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# RASC FORTRAN IV COMPUTER PROGRAM FOR RANKING AND SCALING OF BIOSTRATIGRAPHIC EVENTS 

by
M. Heller, W.S. Gradstein, F.M. Gradstein, and F. Agterberg

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## RASC

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## RASC

The Agterberg - Gradstein method for Ranking and Scaling

- an explanation of the methods and tests
a manual for the implementation of the RASC - computer program
- program input for a sample run
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# RASC FORTRAN IV COMPUTER PROGRAM FOR RANKING AND SCALING OF BIOSTRATIGRAPHIC EVENTS 

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M. Heller, W.S. Gradstein, F.M. Gradstein, and F.P. Agterberg

This two-part Open File consists of text and material on magnetic tape. The text is a manual prepared by M. Heller, W.S. Gradstein, F.M. Gradstein, and F.P. Agterberg followed by a listing of the RASC computer program with symbolic reference maps and output for the Sample Run.

After preparation of the RASC manual, a minor change was made to the original micropaleontological data used for the Sample Run (Late Cretaceous foraminiferal exits in 20 Grand Banks wells). As a result, Sample Run data on the magnetic tape and output for the Sample Run following the listing of the computer program differ slightly from (and supersede) the input and output used for illustration in the manual.

The magnetic tape which is issued separately from this text contains the RASC Fortran IV program followed by the updated Sample Run data.

The magnetic tape (cost $\$ 100.00$ ) may be obtained from the Computer Science Centre, Department of Energy, Mines and Resources, 588 Booth Street, Ottawa, Ontario, KIA OE 4 (Forward cheque or money order in Canadian funds only payable to the Receiver General for Canada to Computer Science Centre).

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## PURPOSE

The RASC-(RAnking and SCaling-) method and the RASC computer-program, written in Fortran IV language, are to be used for:

1. The determination of the average ranking of elements (i.e. stratigraphic events) in different samples (i.e. geological sections) from one assumedly homogeneous population (i.e. geographic region) along an irreversible scale (i.e. a relative timescale). This part is called Ranking.
2. The determination of the average relative distance between these elements along that scale. This part is called Scaling.
3. The detection of heterogeneity in the data by the testing of each sample (geological section) for "normality".

A biostratigrapher can use the program to construct range-charts of average ranges and zonations based on assemblage zones.

## general introduction

The RASC-method and program have been developed for use in biostratigraphy, the science that uses fossils to determine the geological age of rocks. Biostratigraphy is based on Darwinian evolution-principles: In time biological taxa appear, reach a peak of abundance, and disappear, to make place for other taxa.

Ideally, the biostratigrapher likes to follow the Darwinian evolution (lineage) of one taxon. However, the fossil record, as found in sampled outcrops or in wells, never produces a long, continuous set of data. Strata might be geologically reworked or sometimes go undetected. Their fossils might be absent (palaeo-ecological factors, high sedimentation rate) or are poorly preserved.

A more useful method of biostratigraphy is to make a biostratigraphic characterization or zonation of a certain region and to correlate it with characterizations of other regions. Traditionally this was done with the aid of index fossils, but they also might turn out to be rare, for the above-mentioned reasons.

To alleviate these problems, the modern biostratigrapher tends to work in a quantitative way, generally using those stratigraphic events that are most common in the region studied. However, the varying thickness of stratigraphic units (strata representing a certain period of time) precludes the use of fossil information from different geological sections in one region for this quantitative approach. Correlation between sections calls for a relative scale. One way to obtain a relative scale is by averaging the sequences of the events in the individual sections.

RASC has been developed to handle the abundant information needed for this quantitative approach.

The program is based on a method as proposed by Hay in 1972; it considers how often each common event (each event that occurs in at least a specific number of sections) occurs above, simultaneous with, or below each other common event in all geological sections (wells, outcrops) in one assumedly homogenous region. By regrouping these relative orders (ranks) RASC eventually produces a true average sequence for the events. Since this average sequence can also be thought of as an "optimum sequence" the latter term is often used for the result of the 'ranking' procedure.

An event ranked in this way might be the first occurrence in time of a certain fossil. Another event might be the last occurrence in time of that same fossil. The period between the first occurrence and the last occurrence of a fossil is called the fossil's stratigraphic range. With the above ranking exercise a sequence of average ranges can be determined. RASC can thus also be used to produce an average-range-chart.

In practice the biostratigraphic characterization of a certain region doesn't only consist of an average sequence of single events or of an average-range-chart, but also of a sequence (zonation) of "assemblage zones", an assemblage zone being a body of strata characterized by an assemblage of fossils.

It is possible to use the average sequence of stratigraphic events to determine a sequence of fossil-
assemblages along the time-scale. To detect these fossilgroupings it is necessary to estimate distances between the events, which can be done with the use of "cross-overfrequencies".

Events tend to change places with one another in different sections. This is especialy true for benthic fossils, whose occurrences depend greatly on the local palaeoenviromment, in addition to their stage in evolution. RASC uses this crossing-over to determine distances. It should be kept in mind that RASC-distance-estimates can thus not be made when crossing-over does not occur.

It is assumed that the more often the events change places, the closer together they are on the time-scale. This assumption, that cross-over frequencies and relative distances are related, is often used in other science disciplines.

It has turned out to be possible to detect clusters of events along the time-scale in this way, and it might well be possible that these clusters are equivalent to the
above-mentioned fossil-assemblages and that RASC can indeed be used to construct a zonation based on assemblage zones.

To relate the results of RASC to more traditional methods and to the existing literature, the program also provides for a "Unique Event Option", which ranks indicator fossils and marker horizons in the average sequence, even if these unique events were considered to be too rare to participate in the ranking.

The third part of RASC is the normality test, which is a statistical test for the event-positions in each individual section in relation to the optimum sequence. This test has been found necessary because biostratigraphy methods are subject to sample contamination and observer's bias.

This introduction is based on Hay (1972), Gradstein (1982) and Gradstein and Agterberg (in press).

## THE RASC-METHODS AND TESTS

The RASC-program carries out the algorithms for the following methods and tests: Ranking, Scaling (with Marker Horizon and Unique Event Options), and Normality Testing. The methods and tests will be explained in this part of the guide. The original, more extensive, information can be found in Agterberg and Nel $a$ and $b$ (in press).
A. Ranking

The method
The ranking procedure, as carried out by the RASC computer-program, is based on the method as proposed by Hay (1972), but has been considerably changed and shortened. A so-called presorting option has been developed, which shortens the procedure considerably, and provisions have been made to alleviate the problem of "cycling events". As opposed to Hay's method, RASC considers simultaneous events as well.

The RASC-procedure starts with the construction of a cumulative order matrix based on sequences of events observed in the individual sections. Only those events are considered that occur in at least $k_{c}$ wells (common events). $k_{c}$ is the threshold parameter and has to be set prior to ranking (see the appropriate item in "the implementation of the RASC-program").

RASC lists all participating events in the margins of the cumulative order matrix, in the rather arbitrary sequence in which they have actually been identified. It is the objective to reorder these margin-events until their sequence averages all sequences found.

The elements of the matrix are scores, indicating in how many sections one event precedes another event. When for example event A precedes event $B$ in 1 out of 6 sections, and $B$ precedes $A$ in 4 out of 6 sections, the matrix looks as follows:

|  | $A$ | $B$ |
| :---: | :---: | :---: |
| $A$ | $X$ | 1 |
| $B$ | 4 | $X$ |

When $A$ and $B$ occur simultaneously on the 6th section, each score receives an extra 0.5:

|  | $A$ | $B$ |
| :---: | :---: | :---: |
| $A$ | $X$ | 1.5 |
| $B$ | 4.5 | $X$ |

Thus, when a score $s_{A B}$ (or more general $s_{i j}$ ) in the upper right triangle is lower than a score sBA (or more general $s_{j i}$ ) in the lower left triangle, this means that $B(j)$ precedes $A(i)$ more often than $A(i)$ precedes $B(j)$ -- as is the case in the example: In the example the events in the margins were arbitrarily ordened as they were found in the one section in which A precedes B. From the matrix-scores it now appears that the sequence $A-B$ is not the average sequence. Only when $s_{i j}$ in the upper triangle is higher than $\mathrm{siji}_{\mathrm{j}}$ in the lower triangle is the sequence of the events in the margins the true average one. In practice, with many events, many scores sij in the upper triangle are lower than their counterparts sji in the lower triangle, and a regrouping of events is required, until all scores in the upper triangle are higher than, or equal to their counterparts in the lower triangle. Only then are the events in the margins ranked according to their average sequence.

For the first step, the presorting option, RASC derives a simplified matrix $A$ from the cumulative order matrix $S$. The elements ( $a_{i j}$ ) of this matrix are based on the scores sij in the s-matrix in the following way:

When $s_{i j}>s_{j i} \rightarrow a_{i j}=1$
When $s_{i j}<s_{j i} \rightarrow a_{i j}=0$
When $s_{i j}=s_{j i} \neq 0 \rightarrow a_{i j}=0.5$

A simple example of a cumulative order matrix and a resulting $A$-matrix is given here:

|  |  | A | B | C |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | X | 1.5 | 5 |  |  |
| S-matrix: | B | 4.5 | X | 5.5 |  |  |
|  | C | 0 | 1.5 | X |  |  |
|  |  | A | B | C | Sums | Presort Rank Number |
|  | A | X | 0 | 1 | 1 | 2x1/2-0=1 |
| A-matrix: | B | 1 | x | 1 | 2 | $2 \times 2 / 2-0=2$ |
|  | C | 0 | 0 | X | 0 | $2 \times 0 / 2-0=0$ |

When all elements of each row of the A-matrix are added, and the sums are ranked in descending order, the events approach the average sequence.

In practice RASC doesn't just determine the sums of the rows, because, when events $i$ and $j$ never occur together in section, $s_{i j}=s_{j i}=0$ and $a_{i j}=0$, which is a different situation from $s_{i j}\left\langle s_{j i} \rightarrow\right.$ $a_{i j}=0$. In order to distinguish between these two situations RASC determines how often the zeros in the ith row of the s-matrix correspond to $s_{i j}=s_{j i}=0$. When there are $b_{i}$ of these zeros in a row of (N-l) events the Presort Rank Number of event $i$ is determined as follows:
(N-l) $x$ (sum of all $a_{i}$-scores)
(see figure)
$(N-1)-b_{i}$

The next step, the matrix-permutation, is based on Hay's method. RASC lists for this step again the events by their sections, but now in a new sequence based on their Presort Rank Numbers, and constructs a new cumulative order-matrix with the pre-sort sequence of events in its margins (like B-A-C in the example above). Only those events are used in this matrix that occur together in at least $m_{c}$ wells. If they occur together in fewer wells their relative positions are not considered relevant, i.e. when the number of times that two events i and $j$ occur together in a section is called $r_{i j}$, then all scores for which $r_{i j}<m_{c}$ are set equal to 0 and are ignored in the matrix-permutation. $m_{c}$ has to be set prior to ranking (see implementation of the RASC-program) and should be smaller than $k_{c}$.

All scores ( $s_{i j}$ ) of this new matrix $S$ are compared to their counterparts ( $s_{j i}$ ) and whenever a score $s_{i j}$ in the upper triangle is lower than its counterpart sji in the lower triangle, the row i is interchanged with the row $j$ and column i is interchanged with column j.

This procedure is repeated until $s_{i j}>s_{j i}$ for all elements in the upper triangle. The events in the margin are then ranked according to their average sequence. In future explanations this average sequence is often referred to as "optimum sequence".

Cycles in the matrix-permutation
In some cases the matrix-permutation will not produce a solution. Cycles will occur. When for instance events $A, B$ and $C$ get involved in a cycle, this means that $A$ precedes $B$ more often than $B$ precedes $A$ and that $B$ precedes $C$ more often than $C$ precedes $B$, and that $C$ precedes $A$ more often than $A$ precedes $C(\bar{A}-B-C-A)$.

Cycles like these are fairly common and can be interpreted as follows:

1. $A, B$, and $C$ occur simultaneously
2. The data used is so patchy (especially when the $k_{c}$ and $m_{C}$ - values are very low), that no average sequence can be established.

Since the matrix-permutation cannot be carried through when cycles occur, the cycles present a problem. In preceding efforts (cf. Hay 1972) the cycling events were ignored altogether. However, the group of cycling events is also stratigraphic information and should be included in the result.

RASC starts the matrix-permutation, notes when a cycle occurs and which events are involved (cycles of three or four or even five or six events have been found in past uses of RASC), and consequently determines which two of the cycling events have the smallest difference in relative order ( $s_{i j}-s_{j i}$ ). RASC zeros the scores of these two events, which breaks the cycle. If there is more than one pair of events that has the smallest difference in relative order ( $s_{i j}-s_{j i}$ can be the same for several pairs of events), only one of those pairs is zeroed.

In this way the cycles are represented, but by events that in absence of the zeroed scores do have a rank.

Optimum sequence
The final solution for ranking appears in the RASC output directly after the listings (numerical and alphabetical) of all events of the input. It is presented under the heading "Optimum Sequence". Each event in this table receives a "range", which will be explained below. A copy of the Optimum Sequence of the sample run of Chapter $V$ is given in Table 1.

| SEOUENCE | FOSSIL RANGE FOSSIL |
| :--- | :--- | :--- |
| POSITION | NUMBER |


| $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 4 \\ & 5 \\ & 6 \\ & 7 \\ & 8 \\ & 9 \\ & 10 \\ & 11 \\ & 12 \\ & 13 \\ & 13 \\ & 15 \\ & 15 \\ & 17 \\ & 18 \\ & 19 \\ & 20 \\ & 21 \\ & 22 \\ & 23 \\ & 24 \\ & 25 \\ & 26 \\ & 27 \\ & 28 \\ & 29 \\ & 30 \\ & 31 \end{aligned}$ | 5 4 3 105 6 16 8 13 12 12 11 14 14 26 56 20 108 23 55 24 22 54 72 109 109 710 112 3 |  | STENSIOINA POMMERANA <br> GLOBOTRUNCANA ARCA <br> GLOBOTRUNCANA STUARTI <br> LOXOSTOMA GEMMUM <br> GLOBOTRUNCANELLA HAVANENS IS <br> RUGOGLOBIGERINA RUGOSA <br> GLOBIGERINELLOIDES MESSINAE <br> GLOBOTRUNCANA IINNEIANA <br> GLOBOTRUNCANA STUARTIFORMIS <br> GLOBOTRUNCANA FORNICATA <br> GLOBOTRUNCANA CRETACEA <br> GLOBOTRUNCANA ANGUSTICARINATA <br> GAUDRYINA AUSTINANA <br> GLOBOTRUNCANA CORONATA <br> GAVELINELLA MINIMA <br> STENSIOINA EXCULPTA <br> GLOBOTRUNCANA CARINATA <br> SIGALIA DEFLAENSIS <br> GLOBOTRUNCANA CONCAVATA <br> GLOBOTRUNCANA RENZI <br> COARSE AGGLUTINATED SPP <br> GIOBOTRUNCANA IMBRICATA <br> HEDBERGELLA BOSQUENSIS <br> PRAEGLOBOTRUNCANA STEPHANI <br> GLOBOTRUNCANA SCHNEEGANSI <br> PRAEGLOBOTRUNCANA TURBINATA <br> ROTALIPORA CUSHMANI <br> GAVELINOPSIS CENOMANICA |
| :---: | :---: | :---: | :---: |

TABLE 1. Optimum sequence of the sample run, discussed in Chapter V.

Range of the events in the optimum sequence

Each event is given a range in the average (optimum) sequence. This range should not be confused with the stratigraphic range of a taxon-- it only indicates that an event $i$ has been found between the events of the range (h-k). Example: Event 12 of the sample run has position 9 and occurs between positions 8 and 10 , but event $l l$ with position 10 occurs somewhere between positions 9 and 12. The second example means that the order relation of event 11 (position l0) with event 14 (position ll) has not been established, but that 11 does follow 12 (position 9) and precede event 15 (position 12).

The detection of the range of an event in relation to other events takes place in the last $S$-matrix of the matrix-permutations, which is called the "final order relation matrix". If there are scores $s_{i j}$ in the upper triangle that are
(1) ignored, because $x_{i j}<m_{c}$
(2) zeroed, because thè were cycle-participants
(3) "tied", which means that $s_{i j}=s_{j}$ and events $i$ and $j$ are simultañous in the average sequence,
one has not determined whether $j$ has a position either above or below $i$ and vice versa. The position of $j$ is then part of the range of $i$ and the position of $i$ is then part of the range of $j$.

It should be stressed that the event-positions and their ranges do not have a statistical meaning. The positions are average, relative positions and their ranges only indicate if an average sequence can be found between one event and two or more of its neighbours.

## B. Scaling

The scaling part of RASC gives statistical values to the distances between the events in the optimum sequence. These distance estimates result in a slightly different sequence (scaled optimum sequence), and in dendrograms, illustrating the "distances" between the events along a ("time"-) scale.

Several dendrograms appear in the RASC output. Only the dendrogram that appears after the normality test tables contains the unique events, if they were included in the input. If marker horizons were included in the input they appear in all dendrograms (see Marker Horizons and Unique Events).

The RASC scaling method uses so-called cross-over frequencies (how often events change position with one another in different geological sections.), and is based on four assumptions (see below). It includes direct estimates, unweighted indirect estimates, and weighted indirect estimates of the distances between events, the construction of dendrograms, and a final reordering option.

Cross-over frequency
The "cross-over frequency", or rather the non-cross-over-frequency $F_{A B}$ indicates in how many sections A occurs above $B$. When $A$ occurs above $B$ in 9 out of 10 sections $F_{A B}=\frac{9}{10}$ and the "distance" between $A$ and
$B$ is considered to be large. When $A$ occurs above $B$ in 6 out of 10 sections and below $B$ in 4 out of 10 sections, the "distance" is considered to be small. When $F_{A B}=$ 0.5 the "distance" is 0 , since $A$ occurs as often above $B$ as below $B$. When $F_{A B}<0.5, A$ and $B$ are reversed and the "distance" is negative.

RASC uses the final order relation matrix of the ranking-part of RASC to determine the cross-overfrequencies. This implies that scaling has to be preceded by ranking.

## Four assumptions

For the statistical method used in the scaling as carried out by RASC the following assumptions are made.


Fig. 1 Graphic representations of assumptions 1 and 2


Fig 2 Graphic representation of $P_{A B}$ being the probability that $d_{A B} \leq 0$, which is the opposite of assumption $3\left(P_{A B}=\frac{A B}{=}\left(d_{A B} \geq 01\right)\right.$

## Assumption 1

The frequencies of the positions $x_{A}$ and $x_{B}$ etc. each, satisfy normal random (Gaussian) distributions with a mean of $E X_{A}$ and $E X_{B}$ etc. respectively, and a variance of $\sigma^{2}$. The Graphic representation of this assumption can be found in fig. 1.

## Assumption 2

The frequency of the distance between two positions $\left(x_{B}-x_{A}\right.$ or $\left.d_{A B}\right)$ also satisfies a normal distribution, written as $f\left(d_{A B}\right)$, which has a mean of $E X_{B}-E X_{A}$ or $\Delta_{A B}$ and a variance of $2 \sigma \sigma^{2}$. It is the $\Delta A B$ the mean distance between two events, or the distance between two events in the average sequence, that RASC will calculate a value for.

Assumption 2 follows directly from assumption 1 (fig.l)

## Assumption 3

The frequency of $A$ occurring above $B\left(F_{A B}\right)$ equals the statistical probability that $A$ occurs above $B$ $\left(F_{A B}=P_{A B}\right)$. $P_{A B}$ is then also the probability that $d_{A B} \geq 0\left(P_{A B}=P\left(d_{A B} \geq 0\right)\right)$. However, if $P_{A B}$ were the probability that $d_{A B} \leq 0$ (fig. 2), then the parameters $\Delta_{A B}$ and $\sigma^{2}$ of the normal distribution of $d_{A B}$ could be directly calculated from the z-value:

$$
z=\frac{d_{A B}-\Delta_{A B}}{\sigma \sqrt{2}} \rightarrow z=\frac{0-\Delta_{A B}}{\sigma \sqrt{2}}=-\frac{\Delta A B}{\sigma \sqrt{2}}
$$

However, since $P_{A B}=P\left(d_{A B} \geqslant 0\right)$, and since it is known that:

$$
P\left(d_{A B} \leq 0\right)=1-P\left(d_{A B} \geq 0\right)
$$

the $z$-value $\left(z_{B A}\right)$ for the probability that $d_{A B} \leqslant 0$ is the negative of the $z$-value $\left(z_{A B}\right)$ for the probability that $d_{A B} \geq 0\left(P_{A B}\right) \rightarrow z_{A B}=-$
$\mathrm{z}_{\mathrm{BA}}$ It follows that for $\mathrm{P}_{\mathrm{AB}}$
$z_{A B}=\Delta_{A B} \quad \rightarrow \quad \Delta_{A B}=\sigma \sqrt{ } 2 \cdot z_{A B}$
$\sigma \vee 2$

Fig. 3 gives the relationship between $P_{A B}, z_{A B}$, and $\frac{\Delta_{A B}}{\sigma \sqrt{2}}$
$z_{A B}$ can be looked up the tables for the cumulative standard normal distribution, $P_{A B}$ being the cumulative frequency in \%.


Figure 3. Cumulative frequency curve of a normal distribution. The curve indicates how all possible values for $P_{A B}\left(p_{A B}\right)$ relate to the $z$-values of ${ }^{2}$ normal distribution. The dashed lines indicate how one particular value of $P_{A B}$ relates to one particular z-value, which equals, according to assumption 3 , $\Delta_{A B} / \sigma \sqrt{ }$.

## Examples

When $A$ occurs above $B$ in 4 out of 8 sections $F_{A B}=4 / 8=0.5 \rightarrow \overrightarrow{~ F i g . ~} 3$ and Table $z=0 \rightarrow \Delta_{A B}=0$

When $A$ occurs above $B$ in 5 out of 7 sections $F_{A B}=5 / 7 \rightarrow P_{A B}=71 \%$ see Fig. 3 and Table r $z=$ $0.566 \rightarrow \Delta_{A B} \stackrel{A B}{=} 0.566 \cdot \sigma \sqrt{2}$

Assumption 4

$$
\sigma^{2}=0.5
$$

This assumption is wholly arbitrary, but can be made since the distance-scale is relative. It should be kept in mind that the distance-values eventually obtained are based on this arbitrary value, and that only differences between distances bear significance.

Since $\sigma^{2}=0.5 \rightarrow \sigma=10.5 \rightarrow \sigma \sqrt{2}=1 \rightarrow \Delta_{\mathrm{AB}}=\mathrm{z}_{\mathrm{AB}}$

Direct-distance estimate ( $\Delta_{A B}$ )
The scaling procedure as carried out be RASC, starts with the construction of a new matrix (F), which has the events in the optimum sequence in its margins and the "crossover frequency" $F_{i j}$ as its elements. RASC derives this matrix from the final order relation matrix of the ranking (the last s-matrix of the matrix-permutations). For the simple example used in the ranking part, this matrix-conversion can be illustrated, as follows:

|  |  | A | B | C |  |  | A | B | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | X | 4.5 | 5.5 |  | A | X | $\frac{4.5}{6}$ | $\frac{5.5}{7}$ |
| S-matrix | B | 1.5 | X | 5 | F-matrix | B |  | X | $\frac{5}{5}$ |
|  | C | 1.5 | 0 | X |  | C |  |  | X |

(A occurs above B in 4.5 (A occurs above $B$ in sections) 4.5 out of 6 sections)

RASC now converts the $F$-matrix into a $Z$-matrix, to obtain a set of direct estimates for the inter-event distances $\left(\Delta_{A B}=Z_{A B}\right)$

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{AB}}=4.5 / 6 \rightarrow \mathrm{P}_{\mathrm{AB}}=4.5 / 6=75 \% \rightarrow \mathrm{z}_{\mathrm{AB}}=0.674 \\
& \mathrm{~F}_{\mathrm{AC}}=5.5 / 7 \rightarrow \mathrm{P}_{\mathrm{AC}}=5.5 / 7=78 \% \rightarrow \mathrm{z}_{\mathrm{AB}}=0.788 \\
& \mathrm{~F}_{\mathrm{BC}}=5 / 5 \rightarrow \mathrm{P}_{\mathrm{BC}}=5 / 5=100 \% \rightarrow \mathrm{z}_{\mathrm{BC}}=\approx
\end{aligned}
$$

This last value makes sense because $B$ always occurs above $C$, which makes the statistical distance equal to $\approx$ (A situation like this may very well arise when a large number of events over a long time-interval is used. Crossing over between relatively young and relatively old events never takes place of course.)

Since $\approx$ is not a practical value RASC replaces it with an arbitrary number $q$ (or for $P_{A B}=0 \%$ by $-q$ ) and states that whenever $z \geq q$ or $z^{A B}-q, z=q$ or $-q$ and should be ignored..
$q$ can be set by the user. It has to be larger than 1 and smaller than 2 (or smaller than -1 and larger than -2). In past uses of RASC $q$ has been given a value of +1.645 , which corresponds to $P_{A B}=95 \%$ (or 5\%).

It should also be noted that some scores in the original s-matrix were zeroed. They are the scores for which $r_{i j}<m_{c}, r_{i j}$ being how often events $i$ and $j$ occur together and $m_{c}$ being the second threshold parameter set for the matrix-permutations. These zeroed scores are given a z-value of 9.000 and are also ignored. Note that the zeroed cycle-participants have been restored in their original value.

The $z$-matrix for the example is given below. $z_{A B}$ is the value for the direct estimate of the mean distance: $\Delta_{A B}$

|  | A | B | C |
| :---: | :---: | :---: | :---: |
| A | X | 0.674 | 0.788 |
| B |  | X | $1.645 \quad$ (q) |
| C |  |  | X |

Unweighted indirect distance estimate ( $\overline{\Delta_{A B}}{ }^{u}$ )
The z-matrix contains the direct estimates for the statistical distances between events ( $\Delta_{i j}$ ). However, these direct estimates are considered insufficient, since i and $j$ don't very often occur together in a section.

Moreover, there is more information available, since other distance estimates can be used for the estimation of $\Delta_{i j}$. When event $C$ is used for the estimation of $\Delta_{A B}$ ( which is then written as $\Delta_{A B} . C$ or $\triangle_{A B}$ ),
$\Delta_{\mathrm{AB} \cdot \mathrm{C}}=\Delta_{\mathrm{AC}}-\Delta_{\mathrm{BC}}:$
$\Delta_{B C}$
$\Delta_{A C}$


The distances $\Delta_{A C}$ and $\Delta_{B C}$ are calculated from their crossover frequencies $F_{A C}$ and $F_{B C}$. It follows that:

$$
\Delta_{A B, C}=z_{A C}-z_{B C}
$$

When $\mathbf{N}^{*}$ events are used, the distance $\overline{\Delta_{A B}}$ is thus estimated by RASC in ( $N^{*}-1$ ) ways. The following estimate for $\overline{\Delta_{A B}}$ is computed:
$\overline{\Delta_{A B}}=\frac{z_{A B}+\left(z_{A C}-z_{B C}\right)+\left(z_{A D}-z_{B D}\right)+\cdots \cdots}{N^{*}-1}$
Note: $N^{*}$ is not always equal to the total number of events ( N ) in the margins of the z -matrix. Sometimes $z$-values have been set equal to 9.000 or $q$ and are now ignored.

Weighted indirect distance-estimates ( $\overline{\Delta_{A B} W}$ ).
The indirect distance-estimate is based on very diffferent $r_{i j}$ 's--how often event $i$ and event $j$ occur together ${ }^{1}$ In a section. Each $z$-value is based on it's own $r_{i j}$. For this reason RASC computes a weight for the mean distance $\overline{\Delta_{A B}}$ :
$\Delta_{A B}=\frac{w_{A B^{z}}{ }_{A B}+w_{A B} \cdot C^{\left(z_{A C}\right.}{ }^{-z_{B C}}{ }^{1}+W_{A B} \cdot D^{\left(z_{A D}\right.}{ }^{\left.-z_{B D}\right)+\cdots}}{w_{A B}+w_{A B} \cdot C^{+w_{A B}} \cdot D^{+} \cdots \cdots \cdots}$
The values for $w_{A B}$ and $w_{A B .} K^{\prime}$ as used by RASC in the above equation, are primarily determined by $r_{A B}$, $r_{A K}$ and $r_{B K}$. (How often $A$ occurs together with $B^{A B}$ and how of ten $A$ and $B$ occur together with all the other events used for the indirect estimation). The values $w_{A B}$ and $w_{A B}$. will be explained in the box on the next page.

When the weight $W_{A B}$ is assigned in the above way to ${ }^{2}{ }_{A B}$, which is a random variable, the value for $W_{A B}$ is per definition conversely proportional to the variance of $f\left(z_{A B}\right)$ :

$$
W_{A B}=\frac{1}{\overline{\sigma^{2}\left(z_{A B}\right)}}
$$

To relate this value to $r_{A B}$ ' which is the primary factor in determining the weight, one has, to derive the functional relationship of the variance $\sigma^{2}\left(z_{A B}\right)$ with the variance $\sigma^{2}\left(p_{A B}\right)$ of the frequency distribution (binomial) $f\left(p_{A B}\right)$, which has a mean of $P_{A B}$ and $a$ variance of:

$$
\sigma^{2}\left(p_{A B}\right)=\frac{P_{A B}\left(1-P_{A B}\right)}{r_{A B}}
$$

That there is such a relationship can be gleaned from fig. 3. For the exact mathematical derivation one is referred to Braver Hudson (1982). The relationship can be written as follows:

$$
\sigma^{2}\left(z_{A B}\right)=\frac{2 \pi}{e^{-z} A B^{2}} \cdot \sigma^{2}\left(P_{A B}\right)
$$

The value for $w_{A B}$ is now the following:

$$
w_{A B}=\frac{1}{\sigma^{2}\left(z_{A B}\right)}=\frac{r_{A B} \cdot e^{-z_{A B}}}{P_{A B}\left(1-P_{A B}\right) \cdot 2 \pi}
$$

in which $\sigma^{2}\left(z_{A B}\right)$ is the variance of $f\left(z_{A B}\right)$.
$P_{A B}$ equals the observed frequency of $F_{A B}$ ("cross-over" frequency)
$r_{A B}$ indicates how often events $A$ and $B$ occur together
$\mathrm{z}_{\mathrm{AB}}$ is looked up in the table for cumulative standard normal distributions.

The other weight used by RASC, $W_{A B} . K^{\prime}$ has the following value:
$w_{A B \cdot K}=\frac{1}{\sigma^{2}\left(z_{B K}\right)+\sigma^{2}\left(z_{A K}\right)}=\frac{1}{\frac{1}{w_{B K}}+\frac{1}{w_{A K}}}=\frac{w_{B K} \cdot w_{A K}}{w_{B K}+w_{A K}}$

## Standard-deviation

For the (stratigraphic) interpretation of the distance-values in the output it is useful to know the standard-deviations. All computing for $\Delta_{A B}$, the direct distance-estimate, was based on the variance $2 \sigma^{2}$ (standard deviation $\sigma \sqrt{2})$, but the standard deviation of the weighted, indirect estimate $\overline{\Delta_{A B} W}$ is the square root of the following variance:

$$
\sigma^{2}\left(\overline{\Delta_{A B} w}\right)=\frac{w_{A B}\left(z_{A B}-\Delta_{A B W}\right)^{2}+w_{A B} \cdot C^{\left(z_{A C}-z_{B C}+\Delta_{A B W}\right)^{2}+\ldots}}{w_{A B}+w_{A B} \cdot C+\ldots}
$$

## Preliminary solution of scaling

The set of distances between consecutive events, as determined by scaling, appears in the output of the RASCprogram in a table like Table 2. Table 2 is the "weighted distance" table of the sample run. The table of "unweighted distances" is available in the output as an option (see "program inputs", Chapter IV B).

Table 2 lists the inter-event distances in the third column, and the cumulative distances in the fourth column. These cumulative distances use the first event of the scaled sequence as the origin.

The fifth column of Table 2 is called "sum diff 2 values" and it contains the sum of the indirect weighted distance estimates for a certain event-pair: In the case of events $A$ and $B$ it lists $\left.\sum w_{A B} \cdot K^{\left(z_{A K}\right.}{ }^{-z_{B K}}\right)$. The column "weight" contains the $\mathrm{w}_{\mathrm{AB}}$ and the column s.d. contains the standard deviations.

|  |  | FOSSIL DISTANCE .0136 .0091 .0584 .1107 .1651 .2048 .24327 .23773 .0549 .5054 .2196 .0481 $: 3530$ .9876 .9720 .0011 $: 89354$ .83509 .2171 -3006 .0291 .0025 .1080 .3487 .0131 $: 2759$ $: 8785$ .1588 |  |  | WEIGHT |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Table 2. Preliminary solution of scaling as given in the "distance analysis, weighted differences" Table of the Sample Run.

Some of the inter-event distances turn out to be negative (Table 2). This is the result of the (weighted) indirect estimates for. scaling as opposed to the direct unweighted considerations in ranking. In order to obtain all-positive inter-event distances the events will have to be rearranged into a new sequence. This rearranging, or sorting of events, is done on the basis of their cumulative distances. For this purpose RASC takes arbitrarily the position of the first event as the origin of the sequence and calculates the cumulative distance of each event to this origin (Table 2). The rearranging results in a "new sequence".

The "new sequence" of events is given in the RASCoutput in the "events sorted" -- table (Table 3). (The appearance in the output of "events sorted" -tables for unweighted distance estimates is optional. See "program inputs".)

The "events sorted" - tables don't list the standard deviations for reasons that will be explained under the "final reordering option" (this chapter).

| NEW SEQUENCE | DISTANCE FROM IST | $\begin{aligned} & \text { FOSSIL } \\ & \text { PAIRS } \end{aligned}$ | INTER |
| :---: | :---: | :---: | :---: |
|  | S |  |  |
| 105 | 0.0000 | 105-3 | - 0136 |
| 5 | . 0227 | 3- 4 | -0 018 |
| 4 | . 0810 | 4- 6 | . 1107 |
| 6 | 1917 | 6-8 | -1651 |
| 1 | - 556 | 8- 16 | . 2048 |
| 13 | -8049 | 13-12 | - 2432 |
| 12 | -9375 | 12- 14 | - 2873 |
| 14 | 1.2248 | 14-11 | . 0549 |
| $1 \frac{1}{5}$ | 1.279 | 11- 15 | - 5054 |
| 56 | $\frac{1}{2} .0047$ | 56- 26 | - 048 |
| 26 | 2.0528 | 26-20 | . 3530 |
| 20 | 2.4058 | 20-108 | . 3876 |
| 108 | 2.7934 | 108-23 | . 9720 |
| 23 | 3.7654 | 23-55 | . 0011 |
| 5 | 3.7665 | 55-22 | . 5935 |
| 110 | 5.1954 | 110- 70 | - 0754 |
| 70 | 5. 2728 | 10-109 | - 0291 |
| 109 | 5.3019 | 109-54 | . 0025 |
| 54 | 5.3045 | 54- 24 | . 0519 |
| 24 | 5.3564 | 24-72 | . 0561 |
| 112 | 5.4125 | 172-112 | -1610 |
| 71 | 5:7612 | -31-30 | - 0131 |
| 30 | 5:7743 | $30=31$ | - 2759 |
| 31 | 6.0503 | $31-27$ | 8785 |
| 27 131 | 6.9288 | 27-131 | 1588 |

Table 3. The new sequence as given in the "events sorted" table of the Sample Run.

## Dendrograms

With the use of the subroutine DENDRO (adapted from Davis 1973, Program 7.8) RASC now constructs a dendrogram of the (new) sequence of events (fig. 10, next page). Dendrograms of the result of the unweighted distanceestimates are, together with their tables, also available, but their appearance in the output is optional.

The purpose of the dendrogram is to give the user a visual illustration of the inter-event distances. The inter-event distances are indicated with horizontal lines; When a line is long, a distance is great, when a line is short, the distance is small.

The vertical axis of the Dendrogram contains the events (numbers and names) in their established sequence and the distance-value of each event to the next event. On the horizontal axis the inter-event distances are represented on a scale, so that the horizontal lines can be given scaled lengths. To be able to immediately compare the lengths of the horizontal lines, vertical lines are drawn from the endpoint of each horizontal line down to where it meets another horizontal line.

This way the longest horizontal lines indicate (time-) gaps in the sequence (great inter-event distances), while the shorter horizontal lines, in between, represent more or less tightly clustered events. The shorter the lines, the tighter the clustering.

The dendrogram serves thus as an illustration of the clustering of events along the (time-) scale.

## Final reordering option

Since the estimates for successive distances are not statistically independent, the standard deviations for the "new sequence" (table 3) cannot simply be listed. For this reason a final optional routine has been added to the program: "final reordering".

In the final reordering-option RASC repeats all calculations for the distance-estimates, starting with a new $Z$-matrix, that has the new sequence of events in its margins. The purpose is to obtain a new set of standard deviations.

In places where the sequence has changed these new distance-estimates will turn out to be different from he first result of scaling. This is caused by the use of different $z-v a l u e s$ for the indirect distance estimates: When before $z_{C D}-z_{B D}$ was used, now $z_{C E} z_{B E}$ might have to be used.

Some of the new distances might even be negative, in which case the entire procedure will have to be repeated again. RASC allows at most four complete reorderings, after which most of the distances between successive events are positive, and their corresponding standard deviations are known. The only negative distances still left are very small and are an artifact of the algorithm used. The result of the final reordering is the scaled Optimum Sequence.


Fig. 10. Dendrogram of the "Scaled Optimum Sequence" of the sample run. Distance-values along the $y$ axis are distances between an event and the next event. Horizontal lines represent the same distances.

## C. Marker Horizons and Ụnique Events

When marker horizons and unique events are identified as such on the appropriate cards or lines (see "Programs Inputs", Chapter IV), in addition to being entered like all other events, they will be treated in a different way than the other events. A marker horizon, such as a bentonite bed resulting from a vulcanic ash-fall, is treated as a stratigraphic event with 0 variance with regard to its position. A unique event is an event that only occurs in one or two wells but yet constitutes important information to the user (as is the case with an index fossil). The unique event is fitted in the scaled optimum sequence after termination of scaling and reordering.

It is important to note that for a marker horizon to participate it has to occur in more than $k_{c}$ wells, while a unique event is only treated as such when it occurs in fewer than $k_{c}$ wells.

The marker horizons are identified in the dendrograms with a single asterix and the unique events only appear in the last dendrogram (after the normality tests) and is identified with two asterixes.

## Marker Horizons

In order for an event to be treated like a marker horizon the following assumptions are made:

1. A marker horizon is a stratigraphic event with 0 variance with regard to its position, and, 2. The values for the distance between a marker horizon $M$ and a stratigraphic event $A\left(d_{A M}\right)$ are normally distributed with a mean of $\Delta_{A M}$ and a variance of $\sigma^{2}$ (as opposed to $2 \sigma^{2}$ for $d_{A B}$ ).

It follows that:
$z_{A M}=\Delta_{A M} / \sigma \rightarrow \Delta_{A M}=\sigma \cdot z_{A M}=10.5 .2_{A M}$ (assumption 4 of scaling: $\sigma=\sqrt{ } 0.5$ )
$\Delta_{A M . C}=\Delta_{A C}-\Delta_{M C}=z_{A C}-10.5 . z_{M C}$
$\Delta_{A B . M}=\Delta_{B M}-\Delta_{A M}=10.5 \cdot z_{B M}-10.5 \cdot z_{A M}$
The values for the weight $w_{A M}$ and the indirect weight $W_{A M . K}$ or $W_{A B . M}$ have been adjusted accordingly.

If a data base contains two marker horizons, they will theoretically never cross over. A situation in which they do cross over cannot be dealt with by RASC and should be avoided. The appropriate warning is given in the program.

## Unique Events

A unique event $U$ is fitted in the scaled optimum sequence by identifying its neighbors in one of the sections in which it occurs. When its neighbors are $A$ and $B$, and events $A$ and $B$ have scaled optimum positions $X_{A}$ and $X_{B}$ (defined by their cumulative distance from the first event in the scaled optimum sequence), the position of $U$ in the scaled optimum sequence $\overline{X_{U}}$ can be calculated from $X_{A}$ and $X_{B}$ (fig. 4):

| In one Section | Scaled Optimum Sequence |
| :---: | :---: |
| etc. | ORIGIN |
| A | $\vdots$ |
| U-S |  |
| B |  |
|  |  |

Eig.4. A unique event $U$ in one section comnared to the scaled optimum sequence positions of its neighbors $\left(X_{A}, X_{S}, X_{B}\right)$


In reality the situation might be found that $A$ is simultaneous with $K$ and $L$ in the section studied, and that $B$ is simultaneous with, say, event $N$ (fig. 5)


Fig. 5 A unique event $U$ in one section compared to the scaled optimum sequence positions of its neighbors, to estimate a position for the unique event in the scaled optimum sequence ( $\overline{x_{U}}$ )

Instead of $X_{A}$, now an average of $X_{A}, X_{K}$, and $X_{L}$ is used, and instead of $X_{B}$ the average of $X_{B}$ and $X_{N}$. A preliminary estimate for $\overline{X_{U}}$ is then obtained by averaging the average $X_{A}$ and the average $X_{B}$ (fig. 5).

The ultimate estimate for $\overline{X_{U}}$ is obtained by introducing a range $R$ for the preliminary $\overline{X_{U}}: \overline{X_{U}} \pm$ $0.5 R$. This range includes all events encountered within the vicinity of $\overline{X_{U}}$ with a probability greater than $5 \%$ : Since $x_{U}$ is normally distributed with a variance of $6^{2}$, and $\sigma^{2}=0.5$ (assumptions 1 and $4 \rightarrow R=(1.965 . \sigma \Rightarrow 1.38$. In the example $X_{A}, X_{K}$ and $X_{B}$ fall within this range, and the ultimate estimate for $\overline{X_{U}}$ is obtained by averaging $\left.\sqrt{X_{U}}+0.5 R\right)$ with $X_{A}$ and $X_{K}$, and by
averaging $\left(\overline{X_{U}}-0.5 R\right)$ with $X_{B}$, and by adding these averages to $X_{S}$ (or the average of $X_{S}$ and scaled positions of other events that were simultaneous with $S$ and $U$ in the section studied) and dividing the sum through 3.

If the unique event occurs in more than one section, the preceding calculation is carried out for each section separately and the resulting values for $\overline{X_{U}}$ are averaged.
D. Normality testing

Once the scaled optimum sequence of events and the corresponding set of distances between consecutive events has been obtained, RASC provides three options for comparing the event-positions in the individual sections to the positions in the scaled optimum sequence. These options are:

1) Occurrence table, in which the events in the individual sections are recorded in their scaled optimum sequence.
2) Step model table, in which each event in each section receives a penalty score indicating how often it is out of place in that section,
3) Normality tests, which statistically compare the position of each event in each section to its position in the scaled optimum sequence. The "difference-values" found in this way for each event in each section (except for the first and last event of each section) are then together tested against a normal distribution.

The purpose of these comparisons is to detect events that occupy anomalous positions in an individual section, which might be the result of geological reworking, sample contamination or observer's bias. The comparisons can also lead to conclusions about homogeneity of the region studied.

## 1) Occurrence table

The occurrence table is constructed with the scaled optimum sequence of the events in its vertical direction, and the individual sections in the horizontal direction (table 4). Events that occur in a certain section are indicated with an $x$. In this way, the events that occur in a certain section appear in that section according to their optimum sequence. This table is useful for a quick first-glance interpretation of the result.


Table 4. Occurrence Table of the Sample Run.

Each event in a section receives a penalty point each time it is out of place in relation to the other events in that same section, when that section is compared to the scaled optimum sequence. This way each event accumulates in each section a "penalty score" (fig. 6). A high penalty score indicates that an event is likely to be out of place in that section.

| Petrel A-62 | scaled optimum <br> sequence | penalty <br> score |
| :---: | :---: | :---: |
| 4 | 4 | 0.0 |
| 14 | 13 | 2.0 |
| 15 | 14 | 1.0 |
| 13 | 15 | 1.0 |
| 72 | 72 | 0.0 |
| 27 | 27 | 0.0 |
| 131 | 131 | 0.0 |

Fig. 6. Step model comparison of the Petrel A-62 well with the scaled optimum sequence, both of the sample run.

The step model has the followintg draw-back: Events which belong to clusters of events, with many internal inconsistencies, are likely to accumulate high scores, even if they are in their normal places. The step model doesn't provide a way to distinguish between anomalous events and clustering events.
(3) Normality tests
3.1 Test for event-positions in individual sections

The difficulty of giving a definite value to the amount an event-position is out of place in a certain section, is that the positions in individual sections don't have values or distance-values like their counterparts in the scaled optimum sequence do.

Therefore, in order to test a certain event-position in a certain section, the event's neighbours in that section are identified. All three of these events have a position in the scaled optimum sequence, and a value for their cumulative distance from the origin (the first event), of the scaled optimum sequence. These three position-values are compared, by calculating a "second order difference value". When the position values are very different, when the three events should not be neighbours, the "second order difference value" is very much above or below 0. When the three position-values are close, the "second order difference value" approaches 0 , and the events "belong together".

| Scaled Optimum Sequence |  | In a section |  |
| :---: | :---: | :---: | :---: |
|  |  | sequence | Cum. Dist.\| 1 st order diff| 2 nd order diff |
| H | -0.0 | H |  |
| J | -0.6 | M |  |
| K | -1.0 | -L | 1.7 |
| N | -1.5 | $-\mathrm{N}$ | $1.5 \longrightarrow-0.7$ |
| 工 | -1.7 | -J | 0.6 |
| M | -1.8 | 0 |  |
| 0 | -2.0 |  |  |

Fig. 7. Example of calculation of $2^{\text {nd }}$ order diff. of event $N$ in a certain section. Note: hyphens in front of events in the section indicate that those events occurred simultaneous in that section.

The second order difference value is calculated as follows (fig. 7): Event $N$ has, in a certain section, events $J$ and $L$ for its neighbors. The "scaled optimum sequence position" or the "cumulative distance", of $N$ $\left(X_{N}\right)$ is subtracted from the cumulative distance of $J\left(X_{J}\right)$, and the cumulative distance of $L\left(X_{L}\right)$ is subtracted from $X_{N}$, and these differences are subtracted once more.
$2^{\text {nd }}$ order diff. $=\left(X_{J}-X_{N}\right)-\left(X_{N}-X_{L}\right)=(0.6-1.5)-(1.5-1.7)=-0.7$

Since the event-positions in the individual sections are expected to be slightly different (a gradual increase from the origin has to be taken into account), the first order difference is corrected by a small amount when non-simultaneous events are compared.l Simultaneous events don't need this correction.

In the RASC normality test-output a table like fig. 7 appears for each well (table 5). When there are second order differences in the last column of these tables, that exceed certain limits, the event-positions are considered to be anomalous in that section, and receive one or two asterixes: When the $2^{\text {nd }}$ order diff. is higher than 2.326 or lower than -2.326 the event receives one asterix, and when it is higher than 3.290 or lower than -3.290 the event receives two asterixes. 2

1 The small amount was set equal to the difference between highest and lowest position values in the observed sequence divided by the total number of times an event precedes another event without being simultaneous with it.

2 These limits are the ("one-tailed") 95\% and 99\% probability limits of the normal distribution which the $2^{\text {nd }}$ order differences satisfy (fig.8). This distribution has a variance of $4 \sigma^{2}$ because: Each event-position has a variance of $\sigma^{2}$ (assumption 1 of scaling), each difference between event-positions has a variance of $2 \sigma^{2}$ (Assumption 2 of scaling) and the 2nd order diff. then has a variance of $4 \sigma^{2}$. Since $\sigma^{2}=0.5$ (assumption 4 of scaling) the variance is 2 and the standard deviation equals $\sqrt{2}$. The $95 \%$ and $99 \%$ probability limits of this distribution belong then to the 2 nd order diff. values of respectively $295 \% \cdot \sqrt{2}=2.326$ and $z 99 \% \cdot \sqrt{2}=3.290$. For negative 2 nd order diff. values the negatives of these limits are used.

It should be noted that no $2^{\text {nd }}$ order difference values can be determined for the first and the last events of each section.

In practice an anomalous event often turns out to have another anomalous event for its neighbor in a section. This is an artifact of the method. The test cannot distinguish between an anomalous event and its neighbor.

It should also be noted that if there are large hiatuses in an individual section, the events on both sides of such a hiatus will give rise to very high (or very low) $2^{\text {nd }}$ order diff.'s which do necessarily mean that the events positions are anomalous in that section.


Table 5. Normality test output for the Petrel A-62 well of the Sample Run (see Chapter V).

## (3) Normality tests

3.2: The distribution of all 2nd order difference
values of the database

In part 3.1 of the normality test it was stated that a normal frequency distribution can be expected for the second order difference values. Part 2 determines if all second order difference - values as they have been observed in all sections, do together indeed satisfy this normal frequency distribution.


Fig. 8. Normal distribution of $2^{\text {nd }}$ order diff. values divided into 10 classes of equal probability.

For this purpose the area under the expected normal distribution-curve is divided into 10 equal parts (fig. 8). The 10 areas represent 10 equal occurrences, each indicating how often a specific second order difference value is expected to occur. The ten areas are called 10 classes of equal probability and the limits of each class are the second order difference-values that belong to a cumulative frequency of respectively $0.1,0.2$, 0.3, etc. (fig. 8). They are: Class 1/ $\sqrt{2 . z 0.1=-1.813 ~}$ /Class $2 / \sqrt{2} . z_{0.2}=-1.190 /$ Class $3 / \sqrt{2} \cdot z_{0.3}=-\ldots$...etc.

Each of these classes is expected to contain one tenth of all second order differences of the data-base. The "expected occurrence" of each class is thus $1 / 10$ of the total number of second order differences.

By placing all observed second order differences in their appropriate classes one can test if they are indeed normally distributed. For this purpose each second order difference with a value within the limits of a certain class is placed in that class. The number of second order differences thus placed in a class constitutes the "observed occurrence" of that class.

Example: In the example of Table 4 the second order differences (as they were listed in the last column of Table 5) belong to the following classes:

| Class | Dictionary numbers <br> ovents in that class | "observed <br> occurrence" |
| :---: | :---: | :---: |
| 1 | 72 | 1 |
| 2 | 15 | 27 |
| 3 | 14 | 2 |
| 4 | - | 1 |
| 5 | - | 0 |
| 6 | - | 0 |
| 7 | - | 0 |
| 8 | - | 0 |
| 9 | - | 0 |
| 10 | 13 | 0 |

[^0]CLASS NO. OBSERVED EXPECTED DIFFERENCE

| 1 | 30 | 21.500 | 8.500 |
| ---: | ---: | ---: | ---: |
| 2 | 20 | 21.500 | -1.500 |
| 3 | 21 | 21.500 | -.500 |
| 4 | 25 | 21.500 | -6.500 |
| 5 | 22 | 21.500 | .500 |
| 6 | 18 | 21.500 | -300 |
| 7 | 23 | 21.500 | -300 |
| 8 | 13 | 21.500 | -800 |
| 9 | 31 | 21.500 | 9.500 |
| 10 | 21.500 | 9.500 |  |

Table 6.
This table is designed to provide the user with a convenient test for the departure from a normal distribution of the second order difference values: When in each class the "observed occurrence" equals the "expected occurrence", the second order differences are indeed normally distributed.

The degree of significance of any departure is not statistically defined. However, the amount of departure can be reduced by removing some of the anomalous events identified in the first part of the normality tests. Whether the level of departure is significant enough to warrant removal of events and which events are removed are questions that are to be answered by the biostratigrapher on the basis of his/her palaeontological knowledge.

## IMPLEMENTATION OF THE RASC-PROGRAM

## A. Preparation of the Data

To implement the RASC-program for a set of raw data, the following steps have to be taken:

1) Listing of the raw data. The list should contain the section-names, and for each section a sequence of samples with a list of events for each sample (table 7).
2) Production of a dictionary. Each event of the list has to be assigned a unique number. The dictionary has to be key punched on Eortran-cards.
3) Coding of the observed event-sequences in the individual sections, using the numbers as assigned in the dictionary.
4) Unique events and marker horizons have to be identified.
5) Threshold parameters $k_{c}$ and $m_{c}$ have to be established.

## Raw Data

Until now RASC has only been used for the ranking of bio-stratigraphic events, but it could be used for any average ranking of elements along a relative scale. Example: RASC can list the average order in which the 12 yachts of the Bedford Basin Yacht Club have passed the finish in the 20 regattas they participated in last summer. The data should have a relative order, observed in different trials (sections etc.).

In bio-stratigraphy one could presumably make a list of taxa, one following another, but since taxa show overlap in time, it is better to list events that happen to the taxa. At one time a certain taxon, say the planktonic taxon Globotruncana contusa, first appears on the fossil record. "First appearance of Glob. cont." can be listed as an event and given a number, say 1. The same taxon can disappear from the fossil record higher up in the section. "Disappearance" or "Top of Glob. cont." can be listed as another event and given a number, say 2.

The taxon Globotruncana stuarti can thus be involved in events 3 and 4 , and so on.

When RASC is to be used for a certain data-base, it should first be decided which of the events, in which each taxon is involved, can be identified. In the data of the sample run, the above taxa G. contusa and G. stuarti are both listed with their "last" or "top" occurrence. Since the data-base for the sample-run is based on well-cuttingsamples, the event "first appearance" could not be identified. When cores are used, or outcrops are samples, many events can be identified for each taxon (i.e. "top", "subtop", "subbottom", "bottom" etc.).

The user of RASC has to begin by listing all events, in the observed order, by their names for each section or well. An example of such a list is shown in Table 7. The final list shows the observed order of all events in all sections. It should be noted that this RASC program is limited to 25 stratigraphic sections.

| Sample | Event | Dictionary Number (See Table 9) |
| :---: | :---: | :---: |
| $2970{ }^{\prime}$ | Globotruncana arca | 4 |
| $3090^{\prime}$ | G cretacea | 14 |
| $3360^{\prime}$ | G | 15 |
| $3390^{\prime}$ | G linneiana gr. | 13 |
| $4150^{\prime}$ | $G$ aff. renzi | 72 |
| $4320^{\prime}$ | Rotalipora cushmani | 27 |
| $4350^{\prime}$ | $\mathrm{R} \quad$ appenninica | 33 |
|  | praeglobotruncana sp. | 170 |
| $4408{ }^{\prime}$ | Gavelinopsis cenomenica | 131 |
| $4466^{\prime}$ | Favusella washitensis | 169 |

Table 7. Format of raw data listing of the Petrel A-62 well, used in the Sample Run. The events listed are highest stratigraphic occurrences.

## The Dictionary

As an intermediary stage between the user's data and analysis by the RASC program, the purpose of the dictionary is to number each event in a manner compatible with the RASC program. Each distinct event is listed and assigned a unique number. Since the RASC program internally numbers each event in the sequence in which it is entered, the dictionary must list its assigned numbers consecutively and continuously and it must be entered into the computer in that order. How you list your events originally in the dictionary is an arbitrary choice in that there is no requirement for a particular event to receive a specific number. The only requirement is that list then be numbered consecutively. [Note: RASC is restricted to process only 260 numbers or less. One's data base should therefore, not exceed 260 events.]

In practice, when there are large data bases, the dictionary may require extensive editing. The amount of editing can be minimized by alphabetically listing the events before numbering them consecutively. Errors may arise in one of two ways and are handled as follows: 1) An event may be mistakenly assigned two numbers. You should arbitrarily assign one of the numbers to a 'dummy' event, thereby preserving the established numeric ordering. 2) An event may be discovered to have been omitted after the dictionary has been numbered. Simply place the 'discovered' event at the end of the dictionary list (out of alphabetical order) and assign it the next consecutive number. Again it should be pointed out that it is the numeric order of the dictionary list that is important, not the event order.

For the punching (or entry) of the dictionary, individual cards (or lines) are used for each event. Each card (or line) is allowed forty characters starting in column l. Since RASC contains a subroutine for an alphabetical listing of events, alphabetical characters must precede numerical characters. However, it is convenient to have the dictionary numbers punched on the cards to ensure that the sequence of entrance in the file is the same as the sequence of numbers (the numbers are also used for the coding of the individual sequences as will be explained below).

A separate file must be created for the dictionary. The dictionary events are then read into the RASC program from this separate file (see "program inputs").

## List of Coded Sequences

Once all events have been assigned unique numbers in the dictionary, the sequence of events in each section is numerically coded, identifying the events by their assigned numbers and preserving their observed sequences. Events which appear in the same sample of a section (or well) are considered to occur simultaneously. Simultaneous occurrences are identified as such by a dash or hyphen sign between them. For example, in the Petrel A-62 well of the sample run the following order of events has been observed (table 7):
$\begin{array}{lllllllll}4 & 14 & 15 & 13 & 72 & 27 & 33-170 & 131 & 169\end{array}$
The events are ordered from left to right, representing either a stratigraphically upwards or downwards sequence. Which type of sequence is represented is decided in the ordering of the original data. A hyphen
before an event indicates that that event occurs simultaneously with the event immediately preceding it in the sequence. In this example, events 33 and 170 occur simultaneously.

With the completion of the list of coded sequences, the data can now be key-punched and read into the RASC program either by cards or disc file. The format for this procedure is contained in the listing of the RASC computer program.

In key-punching the list of coded sequences, the first punched card contains the name of the section (well) only. Twenty columns are available for the title name. The remaining six cards contain the sequence numbers. The (20I4) format indicates that the 80 columns of the fortran card are divided into 20 units of four columns. Each unit is occupied by an event number, with a maximum of three digits, with or without a hyphen. The numbers of the coded sequence are punched in their respective order from left to right. There are no column spaces between units, consequently each card will contain twenty numbers except for the last number card which may contain less. For example, in key punching the Petrel A-62 well of the sample run, card one gets the name "Petrel A-62" only. Card two is punched as illustrated in fig. 9. Since Petrel A-62 has only 10 events, punching ends at 169 , and the rest of the card is left blank, as is the case with the remaining 5 cards for that well.


Fig. 9. A fortran card containing the coded sequence of the petrel A-62 well of the sample run.

Marker Horizons and Unique Events
If there are events in the dictionary that are to be treated as Marker Horizons or Unique Events, those events first have to be treated as ordinary events (re: the dictionary and the list of coded sequences), but then they should also be identified on separate, designated cards or lines (see program inputs).

Threshold Parameters

Many of the listed events occur in only one or very few sections -- they are not "common". Therefore a threshold parameter $k_{c}$ has to be set. " $k_{c}$ " will allow only those events to participate that occur in at least $k_{c}$ wells. For the matrix-permutation another thresholdparameter is needed, $m_{c}$, which allows only those pairs of events to participate that occur together in at least $m_{c}$ sections. $m_{c}$ should be smaller than, or equal to, $k_{C}$.

It should be kept in mind that, although the threshold parameter is a number of wells, its use limits the number of events and not the number of wells.

For the first run with RASC both threshold parameters have to be chosen arbitrarily. In the RASC-output, under the heading "preprocessing initiated", tables appear that allow for a more rational choosing of values for $k_{c}$ and $m_{C}$. All events are listed by their dictionary numbers versus the number of sections in which each of the events occurs. This table is followed by a shorter version (Table 8) in which the second row indicates how many events occur in a certain number of wells and the third row how many events occur in at least a certain number of wells. Note: The information in both rows is the same but the presentation differs.


Table 8. Second preprocessing table of the Sample Run; this table tabulates the event occurrences and is used to determine $k_{C}$ values.

In the sample run were 76 participating events (the dictionary of 184 events was truncated for the purpose of the sample run), which makes for 76 events that occur in at least 1 well (third row of Table 8), although there are only 21 events that occur only in 1 well ( 2 nd row). There are 10 events that occur in 4 wells ( $2^{\text {nd }}$ row). but there are 40 events that occur in four or more wells. For the sample run $k_{c}$ was set at 5 , which means that 30 events participated in ranking and scaling. $m_{C}$ has to be lower, it was set at 3 .

After the user has entered $k_{c}$ and $m_{c}$ into the
input (see "program inputs"), RASC automatically edits the list of coded sequences to exclude the appropriate events.

## B. Program Inputs

## 1. Run parameters

The first card or line of input into the RASC program contains the list of "Run Parameters". These are as follows: NS, IOCR, INIQ, CRIT, AAA. A complete description of these parameters with their column locations and format is printed in the beginning of the listing of the RASC program. NS is simply the number of sequences or wells in the data (Note: In the present form of RASC NS cannot be higher than 25). The codes IOCR and CRIT designate the $k_{c}$ and the $m_{c}$ threshold parameters respectively. If either unique events or marker horizons are incuded in the data set, the parameter INIQ is set to 1. Otherwise it is set to 0. TOL stands for tolerance. When scores ( $s_{i j}$ ) in the upper triangle of the cumulative order matrix are compared with their counterparts in the lower triangle ( $\mathrm{sij}_{\mathrm{i}}$ ), $\mathrm{s}_{\mathrm{ij}}$ may be lower than $s_{j i}$ by as much as TOL. AAA designates the value for $q$ (see scaling), and is usually set at 1.645 .

## 2. Limits

The second card or line of input contains three "limit" parameters; N, MAX, ITER. Again these are fully described at the beginning of the listing of the RASC program. $N$ is the number of event labels in the dictionary. (Note: In the present form of RASC, $N$ cannot be higher than 260.) MAX is a number, multiple of 20 such that the number of events in the longest sequence is less than or equal to MAX. ITER is the maximum number of matrix-permutations (see chapter IIIA, the second step in Ranking), or "cumulative order matrix-transformations", or "iterations" allowed to be carried out.

## 3. Processing options

Card 3 or line 3 of input gives a list of ten processing options or "control parameters". They are the following: ITAPE, IOMAT, ISRT, IALPHA, ITABl, ISCOKE, ICOMP, ISKIP, IFIN, INOSC.

ITAPE decides if the input is read from cards or tape. When cards are used ITAPE $=0$, when tape is used ITAPE $=1$.

IOMAT controls the printing or suppressing of certain output data. If IOMAT $=1$, printing of order and frequency matrices as well as the intermediate tables takes place. If IOMAT $=0$ they are suppressed.

ISRT gives the user the option to use the presorting process, to shorten computer time. If ISRT $=1$ the presorting option is used. If ISRT $=0$ ranking is carried out by means of matrix permutations only.

IALPHA allows the RASC user to choose between a short version of RASC (ranking only) and a complete version (ranking as well as scaling and normality testing). If IALPHA $=0$ RASC will terminate after ranking. If IALPHA $=1$ scaling analysis will be carried out. If IALPHA $=2$ the individual matrix permutations will be printed as well.

ITABl $=1$ means that the occurrence table which precedes the normality test, will be printed.

ISCORE $=1$ means that the step model will be performed.

ICOMP $=1$ means that the normality test will be carried out.

ISKIP allows the user to skip the matrix permutations, which means that when ISKIP $=1$ the ranking solution will be based on presorting only, which is not sufficient. It is therefore suggested to set ISKIP at 0 .

IFIN stands for the final reordering option. When IFIN = 1 four complete reorderings will be carried out, if necessary, after scaling.

INOSC controls the printing of scaling output. (Note: It controls the printing of certain tables, not the performing of scaling itself.). If INOSC $=1$ the unweighted distance output (tables and dendograms) is suppressed. If INOSC $=2$ weighted as well as unweighted distance output is printed.

## 4. Unique events

Card or line 4 identifies the unique events of the data-set. If INIQ $=1$ on card 1 , up to twenty unique events can be punched on card 4 in 2014 format. If there are no unique events this card or line is left blank. If unique events are included, ICOMP on card or line 3 must be equal to 1. Note: If unique events occur in more than $k_{c}$ (IOCR) wells they will not be treated as such.
5. Marker Horizons

Card or line 5 identifies the marker horizons of the data-set. When $I N I Q=1$ on line or card 1 , up to 20 marker horizons can be entered in 2014 format on card or line 5. If there are no marker horizons this card or line is left blank. Note: The marker horizons have to occur in more than $k_{c}$ (IOCR) wells to participate in ranking and scaling.
6. Data set

The data set contains the list of coded sequences in the format described earlier. This data can be read either from an external tape or file, referred to as tape 10 by RASC, or from cards positioned immediately after card number 5. If ITAPE on card 3 equals 1 this data set will be read off external tape 10. If ITAPE $=0$, cards positioned after card 5 will be used to read in this data.

## 7. Dictionary of events

The format of the dictionary is described on p. 38, 39. The dictionary of events must be placed on a separate file from RASC. It is read into RASC from this separate file, designated as File 99 by the RASC program. The "Read" statement for the dictionary events does not occur until line 514 in the RASC program listing.

To effectively illustrate the procedures involved in RASC, an analysis of actual data was run through the RASC computer program. Tables in the previous chapters were all output from this sample run. The present chapter will describe the input and give a short discussion of the output.

## Data

The stratigraphic information used in the sample run is based on studies by F.M. Gradstein and F. Thomas in 1974-1982 (Gradstein et al. 1975; Gradstein 1977 and unpublished). The data consists of 76 highest stratigraphic occurrences of foraminifera of Late Cretaceous age, as found in 20 exploratory wells in the Newfoundland Basin and on the Grand Banks, N.W. Atlantic. For the purpose of the sample run, the data of an original list of 184 JurassicCretaceous events in 25 wells have been truncated.

In a few cases a distinction has been made between the highest occurrence, or top, of a taxon (as done for all the other taxa), and the peak occurrence (subtop or abundant) of that same taxon. These two events of the same taxon have been given separate dictionary numbers.

Whenever a species-designation was qualified as affinis (aff), it was treated as the nominate species.

Input
A dictionary (table 9) was prepared from the raw data as provided by F. M. Gradstein, and after truncation of most of the wells and the exclusion of some 7 more wells, a list of coded wells (sequences) was constructed (table 10). An example of the raw data for one of the wells has been given in table 7 (Chapter IV). In this table the dictionary numbers were listed also, which has to be done prior to coding of the well. After thus preparing the data, the dictionary and the list of coded sequences was key-punched as described earlier.

| 1 | Globotruncana contusa | 54 | Globotruncana concavata |
| :--- | :--- | :--- | :--- |
| 2 | Fructicose Pseudotextularia | 55 | Globotruncana carinata |
| 3 | Globotruncana stuarti | 56 | Gaudryina austinana |
| 4 | Globotruncana arca | 57 | Globotruncana ventricosa |
| 5 | Stensioina pommerana | 58 | Globigerinelloides aspera |
| 6 | Globotruncanella havanensis | 59 | Rugoglobigerina rotundidorsata |
| 7 | Pseudotextularia elegans | 70 | Hedbergella bosquensis |
| 8 | Globigerinelloides messinae | 71 | Globotruncana schneegansi |
| 9 | Globorotalites michelinianus | 72 | Globotruncana renzi |
| 10 | Arenobulimina americana | 95 | Globotruncana falsostuarti |
| 11 | Globotruncana fornicata | 104 | Globotruneana rosetta |
| 12 | Globotruncana stuartiformis | 105 | Loxostoma gemmun |
| 13 | globotruncana linnaiana | 106 | Arenobulimina d'orbignyi |
| 14 | Globotruncana cretacea | 107 | Kyphopyxa christneri |
| 15 | globotruncana marginata | 108 | Gavelinella minima |
| 16 | Rugoglobigerina rugosa | 109 | Coarse agglutinated spp. |
| 17 | Globorotalites aff. multiseptus | 110 | Globotruncana imbricata |
| 18 | Praebulimina sp. | 111 | Globotruncana primitiva |
| 19 | Aragonia materna kugleri | 112 | Globotruncana helvetica |
| 20 | Globotruncana coronata | 113 | Globotruncana sigali |
| 21 | Reussella szajnochae | 114 | Globotruncana marianosi |
| 22 | Sigalia deflaensis | 115 | Lingulogavelinella turonica |
| 23 | Stensoinia exculpta | 116 | Hedbergella paradubia |
| 24 | Hedbergella amabilis | 128 | Gavelinella tourainensis |
| 25 | Hedbergella spp. | 129 | Praeglobotruncana hagni |
| 26 | Globotruncana angusticoronata | 130 | Praeglobotruncana difformis |
| 27 | Rotalipora cushmani | 139 | Gaudryina pyramidata |
| 28 | Rotalipora greenhornensis | 141 | Hedbergella simplex |
| 29 | Rotalipora deckeri | 140 | Marsonella oxicona |
| 30 | Praeglobotruncana stephani | 168 | Globotruncana gansseri |
| 31 | Praeglobotruncana turbinata | 169 | Favusella washitensis |
| 32 | Grandes hedbergelles | 170 | Praeglobotruncana sp. |
| 33 | Rotalipora appenninica | 173 | Planulina taylorensis |
| 34 | Praeglobotruncana delrioensis | 174 | Epistomina stelligera alveolata |
| 35 | Dorothia aff. filiformis | 175 | Vaginulina texana |
| 48 | Hedbergella planispira | 178 | Neoflabellina rugosa |
| 50 | Agglutinated spp abundant | 184 | Petrel limestone* |
| 53 | Globotruncana conica |  |  |
|  |  |  |  |

Table 9. Dictionary of the 76 Late Cretaceous events of the Sample Run. In the computer output the complete list of 184 Jurassic-Cretaceous events will be printed out; * $=$ unique event.

## ORIGINAL SEQUENCE DATA

$\begin{array}{cccccccccc}\text { BOMAVISTA-CGG } & -53 & -11 & -6 & -16 & 54 & -55 & -17 & -56 & -23\end{array}$
$\begin{array}{cllllllllll}\text { CUMBERLAND } \\ 21 & -165 & -13 & -11 & -6 & -5 & 70 & -24 & -71 & -20 & -54 \\ -16\end{array}$

 | BLUE R-28 | -13 | -12 | -57 | -3 | -21 | -7 | -58 | -6 | -59 | 33 | -30 | -48 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

A GABRIEL C-60 |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 12 | -13 | -57 | -95 | -3 | 16 |







 $\operatorname{FETREL}_{4} \mathrm{~A}^{-62} 15 \quad 13 \quad 72 \quad 27 \quad 33-170 \quad 131 \quad 169$

| HERON | $\mathrm{H}^{-73}$ | -11 | -107 | 17 | -26 | 23 | -20 | 24 | 70 | 55 | 72 | 184 | 27 | -31 | 131 | 169 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |




| JAEGER |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | -109 | -19 | -16 | -5 | 107 | -173 | -15 | -11 | -13 | -178 | -174 |

$\begin{array}{cllllllllll}\text { CORMORANT } \\ 14 & \text { K-13 } \\ -115 & 112 & -30 & -31 & -110 & -115 & -24 & -108 & 27 & -28 & -131\end{array}$


Table 10. List of coded sequences for the Sample Run.

The run parameters for the sample run were chosen as follows:

|  | 20 | (20 wells) |
| :---: | :---: | :---: |
| IOCR | $=5$ | (events had to occur in at least 5 wells) |
| IQ | $=1$ | (we had one unique event or marker horizon) |
| IT | $=3$ | (only scores of events that occur together at least 3 wells were considered for the |
|  |  | matrix permutations) |
| TOL | $=0.0$ | ( $s_{i, j}$ in the upper triangle must be higher than $\mathrm{s}_{\mathrm{i}}$ in the lower triangle) |
| AAA | 1.654 | (when $i$ always occurs above or always occurs below $j$, the "distance" between and $j$ equals $\pm 1.645$ ) |
| N | 184 | (there were 184 events in the dictionary) |
| MAX |  | (there were more than 40 and fewer than 60 events in the longest well). |
| ITER | $=10000$ | (we allowed for 10000 matrix-permutations to take place). |
| Itape | $=1$ | (we had our dictionary and our list of coded sequences on tape). |
| IOMAT | $=1$ | (we wanted to have all matrixes and tables printed). |
| ISRT | $=1$ | (we used the presorting option) |
| IALPGA | $=1$ | (we used ranking as well as scaling) |
| Itabl | $=1$ | (we had the occurrence table made and printed) |
| ISCORE | $=1$ | (we had the step model table printed out) |
| ICOMP |  | (we used the normality tests) |
| ISRIP | $=$ | (we did not skip the matrix-permutations) |
| IFIN | $=1$ | (four complete reordering are carried out, if necessary, following scaling. |
| INOSC | $=2$ | (we had all scaling output printed) |

The information in Tables 9 and 10 is also available on tape and can be used for a sample run by future users.

## Output

The output of the RASC program contains the following eight sections: Input, Preprocessing, Presort Option, Ranking Solution, Scaling Analysis, Final Reordering, Normalilty Tests and Final Dendrogram.

1. Input. This (untitled) section contains input parameters and a print-out of the list of coded sequences (table l0), which is called "original sequence data".
2. Preprocessing. This section contains two tables of event occurrences, which allow for a rational choosing of values for $k_{C}(I O C R)$ and $m_{C}$ (CRIT) (see "threshold parameters", chapter IV). It also contains a new set of coded sequences which are edited to exclude the events that don't occur in $k_{c}$ wells.
3. Presort Option. The "code followed by score" table gives the presort scores (i.e. the Presort Rank Numbers) as calculated from the cumulative order matrix (see Ranking, Chapter IIIA). After the Presort Rank Numbers ("scores") have been grouped in descending order, the original sequence data is edited according to this new "Presort Sequence" and printed.
4. Ranking Solution. This section contlains the cumulative order matrixes before and after the matrixpermutations (if IALPHA had been 2 it had contlained all intermediate matrixes too). In addition it contains a printout of all cycles that occurred during the matrix permutations and which of the scores were zeroed because of the cycles. This part of the print-out begins with the statement that $x$ events were zeroed, but they are the scores that were zeroed on the basis of $\mathrm{m}_{\mathrm{C}}$.

Although the "optimum sequence obtained via ranking" follows directly after the cycles, the table of the Optimum Sequence with the ranges and the anems of the events (as in Table l) only appears after the printout of the dictionary and of the alphabetical list of events.
5. Scaling Analysis. This section contains matrices which are called "upper triangle of normal z-values", and which are a combination of a $Z$-matrix and an F -matrix (see Scaling, Chapter IIIB). The section further contains "distance" - tables and dendrograms, first for unweighted estimations and then for weighted estimations. The weighted output (tables and dendrograms) are explained in Chapter IIIB (scaling) and will be discussed below.
6. Final Reordering. Only the tables and dendrograms (unweighted and weighted) for the result of the last reordering are printed out. However, it says how many reorderings were required ("solution after $x$ iterations", never more than 5, the first + the four complete reorderings that RASC allows for [Chapter IIIB, final reordering]).
7. Normality Tests. This output (Occurrence table, step model table, for each well a normality test table and the comparison table) is explained in the Normality Tests section of Chapter III.
8. Final Dendrogram. This final dendrogram (Fig. 10) comes after the normality tests and is the only dendrogram that contains the unique events, if they were input into the program.

## Discussion of the output

A detailed discussion of the biostratigraphic information contained in the data and analyzed in the Sample Run is beyond the scope of this manual. We limit ourselves to a brief account of the meaning of some of the results.

The first major result of RASC, the Optimum Sequence is shown as Table 1 of this manual. It contains 31 events, most of which are part of standard late Cretaceous Zonations. The top few events have a long range, a result of the lack of data above position zero. The Optimum Sequence can be used for point correlation of well sections and can serve to subjectively study stratigraphic normality of individual wells, using bivariate Shaw-type diagrams. For such a diagram the Optimum Sequence is plotted along the $y$-axis, and the individual well sequence along the $x$-axis. The resulting scatter diagram gives a visual expression of regional biostratigraphic coherence. The Step Model and Normality Test output of RASC also evaluate this feature.

A perusal of this Step Model and the Normality Test output of the Sample Run shows that not many wells have "badly positioned" events. Table 6 shows the data set to be quite "normal".

The dendrogram of Figure 10 contains four or five (stratigraphically meaningful) clusters of Cenomanian through Maestrichtian age. We will have a closer look at the two apparently greater gaps in the scaled sequence: The "distance" between events 20, 108 and 23 and the "distance" between events 31 and 27.

Event 108 (top of Gavelinella minima) has in the Optimum Sequence, after ranking, an exceptionally long Range (Table l). Its position in relation to events $26,56,20$, 23, 55, 24, 22, 72 and 54, could not be determined, either because 108 occurs simultaneous with these events or because 108 did not occur in enough sections together with them. After ranking one can only say that 108 can be placed somewhere between event 15 , on position 12 (top of Globotruncana marginata) and event 109 , on position 23 (top of coarse agglutinated spp.). After scaling event 108 is placed between the top occurrence of Globotruncana coronata and top of Stensonia exculpta, but it should be noted that, while the distance between Glob. coronata and Gav. minima is not well defined (.3876 土.l832), the great distance between Gav. minima and Sten. exculpta is not defined at all (s.d. = 999.999)! To make sure that there is indeed a gap in the sequence between events 20 and 23 it might be better to exclude event 108 from the analysis. A likely conclusion is that Gavelinella minima either has a widely varying stratigraphic top or else has been misidentified in several wells.

The gap between events 31 and 27 is better defined. Event 31 (top of Praeglobotruncana turbinata) has a welldefined position after ranking, between it's predecessor (top of Praeglob. stephani) and it's successor (top of Rotalipora cushmani). The standard deviation of the great "distance" between top of Praeglob. turbinata and top of Rotalipora cushmani is also relatively small (.8785 $\pm$ . 1650). It can be concluded that the RASC analysis of the sample run has revealed a gap in the sequence between the "top of" Rotalipora cushmani and the "top of" Praeglobotruncana turbinata. This gap may in fact be a regional hiatus.

The last page of the standard printout of the RASC program shows the position of marker horizons or of unique events, relative to the scaled sequence. For this Sample Run the Petrel Limestone (dictionary No. 184) was treated as a unique event. It might have been more correct to treat 184 as a marker horizon, but RASC requires marker horizon's to occur in more than $\mathrm{k}_{\mathrm{C}}=5$ and 184 only occurs in two wells. The position of 184 in the final dendrogram is between events 112 and 71 of Turonian age.

This discussion is presented here to show that information from different output tables of RASC should be considered before drawing conclusions about the final dendrogram.

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Gradstein F.M., 1982, Quantitative Biozonations. In: New Concepts and Methods in Stratigraphy; Syllabus of June 1982 CSPG - course, Calgary. (unpublished)

Hay W.W., 1972, Probabilistic Stratigraphy, Ecologae Geologicae Helveticae, (65) 2: p. 255-266.


[^0]:    $2^{\text {nd }}$ In the RASC output a table appears that covers all copy of that table for the sample run:

