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PROVINCIAL INPUT-OUTPUT TABLES

The Balancing Process of the Regional Input-Output Tables

by

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1 - Introduction

Regional input-output tables are, like national tables, subject to a certain number of constraints. If the tables are generated in a top down approach, that is by disaggregating national input-output tables into regional input-output tables covering all regions of a country, then the regional tables are further subject to adding up constraints. Aggregating across regions, for any specific cell, the regional input-output values must indeed give the value of the corresponding cell of the national table. Finally, the external trade of the regions are also subject to specific constraints.

Balancing a complete set of regional tables may therefore represent a tremendous task which needs to be supported by a systematic approach and computerized procedures. Regional input-output tables cannot, of course, be balanced in a purely mechanical fashion using a computerized algorithm. Initial imbalances must be identified and data checked. Then a judgemental analysis must be carried out to modify or reallocate transaction flows based on various extraneous knowledge of the economy and the strengths and weaknesses of economic surveys from which the initial estimates are derived. Nevertheless, once this judgemental exercise has been done, it is likely that many of the constraints listed above will still not be satisfied. A tedious subsequent judgemental exercise can then be attempted to balance the transaction flows so as to satisfy all constraints. However, it is unlikely that any such attempt will be succesfull for large size regional input-output tables as they are produced in Canada. Indeed, the much simpler exercise of balancing the national tables on judgemental ground has proven to be extremely time consuming and very costly in terms of resources.

This certainly explains why previous regional input-output tables in Canada have never been balanced so as to satisfy all constraints. These tables have been constructed so as to satisfy the national constraints first, and then the interregional constraints in a second step. Regional constraints have typically been left aside and hence large imbalances at the regional level between the total supply and the total demand of commodities have been left unresolved. Also, final demand expenditures of the regional input-output tables have not been reconciled with the control totals of final demand categories from the regional income and expenditure accounts. These imbalances have presented serious inconveniences to the data users, particularly to the regional econometric model builders.

This technical note presents a computer assisted approach to the balancing of large size regional input-output tables which takes into account all constraints and still leaves provision for judgemental analysis. The approach consists in exerting the judgemental analysis as described above to the exclusion of any attempt to fully balance the regional tables to their marginal totals. Large initial imbalances are first identified and corrected, as stated above, by checking the data and modifying or reallocating transaction flows on the basis of extraneous information. Then a systematic computerized algorithm can be applied to the data so as to fully balance all tables with respect to all constraints.

The next step consists of measuring the discrepancies between the tables before and after their balancing. This can be done using an automated procedure that calculates some index of structural change, such as the cross entropy index which has been recently applied to the national tables in order to measure their changes through time. All major changes from the initial estimates can then be reassessed based on judgemental expertise. Modifications can be introduced into as many flows as desired in the process without attempting to simultaneously balance the tables. Then the computer algorithm can be applied again and the above process repeated as many times as required until satisfactory final estimates are obtained.

The paper begins by giving a complete description of the various sets of constraints briefly identified above using simple algebra. The systems of constraints are then translated into matrix notation in section 3 for more clarity and later developments. Section 4 analyses the overall process of balancing the regional tables on the national and the regional constraints to the exclusion of the regional commodity balance equations. Section 5 deals with the interregional commodity balance constraints. Section 6 then attacks the regional commodity balance equations taking into account all the previous balancing steps.

2 - The Basic Identities

To formalize all of the above constraints, let us define the following set of variables and indices:

i : commodity number index, $i = 1,\ldots,n$;
j : industry number index, $j = 1,\ldots,m$;
k, p : region number index, $k, p = 1,\ldots,r$;
f : final demand category number index, $f = 1,\ldots,F$;
v_{ijk} : output of commodity i produced by industry j of region k;
u_{ijk} : intermediate input i of industry j of region k;
e_{ifk} : final demand use of commodity i in catogery f in region k (excluding imports and exports);
m_{ik} : international import of commodity i by region k;
x_{ik} : international export of commodity i by region k;
m_{ikp} : import of commodity i by region k from region p;
x_{ikp} : export of commodity i of region k to region p.

The set of "national" constraints on the regional input-output tables are then the following:

$$\sum_k v_{ijk} = v^*_{ij} \tag{2.1}$$

where v^*_{ij} is the national output of commodity i from industry j (we have indicated that this variable was predetermined with a star superscript). Similarly on all other flows we must have:

$$\sum_k u_{ijk} = u^*_{ij} \tag{2.2}$$

$$\sum_k e_{ifk} = e^*_{if} \tag{2.3}$$

$$\sum_k m_{ik} = m^*_i \tag{2.4}$$

$$\sum_k x_{ik} = x^*_i \tag{2.5}$$

To the national constraints we must add the regional constraints:

$$\sum_j v_{ijk} + m_{ik} + \sum_p m_{ikp} = \sum_j u_{ijk} + \sum_f e_{ifk} + x_{ik} + \sum_p x_{ikp} \tag{2.6}$$

That is, for a given region, the supply of a commodity must be equal to its disposition. In (2.6) above, we have implicitly and arbitrarily set x_{ikk} and m_{ikk} equal to zero.

Additionally, the final demand categories' marginal total must balance the control total set in the regional income and expenditure accounts, which means that we must have:

$$\sum_{i} e_{ifk} = e'_{fk} \tag{2.7}$$

Total industry use of inputs must also be set equal to the total value of each industry's outputs. However, this constraint is used in the estimation of the initial flows to set residually the value of the Other Operating Surplus of each industry.

We also have constraints on the interregional flows of goods and services. For every commodity, the import of a region k from another region p m_{ikp} must be equal to the export of that other region x_{ipk} to the region k:

$$m_{ikp} = x_{ipk} \tag{2.8}$$

This set of constraints eliminates half of the interregional flows. These flows may be computed by setting arrays in which the rows give the exports of regions to the other regions (appearing also on the columns). Interregional imports are then given by the columns which, for each region, gives its imports from all other regions.

The marginal totals for these arrays (one for each commodity) are given by summing over the columns and the rows. Summing over the columns first, we have:

$$\sum_{p} m_{ikp} = \sum_{p} x_{ipk} \tag{2.9}$$

That is, the imports of commodity i of region k from other regions must be equal to the exports of all other regions of that commodity to region k. Similarly, summing across the rows gives:

$$\sum_{p} x_{ikp} = \sum_{p} m_{ipk} \tag{2.10}$$

That is the export of a commodity i by a region k to the other regions must be equal to the imports of that commodity by the other regions from region k. From (2.9), we also get:

$$\sum_{p} \sum_{i} m_{ikp} = m^{i}_{k} = \sum_{p} \sum_{i} x_{ikp} = x^{k}_{i} \tag{2.11}$$

That is, the total imports of a region from all other regions m_k^r must be equal to the exports of the other regions to the region k, x_r^k. Similarly, from (2.10) we get:

$$\sum_p \sum_i x_{ikp} = x_k^r = \sum_p \sum_i m_{ipk} = m_r^k \qquad (2.12)$$

Relations (2.9) and (2.10) also implies:

$$\sum_p \sum_k m_{ikp} = m_i^r = \sum_p \sum_k x_{ikp} = x_i^r \qquad (2.13)$$

That is, the interregional imports of commodity i must be equal to the interregional exports of commodity i. In other words, the commercial balance of regions for each commodity must cancel at the national level. Unfortunately, none of the marginal totals from these constraints are known. Therefore, constraints (2.9) to (2.13) are useless in the balancing process and redundant as they all follow from (2.8). We shall nevertheless see below how these marginal totals can be modified to balance the interregional flows jointly with the commodity supply and demand constraints at the regional level.

If we finally assume that the regional distribution of output by commodity and by industry are high quality estimates which must be considered as predetermined, then we have the constraints:

$$v_{ijk} = v'_{ijk} \qquad (2.14)$$

and the non negativity constraints:

$$v_{ijk},\ u_{ijk},\ e_{ijk},\ m_{ijk},\ x_{ijk},\ m_{ik} \text{ and } x_{ik} \geq 0 \qquad (2.15)$$

3 - A matrix notation of the constraints

To simplify the analysis of the above sets of constraints on regional input-output flows, it is better to translate them into a matrix notation. For that purpose we shall set:

$$V'_k = [v'_{ijk}]$$

$$U_k = [u_{ijk}]$$

$$E_k = [e_{ijk}]$$

$$M_k = [m_{ik}] \tag{3.1}$$

$$M_k^p = [m_{ikp}]$$

$$X_k = [x_{ik}]$$

$$X_k^p = [x_{ikp}]$$

From the above definitions, we can write the national variables as:

$$V^* = \sum_k V_k^* \tag{3.2}$$

$$U^* = \sum_k U_k \tag{3.3}$$

$$E^* = \sum_k E_k \tag{3.4}$$

$$M^* = \sum_k M_k \tag{3.5}$$

$$X^* = \sum_k X_k \tag{3.6}$$

Relations (3.2) to (3.6) also set the national constraints (2.1) to (2.5) which have to be satisfied by the regional flows. The constraint (2.6) may be written as:

$$V_k^{T*} i_m + M_k + M_k^p i_p = U_k i_m + E_k i_F + X_k + X_k^p i_p \tag{3.7}$$

where the vectors i's are summation vectors of appropriate dimension. Constraint (2.7) may in turn be written as

$$i_n^T E_k = e_k^{T*} \tag{3.8}$$

where $e_k^{T*} = [e_{fk}^*]$.

Finally, constraint (2.8) is given by:

$$M_k^p = X_p^k \tag{3.9}$$

where in X_p^k $(=[x_{ipk}])$, k is now the variable for fixed p. Of course, the non negativity constraints (2.15) still hold.

4 - Balancing the regional tables

The first step in balancing the regional input-output tables is to determine the interrelationships, if any, among the previous sets of constraints. In other words, when adjusting some given flows in order to satisfy a given set of constraints, we must know which of the other constraints may be affected in the process. Since commodity output flows at the regional level are taken as predetermined and such that constraint (3.2) is satisfied then this constraint becomes irrelevant in the balancing process. In the commodity balance equation (3.7), therefore, only the non output flows can be modified. International imports and exports, intermediate and final uses are all subject to the national constraints (3.3) to (3.6), while interregional imports and exports are subject to (3.9). In addition, final demand uses, which are subject to a national additivity constraint, are also subject to (3.8).

The national constraints can be taken care of easily when balancing (3.7) by treating one region residually. However, this solution presents the disadvantage of not taking into account all the available information and does not either guaranty that the non negativity constraints (2.15) will necessarily be satisfied. A prefered approach would therefore be to increase the number of unknowns rather than to reduce the number of equations. For each variable of (3.7) which can be modified, we will therefore associate a variable λ corresponding to a specific national constraint. Taking all equations of (3.7) into account amounts to take the Schurr product of the matrices corresponding to these variables by parameter matrices Λ. Equation (3.7) will then write:

$$V_k^{T*} i_m + \Lambda_M \bullet M_k + M_k^P i_r = \Lambda_U \bullet U_k i_m + \Lambda_E \bullet E_k i_F + \Lambda_X \bullet X_k + X_k^P i_r \qquad (4.1)$$

The identities (3.2) to (3.6) will now have to be rewritten as:

$$U^* = \Lambda_U \bullet \left(\sum_k U_k\right) \qquad (4.2)$$

$$E^* = \Lambda_E \bullet \left(\sum_k E_k\right) \qquad (4.3)$$

$$M^* = \Lambda_M \bullet \left(\sum_k M_k\right) \qquad (4.4)$$

$$X^* = \Lambda_X \bullet \left(\sum_k X_k\right) \qquad (4.5)$$

According to these revised formulations of the equations, the regional flows which are subject to national constraints can be modified arbitrarily up to a set of balancing constants λ's. If these flows are non negative then the non negativity constraints will be automatically satisfied. The balancing constants will be positive and they will be determined from the national constraint equations. The latter constraints will therefore always be satisfied. Note, however, that modifying the flows arbitrarily in the matrices U_k, E_k, M_k, and X_k in order to satisfy the "row" constraints (4.1) will no longer lead to a full balance as was the case with (3.7). Indeed, these transaction flows can now only be modified up to some constants which will be determined when balancing (4.2) to (4.5). Inserting these constants into (4.1) will disrupt the previously established balance for that equation. This will be discussed later in more detail.

The final demand flows are further subject to the additional regional set of constraints (3.8). These flows can be balanced along both the regional and the national dimensions by adding a new set of variables, $\gamma_{fk} \in \hat{\gamma}_k$, which take into account the regional constraints. Equation (4.1) will then become

$$V_k^{T*} i_m + \Lambda_M \bullet M_k + M_k^P i_r = (\Lambda_U \bullet U_k) i_m + (\Lambda_E \bullet E_k \hat{\gamma}_k) i_F + \Lambda_X \bullet X_k + X_k^P i_r \qquad (4.6)$$

The set of constraints (3.8) will then become

$$i_n^T (\Lambda_E \bullet E_k \ \hat{\gamma}_k) = e_k^{T*} \qquad (4.7)$$

Accordingly, the constraints (4.3) must be modified to:

$$E^* = \Lambda_E \bullet (\sum_k E_k \ \hat{\gamma}_k) \qquad (4.8)$$

Relations (4.7) and (4.8) may be seen as a system of equations which must be solved simultaneously for the unknown adjusting constants. The number of equations matches the number of unknown constants of Λ_E and $\hat{\gamma}_k$. However, the equations are non linear in the variables and can only be solved through an iterative search. A set of constants is initially chosen so as to satisfy the first set of constraints. The second set of constants is chosen so as to satisfy the second set of constraints. This will induce new imbalances with respect to the first set of constraints so that the process has to be repeated until it converges. It does not matter which set of constants is selected first, as in the usual RAS balancing technique, for it can easily be shown by inspection that

$$\Lambda_E \bullet (E_k \hat{y}_k) = (\Lambda_E \bullet E_k) \hat{y}_k \tag{4.9}$$

The solution to (4.7) and (4.8) can in fact be found by applying the RAS algorithm to all elements of a given partition of this system into independent subsets of equations. The elements of the partition are given by slicing the cube formed by the regional matrices on the index f. For each slice, we then have a matrix formed of the f^{th} column in all r regions. This matrix must satisfy the regional totals for the f^{th} columns and the national total for each of the n commodities belonging to the national total. The marginal totals are then made of an n dimensional national vector for category f, e_f^{*} and and r dimensional vector including the totals over commodities of category f in all regions, e_f^{r*}. If we denote by ϵ_f, the f^{th} column of the fXf identity matrix, then these vectors can be obtained as:

$$e_f^{*} = E^{*}\epsilon_f \tag{4.10}$$

and

$$e_f^{r*} = \breve{e}^{T*}(I \otimes \epsilon_f) \tag{4.11}$$

where

$$\breve{e}^{T*} = [e_1^{T*}, e_2^{T*}, \ldots, e_r^{T*}] \tag{4.12}$$

and where $\otimes$ is the Kronecker product.

Equation (4.7) can be rewritten for all regions as

$$\breve{e}^{T*} = i_n^T \left\{ (\Lambda_E^{(1)}, \Lambda_E^{(2)}, \ldots, \Lambda_E^{(r)}) \bullet (E_1, E_2, \ldots, E_r) \begin{bmatrix} \hat{y}_1, & , & , \\ , & \hat{y}_2, & , \\ & & \\ , & , & , \hat{y}_r \end{bmatrix} \right\} \tag{4.13}$$

Setting

$$\hat{\Gamma} = \begin{bmatrix} \hat{y}_1, & , & , \\ , & \hat{y}_2, & , \\ & & \\ , & , & , \hat{y}_r \end{bmatrix} \tag{4.14}$$

$$\breve{E}^T = (E_1, E_2, \ldots, E_r) \tag{4.15}$$

and

$$\breve{\Lambda}_E^T = (i_k^T \otimes \Lambda_E) \tag{4.16}$$

then (4.13) can be written in compact form as follows:

$$\grave{e}^{T*} = i_n^T(\tilde{\Lambda}_E^T \bullet \grave{E}^T \hat{\Gamma}) \qquad (4.17)$$

Combining (4.8) and (4.10), we have

$$e_f^* = \sum_k (\Lambda_E \bullet E_k)\hat{\gamma}_k \epsilon_f \qquad (4.18)$$

$$= \sum_k (\lambda_{Ef} \bullet e_{fk})\gamma_{fk} \qquad (4.19)$$

where λ_{Ef} is the f^{th} column of Λ_E.

But,

$$\sum_k (\lambda_{Ef} \bullet e_{fk}) = \hat{\lambda}_{Ef} \; E_f \qquad (4.20)$$

where E_f is the matrix formed by the f^{th} column of the r regions. Hence (4.19) can be written as

$$e_f^* = \hat{\lambda}_{Ef} \; E_f \; \hat{\gamma}_f \qquad (4.21)$$

where $\hat{\gamma}_f$ is the diagonal matrix containing the elements γ_{fk}. Similarly, combining (4.11) and (4.17) yields:

$$e_f^{*} = i_n^T(\tilde{\Lambda}_E \bullet \grave{E}^T \; \hat{\Gamma})(I \odot \epsilon_f)$$

$$= i_n^T[\lambda_{Ef} \bullet e_{1f}\gamma_{1f}, \lambda_{Ef} \bullet e_{2f}\gamma_{2f}, \ldots, \lambda_{Ef} \bullet e_{rf}\gamma_{rf}]$$

$$= i_n^T(\tilde{\lambda}_{Ef} \; E_f \; \hat{\gamma}_f) \qquad (4.22)$$

As can be seen (4.22) immediately follows from (4.21) as it should since the regional totals are constrained to balance with the national totals for the final demand categories. In other words, (4.22) is redundant and only (4.21) has to be solved. Therefore, as asserted, the subsystems (4.21) can be solved independently for each f by the RAS algorithm.

5 - Solving the interregional flows

As shown above, the set of constraints on the interregional flows reduces to (2.8) or (3.9). This amounts to setting all interregional imports to their interregional export counterparts or vice versa. There are no known control totals on the interregional flows which can be used in the balancing process. Relations such as (2.9) to (2.13) are therefore irrelevant. These relationships will be automatically satisfied if (2.8) or (3.9) are satisfied. Balancing the interregional flows thus consists of reconciling the interregional export figures with the interregional imports figures in order to satisfy the constraints (2.8) or (3.9). This analysis relies heavily on expert judgement, that is on knowledge of the regional economies and knowledge of the strengths and weaknesses of the various surveys. These surveys, as we know, are incomplete and have many weaknesses. In particular there is almost no statistical evidence of interregional flows of services. Some services are known to be traded between the regions while others are known to be local. In general, commodities may be classified into two categories: the traded and the non traded (local) commodities. Zero flows are then attributed to local commodities on an a priori basis.

Similarly, the existing surveys most often give the first destination of shipments as is the case for manufacturing commodities. The ultimate destination is not known but may be quite different. Interregional shipments may be considered as interregional flows. However, "first destination" local shipments may be ultimately shipped to other regions. Hence, interregional shipments of manufacturing commodities may be considered the lower bounds for the interregional flows of these commodities.

Indirect evidence on the interregional flows of goods and services may be derived from production and use data. If the regional use of a commodity exceeds its production and international imports into the region, this is evidence that the commodity is also imported from other regions. The regional commodity balance equation may therefore be used to improve the estimate of the interregional flows as we shall now see in the next section.

6 - The Regional Commodity Balance

Incorporating the interregional flow constraints (3.9) into the regional commodity balance equation (4.6), the latter may now be written as follows:

$$V_k^{T*} i_m + \Lambda_M \bullet M_k + X_p^k i_r = (\Lambda_U \bullet U_k) i_m + (\Lambda_E \bullet E_k \hat{Y}_k) i_F + \Lambda_X \bullet X_k + X_k^p i_r \qquad (6.1)$$

As just stated, in order to balance the supply and demand of commodities at the regional level one may decide to modify the interregional flows. These may also be considered as only one element of the balancing process together with the other components of (6.1). International imports and exports may also be considered in the process as well intermediate and final uses. Production has up to now be considered as predetermined and left ouside of the balancing process. However, as explained in the introduction, the balancing strategy proposed here has two facets. It involves a judgemental analysis and a computerized balancing process. Production data need here to be considered as predetermined only when balancing with a computerized algorithm. Production data may otherwise themselves be considered as objects of the judgemental analysis and modified as a result of that analysis.

Interregional flows may be arbitrarily modified when balancing (6.1) without disrupting their internal balance. However, it must be noted that modifying these flows to satisfy the commodity balance of a region may disrupt the commodity balance equation of other regions as these flows enter into many regional equations. Similarly, the other components of (6.1) cannot be arbitrarily modified as they are already subject to national constraints and, for the final uses, also subject to additional regional constraints. Balancing (6.1) on these flows, therefore implies an iterative procedure whereby the national and other regional constraints will be rebalanced when disrupted. Using a proportional adjustment technique as above implies that we apply the RAS technique to balance the commodity balance equation with the national constraints. We have a nested RAS problem with the final uses data which have to be balanced both nationally and regionally. In order to simplify the balancing process, the final demand uses when balanced as explained in section 4 above may be considered to be predetermined when attempting to reach the commodity balance. It must be stated here, however, that a thourough judgemental analysis of these data in relation to the other components of the commodity balance equation must first be carried out. It may be, for instance, that for a local commodity final demand appears to be larger than the regional production, according to the initial estimates. In that case it will not be possible to balance the commodity equation. Through judgemental analysis, it will have to be decided either to increase the initial production estimate or to reduce the final use of that commodity.

In order to use the interregional flows to balance the regional commodity constraints for all regions simultaneously let us first write all interregional flows in a matrix form as follows:

$$X_r = \begin{bmatrix} x_{11}, x_{12}, \ldots, x_{1r} \\ x_{21}, x_{22}, \ldots, x_{2r} \\ \\ x_{r1}, x_{r2}, \ldots, x_{rr} \end{bmatrix} \qquad (6.2)$$

where the x_{pk}'s are vectors giving the exports of all commodities from region p to region k. Hence X_r is of dimension (r x n) x r. Each row of X_r gives the regional matrices X_k^p. The p^{th} column of X_r gives the component vectors of X_p^k. Each row of X_k^p may be multiplied by a scalar δ_{ik} which amounts to premultiply the matrix X_k^p by a diagonal matrix $\hat{\delta}_k$. Taking all regions together, this amounts to premultiply the matrix X_r by the diagonal matrix $\hat{\Delta}$ defined as:

$$\hat{\Delta} = \begin{bmatrix} \hat{\delta}_1, 0, \ldots, 0 \\ 0, \hat{\delta}_2, \ldots, 0 \\ \\ 0, \ldots, \hat{\delta}_r \end{bmatrix} \qquad (6.3)$$

which gives

$$\hat{\Delta} X_r = \begin{bmatrix} \hat{\delta}_1 x_{11}, \hat{\delta}_1 x_{12}, \ldots, \hat{\delta}_1 x_{1r} \\ \hat{\delta}_2 x_{21}, \hat{\delta}_2 x_{22}, \ldots, \hat{\delta}_2 x_{2r} \\ \\ \hat{\delta}_r x_{r1}, \hat{\delta}_r x_{r2}, \ldots, \hat{\delta}_r x_{rr} \end{bmatrix} \qquad (6.4)$$

Inspecting (6.4) one immediately finds that each column of X_p^k is premultiplied by the diagonal matrix $\hat{\delta}_p$. But this is equivalent to take the Schurr product of X_p^k with the matrix Δ where the latter matrix is given by

$$\Delta = [\delta_1, \delta_2, \ldots, \delta_r] \qquad (6.5)$$

Since this applies to all regions, the commodity balance equation for region k may then be written as:

$$V_k^{T'} i_m + \Lambda_M \bullet M_k + (X_F^k \bullet \Delta) i_F = (\Lambda_U \bullet U_k) i_m + (\Lambda_E \bullet E_k \hat{Y}_k) i_F + \Lambda_X \bullet X_k + \hat{\delta}_k X_k^P i \qquad (6.6)$$

In order to define the strategy needed to solve this complex system, it is best to first look at it commodity wise taking all regions together. Indeed, it can be seen from (6.6) that the balancing problem is decomposable into separate and independent commodity balancing problems: balancing (6.6) over one commodity does not disrupt the regional or national balance of other commodities.

Concentrating on interregional flows we may present them for each commodity in the form of an array X_i. The rows of the array gives the exports of regions to other regions and the columns, their imports from other regions. Diagonal elements are equal to zero.

$$X_i = \begin{bmatrix} x_{11}, x_{12}, \ldots, x_{1r} \\ x_{21}, x_{22}, \ldots, x_{2r} \\ \\ \\ x_{r1}, x_{r2}, \ldots, x_{rr} \end{bmatrix} \qquad (6.7)$$

In (6.7) each element x_{kp} represents the export of commodity i from region k to region p and, at the same time, the import of that commodity by region p from region k. The row margin of X_i is given by the vector $x_i = [x_{ki}]$ and the column margin of the table is given by $m_i = [m_{ki}]$. These margins, as mentioned in section 2 above, are not known before hand so that the RAS technique cannot be applied to the array (6.7) with the marginal totals x_i and m_i. These marginal totals can only be computed from the initial estimates of the interregional transactions included in X_i. However, we may, by introducing the internal shipments within each region on the diagonal of X_i relate the interregional flows to known values from the regional tables. Indeed, the internal shipments within a region, x_{pp}, are just equal to domestic uses (intermediate uses u+ final uses e) minus imports (international imports m + interregional imports m^r). Given that, for a region and a commodity, the commodity balance equation adjusted to national and regional constraints can be written from (4.6) as:

$$q + m + m^r = u + e + x^r + x \qquad (6.8)$$

where x^r is the total interregional exports of the region, then the sum of the elements on a row of X_i is just equal to

$$u + e - m - m^{i} + x^{i} = q - x \tag{6.9}$$

This is an immediate transformation of (6.8). Looking at the sum of each column, we have

$$u + e - m - m^{i} + m^{i} = u + e - m \tag{6.10}$$

The values of the row margins q - x and column margins u + e - m are readily available from the regional input-output tables.

International exports and/or imports can be brought into the balancing process by adding either or both of them to their respective margin and enlarging accordingly the matrix X by these new elements. Doing so, however, implies that international imports and exports will be modified in the balancing process with interregional imports and exports. Since the international flows are subject to national constraints, they will have to be rebalance to the national imports and exports. This iterative process may be repeated until convergence. The end product of this balancing process is a new set of interregional flows which still satisfy constraint (3.9) on interregional flows and which also fully balance the regional commodity balance equations for each commodity and all regions.