. Flow velocity V_c across confining beds, *vs.* confining bed permeability K_c, for various values of gradient I and porosity ϕ_c (viscosity $\mu = 0.5$ centipoise).