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Tables and Type Curves for Analysis of Pump Tests in Leaky Parallel-Channel Aquifers

A. Vandenberg



TECHNICAL BULLETIN NO. 96
(Résumé en français)

**INLAND WATERS DIRECTORATE,
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Abstract

Analysis of drawdown data from pump tests in channel aquifers - as for example, buried bedrock channels - can be cumbersome when leakage effects and boundary effects set in simultaneously. Bukhari *et al.* (1969) have described a method for determining the leakage factor for cases where transmissivity and storativity can be determined from the early part of the drawdown data. Vandenberg (1976) has shown that if the distance measured along the axis of the channel between the pumped well and the observation well is greater than the width of the channel, the time-drawdown curve at the observation well can be closely approximated by assuming linear flow in the aquifer.

This publication contains:

1. Extensive tables for the well function for leaky parallel-channel flow;
2. The corresponding type curves at a scale of 1.85 inches/log cycle, suitable for curve matching;
3. Several sets of type curves for cases where neither the method of Bukhari *et al.* (1969) nor of Vandenberg (1976) can be used;
4. A computer program for plotting type curves for leaky artesian drawdown in a parallel channel for any configuration of pumped well and observation well.
5. Examples illustrating the use of the type curves.

Résumé

L'analyse des données sur le rabattement provoqué par des essais de pompage dans des canaux aquifères, par exemple des canaux de roche de fond engouffrés, peut devenir difficile lorsque les effets de drainage et les effets aux limites apparaissent simultanément. Bukhari et coll. (1969) ont décrit une méthode permettant de déterminer le facteur de drainage lorsque la transmissivité et le coefficient d'emmagasinement peuvent être déduits des premières données sur le rabattement. Vandenberg (1976) a démontré que, dans le cas où la distance mesurée le long de l'axe du canal entre le puits de pompage et le puits d'observation est supérieure à la largeur du canal, la courbe temps-rabattement au puits d'observation peut être approchée de façon assez étroite si l'on suppose que l'écoulement est linéaire au sein de l'aquifère.

La présente publication comprend:

1. des tableaux donnant de nombreuses valeurs de la fonction caractéristique pour un écoulement parallèle dans un canal semi-captif;
2. les courbes types correspondantes, à une échelle de 1.85 pouce/cycle logarithmique, permettant d'établir des correspondances de courbes;
3. plusieurs ensembles de courbes types que l'on peut utiliser lorsque la méthode de Bukhari et coll. (1969) ou celle de Vandenberg (1976) ne s'appliquent pas;
4. un programme informatique permettant de tracer des courbes types pour un rabattement artésien en milieu semi-perméable dans un canal à écoulement parallèle, quelles que soient les positions relatives du puits de pompage et du puits d'observation; et
5. des exemples d'utilisation des courbes types.

Tables and Type Curves for Analysis of Pump Tests in Leaky Parallel-Channel Aquifers

A. Vandenberg

A. INTRODUCTION

Time-drawdown data from constant-rate pump tests in parallel-channel aquifers traditionally have been analyzed on the assumption that the observation well is sufficiently close to the pumped well, in comparison to the distance to the closest image well, to ensure that the early part of the drawdown curve is not significantly affected by the image wells. In that case transmissivity (T) and storativity (S) can be determined separately from the early part of the curve by applying the Theis equation for drawdown in an infinite aquifer. Leakage, however, can only be determined by trial and error, because leakage effects do not become noticeable until later when boundary effects are also prominent, and the usual technique of comparing the drawdown curve with one of Hantush's type curves for the infinite leaky aquifer is thereby excluded.

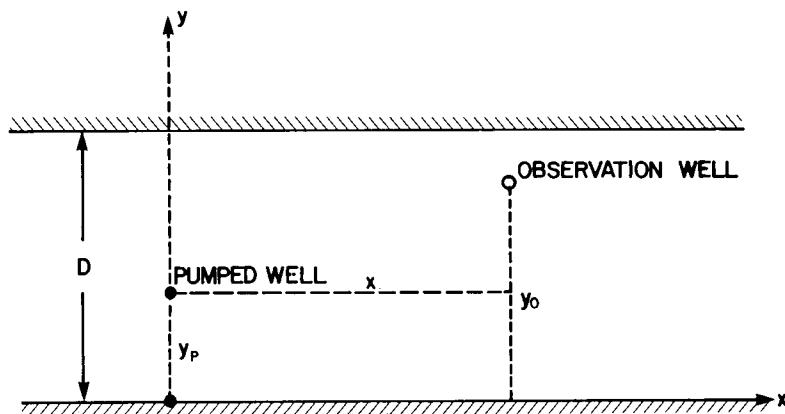


Figure 1. Notation and coordinate system for a pumped well-observation well configuration in a channel aquifer.

Vandenberg (1976), however, has shown that whenever the distance, x , measured along the axis of the channel between the pumped well and the observation well (Fig. 1) is greater than the width of the channel, D , (i.e. $x/D > 1$) the drawdown can be described with sufficient accuracy by

$$s = Qx/(2TD)F(u, x/B) \quad (1)$$

where

s = drawdown;

Q = constant pumping rate;

$$F(u, x/B) = 1/(2\pi^{1/2}) \int_u^{\infty} y^{-3/2} \exp(-y-x^2/4B^2y) dy \quad (2)$$

$$u = x^2 S / (4Tt);$$

t = time since pumping started;

$$B = \text{leakage factor} = (Tb'/K')^{1/2};$$

b' = thickness of the leaky bed;

K' = vertical hydraulic conductivity of the leaky bed;

This result is obtained on the assumption that the pumped well can be replaced by uniform discharge over the entire cross section of the aquifer, and hence the flow is everywhere parallel to the axis of the channel.

Equation (1) admits a type-curve matching technique for the determination of T , S and B from time-drawdown data.

Furthermore, for well configurations to which (1) cannot be applied, i.e. $x/D < 1$, it is still possible to calculate drawdown and to construct type-curves from the exact solution for drawdown in a parallel-channel aquifer:

$$s = Q/(4\pi T) W \left[(u_p, r/B) + \sum_{i=1}^{\infty} W (u_i, r_i/B) \right] \quad (3)$$

where

$$W (u, r/B) = \int_u^{\infty} (1/y) \exp(-y-r^2/4B^2y) dy, \quad (4)$$

the function for radial flow towards a well in a leaky aquifer of infinite extent;

r = the distance between the observation well and the pumped well;

r_i = the distance between the observation well and the i th image well; the locations of the image wells are given in Bukhari *et al.* (1969);

$$u_i = r_i^2 S / 4Tt;$$

$$u_p = r^2 S / 4Tt.$$

However, this requires the construction of a different set of type curves for each particular well configuration.

The purpose of this report is to provide:

1. Extensive tables of $F(u, x/B)$ and the corresponding set of type curves - $\log F(u, x/B)$ as a function of $\log(1/u)$, for various values of x/B - on a scale of 47 mm (1.85 inches/log cycle, which can be used for matching with time-drawdown data plotted on logarithmic paper to the same scale (e.g. K+E No. 467522);
2. Sets of type curves for a number of well configurations for which

$$x/D < 1$$

and for which equation (1), therefore, is not sufficiently accurate; selection of the set closest to the actual field situation will usually give a fairly good estimate of the hydrologic parameters.

3. A listing and users guide for a FORTRAN program which will plot a set of type curves for any given well configuration.

B. TABLES AND TYPE CURVES OF THE LEAKY AQUIFER FUNCTION FOR PARALLEL FLOW

Values of the function $F(u, x/B)$ have been calculated with an absolute error of less than 10^{-5} , for values of u ranging from 10^{-6} to 7 and values of x/B ranging from 0 to 2 (Table 1). Plate 1 (in pocket) gives the corresponding type curves.

For $x/B = 0$ equation (2) becomes (Vandenberg, 1976)

$$F(u) = \exp(-u)/(\pi u)^{\frac{1}{2}} - \operatorname{erfc}(u^{\frac{1}{2}}), \quad (5)$$

which can be called the nonleaky aquifer function for parallel flow. For large values of $1/u$ equation (5) can be approximated by

$$F(u) = (\pi u)^{-\frac{1}{2}}$$

and thus

$$\log F(u) = -\frac{1}{2} \log \pi + \frac{1}{2} \log 1/u$$

Thus the type curve becomes a straight line with a slope of 0.5, a fact which should be borne in mind when drawdown data are matched to the curve.

The entire procedure for determining T, S and B is similar to the technique employed with Hantush's type curves:

1. The time-drawdown data are plotted on logarithmic paper, scale 47 mm (1.85 inches)/log cycle, with time along the horizontal and drawdown along the vertical axis;
2. The drawdown data are matched to one of the type curves, taking care that the axes of the type curve and the drawdown curve remain parallel. The best matching curve immediately gives the value of x/B , and thus of B.
3. A convenient matchpoint is chosen, for example, the point ($1/u = 1$, $F(u, x/B) = 1$), and the corresponding values of t and s are noted.
4. T is calculated from

$$T = (Qx/2sD) F(u, x/B) \quad (6)$$

5. S is calculated from

$$S = 4Ttu/x^2 \quad (7)$$

Equations (6) and (7) imply that all variables and constants are in consistent units; if, for example, metres and minutes are the basic units for length and time, respectively, Q must be expressed in m^3/min , and T will be in m^2/min . Since the minute is a more practical unit of measurement for most pump tests, it has been adopted as the basic time unit for the practical examples in this report, rather than the standard SI unit (second).

In the foregoing it has been assumed that the width and orientation of the channel are both known with sufficient accuracy. If the direction of the channel is known, but not the width, the procedure is the same, except the products TD and SD are calculated instead of T and S

$$TD = (Qx/2s)F(u, x/B)$$

and

$$SD = 4TDtu/x^2.$$

These products can then be used in other applications in much the same manner as T and S are used with aquifers of infinite areal extent.

If the direction of the channel is not known and only one observation well is available, the best that can be done is to use the distance r between pumped well and observation well instead of x , its projection along the direction of the channel. For those cases where $r \gg D$ this will only introduce a small error.

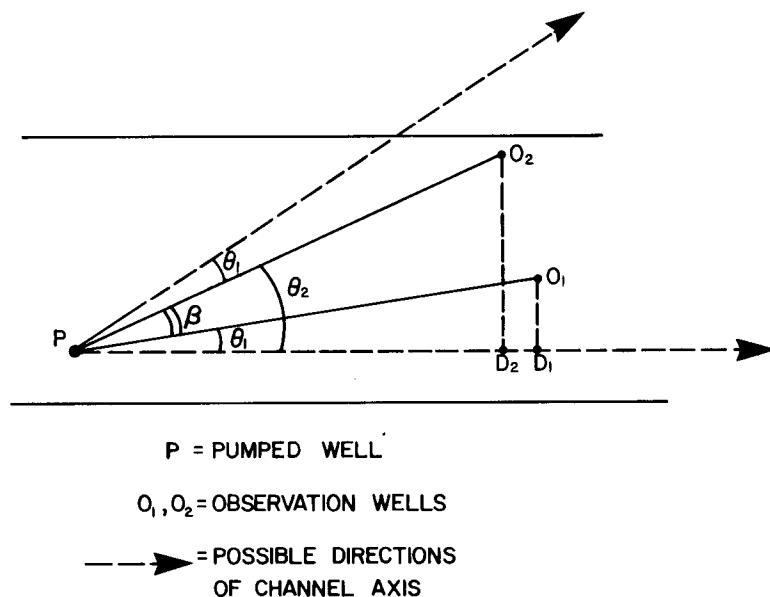


Figure 2. Determining the two possible directions of the channel axis from drawdown in two observation wells.

If two observation wells are available and the three wells are not on a straight line, the channel direction can be determined as one of two choices, as follows (Fig. 2):

1. From the data of observation well O₁ determine by curve matching

$$A_1 = TD/x_1 = (Q/2s_1)F(u, x_1/B)$$

2. From the data of observation well O₂ determine by curve matching

$$A_2 = TD/x_2 = (Q/2s_2)F(u, x_2/B)$$

3. Then

$$A_1/A_2 = x_2/x_1 = r_2 \cos \theta_2 / (r_1 \cos \theta_1)$$

or, since $\beta = \pm (\theta_2 - \theta_1)$, the angle between $P0_1$ and $P0_2$,

$$r_1 A_1 / (r_2 A_2) = \cos \theta_2 / \cos (\theta_2 \mp \beta)$$

from which follows

$$\tan \theta_2 = \pm (r_2 A_2 / r_1 A_1 - \cos \beta) / \sin \beta$$

C. TYPE CURVES OF THE LEAKY AQUIFER FUNCTION FOR NONPARALLEL FLOW

For all well configurations where $x/D < 1$ equation (3) rather than equation (1) should be used to describe drawdown. However, it can be shown that the shape of the time-drawdown curve, plotted on logarithmic paper, depends only on the ratios x/D , y_p/D , y_o/D and r/B , where y_p and y_o are the distances of the pumped well and the observation well, respectively, to one of the channel boundaries (Fig. 1). Thus, if the direction and width of the channel are known for a particular well configuration determined by x/D , y_p/D and y_o/D , a family of type curves can be constructed for various values of r/B , and T , S and B can be determined by matching of the time-drawdown curve to one of the type curves.

Figure 3 shows the 44 configurations corresponding to one of the 16 sets of type curves contained in this report (Plates 2 to 17). Duplications occur (1) because either boundary can be used to define y_p and y_o and (2) because of the principle of reciprocity, which states that if there is a pair of wells of which one is the pumped well and the other the observation well, the time-drawdown curve for the observation well will be the same, regardless of which well is the pumped well. The principle is demonstrated easily for any kind of bounded aquifer which can be represented with the aid of a set of image wells; a general proof for an inhomogeneous aquifer of irregular shape is given by McKinley *et al.* (1968).

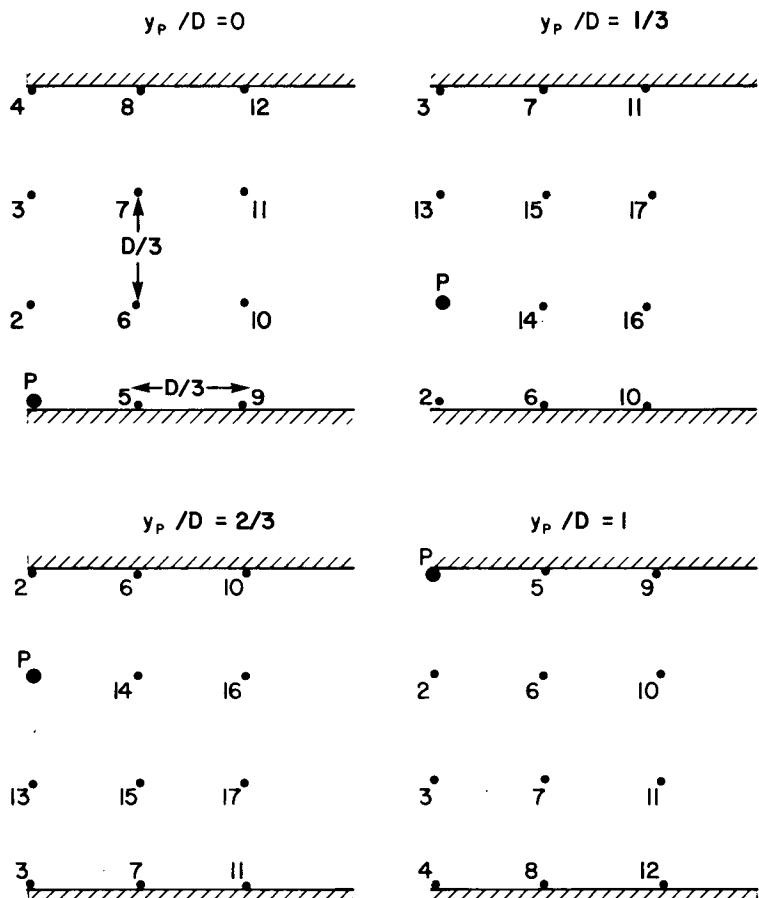
To use Plates 2 to 17, the set of type curves corresponding best with the field configuration is selected first with the aid of Figure 3; next, a curve-matching procedure is carried out similar to that already described for the use of the leaky aquifer curves for parallel flow. In this case, however, B , T and S are calculated from

$$B = r/(r/B),$$

$$T = Q/(4\pi s) \Sigma W(u, r/B)$$

and

$$S = 4Tu/r^2.$$



LEGEND

- P ● PUMPED WELL LOCATION
- 12. ● OBSERVATION WELL LOCATION, AND NUMBER OF THE APPROPRIATE PLATE

Figure 3. Well configurations for which type curves (Plates 2 to 17) are supplied in this report.

D. COMPUTER PROGRAM FOR PLOTTING OF TYPE CURVES OF THE LEAKY AQUIFER FUNCTION FOR NONPARALLEL FLOW

In cases where greater accuracy is needed than can be obtained with the aid of the type curves (Plates 2 to 17), program CURVES, described in this chapter, can be used to produce sets of type curves on a CALCOMP plotter. The graphs will be similar to Plates 2 to 17 and can be made to fit the exact well configuration of the pump test for which time-drawdown data are to be analyzed. The type curves are to be used for curve matching exactly as the type curves described in the preceding section.

Input Data to the Program

The following data must be supplied to the program:

L integer number of values of r/B ; each plot will have L graphs of $\log [\sum W(u_i, r_i/B)]$ versus $\log (1/u)$; L may have any integer value

$$1 \leq L \leq 20$$

DB(L) L values of r/B (dimensionless); all the plots produced in one execution of the program will have the same values of L and $(r/B)_i$, $i = 1 \dots L$

D the width of the channel (length)

YP the y-coordinate of the pumping well (length); the coordinate system is shown in Figure 1.

Y0 the y-coordinate of the observation well (length);

X0 the x-coordinate of the observation well (length);

TST a control parameter (dimensionless); summation of equation (3) stops if the partial sum of 4 consecutive terms is less than TST, or if the ratio of the partial sum of 4 consecutive terms to the total accumulated function value is less than TST.

Input Format

The following table specifies the setup of data cards for use with the program. The FORMAT column gives the FORTRAN format specification.

Card	FORMAT	Columns	Data
1	(I4)	1-4	L
2	(20F4.0)	1-4 5-8 ..	DB (1) DB (2) ..
		77-80	DB (20)
3	(5F5.0)	1-5 6-10 11-15 16-20 21-25	D YP Y0 X0 TST

card No. 3 can be repeated any number of times to produce a number of plots in the same execution run.

4

A blank card to indicate the end of the run.

Printed Output

For each plot the input values from card No. 3 are printed; and if, after 200 iterations, the conditions for terminating the series are not satisfied, a warning message will be printed: "FOR THESE VALUES OF U AND R/B NO CONVERGENCE", followed by the values of u and r/B. The appearance of the message indicates that the accuracy of the curve is poor over the indicated range of u, and since slow convergence of equation (3) occurs only for large values of x/D, it is furthermore an indication that the type curve for linear flow (Plate 1) can be used instead.

Program Listing

The program consists of the main program CURVES, which handles input and output, calls on the CALCOMP plotting routines, and performs all calculations, assisted by subroutine LEAKY for the calculation of the function $W(u,r/B)$ (equation 4). LEAKY in turn calls subroutine NOLEAK for the calculation of the exponential function or well function for the infinite, confined aquifer.

The program listing which follows shows the FORTRAN IV extended language in use at the Dept. of Energy, Mines and Resources, Ottawa, Ontario.

Program Listing

```

PROGRAM CURVES(INPUT,OUTPUT)
C CURVES DRAWS TYPE CURVES FOR THE LEAKY WELL FUNCTION IN PARALLEL
C CHANNEL AQUIFERS ON A SCALE OF 1.85 INCH/LOG CYCLE OR 3 INCH/LOG CYCLICURV 1
C
C DIMENSION BUF(1024),X(102),Y(102),U(100),XA(100),DB(20) CURV 2
C CALL PLOTS(BUF,1024) $ CALL PLOT(2.,2.,-3) CURV 3
C
C IF THE NEXT CARD IS OMITTED SCALE WILL BE 3 INCH/LOG CYCLE,ELSE SCALECURV 4
C WILL BE 1.85 INCH/LOG CYCLE CURV 5
C
C CALL FACTOR(1.85/3.) CURV 6
C READ 700,L,(DB(J),J=1,L) CURV 7
12 READ 500,D,YP,YO,X0,TST CURV 8
C
C D =WIDTH OF CHANNEL CURV 9
C YP=Y-COORDINATE OF PUMPING WELL CURV 10
C YO=Y-COORDINATE OF OBSERVATION WELL CURV 11
C XO=X-COORDINATE OF OBSERVATION WELL CURV 12
C TST=....IF THE SUM OF THE CONTRIBUTIONS OF A GROUP OF 4 IMAGE CURV 13
C WELLS IS LESS THAN TST OR THE RATIO CURV 14
C SUM OF 4 IMAGE WELL CONT./TOTAL ACCUMULATED FUNCTION VALUE CURV 15
C IS LESS THAN TST,NO FURTHER IMAGE WELL CONTRIBUTIONS CURV 16
C ARE CALCULATED CURV 17
C
C IF(D.EQ.0.)GOTO 13 CURV 18
XOS=XO*X0 CURV 19
RS=(YO-YP)*(YO-YP)+XOS CURV 20
PRINT 400,D,YP,YO,X0,TST CURV 21
400 FORMAT(1D,YP,YO,X0,AND TST... *,4F12.2,F12.6//)
500 FORMAT(5F,0)
600 FORMAT(* FOR THESE VALUES OF U AND R/B NO CONVERGENCE*,2E20.5)
700 FORMAT(I4/2DF4.0)
C
C DRAWING OF LOGARITHMIC GRID AND ANNOTATION CURV 22
C
CALL SYMBOL(0.1,13.0,0.21,5H YP=,0.,5) CURV 23
CALL NUMBER(1.2,13.0,0.21,YP,0.,2) CURV 24
CALL SYMBOL(0.1,12.6,0.21,5H YO=,0.,5) CURV 25
CALL NUMBER(1.2,12.6,0.21,YO,0.,2) CURV 26
CALL SYMBOL(0.1,12.2,0.21,5H D=,0.,5) CURV 27
CALL NUMBER(1.2,12.2,0.21,D,0.,2) CURV 28
CALL SYMBOL(0.1,11.8,0.21,5H XO=,0.,5) CURV 29
CALL NUMBER(1.2,11.8,0.21,XO,0.,2) CURV 30
XX=0. $ DO 1 I=1,6 $ CALL PLOT(XX,0.,3) $CALL PLOT(XX,13.5625,2) CURV 31
1 XX=XX+3. $ CALL PLOT(0.,0.,3) $CALL PLOT(15.,0.,2) $P=1.5625 CURV 32
CALL PLOT(0.,P,3) $ DO 2 I=1,5 $CALL PLOT(15.,P,2) $P=P+3. CURV 33
2 CALL PLOT(0.,P,3) $ P=-.25 $ H=.2 CURV 34
CALL SYMBOL(0.,P,H,3H0.1,0.,3) CURV 35
CALL SYMBOL(3.,P,H,1H1,0.,1) CURV 36
CALL SYMBOL(6.,P,H,2H10,0.,2) CURV 37
CALL SYMBOL(7.2,P,H,3H1/U,0.,3) CURV 38
CALL SYMBOL(9.,P,H,3H100,0.,3) CURV 39
CALL SYMBOL(12.,P,H,4H1000,0.,4) CURV 40
CALL SYMBOL(15.,P,H,5H10000,0.,5) CURV 41
P=-.75
CALL SYMBOL(P,1.5625,H,4H0.01,0.,4) CURV 42
CALL SYMBOL(P,4.5625,H,3H0.1,0.,3) CURV 43
CALL SYMBOL(P,7.5625,H,2H1,0.,2) CURV 44
CALL SYMBOL(P,10.5625,H,2H10,0.,2) CURV 45
CALL SYMBOL(P,13.5625,H,3H100,0.,3) CURV 46
P=-1.3
CALL SYMBOL(P,5.5,H,12HSUM H(U,R/B),90.,12) CURV 47
CALL SYMBOL(15.2,13.8,.1,4HRH/B,0.,4) CURV 48
CURV 49
CURV 50
CURV 51
CURV 52
CURV 53
CURV 54
CURV 55
CURV 56
CURV 57
CURV 58
CURV 59
CURV 60
CURV 61
CURV 62
CURV 63
CURV 64
C
C PREPARING X-CORDINATES OF PLOT AND CORRESPONDING VALUES OF U CURV 65
C
C DIF=13.5/99. $ DO 3 I=1,100 $ AI=I-1 $ XA(I)=15.-AI*DIF CURV 66
3 U(I)=10.**(1.-XA(I)/3.) $ DO 4 I=1,L CURV 67
CURV 68
C THE I-LOOP PLOTS A GRAPH FOR EACH VALUE OF R/B CURV 69
C
C KKK=1
DO 5 J=1,100 $ H=0. $ CALL LEAKY(U(J),DB(I),WA) $H=H+WA CURV 70
CURV 71
CURV 72
CURV 73
CURV 74
CURV 75
CURV 76
CURV 77
CURV 78
CURV 79
CURV 80
CURV 81
CURV 82
CURV 83
CURV 84
CURV 85
CURV 86
CURV 87
CURV 88
CURV 89
CURV 90
CURV 91
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CURV 108
CURV 109
CURV 110
CURV 111
CURV 112
CURV 113
CURV 114
CURV 115
CURV 116
CURV 117
CURV 118
CURV 119
CURV 120
CURV 121
CURV 122
CURV 123
CURV 124
CURV 125
CURV 126
CURV 127
CURV 128
CURV 129
C
C THE J-LOOP CALCULATES FUNCTION VALUES FOR 100 VALUES OF U CURV 129
C
C REAL WELL AND FIRST IMAGE WELL CONTRIBUTIONS ARE CALCULATED CURV 129
C SEPARATELY CURV 129
C
C A=(XOS+(YO+YP)*(YO+YP))/RS
ARG1=A*U(J) $ ARG2=SQRT(A)*DB(I)
CALL LEAKY(ARG1,ARG2,WA) $ H=H+WA
DO 6 K=1,200 $ AK=2^K
C
C THE K-LOOP CALCULATES UP TO 200 GROUPS OF 4 IMAGE-WELL CONTRIBUTIONS.CURV 129
C IF THE 200-TH ITERATION DOES NOT OBTAIN A PARTIAL SUM OF 4 IMAGE-WELL CURV 129
C CONTRIBUTIONS LESS THAN TST A WARNING MESSAGE IS WRITTEN TO THE CURV 129
C OUTPUT FILE.CURV 129
C
C SUM=0. $ DO 7 M=1,4 $ I=(M.EQ.1)GA2=YO-YP-AK*D
IF(M.EQ.2)GA2=YO+YP+AK*D
IF(M.EQ.3)GA2=YO+YP-AK*D
IF(M.EQ.4)GA2=YO-YP+AK*D
A=(XOS+GA2*GA2)/RS
ARG1=U(J)*A $ ARG2=SQRT(A)*DB(I) SCALL LEAKY(ARG1,ARG2,R)
7 SUM=SUM+R $ H=H+SUM $ IF(SUM.LT.TST)GOTO 8
IF(ABS(SUM/H).LT.TST)GOTO 8
6 CONTINUE $ PRINT 600,UE(J),DB(I)
8 Y(J)=ALOG10(H)*3.7.5625
IF(Y(J).GT.13.5625)KKK=J+1
IF(Y(J).LT.0.)GOTO 9
5 CONTINUE $ II=100 $ GOTO 10
9 II=J-1
10 DO 11 J=1,II
11 X(J)=XA(J) $ X(II+1)=Y(II+1)=0. $ X(II+2)=Y(II+2)=1.
IF(II.LT.2)GOTO 4
IF(KKK.EQ.0)CALL NUMBER(15.125,Y(1),,1,DB(I),0.,3)
IF(KKK.GT.1)CALL NUMBER(X(KKK),13.6,1,DB(I),45.,3)
4 CALL LINE(X(KKK),Y(KKK),II-KKK+1,1,0,0)
C
C PLOTTING OF THE DIAGRAM SHOWING THE LOCATIONS OF THE WELLS AND THE CURV 129
C BOUNDARIES CURV 129
C
C IF(X0.LT..01)X0=.01
YP=YP/X0 $ YO=Y0/X0 $ XO=0/X0
DS=X0 $ IF(X0.LT.1.)GOTO 61 $ IF(X0.GT.2.5)DS=2.5$ XS=DS/X0
GO TO 62
61 XS=2.5 $ DS=XS*X0
62 YPS=XS*YP $ YOS=XS*YO $ CALL PLOT(.25,7.75,-3)
CALL PLOT(2.5,0.,2)
CALL PLOT(2.5,DS,3) $ CALL PLOT(0.,DS,2)
CALL SYMBOL(0.,YPS,.07,1,0.,-1)
CALL SYMBOL(1.,YPS+.1,10,1HP,0.,1)
CALL SYMBOL(XS,YOS,.07,1,0.,-1)
CALL SYMBOL(XS+.1,YOS+.1,10,1HO,0.,1)
CALL PLOT(20.,-7.75,-3) $ GOTO 12
13 CALL PLOT(0.,0.,999) $ STOP $ END

```

```

SUBROUTINE NOLEAK (U,WU)
IF (U.GT.40.) GOTO 50
A1=.2372905
A1=4.530792
A2=5.1266902
B0=2.4766933
B1=8.6660126
B2=6.1265271
10 IF (U-1.1) 20,20,40
20 TERM=U
WU=-.57721567-ALOG(U)+U
6 A=1.
30 TERM=-A*TERM*U/((A+1.)*(A+1.))
WU=TERM+WU
A=A+1.
IF (ABS(TERM)-1.E-8) 60,60,30
40 WU=EXPI(-U)*(AC+U*(A1+U*(A2+U)))/(B0+U*(B1+U*(B2+U)))/U
GO TO 60
50 WU=1.E-25
60 RETURN
END

```

```

NOLK 1          A(12)=-A(5)
NOLK 2          A(13)=-A(4)
NOLK 3          A(14)=-A(3)
NOLK 4          A(15)=-A(2)
NOLK 5          A(16)=-A(1)
NOLK 6          ARG=ROVB*ROVB*.25
NOLK 7          IF (ARG/U.LE.(-70.)) GO TO 20
NOLK 8          EX=EXP(ARG/U)
NOLK 9          CALL NOLEAK (U,EI)
NOLK 10         FIRST=EX*EI
NOLK 11         GO TO 30
NOLK 12         20 FIRST=0.
NOLK 13         EX=0.
NOLK 14         TERM=0.
NOLK 15         DO 50 I=1,16
NOLK 16         ARG2=A(I)*.5*(1.-EX)+(1.+EX)*.5
NOLK 17         EI2=0.
NOLK 18         IF (ABS(ARG2-1.) LE.1.E-10) GO TO 40
NOLK 19         BLN=ARG/( ALOG(ARG2))
NOLK 20         CALL NOLEAK (BLN,EI2)
NOLK 21         GO TO 50
40 TERM=0.
50 TERM=TERM+EI2*H(I)
TERM=TERM*.5*(1.-EX)
WURB=FIRST+TERM
60 RETURN
70 CALL NOLEAK(U,WURB)
RETURN
END

```

```

LEAK 34
LEAK 35
LEAK 36
LEAK 37
LEAK 38
LEAK 39
LEAK 40
LEAK 41
LEAK 42
LEAK 43
LEAK 44
LEAK 45
LEAK 46
LEAK 47
LEAK 48
LEAK 49
LEAK 50
LEAK 51
LEAK 52
LEAK 53
LEAK 54
LEAK 55
LEAK 56
LEAK 57
LEAK 58
LEAK 59
LEAK 60
LEAK 61
LEAK 62

```

```

|| SUBROUTINE LEAKY (U,ROVB,WURB)
DIMENSION H(16), A(16)
IF (U.GT.10.) WURB=0.
IF (U.LE.10.) GO TO 10
IF (ROVB.EQ.0.) GOTO 70
GO TO 60
10 A(1)=-.9894009
A(2)=-.9445750
A(3)=-.8656312
A(4)=-.7554044
A(5)=-.6178762
A(6)=-.4580168
A(7)=-.2816036
A(8)=-.0950125
H(1)=.0271525
H(2)=.0622535
H(3)=.0951585
H(4)=.1246290
H(5)=.1495960
H(6)=.1691565
H(7)=.1826034
H(8)=.1894506
H(9)=H(8)
H(10)=H(7)
H(11)=H(6)
H(12)=H(5)
H(13)=H(4)
H(14)=H(3)
H(15)=H(2)
H(16)=H(1)
A(9)=-A(8)
A(10)=-A(7)
A(11)=-A(6)

```

```

LEAK 1
LEAK 2
LEAK 3
LEAK 4
LEAK 5
LEAK 6
LEAK 7
LEAK 8
LEAK 9
LEAK 10
LEAK 11
LEAK 12
LEAK 13
LEAK 14
LEAK 15
LEAK 16
LEAK 17
LEAK 18
LEAK 19
LEAK 20
LEAK 21
LEAK 22
LEAK 23
LEAK 24
LEAK 25
LEAK 26
LEAK 27
LEAK 28
LEAK 29
LEAK 30
LEAK 31
LEAK 32
LEAK 33

```

E. EXAMPLES

As examples, some of the data from a series of pump tests near Esterhazy, Sask. are analyzed. The hydrology of the area has been described by Vonhof (1975) and Bourne (1976); for the purpose of this report it is sufficient to state that all the wells are drilled and completed in a 425 m wide preglacial valley, filled with up to 60 m of sand and gravel and buried under 15 m of till (Figs. 4 and 5). Table 2 gives the pertinent data for the pumped well, P, and the two observation wells, O_1 and O_2 .

Drawdown data from O_1 are plotted in Figure 6. Since $x/D > 1$, the type curve of $F(u, x/B)$ can be used for the analysis. The reader may wish to take Plate 1 and go through the matching process to verify that a good fit can be obtained with the curve for which $x/B = 0.4$, thus obtaining

$$B = 580/0.4 = 1450 \text{ m.}$$

The four match-point coordinates selected are

$$F(u) = 1, u = 1, s = 0.125 \text{ m}, t = 56 \text{ min}$$

Thus from equations (6) and (7), respectively

$$T = .227 \times 580 \times 1/(2 \times 0.125 \times 425) = 1.24 \text{ m}^2/\text{min}$$

$$S = 4 \times 1.24 \times 56 \times 1/(580^2) = 8.3 \times 10^{-4}$$

Drawdown in O_2 has been plotted in Figure 7. Since $x/D = 380/425 = .89$ is less than 1, the use of the function $F(u, x/B)$ will give erroneous results. Since y_p/D is very nearly 1/3 and y_o/D is reasonably close to zero, the configuration is intermediate between those of Plate 10 ($x/D = 2/3$) and Plate 1 ($x/D > 1$). Results of curve matching with both of these are presented in Table 3 and may be verified by the reader.

As an example of the use of program CURVES, the family of type curves for the exact well configuration of P and O_2 in the buried channel aquifer has been plotted (Plate 18). It may be noted that the input values - D, X0, Y0, YP - shown on Plate 18 are not the same as those given in Table 2; the reason for this is that the input values were taken from a scaled map; however, the ratios X0/D, Y0/D and YP/D are the same as the ratios x/D, y_o/D and y_p/D .

Results of curve matching of the drawdown data from O_2 with Plate 18 are given in Table 3 and it is seen that the transmissivity so calculated lies between the values

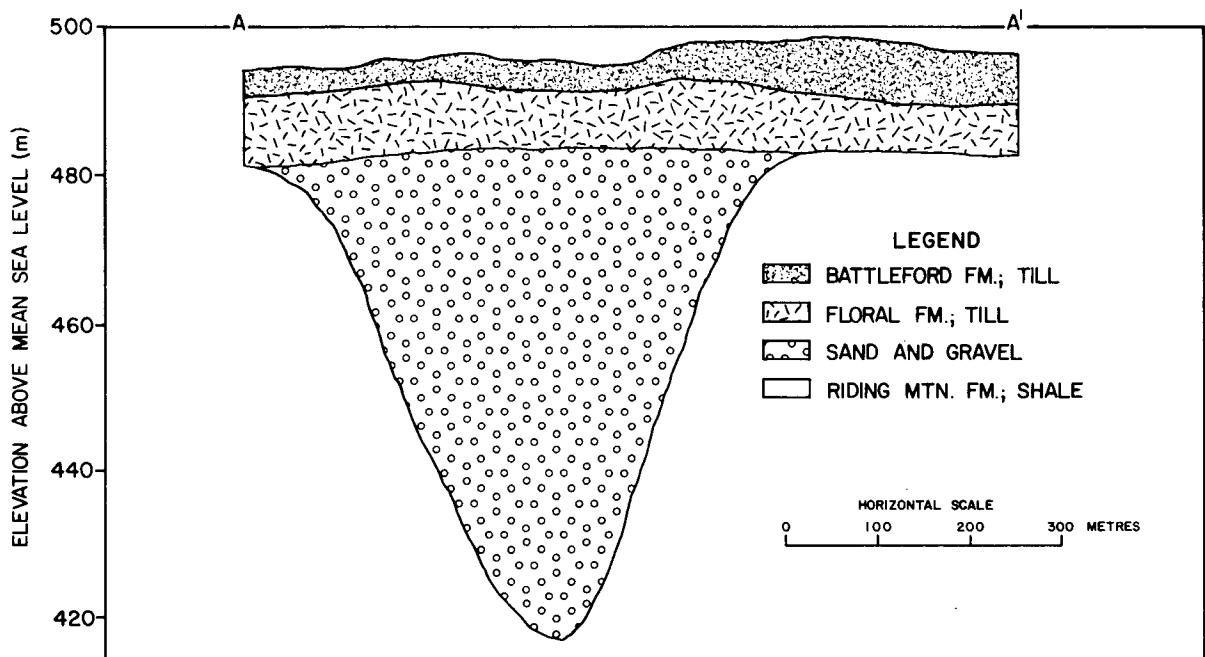


Figure 4. Cross section of buried bedrock channel near Esterhazy, Saskatchewan (after Bourne, 1976).

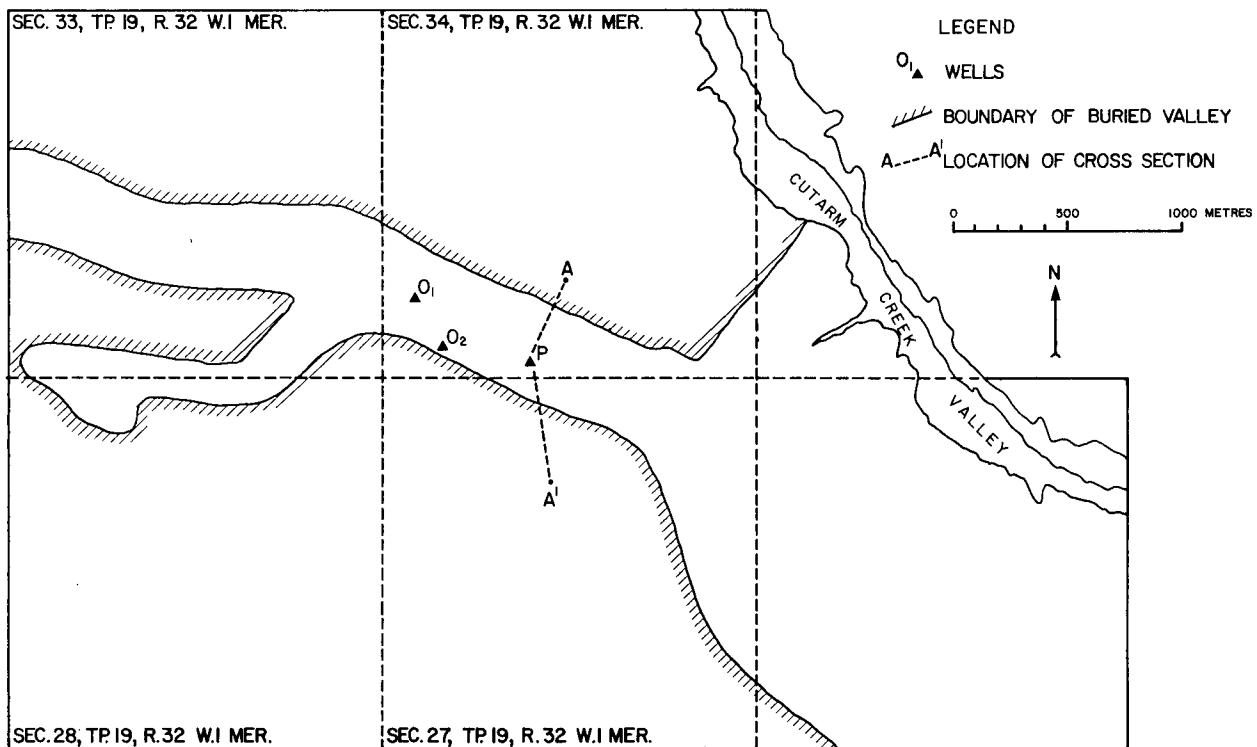


Figure 5. Map of buried bedrock channel near Esterhazy, Saskatchewan (after Bourne, 1976).

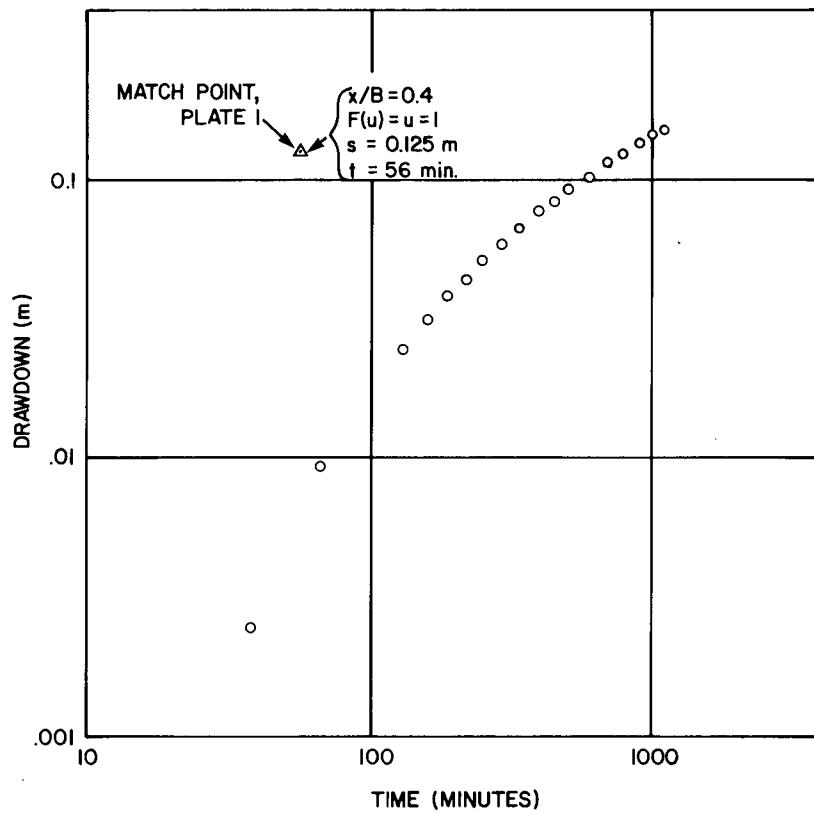


Figure 6. Time-drawdown curve for observation well O_1 .

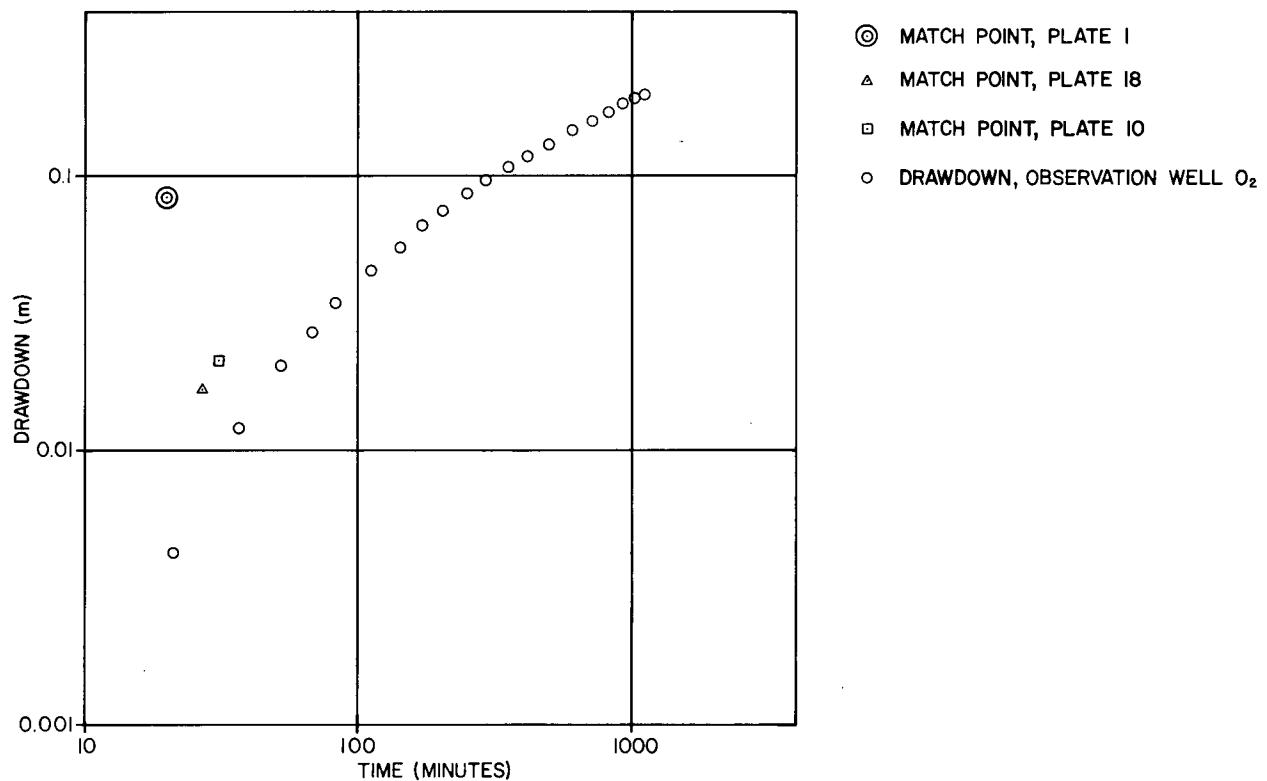


Figure 7. Time-drawdown curve for observation well O_2 .

determined with the aid of Plates 1 and 10. From the values of B, T and b' (thickness of the till = 15 m) the vertical hydraulic conductivity of the till can be determined:

$$K' = Tb'/B^2$$

Thus from the data of O_1 :

$$K' = 1.24 \times 15/(1450)^2 = 8.8 \times 10^{-6} \text{ m/min}$$

and from the data of O_2 :

$$K' = 1.06 \times 15/(1330)^2 = 9.0 \times 10^{-6} \text{ m/min}$$

These values fall in the range of values compiled by Bourne (1976) from various sources for tills of the Canadian Prairies (9×10^{-6} m/min equals .26 Imperial gallons per day per square foot). No effort was made by Bourne (1976) to determine leakage or hydraulic conductivity of the till from the pump-test data.

F. CONCLUSION

The technique outlined in this report, together with the table of function values and the type curves, makes it possible to analyze time-drawdown data to obtain values of transmissivity, storativity and leakage factor from any pumped well-observation well configuration in a channel aquifer by a simple type curve matching procedure. Heretofore, these parameters could only be determined by a cumbersome trial and error method.

G. REFERENCES

- Bourne, D.R., 1976. An electric analog simulation of ground water flow patterns at a potash waste disposal pond located near Esterhazy, Saskatchewan. Unpublished M. Sc. Thesis, U. of British Columbia, Vancouver.
- Bukhari, S.A., A. Vandenberg, and D.H. Lennox, 1969. Iterative analysis: bounded leaky artesian aquifer. Journal of the Irrigation and Drainage Division, Proc. A.S.C.E., Vol. 95, No. IR1, Proc. Paper 6431, pp. 1-14.
- McKinley, R.M., S. Vela and L.A. Carlton, 1968. A field application of pulse testing for detailed reservoir description. Journal of Petroleum Technology, Vol. 20, No. 3, pp. 313-321.
- Vandenberg, A., 1976. Type curves for analysis of pump tests in leaky strip aquifers. Journal of Hydrology. In Press.
- Vonhof, J.A., 1975. Waste disposal problems near potash mines in Saskatchewan, Canada. Proc. Moscow Symposium, August 1971, IASH-AISH Publ. No. 103, p. 210.

Tables 1 to 3

Table 1. Values of the function $F(u, x/B)$.

U	x/B	.00000	.00100	.00200	.00300	.00400	.00500	.00600	.00700	.00800	.00900
.000001	563.19015	519.50058	420.35145	321.03656	247.83256	198.92110	165.66598	141.86053	124.00399	110.11560	
.000002	397.94308	381.92582	340.34591	287.79676	237.52695	196.51863	165.21969	141.79417	123.99667	110.11484	
.000003	324.73598	315.90966	291.89320	258.77784	223.38444	190.75715	163.28535	141.25110	123.86763	110.08909	
.000004	281.09592	275.32759	259.25134	236.05359	209.67730	183.58256	160.02053	139.95659	123.41927	109.95307	
.000005	251.31451	247.17169	235.45698	218.07455	197.52635	176.23331	156.03977	138.02398	122.57749	109.62376	
.000006	229.33081	226.17145	217.15017	203.50979	186.94894	169.21968	151.79230	135.67378	121.38883	109.07396	
.000007	212.24511	209.73352	202.51165	191.44259	177.73992	162.71281	147.53484	133.09370	119.94033	108.32039	
.000008	198.47274	196.41430	190.46420	181.25045	169.67470	156.74279	143.40071	130.41523	118.31649	107.39906	
.000009	187.06489	185.33402	180.32593	172.50206	162.55774	151.28418	139.45328	127.72356	116.58573	106.34953	
.00001	177.41419	175.93855	171.64153	164.89052	156.22911	146.29230	135.71776	125.07148	114.79927	105.20771	
.00002	125.15915	124.63549	123.08783	120.58403	117.23069	113.16421	108.53995	103.52083	98.26640	92.92371	
.00003	102.00954	101.72415	100.87648	99.49153	97.60934	95.28270	92.57430	89.55333	86.29215	82.86294	
.00004	88.20977	88.02430	87.47202	86.56521	85.32370	83.77408	81.94851	79.88345	77.61822	75.19359	
.00005	78.79244	78.65968	78.26378	77.61178	76.71517	75.58946	74.25364	72.72959	71.04134	69.21435	
.00006	71.84093	71.73391	71.43837	70.94079	70.25449	69.38944	68.35793	67.17427	65.85437	64.41533	
.00007	66.43827	66.35810	66.11860	65.72283	65.17581	64.48437	63.65699	62.70364	61.63546	60.46458	
.00008	62.09336	62.01773	61.82158	61.49710	61.04788	60.47887	59.79521	59.00715	58.11985	57.14329	
.00009	58.47616	58.42115	58.25668	57.98439	57.61696	57.12809	56.55240	55.88535	55.13314	54.30262	
.0001	55.42460	55.37763	55.23715	55.00442	54.68151	54.27126	53.77726	53.20375	52.55555	51.83803	
.0002	38.90221	38.88560	38.83585	38.75319	38.63799	38.49075	38.31214	38.10292	37.86403	37.59649	
.0003	31.58327	31.57423	31.54715	31.50209	31.43920	31.35867	31.26073	31.14568	31.01384	30.86561	
.0004	27.22076	27.21489	27.19730	27.16802	27.12712	27.07469	27.01086	26.93575	26.84954	26.75242	
.0005	24.24394	24.23974	24.22715	24.20620	24.17691	24.13935	24.09359	24.03969	23.97776	23.90791	
.0006	22.04676	22.04357	22.03399	22.01805	21.99577	21.96717	21.93231	21.89124	21.84401	21.79069	
.0007	20.33929	20.33675	20.32916	20.31651	20.29882	20.27612	20.24843	20.21580	20.17825	20.13585	
.0008	19.96307	18.96160	18.95478	18.94443	18.92995	18.91136	18.88869	18.86196	18.83119	18.79642	
.0009	17.82324	17.82150	17.81630	17.80762	17.79549	17.77991	17.76090	17.73849	17.71268	17.68351	
.001	16.85908	16.85760	16.85315	16.84575	16.83539	16.82209	16.80585	16.78670	16.76465	16.73972	
.002	11.64088	11.64036	11.63879	11.63618	11.63253	11.62783	11.62209	11.61532	11.60751	11.59868	
.003	9.33153	9.33125	9.33048	9.32898	9.32699	9.32444	9.32133	9.31765	9.31340	9.30860	
.004	7.95628	7.95609	7.95554	7.95462	7.95334	7.95169	7.94967	7.94728	7.94453	7.94141	
.005	7.01871	7.01857	7.01818	7.01753	7.01661	7.01543	7.01399	7.01228	7.01032	7.00810	
.006	6.32731	6.32721	6.32692	6.32642	6.32572	6.32483	6.32373	6.32244	6.32095	6.31926	
.007	5.79050	5.79042	5.79019	5.78979	5.78924	5.78853	5.78767	5.78665	5.78547	5.78413	
.008	5.35823	5.35816	5.35797	5.35765	5.35720	5.35662	5.35591	5.35508	5.35411	5.35302	
.009	5.00052	5.00047	5.00031	5.00004	4.99966	4.99918	4.99859	4.99789	4.99709	4.99617	
.01	4.69822	4.69817	4.69804	4.69781	4.69749	4.69708	4.69657	4.69598	4.69529	4.69452	
.02	3.06895	3.06893	3.06888	3.06880	3.06869	3.06855	3.06838	3.06817	3.06794	3.06767	
.03	2.35458	2.35458	2.35455	2.35451	2.35445	2.35437	2.35428	2.35417	2.35405	2.35390	
.04	1.93304	1.93303	1.93302	1.93299	1.93295	1.93291	1.93285	1.93278	1.93270	1.93261	
.05	1.64825	1.64824	1.64823	1.64821	1.64819	1.64815	1.64811	1.64806	1.64801	1.64795	
.06	1.44013	1.44012	1.44011	1.44010	1.44008	1.44006	1.44003	1.43999	1.43995	1.43990	
.07	1.27999	1.27999	1.27998	1.27997	1.27995	1.27993	1.27991	1.27988	1.27985	1.27981	
.08	1.15219	1.15219	1.15219	1.15218	1.15217	1.15215	1.15213	1.15211	1.15208	1.15205	
.09	1.04739	1.04739	1.04739	1.04738	1.04737	1.04736	1.04734	1.04732	1.04730	1.04728	
.1	.95962	.95962	.95962	.95961	.95960	.95959	.95958	.95956	.95954	.95952	
.2	.50579	.50579	.50579	.50579	.50579	.50578	.50578	.50577	.50577	.50576	
.3	.32451	.32451	.32451	.32451	.32451	.32451	.32451	.32450	.32450	.32450	
.4	.22687	.22687	.22687	.22687	.22687	.22687	.22687	.22687	.22687	.22687	
.5	.16663	.16663	.16663	.16663	.16663	.16663	.16663	.16663	.16663	.16663	
.6	.12641	.12641	.12641	.12641	.12641	.12641	.12641	.12641	.12641	.12641	
.7	.09814	.09814	.09814	.09814	.09814	.09814	.09814	.09814	.09814	.09814	
.8	.07752	.07752	.07752	.07752	.07752	.07752	.07752	.07752	.07752	.07752	
.9	.06208	.06208	.06208	.06208	.06208	.06208	.06208	.06208	.06208	.06208	
1.	.05025	.05025	.05025	.05025	.05025	.05025	.05025	.05025	.05025	.05025	
2.	.00849	.00849	.00849	.00849	.00849	.00849	.00849	.00849	.00849	.00849	
3.	.00191	.00191	.00191	.00191	.00191	.00191	.00191	.00191	.00191	.00191	
4.	.00049	.00049	.00049	.00049	.00049	.00049	.00049	.00049	.00049	.00049	
5.	.00013	.00013	.00013	.00013	.00013	.00013	.00013	.00013	.00013	.00013	
6.	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	.00004	
7.	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	.00001	

***** .00001 .00001 .00001
K1606E0 //// END OF LIST ////
***** K1606E0 //// END OF LIST ////

Table 2. Pertinent data for the Esterhazy pump tests.

	P	o_1	o_2
Pumping rate (m^3/min)	.227		
Distance, y , to south boundary (m)	140	170	25
Distance, r , to pumped well (m)	-	580	400
Projection, x , of distance to pumped well along boundary (m)	-	580	380

Table 3. Drawdown data from Esterhazy well O₂; results of matching with Plate 1, Plate 10 and Plate 18.

	Match point coordinates					T (m ² /min)	S	B (m)
	F or Σ W	u	x/B or r/B	s(m)	t(min)			
Plate 1	1	1	0.25	0.084	19	.227 x 380 x 1/(2 x 0.084 x 425) = 1.21	4 x 1.21 x 19 x 1/(380 ²) = 6.4 x 10 ⁻⁴	380/0.25 = 1520
Plate 10	1	1	0.30	0.021	30	.227 x 1/(4 x π x 0.021) = 0.86	4 x 0.86 x 30 x 1/(400) ² = 6.45 x 10 ⁻⁴	400/0.3 = 1330
Plate 18	1	1	0.3	0.017	26	.227 x 1/(4 x π x 0.017) = 1.06	4 x 1.06 x 26 x 1/(400) ² = 6.9 x 10 ⁻⁴	400/0.3 = 1330

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The attached figures, printed on tracing paper, should replace Figures 6 and 7 in the pocket.

Figure 6. Time-drawdown curve for observation well O₁.

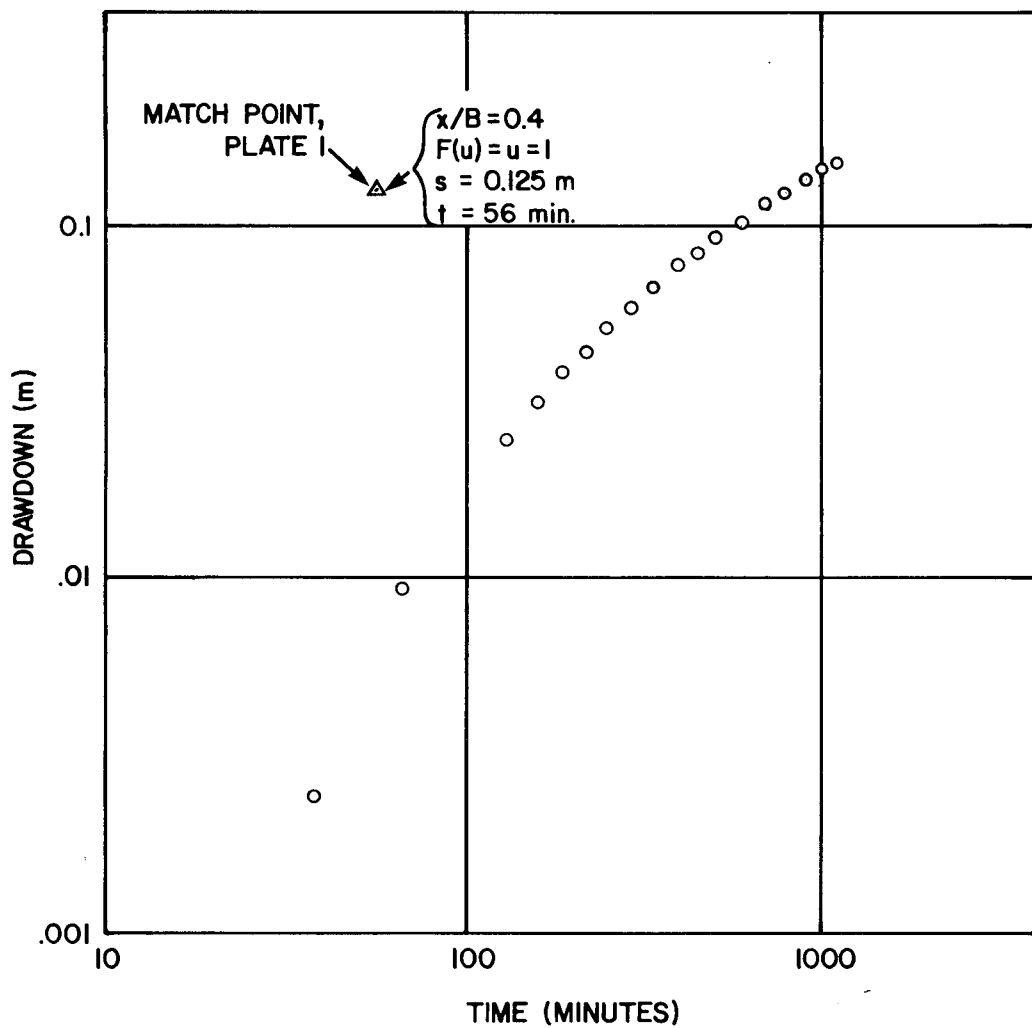


Figure 7. Time-drawdown curve for observation well O₂.

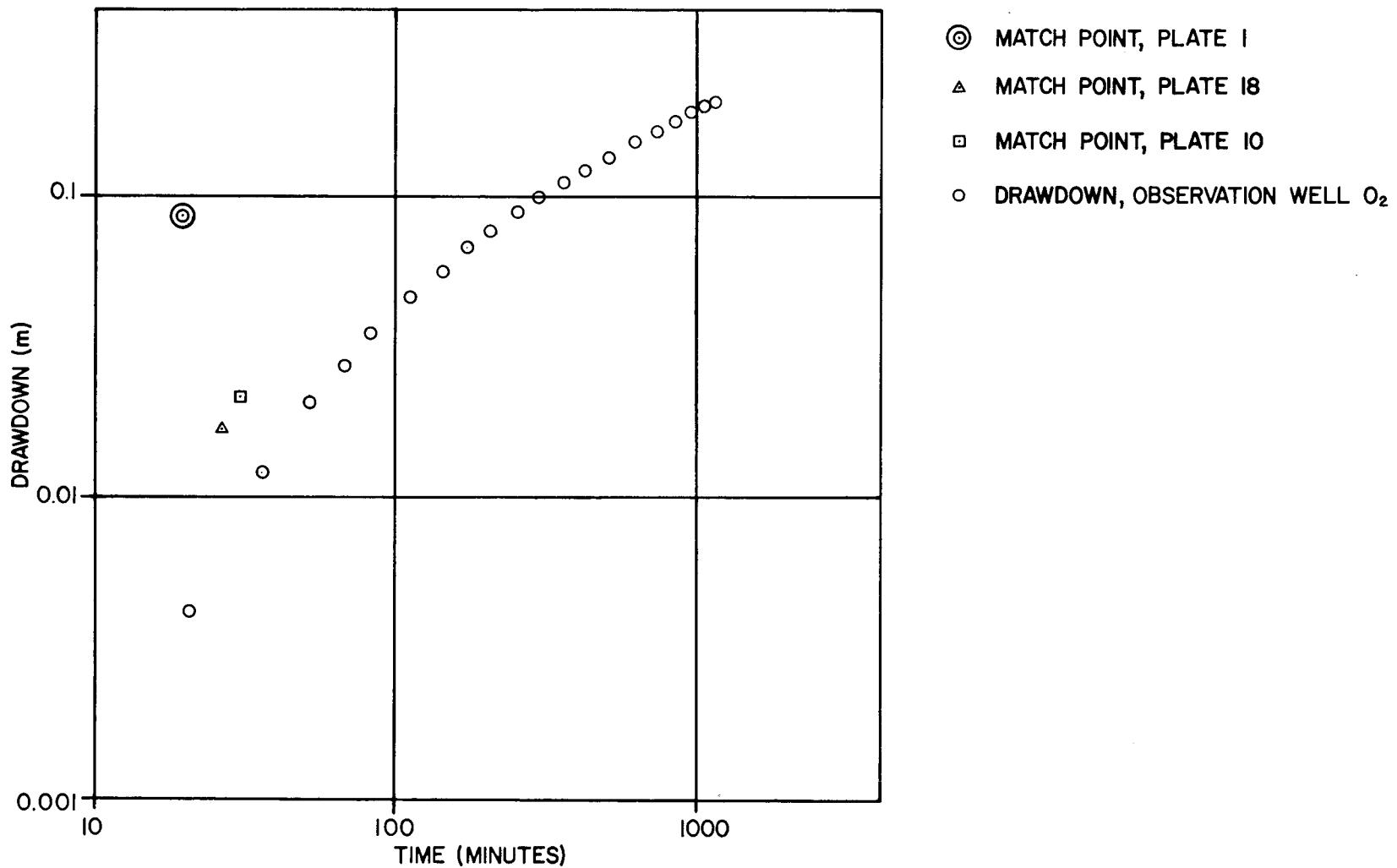


Figure 6. Time-drawdown curve for observation well O₁.

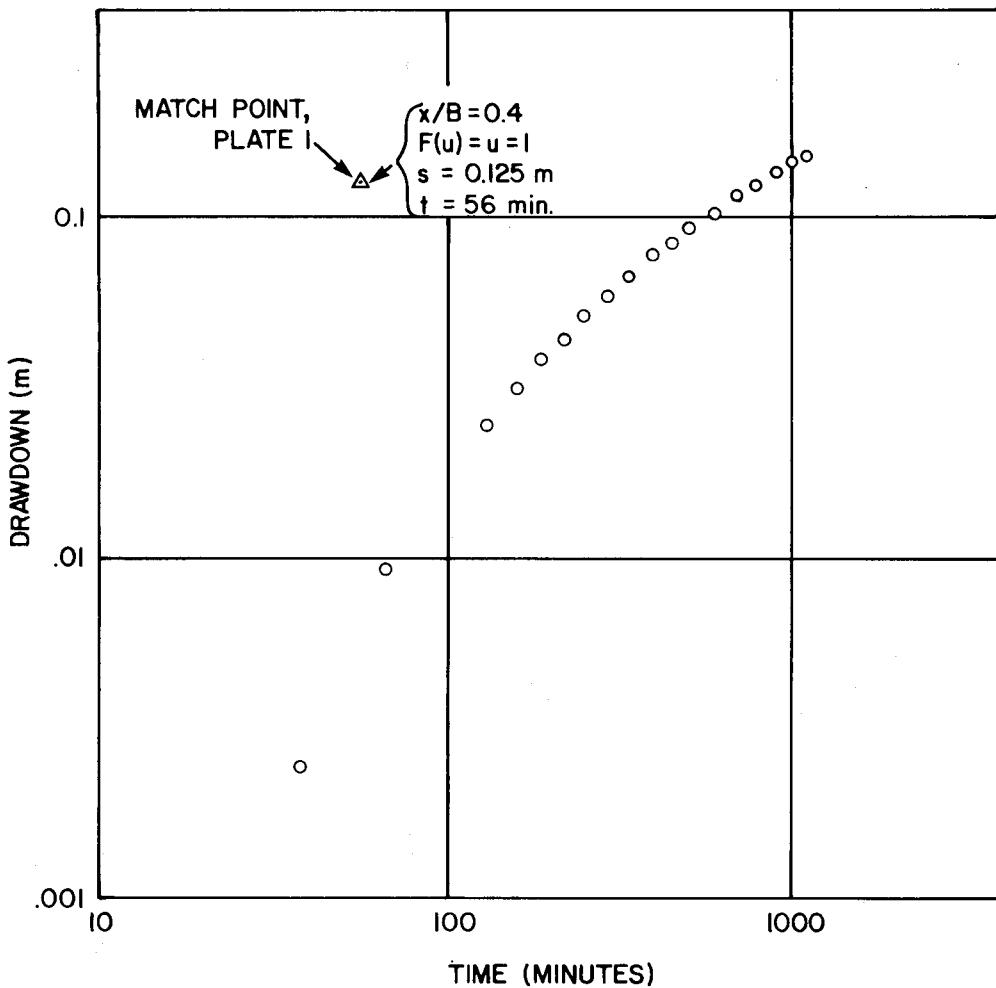


Figure 7. Time-drawdown curve for observation well O₂.

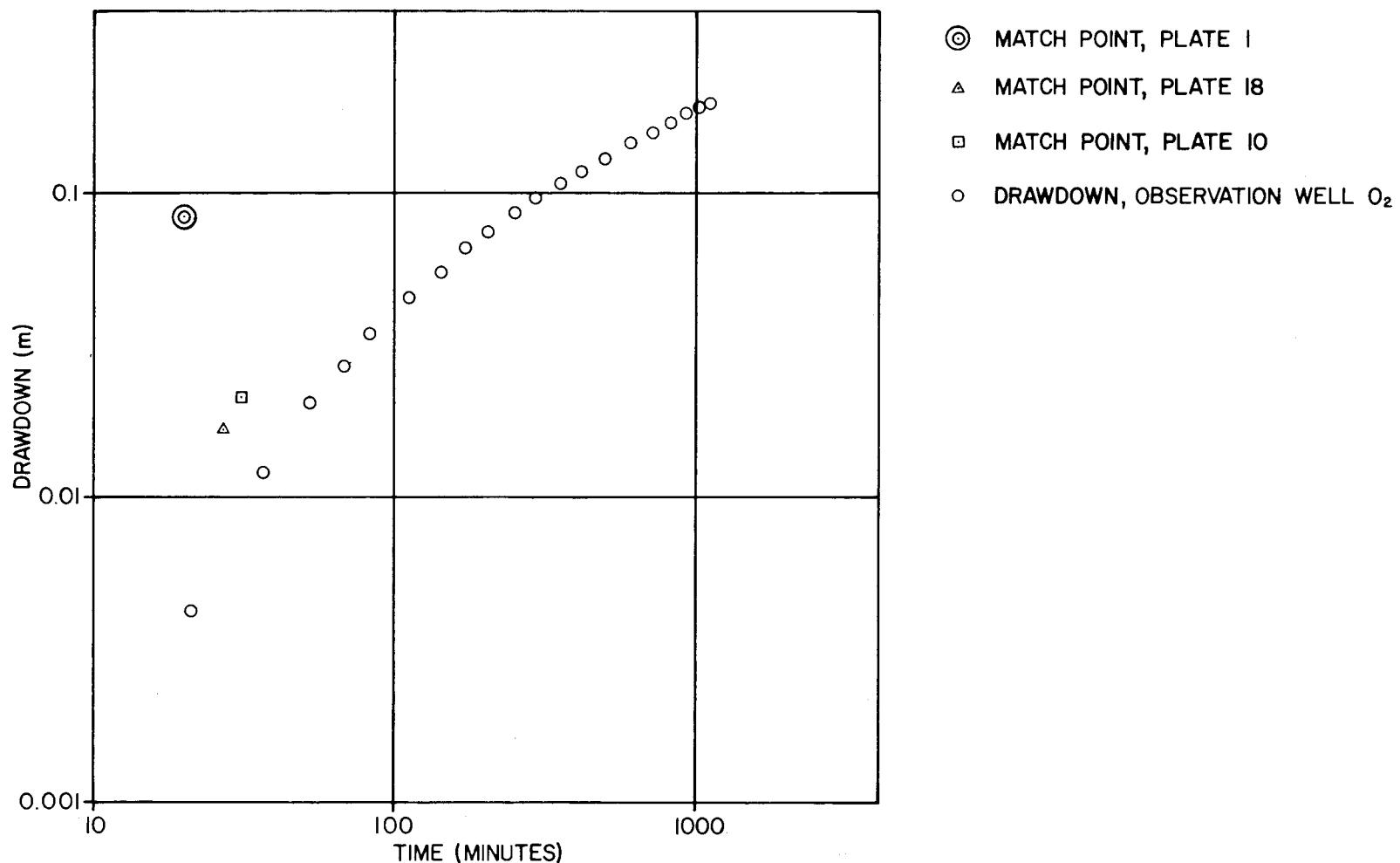


Plate 1. Family of type curves $F(u, x/B)$

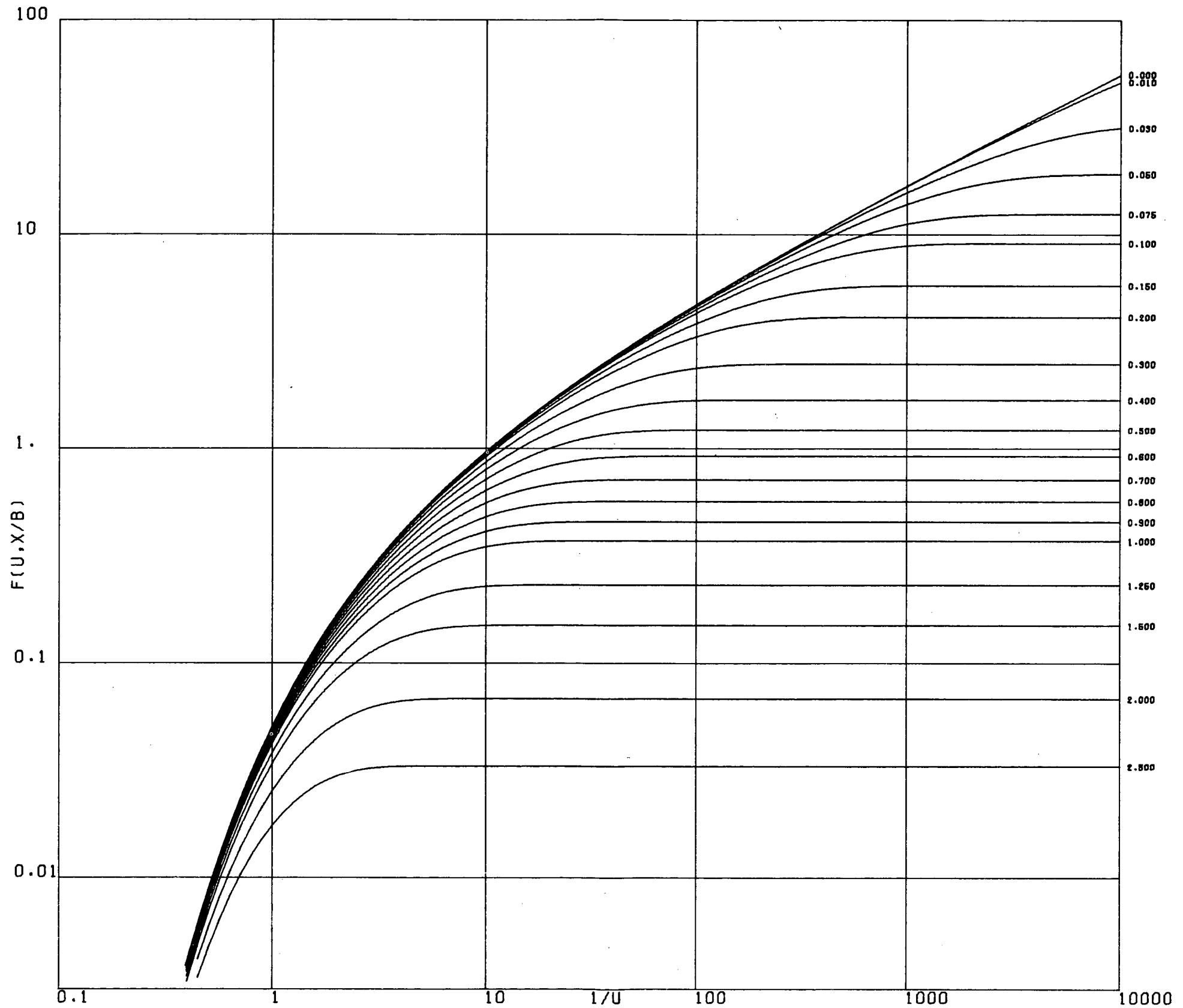


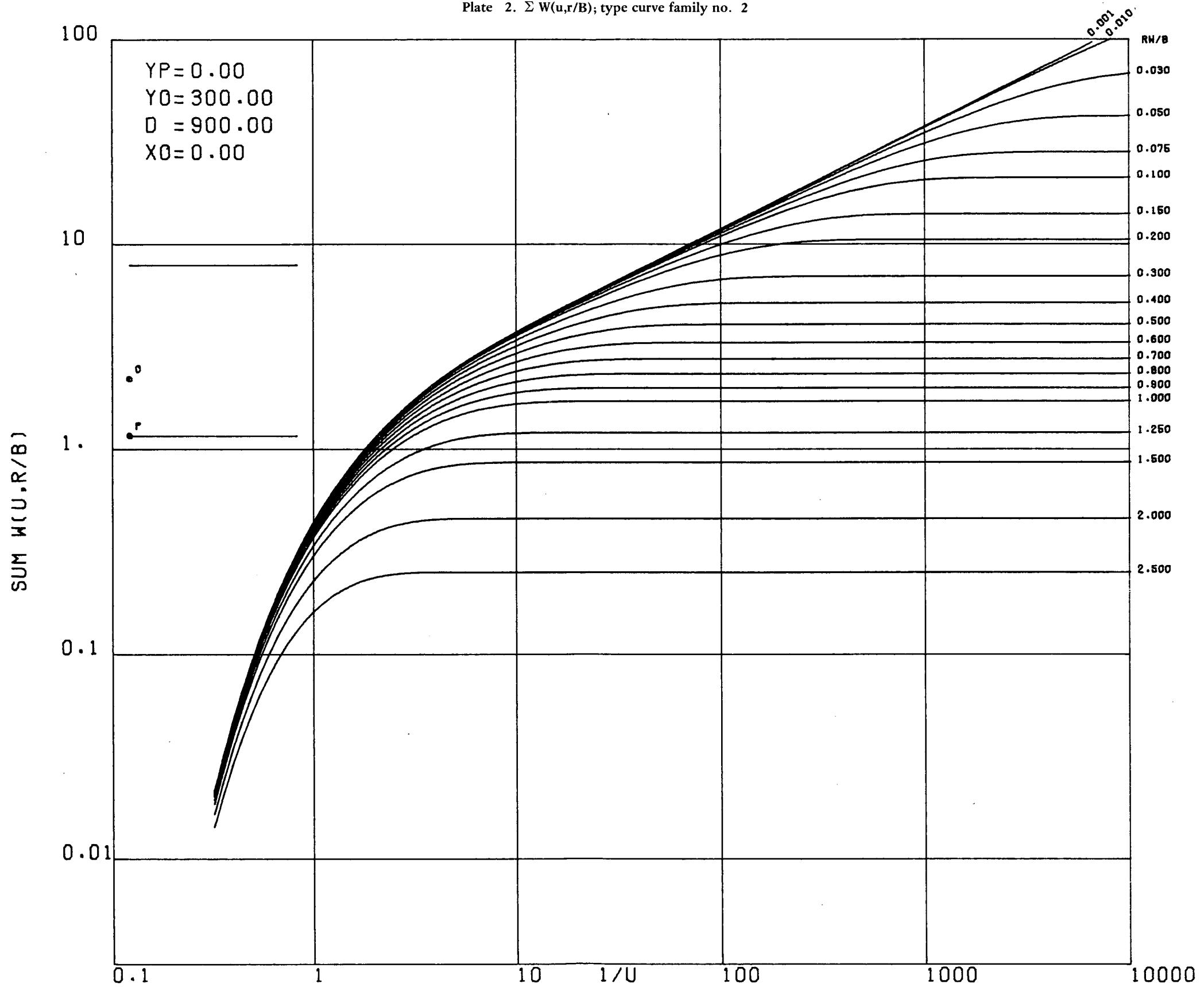
Plate 2. $\Sigma W(u,r/B)$; type curve family no. 2

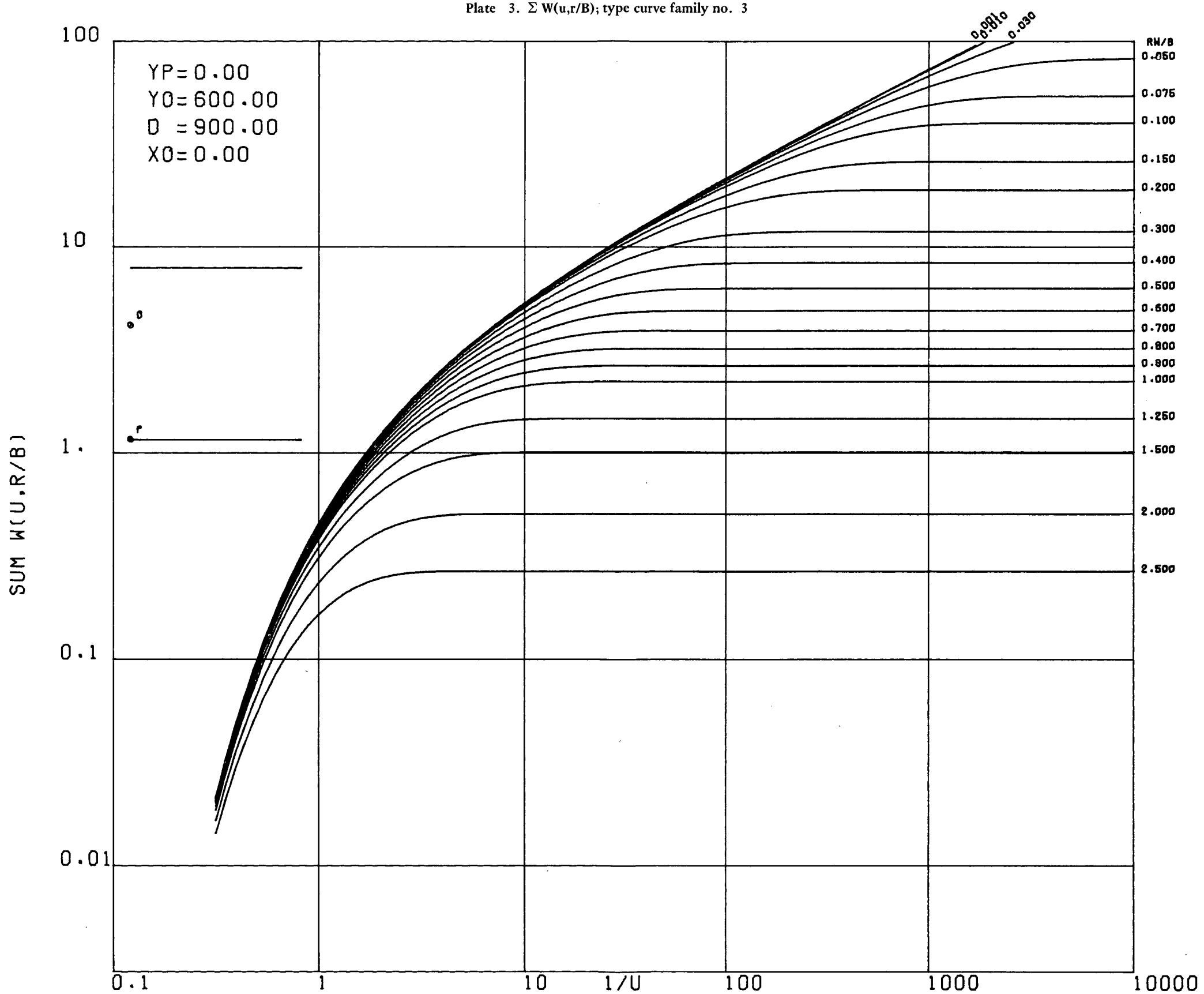
Plate 3. $\Sigma W(u,r/B)$; type curve family no. 3

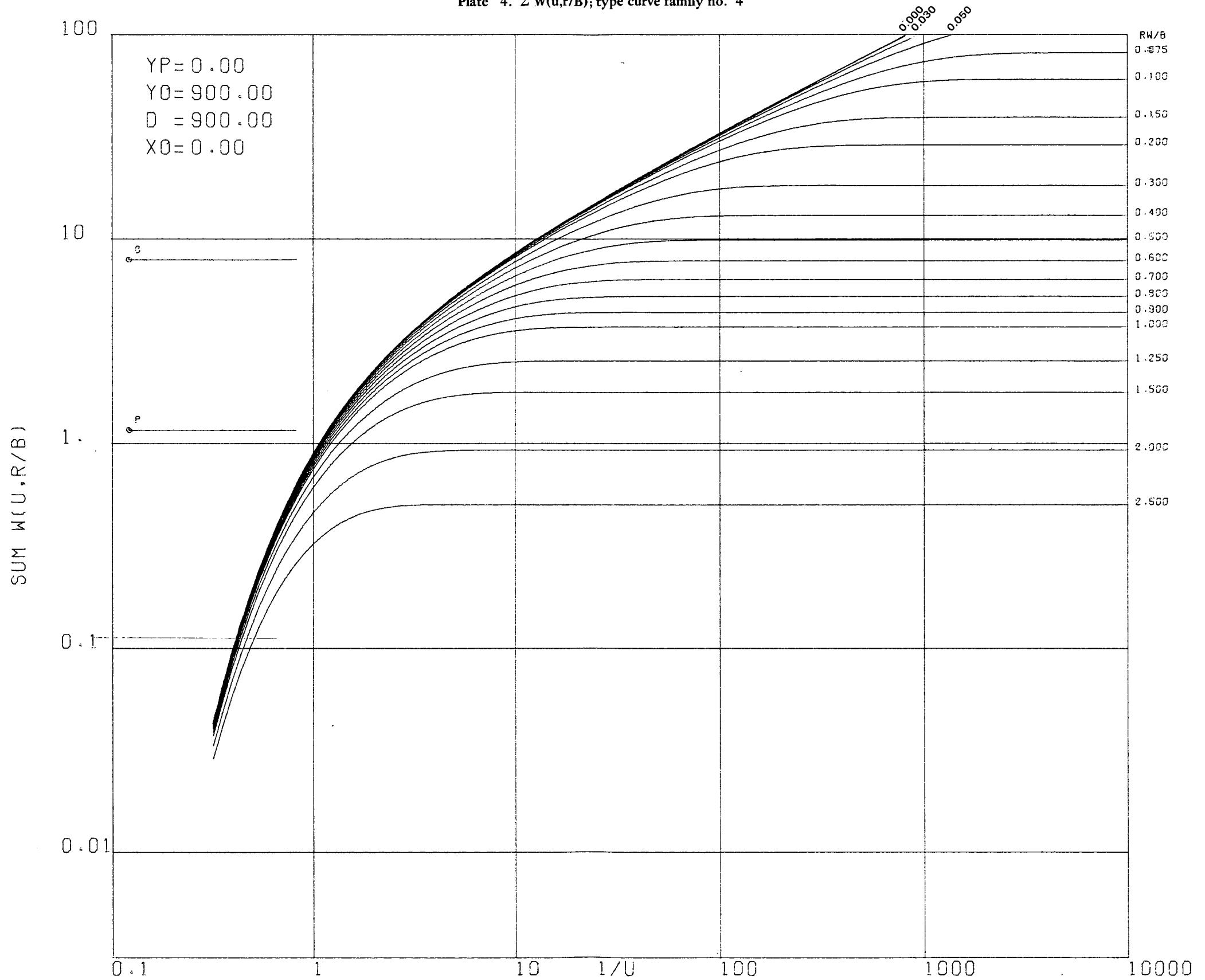
Plate 4. $\Sigma W(u, r/B)$; type curve family no. 4

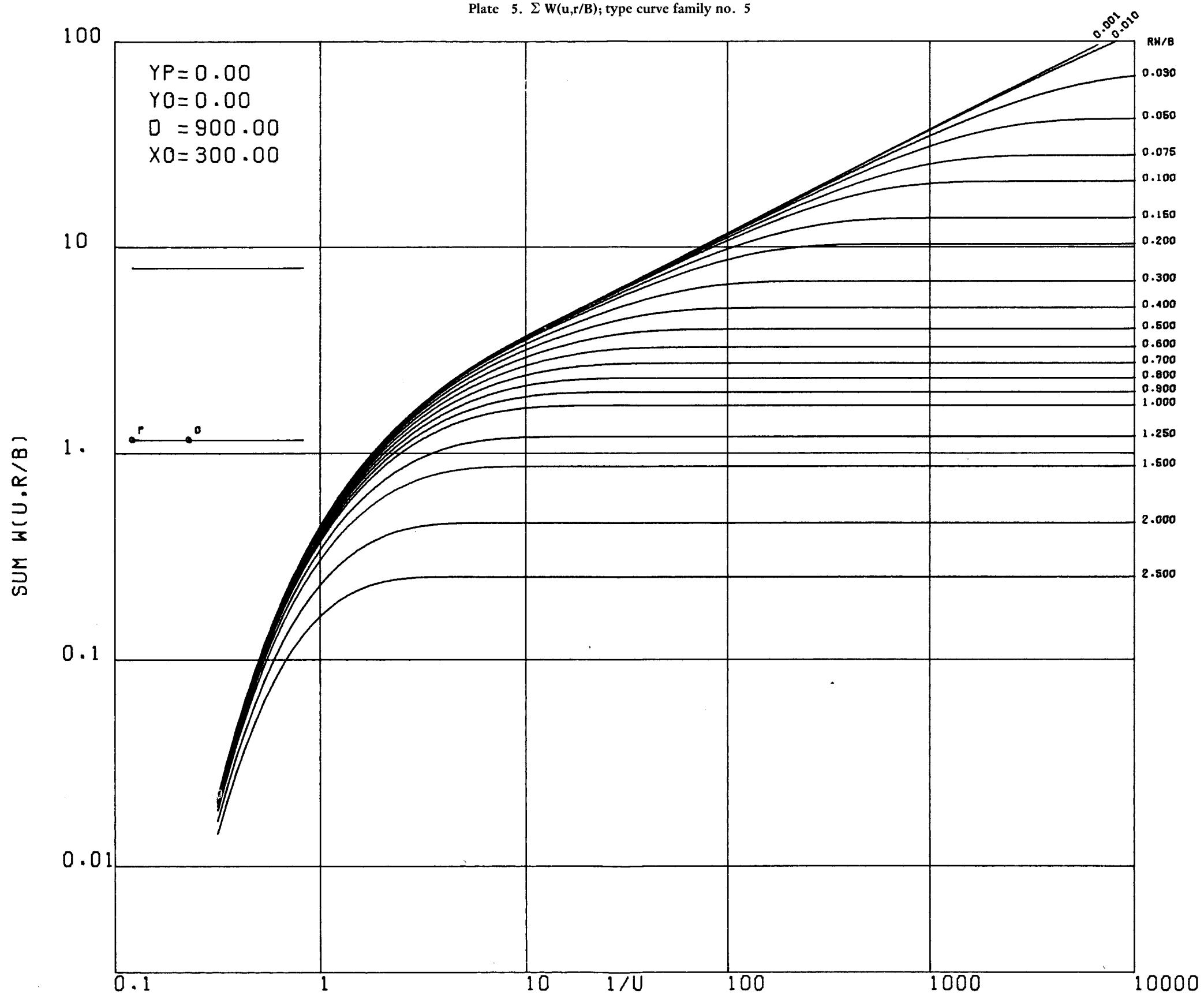
Plate 5. $\Sigma W(u, r/B)$; type curve family no. 5

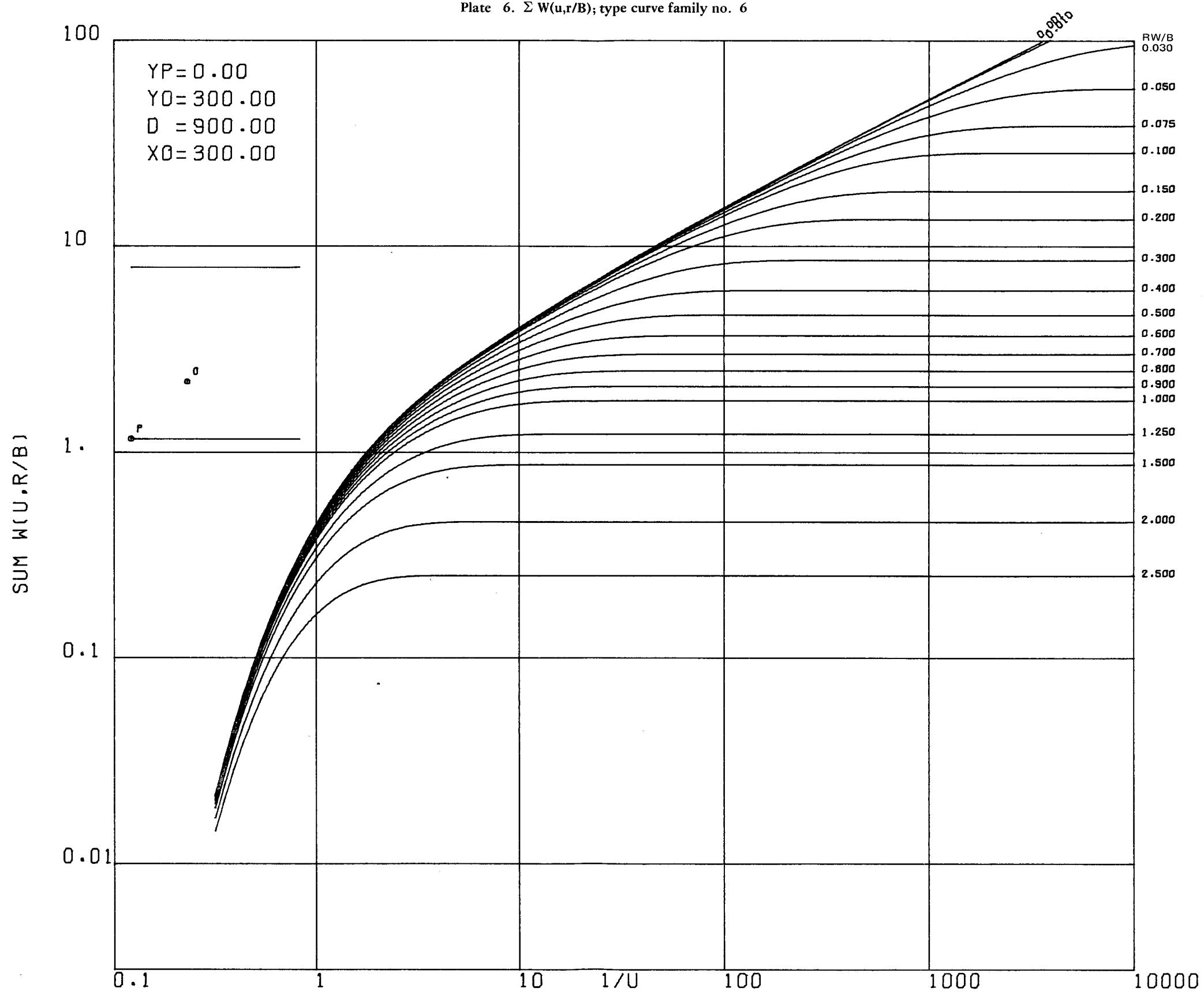
Plate 6. $\Sigma W(u, r/B)$; type curve family no. 6

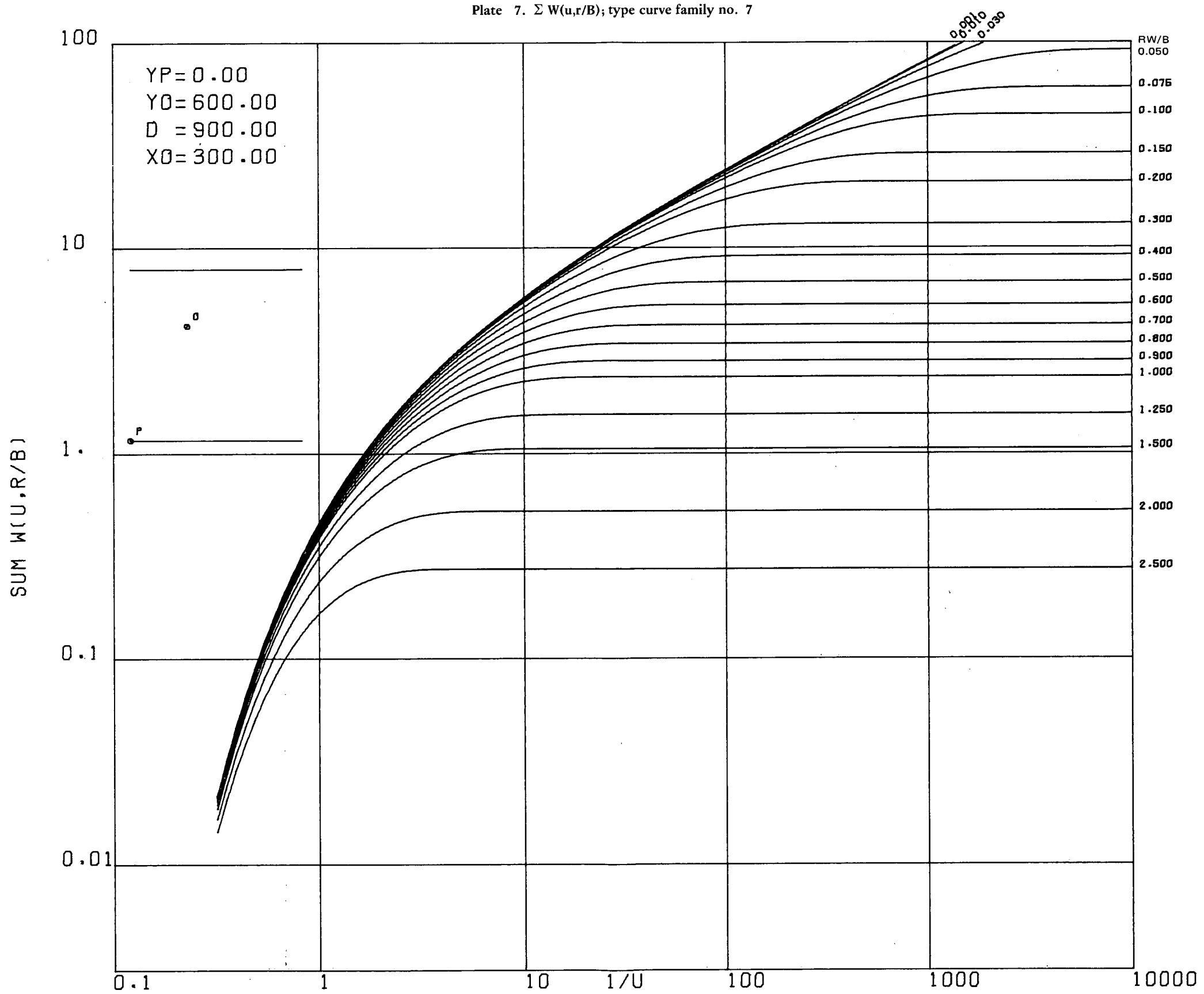
Plate 7. $\Sigma W(u,r/B)$; type curve family no. 7

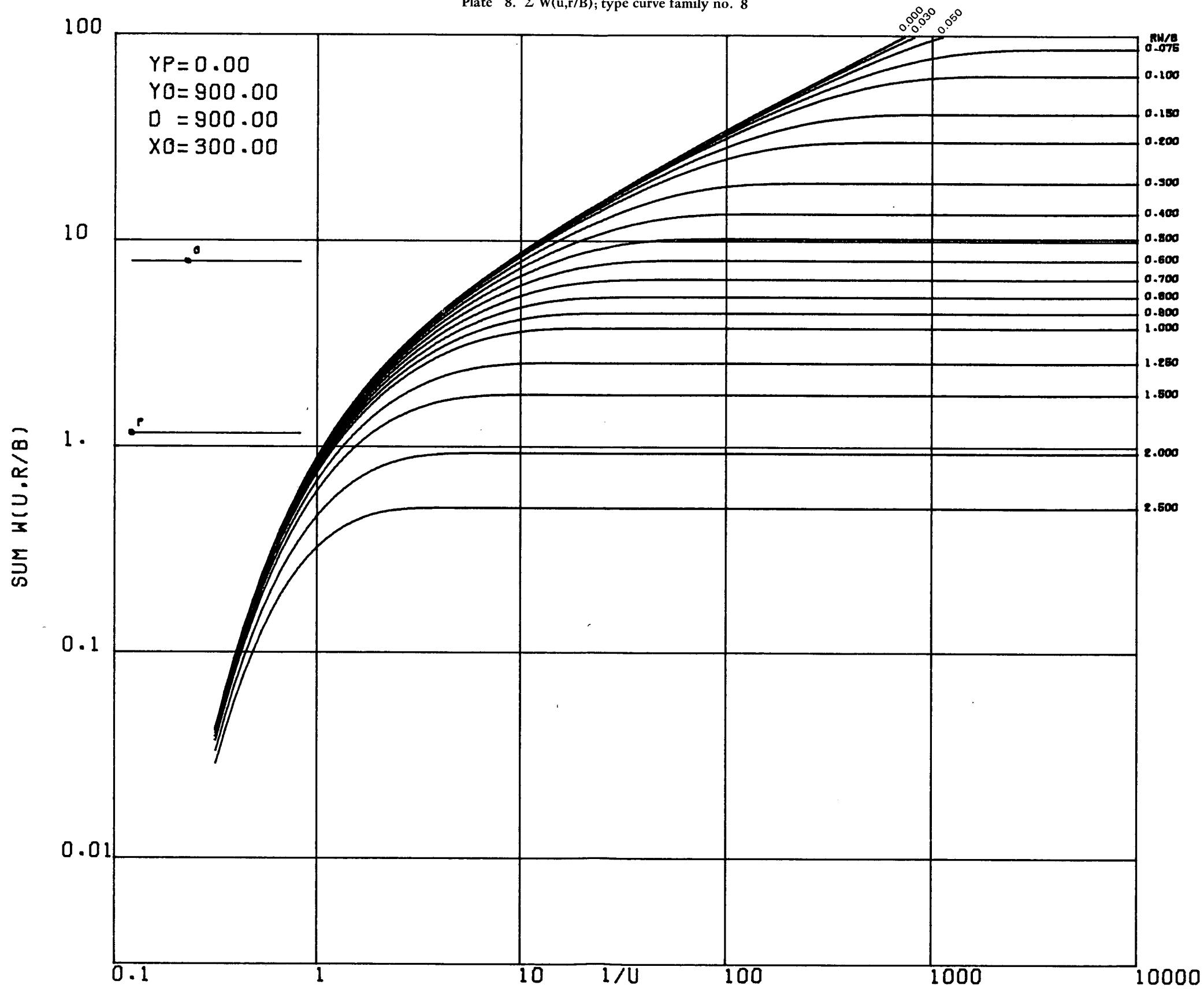
Plate 8. $\Sigma W(u, r/B)$; type curve family no. 8

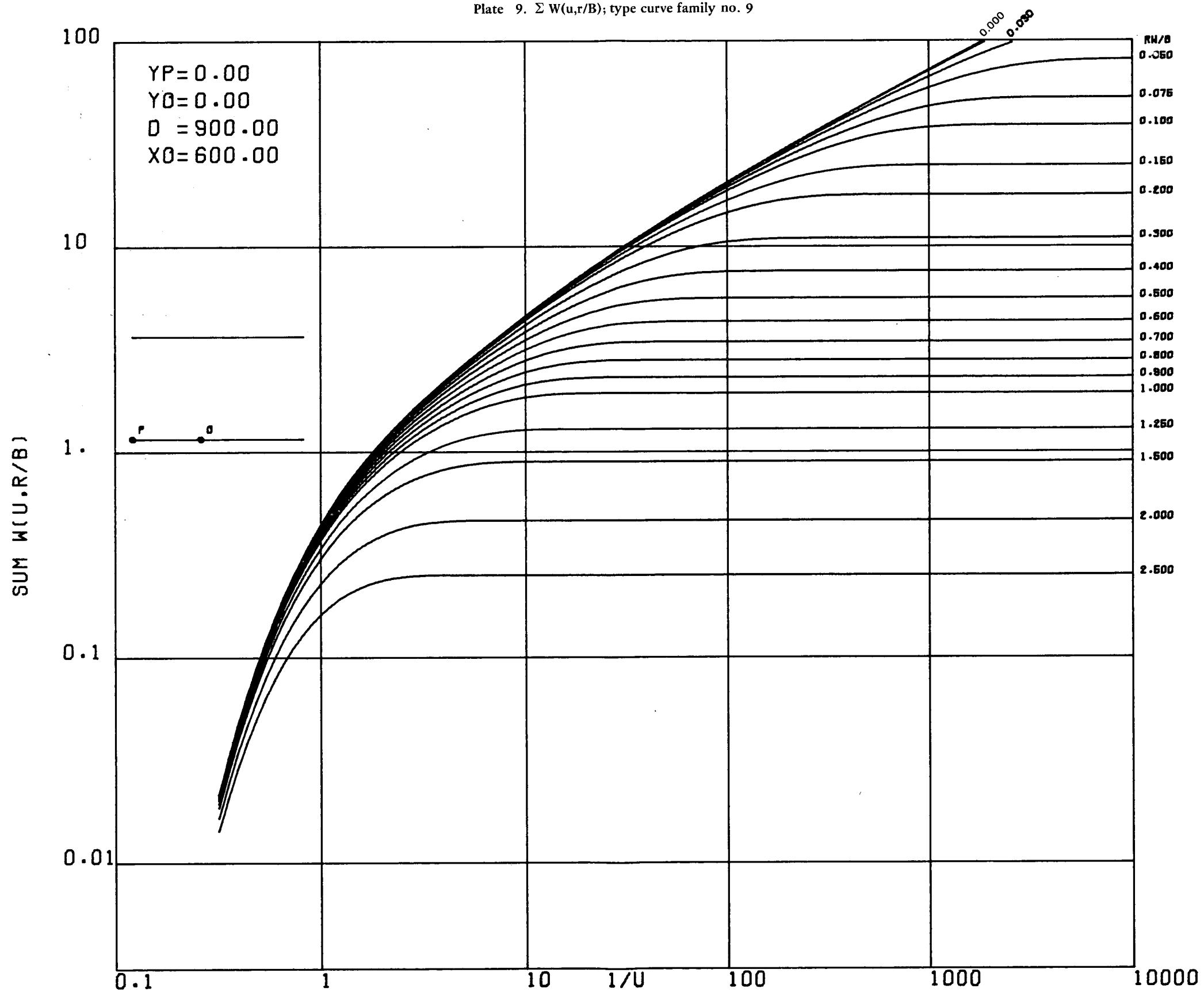
Plate 9. $\Sigma W(u,r/B)$; type curve family no. 9

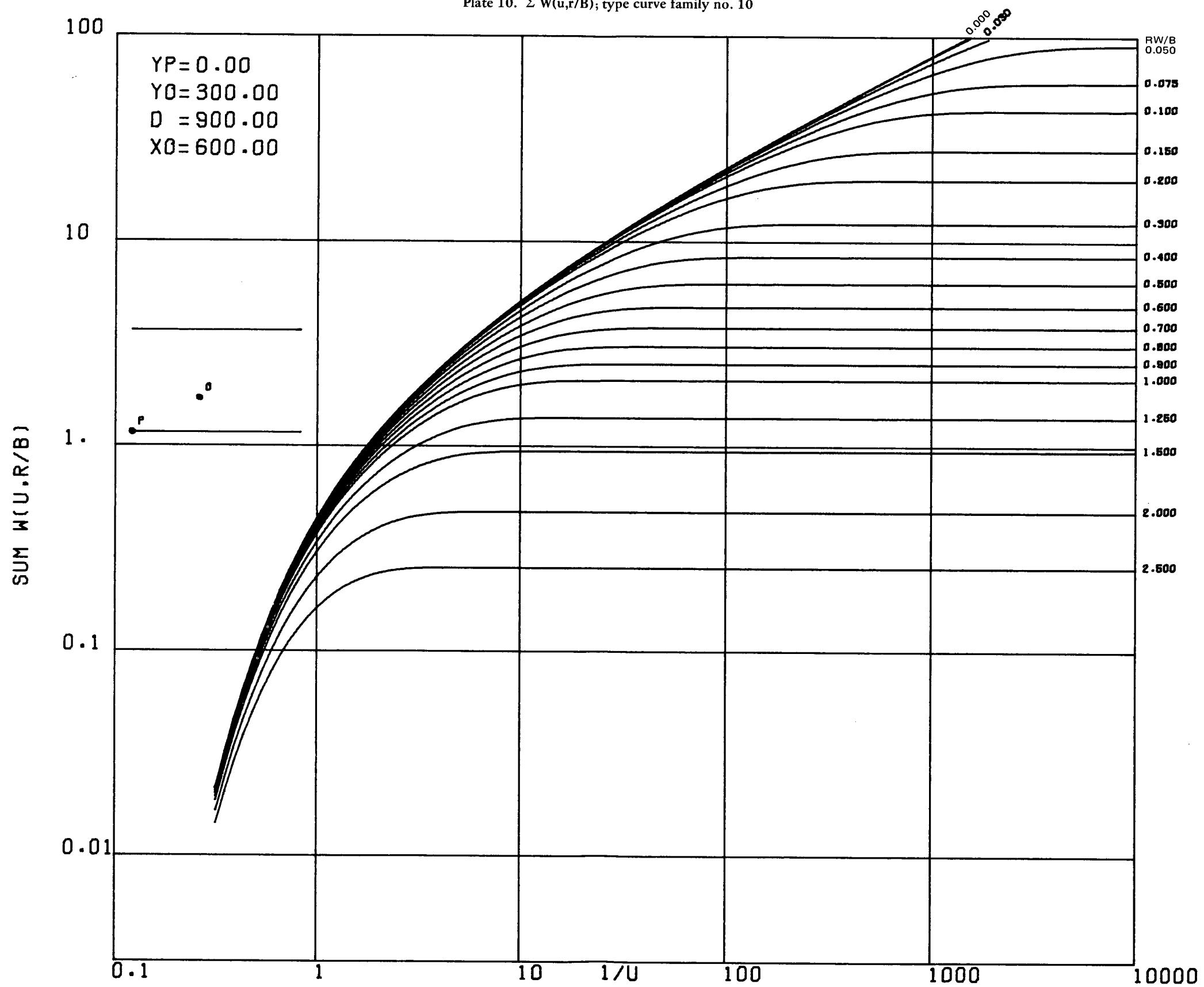
Plate 10. $\Sigma W(u, r/B)$; type curve family no. 10

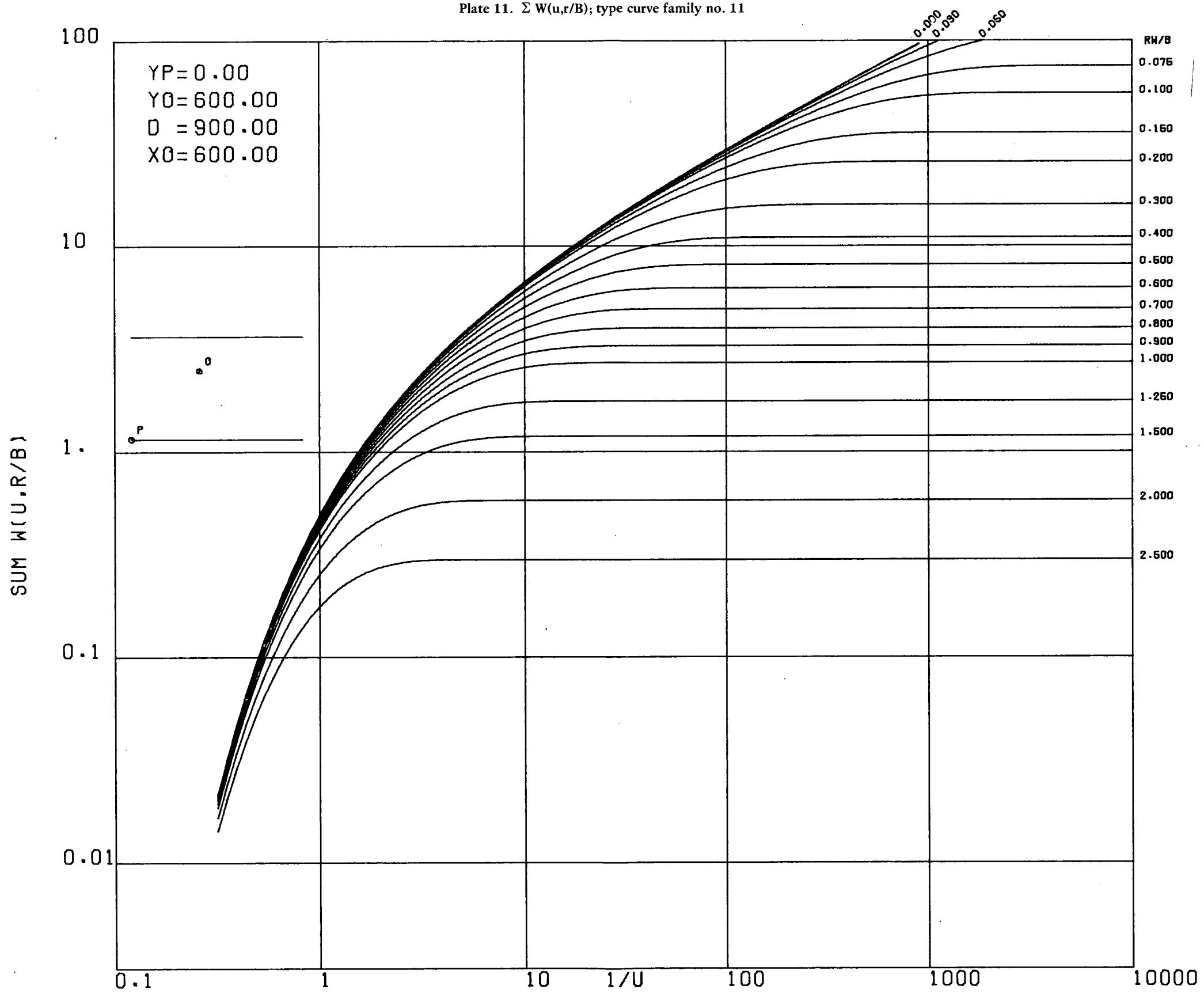
Plate 11. $\Sigma W(u,r/B)$; type curve family no. 11

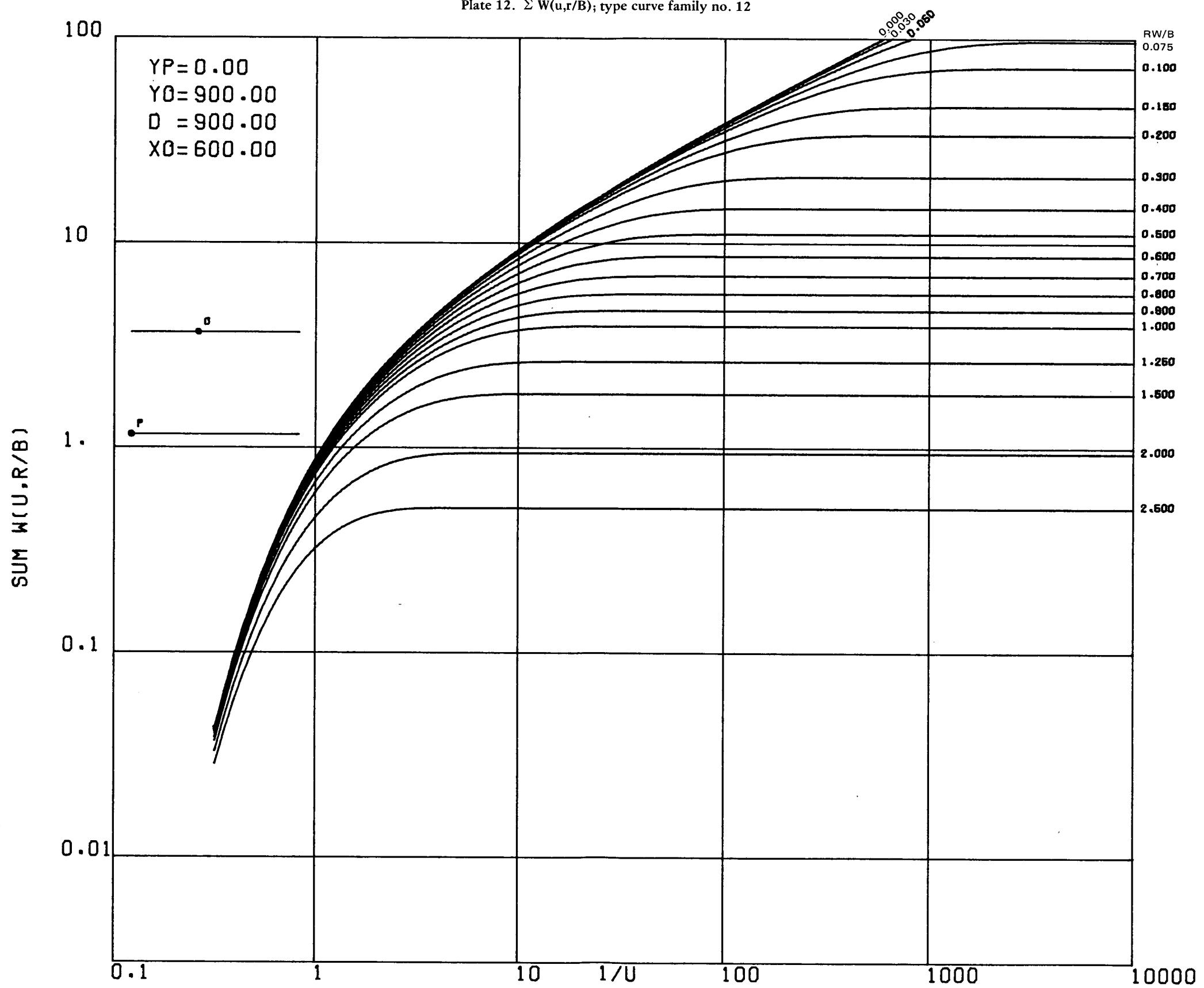
Plate 12. $\Sigma W(u,r/B)$; type curve family no. 12

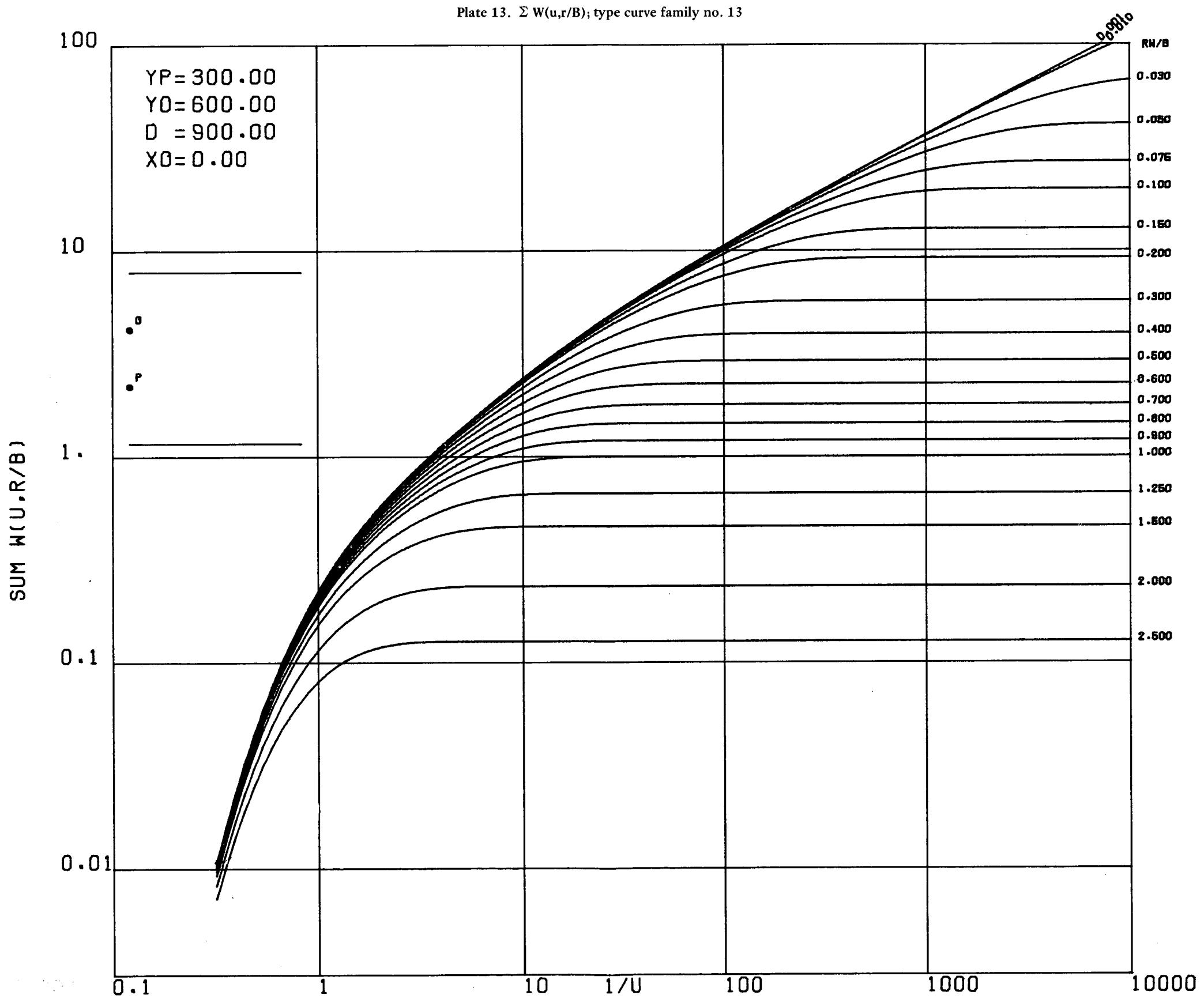
Plate 13. $\Sigma W(u, r/B)$; type curve family no. 13

Plate 14. $\Sigma W(u, r/B)$; type curve family no. 14

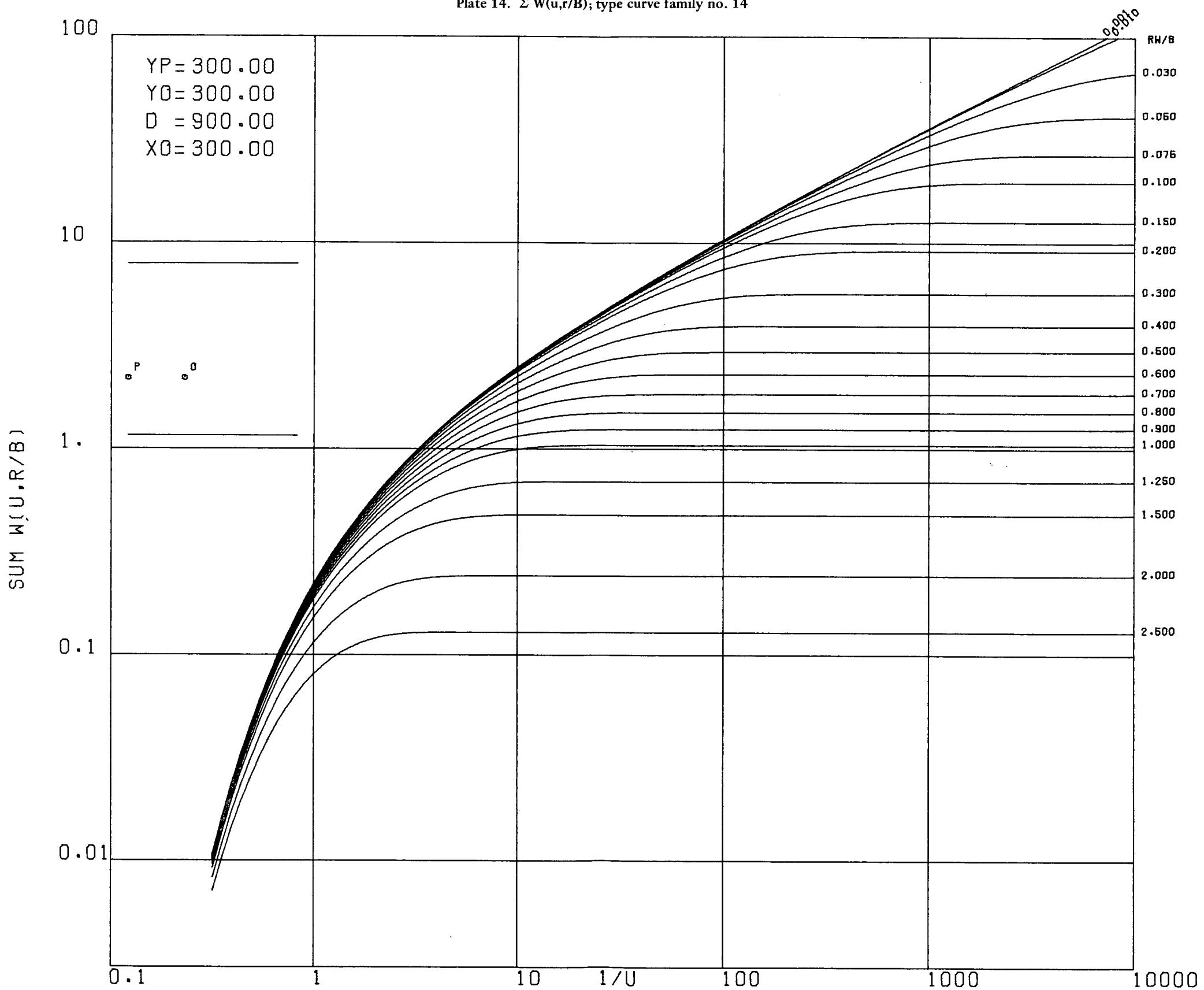


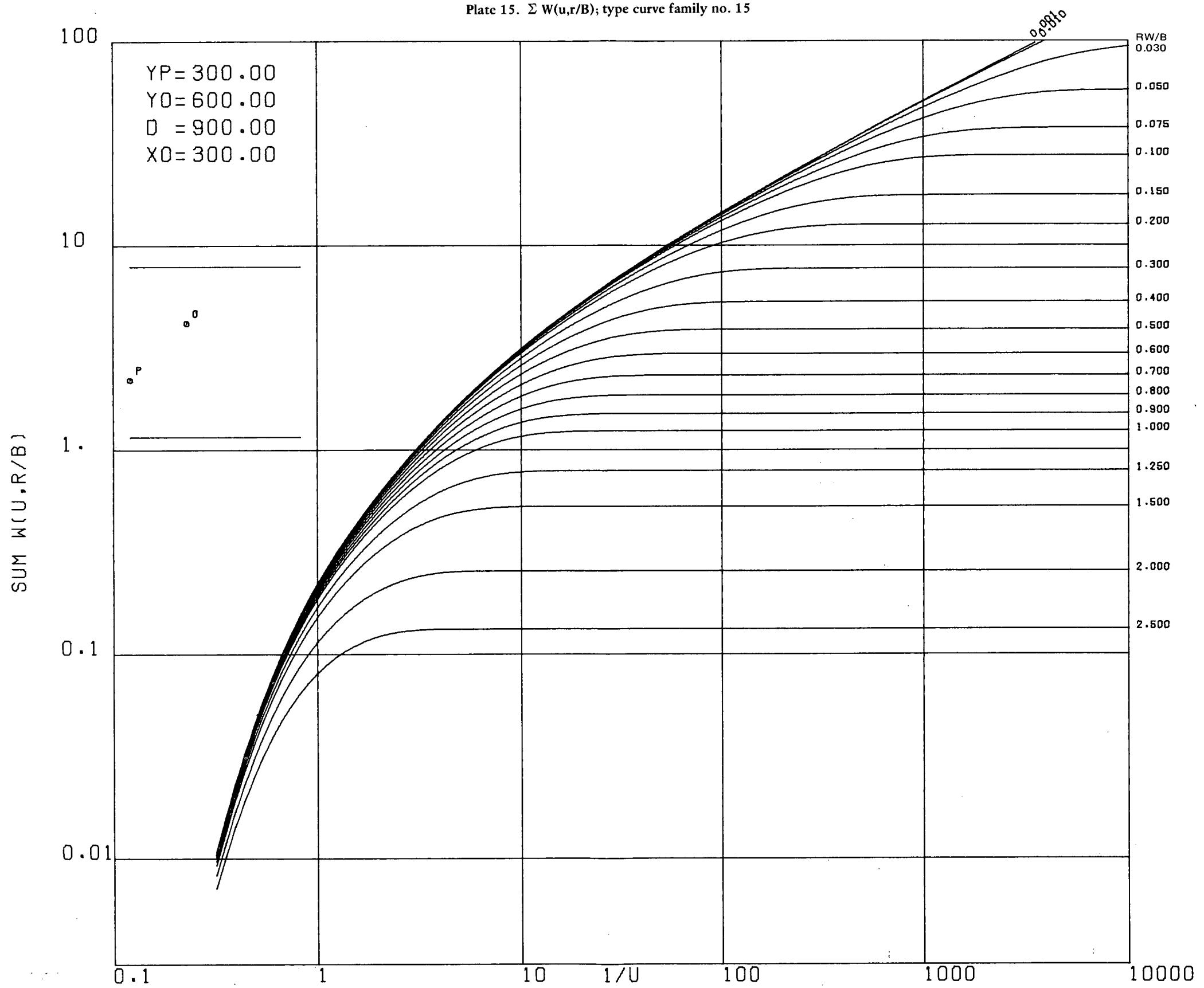
Plate 15. $\Sigma W(u, r/B)$; type curve family no. 15

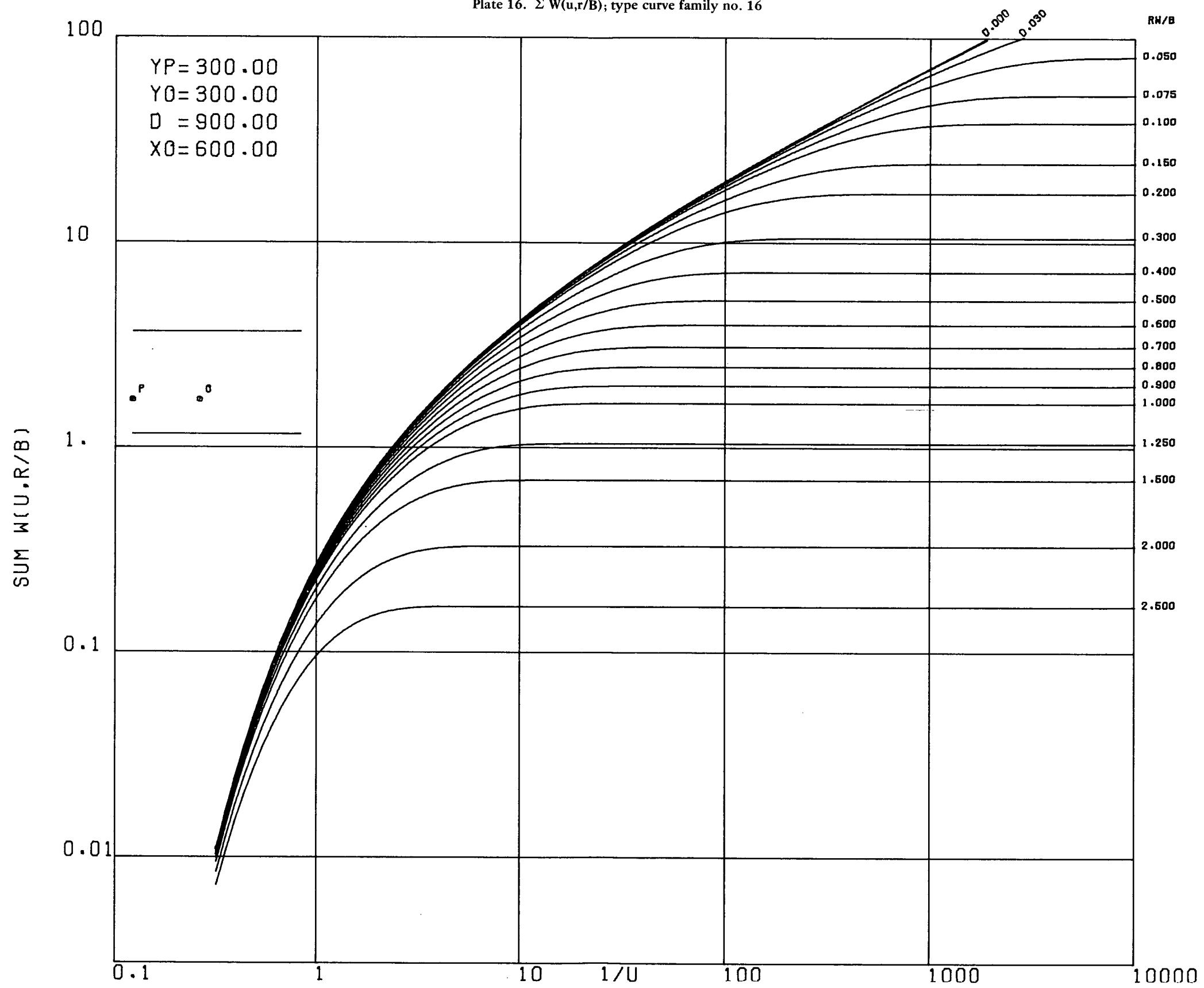
Plate 16. $\Sigma W(u, r/B)$; type curve family no. 16

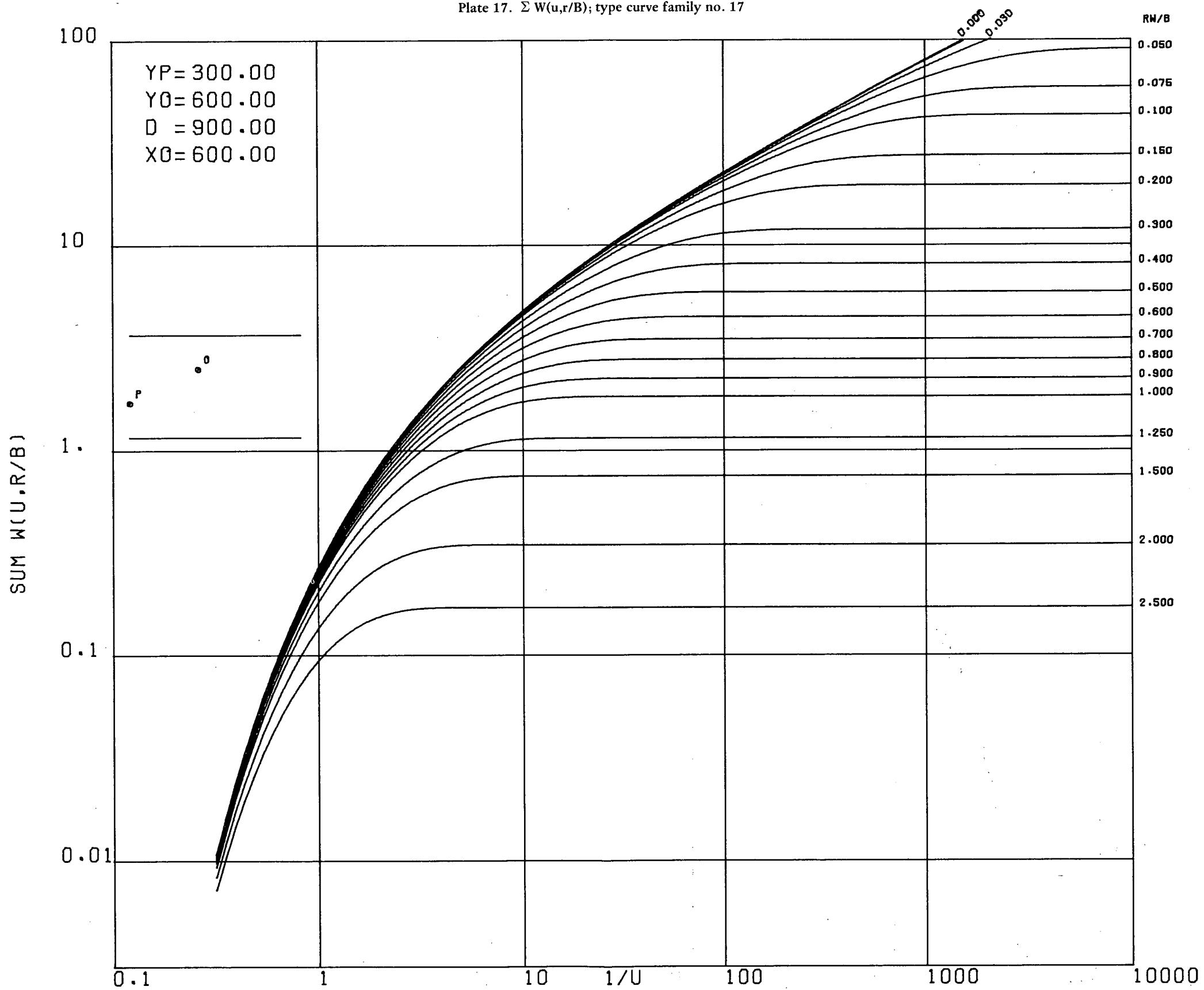
Plate 17. $\Sigma W(u, r/B)$; type curve family no. 17

Plate 18. Family of type curves $\Sigma W(u,r/B)$ for the well configuration of P and O₂ in the buried bedrock channel aquifer near Esterhazy, Saskatchewan.

