FOREST MANAGEMENT NOTE

Note 51 Northwest Region

POLYMORPHIC HEIGHT AND SITE INDEX CURVES FOR THE MAJOR TREE SPECIES IN ALBERTA

In 1989 Cieszewski and Bella developed a new, two-coefficient variable-age–site index (VASI) height model for lodgepole pine in Alberta. This model 1) provided compatible site index (SI) and height-growth estimates, and 2) predicted height growth at any age, without prior knowledge of SI, as a function of any other height and corresponding age.

The VASI model had fewer coefficients than the previously developed models used for the major tree species in Alberta, and also could predict height with greater accuracy and precision (Cieszewski and Bella 1989).

Variable-Age Site Index Equation

The biologically based, nonlinear height-growth model (Cieszewski and Bella 1989) was derived from a process of formulating and testing a biological hypothesis on the polymorphism of lodgepole pine growth, and presented as:

[1] $H(t,h_x,x) =$

$$\frac{h_x + \delta + \sqrt{(h_x - \delta)^2 + \zeta h_x x^{\iota}}}{2 + \zeta t^{\iota} / (h_x - \delta + \sqrt{(h_x - \delta)^2 + \delta h_x x^{\iota}})}$$

where $\delta = 20\beta \mathrm{age}_{SI}^{-1-\alpha}$, $\zeta = 80\beta$, $x^{\iota} = x^{-1-\alpha}$, $t^{\iota} = t^{-1-\alpha}$, and $\mathrm{age}_{SI} = 50$.

After making appropriate substitutions, the equation reads:

[2] $H(t,h_x,x) =$

$$\frac{h_x + 20\beta 50^{-1-\alpha} + \sqrt{(h_x - 20\beta 50^{-1-\alpha})^2 + 80\beta h_x x^{-1-\alpha}}}{2 + \dfrac{80\beta t^{-1-\alpha}}{h_x - 20\beta 50^{-1-\alpha} + \sqrt{(h_x - 20\beta 50^{-1-\alpha})^2 + 80\beta h_x x^{-1-\alpha}}}}$$

Equations 1 and 2 generate biologically sound, nonlinear and polymorphic growth curves as a

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Table 1. Data screening criteria and summaries by species

Screening: Steps	Screening: Criteria	Number of trees rejected: Lodgepole pine	White spruce	Black spruce	Trembling aspen
Initial screening	Missing bh[a] age	107	983	618	1438
	bh age at <50	222	376	219	863
	<3 valid measurements above bh	69	315	286	296
	Missing section	77	41	38	21
	Stump age at bh >5	–[b]	–	–	20
Totals	Rejected	475	1715	1161	2738
	Retained	1360	1261	444	341
Individual tree fitting	Inestimable coefficients	1	2	5	0
Totals	Rejected	1	2	5	0
	Retained	1359	1259	439	341
Visual examination of individual height-over-age plots	SI[c] too low	7	21	3	1
	First age too high	22	31	34	3
	Suppressed growth	82	343	26	51
Totals	Rejected	111	395	63	55
Final	Retained	1248	864	376	286
Dominant and codominant only	Retained	1163	698	282	276

[a]bh = breast height.
[b]Not applicable.
[c]SI = site index.

Table 2. Summary statistics of the data used for fitting height-growth SI[a] curves

Species	Number of trees	bh[b] Age: Avg	SD[c]	Min.	Max.	Height above bh (m): Avg	SD	Min.	Max.	SI above bh (m): Avg	SD	Min.	Max.
Lodgepole pine	1163	98.9	32.83	50.0	260.0	19.22	4.73	7.70	35.50	13.83	3.54	4.40	23.88
White spruce	698	104.6	31.61	50.0	250.0	23.90	5.24	8.69	40.92	14.22	3.43	5.01	24.45
Black spruce	282	94.8	28.35	50.0	190.0	13.96	3.70	6.87	26.50	8.99	2.36	3.98	15.87
Trembling aspen	276	70.0	19.95	50.0	140.0	20.66	3.26	9.04	28.25	17.47	2.83	8.28	24.93

[a]SI = site index.
[b]bh = breast height.
[c]SD = standard deviation.

function of prediction age and height at any other age. When age and height at this age are used to produce a reference point on a height curve (instead of using the standard fixed-age SI), the problem of incompatibility between height and SI predictions is precluded. The reference points of each new curve are, by definition, part of a height curve: this means that height at any age can be used to create a set of base-age invariant height curves, because any point on a height curve will unequivocally define the entire growth curve.

Equation Coefficients

The traditional fixed-age SI was not used in this study, and the term height was given the following two meanings:

1) a computed tree height [$H(t,h_x,x)$] for a prediction age (t); and
2) a known height (h_x) at any other age ($x \neq t$), i.e., this height (h_x) was used in conjunction with this age (x) as a reference point, instead of the SI.

Estimable model coefficients were denoted as α and β. The coefficient for all species were determined by nonlinear regressions, using stem analysis data.

Four Major Trees Species

Because of the excellent performance of the VASI model (eq. 1) with lodgepole pine data, it was decided to establish accurate and precise height-growth curves for all the major tree species in Alberta. Therefore, following the incorporation of individual data for the four species in eq. 2, this paper presents height-growth SI curves for lodgepole pine (*Pinus contorta* var. *latifolia* Engelm.), white spruce [*Picea glauca* (Moench) Voss], black spruce [*Picea mariana* (Mill.) B.S.P.], and trembling aspen (*Populus tremuloides* Michx.).

DATA SOURCES AND COLLECTION

In this study, stem analysis data from dominant and codominant trees were provided by the Alberta Forest Service (AFS), Weldwood of Canada Limited, Hinton Division, and Forestry Canada. The sample covered each species' commercial range in Alberta. The total height of each tree was measured, and sections for ring (age) counts were obtained at stump, at breast height (bh) of 1.3 m, and at fixed intervals (of from 1.0 m to 2.5 m) above bh.

Data Screening Criteria

Initial data screening for all species was based on the following criteria:

1) non-decreasing heights and ages;
2) a minimum of three valid measurements above bh per tree; and
3) a minimum tree bh age of 50 years.

These three constraints screened out of the study up to 65% of the trees (Table 1). Another factor in the initial screening was that aspen data showed much greater variation than conifer data; therefore, a more rigorous screening criterion had to be devised for aspen. Only trees that reached bh by age five were retained in the study (Table 1), because this species has very fast early growth.

In the next step, individual tree data were plotted and screened for suppressed early growth or top damage (Table 1).

Plots of all individual trees were screened for: 1) a minimum SI (which was calculated from average SI minus three standard deviations); 2) a maximum age at which the tree had to reach the first section measurement; and 3) an early growth suppression (Table 1).

The decadal values of height growth, (i.e., heights at age 10, 20, 30, etc. years), were interpolated from the data that passed all of the selection criteria (Table 2). These values were then used in all further analysis.

Table 3 presents distribution of screened data for the new height-growth SI curves by source and species.

Table 3. Screened data sources by species

Source	Lodgepole pine	White spruce	Black spruce	Trembling aspen
AFS	1110	698	282	202
For. Can.	–[a]	–	–	74
Weldwood	53	–	–	–

[a]Not applicable.

DATA ANALYSIS

Equation 2 was chosen for height-growth SI modelling because it has flexibility similar to Monserud's (1984) modified logistic equation currently used by AFS for these same species (Alberta Department of Energy and Natural Resources 1985, 1988), but it solves the problem of compatibility of height versus SI prediction and is simple to use.

Fitting eq. 2 was done using least-square nonlinear regressions performed on the decadal height values. Predicted height was computed as a function of bh prediction age, and another height and its age. The other height and age were always the preceding decadal values of height and its age. Only nonoverlapping decadal periods were fitted, following the procedure described by Borders et al. (1984) for SI-free models derived through the algebraic difference approach. All regression analyses were initially performed using customized SIMPLEX software, and they were then rerun using SAS to obtain additional statistics.

Table 4 presents regression coefficients, their standard errors (SE), and *t*-statistics, as well as standard errors of height predictions (SE of Ht), residual variation coefficients

$$\left(\text{RVC} = \frac{\text{SE}}{\text{Mean Ht}}\right)$$

and R-squared for each species.

Height-Growth Curve Comparisons

New height-growth curves were generated for lodgepole pine, white spruce, black spruce, and trembling aspen (Fig. 1) using eq. 2. To provide comparisons, Figure 1 also includes point estimates, in 10-year steps, of height-growth curves used by the AFS at this time (Alberta Department of Energy and Natural Resources 1985).

Interactive Computer Program

The interactive program shown as Figure 2 is an example of the model's application in the computation of lodgepole pine SI height growth. The same program can be used for other species by changing coefficients a and b in the PARAMETER statement.

Table 4. Regression statistics of the new height-growth SI[a] model

Species	Coefficient	Estimate	SE[b]	*t*-statistic	SE of Ht[c]	RVC[d] (%)	R^2
Lodgepole pine	α	0.20372424	0.0058002482	35.123	0.84	6.3	0.970
	β	97.37473618	1.2735295123	76.461			
White spruce	α	0.3235139	0.0069955077	46.246	1.21	8.1	0.965
	β	260.9162652	4.0621902401	64.230			
Black spruce	α	0.1992266	0.0121355265	16.417	0.66	7.4	0.969
	β	114.8730018	2.5131359871	45.709			
Trembling aspen	α	0.2644606	0.0138721799	19.064	0.69	4.9	0.977
	β	117.3695371	3.2205179969	36.444			

[a]SI = site index.
[b]SE = standard error.
[c]SE of Ht = standard error of height prediction.
[d]RVC = residual variation coefficient.

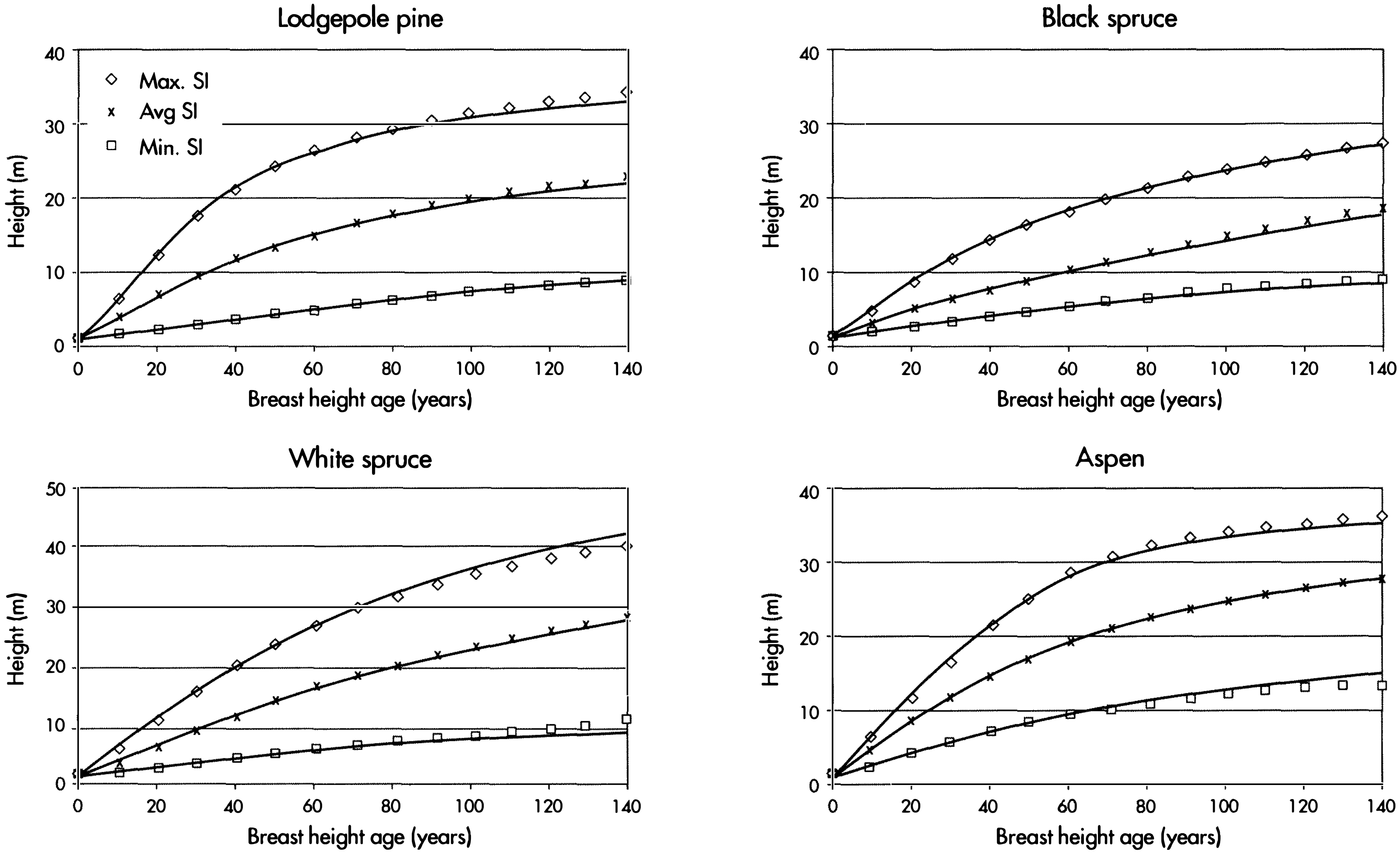

Figure 1. New height-growth curves generated by the VASI height model (solid lines) and height curves currently used by the AFS (symbols)[a] for minimum, average, and maximum SIs of the data available for the major tree species in Alberta.

[a]Source: Alberta Department of Energy and Natural Resources. 1985. Alberta phase 3 forest inventory: yield tables for unmanaged stands. Rep. 60.

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      DOUBLE PRECISION t,a,b,d,z,j,hx,x1,hxRoot,Ht(15)
      CHARACTER*1 Y
      PARAMETER (a=0.20372424D0, b=97.37473618D0)
      WRITE(*,'(//,A,A)')' THIS PROGRAM COMPUTES HEIGHTS ',
     &                     'OF LODGEPOLE PINE IN ALBERTA AS'
      WRITE(*,'(A,A,/)')  '  A FUNCTION OF THE PREDICTION AGE ',
     &                     'AND ANY OTHER AGE AND HEIGHT.'
      GOTO 10
    5 WRITE(*,'(//,A)') ' Please REenter your input...'
   10 WRITE(*,'(/,$,A)')   ' Please enter AGE OF PREDICTION:  '
      READ(*,*) t
      WRITE(*,'($,A)')     ' Enter a KNOWN HEIGHT value:       '
      READ(*,*) hx
      WRITE(*,'($,A)')     ' Enter the KNOWN HEIGHT''S AGE:     '
      READ(*,*) x1
      z = 80*b
      j = -1-a
      d = 20*b*5d1**j
      hxRoot = hx-1.3D0 + DSQRT( (hx-1.3D0-d)**2 + z*(hx-1.3D0)*x1**j )
      pred = ( hxRoot + d ) / ( 2 + z*t**j/(hxRoot-d) ) + 1.3D0
      DO 20 I = 1, 15
   20 Ht(I) = ( hxRoot + d ) / ( 2 + z*(I*10)**j/(hxRoot-d) ) + 1.3D0
      IF ( Ht(5) .GT. 25.3D0 ) THEN
         WRITE(*,'(//,A)') ' Sorry, pine does not grow that high!'
         hxRoot = 24D0 + DSQRT( (24D0-d)**2 + z*24D0*5D1**j )
         pred = ( hxRoot + d ) / ( 2 + z*x1**j/(hxRoot-d) ) + 1.3D0
         WRITE(*,'(/,A,F5.2)') ' MAXIMUM HEIGHT FOR SPECIFIED AGE IS: ',
     &    pred
         GOTO 5
      ELSE IF ( Ht(5) .LT. 5D0 ) THEN
         WRITE(*,'(//,A)') ' Sorry, this tree is too short!'
         hxRoot = 3.7D0 + DSQRT( (3.7D0-d)**2 + z*3.7D0*5D1**j )
         pred = ( hxRoot + d ) / ( 2 + z*x1**j/(hxRoot-d) ) + 1.3D0
         WRITE(*,'(/,A,F5.2,A,/)') ' AT THE SPECIFIED AGE PINE IS MIN.',
     &    pred, 'm TALL!'
         GOTO 5
      END IF
      WRITE(*,'(//,A,I4,1X,A,F5.2,//)') 'Height for age:',INT(t),
     & ' is:   ', pred
      WRITE(*,'(A,F5.2,//)') 'SI of the subject tree is:  ', Ht(5)
      WRITE(*,*) '              DECADAL HEIGHTS OF THE SUBJECT TREE'
      WRITE(*,'(A,15(I4,1X))') 'Age', (I,I=10,150,10)
      WRITE(*,'(A,15(F4.1,1X))')    'Ht. ', (Ht(I),I=1,15)
      WRITE(*,'(//,A,//)') 'Another height? (Y/n)'
      READ(*,'(A1)') Y
      IF ( Y .NE. 'N' .AND. Y .NE. 'n' ) GOTO 10
      WRITE(*,'(A,A1,A)') 'The request was: ', Y, '...    Good bye!'
      STOP
      END
```

Figure 2. Application of the height-growth SI model in an interactive computer program (e.g., lodgepole pine).

SUMMARY

The major points of this study are as follows:

1) These new curves are quite similar in shape to the curves currently used by the AFS, with slight differences showing up in the extremes of site index and age.
2) For each species the equation has only two coefficients, so it is basically simpler than other models currently in use.
3) The main advantage of the equation is that it can predict height directly from a known height at any age for each species, without prior estimation of site index.

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